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Consumer Search with Chain Stores ^{*}

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Abstract

The Stahl model is one of the most applied consumer search models, with many applications and an empirical background. The present paper explores an extension where sellers have asymmetries, which is mostly excluded by the literature. Sellers with heterogeneous numbers of stores are introduced, reflecting a typical market structure. As in the original Stahl model, a market consists of several sellers, and consumers, where some face a cost when sequentially searching. The paper shows that no symmetric Nash equilibrium exists in the extension. Additional results suggest that smallest sellers will be the ones offering the lowest prices, in line with several real world examples provided in the paper. However, profits remain in most cases fixed per store, making a larger firm more profitable, yet with lower quantity sold. The findings suggest that on some level price dispersion will still exist, together with some level of price stickiness, both observed in reality.

Keywords: Sequential Consumer Search, Oligopoly, Asymmetric NE

JEL Classification Numbers: D43, D83, L13.

1 Introduction

Empirical studies, such as Bazucs and Imre [4] or Martin et al. [19], have established that significant price dispersion exists even for homogeneous goods. As the literature suggests,

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this effect is observed in many market structures and is persistent. One of the explanations for this phenomenon is that consumers search for the cheapest price. Since searching is costly, consumers may settle for a slightly higher price, which explains price dispersion. In the literature, many papers deal with search models, for example Burdett and Judd [7], Carlson and McAfee [9], Stahl [21], Varian [23] and Watanabe [24]. These models were developed originally in order to provide a solution to the Diamond Paradox [13], which predicted a complete market failure. The search models vary in scope, length, the stopping condition, and the information revealed during the consumer search. Additional empirical studies, for example Janssen et al. [15], reveal that the model introduced by Stahl in [21] performs very well and predicts correctly the pricing model of 86 out of 87 tested products. Moreover, Baye et al. [3] empirically suggests the existence of the two consumer types predicted by this model. Therefore, this paper will concentrate on the Stahl search model.

An additional phenomenon that can be observed, for example in Bazucs and Imre [4] and Watanabe [24], is a possible correlation between the price offered by a seller and the number of stores possessed by that seller. Namely, the more branches a seller has the higher will be the price offered. Despite the fact that the Stahl model has a variety of extensions, the literature dealing with asymmetries among sellers is not large. This is a very important extension, as in the real world, the number of stores a seller has can vary, for example, see Table 1. Among the few papers on this point is Astorner-Figari and Yankelevitsch [1], where only a model with two sellers is treated. An additional paper is Burdett and Smith [8], where only a single large firm exists and all others are single store. Already in those papers it is noted that the larger firm charges a higher price in expected terms.

The present paper investigates whether this is true in a more general setting than in Astorner-Figari and Yankelevitsch [1] and Burdett and Smith [8]. Here, the Stahl model is extended, and each seller has a predefined, seller-specific number of stores. This reflects a more general setting which is closer to reality. Since, generally, sellers vary in size (or popularity), this aspect should be taken into consideration. For example, when comparing discounters, they typically have a different number of stores, as depicted in Table 1. The site mysupermarket.co.uk compares the prices of various supermarkets in the UK, suggesting that within-chain prices are similar. That is, a given product costs the same in all supermarkets of chain X. Therefore, differing numbers of stores can play a role in the search model setting when prices do not vary within a chain.

The rest of the setting is similar to the original Stahl model [21]. Sellers set the same price for all stores (e.g.: bank offers for saving accounts). The consumers search sequentially and uniformly among stores, and in this setting it can be different than searching uniformly among sellers. This implies that there is a higher chance for a consumer to turn up at a store belonging to the larger seller. Furthermore, if a searcher is unsatisfied with a price,

| | | | | | |
|------|-------|------|-------|-------|------------|
| Aldi | Netto | Lidl | Norma | Penny | Tengelmann |
| 27 | 49 | 36 | 17 | 50 | 89 |

Figure 1: Number of discount stores in Munich, according to kaufda.de, 5.2011

that searcher would refrain from visiting any other store of that same seller. A similar setting was used by Astorner-Figari and Yankelevitsch [1] and Burdett and Smith [8] in the limited versions of the extension.

An important distinction needs to be made regarding the type of the market. When a seller has several stores, is the pricing strategy done at the store level (store good) or at the headquarters level (chain good). An example of a store good is gasoline sold at fuel stations. The name of the chain usually does not imply that the same price would be encountered at all fuel stations of that chain. Examples of a chain good are banks, or cellphone networks. When one looks for a mortgage, usually a bank offer would be the same in all branches of a given bank. For store goods, the original Stahl model is relevant, as observing a price in one store does not have an implication for prices in any other store, which is not the case with chain goods, which reveal the price in all stores of a given chain. This can happen, for example, due to binding advertising, or to a strategic decision of the chain. With a store good, the number of stores plays no role for the price, and there is no connection between price and chain. With a chain good, the chain determines the price for all chain stores, making the size of the chain an important factor. The extension in this paper deals with chain goods, though through examples provided in the paper, some effects of chain size hold also in the store goods case.

The first thing one notices when discussing the extended Stahl model (with heterogeneously sized sellers) is the lack of a symmetric equilibrium, i.e., one where all sellers use the same strategy. The original Stahl model has a unique symmetric equilibrium, as shown in Stahl [21] and the literature does not go far beyond it. For comparison, in the Varian search model (see [23]), it is shown in Baye et al. [5] that there are asymmetric equilibria, but those can be ignored. In the Stahl model, there might be additional equilibria when different settings are considered. For example, it is shown in Baye and Morgan [6] that one can have additional equilibria in commonly known games when the scope is broadened.

1.1 Overview of the Results

The main contribution of this paper is introducing an extension of the Stahl model that includes heterogeneous sellers, and finding the relevant equilibria. Surprisingly, the extra complexity does not make the equilibrium complex. For example, in the case of a unique

smallest seller and unique second smallest seller, the results are in line with Astorner-Figari and Yankelevitsch [1], where all but the two smallest sellers select a reserve price purely. An additional contribution is dealing with the asymmetric equilibria of the model, under a consumer reserve price. Lastly, the paper supports the insight that larger chains, and actually all but the smallest chains, would have higher prices for goods.

The results here imply that when there are at least two smallest sellers, all sellers except the smallest sellers select the reserve price purely. The remaining sellers have a similar equilibrium to the original Stahl model, but with a lower portion of uninformed consumers (all those who visit one of the smallest sellers). This extends the result in Burdett and Smith [8], and shows that the lowest price will be obtained in one of the smaller chains. Moreover, in all equilibria found here, all consumers buy at the first store they visit and no seller will ever set a price above the reserve price. Additional characteristic of the Nash equilibrium (NE) is that expected profits for all sellers are equal to a constant times the chain size (the number of stores). An additional point to note is that the equilibria found here extend the known symmetric equilibrium of the original Stahl model and describe its asymmetric equilibria.

In the case of a single smallest seller, the equilibrium structure slightly changes. Now, it is the smallest and ‘second smallest’ sellers (the ones with the smallest share except the smallest seller) who mix over the entire interval. The second smallest sellers have a mass point at the reserve price. Note that at least one of the second smallest sellers mixes. All larger sellers select the reserve price purely. Note that here the smallest seller has larger profits per store than all other sellers, and all other sellers have the same lower profit per store. The structure and results are similar to Astorner-Figari and Yankelevich [1], where a model with two heterogeneous sized sellers was discussed.

