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Multi-Product Firms, Endogenous Sunk Costs, and Gains from Trade through Intra-Firm Adjustments

Multi-Product Firms, Endogenous Sunk Costs, and Gains from Trade through Intra-Firm Adjustments[*]

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Abstract

In this paper, I investigate welfare gains associated with trade induced intra-firm adjustments of multi-product firms. To disentangle the welfare gains, I split up the R&D portfolio of a multi-product firm into three different channels: i) product innovation, ii) investments in the degree of product differentiation, and iii) process innovation. Trade integration enables firms to exploit economies of scale as innovation requires upfront development costs and encourages firms to spend more on R&D. I derive the indirect utility function and show that consumers benefit from this behavior through a larger product range (love of variety) which is also more differentiated (love of diversity). Furthermore, a larger market is associated with technology upgrading. The resulting cost savings are passed on to consumers, leading to welfare gains from lower prices.

Keywords: International Trade, Multi-Product Firms, Gains from Trade, R&D, Cannibalization Effect, Product Differentiation

JEL Classification: F12, F61, L25

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1 Introduction

In 1942, Joseph Schumpeter argued that innovation activity is carried out by large firms, for whom R&D is endogenous. R&D projects often go hand in hand with high development costs and, therefore, a sufficiently large scale of firm sales is required to cover these costs. Trade liberalization increases the effective size of the market which induces innovation activities through economies of scale. These findings are validated by recent empirical studies. For Canadian and Argentinian firms, Lileeva and Trefler (2010) and Bustos (2011) document that reductions in tariffs lead to investments in productivity-enhancing activities by exporting firms. After trade liberalization, exporters who benefit from the larger market are more technology intensive than nonexporters.

Recent contributions in international trade emphasize the fact that most industries are dominated by firms that produce more than one product.[1] In this paper, I address the R&D portfolio of a multi-product firm (MPF) whereby the focus is to analyze separately different types of research. Firms may invest in product innovation and product differentiation besides investments in production processes. Unbundling these different strands of innovation helps to distinguish between different welfare channels. In contrast to models with single-product firms where gains from trade originate at the industry-level through entry or exit of firms and "between-firm" reallocations of market shares, I highlight intra-firm adjustments as a source for welfare improvements.

Globalization increases the market but also reinforces competition in these markets. The large literature on heterogeneous firms has shown that the latter effect dominates for low-performing firms. I consider large MPFs, therefore I focus on the market size effect of globalization. Rising sales volumes in a larger market raise the returns to the different types of innovation through economies of scale. In my model, a firm weighs the marginal benefit of each type of innovation against the fixed upfront development costs and as the marginal benefit of innovating is increasing in the market size, more investments are encouraged.

A main element of my theory are demand linkages stressed in recent contributions to the international trade literature on MPFs (see for instance: Eckel and Neary (2010) and Dhingra (2013)). In these papers, firms internalize a cannibalization effect when introducing additional varieties to their product portfolio. This means, if varieties within an MPF are horizontally differentiated, adding a new product will create a negative demand externality on all other products of this firm. In my framework, the innovating firm can dampen this negative externality of product innovation by investing in the degree of product differentia-

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[1]Bernard et al. (2010) report the dominance of MPFs. Although MPFs represent a minority of 39 percent of firms, these firms account for 87 percent of output. In a trade context, Bernard et al. (2007) document for the year 2000 that firms that export multiple products account for 99.6 percent of export value.

tion. It is natural to assume that the strength of the cannibalization effect depends on the substitutability of products within the product range of a firm. Adding products differing only slightly from each other, will have a strong cannibalizing impact on existing varieties. However, a product range that spans products which are highly differentiated is less susceptible to cannibalization. To avoid cannibalization among products, firms can invest in new blueprints or product specific attributes such as differences in functional features or design. Furthermore, promotion activities such as advertisement or marketing campaigns help to showcase the differences between products. All these measures come along with fixed costs, however they are implemented to satisfy the consumers desire to choose from a broad and diversified product range.

The key result of the model is that a larger market or trade cost reductions enhance the profit maximizing product range of an MPF and optimal spending in both product differentiation and process innovation. An MPF that widens its product range loses market shares of its existing products through cannibalization. This makes additional spending in product differentiation worthwhile. Furthermore, sunk costs for product differentiation and process innovation are determined endogenously and depend on the level of investment and not on scale and scope of production. Thus, a rising market size enables firms to exploit economies of scale in innovation and gives rise to increasing optimal investment levels as investment costs can be spread over more units of output. Beyond this, I show that returns to both product differentiation and process innovation do not just depend on the size of the market but also on the efficiency of research input utilization. The Global Innovation Index (2013) reports disparities and persistent innovation differences among regions.[2] This is indicative of differences in the scope for product differentiation and the opportunities to reduce production costs between firms in different industries or in developed and less developed countries. The more efficient research input is transformed into research output, the higher will be the equilibrium investment levels and the larger will be the adjustments to globalization. This insight is important to keep in mind when discussing consumers welfare in the context of my model.

On the demand side, I specify quadratic preferences à la Melitz and Ottaviano (2008) and compute the indirect utility function as an appropriate measure for welfare. Consumers benefit from more variety (*love of variety*), *lower prices,* and, notably, from the degree of product differentiation. I refer to this property of the utility function as *love of diversity.* The latter means that consumers value a given product range more when products are more

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[2]The Global Innovation Index is published by the business school INSEAD and the World Intellectual Property Organization (WIPO), a specialized agency of the United Nations. It ranks 141 economies on the basis of their innovation capabilities and results.

differentiated or rephrasing it, the marginal utility of each newly introduced product is increasing in the degree of product differentiation. Having disentangled these three individual welfare channels, I discuss the gains from trade liberalization arising from intra-firm adjustments. Globalization induces an MPF to enlarge and diversify its product range. Given the *love of variety* and *love of diversity* properties of the utility function, this improves consumer welfare. Furthermore, a larger market is associated with technology upgrading. The resulting cost savings are passed on to consumers, leading to welfare gains from lower prices. However, as indicated above, I show that the gains from trade depend on the efficiency of the investments. Conducting the thought experiment of firms innovating in two different scenarios - a developed and a less developed country -, I argue that trade liberalization will improve welfare more when innovation input is converted efficiently in valuable output.

This model is related to the growing trade literature on MPFs with quadratic preferences for differentiated varieties. In Eckel et al. (2011), MPFs invest in the quality of their products. Because of the assumption of flexible manufacturing, firms will invest most in their core product which is sold at the largest scale. Dhingra (2013) analyzes the impact of trade policy on product and process innovation, keeping the degree of product differentiation exogenous. Similar to my model, economies of scale increase optimal spending for process innovation. Separating between internal and external competition, she shows that, in response to trade liberalization, firms will reduce their product range to dampen the cannibalization effect. Firms in my model also try to mitigate cannibalization, however, as I allow for investments in product differentiation, the channel stressed here is a different one.

To model product differentiation, I build on recent contributions with single-product firms by Lin and Saggi (2002), Rosenkranz (2003), and Bastos and Straume (2012). These authors also assume quadratic preferences and derive optimal investment strategies for single-product firms.[3] Firms invest to horizontally differentiate their products from those produced by their rivals. Therefore, the motive for the investment is different in comparison to this paper with MPFs. Lin and Saggi (2002) explicitly point out that in a framework with two single-product firms, investing more in product differentiation also has a negative strategic effect. From a consumer's perspective, also the rival's product seems more differentiated when a firm increases its spending for product differentiation. The resulting increase in the other firm's output hurts the investing firm. Having the same two products produced by one MPF, the MPF internalizes the externality from the investment and, therefore, will differentiate its

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[3]Ferguson (2011) proposes a model with monopolistic competition and CES preferences. In his model, single-product firms invest in horizontal product differentiation to differentiate their product from the products of their rivals. Similar to my model, the author investigates how the size of the market affects the extent of endogenous product differentiation.

products more to avoid cannibalization.[4] In a multi-product Dixit-Stiglitz framework, Lorz and Wrede (2009) endogenize the degree of product differentiation.[5] In a notably different theoretical setup, these authors also evaluate how firms respond to globalization in terms of product variety and diversity. However, the focus of my paper is different, as I split up the R&D portfolio of an MPF to disentangle the welfare gains from globalization.

