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Export and the Labor Market: a Dynamic Model with on-the-job Search

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Abstract

This paper develops a two-sector, two-factor trade model with labor market frictions in which workers search for a job also when they are employed. *On the job search* (OJS) is a key ingredient to explain the response to trade liberalization of sectoral employment, unemployment and wage inequality.

OJS generates wage dispersion and it leads to a reallocation of workers from less productive firms that pay lower wages to more productive ones. Following a trade liberalization the traditional selection effects are more severe than without OJS and the tradable sector experiences a loss of employment, while the opposite is true for the non tradable sector. Starting from autarky, the opening to trade has a positive effect on employment but it increases wage inequality. For an already open economy, a further increase of trade openness can, however, lead to an increase of unemployment. The dynamics of labor market variables is obtained in closed form. The model predicts overshooting at the time of implementation of a trade liberalization, then the paths of adjustment follow a stable transitional dynamics.

JEL Codes: F12, F16, E24

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1 Introduction

Existing theoretical trade models have in common the feature that only unemployed workers search for jobs. In contrast to this literature, empirical studies show that a significant share of worker reallocation is due to workers who search while they are employed, so called *on-the-job* search (hereafter, OJS). In a recent survey Hall and Krueger (2012) show that about 40% of US employed workers are searching on the job. Furthermore, Bjelland et al. (2011) find that employer-to-employer flows represent 30% of separations each quarter in the US. Being consistent with this evidence, I combine the OJS framework that has been introduced by Burdett and Mortensen (1998) and developed in Mortensen (2010) with a Melitz (2003) trade model.

This paper is the first (to the best of my knowledge) that applies the channel of OJS in a trade model. As a result of this mechanism the model generates wage dispersion across identical workers within a sector. OJS introduces a competition channel across firms in the labor market that is absent when only unemployed workers search. In fact, employed workers accept only offers that are better than their current wage. All firms have the same probability to meet a searching worker, but the rate of success per vacancy posted increases with the wage a firm offers. It follows that firms face an increasing labor supply, given a labor demand that shifts upward with productivity (a classical wage bargaining result). The clearing of labor demand and supply at the firm level implies that more productive firms pay higher wages and because of this they employ more workers. Instead, under a vertical labor supply, (without wage dispersion) a higher productivity translates only in a (greater) employment premium.

Three main implications follow as a consequence of OJS. First, the traditional selection effects (increase in average productivity, average profit, employment per firm and a reduction in the number of firms) are more severe than they would have been without OJS. Therefore, neglecting OJS leads one to under estimate the effects of international trade on labor market outcomes. Second, the model predicts that a trade liberalization increases labor market tightness, defined as the ratio of vacancies over job applications. This implies that (i) the distribution of wages across employed workers stochastically dominates the one before the policy implementation. Average productivity is positively affected by the higher labor market tightness and this causes (ii) a loss of employment in the tradable sector.

Although models without wage dispersion capture the increase in labor market tightness, the welfare implications of the first result on the wage distribution are much stronger than simply considering an increase of the average nominal wage. Moreover in a Melitz (2003) framework without wage dispersion the average productivity does not depend on labor market tightness and the effects of an increase in the extensive margin of trade on employment are ambiguous; see Felbermayr et al. (2011a) for a comparison of these results. Third, the model accounts for a positive exporter wage premium and it predicts (*iii*) an increase in wage inequality between-sector and an inverted U shape response of within-sector wage inequality to the degree of trade openness.

There is evidence in favor of the mechanism through which OJS channel works. Bernard et al. (2011) find that exporters are relatively more productive, pay higher wages and employ more workers than non exporters. Indeed, exporters might play a particular role in the labor market because they are at the top of the wage distribution. When employed workers search, exporters are the ones able to attract employees from firms that serve only the domestic market. Taking this point of view, trade exposure induces selection of less productive firms out of the market, not only because output shares are reallocated to more productive firms (that is an intrinsic feature of firm heterogeneity), but also because employees of the least productive firms reallocate to firms that offer relatively higher wages (that is due to OJS).

The model explains the empirical patterns of sectoral employment, unemployment and wage inequality following a trade liberalization. Trefler (2004) studies the impact of the bilateral free trade agreement between US and Canada on employment, productivity, firm dynamics and welfare. The results suggest that the short term effect on employment is negative: 5% of Canadian total manufacturing jobs are lost and the loss rises up to 12% for those industries with the larger tariff reduction. Nevertheless, within ten years the lost employment was offset by employment gains in other manufacturing industries. Goldberg and Pavcnik (2007) document the increase in wage inequality in several developing countries following trade liberalization. The increase in wage inequality is sharper when the country is opening to trade, while it vanishes over time. Felbermayr et al. (2011b) document a spurious positive correlation between unemployment and trade openness. The phenomenon vanishes in the long run once controlling for business cycle variables. Moreover the estimated correlation is either negative and significant or not significant; as in the in the majority of the reviewed studies.

The mechanism of the model sheds light on the key role of OJS for the response of sectoral employment, unemployment and wage inequality following a trade liberalization. When the extensive margin of trade widens, OJS strengthens the competition that firms face in the tradable sector. Selection reduces employment in the tradable sector whereas the additional demand of service (to finance export and entry) increase employment in the service sector, for a given technology. As a consequence, a country is more likely to experience an increase in unemployment when it liberalizes trade in less competitive sectors and the more efficient it is in the production of non tradable services. Wage inequality between workers employed in different sectors increases, because the higher labor market tightness in the tradable sector pushes up wages, while the wage in the non tradable sector is unaffected. The sign of the change in wage inequality across workers within the tradable sector depends on wage response to labor market tightness. The model predicts that an increase in labor market tightness leads to higher wages but the change is larger at the bottom of the wage distribution. When this phenomenon is strong enough, then the wage is a convex function of labor market tightness at the bottom of the distribution. Convexity turns into concavity as the wage grows. As a result, wage inequality increases with a trade liberalization when the share of exporters is relatively small, because there is substantial mass of workers at the bottom of the wage distribution. Further liberalization episodes occur with a larger share of workers employed by exporter firms, leading to a milder response and possibly a decline of wage inequality as employment is more and more concentrated toward exporters.

A further contribution of the paper is the study of transitional dynamics. Melitz and Redding (2013) in a recent survey of the literature argue that there is a lack of understanding for the transitional dynamics following the economy's response to trade liberalization. As an example, the conflict between the short-run adjustment costs and long-run gains from the trade liberalization is clear in the literature I briefly mentioned. In this paper I provide a closed form solution for the transitional dynamics of labor market tightness, sectoral employment, vacancies, unemployment, wages and number of firms.

The analysis remains analytically tractable under the assumption that among a multiplicity of optimal paths for vacancy posting, firms choose the one such that the firm value is constant during the transition. This restriction captures the idea that firms do not post vacancies strategically to temporarily deviate from their steady state value over the transition. From the theoretical point of view, firms do not have an incentive to deviate from this path, because of perfect information. Empirically, this restriction prescribes a positive correlation between hiring and operative profit. A few recent studies investigate the hiring behavior of firms and their findings are consistent with this hypothesis.[1]

The model predicts stable transitional dynamics of labor market variables. The forcing variable is the entry of new firms. The labor market adjusts through job creations and vacancy destruction rates that are endogenously determined by the previous state of the economy and the current resource allocation of service across incumbents and potential entrants. Since vacancies and job applications are posted at the end of a period, the labor market tightness in the tradable sector is predetermined, in the absence of the implementation of a trade liberalization. Because of this rigidity, an unanticipated trade liberalization suddenly changes the number of entrants in the upcoming period, then the labor market tightness jumps to a new value that is not in line with the new steady state level. From this point on, the mass of firms as well as labor market tightness, vacancies and sectoral employment levels monotonically adjust to the new steady state level. Therefore, the model predicts that a trade liberalization is associated with an overshooting (undershooting) of unemployment at the time of policy implementation for those economies that will experience higher (lower) unemployment in the long run.

One way to apply these results to reality is to distinguish between developed economies versus developing economies. The former group is likely to be already open to trade in a wide range of sectors and endowed with better technologies; while the second group is relatively worse in technology and it suddenly opens to trade in relatively more competitive sectors (textile being a classical example). The model's comparative statics predict that a further trade liberalization in a developed economy does not lead to employment gains (possibly unemployment increases) and it is associated with a moderate increase in wage

[1] In a seminal contribution Yashiv (2000) assesses the dynamics of hiring over the business cycle and he finds support for the idea that the correlation of hiring with employment is weak whereas the correlation of hiring with productivity is positive and significant. A first comprehensive survey on hiring activity can be found in Davis (2013). They show that several sources of current expenditure (such as advertisement, screening) are in place at the same time with vacancy posting to quantify the hiring behavior of firms.

inequality (and possibly even a decrease). Instead, developing economies that open to trade experience a decrease of unemployment and a sharp increase in wage inequality. The analysis of the transitional dynamics predicts a larger loss of employment (higher unemployment) in the short run for developed economies, whereas developing economies experience a lower unemployment rate in the short run than in the long run.

The model I propose is related to a recent literature which combines firm heterogeneity with imperfect labor markets, random matching and wage bargaining to investigate the effect of trade on unemployment. Helpman and Itskhoki (2010) develop a two sector model with search and matching frictions in a DMP framework. Felbermayr et al. (2011a) introduce DMP search and matching frictions in the one sector Melitz trade model. In the former, unemployment might increase if the labor market frictions are relatively lower in the tradable sector than in the non tradable sector. In the latter, the effect of a cut in the fixed cost of export causes a decrease in the average productivity of exporters (which gives a positive contribution to employment) and an increase in the average productivity of all incumbent firms (which decreases employment through selection); the net effect on unemployment arises from the balance of the two forces. OJS represents a major departure from these theories that do not feature wage dispersion across firms. Helpman et al. (2010) extend the literature by introducing ex-post match-specific heterogeneity in workers' ability. This channel introduces wage dispersion and the reallocation of workers toward the more productive firms has an effect on wage inequality. Nevertheless, the origin of wage dispersion is different: optimal sorting in Helpman et al. (2010) whereas competition in the labor market through OJS in the framework I propose. This paper is also related to a very recent contribution provided by Felbermayr et al. (2014) in which they discuss the transitional dynamics of firms and labor market following a trade liberalization.

Finally, several aspects of the model have been framed to be consistent with a number of empirical findings on the effect of an increase in trade exposure. First, using matched employer-employee data for the case of Brazil, Helpman et al. (2012) document that half of the variation in total wage inequality is not explained by observed worker and firm characteristics. Moreover, they find robust evidence that the main source of inequality arises within sectors and occupations and between firms. For the case of Germany, Felbermayr et al. (2014) find that at most 11% of total wage inequality is due to observed worker characteristics. On the basis of these findings, I study the change in the wage distribution among identical workers. Second, Eaton et al. (2004) find that the extensive margin in the number of exporter firms explains the majority of the variation in exports. Therefore I discuss the effect of a trade liberalization using the fixed cost to access the foreign market as policy variable.

The remainder of the paper is structured as follows. The next section outlines the model, along the three markets: consumption, labor and service. In section 3 I solve for the unique steady state general equilibrium. In section 4 I study the transitional dynamics. Section 5 discusses the consequences of a trade liberalization. Section 6 concludes.

2 Model

There are two symmetric countries, home and foreign. In each country, there are two sectors, producing a consumption good and service respectively. In the consumption sector a continuum of single product monopolists supply varieties of a differentiated good. In the service sector a continuum of perfectly competitive producers supply a composite good that is used as an input in the consumption sector. Consumption goods are internationally traded, whereas the service is not traded across countries. Labor is the only variable factor of production.

2.1 Consumption sector

Endowments and preferences. The domestic and foreign economy are populated by a continuum of workers. Each worker is endowed with one unit of homogeneous labor that she is willing to rent to firms in exchange for a given wage. Consumption is allocated over a continuum of varieties indexed by i in the set of varieties produced by domestic firms Ω and foreign firms Ω^*. In both countries preferences are represented by a utility function of the CES type. There is not mean to store wealth, therefore consumers do not transfer consumption across periods. The representative household maximizes utility by allocating consumption given the budget constraint. In every period t, the allocation of consumption across varieties satisfies:

$$\begin{aligned} Q_t &= \max_{\{q_t(i)\}_{i\in(\Omega\cup\Omega^*)}>0} \left(\int_0^{\Omega\cup\Omega^*} q_t(i)^{\upsilon}\, di\right)^{\frac{1}{\upsilon}} \\ s.t. &: \int_0^{\Omega\cup\Omega^*} p_t(i)\, q_t(i)\, di \leq Y_t \end{aligned} \tag{1}$$

where $\upsilon \in (0,1)$ is the intensity of taste for variety $\rho = 1/(1-\upsilon) > 1$, is the elasticity of substitution between any two varieties, $p_t^c(i)$ is the price of the variety i of consumption good $q_t(i)$, Y_t is aggregate expenditure. The consumption base price index based on varieties that are consumed in the domestic economy is $P_t = \left[\int_0^{|\Omega|+|\Omega^*|} p_{qt}^c(i)^{-\frac{\upsilon}{1-\upsilon}}\, di\right]^{\frac{\upsilon-1}{\upsilon}}$ and the value of domestic demand is equal to aggregate expenditure $P_t Q_t = Y_t$.