1.2 Existing Literature

The Stahl model has been dealt with extensively in the literature, and is a very popular model. Numerous extensions to the Stahl Model have been introduced, and the various extensions deal with nearly every aspect of the model. Among those are introducing heterogeneous searchers. Examples of such extensions are Chen and Zhang [10] and Stahl [22], where the searchers have different costs for each additional store they visit. They can differ by the search scope, as discussed in Astorner-Figari and Yankelevich [1], where some stores are near, and thus will be searched first. Another extension introducing advertisement costs, has been discussed, for example by Chioveanu and Zhou [11]. There are also models where already the first price is costly, such as Janssen [16], or there is no possibility of freely returning to previously visited store, such as Janssen and Parakhonyak [17]. The literature discusses sequential searches in the model and looks also at non-sequential search, for example in Janssen and Morags-Gonzales [14], or an unknown production cost

as shown in Janssen et al. [18]. Most of the assumptions of the model introduced by Stahl in [21] have been discussed extensively, except for one main assumption, used extensively in the literature. This is the focus on symmetric equilibria, where all sellers select an identical strategy. One of the reasons is mathematical complexity: Carlson and McAfee [9] and Rothshild [20] showed that in symmetric equilibria, a consumer reserve price must exist, and in asymmetric ones it does not have to exist. The reserve price assumption is common in the literature (among others [1, 21, 23]), and therefore, the present paper considers only NE with reserve price, yet justifies the rationality behind it. Nevertheless, one should note that additional equilibria without reserve prices may exist, and fall outside the scope of this paper.

An additional outcome of this model can explain price stickiness, as described for example in Davis and Hamilton [12]. Many equilibria found here have mass points on certain prices. This implies that with some probability the price in the previous round can be the same also in the next round, even though the seller is mixing. In reality it is known that prices do not change too often and are sticky. The results of this model can provide an insight into why this is so: prices selected with mass points can remain unchanged over several periods.

The structure of this paper is as follows: first the proposed extension of the Stahl model is introduced and the difference between the original model and this extended model is discussed to clarify the nature of the extension. Then, the characteristics of the equilibria in the model will be provided, firstly in the case with several smallest firms and then with a unique smallest firm. Then the implications of the results are discussed, examples are provided, together with some suggestions on how those results can be empirically tested.

2 Model

The Stahl model was introduced in Stahl [21]. An extension to it is formally described below. The notation has been adjusted to the recent literature on the Stahl model.

There are N sellers, selling an identical good. Seller i owns n_i stores, where n_i is exogenously given and can differ from seller to seller ¹. We look at the case where not all n_i are equal, as the other case is exactly the original model. The production cost is normalized to 0, and it is assumed that a seller can meet any demand. Furthermore, there are consumers, each of whom wishes to buy a unit of the good. The mass of consumers is normalized to 1. This implies that there are many small consumers, each being strategically insignificant.

¹In the original Stahl Model each seller had a single store.

Besides the size heterogeneity, all sellers are identical. They set their price once at the first stage of the game. If some sellers mix, distributions are selected simultaneously, and only at a later stage can realizations take place.

Consumers are of two types: both value the good at some large value M . A fraction μ of consumers are shoppers, who know where the cheapest price is, and they buy there. In the case of a draw, they randomize uniformly over these stores, spreading equally among the cheapest stores. The rest are searchers, who sample prices. Sampling the price in the first store, randomly and uniformly selected, is free. It is shown in Janssen et al. [16] that if this not the case, then some searchers will avoid purchasing, and the results will be adjusted to exclude those searchers. If the sampled price is satisfactory, the searcher will buy the item. However, if the price is not satisfactory, the searcher will go on to search additional stores sequentially, where each additional search has a cost c . The second (or any later) store is randomly and uniformly selected from the stores of the previously unvisited sellers, and the searcher may be satisfied, or search further. Searchers that are satisfied have a perfect and free recall. This implies buying the item at the cheapest store already encountered, randomizing uniformly in the case of a draw.

The main difference from the original Stahl Model is that the fraction of searchers initially visiting seller i is equal to $\frac{n_i}{\sum_j n_j}$ instead of $1/N$. Further search is also done according to these propensities, and the probability of visiting seller i is n_i divided by the sum of the n_i 's of the sellers the consumer did not visit yet. Thus, at most a single store of a seller is visited by a searcher.

There needs to be consumers of both types, informed and uninformed (namely, $0 < \mu < 1$). If there are only shoppers, this is the Bertrand competition setting, e.g. see Baye and Morgan [6], and if there are only searchers, the Diamond Paradox, Diamond [13], is encountered, both well studied.

Firstly we make a technical assumption on the model. In order to avoid measure theory problems it is assumed that mixing is possible by setting mass points or by selecting distribution over full measure dense subsets of intervals. This limitation allows all commonly used distributions and finite mixtures between such.

Before going on a couple of very basic results are provided:

- Sellers cannot offer a price above some finite bound M . This has the interpretation of being the maximal valuation of a consumer for the good.
- Searchers accept any price below c . The logic behind this is that any price below my further search cost will be accepted, as it is not possible to reduce the cost by searching further.

2.1 Idea of the Extension

The main point of the extension is to capture the fact depicted in Table 1. Heterogeneous sellers have different numbers of stores, and for various reasons keep the price fixed in all of the stores of a given seller. This is not always the case, as some goods do not have to have a price fixed over all stores of a seller. Therefore, one needs to make a distinction between *store goods*, for which each store sets the prices individually, and *chain goods*, which have a fixed price in all of a chain's stores.

A good example for the distinction can be seen in two examples of a seller: a bank and a fuel station. Typically, offers of a given bank do not vary among branches. One would get the same offers for a mortgage, credit, interest rate for deposits, and other banking products no matter the specific branch of the bank one approaches. Clearly, a different bank would make different offers, but usually it is the case that a specific branch of bank 'X' on street 'A' would have the same offers as the branch of bank 'X' in street 'B'. When one looks at fuel stations, one sees the exact opposite. Every single fuel station offers a station-specific price, and it is usually the case that the fuel station of firm 'Y' on street 'A' would have a different price than the fuel station of the same firm 'Y' on street 'B'.

For the fuel station example, the original Stahl model would suffice, as being unsatisfied with a specific station does not imply avoiding the seller completely. However, if after visiting one branch of bank 'X' one does not find a satisfactory offer, there is no reason to visit yet another branch of the same bank. Therefore, present here are both aspects of the extension: the probability of encountering each seller is proportional to the number of stores the seller has, and the fact that at most a single store of a given seller would be visited by a given searcher.

2.2 The Structure of the Game

The game is played between the sellers, searchers, and the shoppers. The time line of the game is as follows:

1. Sellers select pricing strategies and searchers set a reserve price P_M .²
2. Realizations of prices occur for sellers with mixed strategies.
3. Shoppers go and purchase the item at the cheapest store.
4. Each searcher selects a store and observes the price in it.

²All searchers have the same reserve price, see introduction.

5. If the price observed is weakly below P_M , the searcher is satisfied and purchases the item. If not, the search continues.
6. All unsatisfied searchers select one additional store, pay c , and sample the price there.
7. If the price observed is below P_M , the searcher is satisfied and purchases the item. If not, the search continues.
8. ...
9. If a seller has observed all stores and observed only prices above P_M , the seller buys at the cheapest store encountered.

Following the Bayesian structure of equilibrium from Stahl [21] and Burdett and Judd [7], the knowledge structure is as described below. At the time when the reserve price and the strategies are determined, the knowledge of the various agents of the game is as follows:

- Sellers have beliefs regarding the reserve price set by the searchers.
- Searchers have beliefs about which strategies (not realizations) are played by the sellers.

The probability that seller i sells to the shoppers when offering price p is denoted by $\alpha_i(p)$. The expected quantity that seller i sells when offering price p consists of the expected share of searchers that will purchase at that store, plus the probability that that store is the cheapest store multiplied by the fraction of shoppers. This is also the market share of the seller. Let q_i denote the market share of seller i .

Note that the reserve price ensures that the searcher will purchase at the last visited store, unless all stores were searched.

Regarding utilities, note that when sellers make decisions, they can take into consideration only expected utility. This is because their decision is made at the first stage of the game, and cannot be altered afterwards. Therefore, the seller's utility is denoted in ex-ante terms, namely, what is the expected utility for the seller when a certain price is chosen. For searchers, it is possible to pin down the exact utility, since a searcher faced with a price can calculate the benefit exactly without uncertainty. Therefore, a searcher's utility is given in ex-post terms, taken into consideration the exact price paid. Formally, the utilities are as follows:

- Seller i 's utility is the price seller i charges (p_i) multiplied by the expected quantity sold: $p_i q_i$.

- Consumer j 's utility is a large constant M , from which item price and search costs are subtracted. If n_j prices have been observed, and in the end the purchase was made at price p_j , the utility is $M - p - (n_j - 1)c$.

