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Department of Economics
University of Munich

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Multi-Product Offshoring*

Carsten Eckel                 Michael Irlacher
University of Munich,         University of Munich†
CESifo, and CEPR†             

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Abstract

In this paper, we incorporate offshoring of labor-intensive goods in a model with multi-product firms, and explore its implications in partial and general oligopolistic equilibrium. We identify important aspects of this phenomenon and argue that improvements in offshoring opportunities can affect the geographic organization of a firm and its product range. Multi-product firms internalize supply linkages (flexible manufacturing) and demand linkages (cannibalization effect). In partial equilibrium, we find that more products are produced offshore on a larger scale and firms expand their product range with better prospects for offshoring. We identify the cannibalization effect as an important transmission mechanism within multi-product firms and show that the latter effect hits domestic labor demand in addition to the well-known re-location effect. Interestingly in general equilibrium these effects lead to adjustments in domestic factor prices and may cause a partial re-relocation of product lines.

Keywords: Multi-Product Firms, Cannibalization Effect, Product Range, Efficiency-seeking Offshoring, General Oligopolistic Equilibrium.

JEL Classification: F12, F 23, L23


†Department of Economics, D-80539 Muenchen, Germany; tel.: (+49) 2180 - 5824 ; e-mail: Carsten.Eckel@econ.lmu.de; internet: http://www.intecon.vwl.uni-muenchen.de/.

‡Corresponding author. Department of Economics, D-80539 Muenchen, Germany; tel.: (+49) 2180 - 2754; e-mail: Michael.Irlacher@econ.lmu.de; internet: http://www.intecon.vwl.uni-muenchen.de/.
1 Introduction

In the last decades, progress in communication and information technologies has changed the international organization of production. Markets are dominated by large multinational firms that control and manage production lines on a global scale. Global production networks enable firms to benefit from the generally lower labor costs in emerging countries. Against this background, industrialized countries fear a decline of jobs and pressure on wages. Recent academic research has identified two main channels by which offshoring affects domestic labor demand. Firstly, there is a relocation effect from the displacement of tasks that formerly were carried out domestically. Secondly, there are efficiency gains from vertical specialization that benefit domestic workers and increase domestic labor demand.\footnote{The efficiency or productivity effect of offshoring is stressed in recent contributions to the offshoring literature, such as Eckel (2003), Grossman and Rossi-Hansberg (2008), Rodriguez-Clare (2010), and Egger et al. (2013).}

In this paper, we study the consequences of a different kind of offshoring: Offshoring of production lines within multi-product firms (MPFs). Analyzing firms that offer a bundle of horizontally linked products leads to important new insights into the effects of offshoring. Our results are crucially different from the well-known effects of relocating just parts of a production process of a single product. In particular, we argue that the efficiency effect of offshoring can only occur if the production of a single good is linked sequentially in two or more countries. Moreover, at least part of the production stages have to remain in the home country so that domestic employment can benefit from the higher productivity. If, however, the total production line is relocated, the latter effect vanishes. A firm which produces a range of products can decide for each product where it is produced most efficiently. We will show that it is the labor intensity of each product which determines its optimal production location.

The relocation of a complete production process not only prevents the efficiency effect of offshoring but causes a cannibalization effect of offshoring that has not been discussed in the literature. If the products within the product range of an MPF are horizontally differentiated, the introduction of a new product will create a negative demand externality on all other products of this firm. This is typically referred to as the cannibalization effect and plays a big role in our analysis. Giving a firm the opportunity to offshore production will lead to a relocation of labor-intensive products and to an extension of the product range with additional products. We show that both operations will cannibalize output of domestically produced goods and reduce demand for labor in the home country.

A growing literature on MPFs stresses horizontal relationships between products within the boundaries of a single firm and analyzes the effects of globalization on the product range.
of a firm. Bernard et al. (2010) emphasize this as a new margin of firm adjustment, which Eckel and Neary (2010) refer to as intra-firm extensive margin. Within a multi-product framework, we investigate how improvements in the opportunities for offshoring affect the geographic organization and the product range of an MPF. For this purpose, we set up a general oligopolistic equilibrium (GOLE) model with MPFs and enrich this framework by introducing the firm’s opportunity to offshore the production of multiple varieties to a low-wage emerging country. Varieties within a firm’s product line are linked on the cost side through a flexible manufacturing technology, which captures the idea that - besides a core competence - an MPF can expand its portfolio with varieties that are less efficient in production. When producing abroad, a firm can use the same production technology as in the home country but additionally it has to bear offshoring costs.

We derive our results in partial and in general equilibrium. As a main result in partial equilibrium, we find that more products are produced abroad when prospects for offshoring improve. Furthermore, savings from lower offshoring costs lead to an extension of the product portfolio as the opportunity to produce labor-intensive products abroad enlarges the profit maximizing product range of an MPF. In a model where firms internalize demand linkages, rising outputs of foreign-produced varieties and additional varieties in the portfolio are crowding out domestic production, that does not benefit from lower offshoring costs. We stress this cannibalization effect as an important transmission channel that is specific to MPFs. In our model, in addition to the well-established relocation effect, cannibalization hits domestic production. For this reason, the analysis in partial equilibrium clearly indicates that domestic labor demand will decrease in the presence of more offshoring.

In general equilibrium, our analysis highlights adjustments through factor markets as an important transmission channel of external shocks on both the cutoff variety and the product range.\(^2\) With endogenous domestic wages the results are not as clear cut anymore. It is no longer apparent that more products will be produced offshore with falling offshoring costs. We show that the more domestic production benefits from falling domestic wages the more likely is the partial result reversed in general equilibrium. Therefore, our model is able to predict patterns in which firms "re-relocate" entire product lines following a decline in offshoring costs and a delayed fall in wages.

Our paper builds on and extends two strands of the existing literature in international trade on both MPFs and offshoring. Particularly with regard to the connection of both strands, Baldwin and Ottaviano (2001) come up with a multi-product setting where oligopolistic firms may produce some varieties in one country and other varieties in another. However,

\(^2\)The cutoff variety is defined as the product where the firm is indifferent concerning the optimal production location. It is characterized by equal production costs in the home country and the offshore destination.
they explain intra-firm trade patterns akin to reciprocal dumping à la Brander and Krugman (1983) and not via factor price differences across countries. Hence, their approach is not associated with “offshoring” per se. In a recent paper, Yeaple (2012) extends a framework by Bernard et al. (2011) with a proximity-concentration tradeoff. In his setting, firms produce multiple products for multiple countries and choose whether to export from the home country or to manufacture locally. Unlike to our paper, his focus is not on wage differentials between countries but on firm heterogeneity with respect to managerial expertise. Managers deliver expertise to foreign affiliates, which means that firms with a higher manager efficiency tend to build foreign affiliates rather than to export to foreign countries. In an empirical analysis, McCalman and Spearot (2013) examine the role of vertical product differentiation in the decision where to produce a specific variety. Using a dataset of light truck sales in the US, Canada and Mexico, they study the location decision of final assembly. The patterns of offshoring that they find can be explained by the labor intensity in the automobile production. Furthermore, it is consistent with one of our theoretical predictions that foreign output is produced at a lower scale.

We also contribute to the large literature on international production. Our way to determine the cutoff variety between domestic and foreign production is reminiscent of a key contribution to the offshoring literature by Feenstra and Hanson (1996). While in their theoretical model, offshoring takes the form of relocating labor-intensive activities of a single manufactured good, they adopt a more general definition of offshoring in the empirical part.\(^3\) Next to imports of intermediate goods, they further include final goods that are sold under the brandname of a firm in their definition of offshoring. Therefore, this measurement of offshoring is directly related to our way of defining offshoring as the relocation of complete production lines. By including final goods in their measure of offshoring, Feenstra and Hanson do better at explaining wage patterns and employment changes for the United States.\(^4\) Other empirical papers measure offshore activity by the total employment of foreign affiliates. Using this kind of measure, authors typically think of capturing the relocation of vertically related tasks or the replication of domestic production abroad (horizontal FDI). However, we show that this way of measuring international activity is perfectly consistent to what we call multi-product offshoring.

Existing theoretical research on MPFs is concerned mainly with the product market side of the economy. The main question which is tried to be answered is how MPFs absorb international trade. Intra-firm product switching is frequent and contributes like firm entry and

\(^3\)Feenstra and Hanson (1996) refer to this phenomenon as outsourcing.

