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Endogenously-timed Herding and the Synchronization of Investment Cycles

Bernd Süssmuth[¤]
Department of Economics, University of Munich

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Abstract. This paper combines the recent game theoretic approach of endogenous timing of entry to herding models with a macroeconomic model of investment cycles. The integrated description embodies the qualitative results of the myopic herding model in a medium run investment objective of smoothing the capital stock adjustment process. It features a completely disaggregated structure and bears the potential to synchronize individual cyclic investing behaviors. This synchronization via nonlinear feedback from the aggregate activity can serve as an explanation of the inexistent cancelling of heterogeneous sectoral quasi-cycles. The model o®ers an explanatory base for the constitution of the observed strong cyclicality of the aggregate investment series by a multitude of di®erent periodicities and phases on the individual level. Finally, based on recent ¯ndings of the herding literature, the stabilization potential of third parties' information revelation is conjectured.

[¤]correspondence:

SELAPO, University of Munich, Ludwigstr. 28/RG, D-80539 Munich, Germany

phone: ++49 89 2180 2136, fax: ++49 89 336392

e-mail: smuth@rz.uni-muenchen.de

1. Introduction

Since the introduction of the idea by Forrester (1977) several authors[1] have suggested a nonlinear synchronization mechanism in form of `mode-locking' as an explanatory baseline concept for cyclic co-movements between di®erent ¯rms, industrial sectors, economic regions, national economies, or various economic aggregates, see Hillinger and Weser (1988), Mosekilde et al. (1992, 1993), Larsen et al. (1993), Kampmann et al. (1994), Sterman and Mosekilde (1994), Haxholdt et al. (1995), Krugman (1996), Brenner, Weidlich and Witt (1998), Focardi and Marchesi (1998) and Selover and Jensen (1999). All of these contributions have in common to outline the ability of the mode-locking approach to synchronize economic cyclicalities in a deterministic or combined deterministic-stochastic environment, but widely lack a micro-theoretical argumentation behind their models. Therefore a central aim of this paper is to demonstrate that investment delay as a result of myopic herding phenomena, as described in the literature on social learning and investment under uncertainty, is able to serve as the micro-theoretical underpinning of a macroeconomic model of investment cycles' synchronization. The model is built up by combining the short-term investment objective of triggering investment decisions by herding with a medium run objective of smoothing the capital adjustment process. The crude principle of the triggering mechanism consists in the balancing of the option value of waiting and thereby gaining information by observing others' investment activities with the costs of this delay. Herding models with endogenous timing are not only concerned with this triggering of a project, but also with its timing. In other words the myopic triggering problem operates in two dimensions: whether to invest in a project and if so when to invest. This model of timing the triggering of an investment project is used to endogenize the rate of capital adjustment of the medium run investment decision, as described by a second order accelerator equation. The derived model features a completely disaggregated structure. It holds the potential to synchronize individual cyclic investing behaviors via nonlinear feedback from the aggregate activity. This synchronization mechanism can serve as an explanation of the inexistence of the expected cancelling of di®erent individual or sectoral cyclicalities in the °uctuating aggregate investment series that we observe. Furthermore, it is argued that, despite a synchronization through aggregate or key sectoral shocks, less phase locking or synchronization of constituent quasi-cycles and thereby a stabilization of the aggregate behavior can be achieved via third parties' surprise information revelation about the pro¯tability of major investment opportunities.

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[1] Remarkably many of them with an `econo-physics' background.

