



GOVERNANCE AND THE EFFICIENCY  
OF ECONOMIC SYSTEMS  
**GESY**

Discussion Paper No. 467

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license auctions with  
downstream interaction

Cuihong Fan\*  
Byoung Heon Jun\*\*  
Elmar G. Wolfstetter\*\*\*

\* Shanghai University

\*\* Korea University, Seoul

\*\*\* Humboldt-University at Berlin and Korea University, Seoul

July 2014

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

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Speaker: Prof. Dr. Klaus M. Schmidt · Department of Economics · University of Munich · D-80539 Munich,  
Phone: +49(89)2180 2250 · Fax: +49(89)2180 3510

# Optimal bid disclosure in license auctions with downstream interaction\*

Cuihong Fan

Shanghai University of Finance and Economics  
School of Economics<sup>†</sup>

Byoung Heon Jun

Korea University, Seoul  
Department of Economics<sup>‡</sup>

Elmar G. Wolfstetter

Humboldt-University at Berlin and  
Korea University, Seoul<sup>§</sup>

July 2014

## Abstract

The literature on license auctions for process innovations in oligopoly assumed that the auctioneer reveals the winning bid and stressed that this gives firms an incentive to signal strength through their bids, to the benefit of the innovator. In the present paper we examine whether revealing the winning bid is optimal. We consider three disclosure rules: full, partial, and no disclosure of bids, which correspond to standard auctions. We show that more information disclosure increases the total surplus divided between firms and the innovator as well as social surplus. More disclosure also increases bidders' payoff. However, no disclosure maximizes the innovator's expected revenue.

**KEYWORDS:** Auctions, innovation, licensing, information sharing.

**JEL CLASSIFICATIONS:** D21, D43, D44, D45

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\*Comments and suggestions by Emanuelle Auriol and Vijai Krishna are gratefully acknowledged. Research support by the *National Research Foundation of Korea* funded by the Korean Government (NRF-2010-330-B00085), the *Deutsche Forschungsgemeinschaft (DFG)*, *SFB Transregio 15*, "Governance and Efficiency of Economic Systems", and the National Natural Science Foundation of China (Grant: 71371116) is gratefully acknowledged.

<sup>†</sup>Shanghai University of Finance and Economics, School of Economics, Guoding Road 777, 200433 Shanghai, China, Email: cuihongf@mail.shufe.edu.cn

<sup>‡</sup>Sung-Buk Ku An-am Dong 5-1, Seoul 136-701, Korea, Email: bhjun@korea.ac.kr

<sup>§</sup>Institute of Economic Theory I, Humboldt University at Berlin, Spandauer Str. 1, 10178 Berlin, Germany, Email: wolfstetter@gmail.com

# 1 Introduction

An outside innovator auctions the right to use a cost reducing, non-drastic innovation to a firm in a Cournot oligopoly. Should he choose an auction rule that discloses some or all bids prior to the oligopoly game? The present paper explores this issue and determines which of the standard auctions is optimal.

The recent literature on license auctions assumed that the innovator reveals the winning bid and stressed that, in a Cournot oligopoly, this induces firms to signal strength through their bids, which contributes to increase equilibrium bids. However, the literature never examined whether revealing the winning bid is actually optimal.

The analysis of license auctions in oligopoly was initiated by Kamien and Tauman (1986) and others who showed that the interests of the innovator are best served if he auctions a limited number of licenses (see the survey by Kamien, 1992). License auctions were shown to be more profitable than other selling mechanisms such as royalty licensing. One limitation of their analysis was the assumption that firms' cost reductions induced by the innovation are completely known to all firms prior to bidding.

Later, Jehiel and Moldovanu (2000) introduced incomplete information at the auction stage, but maintained the assumption that cost reductions become known after the auction and before the oligopoly game is played. This gap was closed by Das Varma (2003) and Goeree (2003) who assumed that firms can infer the winner's cost reduction only indirectly by observing the winning bid, which gives rise to a signaling issue.<sup>1</sup> They showed that, in a Cournot oligopoly,<sup>2</sup> the incentive to signal strength leads to pointwise higher equilibrium bids than in the benchmark model by Jehiel and Moldovanu (2000) where signaling is irrelevant because the oligopoly subgame is one of complete information.<sup>3</sup>

This comparison of equilibrium bid functions has been taken to imply that disclosing the winning bid increases the innovator's revenue. However, this confuses a comparison of equilibria across two *distinct* models - one in which the uncertainty is removed before the downstream game is played and one in which that uncertainty persists – with an analysis of the effect of bid disclosure *within* the model in which uncertainty persists.

The analysis of bid disclosure rules in license auctions also bears a relationship to the earlier literature on information sharing in oligopoly. That literature assumed

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<sup>1</sup>Similarly, our own contributions to licensing mechanism under incomplete information that award both unrestricted licenses and royalty licenses assumed that the innovator discloses the winning bid (see Fan, Jun, and Wolfstetter, 2013; Fan, Jun, and Wolfstetter, 2014).

<sup>2</sup>If Cournot is replaced by Bertrand competition and goods are imperfect substitutes, bidders have an incentive to signal weakness, which may prevent existence of a monotone increasing equilibrium bid function.

<sup>3</sup>Katzman and Rhodes-Kropf (2008) point out that disclosing the winning bid may adversely affect bidder participation and hence revenue.

that firms can commit to reveal their private information before they draw that information. The main finding was that in a Cournot oligopoly with substitutes firms have an incentive to reveal information concerning their private cost, whereas firms prefer not to reveal information concerning product demand (see Shapiro, 1986; Gal-Or, 1985; Vives, 1984; Vives, 1990). One limitation of that literature is the assumption that firms can commit to reveal information, good or bad, before it becomes available, and that the revealed information is verifiable.

However, directly or indirectly involving an intermediary, such as the innovator who auctions a license, facilitates the information exchange. Indeed, in license auctions the auctioneer can commit to indirectly reveal cost information by choosing an auction rule that reveals some bids or no bid. Information sharing is thus a byproduct of bidding, which also bypasses the verifiability required in the information exchange literature.

In the present paper we consider three unconditional bid disclosure rules: full disclosure which happens to be equivalent to disclosing the winning bid, partial disclosure which is equivalent to disclosing the second-highest bid, and no disclosure. These disclosure rules are intimately linked to standard auction formats, ranging from the Dutch auction, to the English auction, and to standard sealed-bid auctions (either first- or second-price).

Similar to the literature on information exchange in oligopoly we find that more information disclosure increases the total surplus as well as bidders' payoff. However, no disclosure maximizes the innovator's expected revenue. Hence, the different standard auctions are not revenue equivalent, and the innovator is well advised not to disclose any bids. Interestingly, this result is not significantly affected if we allow for a more general conditional disclosure rule in which the winning bid is disclosed only if it exceeds a certain threshold level.