The remainder of the article is organized as follows: In the next section, I present the theoretical model where I start with the optimal consumer behavior in section 2.1. Section 2.2 introduces the second stage of the model where a firm decides on its optimal scale and scope of production. In the first stage in section 2.3, a firm chooses optimal spending in both product differentiation and process optimization. I assume that the firm perfectly anticipates the outcome of the second stage. In section 2.4, I conduct a comparative statics exercise where the focus is on the effects of globalization. Finally, in section 2.5, I disentangle the implications of globalization on consumer welfare. Section 3 concludes and summarizes results. I provide all mathematical derivations in the Appendix.

2   The Model

In this part of the paper, I introduce a two stage model of MPFs. In the *first stage*, an MPF chooses its spending on product differentiation and process innovation, anticipating the effects on optimal scale and scope in the following stage. By investing in process innovation, an MPF can simply lower its production costs. As I have already noted in the introduction, investments in product differentiation mean investing in new blueprints for distinct product features and designs or marketing expenses. These investments make sure that from a consumer's point of view the product portfolio offers a huge variety of unique products. From a firm's perspective, however, these investments are made to reduce cannibalization within its own product range. Both types of investments are costly and require upfront endogenous sunk costs. In the *second stage*, the firm simultaneously chooses the quantity produced of each good and the number of products produced. Production incurs variable costs and a fixed capacity cost for each new production line.

The economy under consideration involves a homogeneous goods industry and a differentiated goods industry. Production in the homogeneous industry is subject to constant

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[4]In the Appendix of this paper, I provide a formal analysis for this result. In a simplified version of the main model, I show how the incentives to innovate in product differentiation differ between single-product firms and MPFs.

[5]In the related industrial organization literature, Lambertini and Mantovani (2009) develop a dynamic model of MPFs and investigate whether there exists complementarity or substituability between investments in product and process innovation.

returns to scale with a unit cost requirement and there is no contingence of R&D. For the sake of simplicity, I characterize the differentiated industry by one active monopoly MPF.[6]

I begin this section with the analysis of consumer preferences and derive optimal demand. Then, the focus is on the optimal behavior of an MPF in the differentiated industry. Solving the two-stage model via backwards induction, I start deriving optimal scale and scope of the firm. Subsequent, in the first stage of the game, I derive two conditions for optimal spending in product differentiation and process innovation. In a comparative statics exercise, I focus on the effects of globalization on optimal R&D expenditures. In particular, I investigate how decreasing transport costs or an increasing market size affect optimal firm behavior. The model is rounded down by a section on the gains from trade arising from the within-firm adjustments. For that purpose, I derive the indirect utility function associated with the underlying preference structure and discuss consumer welfare.

2.1 Consumer Behavior: Preferences and Consumer Demand

In this economy, L consumers maximize their utility defined over the consumption of a homogeneous and a differentiated product. Within the differentiated sector, I further assume that consumers buy a set Ω out of a potential set $\widetilde{\Omega}$ of the differentiated product. The preference structure of a representative consumer follows a quasi-linear specification:[7]

$$U = q_0 + u_1, \tag{1}$$

where q_0 is the consumption of the homogeneous good which is sold at a price $p_0 = 1$. The latter serves as numeraire and absorbs any income effects. Therefore, the following analysis occurs in a partial equilibrium setting. u_1 defines utility in the differentiated sector and displays a standard quadratic form:

$$u_1 = aQ - \frac{1}{2}b\left[(1 - e(s))\int_{i\in\widetilde{\Omega}} q(i)^2 di + e(s) Q^2\right]. \tag{2}$$

In this specification, a and b represent non-negative preference parameters and $q(i)$ denotes per variety consumption with $i \in \widetilde{\Omega}$. Total consumption by the representative consumer is given by $Q \equiv \int_{i\in\widetilde{\Omega}} q(i)di$. The parameter $e(s) \in [0, 1]$ is an inverse measure of product differentiation and is of central interest, as it can be chosen endogenously by a firm. The

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[6]I focus on intra-firm adjustments, so competition between firms plays only a second-order role. For an extension of a similar framework to oligopoly, the interested reader is referred to the Appendix in Eckel et al. (2011).

[7]These preferences combine the continuum quadratic approach to symmetric horizontal product differentiation of Ottaviano, Tabuchi, and Thisse (2002) with the preferences in Neary (2009).

value of $e(s)$ is determined by the level of investment s that an MPF spends on product differentiation. Further assumptions on $e(s)$ will be discussed later on in the model. At this point, however, it is important to notice that lower values of $e(s)$ imply that products are more differentiated and hence less substitutable. The extreme case of $e(s) = 1$ denotes that consumers have no taste for diversity in products and demand depends on aggregate output only.

Consumers maximize utility subject to the budget constraint $q_0 + \int_{i \in \widetilde{\Omega}} p(i)q(i)di = I$, where $p(i)$ denotes the price for variety i and I is individual income. Utility maximization yields the following linear inverse individual demand function:

$$\lambda p(i) = a - b\left[(1 - e(s))q(i) + e(s)Q\right], \tag{3}$$

where λ is the marginal utility of income, the Lagrange multiplier attached to the budget constraint. Given the quasi-linear upper-tier utility, there is no income effect, thereby implying that $\lambda = 1$. Market-clearing imposes that an MPF faces a market demand $x(i)$ that consists of the aggregated demand of all consumers $Lq(i)$ for variety i. For the inverse market demand, I derive

$$p(i) = a - b'\left[(1 - e(s))x(i) + e(s)X\right], \tag{4}$$

where a is the consumers' maximum willingness to pay, $b' \equiv \frac{b}{L}$ is an inverse measure for the market size, and finally, $X \equiv \int_{i \in \Omega} x(i)di$ represents total demand in the differentiated industry. Eq. (4) reveals the price $p(i)$ a consumer is willing to pay for variety i as negatively dependent on a weighted average of the sales of variety i and total output of all available varieties. From Eq. (4), I derive direct demand for variety i as:

$$x(i) = \frac{a}{b'(1 - e(s) + e(s)\delta)} - \frac{1}{b'(1 - e(s))}p(i) + \frac{e(s)\delta}{b'(1 - e(s) + e(s)\delta)(1 - e(s))}\overline{p}, \tag{5}$$

where δ describes the mass of consumed varieties in Ω. The average price of differentiated varieties in the economy is given by $\overline{p} = 1/\delta \int_{i \in \Omega} p(i)di$.

In the model I present hereafter, a firm can invest in the degree of product differentiation which affects the cross elasticity between any two varieties. The cross elasticity of variety i with respect to any other variety j is given by:

$$\varepsilon_{i,j} \equiv \left| \frac{\partial x(i)}{\partial x(j)} \frac{x(j)}{x(i)} \right| = \frac{e(s)}{(1 - e(s))} \frac{x(j)}{x(i)}. \tag{6}$$

It is straightforward to see that investments in the degree of product differentiation (lower values of $e(s)$) reduce the cross elasticity $\varepsilon_{i,j}$ and hence weaken the strength of the cannibal-

ization effect within a firms' portfolio. The lower is the substitutability between varieties, the less does the output of any additional variety reduce the demand for the other products within the portfolio.[8] From a demand perspective, this is the reason why MPFs will invest in the degree of product differentiation in my model.

Lemma 1 *By investing in the degree of product differentiation, an MPF can lower the magnitude of the cannibalization effect through a lower cross elasticity of demand between varieties.*

To conclude this section on preferences, I follow Melitz and Ottaviano (2008) and derive the indirect utility function associated with the quadratic preferences as an appropriate measure for welfare

$$U = I + \frac{\delta}{2b\left(1 - e\left(s\right) + e\left(s\right)\delta\right)}\left(a - \overline{p}\right)^2 + \frac{\delta}{2b\left(1 - e\left(s\right)\right)}\sigma_p^2. \tag{7}$$

The term $\sigma_p^2 = \frac{1}{\delta}\int_0^\delta \left(p\left(i\right) - \overline{p}\right)^2 di$ represents the variance of prices. The demand system exhibits "love of variety", as welfare increases in the product range δ, holding $\overline{p}$ and σ_p^2 constant. Furthermore, welfare decreases in the average price and increases in the variance of prices. In section 2.5, further important properties of Eq. (7) are discussed in detail.