A domestic firm producing the variety i sells the quantity $q_t(i)$ in the domestic market at a price $p_{qt}(i)$ and it exports the quantity $q_t^*(i)$ in the foreign market at a price $p_{qt}^*(i)$. A foreign producer of variety $j \neq i$ sells $b_t^*(j)$ units in the foreign market at a price $p_{bt}^*(j)$ and exports to the domestic market $b_t(j)$ units at a price $p_{bt}(i)$. The aggregate demands for a domestic variety i and an imported variety j both sold in the domestic market are given by:

$$q_t(i) = P_t^{\rho} Q_t \cdot p_{qt}(i)^{-\rho} \quad , \quad b_t(j) = P_t^{\rho} Q_t \cdot p_{bt}(j)^{-\rho} \tag{2}$$

The aggregate demands in the foreign market read $q_t^*(i) = P_t^{*\rho} Q_t^* \cdot p_{qt}^*(i)^{-\rho}$ and $b_t^*(j) = P_t^{*\rho} Q_t^* \cdot p_{bt}^*(j)^{-\rho}$.

Technology. Production employs labor according to a linear technology that is parametrized by the average labor productivity. Firms are heterogeneous, labor productivity is a random variable a following Pareto distribution $T(a)$ bounded over the support $[a_{\min}, a_{\sup}]$ for $a_{\min} < a_{\sup}$ being positive finite real values[2]. One firm is endowed with one productivity level (a) and it produces one and only one variety (i). The production function of a domestic firm is:

$$y(a, l_t) = a l_t \tag{3}$$

where l_t is the demand of labor at time t and $y(a, l_t)$ is output; foreign firms have the same technology.

[2]The choice of a Pareto distribution is common in the literature, although it is not necessary; see the discussion in Melitz (2003) footnote 15. I follow Mortensen (2009) using a bounded productivity distribution to guarantee the properties of the labor market equilibrium without any further assumption on the second derivative of the employment-productivity mapping.

Exports are associated to an additional per unit cost. In order to sell one unit of good in the export market a firm ships $\tau \geq 1$ units. Let $e = \{0, 1\}$ denote the exporter status: $e = 1$ indicates that the firm is an exporter at time t, otherwise $e = 0$. Market clearing at the firm level and feasibility of production require:

$$q_t(a) + e\tau q_t^*(a) = y(a, l_t) \tag{4}$$

Firms maximize profit in each destination market, subject to consumer demand (2) and technology (3). The marginal revenue per unit of production is $\left(1 - \frac{1}{\rho}\right) p_{qt}$ in the domestic market and $\left(1 - \frac{1}{\rho}\right) \frac{p_{qt}^*}{\tau}$ in the foreign market; due to the cost of shipping goods abroad. Equating marginal revenues yields: $p_{qt}^* = \tau p_{qt}$. The difference in prices translates into a difference in demand in the two destination markets: $q_t^* = \tau^{-\rho} q_t$. Total revenue is proportional to revenue in the domestic market, $r_t = (1 + e\tau^{1-\rho}) p_{qt} q_t$. Inverse demand and feasibility of production (4) yield revenue as a function of productivity and employment:

$$r(a, l_t; e) = \left[\left(1 + e\tau^{1-\rho}\right) P_t^\rho Q_t\right]^{\frac{1}{\rho}} (a l_t)^{\frac{\rho-1}{\rho}} \tag{5}$$

conditional on firm exporter status. Notice that, because of C.E.S. preferences and linear technology, the revenue is a log-linear function of employment. This feature will have crucial implications on the wage distribution.

Fixed costs and firm dynamics All incumbent firms purchase f_p units of service (each period) and exporters purchase an additional f_x units of service (each period) to serve the foreign market. Firms that enter the domestic market make an irreversible investment before drawing their productivity. A potential new entrant purchases f_e units of service (once) to enter the market. The sunk cost of entry can be seen as the price of a contingent asset: for any future draw of productivity $a \in [a_{\min}, \infty)$, in case of entry the new firms are alike incumbent firms endowed with the same productivity.[3]

At the beginning of each period, potential entrants and incumbents decide to compete or exit the market, given the productivity they are endowed with. Then firms that are in the market choose to export or not. Let $a^{in} \geq a_{\min}$ be the minimum productivity cutoff above which the value of a domestic firm is larger or equal to zero; and let $a^x > a^{in} \geq a_{\min}$ be the minimum productivity cutoff above which serving the foreign market becomes profitable.

[3]This assumption rules out the role played by firm age.

Exit the market provides zero value. Then, the policies of participation in the domestic and foreign market can be anticipated: domestic firms with $a \geq a^{in}$ serve the domestic market and every firm endowed with $a \geq a^{x}$ exports; firms that are endowed with a productivity $a < a^{in}$ exit the market (endogenous firm exit). In addition, firms could be forced to exit because of an exogenous destruction shock that occurs with probability $\delta_f \in (0,1)$. The mass of firms in the market at time $t+1$ with productivity a or lower $M_{t+1}(a)$ is given by the share of potential entrants that paid the sunk cost in the previous period E_t and at the beginning of period t+1 draw a productivity high enough to make a successful entry, plus the share of previous incumbent firms endowed with a productivity larger than the current cutoff and that are not hit by the destruction shock. The transition equation for the number of firms in the market satisfies:

$$M_{t+1}(a) = \begin{cases} \left[T(a) - T\left(a^{in}\right)\right] E_t + \left(1-\delta_f\right) M_t(a) & a \geq a^{in} \\ 0 & a < a^{in} \end{cases} \tag{6}$$

Similarly, in the foreign economy firm dynamics is described by a^{in*}, a^{x*}, M_t^* and E_t^*.

2.2 Labor market

There are search and matching frictions in the labor market. Workers send job applications and firms post vacancies in order to maximize the discounted lifetime income and profit, respectively. Searching is costly: the loss of value is linear in the number of job applications and vacancies. Matching is random and takes one period: vacancies posted in period t are matched with job applications sent in period t and lead to job reallocation in period $t+1$.

Contracts are not binding: between periods workers can move to other firms and firms renegotiate wages without costs. The wage associated to an offer is the one period payment the worker receives if she decides to accept the offer. Offers are "take it or leave it" and they are not contingent to other offers in the market. Equal treatment across homogeneous employees of the same firm is enforced by law; therefore there is no possibility of intra-firm wage dispersion and firms offer the same wage to insiders and perspective employees they meet on the market. A match ends because either an exogenous destruction shock occurs or the worker accepts a better job offer.[4] There are incentives for workers to search.

[4]Firms do not have an incentive to layoff workers. In order to downsize employment they can offer a lower wage, both to current and perspective employees.

Unemployed workers search in the current period to receive a job offer in the next period. Employed workers search in the current period to receive an alternative job offer and ensure against job destruction or to move to a better paid job.

Time is discrete $t = 0, 1, 2, ...$. At the end of a period $t-1$ the state of the labor market consists of u_{t-1} unemployed workers, n_{t-1} employed workers, v_{t-1} posted vacancies and a distribution of wages w across workers $G_{t-1}(w)$. At the beginning of period t a firm is matched with l_{t-1} workers and it is endowed with ϑ_{t-1} vacancies. At the beginning of a period t firms associate a wage offer to each open vacancy. Setting the wage, firms anticipate the outcome of a bargaining with the marginal worker, according to the scheme developed by Stole and Zwiebel (1996). The aggregation of wage offers yields the wage offer distribution $F_t(w)$. A worker either she is unemployed or she has in hands the wage offer from the employer she has been matched with in the previous period. In both cases a worker has a probability of receiving one job offer from the market. Workers who visit a vacancy observe the job offer and compare with the value of their current status (unemployment or employment at a given wage). The aggregation of acceptance decisions yields the wage distribution across workers $G_t(w)$ and time t allocation of the labor market with u_t unemployed workers, n_t employed workers and v_t vacancies.

Firm dynamics, vacancy and job destruction. Firm exit causes destruction of jobs and vacancies. At the beginning of period t, a share ε_t of v_{t-1} vacancies are destroyed because of endogenous firm exit and a share δ_f are closed due to exogenous firm exit. Similarly, a share ϖ_t of existing n_{t-1} jobs are destroyed because of endogenous exit, a share δ_f because of exogenous firm exit. The entry of new firms is associated with job creation and vacancy posting. A mass of $[1 - T(a^{in})]\ E_{t-1}$ successful entrants draw a productivity $a \geq a^{in}$ and start operating as incumbent firms. Indeed, the number of active firms, jobs and vacancies grow by a factor $\gamma_t = [1 - T(a^{in})]\ E_{t-1}/M_{t-1}$ uniformly on the support $[a^{in}, \infty)$. Following (exogenous and endogenous) firm exit and (endogenous) firm entry, a share $\delta_f + \varepsilon_t - \gamma_t$ of v_{t-1} posted vacancies and a share $\delta_f + \varpi_t - \gamma_t$ of n_{t-1} jobs are destroyed. I define $(1 - \sigma_t)$ and Δ_t as the net destruction rates of vacancies and jobs respectively:

$$(1 - \sigma_t) = \delta_f + \varepsilon_t - \gamma_t \quad , \quad \Delta_t = \delta_f + \varpi_t - \gamma_t \tag{7}$$

where $\sigma_t > 1$ and $\Delta_t < 0$ would indicate a net creation of vacancies and jobs.

Random matching. Unemployed workers send one application per period, whereas employed workers send $\phi \in \{0, 1\}$ applications per period. At the beginning of period t a mass of $\sigma_t v_{t-1}$ open vacancies are matched with $j_{t-1} = u_{t-1} + \phi n_{t-1}$ job applications. Matches are formed according to a a Cobb-Douglas technology with constant returns to scale. The number of matches is given by $(\sigma_t v_{t-1})^{1/2} (j_{t-1})^{1/2}$. The labor market tightness θ_t and the arrival rate of job offers x_t read respectively:

$$\theta_t = \sigma_t (v_{t-1}/j_{t-1}) \quad , \quad x_t = \sqrt{\theta_t} \tag{8}$$

2.2.1 Workers

Let U_t be the value of being unemployed, and $W_t(w)$ be the value of being employed at a wage w in time t. Unemployed workers benefit from the value of home production $\ell \geq 0$ in the current period. In the future with probability x_{t+1} they receive a wage offer in the form of a random draw from the wage offer distribution and choose to accept or reject; otherwise they remain unemployed. The value of being unemployed is:

$$U_t = \ell + \beta \left(x_{t+1} \int_0^\infty \max \{W_{t+1}(z), U_{t+1}\} dF_{t+1}(z) + (1 - x_{t+1}) U_{t+1} \right)$$

where $\beta \in (0, 1)$ is the discount factor.

Employed workers gain a wage w in the current period and suffer a loss of value $\varsigma\phi$ if they search; where the parameter $\varsigma > 0$ indicates the cost of searching for employed workers. In the next period, they receive a wage offer from the market with probability ϕx_t. Between periods, exogenous destruction occurs with probability $\delta = \delta_f + \delta_j - \delta_j \delta_f \in (0, 1)$ otherwise the worker receives a renewal of the contract with the current employer at the wage w'. Workers do not anticipate the consequence of policy implementation, therefore they do not account for endogenous firm destruction. The value of employment is given by:

$$\begin{aligned} W_t(w) \quad = \quad & \max_{\phi=\{0,1\}} w - \phi\varsigma + \beta \{(1-\delta)(1 - \phi x_{t+1}) W_{t+1}(w') + \\ & + (1-\delta) \phi x_{t+1} \int_0^\infty \max\{W_{t+1}(z), W_{t+1}(w')\} dF_{t+1}(z) + \\ & + \delta (1 - \phi x_{t+1}) U_{t+1} + \\ & + \delta \phi x_{t+1} \int_0^\infty \max\{W_{t+1}(z), U_{t+1}\} dF_{t+1}(z) \Big\} \end{aligned}$$

Firms do not change productivity over time, hence for any monotone mapping between wage and productivity, the rank of firms over the wage support is preserved across time.

As consequence, the value of employment is increasing in the wage. A worker reservation wage $w_R \geq 0$ such that $W_t\left(w_t^R\right) = U_t$ does exist. Moreover, a worker who is employed at the reservation wage in the current period w_t^R will earn the future reservation wage w_{t+1}^R in case she will be matched with the same employer. Therefore, the reservation wage reads:

$$w_t^R = \ell + \beta x_{t+1}\left(1-\phi\right)\int_{w_{t+1}^R}^{\infty}\left[W_{t+1}\left(z\right) - U_{t+1}\right] dF_{t+1}\left(z\right) \tag{9}$$

In order to discuss the implications of on-the-job search I study the limit case in which the cost of searching is negligible $\varsigma \to 0$; as in Mortensen (2009). The corner solution $\phi = 1$ is the only feasible solution for every employed worker regardless the current wage. The reservation wage is constant across time and it is equal to the value of leisure: $w^R = \ell$.[5] The worker policies are determined: unemployed workers accept a job offer that higher than the reservation wage; employed workers accept a job offer that pays better than the wage paid by their current employer.

2.2.2 Employment flows

Workers and incumbent firms separate because of exogenous job destruction or because the worker accepts an alternative job offer. The share of employed workers who separate, *separation rate*, is decreasing in firm wage:

$$s_t\left(w\right) = \delta_j + \left(1-\delta_j\right)\phi x_t\left[1 - F_t\left(w\right)\right] \tag{10}$$

Notice that without *on-the-job* search the separation rate will be exogenous and the same across firms.