\(^4\)Feenstra and Hanson (1996) argue that previous studies like Berman et al. (1994) and Lawrence (1994) did not find an impact of offshoring on U.S. wages because of their narrow definition of offshoring.
exit to the evolution of aggregate outcomes in an industry.\textsuperscript{5} The literature differs in the way of modelling the demand for and the decision to supply multiple products and in the assumptions about market structure. Most recent models assume that markets can be characterized by monopolistic competition, in which firms produce a large number of products but are themselves infinitesimal small in scale in the economy (see Arkolakis and Muennder (2010), Bernard et al. (2011), Nocke and Yeaple (2013), and Mayer et al. (2014)). Our model is built along the lines of Eckel and Neary (2010) who set up a different approach and assume that markets are oligopolistic. Their underlying market structure highlights as an important feature the cannibalization effect, which also plays a crucial role in our model.\textsuperscript{6} Next to these demand linkages, Eckel and Neary’s approach incorporates cost linkages between varieties in the form of flexible manufacturing.\textsuperscript{7}

The remainder of the article is structured as follows. Section 2 recaps the basic model of Eckel and Neary (2010) and incorporates offshoring into this framework. Subsequently, we provide comparative static results of falling offshoring costs. Section 3 shows how these results transform when wages are endogenized in general equilibrium. Section 4 concludes and summarizes results. Mathematical derivations and a numerical simulation of our model are presented in the Appendix.

## 2 The Model

To conduct our analysis, we rely on the multi-product framework with flexible manufacturing proposed by Eckel and Neary (2010). We introduce a model where firms on grounds of efficiency seeking can relocate the production of labor-intensive goods abroad. Our setup consists of two countries, Home and Foreign, and a large world market. There is a continuum of identical industries in Home, whereby the output produced in each of these industries is sold on the world market. Foreign is a low wage emerging country and acts as a potential destination for an affiliate. We begin this section with the analysis of one single sector by considering the behavior of the consumers in the world market and the optimal firm behavior in this industry.

\textsuperscript{5}Bernard et al. (2010) report changing product ranges for more than 50 percent of US firms within five years whereby one-half of those firms both added and dropped at least one product.

\textsuperscript{6}The cannibalization effect is also considered in recent articles by Feenstra and Ma (2008) and Dhingra (2013).

\textsuperscript{7}The concept of flexible manufacturing is also used in Milgrom and Roberts (1990), Eaton and Schmitt (1994), Norman and Thisse (1999), and Eckel (2009).
2.1 Consumer Behavior: Preferences and Consumer Demand

We assume that $L^W$ consumers in the world market maximize their utility defined over the consumption of differentiated products. Referring to the model of Eckel and Neary (2010), we maintain the specification of preferences in the form a two-tier utility function.\footnote{These preferences combine the continuum quadratic approach to symmetric horizontal product differentiation of Ottaviano et al. (2002) with the preferences in Neary (2009).} The upper tier is an additive function of a continuum of sub-utility functions over industries $z$, where $z$ varies over the interval $[0, 1]$, given by

$$U[u(z)] = \int_0^1 u(z) \, dz. \quad (1)$$

The representative consumer’s sub-utility is defined over per variety consumption $q(i, z)$ with $i \in \Omega$ and total consumption $Q \equiv \int_{i \in \Omega} q(i, z) \, di$, where $\Omega$ is a set of differentiated goods offered in industry $z$. To be more specific, we assume

$$u(z) = aQ - \frac{1}{2} b \left[ (1 - e) \int_{i \in \Omega} q(i, z)^2 \, di + eQ^2 \right]. \quad (2)$$

Eq. (2) has a standard quadratic form, where $a, b$ denote non-negative preference parameters and $e$ is an inverse measure of product differentiation which lies between 0 and 1. Lower values of $e$ imply that products are more differentiated and hence less substitutable. In the event of $e = 1$, consumers have no taste for diversity in products and demand depends on aggregate output only. Consumers maximize utility in Eqs. (1) and (2) subject to the budget constraint $\int_0^1 \int_{i \in \Omega} p(i, z) q(i, z) \, dizi \leq I$, where $p(i, z)$ denotes the price for variety $i$ in industry $z$ and $I$ is individual income. This yields the following linear inverse individual demand function:

$$\lambda p(i, z) = a - b \left[ (1 - e) q(i, z) + eQ \right], \quad (3)$$

where $\lambda$ is the marginal utility of income, the Lagrange multiplier attached to the budget constraint. Market-clearing imposes that each firm faces a market demand $x(i, z)$ that consists of the aggregated demand of all consumers in the world market $L^W q(i, z)$. For the inverse world market demand, we get

$$p(i, z) = a' - b' \left[ (1 - e) x(i, z) + eX \right], \quad (4)$$

where $a' \equiv \frac{a}{\lambda}$ is the consumers’ maximum willingness to pay and $b' \equiv \frac{b}{X L^W}$ is an inverse measure for the market size. Finally, $X \equiv \int_0^1 x(i, z) \, di$ represents the total volume of varieties produced and consumed in industry $z$. Note that $X$ is defined over the goods actually
consumed with $i \in [0, \delta]$, which is a subset of the potential products $\Omega$. With no quasi-linear term in Eq. (2), the value of $\lambda$ is not constant, which implies that $a'$ and $b'$ are endogenously determined in general equilibrium. However, with a continuum of industries, we may assume that each firm takes these parameters as given. Hence, each firm has market power in its own market but it is small in the economy as a whole. This assumption permits a consistent analysis of oligopoly in general equilibrium. As it has become standard in the literature, we choose the marginal utility of income as the numeraire and set $\lambda$ equal to one (see Neary (2009) for further discussion).

2.2 Firm Behavior: Costs and Technology of MPFs

This section considers technology and optimal firm behavior in industry $z$.\footnote{We concentrate on symmetric industries and drop the industry index $z$ in the following analysis. We consider this index again when we aggregate over all industries and turn to the level of the economy as a whole in general equilibrium.} We focus on intra-firm adjustments, so competition between firms plays only a second-order role. To keep the analysis as simple as possible, we focus on the monopoly case. Extending the analysis to oligopoly is straightforward.\footnote{The interested reader is referred to the Appendix in Eckel et al. (2011).} According to that, each industry $z$ is characterized by exactly one firm whose objective it is to maximize profits by choosing both the scale and scope of production, as well as choosing the optimal location for producing each specific variety. When choosing the optimal location for production, firms seek to reduce costs by producing labor-intensive goods offshore where a comparative advantage exists due to lower wages. For simplicity, we assume no fixed costs for both domestic and foreign production.

In our model, an MPF is characterized by a core competence and flexible manufacturing. Technology is firm-specific and, therefore, it can be applied correspondingly in Home and in Foreign. As in Grossman and Rossi-Hansberg (2008), technology is transferable as a home firm will use its own technology when performing a task abroad. Flexible manufacturing is characterized by one core competence, in which the firm is most efficient in fabrication. Furthermore, an MPF can produce additional varieties with rising marginal costs.

Production costs in our model comprise both a product-specific and a monitoring component (managerial effort) which we assume to be zero for production at home. This assumption implies that the ability to monitor varies with distance. Managerial effort is needed to supervise production and to provide the firm’s technology abroad.\footnote{See for example Grossman and Helpman (2004). They assume that a principal is able to observe the manager’s efforts at a lower cost when the manager’s division is located near to the firm’s headquarters as compared with when it is located across national borders.} By incorporating these costs, we try to capture the more general idea that aggravated monitoring through man-
agers, less skilled workers, worse infrastructure, or inferior contractual enforcement, affect production in emerging countries. In the following analysis, we refer to this cost component as offshoring costs. To put it formally, we assume a Ricardian technology where domestic (foreign) production costs \( c(i) \) \((c^*(i)) \) are given by

\[
c(i) = w\gamma(i) \quad \text{and} \quad c^*(i) = w^*(\gamma(i) + t),
\]

with \( \gamma(i) \) denoting the labor input coefficient for variety \( i \), \( w \) \((w^*) \) being the wage level in Home (Foreign) and finally \( t \) representing the offshoring costs.\(^{12}\) Latter is measured in labor costs and is the same for all products assembled abroad. As we are analyzing the relocation of total production lines and not the relocation of just parts of a production process, the assumption that \( t \) is identical for all offshored varieties seems fair. Technology is firm- and not country-specific, therefore \( \gamma(i) \) is the same in both countries. We assume the following properties: \( \gamma(0) = \gamma^0 \) and \( \frac{\partial c}{\partial i} = \frac{\partial c^*}{\partial i} w > 0. \)