2.2 Firm Behavior: Optimal Scale and Scope of the MPF

As already mentioned above, I start analyzing the optimal firm behavior at the second stage of the model. The firm's objective in this section is to maximize profits by choosing both the scale and scope of production. By doing this, a firm considers R&D investments as given. I assume a cost function $c\left(i, k\right)$ for producing variety i, which depends on the technology level k the firm has chosen. I further suppose that each MPF has access to a continuum of potential varieties, however, due to a fixed cost for new production lines it does not necessarily produce all of them. Denoting the scope of the product portfolio by δ, total profits are given by

$$\pi = \int_0^\delta \left[p(i) - c(i, k) - t\right] x(i) di - \delta r_\delta, \tag{8}$$

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[8]Furthermore, investments in the degree of product differentiation also affect the price elasticity of demand. Referring to Melitz and Ottaviano (2008), I express the price elasticity of demand as $\varepsilon_i \equiv |\left(\partial x\left(i\right)/\partial p\left(i\right)\right)\left(p\left(i\right)/x\left(i\right)\right)| = p\left(i\right)/\left(p^{\max} - p\left(i\right)\right)$, where $p^{\max} \equiv \frac{(1-e(s))a + e(s)\delta\overline{p}}{(1-e(s)+e(s)\delta)}$ is the choke price of the linear demand system. Investments in the degree of product differentiation affect the choke price thus the price elasticity. For a given average price $\overline{p}$ and product range δ, investments in product differentiation reduce the price elasticity and hence relax the firms' internal "competition" between varieties as: $\frac{\partial p^{\max}}{\partial e}\big|_{\overline{p},\delta=const} = -\frac{\delta(a-\overline{p})}{(1-e(s)+e(s)\delta)^2} < 0.$

where t is a uniform trade cost payable by the firm on all the varieties it sells. r_δ represents the fixed cost the firm has to pay for each new production line. Firms simultaneously choose the quantity produced of each good (optimal scale) and the mass of goods produced (optimal scope).

Optimal Scale: Maximizing profits in Eq. (8) with respect to $x(i)$ implies the first-order condition for scale:

$$\frac{\partial \pi}{\partial x(i)} = p(i) - c(i, k) - t - b'\left[(1 - e(s))x(i) + e(s)X\right] = 0, \tag{9}$$

that leads to the optimal output of a single variety

$$x(i) = \frac{a - c(i, k) - t - 2b'e(s)X}{2b'(1 - e(s))}, \tag{10}$$

with $X \equiv \int_0^\delta x(i)\, di$ denoting total firm scale.[9] The negative impact of total firm scale X on the output of a single variety displays the cannibalization effect:

$$\frac{\partial x(i)}{\partial X} = -\frac{e(s)}{(1 - e(s))} < 0. \tag{11}$$

The strength of the cannibalization effect depends on the substitutability of the varieties within the product portfolio of the firm, i.e. the parameter $e(s)$. It is easily verified, that the magnitude of the cannibalization effect in Eq. (11) is increasing in the value of $e(s)$.

For the sake of simplicity and without loss of generality, I impose symmetry on the production costs: $c(i, k) = c(j, k) = c(k)$.[10] Therefore, I rewrite Eq. (10) as:

$$x = \frac{a - c(k) - t}{2b'(1 - e(s) + e(s)\delta)}. \tag{12}$$

Total firm output is then simply given by:

$$X = \frac{\delta(a - c(k) - t)}{2b'(1 - e(s) + e(s)\delta)}. \tag{13}$$

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[9]The second-order condition for this maximization problem is given by: $\frac{\partial^2 \pi}{\partial x(i)^2} = \frac{\partial p(i)}{\partial x(i)} - b'(1 - e) - b'e\frac{\partial X}{\partial x(i)} < 0$.

[10]Eckel and Neary (2010) assume a technology that embodies flexible manufacturing. A firm possesses a core competence and marginal costs for any other variety rise in the distance to the core product. The idea that firms posses one core product is also featured in recent models by Arkolakis and Muendler (2010), Qiu and Zhou (2013), and Mayer et al. (2014).

Note that total output X in Eq. (13) rises in the product range δ, however, output of each variety x is decreasing in δ due to the cannibalization effect. Put it differently, the relationship between total firm output X and the product range δ is a concave function, whereby the slope depends on the degree of product differentiation $e(s)$. Assuming perfectly differentiated products, i.e. $e(s) = 0$, the relation turns out to be linear.[11]

To derive the optimal price, I combine Eq. (12) with the inverse demand function Eq. (4) and get:

$$p = \frac{a + c(k) + t}{2}. \tag{14}$$

Prices are increasing functions of both production and transportation costs.

Optimal Scope: I proceed and consider the firm's choice of its profit maximizing product range. Besides the fixed costs r_δ, the extension of the product portfolio is also limited through the additional cannibalization associated with the launching of further products. I rewrite total profits in Eq. (8) as follows

$$\pi = \frac{\delta (a - c(k) - t)^2}{4b'(1 - e(s) + e(s)\delta)} - \delta r_\delta, \tag{15}$$

where operative profits must be positive, i.e.

$$r_\delta < \frac{(a - c(k) - t)^2}{4b'(1 - e(s) + e(s)\delta)}. \tag{16}$$

Maximizing Eq. (15) with respect to δ implies the respective first-order condition for scope:[12]

$$\frac{\partial \pi}{\partial \delta} = \frac{(1 - e(s))(a - c(k) - t)^2}{4b'(1 - e(s) + e(s)\delta)^2} - r_\delta = 0. \tag{17}$$

From Eqs. (12) and (13), it follows that per variety output decreases but total firm output increases in additional products. An MPF optimally solves this trade-off and adds new varieties until the marginal return of an additional variety equals the fixed cost r_δ of an investment in additional capacity. From inspection of Eq. (17), it is straightforward to see that the marginal return of new varieties is decreasing in the number of products δ. Solving

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[11] Total firm output X is an increasing function of the product range δ as: $\frac{\partial X}{\partial \delta} = \frac{(1 - e(s))(a - c(k) - t)}{2b'(1 - e(s) + e(s)\delta)^2} > 0$. The second derivative is negative: $\frac{\partial^2 X}{\partial \delta^2} = -\frac{e(s)(1 - e(s))(a - c(k) - t)}{b'(1 - e(s) + e(s)\delta)^3} < 0$, which implies a concave relation between the variables X and δ.

[12] The second-order condition is given by: $\frac{\partial^2 \pi}{\partial \delta^2} = -\frac{e(s)(1 - e(s))(a - c(k) - t)^2}{2b'(1 - e(s) + e(s)\delta)^3} < 0$.

for the optimal product range yields:

$$\delta = \frac{(a - c(k) - t)\sqrt{\left(\frac{(1-e(s))}{b'r_\delta}\right)} - 2(1 - e(s))}{2e(s)}.\tag{18}$$

The condition on r_δ stated in Eq. (16) ensures that the product range takes positive values. The optimal product range rises with falling fixed costs r_δ and falling variable costs $c(k)$. Furthermore, I am interested in the effects of globalization on the product range of an MPF. In my model, globalization is captured through falling trade costs t or a rising market size L (i.e. lower values of $b' \equiv \frac{b}{L}$). Inspecting Eq. (18) reveals the multiplicative structure of the fixed costs for product innovation r_δ and the inverse measure for the market size b'. Therefore, an increase in the market size L has the same effect as decreasing fixed costs r_δ. I interpret the term $b'r_\delta$ as the perceived costs of product innovation which are lower in a larger market.

In the first stage of the model, the firm can invest in the degree of product differentiation. Therefore, it is of further interest how this investment affects the optimal product range of an MPF. It can be shown that a larger scope for product differentiation induces the firm to enlarge its product range δ. The reason for this is that lower values of $e(s)$ imply more investments in blueprints for product specific features or marketing which reduce cannibalization among varieties. These investments create a broader spectrum of technological opportunities within a firm can establish more varieties. I summarize these results in the following proposition.

Proposition 1 *Lower values of trade costs t or a larger market size L increase the profit maximizing product range of an MPF, i.e.*

$$\frac{d\delta}{dt} < 0 \ \textit{and} \ \frac{d\delta}{dL} > 0.\tag{19}$$

Furthermore, a rising degree of product differentiation (lower values of $e(s)$) dampens the cannibalization effect and, hence, induces an extension of the product range, i.e.

$$\frac{d\delta}{de} < 0.\tag{20}$$

All derivatives are presented in the Appendix.