The NE of the game, which has a Bayesian form, is as follows:

- No seller can unilaterally adjust the pricing strategy and gain profit in expected terms.
- The reserve price is rational for searchers, and at any price observed they make the optimal decision in expected terms.
- Searchers have a reserve price.
- Searchers have beliefs regarding the strategies played by sellers.
- Seller pricing strategies and searcher beliefs regarding those strategies coincide.
- Seller beliefs regarding the reserve price coincide with the actual reserve price.

A rational reserve price P_M has the following properties:

- After observing a price below P_M , it is optimal to stop searching.
- After observing a price above P_M , it is optimal to continue searching.

Note that the following lemma describes some of the situations where a reserve price of P_M is rational:

Lemma 2.1 *Suppose all sellers offer a price weakly below P_M with probability 1. Also assume that the expected price offered by each sellers is weakly above $P_M - c$. Then P_M is a rational reserve price.*

Proof:

Suppose a searcher observed a price q in the first store. By assumption, $q \leq P_M$.

The expected price in all unvisited stores is a convex combination of the expected values, where none of them falls below $P_M - c$. Therefore, the expected price in unvisited stores is at least $P_M - c$.

From this it follows that a searcher expects to observe a price of at least $E = P_M - c$ when a search is performed. In such a case, the total cost (in expected terms) would be

$E + c$, or more. Since $E + c = P_M \geq q$, it is always (weakly) better to stop after visiting the first store and not search a single additional store.

Using similar reasoning, it would not be optimal to search in multiple stores. \square

Remark 2.1 *Note that additional equilibria, without a searcher reserve price, may exist. As the Stahl model literature concentrates mainly on consumers with a reserve price, these equilibria fall outside the scope of this paper.*

Regarding social welfare, one can say the following:

Remark 2.2 *As the sum of the searcher and seller utilities may differ only in the search cost, any strategy profile where the searchers always purchase the item at the first store visited is socially optimal.*

3 Basic Results

The first thing to note is that no seller would offer a price above the reserve price.

Lemma 3.1 *No seller offers a price above P_M in NE.*

Let p be the highest (or supremum) price in the union of the supports of all sellers in NE, and suppose $p > P_m$. Such a supremum exists as it is assumed that there is a finite bound on prices. Let us distinguish between several cases, based on the number of mass points on p . Remember that in a mixed NE the same expected profit is yielded from all prices a seller has in the support.

Consider a seller with price p in support.

Case 1: no seller has a mass points on p : With probability 1, everyone offers a cheaper price. Searchers would search further, and shoppers would purchase elsewhere. Therefore, in this case, the profit of offering price p , or prices arbitrarily close to it, is arbitrarily close to 0. A deviation to c would be profitable.

Case 2: everyone has a mass point on p : There is a positive probability that all sellers offer price p . This cannot be equilibrium due to undercutting.

Case 3: some sellers have a mass point on p : Similarly to the first case, shoppers would not purchase at a seller offering p . Searchers would search until they encounter a

price below p . Once such a price is encountered, no searcher would purchase the item at price p . Therefore, no consumer would purchase the item at price p , leading to profit of 0. Again, a deviation to c would be profitable.

To sum up, for a seller offering a price $p > P_M$ there is a profitable deviation in all cases. \square

Corollary 3.1 *Any NE is socially optimal. This is since the total utilities of the sellers and consumers sum to a constant as long as the searchers buy at the first store they visit.*

Corollary 3.2 *The mass of searchers visiting seller i is $(1 - \mu)n_i/\sigma n_j$.*

Due to the previous lemma, only the searchers initially visiting seller i would end up purchasing there. This share is given by the number of stores seller i has divided by the total number of stores.

Now a proof for the fact that no symmetric NE exists in the model is provided.

Lemma 3.2 *No symmetric NE (where all sellers choose the same pricing strategy) exists.*

Due to undercutting, no pure NE or symmetric NE with mass points can exist. Suppose there exists a mixed strategy symmetric NE. Denote the pricing strategy in the symmetric NE by s , and let F be its distribution function.

As noted before, no seller sets a price above P_M , as in such a case the highest (or supremum) price in the support yields profit 0.

Suppose that two prices p and q are in the support of s with positive density or mass point. Also assume that there exist two sellers i, j such that $n_i > n_j$.

Note that due to the symmetry of the strategy choice, the probability of a seller's attracting shoppers with price p is equal to $(1 - F(p))^{N-1}$. The next step is to write down the profits of seller i , for both prices. Those must be equal, as mixing is possible only between prices that yield the same expected profit:

$$\pi_i(p) = p[(1 - F(p))^{N-1}\mu + (1 - \mu)n_i/S] = q[(1 - F(q))^{N-1}\mu + (1 - \mu)n_i/S] = \pi_i(q). \quad (1)$$

This contains the expected quantity sold by the seller: probability of being the cheapest times the quantity of shoppers and the quantity of the searchers, which contains only the initial searchers visiting the store (due to corollary 3.1).

Similarly, the profit of seller j is

$$\pi_j(p) = p[(1 - F(p))^{N-1}\mu + (1 - \mu)n_j/S] = q[(1 - F(q))^{N-1}\mu + (1 - \mu)n_j/S] = \pi_j(q). \quad (2)$$

After subtracting the second equation from the first, one obtains

$$((1 - \mu)/S)p[n_i - n_j] = ((1 - \mu)/S)q[n_i - n_j]. \quad (3)$$

Note that $(1 - \mu)/S \neq 0$, and this can be narrowed down. From here, either $p = q$ or $n_i = n_j$, neither can occur due to our assumptions. \square

Next, a distinction is made regarding the number of smallest sellers, as this has a crucial effect on the structure of the NE.

4 Multiple Smallest Sellers

Here we make a distinction between two cases: whether there are several sellers with size n_m (smallest), or whether the smallest seller is unique. In this section, the first case is dealt with. The following theorem provides an insight into what the NE of the model look like. First, we would make a definition describing the smallest sellers.

Definition 4.1 *Let the smallest value of the size parameter be n_m , and the total number of stores $\sum_j n_j$ be S .*

Now we turn to a theorem describing the equilibria in the case where at least two sellers have size n_m .

Theorem 1 *In the model, with multiple smallest sellers, the NE with reserve price P_M looks like this:*

- *All the sellers who have a larger number of stores than n_m select the reserve price P_M as a pure strategy.*
- *The agents with n_m have their support between some price P_L and the reserve price P_M , with no mass points except possibly P_M .*
- $P_L = \frac{n_m(1-\mu)}{(1-\mu)n_m+S\mu}P_M$.
- *Suppose seller i does not have the interval $(p, p') \subset (P_L, P_M)$ in their support. Then the only price above p in the support of seller i is P_M .*

- The profit of seller i is $\text{Const} \cdot n_i$.
- Any price in (P_L, P_M) is in the support of at least two sellers.
- Let I be an interval to the left of P_M . If both sellers i, j have all of I in their supports, then both use the same distribution over I .

The proof is relegated to the Appendix.

4.1 Equilibrium Distribution

In this subsection we will look into the equilibrium described by the theorem. Firstly, the equilibrium strategy for sellers that are not the smallest one is to select a reserve price purely. Sellers with the smallest size have a wider choice. Below is a description of all possible equilibrium strategies for the smallest sellers.

From Theorem 1, for any NE of the Stahl model with a reserve price, there are at most three groups of strategies:

1. Top: A group of sellers (possibly empty) that select P_M as a pure strategy.
2. Bottom: At least two sellers who have the full support of $[P_L, P_M]$ with some NE dependent continuous full support distribution function F .
3. Middle: A group of sellers (possibly empty) with an individual cutoff price, such that below the cutoff price the distribution used is the same F as the Bottom group. Above the cutoff price, the seller has a mass point at the reserve price.

Furthermore, all sellers have the same profit $n_i P_M (1 - \mu) / S$ and have P_M in support.

An illustration of the three types of strategies can be seen in Figure 2.