**Closed Economy:** Without offshoring, optimal firm behavior is composed of maximizing total firm profits both with regard to scale and to scope. Considering the technology assumptions above and denoting the scope of the product portfolio by \( \delta \), profits are given by

\[
\Pi = \int_0^\delta [p(i) - c(i)] x(i) \, di.
\]

Firms simultaneously choose the quantity produced of each good and the mass of products produced. Maximizing profits in Eq. (7) with respect to scale \( x(i) \) implies the first-order condition for scale:

\[
\frac{\partial \Pi}{\partial x(i)} = p(i) - c(i) - b' [(1 - e)x(i) + eX] = 0
\]

that leads to the optimal output of a single variety

\[
x(i) = \frac{a' - w\gamma(i) - 2b'eX}{2b'(1 - e)}
\]

\(^{12}\)Foreign variables are denoted by an asterisk throughout.
with $X \equiv \int_0^\delta x(i)di$ denoting total firm scale.\textsuperscript{13} The negative impact of total firm scale $X$ on the output of a single variety displays the cannibalization effect: $\frac{\partial x(i)}{\partial X} = -\frac{e}{(1-e)} < 0$. An MPF internalizes the effect that increasing output of a certain variety lowers prices for this, as well as, for all other varieties in the firm’s product range. This effect only exists if $e > 0$, i.e. if products are not perfectly differentiated. Furthermore, Eq. (9) shows that, given its total output, a firm produces less of each variety the further away it is from its core competence. Given the symmetric structure of demand, this means that a firm charges higher prices for products that are further away from its core competence (see Eckel and Neary (2010), p.193 for a detailed analysis).

In the next step, we consider the firm’s optimal choice of product line. MPF’s add new products as long as marginal profits are positive. Maximizing Eq. (7) with respect to scope implies the respective first-order condition\textsuperscript{14}

$$\frac{\partial \Pi}{\partial \delta} = [p(\delta) - c(\delta)]x(\delta) = 0. \tag{10}$$

From Eq. (8), we know that the profit on the marginal variety $[p(\delta) - c(\delta)]$ cannot be zero. The firm adds new varieties up to the point where the marginal cost of producing the marginal variety equals the marginal revenue at zero output. The profit maximizing product range implies that the output of the marginal variety $x(\delta)$ is zero. Using Eq. (9) and setting $x(\delta)$ equal to zero yields

$$c(\delta) = a' - 2b'eX. \tag{11}$$

Comparing Eqs. (9) and (11), we see that firms add new varieties to their product portfolio until optimal output of the marginal variety falls to zero. Inspecting Eq. (11) reveals the cannibalization effect which influences the scope of production: $\frac{\partial \delta}{\partial X} = -\frac{b'e}{\partial c(\delta)/\partial \delta} < 0$. Figure 1 illustrates the first-order condition for scope and determines the profit-maximizing product range.

Open Economy: So far, we have implicitly assumed that the offshoring costs $t$ were prohibitively high, so that all production was located in the home country. As globalization

\textsuperscript{13} The second-order condition of this maximization problem is: $\frac{\partial^2 \Pi}{\partial x(i)^2} = \frac{\partial p(i)}{\partial x(i)} - b'(1 - e) - b'e \frac{\partial X}{\partial x(i)} < 0$.

\textsuperscript{14} The second-order condition of this maximization problem is: $\frac{\partial^2 \Pi}{\partial \delta^2} = [p(\delta) - c(\delta)] \frac{\partial x(\delta)}{\partial \delta} < 0$, as $\frac{\partial c(\delta)}{\partial \delta} > 0$ and, thus, $\frac{\partial x(\delta)}{\partial \delta} = -\frac{1}{2b'(1-e)} \frac{\partial c(\delta)}{\partial \delta} < 0$. 


leads to improvements in information technology and reductions in communication costs, we analyze a decrease in the parameter $t$, which implies that firms can enjoy benefits of lower factor prices and thus gains from relocating labor intensive products to a low-wage location. In our model, the motive for offshoring is efficiency-seeking, which means that the necessary condition for offshoring is: $w^* < w$. The sufficient condition for offshoring is that the offshoring costs are below a critical value: $t < t_{\text{crit}}$. The critical value of offshoring costs can be calculated as

$$t_{\text{crit}} = \frac{(a' - 2b e X)(w - w^*)}{w w^*}. \quad (12)$$

It is straightforward to see that the critical value of offshoring costs is rising in the wage differential between Home and Foreign.

In the analysis below, we refer to cases in which offshoring cost are sufficiently low, so there is a fragmentation of production into domestic and foreign-produced varieties. We define $\delta$ as the cutoff variety. For variety $\delta$, the firm is indifferent concerning its optimal production location. Varieties with a lower labor input coefficient than $\delta$ are produced onshore, whereas varieties with a higher labor input coefficient are produced offshore. Combining Eqs. (6) and (9), gives the optimal scale of a foreign-produced variety:

$$x^*(i) = \frac{a' - w^* (\gamma(i) + t)}{2b'eX}. \quad (13)$$

Given that the marginal variety is produced in Foreign, the profit maximizing product range is defined by

$$w^* (\gamma(\delta) + t) = a' - 2b'eX. \quad (14)$$

In the open economy, an MPF faces a third maximization problem, next to optimal scale and scope of production. Now, the firm has also to determine the profit maximizing geographic location of production. Analogous to Eq. (7), total profits in the open economy are given by

$$\Pi = \int_0^{\delta} (p(i) - c(i)) x(i) di + \int_{\delta}^{\delta} (p^*(i) - c^*(i)) x^*(i) di, \quad (15)$$

with the first integral being total profits from domestic production and the second integral being the equivalent for foreign production. With Eq. (8) and total firm output $X$ being composed of domestically and foreign-produced goods as

$$X = \int_0^{\delta} x(i) di + \int_{\delta}^{\delta} x^*(i) di, \quad (16)$$

9
we can rearrange Eq. (15):

$$\Pi = (1 - e)b' \left[ \int_{\tilde{\delta}}^{\delta} x(i)^2 di + \int_{\delta}^{x^*(i)} d\tilde{i} \right] + b'eX^2.$$  \hspace{1cm} (17)

Maximizing Eq. (17) with respect to the optimal cutoff of production $\tilde{\delta}$ leads to

$$x(\tilde{\delta}) = x^*(\tilde{\delta}).$$  \hspace{1cm} (18)

Formal details of the derivation can be found in the Appendix.

**Lemma 1** An MPF chooses the optimal cutoff level of production $\tilde{\delta}$ exactly at that product where optimal scale in Home and in Foreign are the same. Combining Eqs. (9) and (13), this means that for variety $\tilde{\delta}$ the firm is just indifferent concerning the location of production because costs are identical, i.e.

$$w_\gamma(\tilde{\delta}) = w^*(\gamma(\tilde{\delta}) + t).$$  \hspace{1cm} (19)

To visualize our analysis, we illustrate the effects of falling offshoring costs in Figure 2. In Figure 2a), production of the whole portfolio is accomplished in Home as offshoring costs are prohibitively high. In Figure 2b), offshoring cost are below the critical value in Eq. (12). We observe that varieties $i \in [0; \tilde{\delta}]$ are still produced in Home, as their production is efficient enough, so the benefits of lower foreign wages do not prevail the offshoring costs. Production of varieties $i \in [\tilde{\delta}; \delta^{old}]$ is relocated, as these goods can be produced at a lower cost in Foreign. Products $i \in [\delta^{old}, \delta]$ constitute an extension of the firm’s product range. The MPF adds these varieties at the intra-firm extensive margin, whereby these goods would not be offered in case of producing exclusively in Home. The specification of our model suggests that an MPF produces exactly those varieties offshore, where its efficiency is relatively low.

![Insert Figure 2 about here](image-url)

We conclude this section with a graphical illustration of the main properties of our model in Figure 3. The graph portrays optimal scale of production for the entire portfolio across the two production locations. We will use this graph in the next section as a useful tool in the comparative statics analysis. Figure 3 shows that due to the underlying flexible manufacturing technology, output of the core competence is the highest. At the cutoff $\tilde{\delta}$
\( x(\delta) = x^*(\tilde{\delta}) \) the firm switches to foreign production. Therefore, the slope of the curve changes at this point. Finally, the profit maximizing product range is pinned down at \( x^*(\delta) = 0 \).