2.3 Firm Behavior: Optimal R&D Portfolio of an MPF

In this section, I discuss the first stage of the model. I assume that the firm correctly foresees how output levels and product range are determined in the second stage. Firms can invest in cost-reducing process optimization and in a more differentiated product range. To derive the firm's profit function in this stage, I combine the optimal product range in Eq. (18) with the gross profits $\widetilde{\pi}$ in Eq. (15) and come up with the following expression:

$$\Pi = \widetilde{\pi} - kr_k - sr_s, \tag{21}$$

where

$$\widetilde{\pi} = \frac{(a - c(k) - t)\left((a - c(k) - t) - 2\sqrt{(b'r_\delta(1 - e(s)))}\right)}{4b'e(s)}. \tag{22}$$

Recall that k is the level of investment in process optimization and s denotes the investment in product differentiation. Process R&D is conducted at a rate r_k and investing in product differentiation is carried out at a rate r_s. By investing k units in process innovation, a firm can lower its production costs $c(k)$, following the assumptions:

$$\frac{\partial c}{\partial k} \equiv c'(k) < 0 \text{ and } \frac{\partial^2 c}{\partial k^2} \equiv c''(k) > 0. \tag{23}$$

The level of product differentiation $e(s)$ is determined by:

$$\frac{\partial e}{\partial s} \equiv e'(s) < 0 \text{ and } \frac{\partial^2 e}{\partial s^2} \equiv e''(s) > 0, \tag{24}$$

where $e(0) = 1$ and $e(\infty) = 0$. To ensure interior solutions, I further assume that $e'(0) = c'(0) = -\infty$ and $e'(\infty) = c'(\infty) = 0$. I do not impose any specific functional forms of Eqs. (23) and (24) to keep the analysis as general as possible. However, the curvatures of $c(k)$ and $e(s)$ are of special interest as they capture the innovation efficiency of firms. To clarify this issue, I determine the elasticity of $c(k)$ and $e(s)$ with respect to innovation inputs k and s as: $\varepsilon_{c(k)} \equiv |d\ln c/d\ln k| \equiv \left|c'(k)\frac{k}{c(k)}\right|$ and $\varepsilon_{e(s)} \equiv |d\ln e/d\ln s| \equiv \left|e'(s)\frac{s}{e(s)}\right|$. According to that, the percentage change of $c(k)$ and $e(s)$ following an one percentage point increase in k or s, respectively, will be larger, the larger are $|e'(s)|$ and $|c'(k)|$. Having this in mind, I can discuss the implications of differences in the ability to innovate between firms in different countries or industries at a very general level.

[Insert Figure 1 about here]

Figure 1 depicts the case for the response of the differentiation parameter e on the investment level s. It is natural to assume, that there are differences in the scope for product differentiation between firms, for example, in less developed and developed countries. A certain level of development is necessary to create blueprints for a broad and highly differentiated product range. This is illustrated in Figure 1 where the solid line represents a developed country and the dashed line represents a less developed country. The message from the graph is straightforward: firms in a developed country are more efficient in innovation leading to a higher degree of product differentiation for a given level of investment s.[13] Furthermore, the steeper slope (larger $|e'(s)|$) of the solid line indicates that a marginal unit of innovation input will be transformed into more innovation output in the developed country.[14] I will revisit this point when I study the impact of trade liberalization on the innovation choices and the consequent welfare analysis.

In the first stage, the firm maximizes Eq. (21) both with respect to k and s. The first-order conditions for this maximization problem read as follows:

$$\frac{\partial \Pi}{\partial s} = \frac{\partial \widetilde{\pi}}{\partial e} e'(s) - r_s = 0, \tag{25}$$

and

$$\frac{\partial \Pi}{\partial k} = \frac{\partial \widetilde{\pi}}{\partial c} c'(k) - r_k = 0, \tag{26}$$

where

$$\frac{\partial \widetilde{\pi}}{\partial e} < 0 \text{ and } \frac{\partial \widetilde{\pi}}{\partial c} < 0. \tag{27}$$

Operating profits are rising in the degree of product differentiation (lower values of $e(s)$), as this reduces cannibalization and, therefore, lowers competition between the products within the portfolio. Moreover it is straightforward, that operating profits are increasing in lower production costs. The exact expressions for $\frac{\partial \widetilde{\pi}}{\partial e}$ and $\frac{\partial \widetilde{\pi}}{\partial c}$, and a proof that these terms are negative is provided in the Appendix. Furthermore, I provide a simplified version of the model, where I investigate the differences in the incentives to innovate in product differentiation between MPFs and single-product firms.

Eqs. (25) and (26) suggest that the marginal benefits of the two investments essentially consist of two elements. The *first* element is the direct effect of a change in the degree of

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[13] The Global Innovation Index 2013 ranks countries according to their innovation efficiency. Innovation efficiency is calculated as the ratio of the innovation output over innovation input in a country.

[14] Acemoglu and Zilibotti (2001) argue that even if all countries have access to the same set of technologies, there will be large cross-country differences because of varying economic conditions. Their argument is the skill scarcity in developing countries which makes skill-complementary technologies inappropriate. This technology-skill mismatch leads to differences in the efficiency how countries transform innovation inputs into innovation outputs.

product differentiation or the cost parameter on the operating profits $\widetilde{\pi}$ demonstrated in Eq. (27). Most importantly, the magnitude of the direct effect depends on the firm size. This is related to early findings by Schumpeter, who argued that only firms with sufficiently large sales can cover the high fixed costs associated with R&D projects. The *second* element embodies the responsiveness of the differentiation and the cost parameter with respect to investments: $e'(s)$ and $c'(k)$. According to this, the marginal benefit of an investment also depends on the efficiency of transforming research input into output. The larger $|e'(s)|$ and $|c'(k)|$, respectively, the greater is the impact of the marginal unit of investment.

Lemma 2 *The marginal benefit of an investment depends on (i) the total firm size (determined by scale and scope) and (ii) the efficiency of research input utilization.*

The first-order conditions in Eqs. (25) and (26) suggest that it is optimal to invest in process innovation and product differentiation, respectively, until the marginal benefits equal the marginal costs of the investment. Figure 2 illustrates the optimal behavior for the case of investments in product differentiation. The shape of the marginal benefit curve is concave due to the assumptions made on $e(s)$. The optimal investment level s^* is determined at the point where the slope of the investment cost curve is just equal to the slope of the marginal benefit curve. In this point, total profits Π are maximized. Furthermore, it can be seen in the graph that the equilibrium investment levels s^* and k^* increase in the responsiveness of the functions $e(s)$ and $c(k)$, i.e. in their respective slopes $e'(s)$ and $c'(k)$. Eqs. (25) and (26) implicitly determine the equilibrium levels of product differentiation $s^* = s^*(k, \ r_s, \ t, \ r_\delta, \ b')$ and process innovation $k^* = k^*(s, \ r_k, \ t, \ r_\delta, \ b')$. In the next section, I present comparative statics results with respect to the variables in brackets.

[Insert Figure 2 about here]

2.4   Comparative Statics

Having characterized the equilibrium R&D levels, I derive several comparative statics results of the model. I start with the effects of an exogenous change in the investment costs (r_s and r_k) and the capacity costs r_δ followed up with the cross-effects of process innovation on product differentiation and vice versa. Afterwards, the focus is on the effects of economic integration on the equilibrium R&D efforts. As mentioned above, globalization can be captured by both an increase in market size L and a reduction in transportation costs t.

To derive the comparative statics results, I totally differentiate Eqs. (25) and (26). The explicit mathematical expressions for all derivatives are provided in the Appendix. In the

following subsections, I will employ the graphical tool provided in Figure 2 and discuss the results intuitively.

Change in Investment Costs and Capacity Costs: It is straightforward to determine the effects of rising innovation costs on the equilibrium levels of s^* and k^*. Rising rates r_s and r_k increase the costs of R&D and, therefore, reduce equilibrium levels of both process innovation and product differentiation. In Figure 2, this can be illustrated by a rising slope of the investment cost curve in the upper part of the diagram. Consequently, the optimal investment level is shifted to the left hand side.