2. Empirical Indications of Synchronization and Herding

As mentioned above research areas where synchronization of economic cycles is suggested in the literature or might play a role are a) regional or national business cycles, cf. Krugman (1996), Brenner, Weidlich and Witt (1998) or Selover and Jensen (1999), b) different cyclic phenomena like construction cycles and economic long waves, cf. the numerous publications of the research group around John Sterman and Erik Mosekilde, or c) industrial investment cycles on which we will focus in the present paper. Recently, a first attempt to investigate empirically the mode-locking hypothesis in the context of international business cycle co-movements is made in the contribution by Selover and Jensen (1999). They analyze the implication of the mode-lock concept that nonlinear entrainment of the deterministic parts of the observed fluctuations would lead to a setting into phase of the constituting cycles. The crucial point is that this drawing together of different phases does according to a phase-lock model not happen instantaneously, as would be the case for a common shock scenario, but rather develop over time until it would reach its full impact. An adequate technique to check this phenomenon is multivariate spectral analysis. Table 1a displays the findings of Selover and Jensen (1999) for monthly industrial production index series of seven major industrial countries. Their strategy consisted in dividing the sample into three sub-intervals, namely an early period (1958:01 - 1973:09), followed by the turbulent oil price shocks period (1973:10 - 1979:12), and finally a late period (1980:01 - 1995:12). In the focus of their investigations are the early and late period while the period including the two major oil shocks and the change in exchange rate regimes in 1973 is left out. The spectral analytic findings displayed in Table 1a essentially report two main results: Firstly there seems to be a tendency toward a similar periodicity of the national cycles, i.e. there appears to be some synchronization, secondly this synchronization appears to have increased after the turbulent years in the mid and late early seventies. Selover and Jensen (1999) note: \This synchronization, with its convergence in frequency and in phase angles is evidence in support of the mode-locking hypothesis." Table 1b reports features of the distribution of estimates of the phase shift of 450 US SIC[2] 4-digit industrial investment series relative to the aggregate investment series. The phase lead or lag is estimated at the period length corresponding to the maximum of coherency between the respective disaggregated and the aggregate periodicity. Again the sample is divided into early and late period, and a second narrowed definition of the late period is considered to check the robustness of results. The results show that the modal value of estimated phase shifts, covered by around one fifth of all sectors, more than halves from early to late period and tends to take on a zero-value, made up by 40%

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[2]Standard Industrial Classification

(in de¯nition I of late period) up to more than 50% (in de¯nition II of late period) of all sectoral series. The means of the absolute values[3] of estimated phase shifts point in the same direction. Altogether, these ¯ndings give grounds for the phase-locking hypothesis in the context of sectoral investment cycles, too.

Table 1a. Log-di®erence-¯ltered industrial production indexes

Country	Phase (radians)	Period length (months)
Early period		
1958:01 – 1973:09, n= 189		
Canada	+ 3.12	31.5
France	¡ 1.51	31.5
Germany	+ 1.44	47.0
Japan	+ 2.13	63.0
Netherlands	+ 1.68	47.0
UK	¡ 1.87	37.8
US	¡ 3.01	31.5
Late period		
1980:01 – 1995:12, n= 192		
Canada	¡ 1.46	64.0
France	¡ 1.40	48.0
Germany	¡ 0.57	48.0
Japan	¡ 0.98	48.0
Netherlands	¡ 0.65	48.0
UK	¡ 2.26	48.0
US	¡ 1.54	48.0

Source: Selover and Jensen (1999)

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[3]Taking absolute values ensures that phase leads and lags do not cancel out in the course of the computation of mean values.

Table 1b. Baxter/King-¯ltered SIC 4-digit investment series

Distribution of phase shift at maximum of coherency (degrees)

	mode of phase shifts ¹	(absolute phase shifts)
Early period 1962:1 - 1973:4, n= 48	¡ 1.5 (22%)*	4.45
Late period, def. I 1980:1 - 1990:1, n= 41	¡ 0.4 (40%)*	3.97
Late period, def. II 1983:1 - 1990:1, n= 29	+ 0.3 (53%)*	2.89

* values in brackets denote relative frequencies of sectors for which the modal value was estimated.

Data: NBER Manufacturing Productivity Database

As in the case of the phase-lock phenomenon, papers empirically investigating herding behaviors as predicted by theoretical models, i.e. trying to directly test implications of herding models besides empirically con¯rming clustering, are still an exception, see e.g. Graham (1999).[4]

3. The Model

Let us consider $i = 1; ::::; N$ investing industries or ¯rms, each (su±ciently homogeneous to be) represented by a single decision maker i. Our model has two dimensions: a medium run investment objective and a myopic one. We will start our outline with the medium and long run objective of the decision makers, before going over to describe the \moving in the margin", i.e., the myopic behavior of timing the triggering of a certain investment project. The results of the myopic model will have direct implications for the previously outlined medium run model and will be implemented in an integrated model where the rate of capital adjustment is endogenized, i.e., for some of the individuals the adjustment coe±cients will depend on the overall aggregate behavior.

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[4]In contrast to the class of herding models that will be considered in the present paper, Graham (1999) empirically investigates herding models without an endogenously-timing structure.