[Insert Figure 3 about here]

### 2.3 Comparative Statics

We still assume that \( t \) is below its critical value determined in Eq. (12), so the firm engages in foreign production. In the comparative statics, we analyze the effect of better prospects for offshoring on the geographic organization (optimal cutoff) and on the profit-maximizing product range. Furthermore, we investigate the impact of reduced costs of offshoring on the output of domestic and foreign-produced varieties, as well as on total firm output. These endogenous variables of our model \( x(i), x^*(i), \delta, X, \) and, \( \tilde{\delta} \) are determined in Eqs. (9), (13), (14), (16), and, (19) respectively. Totally differentiating this system of equations generates the comparative-static effects of decreasing offshoring costs \( t \).

Recent academic research on MPFs brings forth varying results on the effects of globalization on the product range of a firm. A set of papers, including Eckel and Neary (2010), Bernard et al. (2011), and Mayer et al. (2014) show that MPFs will reduce their product ranges in response to trade liberalization. Increased competition forces firms to drop their worst performing products. In Feenstra and Ma (2008), increasing the market size leads to an expansion of the product range. Very recently, Qiu and Zhou (2013) show that the most productive firms in an economy may expand their product scope after globalization. In this paper, we do not focus on the competition and market size effects of globalization. Globalization does also mean that access to foreign production locations is facilitated. Having the latter interpretation in mind, we can clearly show that the product scope increases in response to globalization.

**Proposition 1** If \( t \) is below the critical value determined in Eq. (12), falling offshoring costs induce an MPF to add new products at the intra-firm extensive margin, i.e.

\[
\frac{d \ln \delta}{d \ln t} = \frac{\Delta_2 t}{\Delta_1 \gamma'(\tilde{\delta}) \tilde{\delta}} < 0, \tag{20}
\]

where: \( \Delta_1 = (1 - e + e\delta) > 0 \) and \( \Delta_2 = \left(1 - e + e\tilde{\delta} \right) \).

This result can be visualized in Figure 2b). A decrease in \( t \) corresponds to a downward shift of the \( c^* \)-curve which indicates an extension of the product range.
In a next step, we want to discuss the effects of globalization on the domestic product range $\tilde{\delta}$. With respect to the large literature on international production, this aspect has been neglected so far in theoretical models. We find that better prospects for offshoring reduce the domestic product range and incentivize a firm to relocate marginal varieties.

**Proposition 2** Falling offshoring costs make foreign production more attractive and thus lead to an efficiency-seeking relocation of production from the high-wage country to the low-wage country, i.e.

$$\frac{d \ln \tilde{\delta}}{d \ln t} = \frac{w^* t}{(w - w^*) \gamma' \left(\tilde{\delta} \right) \tilde{\delta}} > 0. \quad (21)$$

As the wage rate in the home country $w$ is higher than abroad $w^*$, the expression is strictly positive. The magnitude of this effect can be shown to depend on the point elasticity of the cost curve at the marginal variety: $\epsilon_{\gamma}(\tilde{\delta}) \equiv \gamma' \left(\tilde{\delta} \right) \tilde{\delta} / \gamma \left(\tilde{\delta} \right)$. The latter stands for an inverse measure of flexibility of an MPF. High values of $\epsilon_{\gamma}(\tilde{\delta})$ imply that a change in $\delta$ will cause a large change in marginal costs. Hence, the change in the domestic product range following globalization will be smaller, the stronger domestic production costs react to a marginal decrease in $\tilde{\delta}$. To see this, we can rewrite Eq. (21) in $d \ln \tilde{\delta} / d \ln t = 1 / \epsilon_{\gamma}(\tilde{\delta})$ using the indifference condition in Eq. (19). In Figure 2b), a decrease in $t$ corresponds to a downward shift of the $c(i)\^*$-curve which is equivalent to shifting production abroad ($\tilde{\delta}$ falls). Former domestically produced goods are now produced abroad. Referring to previous discussion, the effect is less pronounced in the case of steep cost curves.

So far, we have analyzed within-firm adjustments at the intra-firm extensive margin. In the next step, we focus on the output profiles (intensive margin) of domestically and foreign-produced varieties. Following a fall in $t$, offshore production gets cheaper and, therefore, foreign varieties are produced at a larger scale.

**Proposition 3** If $t$ is below the critical value determined in Eq. (12), falling offshoring costs induce the firm to increase outputs of all foreign-produced varieties, i.e.

$$\frac{d \ln x^*(i)}{d \ln t} = -\frac{w^* t}{2b' \left(1 - e\right) x^*(i)} \frac{\Delta_2}{\Delta_1} < 0. \quad (22)$$

As an important feature in our model, we emphasize demand linkages between varieties in the product portfolio of a firm. Falling offshoring costs do not reduce domestic production costs but indirectly affect domestic output through the cannibalization effect. Rising output of foreign production crowds out domestic production as domestic varieties internalize the cannibalization effect.
Proposition 4  The cannibalization effect induces an MPF to reduce outputs of all domestically produced varieties in consequence of falling offshoring costs, i.e.

\[
\frac{d \ln x(i)}{d \ln t} = \frac{e(\delta - \bar{\delta})}{\Delta_1} \frac{w^* t}{2b'(1-e)x(i)} > 0.
\]  

(23)

In the case of perfectly differentiated varieties, i.e. \(e = 0\), domestic output is independent of foreign production and hence, the derivative in Eq. (23) is zero. With \(e\) being positive, varieties become substitutable and domestic output is crowded out by foreign production. However, it is straightforward to show that despite lower domestic output, total firm output \(X\) is increasing with falling offshoring cost. The positive impact of rising foreign output combined with the extension of the product range outweighs the negative impact of falling domestic output on total firm scale.

Proposition 5  With falling offshoring costs, an MPF increases total firm output because of the higher scale of foreign-produced varieties and the extension of the product portfolio, i.e.

\[
\frac{d \ln X}{d \ln t} = -\frac{w^* (\delta - \bar{\delta}) t}{2b' \Delta_1 X} < 0.
\]  

(24)

Formal details of all the derivations can be found in the Appendix. To illustrate the effects of falling offshoring costs, we draw on the graphical tool developed in Figure 3. In Figure 4, the dotted line represents the situation after the reduction in \(t\). Inspecting this graph reveals two negative effects on domestic production: A relocation effect from shifting production abroad and a cannibalization effect from rising foreign output. The latter effect is a new transmission channel specific to MPFs that we want to highlight. It results from the fact that with lower production costs abroad, output of foreign varieties and the foreign product range will increase. These intra-firm adjustments crowd out the production of domestic varieties which does not benefit from lower production costs abroad. The main comparative static results are indicated by the arrows in Figure 4.

[Insert Figure 4 about here]

2.4 Implications for the Measurement of Offshoring

From a theoretical point of view, the way we are thinking about offshoring as a relocation of production lines within MPFs is novel. However, the manner how offshoring is measured in
the broad empirical literature on international production is similar to our definition. The measure of outsourcing which is used in Feenstra and Hanson (1996) is directly related to our definition, as it includes also final goods next to imported intermediates. The authors argue that this "must be included in any valid measure of outsourcing" (Feenstra and Hanson (1996), p.107). Many other papers that discuss offshoring from an empirical perspective use measurements of offshoring that respond not only to a relocation of vertically related processes, but also respond to what we call multi-product offshoring. Papers such as Head and Ries (2002), Ebenstein et al. (2012), and Becker et al. (2013) measure offshoring activity in an industry by the total employment of foreign affiliates. Using employment in foreign affiliates as a measure for offshoring is perfectly in-line with our model. To underline that measuring offshoring like this could also mean the type of offshoring that we have in mind, we calculate the total employment in foreign affiliates and show how it responds to better offshoring opportunities. In industry $z$, labor demand $l^*$ for foreign-produced varieties is given by

$$ l^* (z) = \int_0^{\delta(z)} \frac{\delta(z)}{\delta(z)} \gamma(i) x^*(i)di. \tag{25} $$

It is determined by the scale and scope of foreign-produced varieties $i \in [\delta; \delta]$. We derive total labor demand in the offshore destination $L^*$ by integrating over all industries $z \in (0, 1)$

$$ L^* = \int_0^1 l^* (z) dz = \int_0^1 \int_0^{\delta(z)} \gamma(i, z) x^*(i, z) didz. \tag{26} $$