Firms meet a searching worker with the same probability x_t/θ_t. But the match becomes a new hiring if and only if the worker accepts the wage offer. Therefore firms are heterogeneous in terms of the success rate of a wage offer. Out of total job applications $j_{t-1} = u_{t-1} + \phi n_{t-1}$, unemployed workers sent $u_{t-1}+ \phi\ \left(\Delta_t + \left(1-\Delta_t\right)\delta_j\right)\ n_{t-1}$ applications and they accept any job offer not below the reservation wage; the residual $\phi\ \left(1-\Delta_t\right)$ $\left(1-\delta_j\right)\ n_{t-1}$ applications are sent by employed workers who will accept only offers that are better than their current wage. The share of vacancies that are successfully filled, *hiring*

[5]This result is not a restriction for the wage distribution. I will show that the minimum of the wage support is endogenous and it is strictly above higher than the reservation wage.

rate, is increasing in firm wage:

$$h_t(w) = \frac{x_t}{\theta_t}\left(\frac{u_{t-1} + \phi\left(\Delta_t + (1-\Delta_t)\,\delta_j\right) n_{t-1}}{j_{t-1}} + \right. \\ \left. + \frac{\phi\,(1-\Delta_t)\,(1-\delta_j)\, n_{t-1}}{j_{t-1}} G_t(w)\right) \tag{11}$$

Once more, *on-the-job* search is necessary to obtain heterogeneity across firms in their hiring rate. Unemployment is given by previous unemployed workers and new unemployed workers who did not receive job offers:

$$u_t = (1-x_t)\,u_{t-1} + (1-\phi x_t)\left(\Delta_t + (1-\Delta_t)\,\delta_j\right) n_{t-1} \tag{12}$$

Equation (12) is the law of transition for unemployment.

In order to determine the wage distribution across workers I follow Burdett and Mortensen (1998) decomposing employment flows per wage level. Consider the group of workers matched with a firm that pays a wage w or lower: $G_t(w)\ n_t$. In the upcoming search period a share $(1-\delta_j)\ (1-\Delta_t)$ stays matched with a firm that pays a wage w or lower unless she finds a better offer, indeed with probability $1-\phi x_{t+1}\,(1-F_{t+1}(w))$. The inflow from previous unemployed workers u_t consists of those who receive a job offer at a wage w or lower: $u_t x_{t+1} F_{t+1}(w)$. The inflow from employed workers n_t consists of those who separated $\left(\Delta_{t+1} + (1-\Delta_{t+1})\,\delta_j\right) n_t$ and in the upcoming period receive a job offer at a wage w or lower. Total inflow in the subgroup of workers who are employed at a wage w or lower is given by: $[u_t + \phi\left(\Delta_{t+1} + (1-\Delta_{t+1})\,\delta_j\right) n_t]\ x_{t+1}F_{t+1}(w)$. The number of workers matched at a wage w or lower in the next period is given by:

$$G^c_{t+1}(w)\,n^c_{t+1} = G_t(w)\,n^c_t\,(1-\Delta_{t+1})\,(1-\delta_j)\left(1 - x^c_{t+1}\left(1-F^c_{t+1}(w)\right)\right) + \\ + [u^c_t + \left(\Delta_{t+1} + (1-\Delta_{t+1})\,\delta_j\right) n^c_t] x^c_{t+1} F^c_{t+1}(w) \tag{13}$$

Equation (13) is the law of transition for the wage distribution across workers.

2.2.3 Firms

Incumbent firms in period t are characterized by the pair of values: productivity a and previous employment l_{t-1}. A firm of type $\{a, l_{t-1}\}$ realizes the state of the economy at the beginning of time t and makes two decisions: it sets the wage (through which it fixes employment in the current period) and chooses the number of vacancies to issue in the market (through which it affects employment in the next period).

Wage. The wage determination is the outcome of an intra-temporal problem, through which current wage and current employment are determined simultaneously at the beginning of a period. Before matches are formed, firms specify a wage offer, the same for all the vacancies they have on the market and they commit to that wage for the current period. Then worker reservation policies are understood. Since offers are not contingent and there is not recall, firms match only with workers who are willing to accept the proposed wage. Indeed, firms that offer higher wages are more likely to hire employed workers and face a lower separation rate among their employees.

Making the offer, firms anticipate the outcome of a bilateral bargaining with the marginal worker on the nominal wage. A bargaining stage starts with one player who makes an offer, that consists of a price per unit of labor provided in the current period; with probability $\mu \in (0, 1)$ the worker makes the first move. The other player replies with a counteroffer, or she accepts or she breaks the bargaining. A counteroffer leads the bargaining to the next stage. When an offer is accepted, the two parties commit immediately to the agreement. In case the two parties break the negotiation, the firm loses the profit due to the marginal worker in the current period, whereas the worker gains the current value of unemployment $\ell \geq 0$ and loses the wage the firm pays in the current period $w_t > 0$. Let $\pi_t = r\left(a, l_t; e\right) - w_t l_t - f_p - e f_x$ be the profit before the hiring cost, then the equilibrium bargained wage satisfies: $\frac{\partial \pi_t}{\partial l_t} = \frac{1-\mu}{\mu}\left(w_t - \ell\right)$. The partial derivative of firm profit with respect to employment reads: $\frac{\partial \pi_t}{\partial l_t} = \frac{\partial r(a,l_t;e)}{\partial l_t} - \frac{\partial w_t}{\partial l_t} l_t - w_t$. Therefore the wage is the particular solution of the ordinary differential equation $\frac{\partial w_t}{\partial l_t} + \frac{1}{\mu}\frac{1}{l_t} w_t = \left(\frac{\partial r(a,l_t;e)}{\partial l_t} + \frac{1-\mu}{\mu}\ell\right)\frac{1}{l_t}$.[6] A firm endowed with productivity a and matched with l_t workers offers a wage that is a linear function of revenue per worker:

$$w\left(a, l_t; e\right) = \mu \frac{\rho - 1}{\rho - \mu} \frac{r\left(a, l_t; e\right)}{l_t} \tag{14}$$

Hereafter the dependence of the wage from the exporter status is omitted when it is not necessary, for simplicity in the notation.

[6]Let $I = \int \frac{1}{\mu}\frac{1}{l_t} dl_t$ and multiply both sides for the factor $\exp^I = l_t^{\frac{1}{\mu}}$. Integrating it yields the general solution: $w_t = (1-\mu)\ell + l_t^{-\frac{1}{\mu}} \int \frac{\partial r_t}{\partial l_t} l_t^{\frac{1-\mu}{\mu}} dl_t + C$, where C is the constant of integration. The revenue function (5) yields $l^{-\frac{1}{\mu}} \int \frac{\partial r}{\partial l} l^{\frac{1-\mu}{\mu}} dl = \mu \frac{\rho-1}{\rho-\mu}\frac{r}{l}$. The particular solution is the one that passes through the point $w\left(a, 0\right) = 0$.

Vacancies. The choice on the number of vacancies is taken at the end of the period as the outcome of an inter-temporal optimality problem. For the case in which the wage offer distribution and the wage distribution depend on time only through the labor market tightness, the inter-temporal problem can be formulated recursively. Then $F_t(w) = F(w(a,l_t),\theta_t)$ and $G_t(w) = G(w(a,l_t),\theta_t)$, therefore separation and hiring rates are respectively $s_t(w) = s(w(a,l_t),\theta_t)$ and $h_t(w) = h(w(a,l_t),\theta_t)$. The endogenous aggregate state of the recursive firm problem is θ_t, whereas productivity a and firm employment l_t compose the vector of the individual state variables.

The number of vacancies $\vartheta_t > 0$ is the control variable of the inter-temporal optimality problem. Firms choose the level of employment period by period such that the lifetime stream of profit is maximized. The value of a firm endowed with productivity a at time t is given by:

$$\Pi(a,l_t;\theta_t) = \max_{\vartheta_t \in [0,\vartheta_t^{max}]} \pi(a,l_t) - k\vartheta_t + \beta(1-\delta_f)\Pi(a,l_{t+1};\theta_{t+1})$$

subject to the law of motion of employment,

$$l_{t+1} = [1 - s(w(a,l_{t+1}),\theta_{t+1})]\,l_t + h((w(a,l_{t+1}),\theta_{t+1})\,\vartheta_t \tag{15}$$

and to the transition rule for the labor market tightness in the consumption sector: $\theta_{t+1}^c = \sigma_{t+1}\frac{v_t^c}{u_t^c+n_t^c} = \sigma_{t+1}\theta_t^c$. Notice that vacancy and job endogenous destruction rates $\sigma_{t+1} = 1-\delta_f+\left[1-T\left(a_{t+1}^{in}\right)\right]\frac{E_t}{M_t}$ and $\Delta_{t+1} = \delta_f - \left[1-T\left(a_{t+1}^{in}\right)\right]\frac{E_t}{M_t}$ are the only channels through which the aggregate uncertainty about the productivity cutoff in the next period a_{t+1}^{in} enters the firm problem. Firms are rational forward looking agents with perfect information, then by assumption the productivity cutoff at the end of every period t (i.e. after the entry and exit decisions are made) is equal to the future productivity cutoff $a_{t+1}^{in} = a_t^{in} = a^{in}$. Therefore, the labor market tightness is predetermined. The right extreme of the compact feasibility set is $\vartheta_t^{max} = \frac{\pi_t}{k}$ as it is implied by the fact that firms are financially constrained. The necessary first order condition for an interior solution requires that the expected cost of filling a vacancy is equal to the expected discounted value of the marginal job:

$$\frac{k}{h(w(a,l_{t+1}),\theta_{t+1})} = \beta(1-\delta_f)\,J(a,l_{t+1};\theta_{t+1}) \tag{16}$$

where $J(a,l_t;\theta_t) = \frac{\partial\Pi(a,l_t;\theta_t)}{\partial l_t}$. Since the cost of holding a vacancy is linear, the choice of an optimal number of vacancies is unrestricted. In steady state, firms issue the vacancies that are sufficient to keep employment at the steady state level. Let w and l be respectively the steady state wage and employment of a firm endowed with productivity a, then the steady state vacancies $\vartheta(a,l)$ are such that hirings are equal to separations: $h(w,\theta)\vartheta(a,l) = s(w,\theta)l$.

When the dynamics is of concern, the common approach in the literature is either to set the vacancies such that firm employment jumps to the steady state level (as in Felbermayr et al. (2011a) and implicitly in Melitz (2003)) or to specify an ad hoc cost function of hiring, as it is discussed in Coşar et al. (2010).[7] In the context of this paper, I restrict the discussion to the unique policy that belongs to the set of feasible and optimal paths $\left\{\vartheta_t : \vartheta_t \in \left(0, \frac{\pi_t}{k}\right) \forall t = 0, 1, ...\right\}$ such that the flow value of a firm $\pi(a, l_t) - k\vartheta(a, l_t)$ does not deviate from its steady state value $\pi(a,l) - k\vartheta(a,l)$:

$$\vartheta(a, l_t) = \vartheta(a,l) + \frac{\pi(a, l_t) - \pi(a,l)}{k} \tag{17}$$

Intuitively, policy (17) prescribes that positive (negative) deviations of the firm value from the steady state are used to finance more (less) vacancies with respect to the steady state allocation.

From the theoretical point of view, the policy (17) makes the discussion of firm dynamics tractable: out of the steady state the value of each firm is stable and it only depends on the idiosyncratic productivity a, which is the sufficient statistics to determine entry, exit and export decisions as in the Melitz (2003) framework. This gain of tractability comes at the cost of ruling out from the discussion those feasible plans that prescribe a fluctuation of firm value around this stable path. The cost of this restriction appears to be acceptable given that in this framework firms do not have incentives to strategically transfer value across periods anyhow.[8] In particular, notice that the evolution of labor market tightness hits all firms in the same way, as the probability at which a worker visits a vacancy is the

[7] Assuming a convex cost function for vacancy posting would be sufficient to restrict the solution; see the discussion in Felbermayr et al. (2011a) footnote 18. A convex cost in this context will go through the analysis (see Mortensen (2009)). Nevertheless, it wont change the direction of worker reallocation (from low productive firms to high productive firms) and it will be costly in terms of interpretation, as it would make firms heterogeneous also in the cost of posting the marginal vacancy.

[8] Workers and firms are forward looking under perfect information, the only sources of uncertainty (job and firm destruction shocks) cannot be ensured, there is no market for assets, no technological progress neither productivity or demand shocks and firm age or growth rate do not play any role.

same across firms. What is specific to the firm is how it responds to labor market conditions by setting the wage. This is an intra-temporal decision that accounts for the value of the marginal job in the current period only and it does not commit the firm across periods.

The model is therefore silent on the dynamics of firm value during the transition, in exchange for a simple and time consistent discussion of the industry dynamics and the evolution of employment, vacancies and wages.

2.3 Service sector

The demand of service in the economy is given by:

$$S_t^d = \left(f_p + \frac{1 - T\left(a^x\right)}{1 - T\left(a^{in}\right)} f_x \right) M_t + f_e E_t \tag{18}$$

The production of service is performed by a representative firm with a linear technology that employs n_t^s workers with labor productivity a^s. The supply of service is:

$$S_t^s = a^s n_t^s \tag{19}$$

Productivity dispersion and firm dynamics do not play any role in the service sector.[9] The bargained wage is a convex combination of reservation wage and average revenue:

$$w_t^s = (1 - \mu)\, \ell + \mu p_t^s a^s \tag{20}$$

The wage distributions in the service sector are degenerate: equal 1 for $w = w^s$ and zero otherwise. The representative firm in the service sector makes zero profit:

$$p_t^s S_t = w_t^s n_t^s + k v_t^s \tag{21}$$

where v_t^s are the vacancies yield in the service sector.

3 Equilibrium

A symmetric general equilibrium is defined. Endowments and technologies are assumed to be the same across countries. As a consequence, the foreign economy replicates the domestic economy and a solution is discussed for the latter only. The key variables for the

[9]The labor market allocation in the service sector is characterized by zero probability of firm destruction $\delta_f = 0$ and no endogenous job or vacancy creation due to firm entry, $\Delta_t = 0$ and $\sigma_t = 1$.

analysis are the cutoff productivity a^{in} and labor market tightness θ^c; the service is the numeraire good $p^s = 1$.