3.1. The Medium-Term Investment Objective. For the moment, the long run components of the model are assumed to be exogenously given and left out of the explanation. This holds true for the desired fixed capital stock K_i^π of a firm as well as for its time derivative. They are regarded as products of long-term planning and assumed to be exogenously given. Therefore the analysis is limited to the minimization of costs due to capital deficit and adjustment processes arising in the medium-run investment behavior. All costs are formulated in terms of deviations k_i of the actual capital K_i from its desired value K_i^π; i.e., $k_i = K_i^\pi - K_i$, $k_i^0 = \frac{d}{dt}(K_i^\pi - K_i)$, etc. These costs may be expressed as a sum of contributions each quadratic in k_i, $k_i^0 = i_i$ and $k_i^{00} = i_i^0$, where $i_i = k_i^0$ denotes the individual investment deficit. The microeconomic underpinning of the medium run model of this section is an intertemporal optimization calculus in the presence of these three quadratic adjustment cost components outlined in more detail in the following three assumptions:

Assumption 1. Costs arise due to the individual capital deficit or excess of capital: $k_i > 0$ expresses excess capital of an industry leading to inflexibility and enhanced depreciation causing costs.
$k_i < 0$ stands for an underequipment with capital, i.e., a capital deficit of a firm. This leads to missing production possibilities and excessive capacity utilization.

Assumption 2. Costs arise due to changes of the individual capital stock:
$k_i^0 > 0$ and $k_i^0 < 0$ mark situations where imperfect substitutability with other production factors leads to costs or, in the case of close to perfect substitutability, leads to a sufficient use of the other input factors and thereby cause costs.

Assumption 3. Costs arise due to changes of the individual investment strategy:
$k_i^{00} = i_i^0 > 0$ and $k_i^{00} = i_i^0 < 0$ reflect changes in the medium run investment strategy of a firm. Changes of contractual commitments and supplier's arrangements are cost-bearing consequences of this behavior.

Taking all the components expressed in these assumptions into account we can formulate the following cost function for investor i:[5]

$$C_i\left(k_i; k_i^0; k_i^{00}\right) = \beta k_i^2 + \gamma_i\left(k_i^0\right)^2 + \delta_i\left(k_i^{00}\right)^2. \tag{1}$$

Every firm determines the time paths of her production capital in such a way that the present value of all potential medium-term cost components, i.e. (1) discounted with an appropriate discount rate, is minimized. Since we are concerned with a

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[5]The derivation and development of the model widely follows Hillinger, Reiter and Weser (1992).

medium-term objective focusing, e.g., on labor force's training costs (assumption 2) or institutional costs due to changing suppliers (assumption 3), etc., we are confronted with a situation of imperfect foresight. Hence, the relevant discount factor for an intertemporal optimization calculus is the medium-term discount rate $\frac{1}{2}^n$, where naturally $\frac{1}{2}^n > \frac{1}{2}$, i.e., the long-term discount rate $\frac{1}{2}$ (as reflected in the market discount rate) usually takes on lower values than $\frac{1}{2}^n$:

$$\min_{k_i(t)} \int_{t_0}^{1} \left(\circledR k_i^2 + {}^-_i \left(\dot{k}_i^0 \right)^2 + {}^\circ_i \left(\dot{k}_i^\infty \right)^{2\flat} \right) e^{\frac{1}{2}^n (t_0 i\, t)} dt. \tag{2}$$

Given the initial values for the individual capital and investment components $K_i(t_0)$ and $I_i(t_0)$, the relevant transversality conditions are:

$$\lim_{t! \, 1} i_i e^{\frac{1}{2}^n (t_0 i\, t)} = k_i^0 e^{\frac{1}{2}^n (t_0 i\, t)} = 0,$$
$$\lim_{t! \, 1} i_i^0 e^{\frac{1}{2}^n (t_0 i\, t)} = k_i^\infty e^{\frac{1}{2}^n (t_0 i\, t)} = 0, \tag{3}$$
$$\lim_{t! \, 1} i_i^\infty e^{\frac{1}{2}^n (t_0 i\, t)} = k_i^{\infty} e^{\frac{1}{2}^n (t_0 i\, t)} = 0.$$

These transversality conditions ensure that investment and its time derivatives do not take place faster than firms discount the future medium-term time horizon. From (2) and (3), we are able to derive the following Euler equation for $k_i(t)$ by means of standard techniques of variational analysis:

$$\circledR k_i + {}^-_i \left(\frac{1}{2}^n i\, \frac{d}{dt} \right) k_i^0 + {}^\circ_i \left(\frac{1}{2}^n i\, \frac{d}{dt} \right)^2 k_i^\infty = 0. \tag{4}$$

We find the characteristic polynomial of this fourth order differential equation in k_i to be:

$$P(x) = \circledR + {}^-_i \left(\frac{1}{2}^n i\, x \right) x + {}^\circ_i \left(\frac{1}{2}^n i\, x \right)^2 x^2. \tag{5}$$