Interestingly, in our analysis two kinds of signaling effects occur, to which we refer as first- and second-order signaling effects. The *first-order* signaling effect is the effect of player A's observed action on player B's belief about A's type; the *second-order* signaling effect is the effect of A's observed action on B's belief about A's belief about B's type. Whereas disclosing the winning bid entails a first-order signaling effect on the losers of the auction, disclosing the highest losing bid entails a second-order signaling effect on the winner of the auction because it allows the winner to update his beliefs concerning the beliefs of the highest losing bidder.

The role of bid disclosure rules has also been brought up in Hafalir and Krishna (2008) and Lebrun (2010) who consider an asymmetric first-price auction with resale. There, an auction may also be followed by downstream interaction, a resale,<sup>4</sup> and bid disclosure affects the equilibrium. However, whereas in our model, different disclosure rules entail different expected payoffs, in the auction with resale,

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<sup>4</sup>Whereas in our model the downstream interaction occurs independent of the outcome of the auction, in auctions with resale the downstream transaction occurs only if the outcome of the initial

the choice of disclosure rule neither affects the allocation nor expected payoffs (as shown by Lebrun, 2010), although it significantly affects the nature of the equilibrium.<sup>5</sup>

The plan of the paper is as follows. The model is stated in Section 2. We analyze the relevant duopoly subgames and the bidding games under full, partial, and no information disclosure in Sections 3 to 5. In Section 6 we provide intuitive interpretations for the differences between equilibrium bid functions across disclosure rules. In Section 7 we show that the different disclosure rules can be ranked unambiguously according to the total surplus (to be shared by firms and the innovator), bidders' payoffs, and the innovator's revenue, and we identify the optimal standard auction. Finally, in Section 8 we extend our analysis to allow for conditional disclosure rules and confirm the robustness of our results.

## 2 The Model

Suppose an outside innovator employs a standard auction to sell the exclusive right to use a non-drastic innovation to one of two firms. The innovator sets a disclosure rule that commits him to reveal some or all or no bids. After the outcome of the auction has been disclosed, the two firms play a Cournot duopoly game.

Because all standard auction formats are revenue equivalent (provided the auctioneer discloses the same information), we focus, without loss of generality, on first-price auctions.<sup>6</sup> We will, however, see that standard auctions are not revenue equivalent due to differences in the implied information disclosure.

Three unconditional bid disclosure rules are considered: the innovator either discloses

- the winning bid (full disclosure<sup>7</sup>), or
- only the losing bid (partial disclosure), or
- neither the winning nor the losing bid (no disclosure).

(In addition, we generalize and allow for conditional disclosure rules in which the winning bid is disclosed if and only if it exceeds a certain threshold level.)

The timing of the licensing game is as follows: 1) The innovator announces the bid disclosure rule. 2) Firms simultaneously submit their bids. 3) The innovator awards

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auction is inefficient. Inefficiency is notorious in asymmetric first-price auctions and does not occur in our symmetric framework.

<sup>5</sup>Whereas in our analysis a separating equilibrium exists for all possible bid disclosure rules, in auctions with resale a separating equilibrium exist only if either no bid or only the winning bid is disclosed (see Ch. 4, Krishna, 2002) and Hafalir and Krishna (2008) .

<sup>6</sup>Of course, one cannot have a second price auction with no information disclosure.

<sup>7</sup>Revealing the winning bid is as informative as revealing the winning *and* the losing bids.

the license to the highest bidder who pays his bid (the losing bidder pays nothing), and discloses information concerning bids according to the announced disclosure rule. 4) Firms play a homogeneous goods Cournot duopoly game.

Prior to the innovation, firms have the same unit cost  $c < 1/2$ . Using the innovation reduces unit costs by an amount  $x_i$  that depends on who uses it. Potential cost reductions are firms' private information, unknown to their rival and to the innovator. They are *i.i.d.* random variables, drawn from the *c.d.f.*  $F : [0, c] \rightarrow [0, 1]$ ,  $c > 0$ , with positive *p.d.f.* everywhere.

Firms and the innovator are risk neutral, inverse market demand is linear in aggregate output,  $Q$ ,  $P(Q) := 1 - Q$ , and the probability distribution of cost reductions  $F$  is the uniform distribution. The simplifying assumption of linear demand is also commonly used in the information sharing in oligopoly literature to be able to obtain closed-form solutions of expected payoffs (see, for example Shapiro, 1986; Vives, 1990; Raith, 1996).

### 3 Full disclosure

If equilibrium bid functions are strictly increasing (which we will confirm later), observing the winning bid reveals the winner's cost reduction to the losing bidder. The loser's cost is common knowledge. Therefore, if the innovator discloses the winning bid the innovator has revealed all relevant information, which is why we refer to this as full information disclosure.

The disclosure of the winning bid has a first-order signaling effect because it enables the loser to update his prior beliefs about the cost reduction of the winner. Bidders may thus have an incentive to strategically inflate their bids in order to signal strength. Of course, in equilibrium such "misleading" signaling is deterred.

In the following we solve the equilibrium expected payoffs of firms and the innovator. For this purpose we need to find the equilibrium bid function and firms' equilibrium profits in the duopoly subgames.

We employ the following procedure to solve the equilibrium bid function  $\beta_f$ . Consider one firm that unilaterally deviates from equilibrium bidding. We then state conditions concerning the  $\beta_f$  function that make deviations unprofitable. These conditions yield a unique  $\beta_f$  function.

Unilateral deviations from equilibrium bidding lead into duopoly subgames that are off the equilibrium path. Therefore, in order to compute the payoff of the firm that deviates from equilibrium bidding, we must first solve all duopoly subgames, on and off the equilibrium path.

### 3.1 Downstream duopoly “subgames”

Consider a firm, say firm 1, that had drawn the cost reduction  $x$  but bid  $\beta_f(z)$ , as if it had drawn cost reduction  $z$ , while the other firm had played the strictly increasing equilibrium bidding strategy,  $\beta_f$ . In the continuation duopoly game, the following “subgames” occur, depending upon the pretended cost reduction of firm 1,  $z$ , and the cost reduction parameter of firm 2, denoted by  $y$ .

#### 3.1.1 When firm 1 won the auction ( $z \geq y$ )

In that case firm 1 privately knows that its cost reduction is  $x$ , whereas firm 2 (the loser) believes that firm 1’s cost reduction is  $z$ . Therefore, firm 2 believes to play a duopoly subgame with unit costs  $(c_1, c_2) = (c - z, c)$ . Denote the associated equilibrium strategies of the game that the loser believes to play by

$$(q_W^*(z), q_L^*(z)) = \left( \frac{1 - 2c_1 + c_2}{3}, \frac{1 - 2c_2 + c_1}{3} \right). \quad (1)$$

Firm 1 anticipates that the loser plays  $q_L^*(z)$ . But because firm 1 privately knows that its cost reduction is  $x$  rather than  $z$  it plays the best reply:

$$q_W^f(x, z) = \arg \max_q (1 - q - q_L^*(z) - c + x)q = \frac{2 - 2c + 3x + z}{6}. \quad (2)$$

The reduced form profit function of firm 1, conditional on winning, is

$$\pi_W^f(x, z) := q_W^f(x, z)^2. \quad (3)$$

#### 3.1.2 When firm 1 lost the auction ( $z < y$ )

In that case firms play a duopoly subgame with unit costs  $(c_1, c_2) = (c, c - y)$ , with equilibrium strategies  $(q_L^*(y), q_W^*(y))$ .