By substituting for $x(i)^*$ and evaluating the integral, we come up with the following equation

$$ L^* = \left( \delta - \tilde{\delta} \right) \left( (a^* - 2b^*eX - w^*t) \mu_{\gamma}^* - w^* \mu_{\gamma^*}^* \right) \frac{2b^*}{2b^* \left( 1 - e \right)}, \tag{27} $$

where $\mu_{\gamma}^* \equiv \frac{1}{\delta - \tilde{\delta}} \int_{\delta}^{\tilde{\delta}} \gamma (i) di$ is the mean labor input of foreign-produced varieties and $\mu_{\gamma^*}^* \equiv \frac{1}{\delta - \tilde{\delta}} \int_{\delta}^{\tilde{\delta}} \gamma (i)^2 di$ is the second moment around zero of the distribution of labor requirements. We totally differentiate Eq. (27) and analyze again the effects of better prospects for offshoring:

$$ \frac{d \ln L^*}{d \ln t} = - \frac{w^* t}{2b^* (1 - e) L^*} \left\{ \left( \frac{\delta - \tilde{\delta}}{\Delta_1} \right) \mu_{\gamma}^* \Delta_2 + \frac{\gamma (\delta) - \gamma (\tilde{\delta})}{(w - w^*) \epsilon_{\gamma(\tilde{\delta})}} \right\} < 0. \tag{28} $$

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The latter expression clearly indicates that the total employment of foreign affiliates is increasing in falling offshoring costs. Therefore, measuring offshore activity by total employment of foreign affiliates captures the type of offshoring that we have in mind.

Lemma 2 Falling offshoring costs increase total employment in the offshoring destination.

3 General Equilibrium

The previous section analyzed the effects of falling offshoring costs on the product range, per variety output, total firm output and the optimal location of production. Up to this point, the approach was partial, since we did not consider endogenous changes in wages. Our analysis in partial equilibrium clearly yields a fall in domestic production, because, on the one hand, per variety output of domestic varieties gets crowded out and, on the other hand, varieties close to the cutoff are relocated with falling offshoring costs. In the next steps, we focus on new insights into the labor market effects from offshoring which arise from the framework that we have presented so far. For this purpose, we introduce a simple labor market and show how domestic labor demand is affected by multi-product offshoring. Subsequently, we analyze again the comparative statics exercise of falling offshoring costs under consideration of labor market clearing.

3.1 Labor Market Clearing

In this section, we turn to the level of the economy as a whole and explore the general equilibrium effects of falling offshoring costs. To simplify the analysis, we assume that all industries are identical. In a first step, we need to specify how wages are determined. We assume a total labor supply $L^S$, that is supplied inelastically by the households in Home. Domestic labor demand in industry $z$ is given by

$$l(z) = \int_0^{\tilde{\delta}(z)} \gamma(i)x(i)di.$$  \hspace{1cm} (29)

It is determined by the scale and scope of domestically produced varieties $i \in [0; \tilde{\delta}]$. We derive total labor demand $L$ in our economy by integrating over all industries $z \in (0,1)$:

$$L = \int_0^1 l(z)dz = \int_0^{\tilde{\delta}(z)} \gamma(i,z)x(i,z)idz.$$  \hspace{1cm} (30)
Our main interest in this section is to determine the labor market effects of offshoring. In the previous section we have identified two effects of falling offshoring costs: A relocation effect and a cannibalization effect. The relocation effect affects the marginal variety $\delta$ and the cannibalization effect affects the scale of domestic production $x(i)$. Totally differentiating domestic labor demand in Eq. (30) with respect to $t$ yields:

$$\frac{d\ln L}{d\ln t} = \frac{\tilde{\delta}}{L} \left\{ \gamma(\tilde{\delta}) x(\tilde{\delta}) \frac{d\ln \tilde{\delta}}{d\ln t} + \mu'_\gamma x(i) \frac{d\ln x(i)}{d\ln t} \right\} > 0,$$

(31)

with $\mu'_\gamma \equiv 1/\tilde{\delta} \int_0^{\gamma} \gamma(i)di$ being the mean labor input of domestically produced varieties. The first part of Eq. (31) describes the relocation effect and the second part stands for the cannibalization effect. Latter effect is new and is specific to MPFs. With falling offshoring costs, scale of foreign production rises because of lower production costs abroad. This behavior cannibalizes domestic production and reduces domestic labor demand.

**Lemma 3** For a given domestic wage, falling offshoring costs reduce domestic demand for labor through two channels. A relocation effect leads to a shift of labor-intensive domestic products abroad. Furthermore, domestic production internalizes a cannibalization effect of rising foreign output and is crowded out.

In equilibrium, wages must adjust to ensure that total labor supply $L^S$ equals total labor demand determined by the cutoff of domestic production $\tilde{\delta}$ in all industries $z \in (0, 1)$. This is reflected by the following labor-market equilibrium condition for the home country:

$$L^S = \int_0^1 l(z) dz = \int_0^{\tilde{\delta}(z)} \gamma(i, z)x(i, z)didz.$$

(32)

We can now substitute for $x(i)$ and evaluate the integral to obtain

$$L^S = \frac{\tilde{\delta} \left[ (a' - 2b'eX) \mu'_\gamma - w\mu''_\gamma \right]}{2b'(1-e)},$$

(33)

where $\mu''_\gamma \equiv 1/\tilde{\delta} \int_0^{\gamma} \gamma(i)^2 di$ stands for the second moment around zero of the distribution of labor requirements. Combining Eq. (33) with the system of equations from the analysis in partial equilibrium, we can use the respective equations for investigating how firm-level

\[\text{From inspection of propositions 2 and 4, we know that: } \frac{d\ln \tilde{\delta}}{d\ln t} > 0 \text{ and } \frac{d\ln x(i)}{d\ln t} > 0.\]
adjustments respond to declining offshoring costs with endogenous wages. We derive the comparative statics results by totally differentiating all equations of the system. Formal details of all the derivations can be found in the Appendix.

3.2 Comparative Statics in General Equilibrium

One important issue in general equilibrium which we want to analyze in the first place, is the effect of better prospects for offshoring on domestic factor prices $w$. In the previous sections, we have identified two negative impacts of offshoring on domestic labor demand: The relocation and the cannibalization effect. However, in equilibrium, total labor supply must equal total demand for labor. To ensure this equality, domestic wages must fall.

**Proposition 6** With falling offshoring costs, ceteris paribus, foreign production gets more attractive. To ensure labor market clearing in equilibrium, there are adjustments on the labor market in the form of falling domestic wages, i.e.$^{16}$

$$
\frac{\Delta w}{w^* t} \frac{d \ln w}{d \ln t} = \left\{ e (w - w^*) \gamma' \left( \tilde{\delta} \right) \left( \delta - \tilde{\delta} \right) \delta \mu' + \Delta_1 w^* \left[ \gamma(\delta) - \gamma \left( \tilde{\delta} \right) \right] \gamma \left( \tilde{\delta} \right) \right\} > 0. \tag{34}
$$

Considering these labor market adjustments reveals that in general equilibrium falling offshoring costs not only make foreign production cheaper but also reduce production costs in the home country. The latter has important implications on the main variables of interest in our model, which we point out in the following.

With lower production costs in both countries, it is apparent that an MPF will increase its total scale:

$$
\frac{d \ln X}{d \ln t} = - \frac{w^* (\delta - \tilde{\delta}) t}{2b' \Delta_1 X} - \frac{w^* \delta \mu'_*}{2b' \Delta_1 X} \frac{d \ln w}{d \ln t} < 0. \tag{35}
$$

The mathematical derivation and an expression where the change in wages is substituted can be found in the Appendix. Eq. (35) is the general equilibrium equivalent of Eq. (24). Comparing both equations immediately points out that due to the adjustment of factor prices (represented by the second fraction), the general equilibrium effect will be of greater magnitude than the partial equilibrium effect (represented by the first fraction). Within our framework, a larger firm scale $X$ enhances cannibalization between varieties. Caused by falling domestic wages, the latter effect leads to a new channel that we have to consider when analyzing the repercussions of falling offshoring costs on the product range of a firm.