The effects of varying capacity costs r_δ on the investment levels can be interpreted like the effects of a changing product range. From Eq. (18), it follows that rising capacity costs r_δ will reduce the profit maximizing product range. The latter reduces total firm size. Keeping this coherence in mind, it is obvious that rising fixed costs r_δ reduce both the optimal levels of product differentiation and process innovation as the gains from these investments are reduced. More precisely, investments in differentiation are cut back, as within a smaller product range, the cannibalization effect is less fierce. Furthermore, due to a lower firm-level output, the benefits from a better technology level are reduced, leading to less process innovation. Expressed in Figure 2, this means a lower slope of the marginal benefit curve which again shifts the optimal investment level to the left hand side. I summarize these findings in the following proposition.

Proposition 2 *Rising investment costs (r_s and r_k) and capacity costs (r_δ) reduce optimal levels of investment in both product differentiation and process innovation, i.e.*

$$\frac{ds^*}{dr_s} < 0; \; \frac{dk^*}{dr_k} < 0, \tag{28}$$

and

$$\frac{ds^*}{dr_\delta} < 0; \; \frac{dk^*}{dr_\delta} < 0. \tag{29}$$

Cross - Effects: In the next step, I investigate the interaction between process innovation and product differentiation. As in the related industrial organization literature with single product firms (see for example Lin and Saggi (2002)), I find a two-way complementarity in which the investment in one branch of research makes the other more attractive. On the one hand, Eq. (18) shows that a better technology induces an MPF to add more products to its portfolio. The latter intensifies cannibalization and incentivizes a higher level of product differentiation. On the other hand, more differentiated products enable a higher sales volume and, therefore, enhance the incentives to invest in a better technology.

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Proposition 3 *Firms invest more in product differentiation when they can undertake process innovation and vice versa, i.e.*

$$\frac{ds^*}{dk} > 0 \ \ and \ \ \frac{dk^*}{ds} > 0. \tag{30}$$

Globalization: I conclude this section with inspecting the effects of trade liberalization on the R&D efforts of MPFs. In my framework, globalization is modelled as a reduction in trade costs t or alternatively, following Krugman (1979), as an increase in the market size L (recall: the demand parameter $b' \equiv \frac{b}{L}$ is an inverse measure of market size which is decreasing in the number of consumers). Considering proposition 1, I know that a larger market encourages a firm to add additional products to its portfolio. An MPF which widens its product range cannibalizes market shares of its existing products. This makes additional spending on product differentiation attractive. At the same time, total firm output X rises after trade liberalization which raises investments in both product differentiation and process innovation through economies of scale. In Figure 3, this is illustrated through a steeper marginal benefit curve which leads to a higher equilibrium spending on product differentiation s_1^*.[15]

[Insert Figure 3 about here]

Economic integration can also be interpreted as a process of reducing trade costs. Similar to an increase in market size, lower transportation costs enlarge total firm output and induce higher equilibrium investments in both types of R&D.

The fact that investments of firms are positively correlated to the market size is in line with recent contributions in the literature. Lileeva and Trefler (2010) and Bustos (2011) underline how rising revenues after trade liberalization induce exporters to invest in better technologies. Concerning the investment in product differentiation, my findings are related to models with endogenous investments in quality, such as Antoniades (2012) for single-product firms and Eckel et. al (2011) for MPFs. In these papers, spending in quality is an endogenous sunk cost and is increasing in firm scale. Firms choose expenditures subject to the size of the market as with more consumption the investment costs can be spread over more units of output.

Studying the impact of trade liberalization on the equilibrium investment levels reveals a coherence between the magnitude of the effects and the efficiency of the investment input utilization. The more efficient are firms in a country or an industry in transforming innovation inputs into innovation outputs, i.e. the larger are $|e'(s)|$ and $|c'(k)|$, the larger

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[15]The same graph could be drawn for process innovation.

is the magnitude of the effects of globalization. I summarize these results in the following proposition.

Proposition 4 *Rising market size L and falling trade costs t enhance the equilibrium levels of both types of investments, i.e*

$$\frac{ds^*}{dL} > 0; \ \frac{dk^*}{dL} > 0, \tag{31}$$

and

$$\frac{ds^*}{dt} < 0; \ \frac{dk^*}{dt} < 0. \tag{32}$$

The magnitude of these effects is amplified when innovation inputs are in efficient use.

2.5 Welfare

This section builds on the comparative static results concerning globalization and studies the impact on consumer welfare. Following Melitz and Ottaviano (2008), the indirect utility stated in Eq. (7) serves as an appropriate measure for welfare. In the simplified model setup with symmetric varieties, indirect utility is reduced as follows:

$$V = I + \frac{\delta}{2b\left(1 - e\left(s\right) + e\left(s\right)\delta\right)}\left(a - p\right)^2. \tag{33}$$

I start the welfare analysis by discussing important properties of the indirect utility function in Eq. (33). By doing this, I identify the distinct channels through which innovation affects consumers welfare. Having unbundled the different welfare channels, I focus again on the variables concerning globalization and discuss the welfare gains from trade.

Properties of the Indirect Utility Function   At a first glance and not strikingly, welfare is higher the lower is the price level p. Furthermore, as already mentioned, the indirect utility function displays "love of variety", i.e.

$$\frac{\partial V}{\partial \delta}\big|_{p,e=const} = \frac{\left(1 - e\left(s\right)\right)}{2b\left(1 - e\left(s\right) + e\left(s\right)\delta\right)^2}\left(a - p\right)^2 > 0. \tag{34}$$

For a given price and degree of product differentiation, consumer welfare is increasing in the number of available products δ. As the degree of product differentiation is endogenously chosen by a firm in my model, I am interested in the role of product differentiation for consumer welfare. It is straightforward to show that the marginal utility of an additional

product is increasing in the degree of product differentiation:

$$\frac{\partial V}{\partial \delta \partial e} = -\frac{(2 - e(s))\delta - (1 - e(s))}{2b(1 - e(s) + e(s)\delta)^3}(a - p)^2 < 0. \tag{35}$$

Furthermore, I can show that for a given product range δ, utility is increasing when products are more differentiated:[16]

$$\frac{\partial V}{\partial e}\Big|_{p,\delta=const} = -\frac{\delta(\delta - 1)}{2b(1 - e(s) + e(s)\delta)^2}(a - p)^2 < 0. \tag{36}$$

I call this attribute of the utility function "love of diversity". In addressing the question to what extent globalization matters for consumer welfare in my framework, this property of the utility function is central. From previous discussion, it is obvious that consumers value a given product range more when products are more differentiated.

Lemma 3 *Welfare increases in the number of available products ("love of variety"), the degree of product differentiation ("love of diversity"), and decreases in the price level.*

Welfare Gains from Trade   With the properties of the welfare function fully characterized, I proceed discussing some of the key implications of the theory. In the previous section, I have analyzed the impact of trade liberalization on the product range and the endogenous choice of R&D expenditures. I recap the key comparative statics results and highlight their implications for consumer welfare.

My theoretical model suggests an extension of the product range following an increase in the market size or alternatively lower trade costs. Qualitatively, this result is in line with trade models with single-product firms such as Krugman (1980) and Melitz (2003) where the transition from autarky to trade induces entry of firms. Firm entry increases the number of available products in the market which gives rise to gains from trade through the well-known "love of variety" nature of the utility function. However, worthwhile to mention at this point is the different source of gains from trade in this model stemming from adjustments within the firm in contrast to entry of firms at the industry level.

The endogenous choice of investment levels in product differentiation and process innovation is the key element of this theory. Trade liberalization enables firms to exploit economies of scale in innovation and increases incentives to invest in R&D as in a larger market fixed

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[16]Welfare gains have diminishing returns with respect to product differentiation: $\frac{\partial V^2}{\partial^2 e}\Big|_{p,\delta=const} = \frac{\delta(a-p)^2(\delta-1)^2}{b(1-e(s)+e(s)\delta)^3} > 0$. Substituting information from Eqs. (4) and (12) in the indirect utility Eq. (33), I compute the total derivative as follows: $\frac{dV}{de} = \frac{(a-c(k)-t)^2}{8b([(1-e(s)+e(s)\delta)])^2}\left((1 - e(s))\frac{d\delta}{de} - \delta(\delta - 1)\right) < 0$. Recall from proposition 1 that $\frac{d\delta}{de} < 0$.

investment costs can be spread over a larger scale of output. Therefore, my formal analysis reveals increasing spending in both types of investments after trade liberalization. This enhances welfare via two separate channels. The central welfare channel in this model is what I called "love of diversity". Given the opportunity to serve a larger market, an MPF will spend more resources on research for new blueprints or product specific attributes. For the consumer, this increases welfare as it leads to products with new functional features or a new design and thus the opportunity to choose from a broader product range. Technically, a higher degree of product differentiation enlarges the marginal utility of each new product and thus enhances welfare. Furthermore, consumers enjoy lower prices as MPFs increase investments in better processes. As firm size grows in a larger market, a better technology becomes more valuable. The resulting cost savings are passed on to consumers, leading to welfare gains from lower prices (compare Eq. (14)).