Hence, the reduced form profit function of firm 1 conditional on losing is

$$\pi_L^f(y) := q_L^*(y)^2. \quad (4)$$

### 3.2 Equilibrium bid strategy

Using the above solution of the duopoly subgames, the expected payoff of a bidder with cost reduction  $x$  who bids as if his cost reduction were equal to  $z$ , while his rival follows the equilibrium strategy  $\beta_f$ , is:

$$\Pi_f(x, z) = F(z) \left( \pi_W^f(x, z) - \beta_f(z) \right) + \int_z^c \pi_L^f(y) dF(y). \quad (5)$$

For  $\beta_f$  to be an equilibrium, it must be such that  $x = \arg \max_z \Pi_f(x, z)$ . Using the first-order condition, one must have:

$$F'(x) \left( \pi_W^f(x, x) - \beta_f(x) \right) + F(x) \left( \partial_z \pi_W^f(x, x) - \beta_f'(x) \right) - F'(x) \pi_L^f(x) = 0, \quad (6)$$

which can be written in the form:

$$(\beta_f(x)F(x))' = F'(x) \left( \pi_W^f(x, x) - \pi_L^f(x) \right) + F(x) \partial_z \pi_W^f(x, x). \quad (7)$$

Integration of (7) yields:

$$\begin{aligned} \beta_f(x) &= \int_0^x \left( \pi_W^f(y, y) - \pi_L^f(y) \right) \frac{F'(y)}{F(x)} dy + \int_0^x \partial_z \pi_W^f(y, y) \frac{F(y)}{F(x)} dy \\ &= \frac{21(1-c)}{54}x + \frac{5}{27}x^2 \end{aligned} \quad (8)$$

Because  $\partial_{zx} \Pi_f(x, z) = (2-2c+3x+2z)/6c > 0$ , the function  $\Pi_f(x, z)$  is pseudoconcave; hence, the first-order conditions yield global maxima. Moreover,  $\beta_f(x)$  is strictly increasing which confirms the assumed monotonicity. Therefore,  $\beta_f(x)$  is the equilibrium bid function.

The equilibrium requirement (7) has a nice interpretation: whereas its RHS states the marginal benefit of a higher  $z$  its LHS states its marginal cost. In equilibrium, the bid function must be such that the marginal benefit equals the marginal cost, so that it does not pay to deviate from bidding  $\beta_f(x)$ , for all  $x$ .

The marginal benefit has two components: as  $z$  is increased, it becomes more likely to win rather than lose the auction (first term) and, in the event of winning, the rival is led to believe that he faces a stronger player, with a higher cost reduction, which makes him reduce his output – to the benefit of the winner. The latter reflects the fact that signaling strength confers a strategic advantage in the event of winning.

## 4 Partial disclosure

We now consider the case of partial information disclosure. Because revealing only the winning bid implies full information disclosure, partial information disclosure means disclosing only the losing bid.

If equilibrium bid functions are strictly increasing (which we will confirm later), the losing bid informs the winner about the loser's assessment of the winner's cost reduction. Therefore, revealing the losing bid has a second-order signaling effect.

A higher losing bid indicates to the winner that he is seen as stronger, which has an adverse effect on the loser's profit. Taking this into account, bidders have an incentive to strategically deflate their bids in order to “hide” the extent to which losing makes them more pessimistic.



In the following we solve the equilibrium expected payoffs of firms and the innovator. For this purpose we need to find the equilibrium bid function and firms' equilibrium expected profits in all duopoly subgames.

We employ the following procedure to solve the equilibrium bid function  $\beta_p$ . Consider one firm that unilaterally deviates from equilibrium bidding. We then state conditions concerning the  $\beta_p$  function that make deviations unprofitable. These conditions yield a unique  $\beta_p$  function.

Unilateral deviations from equilibrium bidding lead into duopoly subgames that are off the equilibrium path. Therefore, in order to compute the payoff of the firm that deviates from equilibrium bidding, we must first solve all duopoly subgames, on and off the equilibrium path.

#### 4.1 Downstream duopoly “subgames”

Suppose firm 1 has drawn the cost reduction  $x$  but bids  $\beta_p(z)$ , as if it had drawn cost reduction  $z$ , while firm 2 has played the strictly increasing equilibrium bid strategy,  $\beta_p$ . In the continuation duopoly game, the following “subgames” occur, depending upon the pretended cost reduction of firm 1,  $z$ , and the cost reduction of firm 2,  $y$ .

##### 4.1.1 When firm 1 won the auction ( $z > y$ )

In that case firm 1 privately knows that its cost reduction is  $x$ , whereas firm 2 (the loser) believes that firm 1's cost reduction is in the set  $(y, c]$ , and firm 1 knows this because it observes  $\beta_p(y)$ . Denote the equilibrium strategies by  $(q_W^p(x, y), q_L^p(y))$ . They must solve the following conditions:

$$q_W^p(x, y) = \arg \max_q q (1 - q - q_L^p(y) - c + x) \quad (9)$$

$$q_L^p(y) = \arg \max_q q \int_y^c (1 - q - q_W^p(x, y) - c) \frac{dF(x)}{1 - F(y)}. \quad (10)$$

This yields the equilibrium strategies and the reduced form profit function of firm 1, conditional on winning:

$$q_W^p(x, y) = \frac{1}{12} (4 - 3c + 6x + y) \quad (11)$$

$$q_L^p(y) = \frac{1}{6} (2 - 3c - y) \quad (12)$$

$$\pi_W^p(x, y) = q_W^p(x, y)^2. \quad (13)$$

##### 4.1.2 When firm 1 lost the auction ( $y > z$ )

In that case firm 1 believes that firm 2's cost reduction is in the set  $(z, c]$ , and firm 2 knows this.