$^{16}$ The term $\Delta = \left\{ (1 - e) + e (\delta - \tilde{\delta}) \delta \mu'_* + e \delta^2 \sigma^2 \right\} (w - w^*) \gamma' \left( \tilde{\delta} \right) + \Delta_1 \gamma \left( \tilde{\delta} \right) w^* \left[ \gamma(\delta) - \gamma \left( \tilde{\delta} \right) \right] \gamma \left( \tilde{\delta} \right)$ is the determinant of the system of equations. It is positive which ensures that the equilibrium is unique and stable.
We illustrate this channel in the following equation:

\[
\frac{d \ln \delta}{d \ln t} = -\frac{\Delta_2 t}{\Delta_1 \gamma'(\delta) \delta} + \frac{e \tilde{w} \mu_y'}{\Delta_1 w^* \gamma'(\delta) \delta} \frac{d \ln w}{d \ln t} < 0,
\] 

(36)

where the first part of Eq. (36) represents the partial effect which is clearly of a negative sign. The second part of Eq. (36) is the additional channel in general equilibrium arising from the adjustment of wages. This effect is positive and, therefore, works in the opposite direction as it induces the firm to increase its total output \(X\). Inspecting Eq. (36) reveals that the general equilibrium effect is switched off for \(e\) being zero. With products being perfectly differentiated, there is no cannibalization of the rising total firm output \(X\) on the marginal varieties within the product range. However, we can analytically show that the result from partial equilibrium is reconfirmed even for \(e > 0\). Therefore, the adjustments in general equilibrium only have a dampening effect on the product range which is driven by the intensity of cannibalization determined by the differentiation parameter \(e\). A proof for this result is provided in the Appendix.

**Proposition 7** Falling offshoring costs reduce costs in both production sites and hence enlarge total firm output \(X\) to a larger extend compared to partial equilibrium. Latter result dampens but does not reverse the effect of falling offshoring costs on the product range in general equilibrium. The dampening effect depends on the strength of cannibalization.

In the next step, we focus our attention on the optimal geographic organization of an MPF. Regarding the optimal cutoff of production, we identify two opposing forces in general equilibrium following a fall in the parameter \(t\). On the one hand, there is the direct effect of lower offshoring costs which tends to shift production abroad (observed effect in partial equilibrium, see Eq. (21)). On the other hand, we find decreasing domestic wages which brings forth an incentive to pull back production into the home country. The latter causes an ambiguity on the total effect of falling offshoring costs in general equilibrium which can be seen in the following derivative:

\[
\frac{\Delta}{w^* t} d \ln \tilde{\delta} = \Delta_1 \mu''_y - e \left[ \left( \delta - \tilde{\delta} \right) \gamma \left( \delta \right) + \tilde{\delta} \mu'_y \right] \mu'_y \geq 0.
\]

(37)

We now focus on this ambiguity and investigate the causes that lie behind it. To begin with, Eq. (37) is positive for \(e\) being zero. With perfectly differentiated products, domestic varieties do not internalize cannibalization through rising outputs of foreign varieties (compare Eq. (23)). Thus, there is no reducing force on domestic labor demand via a lower scale of domestically produced varieties (the cannibalization effect of offshoring, stressed in
Eq. (31)). Thereby, to ensure labor market clearing, domestic wages will decline less and the wage-effect will not dominate the better opportunities to offshore. With $0 < e < 1$, there is the possibility that Eq. (37) gets negative, i.e. with falling offshoring costs even more products are produced in Home. This happens if the general equilibrium adjustment of factor prices prevails the foreign cost reduction via lower offshoring costs.

To get some further intuition for this ambiguity, we investigate the effect of an exogenous change in domestic wages on domestic output. Differentiating optimal scale $x(i)$ in Eq. (9) with respect to the wage rate $w$ yields:

$$\frac{d \ln x(i)}{d \ln w} = \frac{w}{2b'(1-e)x(i)} \left[ \frac{e\ddot{\gamma} - \Delta_1}{\Delta_1} \right] \leq 0. \quad (38)$$

The algebraic sign of Eq. (38) behaves ambiguously. Outputs of varieties with a labor input coefficient $\gamma(i)$ far below the average $\mu'_\gamma$ may even fall with falling wages (i.e. for these varieties $\frac{d \ln x(i)}{d \ln w} > 0$). The reason for this is that varieties which are very efficient in production require just sparse labor input and hence, benefit slightly from falling wages. However, these varieties fully internalize the cannibalization effect through rising outputs of labor-intensive varieties which benefit a lot from lower factor prices.\(^{18}\) Latter results imply that varieties benefit more from falling wages the higher is their respective labor input. These insights are important features of our model which can help to explain the ambiguous effects of lower offshoring costs on the cutoff variety $\ddot{\delta}$.

From the total derivatives of our system of equations, we obtain two equations in $\frac{d \ln \ddot{\delta}}{d \ln t}$ and $\frac{d \ln w}{d \ln t}$, given by:

$$\frac{d \ln \ddot{\delta}}{d \ln t} = \frac{w't}{(w - w^*)} \gamma'(\ddot{\delta}) \ddot{\delta} - \frac{w'_\gamma(\ddot{\delta})}{(w - w^*)} \gamma'(\ddot{\delta}) \ddot{\delta} \frac{d \ln w}{d \ln t} \geq 0 \quad (39)$$

and

$$\frac{d \ln \ddot{\delta}}{d \ln t} = -\frac{w^t(\delta - \ddot{\delta})}{\Delta_1 \Delta_3 \gamma(\ddot{\delta})} e\mu'_\gamma + \frac{w}{\Delta_3 \gamma(\ddot{\delta})} \left( \mu'' - \frac{e\ddot{\gamma} \mu'^2}{\Delta_1} \right) \frac{d \ln w}{d \ln t} \geq 0. \quad (40)$$

---

\(^{17}\)The interested reader finds the effects of an exogenous change in domestic wages on all endogenous variables in the Appendix.

\(^{18}\)The condition for the output of the core competence to fall with falling wages is: $\gamma(0) < \frac{e\ddot{\gamma} \mu'_\gamma}{\Delta_1}$. The cutoff variety $\ddot{\delta}$ has the highest labor input coefficient $\gamma(\ddot{\delta})$ in the domestic product range. The output of this variety $x(\ddot{\delta})$ rises with falling wages: $\frac{d \ln x(\ddot{\delta})}{d \ln w} < 0$.

\(^{19}\)\(\Delta_3 = \left( (a' - 2b'eX) - w\gamma(\ddot{\delta}) \right) > 0\). From the first-order condition of scope it becomes obvious that this expression is positive.
Eq. (39) follows immediately from the determination of the profit maximizing cutoff in Eq. (19). Eq. (40) is derived after some mathematical conversion from the labor market clearing condition in Eq. (33). Inspecting Eq. (39) reveals that the partial equilibrium result (the first part of the expression) is more likely to be reversed, the higher the adjustment in wages is weighted. From the analysis of Eq. (38), we know that varieties with high labor inputs will benefit more from reductions in factor prices. This insight can be reapplied to Eq. (39), where we observe the wage effect to be of greater impact, the higher is the labor input at the marginal variety $\gamma \left( \tilde{\delta} \right)$. By analogy, we apply this intuition to Eq. (40), where it becomes apparent that the higher is the mean labor input of domestic production $\mu'_\gamma$, the more likely the domestic wage reduction outweighs the cost advantage through lower offshoring costs. We summarize these insights in the following proposition.

**Proposition 8** In partial equilibrium, lower offshoring costs $t$ lead to a distinct fall in $\tilde{\delta}$ (i.e. $\frac{d \ln \tilde{\delta}}{d \ln t} > 0$). This result does not necessarily hold in general equilibrium which implies that it is possible that even more products are produced onshore with better opportunities of offshoring (i.e. $\frac{d \ln \tilde{\delta}}{d \ln t} < 0$). This ambiguity is caused by the general equilibrium result of falling domestic wages. We show that the result in partial equilibrium is more likely to be reversed, the higher are the benefits of falling wages in the domestic production.

We conclude this section by illustrating the ambiguity on $\tilde{\delta}$ in a $\left\{ \frac{d \ln \tilde{\delta}}{d \ln t}, \frac{d \ln w}{d \ln t} \right\}$ space. Figure 5 illustrates Eqs. (39) and (40). For Eq. (39), the slope is clearly negative, whereas the slope of Eq. (40) depends on the sign of $\left( \mu''_\gamma - \frac{\epsilon \tilde{\delta}(\mu'_\gamma)^2}{\Delta_1} \right)$.