These possible distributions follow directly from the fact that sellers must have the same distribution on intervals (Lemma A.9 and the corollaries) and that any point in between P_L and P_M needs to be in the support of at least two sellers (Corollary A.2).

Remark 4.1 *For any combination where the middle group is empty and the bottom group has at least two sellers, there exists a corresponding NE. To see this, simply adjust the shoppers' share to reflect the game where only searchers visiting the mixing sellers are in the game.*

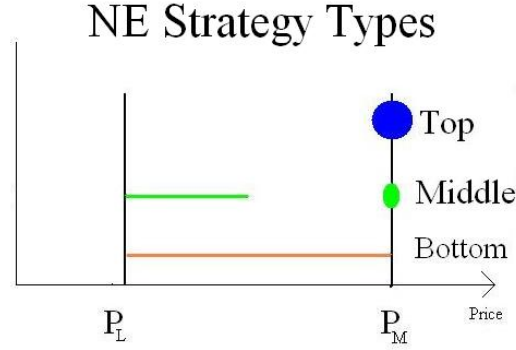


Figure 2: The three types of strategies available in an NE of the model

Remark 4.2 *Note that this result extends the original model equilibrium, as all sellers can have the same size in the model discussed here. When all sellers have the same size and choose the bottom strategy, the resulting equilibrium would be symmetric and identical to that of the original model. When some choose the middle or top strategies, an asymmetric equilibrium of the original model is obtained.*

Now it is possible to elaborate on the structure of the F which is used in equilibrium by sellers, and on what reserve price can be used. Suppose that in equilibrium we have B sellers with the ‘bottom’ strategy (mixing over the entire support), T sellers with the ‘top’ strategy (pure reserve price), and $M = \{1, 2, \dots, m\}$ sellers with the ‘middle’ strategy (a cutoff price strategy), with the corresponding cutoff prices of cp_1, cp_2, \dots, cp_m and corresponding mass points at the reserve price with masses a_1, a_2, \dots, a_m .

Denote the set of sellers with cutoff point above some price p by $L(p)$. Additionally, let us denote by μ^* the adjusted share of shoppers. That is, $1 - \mu^*$ is the mass of searchers visiting one of the smaller sellers. All other searchers purchase the item for P_M from one of the larger sellers.

From the structure of the equilibrium, all sellers have equal profit per store, and since all have the size n_m , they have equal profits. Furthermore, all sellers have P_M in their support and the reserve price attracts no shoppers. Therefore, the profit for all sellers is

$$\pi = n_m P_M (1 - \mu^*) / S. \quad (4)$$

For any price p , the expected profit needs to be equal to the expression above. At price p , if seller i is the cheapest, there is some probability, $\alpha_i(p)$, of their attracting shoppers. This can be calculated as follows:

- For each seller $j \neq i$, calculate the probability that j offers a price above p .

- Multiply these probabilities.
- All larger sellers select P_M purely and therefore have no effect.

Let p be a price in (P_L, P_M) . For group B , this probability is clearly equal to $1 - F(p)$. For group T it is zero. For group M , we need to distinguish two cases: either the seller is in $L(p)$ and the probability is $1 - F(p)$, or not, and then it is equal to $a(p)$. Combining the cases, we get that the expression for the expected profit is

$$\pi = n_m P_M (1 - \mu^*) / S = p [n_m (1 - \mu^*) / S + \mu \left(\prod_{j \in B \cup L(p)} (1 - F(p)) \prod_{j \in M \setminus L(p)} (a_j) \right)]. \quad (5)$$

As F is the same, we can simplify:

$$p [n_m (1 - \mu^*) / S + \mu^* ((1 - F(p))^{b+|L(p)|} \prod_{j \in M \setminus L(p)} a_j)] = P_M n_m (1 - \mu^*) / S. \quad (6)$$

Extracting $F(p)$ from this equation yields

$$F(p) = 1 - \sqrt[b+|L(p)|]{\frac{n_m}{S \mu^*} \left(\frac{P_M}{p} - 1 \right) \frac{1 - \mu^*}{\prod_{j \in M \setminus L(p)} a_j}}. \quad (7)$$

Note that at if seller j has a cutoff price at p , it must be the case that $a_j(p) = 1 - F(p)$, and therefore F will be continuous. It will also be differentiable at all points that are not cutoff prices. Therefore, it is still possible to calculate the density and in some cases the expected value explicitly, as the number of cutoff prices is finite. The last step, based on Lemma 2.1, requires finding the expected value $E(F)$, and setting the reserve price at $E(F) + c$. This step is technical and the expressions involved cannot be generally calculated, thus it is not done here for the general case, yet examples with some specific cases are provided.

4.2 Examples of Equilibria

Consider three sellers, and consumers of which $1/6$ are shoppers. One-half of the consumers are searchers initially visiting one of the stores, and $1/6$ of the consumers are searchers initially visiting each of the two others. The corresponding number of stores is, for example, 3, 1, 1.

The following asymmetric NE exists:

- The searchers have a reserve price of $P_M = c / (1 - \ln 2) > c$.
- The seller with the larger number of stores offers the reserve price as a pure strategy.

- The other two sellers use the same continuous distribution function on $[P_M/2, P_M]$.
- The distribution function for the two mixing sellers is $F(p) = 2 - P_M/p$.

One needs to check that no seller wishes to deviate. Firstly, note that prices above P_M or below $P_L = P_M/2$ are not profitable for all sellers. Already at P_L there is a probability 1 of selling to shoppers and there is no need of a further discount. Prices above P_M would leave the seller with a quantity sold of zero.

The profit for the pure strategy seller is $P_M/2$ and for the other two it is $P_M/6$ when offering the price P_M .

The profit of the mixed strategy seller when offering a price $p \in [P_L, P_M)$ is

$$p\left(\frac{1}{6} + \frac{1 - F(p)}{6}\right) = \frac{p}{6}\left(2 - 2 + \frac{P_M}{p}\right) = \frac{P_M}{6}. \quad (8)$$

Therefore, the mixing agents are indeed indifferent between the prices in the interval.

Lastly, one needs to show that the pure strategy agent would not deviate to a lower price. The resulting profit, from offering a lower price p , would be

$$p\left(\frac{1}{2} + \frac{(1 - F(p))^2}{6}\right) \leq p\left(\frac{1}{2} + \frac{1 - F(p)}{6}\right) < p\left(\frac{1}{2} + \frac{1 - F(p)}{2}\right) = \frac{P_M}{2}. \quad (9)$$

The first inequality is due to $F(p)$'s lying between 0 and 1. The second strict inequality is from the fact that $1 - F(p)$ is strictly positive when $p < P_M$. The third equality follows from Equation 8.

It is clear that the pure strategy agent has no incentive to deviate to a different strategy, and therefore this is an NE.

Note that the expected price of each seller is at least $P_M - c$. Furthermore, no seller offers a price above P_M . Therefore, the reserve price P_M is rational, by Lemma 2.1.

4.3 Additional Examples

It is possible to construct additional examples for the original model as follows: add to a symmetric NE setting an additional seller that charges purely a reserve price. Then, by adjusting the fractions of the searchers, as in the example above, an NE will be obtained. There exists an NE where the sellers with the lowest number of stores ignore the searchers visiting one of the larger sellers and obtain among them a symmetric (for example) NE, and the other sellers set a pure strategy of P_M . The way to show that the profiles are NE is similar to the example above, for example, by having more than one agent selecting the reserve price as a pure strategy, or a seller's having a cutoff price.

5 Unique Smallest Seller

So far, the case where the smallest seller was unique has been omitted. This case is treated here. In this structure, there exists an NE, however these differ slightly in structure from the previous case. In Astorner-Figari and Yankelevich [1], the Stahl model with two different sized sellers is discussed, but here this is done in a more general setting with more sellers. The NE below stands in line with two sellers' behaving according to the results in Astorner-Figari and Yankelevich [1], while the rest offer the reserve price purely.