$P(x)$ is obviously quadratic in $y = \left(\frac{1}{2}^n i\, x \right) x$, so that applying the quadratic formula, we get the following two solutions for (5) in terms of y:

$$y_{1;2} = \frac{1}{2^\circ_i} \left(i\, {}^-_i \S \sqrt{{}^{-2}_i i\, 4\circledR^\circ_i} \right). \tag{6}$$

Solving $y = \left(\frac{1}{2}^n i\, x \right) x = \frac{1}{2}^n x\, i\, x^2$ for x and substituting (6) in, leads us to the following potential solutions of (5) that fulfill the transversality conditions:

$$x_{1;2} = \frac{\frac{1}{2}^n}{2} i\, \sqrt{ \frac{(\frac{1}{2}^n)^2}{4} + \frac{1}{2^\circ_i} \left(i\, {}^-_i \S \sqrt{{}^{-2}_i i\, 4\circledR^\circ_i} \right) }$$

$$= \frac{\frac{1}{2}\eta^2}{2} 4 1 \mathbf{j} \left[1 + \frac{4}{2^{\circ}{}_i \, (\frac{1}{2}\eta)^2} - \mathbf{j} \S \sqrt{\bar{}^2{}_i \mathbf{j} \; 4 \circledR{}^{\circ}{}_i} \; 5 \right]^3 \tag{7}$$

$$= \frac{\frac{1}{2}\eta^2}{2} 4 1 \mathbf{j} \left[1 + \frac{2}{^{\circ}{}_i \, (\frac{1}{2}\eta)^2} - \mathbf{j} \S \sqrt{\bar{}^2{}_i \mathbf{j} \; 4 \circledR{}^{\circ}{}_i} \; 5 \right]^3$$

Obviously, the solutions are oscillatory for $\bar{}^2{}_i < 4\circledR{}^{\circ}{}_i$, i.e., su± ciently large values of the cost parameters associated with discrepancies in the individual capital stock and with changes in the investment strategy. Approximately, we can simplify equation (7) as:[6]

$$x_{1;2} \; \frac{1}{4} \; \frac{\frac{1}{2}\eta}{2} \left[1 \mathbf{j} \left(1 + \frac{1}{^{\circ}{}_i \, (\frac{1}{2}\eta)^2} - \mathbf{j} \S \sqrt{\bar{}^2{}_i \mathbf{j} \; 4 \circledR{}^{\circ}{}_i} \right)^\# \right]$$

$$\frac{1}{4} \; \frac{1}{2^{\circ}{}_i \frac{1}{2}\eta} - \mathbf{j} \S \sqrt{\bar{}^2{}_i \mathbf{j} \; 4 \circledR{}^{\circ}{}_i} \; . \tag{8}$$

The values of $x_{1;2}$ given by (8) are the roots of the following polynomial:

$$\circledR{} + \bar{}_i \frac{1}{2}\eta \, x + ^{\circ}{}_i \, (\frac{1}{2}\eta)^2 \, x^2, \tag{9}$$

which can be regarded as the characteristic polynomial of the di®erential equation

$$\circledR{} k_i + \bar{}_i \frac{1}{2}\eta \, k_i^0 + ^{\circ}{}_i \, (\frac{1}{2}\eta)^2 \, k_i^{00} = 0. \tag{10}$$

Finally, substitution of the relationship $k_i = K_i^{\pi} \mathbf{j} \, K_i$ leads by setting $(K_i^{\pi})^0 = (K_i^{\pi})^{00} = 0$, according to our assumption about long run variables, to:

$$I_i^0 = K_i^{00} = \frac{\circledR{}}{^{\circ}{}_i \, (\frac{1}{2}\eta)^2} (K_i^{\pi} \mathbf{j} \, K_i) \mathbf{j} \; \frac{\bar{}_i}{^{\circ}{}_i \frac{1}{2}\eta} K_i^0 = a_i (K_i^{\pi} \mathbf{j} \, K_i) \mathbf{j} \; b_i K_i^0, \tag{11}$$

where $a_i = \frac{\circledR{}}{^{\circ}{}_i \, (\frac{1}{2}\eta)^2}$ and $b_i = \frac{\bar{}_i}{^{\circ}{}_i \frac{1}{2}\eta}$:

In conclusion, the second order accelerator equation (11) re°ects the inertia of the investment process due to institutional medium-term frictions. It expresses the acceleration K_i^{00} of the individual capital stock K_i or, in other words, the rate of change of individual investment behavior I_i^0 in the medium run. Accordingly, parameter a_i is mainly responsible for the rate at which individual investment is adjusted. While parameter a_i is mainly responsible for the period length of the cyclic series described by equation (11), parameter b_i determines the rate of damping.