A wage dispersion equilibrium of the labor market consists of: (i) a pair of functions that define the wage support $w_0(\theta^c) : (0,1) \to \mathbb{R}^+$, $w_1(\theta^c) : (0,1) \to \mathbb{R}^+$, $w_1(\theta^c) \geq w_0(\theta^c)$ and a pair of cumulative density functions $F^c(w;\theta^c) : [w_0, w_1] \to [0,1]$ and $G^c(w;\theta^c) : [w_0, w_1] \to [0,1]$; (ii) a monotonic increasing wage-productivity assignment $\omega(a;\theta^c) : [a^{in}, a_{sup}] \to [w_0, w_1]$; (iii) a labor market tightness θ^c; such that for a given level of aggregate employment in the consumption sector n^c: (a) workers and firms split the surplus of a match according to the bargaining rule (14) and the value of a job is equal to the expected cost of hiring a worker (16); (b) in the consumption sector, the labor demand and supply clear at the firm level; (c) employment in the consumption sector aggregate up to n^c.

3.1 Wage distribution

In steady state, there is no either job creation $\Delta = 0$ or vacancy destruction $\sigma = 1$ due to policy implementation. As a result, hiring and separation satisfy: $h^c(w,\theta^c)\, s^c(w,\theta^c) = \frac{x^c}{\theta^c}\delta_j$. The product of the two rates is constant across firms and indeed across wage levels: $s(w)\frac{dh(w)}{dw} = -\frac{ds(w)}{dw}h(w)$. The envelope condition applied to the firm inter-temporal problem yields the value of a job, which in steady state reads:

$$J(w,l;\theta^c) = \frac{\frac{\partial \pi}{\partial l}}{1-\beta(1-\delta_f)\left(1-s^c(w;\theta^c)-2\frac{\partial s^c(w;\theta^c)}{\partial w}\frac{\partial w}{\partial l}l\right)} \tag{22}$$

where w satisfies (14). The bargaining between firms and workers implies that the marginal profit is equal to the wage $\frac{\partial \pi}{\partial l} = \frac{1-\mu}{\mu}(w-\ell)$. The change in the wage due to an increase in employment evaluated at the equilibrium revenue reads $\frac{\partial w}{\partial l}l = -\frac{1}{\rho}(w-(1-\mu)\ell)$. The value (22) and the optimality condition (16) determine the *Job Creation* equation:

$$\frac{\partial s^c(w;\theta^c)}{\partial w}s^c(w;\theta^c)\,x^c c = -\frac{\rho}{2}\frac{(w-\ell)-(s^c(w;\theta^c)+b)\,s^c(w;\theta^c)\,x^c c}{w-(1-\mu)\ell} \tag{23}$$

where $b = \frac{1-\beta\left(1-\delta_f\right)}{\beta\left(1-\delta_f\right)} > 0$ is a discount factor and $c = \frac{k}{\delta_j}\frac{\mu}{1-\mu} > 0$ parametrizes labor market frictions. The job creation condition (23) is an ordinary differential equation in the wage, that can be solved for a given level of labor market tightness θ^c. There exists at least one continuous function $\hat{s}^c(w,\theta^c)$ that solves the job creation condition (23).[10] For a given pair

[10]The job creation (23) can be written as an ordinary differential equation of the Abel type in terms of the variables $z(w;\theta^c) = \frac{1}{s^c(w;\theta^c)} \in \left[\frac{1}{\delta_j+(1-\delta_j)x^c}, \frac{1}{\delta_j}\right]$ and $w \in \mathbb{R}^+$: $z'_w = c_3(w)\,z^3 - c_2(w)\,z^2 - c_1(w)\,z$,

of wage and separation rate the solution to the job creation condition $\hat{s}^c(w;\theta^c)$ is unique, although it is not convenient to work on it analytically.[11] Nevertheless, several properties of the solution $\hat{s}^c(w;\theta^c)$ can be inferred considering that it has to satisfy the job creation condition (23), the definition of separation rate (10) and that $F^c(w;\theta^c)$ has the properties of a cumulative density function across wages. Indeed, $\hat{s}^c(w;\theta^c)$ is a positive, decreasing and concave function of the wage: $\frac{\partial \hat{s}^c(w;\theta^c)}{\partial w} \leq 0$, $\frac{\partial^2 \hat{s}^c(w;\theta^c)}{\partial w^2} \leq 0$. At a given wage, the separation rate increases with the probability that a worker receives a wage offer: $\frac{\partial \hat{s}^c(w;\theta^c)}{\partial \theta^c} \geq 0$. A further increase in the probability of finding a job reduces the drop in the separation rate due to the increase in the wage: $\frac{\partial^2 \hat{s}^c(w;\theta^c)}{\partial w \partial \theta^c} \geq 0$. The wage offer distribution is derived by inverting the definition of separation rate (10):

$$F^c(w;\theta^c) = 1 - \frac{\hat{s}^c(w;\theta^c) - \delta_j}{(1-\delta_j)\, x^c} \tag{24}$$

Proposition 1 characterizes the properties of the wage offer distribution.

Proposition 1: The c.d.f. of the wage offer distribution $F^c(w;\theta^c)$ is continuous, convex in the wage and the density $\frac{\partial F^c(w;\theta^c)}{\partial w}$ is bounded above. Moreover, the wage offer distribution at higher labor market tightness first order stochastically dominates the wage offer distribution at lower labor market tightness.

Proof. *Let F^c be a c.d.f. then $F^c \in [0,1]$ and $\frac{\partial F^c}{\partial w} \geq 0$. The definition of separation rate (10) implies $s^c > 0$ and $\frac{\partial s^c}{\partial w} = -(1-\delta_j)\, x^c \frac{\partial F^c}{\partial w} \leq 0$. The density of wage offers has to be bounded above to guarantee that $w > \ell > 0$ for every $\ell \geq 0$ and w to be finite:*

$$\frac{\partial F^c(w)}{\partial w} s^c(w) < \frac{\rho}{2} \frac{1-\mu}{\mu} \frac{\delta_j}{k} \frac{1}{(1-\delta_j)} \frac{1}{\theta^c}$$

Clearly the density is continuous and finite, as $\delta_j, \theta^c, s^c(w) \in (0,1)$.

If the compact set $[w_0, w_1]$ is the support of the c.d.f. F^c then for every pair of values $w' < w'' \in [w_0, w_1]$ the following condition holds:

$$0 < \frac{\partial F^c(w_0)}{\partial w} s^c(w_0) < \cdots < \frac{\partial F^c(w')}{\partial w} s^c(w') < \frac{\partial F^c(w'')}{\partial w} s^c(w'') < \cdots < \frac{\partial F^c(w_1)}{\partial w} s^c(w_1)$$

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where $-\frac{\partial s^c(w;\theta^c)}{\partial w} \frac{1}{s^c(w;\theta^c)^2} = \frac{\partial z(w;\theta^c)}{\partial w}$ and the coefficients are $c_3(w) = \frac{\rho}{2} \frac{1-\mu}{\mu} \frac{x^c}{\theta^c} \frac{\delta_j}{k} \frac{w-\ell}{w-(1-\mu)\ell}$, $c_2(w) = \frac{\rho}{2} \frac{1-\beta(1-\delta_f)}{\beta(1-\delta_f)} \frac{1}{w-(1-\mu)\ell}$ and $c_1(w) = \frac{\rho}{2} \frac{1}{w-(1-\mu)\ell}$; all positive and finite for $w \geq \ell$. The right hand side of the differential equation is continuous on the open domain $\mathbb{R}^+ \times (1,\infty)$. *Peano existence theorem* guarantees that there is at least one integral curve of the differential equation that passes through a given point (z_0, w_0).

[11]The methodology that has recently been proposed by Panayotounakos and Zarmpoutis (2011) shows how to compute the unique exact solution $\hat{z}^c(w;\theta^c)$ of the Cauchy problem.

for every given value of labor market tightness $\theta^c \in (0,1)$. The necessity of such inequality is clear by looking at the fraction that expresses the wage gap $w-\ell$. In order to preserve the wage sorting the density of the wage offer has to be increasing $\frac{\partial^2 F^c(w)}{\partial w^2} > 0$ to more than compensate the fall in the separation rate. The deviation in $\hat{s}^c(w;\theta^c)$ due to labor market tightness has the same sign of the change in the separation rate due to the probability of finding a job x^c; given (8). The partial change in the separation rate due to a marginal increase in the probability of receiving an offer is:

$$\frac{\partial \hat{s}^c(w,x^c)}{\partial x^c} = (1-\delta_j)\left[(1-F^c(w,x^c)) - x^c \frac{\partial F^c(w,x^c)}{\partial x^c}\right] \geq 0$$

from (10). Compute the interaction term:

$$\frac{\partial^2 \hat{s}^c(w,x^c)}{\partial x^c \partial w} = (1-\delta_j)\left[-\frac{\partial F^c(w,x^c)}{\partial w} - x^c \frac{\partial^2 F^c(w,x^c)}{\partial x^c \partial w}\right] \geq 0$$

Since $\frac{\partial F^c(w,x^c)}{\partial w} > 0$ then $\frac{\partial^2 \hat{s}^c(w,x^c)}{\partial x^c \partial w} \geq 0$ implies $\frac{\partial^2 F^c(w,x^c)}{\partial x^c \partial w} < 0$. The density of the wage offer distribution is lower at each wage level, then the same has to be true for the c.d.f. $\frac{\partial F^c(w,x^c)}{\partial x^c} < 0$. □

Proposition 2 characterizes the properties of the wage support. Consistently with Mortensen (2010) I discuss the equilibrium allocation when there is no mass point at the minimum of the wage support.

Proposition 2: The minimum of the wage support $w_0(\theta^c) > \ell$, the maximum of the wage support $w_1(\theta^c) > w_0(\theta^c)$ and the length of the wage support $w_1(\theta^c) - w_0(\theta^c)$ are increasing in labor market tightness θ^c. Moreover, if there is no mass point on the wage distribution $\frac{\partial F^c(w_0,\theta^c)}{\partial w} = 0$, the minimum wage in the consumption sector is given by:

$$w_0(\theta^c) = \ell + \frac{\mu}{1-\mu}\frac{k}{\delta_j}(b+s_0^c)\,x^c s_0^c \tag{25}$$

where $s_0^c = \delta_j + (1-\delta_j)\,x^c$ and notice that $w_0(0) = \ell$.

Proof. *Invert the job creation (23) to obtain the wage:*

$$w = \ell + \frac{(s^c+b)\,x^c s^c - \mu\ell\,\frac{2}{\rho} \times \frac{\partial s^c}{\partial w} s^c x^c}{\frac{1-\mu}{\mu}\frac{\delta_j}{k} + \frac{2}{\rho} \times \frac{\partial s^c}{\partial w} s^c x^c}$$

The condition $\frac{\partial F^c(w_0,\theta^c)}{\partial w} = 0$ implies $\frac{\partial s^c(w_0,\theta^c)}{\partial w} = 0$. The substitution in the job creation condition (23) yields a closed form solution for the minimum of wage support:

$$w_0(\theta^c) = \ell + \frac{\mu}{1-\mu}\frac{k}{\delta_j}(b+s_0^c)\,x^c s_0^c$$

where by definition $F^c(w_0,\theta^c) = 0$ implies $s_0 = \delta_j + (1-\delta_j)\,x^c$, then $w_0(\theta^c) > \ell$ is increasing in θ^c. From (23) the signs of the following partial derivatives are determined $\frac{\partial}{\partial \theta^c}\left(\frac{\partial s^c}{\partial w}\right) \geq 0$ and $\frac{\partial}{\partial \theta^c}\left(\frac{\partial s^c}{\partial w} s^c x^c\right) \leq 0$. The two inequalities are sufficient to guarantee that the maximum of the wage support,

$$w_1(\theta^c) = \ell + \frac{(\delta_j+b)\,x^c\delta_j - \mu\ell\,\frac{2}{\rho}\,\frac{\partial s^c}{\partial w}\delta_j x^c}{\frac{1-\mu}{\mu}\frac{\delta_j}{k} + \frac{2}{\rho}\,\frac{\partial s^c}{\partial w}\delta_j x^c}$$

is increasing in the labor market tightness θ^c. □

In steady state inflows and outflows of workers from and to the same wage group has to balance, such that $n_{t-1} = n_t$, $u_{t-1} = u_t$ and there is not endogenous job destruction $\Delta = 0$. The distribution of wages across workers (13) in steady state reads:

$$G^c(w; \theta^c) = \frac{\delta_j F^c(w; \theta^c)}{\delta_j + (1 - \delta_j) x^c [1 - F^c(w; \theta^c)]} \tag{26}$$

Notice that $G^c(w; \theta^c) = \delta_j \frac{F^c(w;\theta^c)}{\hat{s}^c(w;\theta^c)}$ then it is immediate to show that it inherits all the properties that have been discussed for the wage offer distribution: $\frac{\partial G^c}{\partial w} \geq 0$, $\frac{\partial^2 G^c}{\partial w^2} \geq 0$, $\frac{\partial G^c}{\partial \theta^c} \leq 0$ and $\frac{\partial^2 G^c}{\partial w \partial \theta^c} \leq 0$. Results of Propositions 1 and 2 hold for any value of leisure $\ell \geq 0$. Without loss of generality and for the sake of tractability, hereafter the value of leisure is set to zero $w^R = \ell = 0$.