Denote the smallest seller by m and (one of) the second smallest seller by j , with the corresponding number of stores n_m and n_j . Denote the shares of the consumers that visit seller i as searchers (note Corollary 3.1) as follows.

$$Src_i = \frac{n_i}{S}(1 - \mu) \quad (10)$$

Proposition 5.1 *There exists an NE with a reserve price P_M such that*

1. *All sellers except m and j select P_M purely.*
2. *The lowest price in the union of sellers' support is $P_L = P_M(\frac{Src_j}{\mu + Src_j})$.*
3. *m and j mix on the entire interval (P_L, P_M) .*
4. *The distributions are as below, and j has a mass point at P_M .*

$$F_m(p) = 1 - \frac{Src_j}{\mu} \left(\frac{P_M}{p} - 1 \right) \quad (11)$$

$$F_j(p) = \left(1 - \frac{P_L}{p} \right) \left(1 + \frac{Src_m}{\mu} \right) \quad (12)$$

$$P_M = \frac{c}{1 - \ln\left(\frac{Src_j + \mu}{Src_j}\right) \cdot \frac{Src_j}{\mu}} \quad (13)$$

$$F_j(p) \leq F_m(p) \forall p \quad (14)$$

Proof:

First check that there is no deviation by sellers m and j :

It is easy to verify that there is a constant profit for sellers m and j in the interval, since

$$\pi_m(p) = ((1 - F_j(p))\mu + Src_m) \quad (15)$$

$$\pi_j(p) = ((1 - F_m(p))\mu + Src_j). \quad (16)$$

The market share consists only of searchers initially visiting the stores of a given seller, and possibly of the shoppers. The probability that a seller attracts shoppers with price p is as follows: for seller m is $1 - F_j(p)$, and for j is $1 - F_m(p)$. It is simply the probability to be the cheapest seller, meaning that the other seller has price above p .

Offering prices below P_L are not profitable, as already in P_L one attracts the shoppers with probability 1. Prices above P_M will not be offered due to Corollary 3.1. Therefore, m and j have no profitable deviation.

A seller k who is not m or j would similarly refrain from selecting prices below P_L or above P_M . Deviating to a price $p \in (P_L, P_M)$ would yield the profit

$$\begin{aligned} \pi_k(p) &= p((1 - F_m(p))(1 - F_j(p))\mu + Src_k) < \\ & p((1 - F_m(p))\mu) + p(Src_k). \end{aligned}$$

Seller j has price p in support and therefore

$$\pi_j = p((1 - F_m(p))\mu + Src_j) = P_M Src_j. \quad (17)$$

Combining these equations yields

$$\pi_k(p) < \pi_j - Src_j p + p Src_k = P_M Src_j + p(Src_k - Src_j). \quad (18)$$

Note that the size of k is at least the size of j , implying that $Src_k \geq Src_j$. Note also that $p < P_M$. Therefore, the profit of seller k when offering price $p \in (P_L, P_M)$ is below $P_M Src_k$. However, this profit is obtained by k when offering P_M , and therefore, there is no profitable deviation from P_M .

Lastly, the reserve price is rational. Note that if one compares the derivatives of F_m and F_j ,

$$f_m = \frac{Src_j P_m}{\mu p^2} \quad (19)$$

$$f_j = \frac{(\mu + Src_m) P_L}{\mu p^2}. \quad (20)$$

Using the facts that $Src_m < Src_j$ and $P_M Src_j = P_L(\mu + Src_j)$, it is easy to see that $f_m(p) > f_j(p)$ for any price in (P_L, P_M) . As the distribution of j has a mass point at the maximal price of P_M , the expected value of F_m is smaller than that for F_j .

From Lemma 2.1, in order for the reserve price to be rational, $E(F_m)$ needs to be at least $P_M - c$, which occurs with equality, due to the structure of P_M :

$$E(F_m(p)) = \int_{P_L} P_M p f_m(p) = \frac{Src_j P_m}{\mu} \int_{P_L} P_M \frac{1}{p} = \frac{Src_j P_m}{\mu} \cdot \ln \left(\frac{Src_j + \mu}{Src_j} \right) = P_m - c. \quad (21)$$

□

Note that here, all sellers but m have the same profit per store and a mass point at P_M . The smallest seller has a larger profit per store, and offers more generous discounts. Additional equilibria may exist where several of the smallest sellers after m also mix, with identical distribution (but different than the distribution of m). This is in line with the results pointed out in Astorner-Figari and Yankelevich [1]. The next step is to determine the general structure of an NE in such case. This is given in the theorem below.

Theorem 2 *In the case of a unique smallest seller, the NE with a reserve price P_M of the game is as follows.*

- *All sellers with size above n_j select the reserve price purely.*
- *The lowest price in the union of all supports is $P_L = P_M(\frac{Src_j}{\mu + Src_j})$.*
- *Seller m mixes with a continuous, dense distribution function F_m over (P_L, P_M) .*
- *Some sellers with size n_j also mix over the entire interval with a continuous dense F_j , such that $F_j(p) < F_m(p)$ for all $p \in (P_L, P_M)$, and in addition have a mass point at P_M .*
- *All sellers except m have the same profit per store, and m has a higher profit per store.*
- *Let I be an interval to the left of P_M . If sellers $i, j \neq m$ both have all of I in their support, then both use the same distribution over I .*
- *Any price in (P_L, P_M) is in support of at least two sellers.*

The proof is relegated to the Appendix.

The calculation of the equilibrium distribution is along similar lines to Proposition 5.1 and subsection 4.1.

Remark 5.1 *The strategy choice of sellers with the second smallest size is similar to that of the smallest sellers in the previous case. However, all such sellers will have a mass point at the reserve price.*

6 Discussion

Here is a short discussion of the model and results. First, the structure and economic motivation on the results will be provided. Then, a couple of situations will be shown where the importance of the extension becomes clear. Then, a couple of empirical tests will be suggested for verifying whether the results hold in the lab. Lastly, some points for future research will be suggested.

6.1 NE Structure

The structure of the NE is in line with the results in Astorner-Figari and Yankelevich [1] and Burdett and Smith [8]. Here, a wider setting for seller sizes is applied. In this extension, we see that indeed larger firms find no incentive to compete for shoppers. The reason behind this can be seen easily from the profit structure. The profit of seller i is

$$\pi_i(p) = p\alpha_i(p) + p(1 - \mu)n_i/S, \quad (22)$$

where α is the probability of attracting shoppers when offering price p .

The profit consists of two components: the expected profit from shoppers and the profit from searchers. Setting a lower price has two effects: one is that it increases the probability of attracting shoppers (higher α), but on the other hand, it reduces the informational rent from the searchers (due to a lower p). Note that the first, positive, effect is more size-independent (the probability of being the cheapest increases the same, no matter your size), whereas the second effect is size-dependent and is more significant for larger sellers. Therefore, a larger seller will find it less attractive to offer discounts. In the case of at least two smallest sellers, these will compete one with the other, and larger sellers will not even bother to enter the ‘shopper market’, but will stick to the reserve price. Now suppose that there is a unique smallest seller. The smallest seller will compete for shoppers with some of the ‘second smallest’ sellers. Sellers above that ‘second smallest’ size will stay out.

The three types of strategies for the smallest sellers have some economic motivations. The mixing seller wishes to compete for the shoppers when the pure reserve price seller does not to bother with shoppers. That kind of behavior is common in the economic world, and not in all cases will all compete as predicted by the symmetric NE. If only a single seller decides to compete, monopolistic profits will accrue, which would attract additional competitors, and therefore, in NE, at least two sellers will compete for the shoppers. As suggested in Astorner-Figari and Yankelevich [1], in the case of a unique smallest seller (their setting is of two sellers with different sizes), the smaller seller will offer lower prices

with a higher probability, due to their lower share of the visiting searchers. Therefore, in a competition with second smallest firms, the smallest seller has an advantage.

The cutoff price is for sellers that do not wish to be bothered with small probabilities. There are several effects that may cause a seller to refrain from bothering about events with sufficiently low probabilities, for example, see Barron and Yechiam [2]. Such a seller will compete for shoppers, but only at prices that yield the benefit of getting the shoppers with a high enough probability. When the probability of attracting shoppers is lower than this individual threshold, the seller prefers to refrain from the shopper market, but instead selects the reserve price with a mass point.

These three strategies, in addition to the size implication, explain the behavior of sellers in the NE of this model.