By the above reasoning (reversing the roles of firms 1 and 2) we find that the equilibrium strategy of firm 1 is  $q_L^p(z)$  and that of firm 2 is  $q_W^p(y, z)$ . Therefore, the reduced form profit function of firm 1, conditional on losing, is

$$\pi_L^p(z) = q_L^p(z)^2. \quad (14)$$

## 4.2 Equilibrium bid strategy

Using the above solution of the duopoly subgames, the expected payoff of a bidder with cost reduction  $x$  who bids as if his cost reduction were equal to  $z$ , while his rival follows the equilibrium strategy  $\beta_p$ , is:

$$\Pi_p(x, z) = \int_0^z (\pi_W^p(x, y) - \beta_p(z)) dF(y) + (1 - F(z))\pi_L^p(z). \quad (15)$$

For  $\beta_p$  to be an equilibrium, it must be such that  $x = \arg \max_z \Pi_p(x, z)$ . Using the first-order condition, one must have:

$$F'(x) (\pi_W^p(x, x) - \beta_p(x)) - \beta_p'(x)F(x) + (1 - F(x))\pi_L^{p'}(x) - F'(x)\pi_L^p(x) = 0.$$

which can be written in the form:

$$(\beta_p(x)F(x))' = F'(x) (\pi_W^p(x, x) - \pi_L^p(x)) + (1 - F(x))\pi_L^{p'}(x). \quad (16)$$

Integration yields

$$\begin{aligned} \beta_p(x) &= \int_0^x (\pi_W^p(y, y) - \pi_L^p(y)) \frac{F'(y)}{F(x)} dy + \int_0^x \pi_L^{p'}(y) \frac{1 - F(y)}{F(x)} dy \\ &= \frac{24c - 9c^2}{432} + \frac{132 - 123c}{432}x + \frac{37}{432}x^2. \end{aligned} \quad (17)$$

Because  $\partial_{zx}\Pi_p(x, z) = 4 - 3c + 6x + z/12c > 0$ , the function  $\Pi_p(x, z)$  is pseudoconcave. Hence, the first-order conditions yield global maxima. Moreover,  $\beta_p(x)$  is strictly increasing which confirms the assumed monotonicity. Hence,  $\beta_p(x)$  is the equilibrium bid function.

The equilibrium requirement (16) has the following interpretation: its RHS states the marginal benefit of raising one's bid; the LHS states its marginal cost. In equilibrium, the bid function must be such that the marginal benefit equals the marginal cost, so that it does not pay to deviate from bidding  $\beta_p(x)$ , for all  $x$ .

The marginal benefit has two components: as  $z$  is increased from  $z$  to  $z' > z$  1) it becomes more likely to win rather than lose the auction (this is captured by the first term); 2) in the event of losing, the set of rivals' types (who win) is changed from  $(z, c]$  to  $(z', c]$ ; therefore, firm 1 infers that it faces a stronger rival whose average output is greater, which reduces firm 1's expected profit.

Like in the case of full disclosure, bidding has a signaling aspect. However, unlike in the case of full disclosure, a signal is sent only in the event of losing (rather than winning), the signaling effect is a second-order (rather than first-order) effect, and signaling entails an incentive to strategically deflate (rather than inflate) bidding.

## 5 No disclosure

If bids are not disclosed, updating of prior beliefs occurs only in response to winning and losing. The loser can infer a lower bound of the winner's cost reduction, and the winner can draw an inference concerning the loser's belief about the winner's cost reduction. These updated beliefs affect the play in the continuation duopoly subgames.

In order to solve the equilibrium bid function we first need to solve the duopoly subgames that may occur if a bidder unilaterally deviates from equilibrium bidding, on and off the equilibrium path of the bidding game.

Firms play output strategies conditional on either winning or losing.

### 5.1 Duopoly subgame on the equilibrium path of the game

Consider a firm with cost reduction  $x$  that faces a rival with (unknown) cost reduction  $y$ . Denote the equilibrium output strategies *on the equilibrium path* by  $q_W^{n^*}(x), q_L^{n^*}(x)$ . They must solve the following requirements:

$$q_W^{n^*}(x) = \arg \max_q q \int_0^x \left(1 - q - q_L^{n^*}(y) - c + x\right) \frac{F'(y)}{F(x)} dy \quad (18)$$

$$q_L^{n^*}(x) = \arg \max_q q \int_x^c \left(1 - q - q_W^{n^*}(y) - c\right) \frac{F'(y)}{(1 - F(x))} dy. \quad (19)$$

As one can easily confirm, these conditions have a linear solution:

$$q_W^{n^*}(x) = \frac{1}{45} (15 - 11c) + \frac{8}{15}x \quad (20)$$

$$q_L^{n^*}(x) = \frac{1}{45} (15 - 23c) - \frac{2}{15}x. \quad (21)$$

### 5.2 Duopoly subgames off the equilibrium path

Consider a firm with cost reduction  $x$  that unilaterally deviated from equilibrium bidding and bids  $\beta_n(z)$ , whereas the rival followed the equilibrium bid strategy. If that firm won the auction, its equilibrium output strategy,  $q_W^n(x, z)$ , solves the condition:

$$q_W^n(x, z) = \arg \max_q q \int_0^z \left(1 - q - q_L^{n^*}(y) - c + x\right) \frac{F'(y)}{F(z)} dy. \quad (22)$$

Whereas if it lost the auction, its equilibrium output strategy,  $q_L^n(z)$ , solves the condition:

$$q_L^n(z) = \arg \max_q \int_z^c \left(1 - q - q_W^*(y) - c\right) \frac{F'(y)}{(1 - F(z))} dy = q_L^*(z). \quad (23)$$

This yields:

$$q_W^n(x, z) = \frac{1}{45} (15 - 11c) + \frac{1}{2}x + \frac{1}{30}z \quad (24)$$

$$q_L^n(z) = \frac{1}{45} (15 - 23c) - \frac{2}{15}z \quad (25)$$

$$\pi_W^n(x, z) = q_W^n(x, z)^2 \quad (26)$$

$$\pi_L^n(z) = q_L^n(z)^2. \quad (27)$$

### 5.3 Equilibrium bid strategy

Using the above solution of the duopoly subgames, the expected payoff of a bidder with cost reduction  $x$  who bids as if his cost reduction were equal to  $z$ , while his rival follows the equilibrium strategy  $\beta_n$ , is:

$$\Pi_n(x, z) = F(z) (\pi_W^n(x, z) - \beta_n(z)) + (1 - F(z)) \pi_L^n(z). \quad (28)$$

For  $\beta_n$  to be an equilibrium, it must be such that  $x = \arg \max_z \Pi_n(x, z)$ . Using the first-order condition, one must have:

$$F'(x) (\pi_W^n(x, x) - \beta_n(x)) + F(x) (\partial_z \pi_W^n(x, x) - \beta_n'(x)) - F'(x) \pi_L^n(x) + (1 - F(x)) \pi_L^{n'}(x) = 0,$$

which can be written in the form:

$$(\beta_n(x)F(x))' = F'(x) (\pi_W^n(x, x) - \pi_L^n(x)) + F(x) \partial_z \pi_W^n(x, x) + (1 - F(x)) \pi_L^{n'}(x). \quad (29)$$

Integration of (29) yields:

$$\begin{aligned} \beta_n(x) &= \int_0^x (\pi_W^n(y, y) - \pi_L^n(y)) \frac{F'(y)}{F(x)} dy + \int_0^x \partial_z \pi_W^n(y, y) \frac{F(y)}{F(x)} dy + \int_0^x \pi_L^{n'}(y) \frac{1 - F(y)}{F(x)} dy \\ &= \frac{4c(15 - 11c)}{675} + \frac{375 - 347c}{1350}x + \frac{4}{45}x^2. \end{aligned} \quad (30)$$

Because  $\partial_{zx} \Pi_n(x, z) = (30 - 22c + 45x + 6z)/90c > 0$ , the function  $\Pi_n(x, z)$  is pseudoconcave; hence, the first-order conditions yield global maxima. Moreover,  $\beta_n(x)$  is strictly increasing, which confirms the assumed monotonicity. Therefore,  $\beta_n(x)$  is the equilibrium bid function.