To unbundle the different channels analytically, I substitute Eqs. (4) and (12) into Eq. (33) and rewrite the indirect utility function as follows:

$$V = I + \frac{\delta \left(a - c\left(k\right) - t\right)^2}{8b\left[\left(1 - e\left(s\right) + e\left(s\right)\delta\right)\right]}.$$ (37)

By totally differentiating Eq. (37), I identify the three distinct channels which were discussed above. The following expression represents the gains from trade induced by an increase in the market size L:[17]

$$\frac{dV}{dL} = \underbrace{\frac{\left(1 - e\left(s\right)\right)bx^2}{2L^2}\frac{d\delta}{dL}}_{>0:\ \text{Love of variety}} \underbrace{- \frac{2b\left(\delta - 1\right)\left(a - c\left(k\right) - t\right)X}{L}e'\left(s\right)\frac{ds}{dL}}_{>0:\ \text{Love of diversity}} \underbrace{- \frac{X}{2L}c'\left(k\right)\frac{dk}{dL}}_{>0:\ \text{Lower prices}} > 0.$$ (38)

Inspection of Eq. (38) clearly reveals that consumers in a larger market are better off. It highlights three distinct channels of gains from trade and shows how trade liberalization affects welfare through within-firm adjustments. I derive the analogous expression for falling trade costs t in the Appendix.

Preceding discussion suggested that consumers in a larger market or in a market with lower trade costs, ceteris paribus, are better off. The magnitude of the gains from trade naturally depends on the increase of investment levels after trade liberalization. Furthermore, inspection of Eq. (38) reveals that the welfare gains also depend on the efficiency of research input utilization determined by $e'\left(s\right)$ and $c'\left(k\right)$. If trade induced investments (i.e. $\frac{ds}{dL}$ and $\frac{dk}{dL}$) do not generate more differentiated products or a better production technology because

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[17] To determine the sign of the derivative in Eq. (38), recall that $e'\left(s\right) < 0$ and $c'\left(k\right) < 0$. Furthermore, it follows from proposition 1: $\frac{d\delta}{dL} > 0$ and from proposition 4: $\frac{ds}{dL} > 0$ and $\frac{dk}{dL} > 0$.

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of inefficient innovation (low values of $|e'(s)|$ and $|c'(k)|$), the welfare gains from an increase in the market size will be low. The latter implies that gains from trade will be larger in a developed country where due to better technological conditions the marginal benefit of each unit of investment is higher. I summarize these results in the following proposition.

Proposition 5 *Table 1 summarizes the welfare gains from trade integration through the different elements of the welfare function. Economies of scale after trade liberalization lead to larger welfare gains in countries where the technological stage of development allows an efficient use of research input.*

Table 1: Welfare Effects of Globalization

Effects of $L \uparrow$ or $t \downarrow$:	Welfare channel:
Product range: $\delta \uparrow$	Love of variety
Product differentiation: $s^* \uparrow$; $e(s) \downarrow$	Love of diversity
Process innovation: $k^* \uparrow$; $c(k) \downarrow$	Lower prices

3 Conclusion

In this paper, I focus on the gains from trade associated with intra-firm adjustments. There is indeed recent evidence that innovating firms account for a large fraction of the productivity and variety gains at a sector-level. To distinguish between the different welfare channels, I construct a multi-product framework in which a firm invests in different types of research. Trade liberalization provides welfare gains which originate at the firm-level because firms exploit economies of scale in innovation. An MPF weighs the marginal benefit of each type of innovation against the fixed upfront development costs. The market size effect of globalization accompanied by rising sales volumes raises the returns to the different types of innovation. Consumers benefit from a larger product portfolio (love of variety) of more differentiated products (love of diversity). Furthermore, a larger firm output encourages technology-upgrading. Consumers benefit from the investment in a cost-reducing technology through lower prices.

The key element of this theory is the investment in the degree of product differentiation and the consequent welfare gains through more differentiated products. In most studies, product differentiation is a main component of the industry structure which is treated as an exogenous variable. By endogenizing the degree of product differentiation, I highlight an additional channel in which globalization may affect product variety. I show that adding additional products in a larger market encourages MPFs to invest in a more diversified

product range. In contrast to single-product firms, MPFs have higher incentives to invest in product differentiation in order to reduce the cannibalization effect. Consumers enjoy additional gains as the marginal benefit of any new variety rises in the degree of product differentiation. This implies that consumer welfare is not only determined by the absolute number of available products but by the individual product features that distinguish these varieties. Consumers value choosing from a broad and diversified product range. Therefore, investments in the diversity of the available products is an important aspect when analyzing consumer welfare. Finally, I have shown that welfare improvements through economies of scale depend on the efficiency of innovation input utilization. The better research input is transformed into research output, the higher will be the equilibrium investment levels and the larger will be the gains from trade through intra-firm adjustments.

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5 Appendix

5.1 Proof of Proposition 1

In the model presented above, globalization is captured by lower trade costs t or a larger market size L (lower values of b'). Though it is straightforward to see the effects of globalization on firm scope in Eq. (18), I present the derivatives for completeness. Differentiating Eq. (18) with respect to t and b' yields:

$$\frac{d\delta}{dt} = -\frac{\sqrt{\left(\frac{(1-e(s))}{b'r_\delta}\right)}}{2e(s)} < 0 \tag{39}$$

and

$$\frac{d\delta}{db'} = -\frac{(a-c(k)-t)\sqrt{\left(\frac{(1-e(s))}{b'r_\delta}\right)}}{4b'e(s)} < 0. \tag{40}$$

The second part of proposition 1 considers the effect of the degree of product differentiation on the optimal product range. Differentiating Eq. (18) with respect to $e(s)$ yields:

$$2\delta\frac{d\ln\delta}{d\ln e} = -\left(\frac{(a-c(k)-t)}{2}\sqrt{\left(\frac{1}{b'r_\delta(1-e)}\right)} + 2(\delta-1)\right) < 0. \tag{41}$$

5.2 Optimal Firm Behavior in the First Stage

Maximizing Eq. (21) with respect to s and k leads to the first-order conditions in Eqs. (25) and (26). The explicit expressions for $\frac{\partial\widetilde{\pi}}{\partial e}$ and $\frac{\partial\widetilde{\pi}}{\partial c}$ in Eq. (25) are given by

$$\frac{\partial\widetilde{\pi}}{\partial e} = -\frac{(a-c(k)-t)\left(\delta-\frac{1}{2}\right)}{2e(s)}\sqrt{\left(\frac{r_\delta}{b'(1-e(s))}\right)} < 0, \tag{42}$$

and

$$\frac{\partial\widetilde{\pi}}{\partial c} = -\frac{(a-c(k)-t)-\sqrt{(b'r_\delta(1-e(s)))}}{2b'e(s)} < 0. \tag{43}$$

To derive Eq. (42), differentiate $\widetilde{\pi}$ in Eq. (22) with respect to $e(s)$ and substitute information from Eq. (18).

5.3 Comparative Statics

In this part of the Appendix, I provide all analytical results for section 2.4. To derive my results, I totally differentiate the following first-order conditions:

$$\frac{\partial \Pi}{\partial s} = -\frac{(a - c(k) - t)\left((a - c(k) - t) - (2 - e(s))\sqrt{\left(\frac{b'r_\delta}{(1 - e(s))}\right)}\right)}{4b'e(s)^2}e'(s) - r_s = 0, \quad (44)$$

and

$$\frac{\partial \Pi}{\partial k} = -\frac{(a - c(k) - t) - \sqrt{(b'r_\delta(1 - e(s)))}}{2b'e(s)}c'(k) - r_k = 0. \quad (45)$$

Throughout the analysis, I apply the following second-order conditions:

$$\frac{\partial^2 \Pi}{\partial s^2} = \frac{\partial^2 \widetilde{\pi}}{\partial e^2}e'(s) + \frac{\partial \widetilde{\pi}}{\partial e}e''(s) < 0, \quad (46)$$

and

$$\frac{\partial^2 \Pi}{\partial k^2} = \frac{\partial^2 \widetilde{\pi}}{\partial c^2}c'(k) + \frac{\partial \widetilde{\pi}}{\partial c}c''(k) < 0. \quad (47)$$

To determine the signs of the following derivatives, recall that $e'(s) < 0$ and $c'(k) < 0$.