3.2 Employment, wage, vacancies and profit

Let $l^s(w, \theta^c)$ be the number of workers who are employed by a firm that pays a wage w when the labor market tightness is θ^c. By definition, the employment identity reads: $n^c\ G^c(w, \theta^c) = \int_{w_0}^{w} l^s(z, \theta^c)\ dz$. Therefore, differentiating over the wage support yields: $l^s(w, \theta^c) = n^c \frac{\partial G^c(w,\theta^c)}{\partial w}$ for every $w \in [w_0(\theta^c), w_1(\theta^c)]$. Inverting the wage equation (14) yields the labor demand $l^d(w, a)$ as proportional to $a^{\rho-1} w^{-\rho}$. Figure 1 shows the equilibrium allocation of wage and employment at the firm level. The wage equation (14) determines the firm *labor demand*, that is decreasing relationship between employment and wage. The demand schedule is increasing in productivity; the solid line refers to a given firm and the dash line refers to a more productive one. Under wage dispersion firms face an increasing labor supply. This is the result of the searching attitude of workers. Without *on-the-job* search the separation rate and hiring rate would have been the same across firm. Both the wage offer distribution across firms and the wage distribution across workers would have been degenerate. Under that scenario the labor supply would have been vertical at the one equilibrium wage. The non degenerate, continuous and convex c.d.f. of the wage distribution is responsible for an increasing relationship between employment and wage, that is the firm *labor supply* which does not depend on productivity. More productive firms pay better wages $\frac{\partial \omega(a,\theta^c)}{\partial a} > 0$ and employ more workers $\frac{\partial \lambda(a,\theta^c)}{\partial a} > 0$.

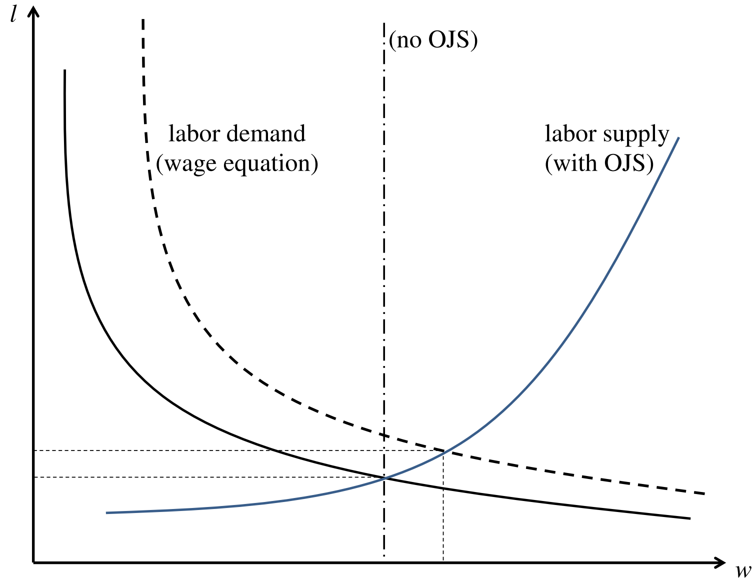

Figure 1: **Wage and employment assignment**. Because of *on-the-job* search, firms with higher productivity a pay better wages $w = \omega(a;\theta^c)$ and hire more workers $l = \lambda(a;\theta^c)$. Without *on-the-job* search the adjustment would take place through employment only.

Analytically, a monotonic wage mapping $w = \omega(a, \theta^c)$ such that $\frac{\partial w}{\partial a} \geq 0$ implies that differentiating the employment identity over productivity yields:

$$l^s(\omega(a, \theta^c), a) = n^c \frac{\partial G^c(\omega(a, \theta^c), \theta^c)}{\partial w} \frac{\partial \omega(a, \theta^c)}{\partial a} \tag{27}$$

for every $a \in [a^{in}, a_{sup}]$. A firm endowed with the cutoff productivity a^{in} offers the lowest wage w_0, it is matched with the lowest employment $l_0 = \lambda(a^{in}, \theta^c)$ and it yields the lowest number of vacancies ϑ_0, which has to be strictly above zero in steady state. Moreover, since exit is not costly, the profit of the least productive firm has to be zero, then: $\frac{1}{\gamma} w_0 l_0 = k\vartheta_0 + f_p$. The vacancy per employee ratio is $\frac{\vartheta_0}{l_0} = \frac{x^c}{\delta_j} s_0^2$. The minimum firm size is given by: $l_0 = \frac{f_p}{\frac{w_0}{\gamma} - \frac{k}{\delta_j} x^c s_0^2}$, which is a decreasing function of labor market tightness, once substituting for the minimum wage (25).

Output of the least productive firm is then determined $y_0 = q_0 = a^{in} l_0$, so does the revenue $r_0 = \frac{1+\gamma}{\gamma} w_0 l_0$, then the price $p_0 = \frac{1+\gamma}{\gamma} \frac{w_0}{a^{in}}$. Finally the properties of the demand system imply $P^\rho Q = p_0^\rho q_0$, then $\left(\frac{1}{\mu} \frac{\rho-\mu}{\rho-1}\right)^{-\rho} P^\rho Q = \left(\frac{w_0}{a^{in}}\right)^\rho a^{in} l_0$. Inverting the wage equation (14) yields the labor demand:

$$l^d(\omega(a, \theta^c), a) = \left(1 + e\tau^{1-\rho}\right) \left(\frac{a}{a^{in}}\right)^{\rho-1} \left(\frac{\omega(a, \theta^c)}{w_0}\right)^{-\rho} l_0 \tag{28}$$

The clearing of labor demand (28) and labor supply (27) yields a separable differential equation of the wage on productivity that passes through the point $w\left(a^{in},\theta^{c}\right)=w_{0}\left(\theta^{c}\right)$.

In order to simplify the exposition, define the continuous function $\tilde{w}=W\left(w,\theta^{c}\right)=\int w^{\rho}\frac{\partial G^{c}(w,\theta^{c})}{\partial w}dw$, with partial derivatives $W'_{w}>0$, $W'_{\theta}<0$. Then define the continuous function $\tilde{a}=A\left(a,e\right)=\frac{\left(1+e\tau^{1-\rho}\right)a^{\rho}-a^{in\ \rho}}{\left(1+\tau^{1-\rho}\right)a_{sup}^{\rho}-a^{in\ \rho}}\in[0,1]$, with partial derivatives $A'_{a}>0$, $A'_{e}>0$. The wage assignment $\omega\left(a,\theta^{c}\right)$ satisfies:

$$\tilde{w}\left(a,\theta^{c}\right)=\left(1-\tilde{a}\right)\tilde{w}_{0}\left(\theta^{c}\right)+\tilde{a}\tilde{w}_{1}\left(\theta^{c}\right) \tag{29}$$

where $\tilde{w}_{0}=W\left(w_{0}\left(\theta^{c}\right),\theta^{c}\right)$ and $\tilde{w}_{1}=W\left(w_{1}\left(\theta^{c}\right),\theta^{c}\right)$. The wage transformation $\tilde{w}$ is a convex combination of $\tilde{w}_{1}>\tilde{w}_{0}$. The productivity level works as a pointer from the productivity support of incumbent firms $[a^{in},a_{sup}]$ to the wage support $[w_{0},w_{1}]$. The inverse of W with respect to its first argument for a given level of θ^{c} yields the wage, hence employment, as an increasing function of productivity a and increasing in the exporter status e. Notice that employment is an increasing function of productivity if and only if the wage elasticity to productivity is sufficiently lower than one: $\frac{\partial w}{\partial a}\frac{a}{w}\leq\frac{\rho-1}{\rho}$. Indeed, the wage is a concave function of productivity.

The number of vacancies issued at a wage w is: $\vartheta^{s}\left(w,\theta^{c}\right)=v^{c}\frac{\partial F^{c}(w,\theta^{c})}{\partial w}$. Indeed, firms that pay better wages are also the ones that issue more vacancies. This result is due to employment. Although separation per hiring rate $\frac{s(w)}{h(w)}$ is lower at better wages, employment increases with the wage, such that more vacancies are needed to keep employment in steady state. The monotonicity of the wage-productivity mapping implies that more productive firms issue more vacancies.

Computing the firm profit, notice that the revenue is proportional to labor cost $r=\frac{1+\gamma}{\gamma}wl$; where the ratio $\gamma=\frac{\rho-1}{\rho}\frac{\mu}{1-\mu}>0$ measures the degree of competition. Total profit for a firm with exporter status e, that is endowed with productivity a, when the labor market tightness is θ^{c} reads:

$$\begin{aligned}
\Pi\left(a,\theta^{c};e\right) &= \frac{1}{\gamma}w\left(a,\theta^{c};e\right)l\left(a,\theta^{c};e\right)-k\vartheta\left(a,\theta^{c};e\right)-f_{p}-ef_{x}\\
&= \left(\frac{1}{\gamma}w\left(a,\theta^{c};e\right)-\frac{k}{\delta_{j}}\frac{\theta^{c}}{x^{c}}s^{c}\left(\omega\left(a,\theta^{c};e\right),\theta^{c}\right)^{2}\right)l\left(a,\theta^{c};e\right)-f_{p}-ef_{x}\\
&\geq \left(1+e\tau^{1-\rho}\right)\left(\frac{a}{a^{in}}\right)^{\rho-1}\left(\frac{\omega\left(a,\theta^{c};e\right)}{w_{0}}\right)^{1-\rho}\left(\frac{w_{0}l_{0}}{\gamma}-k\vartheta_{0}\right)-f_{p}-ef_{x}\\
&= \left[\left(1+e\tau^{1-\rho}\right)\left(\frac{a}{a^{in}}\right)^{\rho-1}\left(\frac{\omega\left(a,\theta^{c};e\right)}{w_{0}}\right)^{1-\rho}-1\right]f_{p}-ef_{x}
\end{aligned} \tag{30}$$

where the inequality holds strictly for $w > w_0$.[12] Firms that pay better wages make higher profits, both because profit per worker is higher and because they employ more workers. Indeed, more productive firms make higher profits.

The dependence of profit on labor market tightness passes through the wage ratio w/w_0. The labor demand (28) implies that w/w_0 reacts to a change in labor market tightness in the opposite direction than the employment ratio $l(w)/l_0$; everything else being constant. Given the labor supply (27) and the properties of the wage distribution, the employment ratio $l(w)/l_0$ is increasing in labor market tightness; indeed the same has to be true for profit. At the same time, low productive firms rise the wage relatively more than high productive firms.

Notice that this argument does not apply to absolute values. The minimum wage w_0 and minimum employment l_0 are respectively increasing and decreasing functions of labor market tightness. Workers reallocate from firms that pay low wages to better ones. Therefore, when the probability of receiving a job offer increases, employment decreases for those firms that pay lower wages and increases at the top of the wage distribution.

3.3 General equilibrium

The aggregation of price index and quantity index over M domestic producers and $(1+p_x)$ varieties in the domestic market can be performed as in Melitz (2003).[13] It follows that $P^\rho Q = \bar{p}\bar{q} = \bar{r}$ does not depend on the mass of firms. Instead the identity implied by the demand system $P^\rho Q = p_0^\rho q_0$ and the employment level $l_0 = \frac{f_p}{\frac{w_0}{\gamma} - \frac{k}{\delta_j} x^c s_0^2}$ yield the average revenue $\bar{r} = \left(\frac{1}{\mu}\frac{\rho-\mu}{\rho-1}\right)^\rho \left(\frac{w_0}{a^{in}}\right)^\rho a^{in} l_0$, which is decreasing in the productivity cutoff and increasing in labor market tightness. The right hand side in (37) is decreasing in productivity cutoff and increasing in labor market tightness if the (positive) percentage change in the average revenue offsets the (positive) change in average wage across workers.

The expenditure of domestic residents on consumption goods is equal to the total revenue of domestic firms, due to domestic sales plus the revenue of foreign exporter firms

[12] In the proof of Proposition 2 I derived an expression for the wage. Rearranging terms it can be shown that $w > \gamma \frac{k}{\delta_j} \frac{\theta^c}{x^c} s^c(w)^2$.

[13] Computing average productivity, the variable $a^{\rho-1}$ is weighted by the ratio $\left(\frac{w(a)}{w(\bar{a})}\right)^{1-\rho}$; which is one in Melitz (2003)

on the domestic market. As trade is balanced, the value of domestic imports is equal to the value of foreign exports. Total expenditure in the domestic economy is proportional to the revenue on the domestic market made by the firm endowed with average productivity: $PQ = (1 + p_x) M\bar{r}$.

Furthermore, I define two average measures to simplify the following discussion. Average payments to workers in the consumption sectors are $\mathbb{E}[wl]$, for $\mathbb{E}[wl] = \int_{a^{in}}^{a_{sup}} w(z, \theta^c)\, l(w(z, \theta^c), a) \frac{dT(z)}{1-T(a^{in})}$. Notice that $\mathbb{E}[wl] = \frac{\gamma \bar{r}}{1+\gamma}$ as a result of the bargaining scheme (14). Payments can also be defined in terms of the average wage across workers in the consumption sector $\mathbb{E}[w^c] = \int_{w_0}^{w_1} w dG(w)$.

Employment per firm. Consistency implies that total labor income in the consumption sector satisfies the identity $\mathbb{E}[w^c] n^c = \mathbb{E}[wl] M$. This condition yields average employment per firm in the consumption sector:

$$\frac{n^c}{M} = \frac{\gamma}{1+\gamma} \frac{\bar{r}}{\mathbb{E}[w^c]} \tag{31}$$

which is decreasing in productivity cutoff a^{in} (since $\rho > 1$) and increasing in labor market tightness (as the ratio $\frac{w_0}{w}$ does).

Service per firm. In the service sector, the clearing of labor demand and supply (18)-(19) implies,

$$\frac{a^s n^s}{M} = f_p + p_x f_x + \frac{\delta_f f_e}{1 - T(a^{in})} \tag{32}$$

where $p_x = \frac{1-T(a^x)}{1-T(a^{in})}$ is the share of incumbent firms that export and the mass of entrants $(1 - T(a^{in})) E$ replaces firms that exit because of exogenous destruction shock $\delta_f M$. Given (31) and (32), the two aggregate constraints close the equilibrium.