We take away from the graph that the more do domestic wages respond to changes in the offshoring costs, the more likely is a result contrary to the partial equilibrium case (an intersection of the two curves below the x-axis). By all means $\frac{d \ln \tilde{\delta}}{d \ln t} < 0$, for $\mu''_\gamma \leq \frac{\epsilon \tilde{\delta}(\mu'_\gamma)^2}{\Delta_1}$, which implies Eq. (40) to be horizontal or to be negatively sloped. If $\mu''_\gamma > \frac{\epsilon \tilde{\delta}(\mu'_\gamma)^2}{\Delta_1}$, Figure 5 illustrates that the algebraic sign of $\frac{d \ln \tilde{\delta}}{d \ln t}$ can be both negative or positive.

In the Appendix, we provide a numerical simulation of our model where we show that in fact, the result in partial equilibrium can be reversed in general equilibrium. Assuming specific parameter values and a linear cost function, we are able to document that there are cases in which $\frac{d \ln \tilde{\delta}}{d \ln t} < 0$. 

[Insert Figure 5 about here]
4 Conclusion

Although globalization of production has been discussed extensively in the literature, there is not yet a framework to study the relocation of whole varieties within the boundaries of a firm. In this chapter, we show that the relocation of entire production lines leads to new insights into the labor market outcomes of offshoring. Reversing the assumptions that processes within a firm are vertically related and that part of the production of a variety stays in the home country we have highlighted new multi-product specific transmission channels of offshoring. We set up a general oligopolistic equilibrium model of MPFs and offshoring, which allows us to study the consequences of globalization in the sense of declining costs of offshoring. We show that better prospects for offshoring affect the geographic organization and the product range of an MPF. Giving a firm the opportunity to offshore the production of labor-intensive products will lead to a broader product range. Considering the offshoring impacts on domestic employment, we highlight the cannibalization effect of foreign on domestic output, which hits domestic employment next to the well established relocation effect. Having wages endogenized, our model suggests ambiguous tendencies on the cutoff of production. The more do domestic wages respond to changes in offshoring costs and the higher are the benefits from lower wages in domestic production, the more likely is an even extended domestic production in an economy with increasing globalization. Therefore, our model is able to predict patterns in which firms re-relocate entire product lines following globalization and a decline in offshoring costs.

One issue we did not consider in our model is welfare of workers. As our specification considers domestic production only and consumption takes place on a third market, workers suffer from declining wages and do not benefit from lower prices of final goods. Due to this construction it does not make sense to assess welfare as we can not make any statements concerning the real wages in our model.
References


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5 Appendix

5.1 Proof of Lemma 1

In the open economy scenario, an MPF has to determine the profit maximizing geographic location of production. In the following, we will sketch this maximization problem. From the first-order condition for scale in Eq. (8), we know:

\[ p(i) - c(i) = b'(1 - e)x(i) + b'eX, \]  

(A1)

which inserted in the open economy total profits in Eq. (15) leads to

\[ \frac{\Pi}{b'} = (1 - e) \int_0^\delta x(i)^2 di + eX \int_0^\delta x(i) di + (1 - e) \int_\delta^{\tilde{\delta}} x^*(i)^2 di + eX \int_\delta^{\tilde{\delta}} x^*(i) di. \]  

(A2)

Given that \[ X = \int_0^{\tilde{\delta}} x(i) di + \int_\delta^{\tilde{\delta}} x^*(i) di, \] we derive Eq. (17). To identify a condition for an optimally chosen cutoff variety \( \tilde{\delta} \), we maximize Eq. (17) with respect to \( \tilde{\delta} \). This implies the following first-order condition:

\[ \frac{1}{b'} \frac{d\Pi}{d\tilde{\delta}} = (1 - e) \left[ \int_0^\delta 2x(i) \frac{dx(i)}{d\tilde{\delta}} di + \int_\delta^{\tilde{\delta}} 2x^*(i) \frac{dx^*}{d\tilde{\delta}} di \right] \]  

(A3)

\[ + (1 - e) \left[ x(\tilde{\delta})^2 - x^*(\tilde{\delta})^2 \right] + (1 - e)x^*(\tilde{\delta}) \frac{d\delta}{d\tilde{\delta}} + 2eX \frac{dX}{d\tilde{\delta}} = 0. \]

With \( x^*(\delta) = 0 \), \( \frac{dx(i)}{d\delta} = \frac{dx^*(i)}{d\tilde{\delta}} = -\frac{e}{1 - e} \frac{dx}{d\tilde{\delta}} \), and some mathematical conversion, we derive

\[ \frac{1}{b'} \frac{d\Pi}{d\tilde{\delta}} = (1 - e) \left[ x(\tilde{\delta})^2 - x^*(\tilde{\delta})^2 \right] = 0. \]  

(A4)

5.2 Comparative Statics in Partial Equilibrium

In the following, we show how to derive the comparative static results of the model. In our model, the equilibrium is determined by the following system of equations:

\[ w_\gamma(\tilde{\delta}) = w^*(\gamma(\tilde{\delta}) + t) \]  

(A5)
\[
x(i) = \frac{a' - w\gamma(i) - 2b'eX}{2b'(1 - e)} \quad (A6)
\]

\[
x^*(i) = \frac{a' - w^*(\gamma(i) + t) - 2b'eX}{2b'(1 - e)} \quad (A7)
\]

\[
X = \int_0^\tilde{\delta} x(i)di + \int_{\tilde{\delta}}^{\delta} x^*(i)di \quad (A8)
\]

\[
w^*(\gamma(\delta) + t) = a' - 2b'eX \quad (A9)
\]

We can reduce this system of equations to two equations in \(\tilde{\delta}\) and \(\delta\). In a first step, we substitute Eqs. (A6) and (A7) in Eq. (A8) and derive total output as

\[
X = \frac{1}{2b'\Delta_1} \left\{ a'\delta - w \int_0^{\tilde{\delta}} \gamma(i)di - w^* \left[ \int_{\tilde{\delta}}^{\delta} \gamma(i)di + t \left( \delta - \tilde{\delta} \right) \right] \right\}. \quad (A10)
\]

In a second step, we combine the latter expression with Eq. (A9) which leads to

\[
w^*(\gamma(\delta) + t) = a' - \frac{e}{\Delta_1} \left\{ a'\delta - w \int_0^{\tilde{\delta}} \gamma(i)di - w^* \left[ \int_{\tilde{\delta}}^{\delta} \gamma(i)di + t \left( \delta - \tilde{\delta} \right) \right] \right\}. \quad (A11)
\]

Eqs. (A5) and (A11) constitute two equations in two endogenous variables: \(\tilde{\delta}\) and \(\delta\). By totally differentiating this system of equations, we derive our results in partial equilibrium. We show the total derivatives of Eqs. (A5) and (A11) in the next section of this Appendix.

### 5.3 Comparative Statics in General Equilibrium

In general equilibrium, we add the labor market clearing condition to our system of equations from the previous section. By substituting Eq. (A9) into the labor market clearing condition in Eq. (33), we derive

\[
L = \frac{(w^*(\gamma(\delta) + t))\tilde{\delta}\mu'_\gamma - w\tilde{\delta}\mu''_\gamma}{2b'(1 - e)}. \quad (A12)
\]

The combination of Eqs. (A5), (A11), and (A12) determines the general equilibrium of our model. In the total derivatives, we take into account that domestic wages are endogenously determined in the domestic labor market. For deriving the following results, note
that $\frac{d}{d\delta}(\delta\mu'_{\gamma}) = \gamma(\tilde{\delta})$ and $\frac{d^2}{d\delta^2}(\delta\mu''_{\gamma}) = \gamma(\tilde{\delta})^2$. Totally differentiating the three equilibrium conditions Eqs. (A5), (A11), and (A12), with the results written as a matrix equation, we can analyze a change in the offshoring cost $t$ as follows:

$$\begin{bmatrix}
0 & (w - w^*) \gamma'(\tilde{\delta}) & \gamma(\tilde{\delta}) \\
\Delta_1 & 0 & -e\delta\mu'_{\gamma} \\
\tilde{\delta}\mu'_{\gamma} & w^* [\gamma(\delta) - \gamma(\tilde{\delta})] & \gamma(\tilde{\delta}) & -\tilde{\delta}\mu''_{\gamma}
\end{bmatrix}
\begin{bmatrix}
\gamma'(\delta) d\ln\delta \\
\gamma(\delta) d\ln\delta \\
\delta d\ln\delta \\
\gamma(\delta) d\ln\delta
\end{bmatrix}
= \begin{bmatrix} 1 \\
-\Delta_2 \\
-\tilde{\delta}\mu'_{\gamma}
\end{bmatrix}. \quad (A13)$$

The terms $\Delta_1$ and $\Delta_2$ are defined in Eq. (20) and are strictly positive. Using $\sigma^2_{\gamma} = \mu''_{\gamma} - \mu'_{\gamma}^2$, we can show that the determinant of the coefficient matrix $\Delta$ is positive:

$$\Delta = \left\{ (1 - e) + e(\delta - \tilde{\delta}) \tilde{\delta}\mu''_{\gamma} + e\tilde{\delta}^2\sigma^2_{\gamma} \right\} (w - w^*) \gamma'(\tilde{\delta}) + \Delta_1 \gamma(\tilde{\delta})^2 w^* [\gamma(\delta) - \gamma(\tilde{\delta})] > 0. \quad (A14)$$

In the following, we provide the solutions of the comparative statics exercise which we use in the general equilibrium part of our model.