The equilibrium requirement (29) has the following interpretation: the RHS states the marginal benefit of increasing one's bid and the LHS states its marginal cost. In equilibrium, the bid function must be such that the marginal benefit equals the marginal cost, so that it does not pay to deviate from equilibrium bidding  $\beta_n(x)$ , for all  $x$ .

The marginal benefit has three components: as  $z$  is increased to  $z'$ , 1) it becomes more likely to win rather than lose the auction (this is captured by the first term), 2) in the event of winning, the set of rival's types (who lose) is increased from  $[0, z]$  to  $[0, z']$ ; because losers' output is decreasing in their type parameter, it follows that the rival's average output diminishes, to the benefit of the winner (this is captured by the second term); 3) in the event of losing, the set of rivals' types (who win) is reduced from  $(z, c]$  to  $(z', c]$ ; therefore, one infers that one faces a rival who is on average stronger and produces higher output, which reduces the own expected profit.

## 6 Comparison of equilibrium bid functions

We now summarize and interpret the relationship between the equilibrium bid functions.

**Proposition 1.** *No disclosure implies more aggressive bidding than partial disclosure,  $\beta_n(x) > \beta_p(x)$ , whereas  $\beta_n(x) > \beta_f(x)$  for  $x$  below a threshold level  $\hat{x}$  and  $\beta_n(x) < \beta_f(x)$  for all  $x > \hat{x}$ .*<sup>8</sup>

*Proof.* 1) Compute  $\phi(x) := \beta_n(x) - \beta_p(x)$ , which is a strictly convex function of  $x$  that is decreasing and positive valued at  $x = c$ . Hence,  $\phi(x) > 0$  for all  $x \in [0, c]$ .

2) Compute  $\psi(x) := \beta_n(x) - \beta_f(x)$ , which is a strictly concave function of  $x$  that is decreasing and positive valued at  $x = 0$  and negative valued at  $x = c$ . Hence,  $\psi(x) = 0$  has exactly one root  $\hat{x} \in (0, c)$ .  $\square$

The relationship between equilibrium bid functions is illustrated in Figure 1.

The relationship between the bid functions can be interpreted by comparing the distinct terms of the bid functions. All bid functions have in common a component that reflects the profit premium of winning the license,  $E(\pi_W(X_{(2)}) - \pi_L(X_{(2)}) | X_{(2)} < x)$ .<sup>9</sup> This profit premium differs across disclosure rules. Focusing on the most relevant comparison between full and no disclosure, we find that this profit premium is higher under no disclosure than under full disclosure. This contributes to make  $\beta_n(x)$  greater than  $\beta_f(x)$ .

<sup>8</sup>Similarly,  $\beta_p$  intersects  $\beta_f$  from above exactly once.

<sup>9</sup> $X_{(2)}$  denotes the second highest order statistics of the sample of two *i.i.d.* random variables.

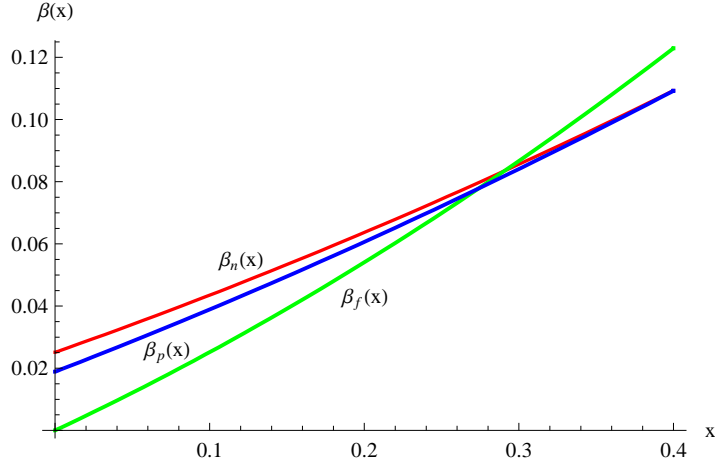


Figure 1: Equilibrium bid functions (assuming  $c = 0.4$ )

Interestingly, if the winning bid is not revealed, the profit premium is positive even for  $x = 0$ , which reflects in the positive intercepts of  $\beta_n$  and  $\beta_p$  in Figure 1. The reason for this paradoxical property is that the winner benefits from the uncertainty of the loser about the winner's cost reduction. Moreover, at  $x = 0$  the profit premium is higher under no than under partial disclosure. The reason is as follows: under partial disclosure the loser's type becomes common knowledge. Starting from no disclosure, the loser with the lowest cost reduction equal to 0 would like to inform the winner about his type because that would induce the winner to produce less output. Therefore,  $\pi_L^p(0) > \pi_L^n(0)$  and  $\pi_W^p(0,0) < \pi_W^n(0,0)$ . This explains why for low cost reduction the profit premium is higher under no disclosure than under partial disclosure, which contributes to make  $\beta_n(0) > \beta_p(0)$ , as depicted in Figure 1.

The  $\beta_f$  function has one other term which reflects the benefit from signaling strength that contributes to increase  $\beta_f$ . Whereas the  $\beta_n$  function has two other terms. Both of these terms reflect the benefit and cost of experimentation. Specifically, the term  $\partial_z \pi_W^n(x,x)F(x)$  represents the fact that, in the event of winning, a slightly inflated bid,  $\beta_n(z) > \beta_n(x)$ , informs the bidder that he faces a rival (the loser) whose type set is increased from  $[0,x]$  to  $[0,z]$ . Because the loser's output is decreasing in his type parameter, the outcome of this experimentation tells the bidder that his rival's average output is lower, which is why this term is positive. Similarly, the term  $\pi_L^{n'}(x)(1-F(x))$  represents the fact that in the event of losing a slightly inflated bid informs the bidder that he is facing a rival (the winner) whose type set is reduced from  $(x,c]$  to  $(z,c]$ . Because the winner's output is increasing in his type parameter, the outcome of this experimentation tells the bidder that his rival's average output is higher, which is why this term is negative.

Computing the difference between the signaling term in the  $\beta_f$  function and the sum of terms that reflect the benefit and cost of experimentation in the  $\beta_n$  function,

one finds that the signaling term exceeds the experimentation terms and that the difference between these terms is increasing in  $x$ . This explains why  $\beta_f(x) > \beta_n(x)$  for high  $x$ .