Economy budget constraint. Total payments to workers are equal to total expenditure:

$$\left[(1 + p_x) - \frac{\gamma}{1+\gamma}\right] \bar{r} M - \mu a^s n^s = 0 \tag{33}$$

where $(1 + p_x) M\bar{r}$ is total revenue and $\mathbb{E}[wl] M = \frac{\gamma}{1+\gamma} \bar{r} M$ is total labor income in the consumption sector and $w^s = \mu a^s$ is the wage in the service sector.

Labor market clearing. The number of workers who are employed or unemployed in either of the two sectors have to sum up to total workforce:

$$\left[1 + \left(\frac{1-x^c}{x^c}\right) \delta_j\right] n^c + \left[1 + \left(\frac{1-x^s}{x^s}\right) \delta_j\right] n^s = N \tag{34}$$

where the steady state number of unemployed workers who search in the two sectors is given by $u^c = \left(\frac{1-x^c}{x^c}\right)\delta_j n^c$ and $u^s = \left(\frac{1-x^s}{x^s}\right)\delta_j n^s$; according to (12). The system of (32) and (33) determines the share of exporters over incumbent firms p_x, which is increasing in a^{in} and f_x and decreasing in $\bar{r}$, then in labor market tightness θ^c. The linear system of 3 equations in 3 unknowns (31), (33) and (34) determines n^s, n^c, M and completes the characterization of the steady state equilibrium, for a given pair of values $\{a_{in}, \theta^c\}$. Sections 3.4 and 3.5 define the two equilibrium conditions that characterize productivity cutoff and labor market tightness.

Finally notice that for a given level of wage $w^s = \mu a^s$, the zero profit condition in the service sector fixes the level of vacancies: $v^s = \frac{(1-\mu)a^s}{k} n^s$. Labor market tightness in the service sector is the unique positive solution of the quadratic equation: $x^{s\ 2} + \frac{\delta_j}{1-\delta_j} x^s - \frac{1-\mu}{1-\delta_j}\frac{a^s}{k} = 0$. Indeed the steady state value of labor market tightness in the service sector only depends on productivity a^s and labor market frictions k, δ_j, μ.

3.4 Firm dynamics equilibrium

The first equilibrium condition is given by the locus of points of cutoff productivity a^{in} and labor market tightness θ^c such that firms optimally choose to stay in the market or exit, given the optimal decision of exporting and free entry. The average profit of an incumbent reads:

$$\bar{\Pi} = \int_{a^{in}}^{a^x} \Pi\left(z;\theta^c;0\right) \frac{dT\left(z\right)}{1-T\left(a^{in}\right)} + \int_{a^x}^{a^{sup}} \Pi\left(z;\theta^c;1\right) \frac{dT\left(z\right)}{1-T\left(a^{in}\right)} \tag{35}$$

Equation (35) is the *zero profit condition*. Under usual regularity conditions on the productivity distribution, the average profit is a decreasing function of the productivity cutoff.[14] The properties of the profit function (30) imply that firms that remain after a tightening of the labor market are characterized by higher profit.

If entry is optimal then the expected present discounted value of entry has to be neither strictly larger than the sunk cost of entry f_e or strictly lower that the sunk cost of entry. The flow value a potential entrant expects to gain is equal to the probability of making a successful entry $1-T\left(a^{in}\right)$ times the average profit of an incumbent firm $\bar{\Pi}$. The *free entry*

[14]SeeMelitz (2003), footnote 15.

condition for optimal entry reads:

$$\bar{\Pi} = \frac{1 - \beta (1 - \delta_f)}{1 - T (a^{in})} f_e \tag{36}$$

In the space defined by productivity cutoff and average profit, the zero profit condition (35) is a decreasing schedule, whereas the free entry condition (36) is increasing in a^{in}. Figure 2 shows the unique intersection, that determines the value of the cutoff productivity a^{in} such that the firm dynamics is in equilibrium, given level of labor market tightness. After

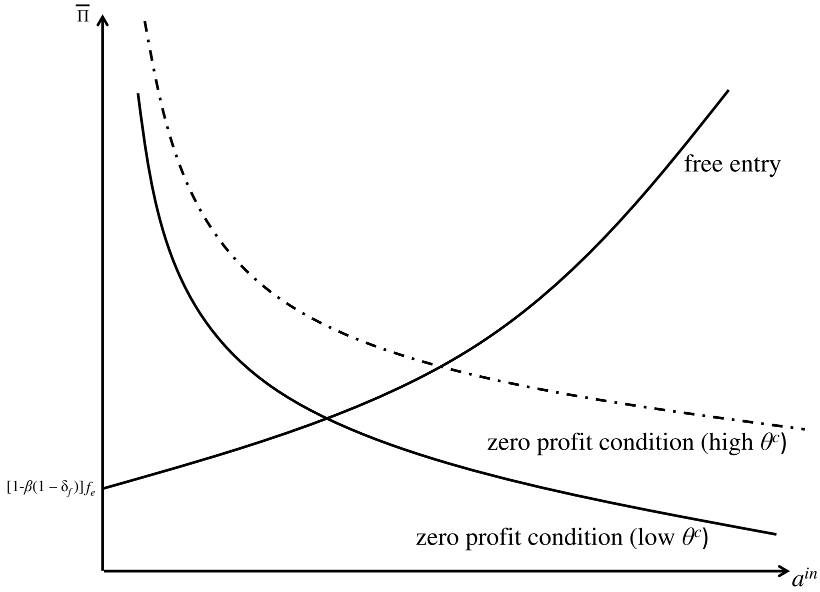

Figure 2: **Firm Dynamics Equilibrium** in the consumption sector. For a given labor market tightness θ^c there exists a unique value of productivity cutoff a^{in} such that forward looking entry, exit and export decisions maximize firm value under free entry.

a tightening of the labor market the zero profit condition lies above the original schedule. Everything else being constant a tighter labor market determines a higher productivity cutoff and indeed it leads to a tougher selection of incumbent firms.

3.5 Labor market equilibrium

The second equilibrium condition is given by the locus of points of cutoff productivity a^{in} and labor market tightness θ^c such that the average employment per firm coincides with the ratio in aggregate variables $\frac{n^c}{M}$. The identity:

$$\int_{a^{in}}^{a^x} l(z; \theta^c; 0) \frac{dT(z)}{1 - T(a^{in})} + \int_{a^x}^{a^{sup}} l(z; \theta^c; 1) \frac{dT(z)}{1 - T(a^{in})} = \frac{n^c}{M} \tag{37}$$

defines the labor market equilibrium, where the ratio $\frac{n^c}{M}$ is a general equilibrium outcome (31). The left hand side in (37) is determined aggregating labor income in the consumption sector. The ratio $\frac{n^c}{M}$ is decreasing in productivity cutoff, under the same assumptions on the productivity distribution that hold for the zero profit condition and it is increasing in labor market tightness, with convexity due to $\rho > 1$.

The right hand side in (37) is obtained aggregating employment at the firm level $l(a, \theta^c)$, that is a non monotonic function of labor market tightness. A higher labor market tightness increases the probability that a worker quits a low paid job to match with better employers. Under the assumption that the productivity distribution is sufficiently positively skewed then the contraction of employment for firms at the left of the productivity distribution dominates and expected employment per firm in the left hand side of (37) falls as the labor market tightness increases. The pace at which average employment falls is increasing in θ^c, because (for a given a^{in}) employment becomes more and more concentrate at the top performer firms.

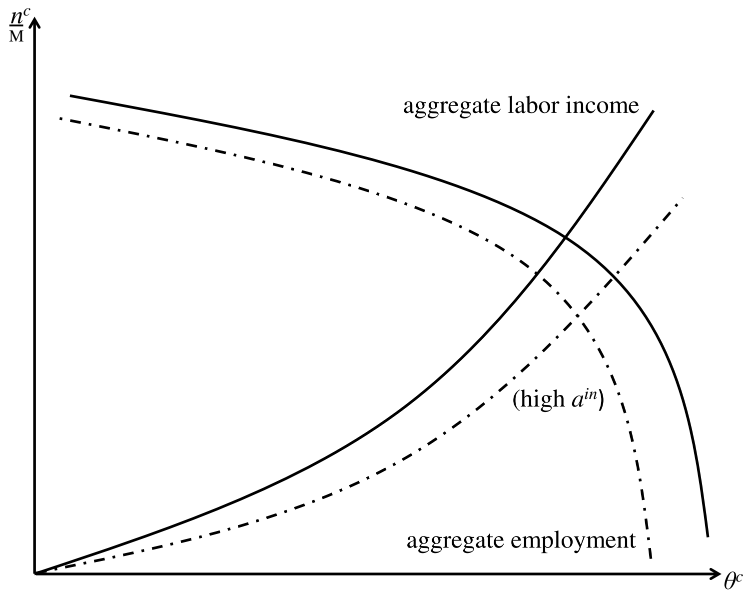

Figure 3: **Labor Market Equilibrium** in the consumption sector. For a given productivity cutoff a^{in} there exists a unique value of labor market tightness θ^c such that average employment per firm satisfies both aggregation of labor income and employment.

Figure (3) describes the labor market equilibrium, for a given level of productivity cutoff (solid lines) and a higher level (dashed lines). The r.h.s. of (37) is labeled as the schedule that originates from aggregating labor income, whereas the l.h.s. is the result of computing expected employment across firms.

4 Dynamics

Let $t = p$ be the time of a policy implementation. The dynamic equilibrium consists of the sequence at time $t-p = 0, 1, \ldots$ of the same variables that compose steady state equilibrium of the third paragraph; such that: in the consumption sector, (a) entry, exit and export decisions maximize firm value subject to output and labor demand and supply market clearing for every $a \geq a^{in}$ and under free entry, (b) the mass of firms evolves according to (6), employment flows are governed by the law of motion (15), given the separation rate and the hiring rate (10)-(11), the net destruction rate of vacancies σ_t and jobs Δ_t defined in (7), (c) wage satisfies the bargaining rule (14) and (d) vacancies are posted according to the policy (17); in the service sector (e) there is zero profit, output and labor demand and supply clear; at the aggregate level (f) the sum of employment plus unemployment in the two sectors is equal to total workforce.

The discussion of the dynamics is organized in three steps. The economy is assumed to be in the initial steady state, indexed by the subscript I. Suddenly an unanticipated and permanent policy is implemented (represented as a change in the parameter values) and the economy is taken away from the initial steady state to a new allocation; that will be indexed by the subscript P to refer to the time of policy implementation. Entry of firms and reallocation of workers drive the economy to the new steady state; indexed by N.

Workers who are employed at time $t+1$ in the consumption sector are those who do not separate, either because of endogenous or exogenous destruction $(1-\Delta_{t+1})(1-\delta_j)\, n_t^c$ plus the share x_{t+1}^c of previously employed workers who separated $[1-(1-\Delta_{t+1})(1-\delta_j)]\, n_t^c$ and unemployed workers u_t who find a job offer. The same argument holds in the service sector. Workers who lose their job or were unemployed at time t and do not receive a job offer are unemployed at time $t+1$. The system,

$$\begin{aligned}
n_{t+1}^{c} &= \left[\left(1-x_{t+1}^{c}\right)\left(1-\Delta_{t+1}\right)\left(1-\delta_{j}\right)+x_{t+1}^{c}\right] n_{t}^{c}+x_{t+1}^{c} u_{t}^{c} \\
u_{t+1}^{c} &= \left(1-x_{t+1}^{c}\right) u_{t}^{c}+\left[\Delta_{t+1}+\left(1-\Delta_{t+1}\right) \delta_{j}\right]\left(1-x_{t+1}^{c}\right) n_{t}^{c} \\
n_{t+1}^{s} &= \left[\left(1-x_{t+1}^{s}\right)\left(1-\delta_{j}\right)+x_{t+1}^{s}\right] n_{t}^{s}+x_{t+1}^{s} u_{t}^{s} \\
u_{t+1}^{s} &= \left(1-x_{t+1}^{s}\right) u_{t}^{s}+\delta_{j}\left(1-x_{t+1}^{s}\right) n_{t}^{s}
\end{aligned} \tag{38}$$

describes the dynamics of employment flows for every $t \geq p$.

The total number of vacancies issued in the consumption sector is obtained as aggregation at the firm level. The policy for vacancy posting (17) links the dynamics of vacancies to the path of variable profits. The structure of fixed costs is directly affected by the policy implementation, but it remains unchanged from the time of policy implementation on. Potential firms and incumbents confirm the same entry, exit and export decisions, given the set of policy parameter values. Indeed, after the policy implementation the gap between variable profit and total profit is constant over time, for a given productivity value and for the average across productivity values as well. The expected total profit of an incumbent firm is fixed by the free entry condition (36) such that: $\bar{\Pi}_t = \bar{\Pi}_N$ for every $t \geq p$. Indeed, the average variable profit across incumbent firms is constant over time $\bar{\pi}_t = \bar{\pi}_N$. It follows that also the ratio of total vacancies per firm has to be constant over time and equal to the average vacancies posted in steady state. For every $t \geq p$, the dynamics of total vacancies in the consumption sector v_t^c is driven by the evolution of the mass of firms:

$$\begin{aligned}
v_{t+1}^{c} &= M_{t+1} \int_{a_{N}^{in}}^{\infty} \frac{\vartheta^{c}\left(z, \theta_{N}^{c}\right) d T(z)}{1-T\left(a_{N}^{in}\right)} \\
v_{t+1}^{s} &= \frac{1-\mu}{k} a^{s} n_{t+1}^{s}
\end{aligned} \tag{39}$$

whereas the zero profit condition yields total vacancies in the service sector v_t^s.