Proof of Proposition 2 The following derivatives show how equilibrium investment levels s^* and k^* respond to changes in investment and capacity costs.

Change in Investments Costs:

$$\frac{ds^*}{dr_s} = \frac{1}{\frac{\partial^2 \Pi}{\partial s^2}} < 0 \quad (48)$$

$$\frac{dk^*}{dr_k} = \frac{1}{\frac{\partial^2 \Pi}{\partial k^2}} < 0 \quad (49)$$

Change in Capacity Costs:

$$\frac{ds^*}{dr_\delta} = -\frac{(a - c(k) - t)\left((2 - e(s))\sqrt{\left(\frac{b'}{r_\delta(1 - e(s))}\right)}\right)}{8b'e(s)^2\frac{\partial^2 \Pi}{\partial s^2}}e'(s) < 0 \quad (50)$$

$$\frac{dk^*}{dr_\delta} = -\frac{1}{4e(s)\frac{\partial^2 \Pi}{\partial k^2}}\sqrt{\left(\frac{(1 - e(s))}{b'r_\delta}\right)}c'(k) < 0 \quad (51)$$

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Proof of Proposition 3 To determine the interaction between process innovation and product differentiation (cross-effects), I totally differentiate Eq. (44) with respect to k and Eq. (45) with respect to s. I combine the derivatives with information from Eq. (18) to show the positive signs.

$$\frac{ds^*}{dk} = -\frac{e'(s)\,c'(k)}{2b'e(s)\frac{\partial^2\Pi}{\partial s^2}}\left(\frac{(a-c(k)-t)}{2e(s)} + \sqrt{\left(\frac{b'r_\delta}{(1-e(s))}\right)}\left(\delta - \frac{1}{2}\right)\right) > 0 \qquad (52)$$

$$\frac{dk^*}{ds} = -\frac{e'(s)\,c'(k)}{2b'e(s)\frac{\partial^2\Pi}{\partial k^2}}\left(\frac{((a-c(k)-t))}{2e(s)} + \sqrt{\left(\frac{b'r_\delta}{(1-e(s))}\right)}\left(\delta - \frac{1}{2}\right)\right) > 0 \qquad (53)$$

Proof of Proposition 4 Globalization is captured by an increase in the size of the market L (recall: $b' \equiv \frac{b}{L}$) or by falling trade costs t. The following derivatives show how equilibrium values of s^* and k^* respond to changes in these parameters.

Change in Market Size: Again, I totally differentiate Eq. (44) and then substitute information from Eq. (18) to show that the sign of the following derivative is clearly negative:

$$\frac{ds^*}{db'} = -\frac{(a-c(k)-t)}{4b'^2 e(s)\frac{\partial^2\Pi}{\partial s^2}}\left(\frac{(a-c(k)-t)}{2e(s)} + \delta\sqrt{\left(\frac{b'r_\delta}{(1-e(s))}\right)}\right)e'(s) < 0. \qquad (54)$$

From inspection of Eq. (22), I know that the following expression is clearly negative:

$$\frac{dk^*}{db'} = -\frac{2(a-c(k)-t) - \sqrt{(b'r_\delta(1-e(s)))}}{4b'^2 e(s)\frac{\partial^2\Pi}{\partial k^2}}c'(k) < 0. \qquad (55)$$

Change in Trade Costs:

$$\frac{ds^*}{dt} = -\frac{e'(s)}{2b'e(s)\frac{\partial^2\Pi}{\partial s^2}}\left(\frac{((a-c(k)-t))}{2e(s)} + \sqrt{\left(\frac{b'r_\delta}{(1-e(s))}\right)}\left(\delta - \frac{1}{2}\right)\right) < 0 \qquad (56)$$

$$\frac{dk^*}{dt} = -\frac{c'(k)}{2b'e(s)\frac{\partial^2\Pi}{\partial k^2}} < 0 \qquad (57)$$

5.4 Welfare

In the main body of this paper, I present the disentangled welfare gains of an increase in the market size L. For the sake of completeness, I also present the explicit expression for a

change in the trade cost parameter t.

$$\frac{dV}{dt} = \underbrace{\frac{(1 - e(s)) \, x^2 b}{2L^2} \frac{d\delta}{dt}}_{<0: \text{ Love of variety}} \underbrace{- \frac{2b(\delta - 1)(a - c(k) - t) \, X}{L} e'(s) \frac{ds}{dt}}_{<0: \text{ Love of diversity}} \underbrace{- \frac{X}{2L} \left(1 + c'(k) \frac{dk}{dt} \right)}_{<0: \text{ Lower prices}} < 0$$

(58)

5.5 A Benchmark Model with Two Products and Endogenous Product Differentiation

In this part of the Appendix, I formulate a simplified version of the model with only two varieties being produced. These varieties are offered by either two single-product firms that compete in a Cournot fashion or by one MPF. Using this simple model, I want to show how the incentives to innovate in the degree of product differentiation differ according to whether the two products are offered by one MPF or by two single-product firms. As the focus is on innovation in product differentiation, I abstract from the two other strands of R&D considered in the main model. The following executions will be held rather scarce as the model presented here is based on the model in the main body.

Preferences   As in the main model, the preference structure of a representative consumer follows a quasi-linear specification:

$$U = q_0 + a(q_1 + q_2) - \frac{1}{2}b\left(q_1^2 + q_2^2 + 2e(s)q_1q_2\right)$$

(59)

where $e(s) \in [0,1]$ and q_0 is the consumption of the homogeneous good. The specification in Eq. (59) is the two goods case equivalent to the preference structure in the main model. Utility maximization gives rise to the following market demand system:

$$p_1 = a - b'(x_1 + e(s)x_2) \text{ and } p_2 = a - b'(x_2 + e(s)x_1),$$

(60)

where market-clearing imposes that: $x_1 \equiv Lq_1$ and $x_2 \equiv Lq_2$. As I am not interested in market size effects, I assume in the following that $b = L = 1$.

Optimal Firm Behavior   I consider two scenarios in this section. In the first scenario, two firms, each producing one variety, invest in the degree of product differentiation. In the second scenario, an MPF produces the two varieties and conducts the investment. In both scenarios, the marginal cost of production equals c and r_δ denotes the fixed capacity cost. As in the main model, firm i can invest s units in the degree of product differentiation $e(s)$.

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The investments follow the assumptions made in Eq. (24). Again, investments in the degree of product differentiation are conducted at a rate r_s.

As it has already been pointed out by Lin and Saggi (2002), for Cournot firms, investments in the degree of product differentiation have two conflicting effects on profits. For the profits of firm i, this means

$$\frac{d\pi_i}{ds_i} = \underbrace{\frac{\partial \pi_i}{\partial e} e'(s_i)}_{\text{direct effect: } >0} + \underbrace{\frac{\partial \pi_i}{\partial x_j}\frac{dx_j}{\partial e} e'(s_i)}_{\text{indirect effect: } <0}. \tag{61}$$

The direct effect in Eq. (61) is positive. It captures the increase in the demand for the own product following investments in the degree of product differentiation. However, there is also a negative indirect effect from the investment. This strategic effect occurs because also firm j benefits from the investment of firm i. The resulting increase in the output of the competing firm j hurts the innovating firm i. The latter effect displays the crucial difference to the case of an MPF. An MPF internalizes the strategic effect as it produces both varieties. Therefore, it will invest more in product differentiation as, on the one hand, it can increase output of both varieties and, on the other hand, the cannibalization effect across the two varieties is dampened. Obviously, the positive effect of investments in product differentiation on product innovation that was mentioned in proposition 1 cannot occur as the number of products is exogenously given.