The relationship between the equilibrium bid functions does not indicate immediately which disclosure rule is optimal. However, the disclosure rules can be ranked unambiguously.

## 7 Optimal unconditional bid disclosure

We now analyze which of the above disclosure rules is optimal for the innovator and examine whether there is a conflict of interest between firms and the innovator. Because different disclosure rules affect the size of the total surplus, firms' preference order is not necessarily the opposite of the innovator's preference, and we also provide the ranking by total surplus.

The innovator's equilibrium expected revenues in the different regimes,  $R_i := \int_0^c \beta_i(x) 2F(x) dF(x)$ ,  $i \in \{f, p, n\}$ , are:

$$R_f = \frac{1}{54} (14c - 9c^2) \quad (31)$$

$$R_n = \frac{1}{2025} (555c - 389c^2) \quad (32)$$

$$R_p = \frac{1}{864} (224c - 145c^2). \quad (33)$$

The corresponding equilibrium expected profits of firms,  $\Pi_i^* := \int_0^c \Pi_i(x, x) dF(x)$ ,  $i \in \{f, p, n\}$ , are:

$$\Pi_f^* = \frac{1}{54} (6 - 15c + 14c^2) \quad (34)$$

$$\Pi_p^* = \frac{1}{288} (32 - 80c + 73c^2) \quad (35)$$

$$\Pi_n^* = \frac{1}{4050} (450 - 1155c + 1073c^2). \quad (36)$$

Using the above results, and the fact that the total surplus is equal to  $S_i := R_i + 2\Pi_i^*$ , it is easy to confirm the following rankings of disclosure rules.

**Proposition 2.** *The disclosure rules are ranked as follows:*

$$R_n > R_f > R_p \quad (\text{innovator's revenue ranking}) \quad (37)$$

$$\Pi_f^* > \Pi_p^* > \Pi_n^* \quad (\text{firms' ranking}) \quad (38)$$

$$S_f > S_p > S_n \quad (\text{surplus ranking}). \quad (39)$$

Evidently, more information improves efficiency. While firms also prefer more information, the innovator most prefers the least efficient regime of no information disclosure. This indicates a sharp conflict of interest, except that all parties agree that full disclosure is preferable to partial disclosure. Of course, the latter could not occur if the total surplus were not affected by the disclosure rule.

The ranking of disclosure rules by total surplus can be interpreted as follows. The expected total surplus can be written as  $S = E((1 - Q)Q) - E(cq_L + (c - X_{(1)})q_W)$ ,  $Q := q_W + q_L$ , which in turn can be rewritten as:<sup>10</sup>

$$S = (1 - E(Q))E(Q) - \text{Var}(Q) - \bar{C}, \quad \bar{C} := E(cq_L + (c - X_{(1)})q_W). \quad (40)$$

Using the above solution of  $q_W^i, q_L^i$  for  $i \in \{n, p, f\}$  one finds that the expected value of aggregate output is the same for all three disclosure rules, whereas the variance of aggregate output and the expected value of aggregate cost decrease as more information is disclosed:<sup>11</sup>

$$E(Q^n) = E(Q^p) = E(Q^f), \quad \text{Var}(Q^n) > \text{Var}(Q^p) > \text{Var}(Q^f), \quad \bar{C}^n > \bar{C}^p > \bar{C}^f.$$

This explains why the expected value of total surplus increases as more information is disclosed.<sup>12</sup> Moreover, because average output is not affected by the disclosure rule, consumer surplus is also unaffected. Therefore, the ranking of total surplus extends to the ranking of social surplus.

The intuition for the rankings of disclosure rules from the perspective of the innovator and bidders is less transparent. However, using the surplus and the innovator's revenue rankings it is easy to see why bidders least prefer the disclosure rule that is most preferred by the innovator, as follows:<sup>13</sup>

$$\Pi_n^* = \frac{1}{2}(S_n - R_n) > \frac{1}{2}(S_f - R_n) > \frac{1}{2}(S_f - R_f) = \Pi_f^*. \quad (41)$$

Disclosure rules are intimately connected to auction formats. In an open, descending bid (Dutch) auction the highest bid is automatically revealed to bidders, and in an open, ascending bid (English) and second-price sealed-bid auction the second highest bid is revealed to bidders, whereas in a first-price sealed-bid auction bids are invisible (unless the auctioneer chooses to disclose information). Because the considered auction formats are revenue equivalent if one controls for the disclosed information, we find the following revenue ranking of auction formats which indicates that revenue equivalence fails.

<sup>10</sup>Here,  $X_{(1)}$  denotes the largest order statistic of the sample of the two cost reductions.

<sup>11</sup>Specifically,  $E(Q) = (6-4c)/9$ ,  $\text{Var}(Q^f) = c^2/162$ ,  $\text{Var}(Q^p) = \text{Var}(Q^f) + 5c^2/864$ ,  $\text{Var}(Q^n) = \text{Var}(Q^p) + 19c^2/21600$ ,  $\bar{C}^f = (4c-5c^2)/9$ ,  $\bar{C}^p = \bar{C}^f + c^2/144$ ,  $\bar{C}^n = \bar{C}^p + c^2/2160$ .

<sup>12</sup>This interpretation is similar to Shapiro (1986) who compared the incentives for full vs. no disclosure assuming that firms can commit in advance to exchange verifiable information.

<sup>13</sup>Note, viewed from behind the "veil of ignorance", bidders' expected payoff is equal to one half of what is left of the total surplus after deducting the innovator's expected revenue.



**Corollary 1.** *The revenue ranking of standard auction formats in terms of the innovator's revenue is:*

$$1\text{-st price sealed-bid} \succ \text{Dutch} \succ \text{English or 2-nd price sealed-bid} . \quad (42)$$

*Hence, due to differences between the implied information disclosure, the standard auctions are not revenue equivalent and the 1-st price sealed-bid auction is the optimal auction for the seller.*

## 8 Extension: Conditional disclosure

A comparison of the equilibrium bid functions plotted in Figure 1 indicates that full disclosure induces the highest winning bids for high values of the winner's cost reduction, whereas no disclosure yields the highest winning bids for low values of the winner's cost reduction. This suggests that the innovator may wish to apply a conditional disclosure rule and disclose the winning bid if the winner's cost reduction is above a certain threshold level,  $\gamma$ , and not disclose any bid otherwise. However, if such a conditional disclosure rule is applied, the equilibrium bid functions will also be affected. Therefore, it is not clear without detailed analysis whether switching to such a conditional disclosure rule is profitable for the innovator.

In Appendix A we spell out the detailed analysis of the game subject to conditional disclosure. There, we first solve the game for a given threshold level  $\gamma$  and then compute the optimal  $\gamma$ . Of course, "no disclosure" and "full disclosure" are special cases of conditional disclosure, obtained by setting  $\gamma = c$  and  $\gamma = 0$ , respectively. Also, recall that "partial disclosure" is less profitable for the innovator than "no disclosure" and "full disclosure" (by Proposition 2). Therefore, the optimal disclosure rule is simply the optimal conditional disclosure rule.