The wage offer distribution is specified period by period integrating the policy for vacancy posting (17) over the productivity support. Firm productivity does not change over time and the wage mapping is continuous, monotone and rank preserving. Hence $F^c\left(\omega\left(a, \theta_{t+1}^c\right), \theta_{t+1}^c\right)$ is the share of wage offers that are open at the beginning of time $t+1$ (indeed, issued at the end of time t) by a firm with productivity lower or equal to a. For every $t \geq p$, the transition equation for the wage offer distribution results:

$$F^{c}\left(\omega\left(a, \theta_{t+1}^{c}\right), \theta_{t+1}^{c}\right)=\frac{M_{t+1}(a)}{\sigma_{t+1} v_{t}^{c}} \int_{a_{N}^{in}}^{a} \frac{\vartheta^{c}\left(z, \theta_{t}^{c}\right) d T(z)}{T(a)-T\left(a_{N}^{in}\right)} \tag{40}$$

where $M_{t+1}(a)$ satisfies the transition equation for the mass of firms (6) and the service market clearing (18)-(19). The change of variable $a = \omega^{-1}\left(w, \theta^c_{t+1}\right)$ in equation (40) yields the law of transition for the wage offer distribution. The law of transition for the wage distribution (13) completes the characterization of the equilibrium out of the steady state.

4.1 Policy implementation

Firm entry and exit are forward looking, under free entry and perfect information. As a consequence, at the time of policy implementation both incumbents and new entrants perfectly foresee the new productivity cutoff $a^{in}_N > a^{in}_I$ and optimally take their exit, entry and export decisions such that $a^{in}_P = a^{in}_N$ and $a^x_P = a^x_N$. Equation (6) accounts for the mass of firms immediately after policy implementation:

$$M_P = \frac{1 - T\left(a^{in}_N\right)}{1 - T\left(a^{in}_I\right)} M_I \tag{41}$$

where we used the steady state relationship $\left[1 - T\left(a^{in}_I\right)\right] E_I = \delta_f M_I$.

The net destruction rates of vacancies σ_P and jobs Δ_P are defined in (7). The endogenous rate of vacancy and job destruction at the time of policy implementation are respectively $\varepsilon_P = F^c\left(\omega\left(a^{in}_N, \theta^c_I\right), \theta^c_I\right) - F^c\left(\omega\left(a^{in}_I, \theta^c_I\right), \theta^c_I\right)$ and $\varpi_P = G^c\left(\omega\left(a^{in}_N, \theta^c_I\right), \theta^c_I\right) - G^c\left(\omega\left(a^{in}_I, \theta^c_I\right), \theta^c_I\right)$ at the time of policy implementation and zero otherwise. Vacancy and job creation rates due to firm entry are equal to $\left[1 - T\left(a^{in}_N\right)\right] \frac{E_I}{M_I}$. As in the initial steady state $\left[1 - T\left(a^{in}_I\right)\right] E_I = \delta_f M_I$ then when policy implementation hits, vacancies and job destruction rates jump to the values: $\sigma_P = 1 - \delta_f - \varepsilon_P + \frac{1-T\left(a^{in}_N\right)}{1-T\left(a^{in}_I\right)}\delta_f$ and $\Delta_P = \delta_f + \varpi_P - \frac{1-T\left(a^{in}_N\right)}{1-T\left(a^{in}_I\right)}\delta_f$. Labor market tightness and probability of finding a job jump respectively to $\theta^c_P = \frac{\sigma_P v^c_I}{u^c_I + n^c_I} = \sigma_P \theta^c_I$, $x^c_P = \sqrt{\sigma_P}\ x^c_I$ as it is implied by (8). In the service sector there is no either endogenous destruction or firm destruction, then $x^s_P = x^s_I$.

Workers foresee the new equilibrium allocation. Indeed, immediately after policy implementation the total job applications to the two sectors jump to their steady state levels and they remain constant during the transition: $u^c_t + n^c_t = u^c_N + n^c_N$ and $u^s_t + n^s_t = u^s_N + n^s_N$ for every $t \geq p$.[15] Employment in the two sectors at the time of policy implementation

[15] Notice that the claim refers to the aggregate flow of job applications, as it can be derived from (39). The model is silent on which worker applies to which sector.

reads:

$$n_P^c = [(1 - x_P^c)(1 - \Delta_P)(1 - \delta_j)]\, n_I^c + x_P^c \,(u_N^c + n_N^c) \tag{42}$$

$$n_P^s = [(1 - x_P^s)(1 - \delta_j)]\, n_I^s + x_P^s \,(u_N^s + n_N^s) \tag{43}$$

Finally, the service market clearing determines the number of potential entrants that pay the sunk cost after policy implementation: $E_P = \frac{a^s}{f_e} n_P^s - \left(\frac{f_p}{f_e} + \frac{1-T\left(a_N^x\right)}{1-T\left(a_N^{in}\right)} \frac{f_x}{f_e} \right) M_P$.

4.2 Transitional Dynamics

The law of motion of the mass of firms (6) and the market clearing condition for services (18)-(19) give the transition equation for the mass of firms:

$$M_{t+1} = c_{Mn}\, n_t^s - c_{MM}\, M_t \quad , \quad c_{Mn} \in \left(0, \frac{a^s}{f_e}\right), \quad c_{MM} > -(1 - \delta_f) \tag{44}$$

where $c_{Mn} = [1 - T(a_N^{in})] \frac{a^s}{f_e}$ and $c_{MM} = [1 - T(a_N^{in})] \frac{f_p}{f_e} + [1 - T(a_N^x)] \frac{f_x}{f_e} - (1 - \delta_f)$. The evolution of employment in each of the two sectors is given by a first order difference equation with variable coefficients:

$$n_{t+1}^c = c_{nc}(t+1)\; n_t^c + x_{t+1}^c (u_N^c + n_N^c) \quad , \quad c_{nc}(t+1) \in (0,1) \tag{45}$$

$$n_{t+1}^s = c_{ns}(t+1)\; n_t^s + x_{t+1}^s (u_N^s + n_N^s) \quad , \quad c_{ns}(t+1) \in (0,1) \tag{46}$$

where $c_{nc}(t+1) = \left(1 - x_{t+1}^c\right)(1 - \Delta_{t+1})(1 - \delta_j)$ and $c_{ns}(t+1) = \left(1 - x_{t+1}^s\right)(1 - \delta_j)$. The total number of vacancies posted in the consumption sector v_t^c is M_t times the number of vacancies posted by the average firm, according to the policy (17). Vacancies and job destruction rates are: $\sigma_{t+1} = 1 - \delta_f + [1 - T(a_N^{in})] \frac{E_t}{M_t}$ and $\Delta_{t+1} = \delta_f - [1 - T(a_N^{in})] \frac{E_t}{M_t}$. The probability of finding a job in either of the two sectors is predetermined during the transition: $x_{t+1}^c = \sqrt{\frac{\sigma_{t+1} v_t^c}{u_N^c + n_N^c}}$ and $x_{t+1}^s = \sqrt{\frac{1-\mu}{k} \frac{a^s}{u_N^s + n_N^s} n_t^s}$. The number of potential entrants that pay the cost at time $t+1$ is implied by service market clearing $E_{t+1} = \frac{a^s}{f_e} n_{t+1}^s - \left(\frac{f_p}{f_e} + \frac{1-T\left(a_N^x\right)}{1-T\left(a_N^{in}\right)} \frac{f_x}{f_e} \right) M_{t+1}$.

The system of equations (44)-(46) characterizes the transitional dynamics. The mass of firms over time is described by the system of the first order difference equation with constant coefficient (44) and employment in the service sector (46). Notice that the latter does not depend on M_t. The dynamics of the system is driven by the two coefficients c_{MM} and c_{ns}. If the sunk cost of entry is large enough $\frac{f_p}{f_e} + \frac{f_x}{f_e} \geq (2 - \delta_f)$ then the evolution of the mass of firms is stable. Employment in the service sector (46) evolves according to a non linear difference equation in n_t^s. The locus of points that describes n_{t+1}^s is a positive, increasing and concave function of n_t^s. The dynamics of employment in the service sector is stable. The coefficient $c_{ns}(t+1)$ remains bounded within the interval $(0,1)$. Employment in the consumption sector evolves according to two forces: total vacancies v_t^c and destruction rates, σ_{t+1} and Δ_{t+1}. Nevertheless, the coefficient $c_{nc}(t+1)$ is always positive and bounded below one. Indeed the dynamics of employment in the two sectors converges monotonically to the new steady state.

5 The effect of a trade liberalization

Let the economy be in steady state equilibrium. For a sufficiently large fixed cost f_x the export cutoff a^x is strictly larger that the productivity cutoff a^{in}. Indeed, only the firms endowed with productivity $a \geq a^x$ select into the export market. Given this initial allocation, I discuss the effect of an unanticipated and permanent cut of the fixed cost of export f_x.

Consider a partial equilibrium analysis of firm dynamics. A lower fixed cost of export determines an increase of the average profit of incumbent firms, (35). The new zero profit condition lies above the original one, given the same level of labor market tightness. The free entry condition is unchanged. Figure (4) shows the *selection effect*, a feature that is common to models that nest the Melitz (2003) framework. The productivity cutoff increases, the least productive firms are selected out of the market and the average profit across incumbent firms increases.

Figure 5 shows the labor market equilibrium. For a given level of a^{in} the new aggregate employment schedule for lower f_x lies above the original one. Everything else being constant, the additional demand for labor due to new exporter makes the labor market becomes tighter. Following the expansion in the extensive margin of trade, the average employment per firm increases.

Following the increase on labor market tightness, firms will update their wage. In models without wage dispersion there is no reallocation of workers across firms other than the one implied by the reallocation of output share. In this model, workers reallocate follow-

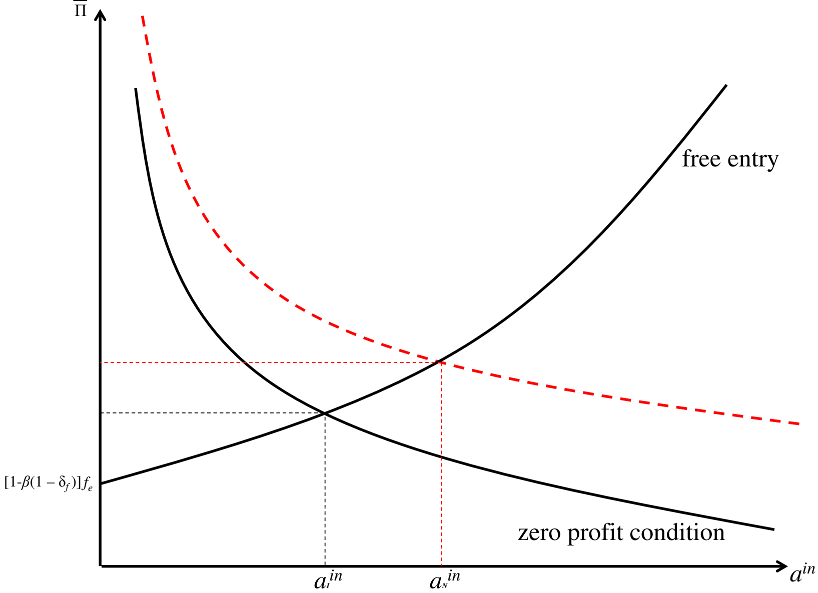

Figure 4: **Selection**. A fall in the fixed cost of export f_x determines an increase in the productivity cutoff a^{in} and average profit $\bar{\Pi}$.

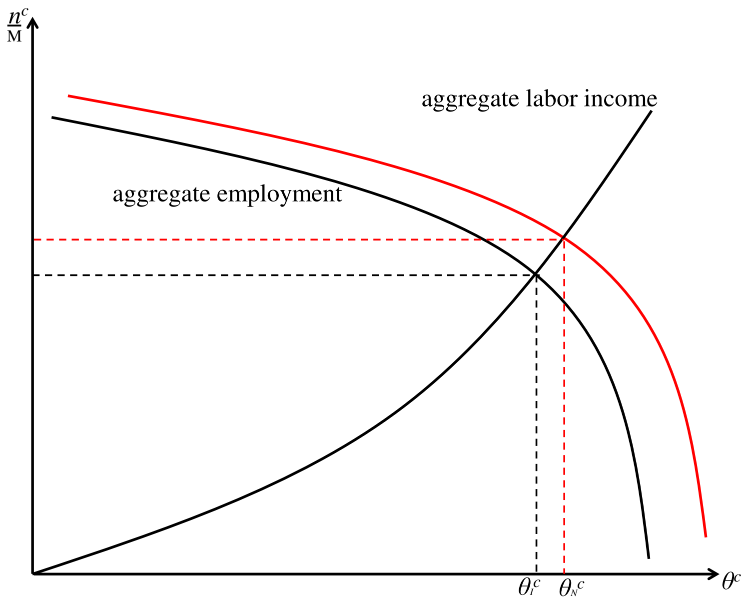

Figure 5: **Trade liberalization and labor market tightness**. Holding constant the productivity cutoff, the additional demand of labor from new exporters tightens the labor market.

ing the change in relative wages across firms. The burden of the competition in the labor market is more severe for relatively low productive firms, since they have to rise the wage relatively more (everything else being constant the ratio $\frac{w_0}{w}$ is increasing in θ^c). This asymmetry determines a further shift up of the zero profit condition. In the new steady state equilibrium, the average profit and the productivity cutoff will be higher than in a model without wage dispersion. Figure 6 shows the additional competition that is captured by this model as a result of on-the-job search. These results are summarized in the following

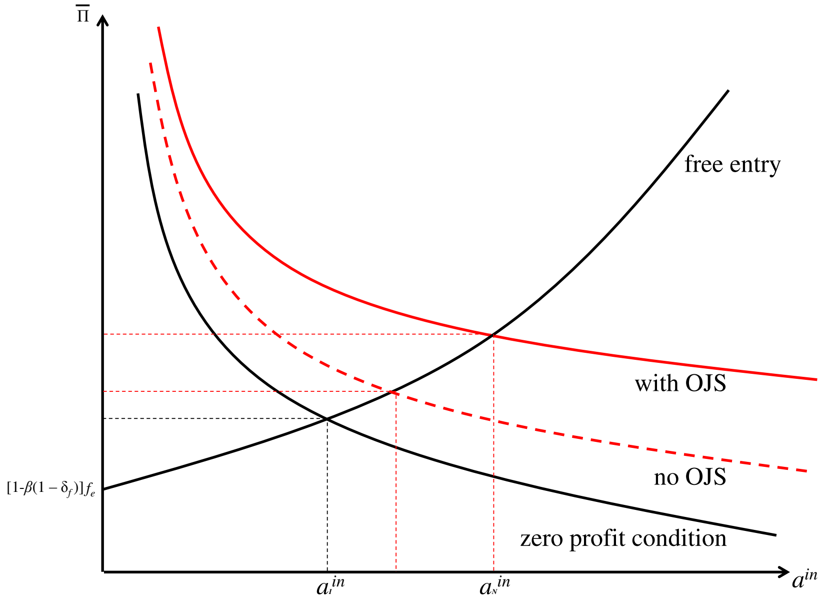

Figure 6: **Competition in the labor market**. After a fall in the fixed cost of export f_x, relatively more vacancies are yields by relatively more productive firms that offer relatively higher wages. Least productive firms are forced to rise the wage relatively more.

statement.