Equilibrium Investment Levels in the Two Scenarios In the case with two single-product firms, profits of firm i in the second stage are given by

$$\pi_i = (p_i - c)\, x_i - r_\delta. \tag{62}$$

The demand system in Eq. (60), implies the following equilibrium profits under Cournot competition:

$$\pi_i = \left(\frac{(a-c)}{(2 + e(s))}\right)^2 - r_\delta. \tag{63}$$

Firm i can invest s units (at a rate r_s) in the degree of product differentiation $e(s)$, whereby the investment follows the assumptions made in Eq. (24). Therefore, profits of firm i in the R&D stage are given by:

$$\Pi_i = \pi_i - s_i r_s. \tag{64}$$

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The respective first-order condition for this maximization problem is

$$\frac{\partial \Pi_i}{\partial s_i} = \frac{\partial \pi_i}{\partial e} e'(s_i) - r_s = 0, \tag{65}$$

where $\frac{\partial \pi_i}{\partial e} = -\frac{2(a-c)^2}{((2+e(s)))^3} < 0$ and $e'(s_i) < 0$.

In the second scenario, the two products are offered by one MPF. The profits of the firm in the second stage are given by:

$$\pi = (p_1 - c) x_1 + (p_2 - c) x_2 - 2r_\delta. \tag{66}$$

Calculating the equilibrium profits yields:

$$\pi = \frac{(a-c)^2}{2(1+e(s))} - 2r_\delta. \tag{67}$$

In the R&D stage, the firm maximizes the following profit function

$$\Pi = \pi - sr_s \tag{68}$$

with respect to an optimal investment level s. The first-order condition is given by

$$\frac{\partial \Pi}{\partial s} = \frac{\partial \pi}{\partial e} e'(s) - r_s = 0, \tag{69}$$

where $\frac{\partial \pi}{\partial e} = -\frac{(a-c)^2}{2((1+e(s)))^2} < 0$.

Eqs. (65) and (69) implicitly determine the optimal investments for the two scenarios. From inspection of these equations, one observes that an MPF will invest more in the degree of differentiation as $\left|\frac{\partial \pi}{\partial e}\right|_{MPF} > \left|\frac{\partial \pi}{\partial e}\right|_{SPF}$. Since $((2+e(s)))^3 > 4((1+e(s)))^2$ for all $e(s) \in [0, 1]$, it follows that the marginal benefit for product differentiation is higher for MPFs. The latter implies that in the optimum: $|e'(s)|_{MPF} < |e'(s)|_{SPF}$. Figure 4 illustrates again the properties of the function $e(s)$ and helps with the interpretation of the result.

[Insert Figure 4 about here]

Lemma 4 *Even in the case of an exogenous product range of two products, MPFs will invest more in the degree of product differentiation than single-product firms. For a single-product firm, the marginal benefit of the investment is lower, as the investment is accompanied by a negative strategic effect. An MPF internalizes this effect as it produces both varieties.*

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Therefore, it will invest more in product differentiation as, on the one hand, it can increase output of both varieties and, on the other hand, the cannibalization effect across these two varieties is dampened.

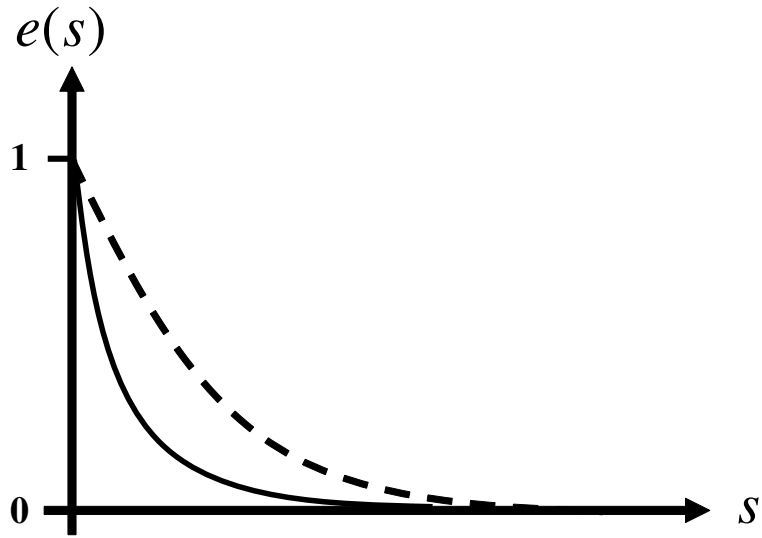Figure 1: Differences in the Ability to Innovate

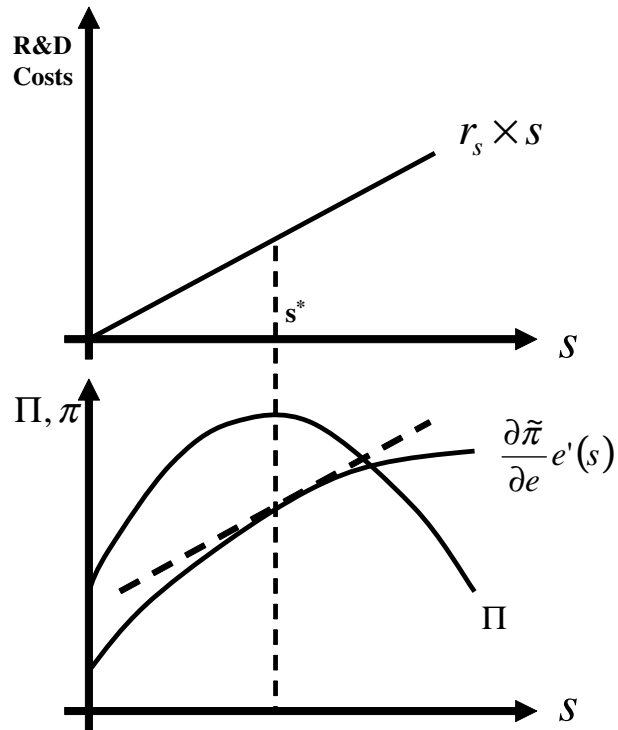$e(s)$

1

0

s

Figure 2: Optimal Degree of Product Differentiation

R&D Costs

$r_s \times s$

s^*

s

Π, π

$\dfrac{\partial \tilde{\pi}}{\partial e} e'(s)$

Π

s

Figure 3: Economies of Scale in Innovation

R&D
Costs

$r_s \times s$

s^* s^*_1

S

$\Pi, \tilde{\pi}$

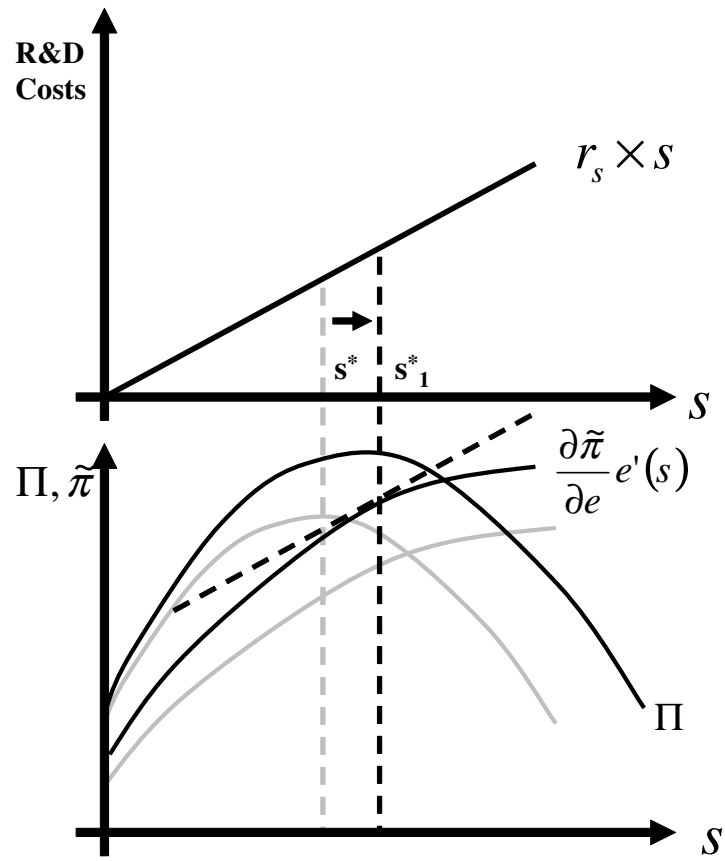$\dfrac{\partial \tilde{\pi}}{\partial e} e'(s)$

Π

S

Figure 4: Optimal Degree of Product Differentiation (SPF vs. MPF)