The results of our analysis are illustrated in Figure 2. There, the solid curve is a plot of the optimal threshold level  $\gamma(c)$ . Evidently, for all cost parameters  $c < \hat{c} := 5/14$ ,  $\gamma(c) = c$ , i.e., the unconditional "no disclosure" is optimal for the innovator. Whereas, if  $c > \hat{c}$ ,  $\gamma(c) \in (0, c)$ , i.e., it is optimal for the innovator to disclose the winning bid if the winner's cost reduction is between  $\gamma(c)$  and otherwise to not disclose any bid. For example, if  $c = c'$ , the winner's cost reduction is disclosed only if it is in the narrow range between  $\gamma(c')$  and  $c'$ , which is indicated by the solid vertical line.

Altogether this indicates that allowing for conditional disclosure has only a relatively minor impact because typically "no disclosure" ( $\gamma(c) = c$ ) is optimal and even if  $\gamma(c) \in (0, c)$ , the winning bid is disclosed only in a narrow range of the winner's cost reductions.

Finally we mention that conditional disclosure is feasible only if the auctioneer employs a sealed bid auction. In an ascending-bid (English) auction, the innovator

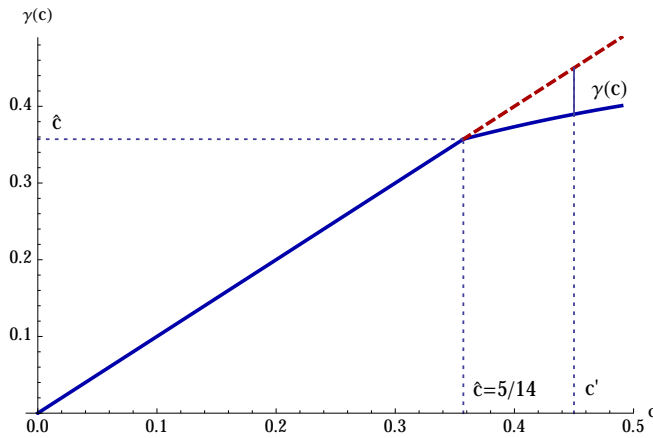


Figure 2: The optimal threshold level for conditional disclosure  $\gamma(c)$

cannot possibly reveal the winning bid because he cannot even observe it, and in a descending-bid (Dutch) auction, the winning bid is automatically revealed.

## 9 Conclusions

The present paper contributes to the literature on license auctions with downstream interaction among bidders and the literature on information exchange in oligopoly.

Compared to the information exchange literature, we show that in a license auction information exchange is implied by the publication of bids. Information exchange does not require that firms commit to exchange information, good and bad, and that information is verifiable by the recipient. The innovator can easily commit to administer the exchange of information simply by choosing a particular auction rule, such as an open, descending-bid (Dutch) auction. The innovator may thus be viewed as a mediator who indirectly administers the information exchange between bidders by choosing a particular open bid auction format. If the innovator were an impartial mediator who pursued the interests of bidders, he would apply an open auction format that reveals the winning bid. However, as the innovator pursues his own agenda, it is in his own best interest not to reveal any bid.

Compared to the literature on license auctions with downstream interaction, we show that revealing the winning bid is not optimal for the innovator. True, revealing the winning bid gives rise to a signaling benefit that contributes to increase bids. However, this is not all that matters to determine which disclosure rule maximizes the innovator's expected revenue. Not revealing any bid considerably increases the premium of winning, because it makes winning valuable even if the winner has a minimal cost reduction. Moreover, if no bid is revealed, there is an experimentation benefit that contributes to increase equilibrium bids. Altogether, these effects on

bidding are sufficiently high to outweigh the signaling benefit of disclosing the winning bid. In addition we find that these results are not significantly affected if we allow for more general conditional disclosure rules.

## A Appendix

Here we spell out the extension of our analysis to conditional disclosure rules, which was summarized in Section 8. Conditional disclosure means that the innovator discloses the winning bid if the winner's cost reduction is at or above the threshold level  $\gamma \in [0, c]$  and does not disclose any bid otherwise. We analyze the equilibrium for a given  $\gamma$ , and then find the optimal  $\gamma$  that maximizes the innovator's expected revenue.

In order to find the equilibrium bid function for a given  $\gamma$ , we use the following procedure: 1) First, we construct the bid function  $\beta_n$  that makes "small" unilateral deviations from bidding  $\beta_n(x)$  unprofitable, for all  $x < \gamma$ . Similarly, we construct the bid function  $\beta_f$  that makes "small" deviations from bidding  $\beta_f(x)$  unprofitable for all  $x \geq \gamma$ . Thereby "small" means that the deviation does not induce a change in disclosure regime, either from full disclosure ( $f$ ) to no disclosure ( $n$ ) or from  $n$  to  $f$ . Second, we show that the bid function:

$$\beta(x) = \begin{cases} \beta_n(x) & \text{if } x < \gamma \\ \beta_f(x) & \text{if } x \geq \gamma \end{cases} \quad (\text{A.1})$$

makes "large" deviations, that may induce a regime change, also unprofitable. Therefore,  $\beta$  is the equilibrium bid function.

### A.1 Downstream duopoly "subgames"

Unlike in the analysis in the main text, when the winning bid has not been disclosed, the loser infers that the winner's cost reduction is below  $\gamma$ , and this is common knowledge. Therefore, one must distinguish between the duopoly subgames with full disclosure and no disclosure. The subgames with full disclosure are the same as in the main text, whereas the subgames with no disclosure are slightly changed, due to the fact that the loser infers that the winner's cost reduction is below  $\gamma$ .

Because the analysis is a straightforward adaptation of the analysis in the main text, we only state the equilibrium outputs and profits for the subgames with no disclosure:

$$q_W^{n*}(x) = \frac{15(1-c) + 4\gamma + 24x}{45}, \quad q_L^{n*}(x) = \frac{15(1-c) - 8\gamma - 6x}{45} \quad (\text{A.2})$$

$$q_W^n(x, z) = \frac{30(1-c) + 8\gamma + 45x + 3z}{90}, \quad q_L^n(z) = \frac{15(1-c) - 8\gamma - 6z}{45} \quad (\text{A.3})$$

$$\pi_W^n(x, z) = q_W^n(x, z)^2, \quad \pi_L^n(z) = q_L^n(z)^2. \quad (\text{A.4})$$

## A.2 Equilibrium bid strategy

**Bid function  $\beta_n$**  Using the above solution of the duopoly subgames, the expected payoff of a bidder with cost reduction  $x$  who bids as if his cost reduction were equal to  $z < \gamma$ , while his rival follows the equilibrium strategy  $\beta$ , is:

$$\Pi_n(x, z) = F(z) \left( \pi_W^n(x, z) - \beta_n(z) \right) + \left( F(\gamma) - F(z) \right) \pi_L^n(z) + \int_\gamma^c \pi_L^f(y) dF(y).$$