Proposition 3: A lower fixed cost to access the foreign market determines in the tradable sector: (i) a higher average productivity, (ii) a tighter labor market, (iii) higher average profit, (iv) higher average employment per firm and (v) higher average wage, both across workers and across firms.

Proof. *Results (i)-(iv) are implied by the properties of the firm dynamics equilibrium (35)-(36) and the labor market equilibrium (37). The result (v) is implied by the properties of the wage support and the wage distribution (Propositions 1 and 2) and the fact that the wage-productivity assignment* $\omega(a, \theta^c)$ *is increasing in both arguments.* $\square$

The comparative statics analysis is performed by solving the system (31), (33) and (34), that yields the mass of firms M and employment levels in the two sectors n^c and n^s:

$$\begin{aligned} M &= \frac{N}{\eta(\theta^c)\frac{\gamma}{1+\gamma}\frac{\bar{r}}{\mathbb{E}[w]}+\eta(\theta^s)\frac{\chi\bar{r}}{\mu a^s}} \\ n^c &= \frac{\gamma}{1+\gamma}\frac{\bar{r}}{\mathbb{E}[w]}M \\ n^s &= \frac{\chi\bar{r}}{\mu a^s}M \end{aligned} \tag{47}$$

where $\chi = (1+p_x) - \frac{\gamma}{1+\gamma} > 0$ accounts for the extensive margin of export and $\eta(\theta^i) = 1 + \left(\frac{1-x^i}{x^i}\right)\delta_j > 1$ is the ratio of workforce over employment at the sectoral level $\frac{n^i+u^i}{n^i}$ for $i = c, s$. From the discussion in the third section and the results stated in Proposition 3 we know that both $\frac{\bar{r}}{\mathbb{E}[w]}$ and $\chi\mathbb{E}[w]$ increase after a fall in the fixed cost of export. There exists a value $\delta_j^\star > 0$ such that when the exogenous destruction of a job is small enough $0 < \delta_j < \delta_j^\star$ the change in the three aggregate variables is unambiguously determined by looking at the change in $\frac{\bar{r}}{\mathbb{E}[w]}$ and $\chi\mathbb{E}[w]$. Proposition 4 discusses the unambiguous results under this scenario.

Proposition 4: For a sufficiently low value of the exogenous job destruction probability $0 < \delta_j < \delta_j^\star$, a trade liberalization determines: (i) a reduction in the number of domestic producers, (ii) a loss of employment in the tradable sector, (iii) an increase of employment in the service sector. A higher exogenous job destruction rate mitigates the reduction in firms and employment in the consumption sector and it amplifies the increase of employment in the service sector.

Proof. *Let $\delta_j \approxeq 0$ and recall that steady state value of labor market tightness in the service sector is not affected by the trade liberalization, indeed $\eta(\theta^s)$ is unchanged. Then it is immediate to recognize to conclude that the increase in $\frac{\bar{r}}{\mathbb{E}[w]}$ and $\chi\mathbb{E}[w]$ shrinks the number of incumbent producers M. Substituting M in the expression for n^c and n^s, then dividing by the numerator yields the conclusion that the rise in $\chi\mathbb{E}[w]\uparrow$ determines the loss of employment in the consumption sector $n^c\downarrow$ and the increase of employment in the service sector $n^s\uparrow$.*

When $\delta_j > 0$ then the increase in labor market tightness lessens the value of $\eta(\theta^c)$. Therefore, everything else being constant, the denominator in the expressions for the mass of firms M and employment in the consumption sector n^c shrinks relative to the case with $\delta_j \approxeq 0$. In the expression for employment in the service sector n^s the positive contribution of $\chi\mathbb{E}[w]$ is weighted more by the marginal fall in $\eta(\theta^c)$.

□

Unemployment. Let the value $\delta_j^{\star\star} > 0$ be such that when $\delta_j = \delta_j^{\star\star}$ the change in the mass of domestic firms M after a fall in the fixed cost f_x is null. Under this scenario employment in both sectors increase. Hereafter I define $\delta_j^{\star}$ as the value such that when $\delta_j = \delta_j^{\star}$ the change in employment n^c after a fall in the fixed cost f_x is null. Then the following sorting applies $0 < \delta_j^{\star} < \delta_j^{\star\star}$. Notice that when Proposition 4 holds, then a necessary (not sufficient) condition for the increase in total unemployment is matched. Total unemployment $u = N - n^c - n^s$ increases only if the total number of firms in the consumption sector M decreases. Given the necessary condition $\delta_j < \delta_j^{\star}$, I discuss in the following Proposition under which conditions the economy experiences an increase of total unemployment after the trade liberalization.

Proposition 5: For a sufficiently low degree of competition across firms in the output market of the tradable sector $1 < \rho < \rho^{\star}$ and a sufficiently high productivity in the service sector $a^s > a^{s\star} > 0$, a trade liberalization determines an increase in total unemployment.

Proof. *Let total unemployment be $u = N - n^c - n^s$. Then the change in total unemployment is given by:*

$$\frac{du}{df_x} = -\left[\frac{dA}{df_x}\left(\eta\left(\theta^c\right)A + \eta\left(\theta^s\right)B - \eta\left(\theta^c\right)\right) + \frac{dB}{df_x}\left(\eta\left(\theta^c\right)A + \eta\left(\theta^s\right)B - \eta\left(\theta^s\right)\right) - \frac{d\eta\left(\theta^c\right)}{df_x}A\right]$$

where $A = \frac{\gamma}{1+\gamma}\frac{\bar{r}}{\mathbb{E}[w]} > 0$ and $B = \frac{\chi\bar{r}}{\mu a^s} > 0$. From the corresponding definitions $\eta\left(\theta^c\right), \eta\left(\theta^s\right) > 1$ and Proposition 4 implies $\frac{dA}{df_x} > 0$, $\frac{dB}{df_x} > 0$ and $\frac{d\eta(\theta^c)}{df_x} < 0$. A necessary condition for $\frac{du}{df_x} \geq 0$ is that either $\gamma < \frac{\mathbb{E}[w]/\bar{r}}{1-\mathbb{E}[w]/\bar{r}} = \gamma^{\star}$, then $\rho < \frac{\frac{\mu}{1-\mu}}{\frac{\mu}{1-\mu}-\gamma^{\star}} = \rho^{\star} > 1$ or $a^s > \frac{\chi}{\mu}\bar{r} = a^{s\star} > 0$ or both; otherwise $\frac{du}{df_x} < 0$. When $\rho \to 1$, i.e. $\gamma \to 0$ and $a^s \to \infty$ then $\frac{du}{df_x} = 0$. It follows that there is a subset of the two-dimensional real space $(1, \rho^{\star}) \times (a^{s\star}, \infty)$ in which $\frac{du}{df_x} \geq 0$. Both $\mathbb{E}[w]$ and $\bar{r}$ are monotonic increasing functions of a^{in} and θ^c, then the region of the parameter space in which $\frac{du}{df_x} \geq 0$ can be approximated numerically in the neighborhood of the point $\left(a_I^{in}, \theta_I^c\right)$. $\square$

The intuition behind the result in Proposition 5 suggests that a country is more likely to experience an increase of unemployment when it liberalizes trade in less competitive sectors and the more efficient is the production of non tradable services (which are an input for the tradable sector). This is more likely the case for a developed economy, which is already open to trade and endowed with better technologies. Instead, developing economies that are relatively worse in technology and suddenly open to trade in relatively more competitive sectors (textile being a classical example) are more likely to experience gains in employment.

Wage inequality. The change in wage inequality is the balance of three forces. Following the increase in labor market tightness $\theta^c \uparrow$, in the consumption sector (i) the wage support becomes wider and (ii) it shifts to the right with an increase of the average wage across workers. In the service sector the wage remains constant, indeed (iii) the wage gap between employed workers in different sectors increases.

It is immediate to conclude that wage inequality increases between workers employed in different sectors. Instead, the analysis of within sector wage inequality might lead to ambiguous results. Nevertheless, Proposition 1 already discussed the first order stochastic dominance of the wage distribution at lower labor market tightness. Therefore, under this criteria, a worker who aims to maximize labor income conditionally on being employed (at a random firm) before and after the policy implementation would be in favor of the trade liberalization. The predictions of the model in terms of wage inequality are discussed in the following proposition:

Proposition 6: After a trade liberalization (i) wage inequality between workers employed in the tradable versus the non tradable sector increases, (ii) the change in wage inequality within the tradable sector has the sign of the covariance $cov\left(\frac{w}{\mathbb{E}[w]}, \frac{dw}{d\theta^c}\right)$ over the wage support, (iii) employed workers randomly assigned to a firm are better off after the trade liberalization.

Proof. Point (i) is trivial, point (iii) follows directly from Proposition 1. The effect of a trade liberalization on the wage distribution across employees of the consumption sector passes through the positive deviation in labor market tightness. Let me consider the variance of wages among identical workers as a measure of wage inequality, then I am interested in the sign of:

$$\frac{d}{d\theta^c}\left(\mathbb{E}\left[w^2\right] - \mathbb{E}\left[w\right]^2\right) = 2\mathbb{E}\left[w\frac{dw}{d\theta^c}\right] - 2\mathbb{E}\left[w\right]\mathbb{E}\left[\frac{dw}{d\theta^c}\right]$$

where $\frac{dw}{d\theta^c}$ *is a random variable across workers, as a function of the wage* w*. Substituting for* $\mathbb{E}\left[w\frac{dw}{d\theta^c}\right] = \mathbb{E}\left[w\right]\mathbb{E}\left[\frac{dw}{d\theta^c}\right] + cov\left(\frac{w}{\mathbb{E}[w]}, \frac{dw}{d\theta^c}\right)$ *yields the result.* □

In order to interpret the results in Proposition 6, notice that the discussion in the third section showed that an increase in labor market tightness leads to higher wages and the change is sharper at the bottom of the wage distribution. This result is in line with the conjecture that the wage is a convex function of labor market tightness at the bottom of the distribution and the convexity turns into concavity as the wage grows to the right side of the support. Under this interpretation, when the population of workers is relatively more concentrated at the bottom of the wage distribution $cov\left(\frac{w}{\mathbb{E}[w]}, \frac{dw}{d\theta^c}\right) > 0$, but the covariance decreases and possibly becomes negative when the mass of workers on the right end side becomes large enough. Thanks to on-the-job search the model predicts that a trade liberalization episode reallocates workers from firms that pay low wages to better one. Therefore this mechanism accounts for the well-known result that the relationship between wage inequality and trade openness has an inverted U shape.

6 Conclusion

In this paper I develop a new framework to analyze the impact of trade liberalization on firm selection, employment and wage distribution. Accounting for OJS in a Melitz (2003) trade model I show that the classical selection effects due to international trade are magnified. Moreover, OJS plays a key role to assess the impact of wider trade exposure on sectoral employment, unemployment and wage distribution.

The model predicts that an unanticipated trade liberalization determines tougher competition, both in the output market (higher productivity) and in the labor market (which becomes more tight). As a consequence, (i) employed workers are better off, (ii) the economy experiences a loss of employment in the tradable sector and (iii) wage inequality between-sector increases, whereas within-sector wage inequality responds to trade openness following an inverted U shape. The response of unemployment to trade liberalization depends on the degree of competition in the tradable sector and productivity in the non tradable sector. Unemployment increases when the liberalization occurs in less competitive sectors and the more the country employs efficient technologies in the production of non tradable services. Wage inequality increases when a country opens to trade, but the effect vanishes as the degree of trade openness increases.

Furthermore I characterize the transitional dynamics of the labor market following a trade liberalization. At the time of policy implementation the labor market tightness jumps to a level that is not in line with the long run equilibrium. Then, the model reaches the new steady state following stable paths. As a consequence, those economies that experience higher (lower) unemployment in the long run suffer (benefit) even more in the short run.

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