6.2 Real World Examples

The first example uses the data from Table 2, which is from Bazucs and Imre [4]. That paper discusses the pricing of a homogeneous good (milk) in 8 discounters in Hungary over a period of five years (2004–2008). The data as to the numbers of stores was not available in the paper, and so the current numbers of stores were obtained from company profiles (Jan 2012), with two exceptions. One of the supermarket chains, PLUS, was purchased by another, InterSpar, after the relevant period ³. Therefore, the number of PLUS stores (around 170 stores) was subtracted from the current number of InterSpar stores. The current number of stores serves as an indicator of the number of stores in the research period, and is divided into several distinct groups by size. This provides enough insight, and the idea that smaller chains are usually cheaper. Note that there can be additional factors (such as the location of the stores) affecting the price. However, one factor can indeed be the chain size.

| Chain name | Number of stores in Hungary | Avg. price of milk |
|------------|-----------------------------|--------------------|
| InterSpar* | 100–200 | 182 HUF |
| Cora | Below 20 | 198 HUF |
| Match | Below 20 | 200 HUF |
| Tesco | 200–400 | 205 HUF |
| Auchan | Below 20 | 211 HUF |
| CBA | Above 500 | 213 HUF |
| Plus* | 100–200 | 230 HUF |
| COOP | Above 500 | 240 HUF |

³For example, see <http://www.portfolio.hu/en/tool/print/2/14361>

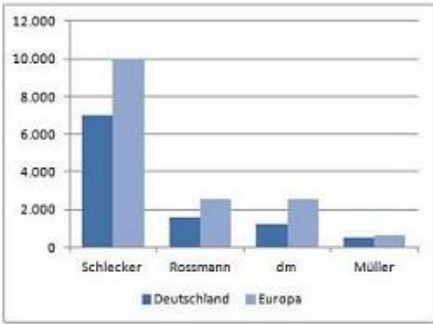


Figure 3: Number of stores in various drug store chains

Another example is from drug stores in Germany. GKL⁴ research found that Schlecker is 10%–20% more expensive than its competitors. The number of stores in January 2012 of the various chains, as collected by GKL, in Germany and Europe is given in Figure 3. Again we see a tendency that the largest chain is the more expensive one. Again, additional factors may have an effect here, but it seems that chain size plays a role when determining prices.

To conclude, it seems that there should be some positive correlation between chain size and price here too. A similar effect was observed in a more limited setting in Astorne-Figari and Yankelevich [1] and Burdett and Smith [8]. Additional empirical research checking this connection explicitly should be able to determine how strong it is.

6.3 Empirical Tests and Policy Suggestions

The structure of the NE allows running several empirical tests on a database containing pricing and chain size data. In most of the NE found here, some sellers select the reserve price with a mass point. This implies that the reserve price will be more commonly selected. Similarly, larger discounts will be rarer, as the reserve price will be more common. Furthermore, one should see a correlation between chain size and price. Moreover, in a dynamic setting, there should be some price stickiness, as mass points exist.

When examining the probability for a consumer to encounter a lower price, it is clearly visible that the larger is the variance in store sizes, and the rarer are the smallest stores, the closer to the reserve price is the expected price paid. This is an additional factor to examine, and it suggests an interesting policy decision. If the regulator wishes to reduce the prices of goods, reducing the variance among the selling firms can reduce the price, as there exists an NE where fewer sellers select the reserve price purely.

⁴Conducted in Jan. 2012 by GKL, comparing prices in 1700 stores in Germany. Media report for example here: <http://www.handelsblatt.com/unternehmen/handel-dienstleister/gkl-analyse-schlecker-deutlich-teurer-als-die-konkurrenz/6254086.html>

6.4 Future Research

The results here raise several important questions, which leave room for fruitful future research. Firstly, the assumption here is that a reserve price exists. There may be additional NE without a reserve price, and an interesting question is whether such exist and what do they look like. This will allow fully characterizing all NE of the model and fully explaining the behavior of sellers.

An additional question is related to the determination of the reserve price. What is the full set of reserve prices under a certain setting, as here the lemma provides only a sufficient condition for its rationality. Moreover, which reserve prices will the consumers set in order to minimize their price. Similarly, how would sellers react to increasing the reserve price—by sticking to reserve price, or mixing over the entire interval, or perhaps a mixture of all three strategies, including some specific cutoff prices? This question of consumer welfare and seller welfare will provide an important insight into the behavior of these groups, and can guide policy decisions for a regulator in order to induce lower or higher prices.

Additional extensions can be imposing the price to be similar in all stores of a seller but not identical. On the consumer side, introducing reputation effects, where a high price has a negative effect, but not total disregard, on future search probability. What would be the equilibria when these, or other extensions, are introduced?

The Stahl model is a very important tool and the model is being used and applied in numerous papers. The author hopes that this paper provides an additional important insight which will make the Stahl model more applicable and more realistic.

A Proof of Theorem 1

Some of the results will be applicable also for the proof of Theorem 2.

Remark A.1 *Remember that in any NE, a seller i using a mixed strategy is indifferent among all strategies with positive density. Thus, seller i would receive the same expected profit from all pricing strategies with positive density.*

A.1 Mass Points and Highest Offered Price

Lemma A.1 *In any NE there are no mass points at any price below P_M that can attract shoppers with positive probability.*

Suppose $q < P_M$ and seller i has a mass point at q . Assume further that there is a positive probability that a shopper would purchase the item at price q or above.

Firstly note that due to undercutting, no other seller $j \neq i$ has a mass point at price q .

Note that at price q , the chance of attracting shoppers drops discontinuously for any seller $j \neq i$. Therefore, if seller j has the interval $(q, q + \varepsilon)$ for some small $\varepsilon > 0$ in support, a deviation to q would increase the probability of attracting shoppers. Since $q < P_M$, relevant prices attract searchers, and the share of searchers would not be affected. Therefore, such a deviation is profitable for seller j and would not occur in equilibrium.

From this it follows that there exists $\varepsilon > 0$ such that no seller except i has $(q, q + \varepsilon)$ in support. Furthermore, there are no mass points at q , except the one i has. Thus, seller $j \neq i$ would never offer a price in $[q, q + \varepsilon)$. From this it follows that seller i can set the mass point not at q , but rather at a higher price in $(q, q + \varepsilon)$, with the same probability of attracting shoppers, and higher expected profit. This contradicts the fact that initially we had an NE. \square

Lemma A.2 *In any NE, all sellers have P_M as the supremum point of their strategy support.*

From Corollary 3.1, it cannot be higher than P_M .

Suppose that the supremum price in the support of seller i is $p < P_M$. Further assume that seller i has the (weakly) lowest support supremum. For any price above p and below P_M , the probability of selling to shoppers is 0. Therefore, in equilibrium, no seller would have a price $q \in (p, P_M)$ in their support.

No sellers can have a mass point at p , as in such a case undercutting would be profitable. From Lemma A.1, seller i has no mass point at price p . It follows that the probability of selling to shoppers at price p is 0 for seller $j \neq i$. Hence seller j would have no mass point at p , as P_M yields a higher profit. Therefore, a profitable deviation exists for seller i , where i selects prices arbitrarily close to P_M instead of prices arbitrarily close to p . \square

This lemma, combined with Corollary 3.1 and Lemma A.1, shows that there can be no mass points at any price except for P_M .

A.2 Single Interval

Lemma A.3 *In any NE, there exists an interval I such that the union of the sellers' strategies is contained in I and dense in it.*

Suppose there exists an interval $[a, b]$ (where $a < b < P_M$) such that sellers select prices only below a and above b , and there exist prices both below a and above b in the union of the supports. Let p be the highest price below a that is in the union of the supports of the sellers. Let us examine a deviation from p and prices just below it to b .

Note that there are no mass points between p and a , and that the distribution mass of all sellers is arbitrarily small there. Since the probability for someone's selecting a price arbitrarily close to p and below p is arbitrarily small, the decrease of the probability of selling to shoppers is arbitrarily small. From lemma A.1, at most a single seller has a mass point at p . Let seller i be a seller with mass point at p , or if no seller has mass points there, seller i has p in support.