**Effect on Domestic Wages:**

$$\frac{wd\ln w}{w^*td\ln t} = \frac{1}{\Delta} \left\{ (w - w^*) \gamma'(\tilde{\delta}) e(\delta - \tilde{\delta}) \tilde{\delta}\mu'_{\gamma} + \Delta_1 w^* [\gamma(\delta) - \gamma(\tilde{\delta})] \gamma(\tilde{\delta}) \right\} > 0 \quad (A15)$$

**Effect on Product Range:**

$$\frac{\gamma'(\delta) d\ln\delta}{td\ln t} = \frac{1}{\Delta} \left\{ 1 \begin{bmatrix} 1 & (w - w^*) \gamma'(\tilde{\delta}) & \gamma(\tilde{\delta}) \\
-\Delta_2 & 0 & -e\delta\mu'_{\gamma} \\
-\tilde{\delta}\mu'_{\gamma} & w^* [\gamma(\delta) - \gamma(\tilde{\delta})] & \gamma(\tilde{\delta}) & -\tilde{\delta}\mu''_{\gamma}
\end{bmatrix} \gamma'(\delta) d\ln\delta \right\} < 0 \quad (A17)$$
Effect on Total Output: Totally differentiating Eq. (A10) and using information from Eq. (A16) yields

\[
\frac{2b'\Delta_1 X}{w^*t} \frac{d \ln X}{d \ln t} = -\left(\delta - \bar{\delta}\right) - \frac{\bar{\delta} \mu'_\gamma}{\Delta} \left\{ (w - w^*) \gamma'(\bar{\delta}) e\left(\delta - \bar{\delta}\right) \bar{\delta} \mu'_\gamma + \Delta_1 w^* \left[ \gamma(\delta) - \gamma(\bar{\delta}) \right] \gamma(\bar{\delta}) \right\} < 0.
\] (A19)

Effect on Cutoff Variety:

\[
\frac{\delta d \ln \bar{\delta}}{w^* td \ln t} = \frac{1}{\Delta} \left| \begin{array}{ccc} 0 & 1 & \gamma(\bar{\delta}) \\ \Delta_1 & -\Delta_2 & -e\bar{\delta} \mu'_\gamma \\ \bar{\delta} \mu'_\gamma & -\bar{\delta} \mu'_\gamma & -\bar{\delta} \mu''_\gamma \end{array} \right|
\] (A20)

Using again \(\sigma^2 = \mu''_\gamma - \mu'^2_\gamma\), we derive the following result:

\[
\frac{\ln \bar{\delta}}{w^* td \ln t} = \frac{1}{\Delta} \left( \Delta_1 \mu''_\gamma - e \left[ \left(\delta - \bar{\delta}\right) \gamma(\bar{\delta}) + \bar{\delta} \mu'_\gamma \right] \mu'_\gamma \right) \geq 0.
\] (A21)

5.4 Effects of an Exogenous Change in Domestic Wages

This section keeps offshoring costs \(t\) constant and considers responses of the system of endogenously determined variables in Eqs. (9), (13), (14), (16), and (19) to changes in the domestic wage rate. Totally differentiating this system of equations generates the following comparative statics results. It is apparent that with falling domestic wages total firm output will increase, i.e.

\[
\frac{d \ln X}{d \ln w} = -\frac{w \bar{\delta} \mu'_\gamma}{2b'\Delta_1 X} < 0.
\] (A22)

This effect gets larger the more domestic varieties benefit from falling wages, i.e. the higher is \(\mu'_\gamma\), and the more domestic varieties are produced onshore, i.e. the higher is \(\bar{\delta}\).

Changes in domestic factor prices clearly affect the cutoff variety \(\bar{\delta}\) as it is determined by the equality of production costs on- and offshore. With Home becoming a more attractive production site, more varieties will be manufactured domestically, i.e.

\[
\frac{d \ln \bar{\delta}}{d \ln w} = -\frac{w}{(w - w^*) \gamma'(\bar{\delta})} \frac{\gamma(\bar{\delta})}{\bar{\delta}} < 0.
\] (A23)

Akin to the previous result, we find that this effect gets stronger the more the cutoff variety benefits from falling wages in terms of a higher marginal labor requirement \(\gamma(\bar{\delta})\).
varieties not being perfectly differentiated (i.e. \( e > 0 \)), foreign scale gets crowded out

\[
\frac{d \ln x^*(i)}{d \ln w} = \frac{w}{2b'(1-e)x^*(i)} \frac{\tilde{e} \mu'_\gamma}{\Delta_1} > 0,
\]

and the product range decreases as marginal varieties undergo cannibalization

\[
\frac{d \ln \delta}{d \ln w} = \frac{\tilde{e} \delta w \mu'_\gamma}{\Delta_1 w^* \gamma'(\delta) \delta} > 0.
\]

The cannibalization effect becomes stronger the more domestic production benefits from falling wages (i.e. the higher \( \tilde{\delta} \) and \( \mu'_\gamma \)). Figure 6 illustrates all effects.

### 5.5 Numerical Example with a Linear Cost Function

In this section, we round down our analysis in general equilibrium with a numerical simulation, where we focus on the ambiguity of the effect of falling offshoring costs \( t \) on the cutoff variety \( \tilde{\delta} \). For specific parameter values and a linear cost function, Table 1 summarizes results for different degrees of product differentiation. Results once again underline the issue of cannibalization in this framework. We observe a falling total firm output \( X \) and a falling product range \( \delta \) with rising substitutability between varieties (higher values of \( e \)). Referring to proposition 8 in the main body, it is important to mention that Table 1 shows a specific case where partial equilibrium results with respect to the cutoff variety \( \tilde{\delta} \) get reversed in general equilibrium, i.e. \( \frac{d \tilde{\delta}}{d t} < 0 \). In this parameterization with an underlying linear cost function, we find more varieties being produced onshore with falling offshoring costs. As explained before, this result is due to the prevailing effect of falling domestic wages in comparison to the better prospects for offshoring.

<table>
<thead>
<tr>
<th>Product differentiation ( e )</th>
<th>( w )</th>
<th>( X )</th>
<th>( \delta )</th>
<th>( \tilde{\delta} )</th>
<th>( \frac{d \tilde{\delta}}{d t} )</th>
<th>( \frac{d \delta}{d t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>8.70</td>
<td>121.73</td>
<td>1.36</td>
<td>22.13</td>
<td>-0.688</td>
<td>-0.495</td>
</tr>
<tr>
<td>0.5</td>
<td>8.14</td>
<td>36.13</td>
<td>1.77</td>
<td>8.86</td>
<td>-1.081</td>
<td>-0.003</td>
</tr>
<tr>
<td>0.9</td>
<td>9.01</td>
<td>22.96</td>
<td>1.14</td>
<td>2.91</td>
<td>-0.370</td>
<td>-0.143</td>
</tr>
</tbody>
</table>

Notes: Parameter values are: \( a' = 100, b' = 2, L^W = 20, w^* = 3.5 \), and \( t = 2.5 \).

For this calculation, we assume a linear cost function: \( \gamma(i) = \gamma_0 + \gamma_1 i = 1 + 0.5i \).

**Lemma 4** By assuming a linear cost function within this framework, we can show that there is the possibility that an MPF produces even more varieties domestically when it faces better prospects for offshoring.
Figure 1: Profit Maximizing Product Range

Figure 2: Effects of Falling Offshoring Costs
Figure 3: Output Schedule

Figure 4: Output Schedule and Comparative Statics
Figure 5: Cutoff Variety in General Equilibrium

Figure 6: Exogenous Decrease in Domestic Wages