For  $\beta_n$  to be an equilibrium, it must be such that  $x = \arg \max_z \Pi_n(x, z)$ . Using the first-order condition, one must have:

$$\left( \beta_n(x) F(x) \right)' = F'(x) \left( \pi_W^n(x, x) - \pi_L^n(x) \right) + F(x) \partial_z \pi_W^n(x, x) + \left( F(\gamma) - F(x) \right) \pi_L^{n'}(x).$$

Integration yields:

$$\begin{aligned} \beta_n(x) &= \int_0^x \left( \pi_W^n(y, y) - \pi_L^n(y) \right) \frac{F'(y)}{F(x)} dy + \int_0^x \partial_z \pi_W^n(y, y) \frac{F(y)}{F(x)} dy \\ &\quad + \int_0^x \pi_L^{n'}(y) \frac{F(\gamma) - F(y)}{F(x)} dy \\ &= \frac{4\gamma(15(1-c) + 4\gamma)}{675} + \frac{375(1-c) + 28\gamma}{1350} x + \frac{4}{45} x^2. \end{aligned} \quad (\text{A.5})$$

**Bid function  $\beta_f$**  Similarly, the expected payoff of a bidder with cost reduction  $x$  who bids as if his cost reduction were equal to  $z > \gamma$ , while his rival follows the equilibrium strategy  $\beta$ , is:

$$\Pi_f(x, z) = F(z) \left( \pi_W^f(x, z) - \beta_f(z) \right) + \int_z^c \pi_L^f(y) dF(y).$$

For  $\beta_f$  to be an equilibrium, it must be such that  $x = \arg \max_z \Pi_f(x, z)$ . Using the first-order condition, one must have:

$$\left( \beta_f(x) F(x) \right)' = F'(x) \left( \pi_W^f(x, x) - \pi_L^f(x) \right) + F(x) \partial_z \pi_W^f(x, x).$$

Integration yields:

$$\begin{aligned} \beta_f(x) &= \frac{\beta_f(\gamma) F(\gamma)}{F(x)} + \int_\gamma^x \left( \pi_W^f(y, y) - \pi_L^f(y) \right) \frac{dF(y)}{F(x)} + \int_\gamma^x \partial_z \pi_W^f(y, y) \frac{F(y)}{F(x)} dy \\ &= \beta_f(\gamma) \frac{\gamma}{x} + \frac{x - \gamma}{54x} \left( 21(1-c)(x + \gamma) + 10(x^2 + \gamma x + \gamma^2) \right) \end{aligned} \quad (\text{A.6})$$

where, using the indifference condition  $\Pi_f(\gamma, \gamma) = \Pi_n(\gamma, \gamma)$ ,

$$\beta_f(\gamma) = \frac{107(1-c)}{270} \gamma + \frac{386}{2025} \gamma^2. \quad (\text{A.7})$$

So far we have only required that “small” unilateral deviations in  $z$  from  $x$ , that do not induce a change in the disclosure regime (either from  $f$  to  $n$  or from  $n$  to  $f$ ), are not profitable. We now show that the above bid functions, that have been designed to rule out profitable small deviations, assure that “large” deviations are not profitable either.

For this purpose denote firms’ expected profit by

$$\Pi(x, z) = \begin{cases} \Pi_f(x, z) & \text{if } z \geq \gamma \\ \Pi_n(x, z) & \text{if } z < \gamma. \end{cases} \quad (\text{A.8})$$

We consider the two profiles of “large” deviations:  $x < \gamma < z$  and  $x > \gamma > z$ , and show that in both cases  $\Pi(x, z) < \Pi(x, x)$ .

In order to prepare the proof, note that  $\Pi_f(\gamma, \gamma) = \Pi_n(\gamma, \gamma)$ , and that

$$\begin{aligned} \partial_{xz}\Pi_f(x, z) &= \frac{2(1-c) + 3x + 2z}{6c} > 0, \quad \text{and} \\ \partial_{xz}\Pi_n(x, z) &= \frac{30(1-c) + 45x + 6z + 8\gamma}{90c} > 0. \end{aligned}$$

Thus, one has

$$\partial_z\Pi_f(x, z), \partial_z\Pi_n(x, z) < 0 \quad \text{for } z > x, \quad \text{and} \quad (\text{A.9})$$

$$\partial_z\Pi_f(x, z), \partial_z\Pi_n(x, z) > 0 \quad \text{for } z < x. \quad (\text{A.10})$$

Moreover,

$$\Pi_f(x, \gamma) - \Pi_n(x, \gamma) = \frac{2\gamma^2(x - \gamma)}{45c}. \quad (\text{A.11})$$

Hence, by combining all of the above,

$$\Pi(x, x) > \Pi_n(x, \gamma) > \Pi_f(x, \gamma) > \Pi_f(x, z) = \Pi(x, z) \quad \text{for } x < \gamma < z \quad \text{and} \quad (\text{A.12})$$

$$\Pi(x, z) < \Pi_n(x, \gamma) < \Pi_f(x, \gamma) < \Pi_f(x, x) = \Pi(x, x) \quad \text{for } x > \gamma > z. \quad (\text{A.13})$$

This completes the proof that  $\beta$  is the equilibrium strategy.

### A.3 The innovator’s expected revenue

The innovator’s expected revenue is equal to:

$$\begin{aligned} R(\gamma) &= \int_0^\gamma \beta_n(x) 2F(x) dF(x) + \int_\gamma^c \beta_f(x) 2F(x) dF(x) \\ &= \frac{(14-9c)c}{54} + \frac{2(1-c)}{135c} \gamma^2 + \frac{22}{2025c} \gamma^3 - \frac{29}{1350c^2} \gamma^4. \end{aligned} \quad (\text{A.14})$$

which is a polynomial function of  $\gamma$  of order 4.  $\partial_\gamma R$  is a cubic function of  $\gamma$  and the first order condition  $R'(\gamma) = 0$  has three roots:  $\gamma_1 = 0$ , and  $\gamma_2, \gamma_3$ :

$$\gamma_2 = \frac{1}{58} \left( 11c - \sqrt{c(1160 - 1039c)} \right) \quad (\text{A.15})$$

$$\gamma_3 = \frac{1}{58} \left( 11c + \sqrt{c(1160 - 1039c)} \right). \quad (\text{A.16})$$

Evidently,  $\gamma_2 < 0$  and  $R(\gamma_3) > R(\gamma_1)$ ; hence, the only solution is  $\gamma_3$ . Hence, the innovator's revenue is maximized at  $\gamma(c) = \min\{c, \gamma_3\}$ . Because  $c < \gamma_3$  if and only if  $c < (5+4d)/14$ , we conclude that the innovator's revenue is maximized at

$$\gamma(c) = \begin{cases} c, & \text{if } c < 5/14 \\ \gamma_3, & \text{otherwise.} \end{cases} \quad (\text{A.17})$$

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