The profit of seller i from raising the price from p to $b - \varepsilon$ is much higher than an arbitrarily small loss in probability to attract shoppers. The gain is at least $n_i(b - p)(1 - \mu)/n$, as the searchers pay strictly more after the deviation. Remember that searchers' behavior does not change, as the prices are below P_M . Therefore, if the support is not continuous, there is a profitable deviation, contradicting the NE condition. \square

Corollary A.1 *In any NE, there exists an interval $I = [A, P_M]$ such that in any NE strategy profile the sellers randomize continuously over I , and possibly some sellers set mass points at P_M , where A is some positive number.*

Lemma A.4 *The previous lemma holds also for two sellers. This means that any interval has a non-empty intersection with the support of at least two sellers.*

Suppose that all points in an interval $[p, p']$ (where $p < p' < P_M$) are selected by a unique seller denoted by i . Then there exists a profitable deviation for i consisting of setting a mass point at p' instead of selecting the original distribution over the interval. \square

Corollary A.2 *In any NE, any interval between A and P_M has points in the support of at least two sellers.*

A.3 Profit per Branch for Multiple Smallest Firms

Seller profit divided by the number of stores (n_i) is referred to as the seller's 'Profit per Branch' (PPB), and is denoted by $\hat{\pi}$. The main point is that in NE, each seller has a fixed PPB when mixing over all strategies with positive density.

Let F_i be the distribution describing the equilibrium mixed strategy of seller i .

Lemma A.5 *At least $N - 1$ sellers have the same PPB.*

If all sellers have a mass point at P_M , undercutting is possible. Therefore, in any NE, at least one seller does not have a mass point at P_M .

A seller who does not have a mass point at P_M would, with probability 1, offer a price below P_M . Thus, all sellers who have a mass point at P_M have probability 0 of selling to shoppers at that price.

Therefore the PPB for sellers with a mass point at P_M is

$$\hat{\pi} = \pi/n_i = P_M(1 - \mu)/S. \quad (23)$$

As the support supremum of all sellers is P_M , Lemma A.2 implies that one of the following two cases must hold in NE (remember that if all have mass points at P_M , undercutting is possible).

- At least two sellers do not have a mass point at P_M .
- A single seller does not have a mass point at P_M .

In the latter case, the lemma holds, as all sellers with mass point at P_M have the same PPB.

Let us now turn to the first case, and denote two of the sellers without mass points at P_M by i and j . Seller i 's probability of attracting shoppers is

$$\prod_{k \neq i} (1 - F_k(p)) \leq 1 - F_j(p). \quad (24)$$

As p goes to P_M , $1 - F_j$ goes to 0. Similarly, for seller j it is bounded by $1 - F_i$. Therefore, in this case, for prices arbitrarily close to P_M , the probability of attracting shoppers is arbitrarily close to zero. Therefore, the PPB of seller i , and the PPB all sellers is

$$\hat{\pi} = \pi/n_i = P_M(1 - \mu)/S. \quad (25)$$

□

Corollary A.3 *If at least two sellers do not have a mass point at P_M , then all sellers have the same PPB.*

Corollary A.4 *If only one seller, denoted by i , does not have a mass point at P_M , their PPB will not be lower than all other sellers.*

Seller i can always deviate to P_M and obtain PPB of at least:

$$\hat{\pi} = \pi/n_i = P_M(1 - \mu)/S \quad (26)$$

This profit is obtained only from searchers, and additional profit can be obtained from shoppers.

Lemma A.6 *Suppose the smallest seller is not unique. In any NE, all sellers have the same PPB. That is, $\frac{\pi_i}{n_i}$ is equal for all sellers.*

From Lemma A.5, we can concentrate on the case when seller i is the only seller who does not have a mass point at P_M , and has a higher PPB than all other sellers. Note that seller i can always deviate to P_M , and have the same PPB as all other sellers.

Let p_i be the lowest price in the support of i . Assume further that there exists a seller denoted j with $n_j \leq n_i$ ⁵. Since the lowest n_i is not unique (by assumption), such j always exists.

Recall that p_i the lowest (infimum) price seller i can offer. The profit of seller i offering p_i is as follows (Note that all searchers visit exactly one store, by Corollary 3.1).

$$\pi_i(p_i) = p_i((1 - F_j(p)) \prod_{k \neq i, j} (1 - F_k(p_i)))\mu + (1 - \mu)n_i/S \quad (27)$$

The profit of seller j is

$$\pi_j(p_i) = p_i((1 - F_i(p)) \prod_{k \neq i, j} (1 - F_k(p_i)))\mu + (1 - \mu)n_j/S. \quad (28)$$

If one calculates the PPB for the two sellers, it will be equal to

$$\hat{\pi}_i = p_i(1 - F_j(p)) \left[\prod_{k \neq i, j} (1 - F_k(p_i)) \frac{\mu}{n_i} \right] + (1 - \mu)p_i/S \quad (29)$$

$$\hat{\pi}_j = p_i(1 - F_i(p)) \left[\prod_{k \neq i, j} (1 - F_k(p_i)) \frac{\mu}{n_j} \right] + (1 - \mu)p_i/S. \quad (30)$$

Since $0 = F_i(p_i) \leq F_j(p_i)$ and $1/n_i \leq 1/n_j$, one gets that the expression $\hat{\pi}_j$ is weakly higher than that of $\hat{\pi}_i$.

⁵Here is the only place where the multiple smallest sellers assumption is required

Therefore, the PPB of seller j is weakly higher, contradicting our assumption.

Therefore, all sellers must have the same PPB. \square

Definition A.1 *Let P_L be the lowest possible price which can be offered in equilibrium.*

Remember that no mass points are used at prices below P_M . Since P_M is the highest price, it is clear that $P_L < P_M$. Therefore, P_L attracts shoppers with probability 1. Thus, if P_L is offered by seller i , P_L needs to satisfy

$$P_L(\mu/n_i + (1 - \mu)/S) = P_M(1 - \mu)/S. \quad (31)$$

Lemma A.7 *In NE, only sellers with the smallest number of stores have P_L in their support.*

Let i, j be two sellers satisfying $n_i < n_j$.

Suppose that seller j is the seller with P_L in support. Therefore, the PPB of seller j is

$$\hat{\pi}_j(P_L) = P_L(\mu/n_j + (1 - \mu)/S) = P_M(1 - \mu)/S. \quad (32)$$

Since $n_i < n_j$, if seller i were to offer P_L , their PPB would be

$$\hat{\pi}_i(P_L) = P_L(\mu/n_i + (1 - \mu)/S). \quad (33)$$

Since $n_i < n_j$, we get that the PPB of seller i would be larger than that of seller j . Since in any NE, the PPBs of i and j are equal, seller i does not have P_L in support. However, a deviation to P_L is possible and would increase the PPB (and clearly the profit), contradicting the NE assumption. \square

A.4 Attracting Shoppers

Definition A.2 *Let $\alpha_i(p)$ denote the probability that p is the cheapest price, if seller i selects it. Explicitly: what is the probability of seller i 's selling to shoppers, given that i has selected price p ? As the distribution is with no mass points except (maybe) P_M , one can define $\alpha_i(p)$ as follows.*

$$\beta_j(p) = 1 - F_j(p) \quad (34)$$

$$\alpha_j(p) = \prod_{j \neq i} \beta_j(p) \quad (35)$$

Lemma A.8 *Suppose seller i has price p in support. Then, for each seller j , $\alpha_i(p)/n_i \geq \alpha_j(p)/n_j$.*

Note that the PPB of seller i is

$$\hat{\pi}_i(p) = p(\mu\alpha_i(p)/n_i + (1 - \mu)/S) \quad (36)$$

Suppose that $\alpha_i(p)/n_i < \alpha_j(p)/n_j$. Then, a seller j who offers price p has a PPB of

$$\hat{\pi}_j(p) = p(\mu\alpha_j(p)/n_j + (1 - \mu)/S) > p(\mu\alpha_i(p)/n_i + (1 - \mu)/S) = \hat{\pi}_i(p). \quad (37)$$

A seller j with price p in their support would get a higher *PPB* than i , contradicting Lemma A.6. A seller j who does not have p in support can deviate and increase their PPB, which is equal to the PPB of seller i . \square

Lemma A.9 *Consider an NE strategy profile with a reserve price P_M . Let I be an interval which does not include P_M . Suppose seller i has all of the interval I in support, and seller j does not. Then in the support of seller j there are no prices above I except P_M .*

The probability of attracting shoppers is

$$\alpha_i(p) = (1 - F_j(p)) \prod_{k \neq i, j} (1 - F_k(p)). \quad (38)$$

Writing out and comparing the α divided by the number of stores yields

$$\frac{\alpha_i(p)}{n_i} = \prod_{k \neq i, j} (1 - F_k(p)) \frac{(1 - F_j(p))}{n_i} \quad (39)$$

$$\frac{\alpha_j(p)}{n_j} = \prod_{k \neq i, j} (1 - F_k(p)) \frac{(1 - F_i(p))}{n_j}. \quad (40)$$

By Lemma A.8, the first expression is weakly larger than the second, as only the highest α/n can have the price in support. This implies that

$$\frac{(1 - F_j(p))}{n_i} \geq \frac{(1 - F_i(p))}{n_j}. \quad (41)$$

It follows that in I , seller i has the highest α/n . Assume that there exists a price $p' < P_M$ which is the lowest price above I that can be in the support of seller j . This implies that at price p' , seller j has a maximal α/n . Note that since the prices between $p \in I$ and

p' cannot be selected by j , since $F_j(p') = F_j(p)$, as j does not select prices in between. However, F_i has increased in I , as seller i has some mass over I . This implies that the inequality (41) holds also for p' as it does for prices in I , as an element on the left hand side was increased, and right hand side remained constant. Thus, at p' , it holds strictly.

$$\frac{(1 - F_j(p'))}{n_i} > \frac{(1 - F_i(p'))}{n_j} \quad (42)$$

Remark A.2 *The reason that the result holds for prices below P_M is that in the course of the proof, a division by $\prod_{k \neq i, j} (1 - F_k(p))$ was applied, and it needs to be positive. This happens at any price below P_M .*

The inequality above implies that seller j cannot have density on price p' , as seller i has a higher α/n , for $p < P_M$. Thus, if seller j does not select some interval in support union, j will not select any price above it except possibly P_M . \square

Corollary A.5 *In any NE of the game, all sellers that do not have the lowest number of stores will select P_M as a pure strategy.*

This is since such sellers cannot offer the price P_L with sufficiently high PPB, as required by Lemma A.7.

Corollary A.6 *The last lemma and the last corollary hold for any two sellers with the same PPB, and for any seller who does not have the smallest stores number. This holds even if the smallest seller is unique.*

Corollary A.7 *Consider an NE with P_M as a reserve price, with a single smallest seller. Let I be an interval which does not include P_M . Consider two sellers i, j with the same size above n_m . Suppose the sellers i, j have all of I in their support. Then, they must do so with the same density.*

Note that the equality of PPBs must hold for those two sellers, and therefore α_i and α_j in the interval cannot differ. If this was the case, the one with lower density could not select some prices in I , as that seller would need to jump to P_M .

From the lemmas so far, we can conclude the following.

- All sellers with size above n_m select P_M purely.

- All sellers have same profit per store.
- Searchers buy at the first store they visit, as all sellers always offer prices weakly below P_M .
- If seller i does not have the interval $(p, p') \subset (P_L, P_M)$ in their support, then the only price above p the seller i has in support is P_M . (This follows from Lemma A.9.)
- Let I be an interval which does not include P_M . If two sellers have in support all of interval I , then both do so with the same density.
- Prices in (P_L, P_M) need to be in the support of at least two sellers.

This completes the proof of Theorem 1

B Proof of Theorem 2

In this section I provide the proof of Theorem 2. Note that some of the results for Theorem 1 also hold here. Specifically, the ones in subsections A.1 and A.2.

As before, the smallest number of stores of any seller is denoted by n_m , and is the parameter of seller m . The next smallest size is denoted by n_j and is the parameter of seller j .

Following Lemma A.6, either all sellers have (except for m) the same *PPB*, when m has a weakly higher *PPB*. From Corollary A.6 we have that among sellers with the same *PPB*, only sellers with the smallest size would offer prices below P_M . Therefore, all sellers with size above n_j (the second smallest size) offer P_M purely. Furthermore, P_L is determined by the second smallest sellers, as at least two sellers must ‘cover’ every interval (Corollaries A.2 and A.6).

Let P_L denote the lowest price in the support union.

Lemma B.1 *In any NE, the PPB of all sellers except m is equal to $P_M \frac{1-\mu}{S}$. The PPB of seller m is strictly higher. Furthermore, all sellers except m have a mass point at P_M .*

Here also Corollary A.2 holds. Thus, some sellers mix, and at least two have in their support the price P_L , where some seller $i \neq m$ is one of those. Comparing possible *PPBs* when offering price P_L of sellers i and m yield the following equations.

$$PPB_i = P_L(\mu/n_i + (1 - \mu)/S) \quad (43)$$

$$PPB_m = P_L(\mu/n_m + (1 - \mu)/S) \quad (44)$$

Since $n_m < n_i$ by definition, it is clear that an m offering this price will have a higher PPB. This implies that an m offering P_L must have a higher PPB than all other sellers.

If m does not have P_L in support, a deviation to P_L increases the PPB above that of i . Therefore, in equilibrium, the PPB of seller m is strictly larger than the PPB of all other sellers.

Note that if at least two sellers i, j do not have a mass point at P_M , then all sellers have same PPB. As the price increases to P_M , seller i 's probability of attracting shoppers is

$$\prod_{k \neq i} (1 - F_k(p)) \leq 1 - F(j). \quad (45)$$

As p goes to P_M , $1 - F_j$ goes to 0. Similarly, for seller j , it is bounded by $1 - F_i$. Therefore, exactly one seller does not have a mass point at P_M . \square

Lemma B.2 *m has a higher probability of offering a discount. Namely, $F_j < F_m$ in (P_L, P_M) .*

Proof:

Note that

$$\alpha_i(p) = \prod_{k \neq i} (1 - F_k(p)) = \frac{\prod (1 - F_k(p))}{1 - F_i(p)}. \quad (46)$$

Therefore, the larger α will have the larger F . Note that j sells to no shoppers at price P_M and m sells to all shoppers at price P_L . When we write the profit expressions for sellers j and m , we get

$$\pi_m(p) = p(\mu\alpha_m(p) + \frac{n_m(1 - \mu)}{S}) = P_L \left(\mu + \frac{n_m(1 - \mu)}{S} \right) \quad (47)$$

$$\pi_j(p) = p(\mu\alpha_j(p) + \frac{n_j(1 - \mu)}{S}) = P_M \left(\frac{n_j(1 - \mu)}{S} \right). \quad (48)$$

For simplicity, put $Src_i = (1 - \mu)n_i/S$

Extracting the expressions for α_m and α_j yields

$$\alpha_m = \frac{(P_L - p)Src_m + P_L\mu}{p\mu} \quad (49)$$

$$\alpha_j = \frac{P_M - p}{p\mu} Src_j. \quad (50)$$

Comparing $\alpha \cdot p$, we get

$$p \cdot \alpha_m = \frac{(P_L - p)Src_m + P_L\mu}{\mu} = Const_m - p \cdot Src_m \quad (51)$$

$$p \cdot \alpha_j = \frac{P_M - p}{\mu} Src_j = Const_j - p \cdot Src_j. \quad (52)$$

Note that $\alpha_m(P_L) = \alpha_j(P_L) = 1$. Comparing their derivatives, one gets that $\alpha_m > \alpha_j$ for prices in (P_L, P_M) , since $Src_m < Src_j$. Since

$$\alpha_j(p) \cdot (1 - F_j(p)) = \alpha_m(p) \cdot (1 - F_m(p)), \quad (53)$$

we obtain that $(1 - F_j(p)) > (1 - F_m(p))$ or $F_j(p) < F_m(p)$, as required. \square

Combining the lemmas, we obtain Theorem 2.

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