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[6]Actually, we apply $\sqrt{1 + x} \; \frac{1}{4} \; 1 + \frac{x}{2}$ assuming su± ciently small values of x.

3.2. The Short-Term Investment Behavior. The early and mid 1990s have seen a growing interest of the economic community in herding phenomena. Pioneered by the works of Scharfstein and Stein (1990), Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992), where herd externalities depended on the ordering or queuing of players, the latest contributions focus on endogenous timing and information revelation in herding models of investment, see e.g. Chamley and Gale (1994), Gul and Lundholm (1995) or Sgroi (1999). A survey of these models is given in Gale (1996a). This recent body of literature leads to the suggestion that the rate a_i at which an individual changes investment behavior for strategic reasons is not constant, at least myopically. Informational cascades can cause longer lasting strategic delay of investments able to a®ect the medium-term behavior and eventually also lead to aggregate °uctuations. The few existing models in this vein that build the bridge to aggregate cycles are the ones by Gale (1996b), Gonz§lez (1997) and Chalkley and Lee (1998). These authors focus on coordination failures among agents either based on production or information externalities leading to the generation of genuine business cycles. Besides they all put a lot of emphasis on the explanation of potential asymmetry in business cycle time series caused by strategic delay. Their models relate to some extent to the approach that is adopted here, but none of them explicitly applies it to explain the synchronization of di®erent individual cycles underlying the aggregate investment series. In general, the crucial reasoning behind endogenously-timed herding is that the short run timing of an individual investment decision is not independent of the overall investment behavior of the other agents in the respective sector or economy. This channel of aggregate in°uence arises due to uncertainty about the pro¯tability of future investment projects: The individual strategy consists in weighing up the option value of waiting and thereby gaining information by observing others' actions with the costs of this delay. The following subsection outlines the myopic investment behavior on the basis of a basic herding scenario. The outline is in its main parts adopted from Gale (1996a) and Sgroi (1999). In the course of this section, the qualitative main results and implications for the medium-term investing activity are summarized in the form of propositions[7] to be later on embodied in a game-like integrated model.

The Basic Herding Model of Investment. Initially we are considering N = 2 agents, although all results can and will be generalized to the multi-agent case. These agents face a myopic two-sided investment decision problem: whether and if so when to run a certain investment project. This project has a speci¯c value equaling the state of the world, w, which in the simple base case is assumed ¯xed at the beginning of time. Let us index the myopic time horizon by t 2 T_{++}, e.g. days,

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[7]Formal proofs of these propositions are given in the Appendix of the paper.

weeks or months, i.e. for reasons of simpli¯cation, we assume to \move discretely in the margin" of a continuous time world. Gul and Lundholm (1995) have shown that in the more complex framework of continuous time, the results of the herding model of investment remain qualitatively the same. Put it di®erently the reason for gradual investment, i.e. strategicaly delayed investment and disinvestment, in this kind of models is due to the informational externality and is not an artifact of a (quasi-)discrete-time framework. This was already noted and shown in Caplin and Leahy (1993). Agents do not directly observe w, instead they receive a signal, θ, at t = 1. In the following, we use superscript to index agents and subscript to index time, so θ^i_t is the signal of agent i $\in$ f1;2g at time t. We will use i and j to denote our two agents. Assume for the sake of simplicity that the signals θ^i and θ^j are independent and identically drawn from a uniform distribution with range [¡1;1], so $\theta^i \sim U[¡1;1]$ for i $\in$ f1;2g. These signals do not change over time, and the state of the world w is set equal to the sum of all signals, $w = \theta^i + \theta^j$. Actions are de¯ned as: $x^i = 1$ ( ) \invest"; and $x^i = 0$ ( ) \do not invest". An agent can observe his own signal, but not the signal of the other agent. In each period actions are made simultaneously, so the two agents cannot observe each others' actions. However, in period 2, the agent will know the action that the other agent performed in period 1, and through the observed choice of activity some information about the nature of the other agent's signal may be revealed. We abstract from pre-play communication that would give the agents a possibility to meet and reveal their signals. The ¯nal ingredient of our basic setting are payo®s, π^i_t, where t $\in T_{++}$ and i $\in$ f1;2g, discounted strictly by a short-term rate δ[8]

$$\pi^i_t = \begin{cases} (\delta)^{t_i-1} w & \text{if} \quad x^i = 1 \\ 0 & \text{if} \quad x^i = 0 \end{cases} \cdot \tag{12}$$

To solve the short-term decision problem, we consider the problem faced by agent i: whether, and if so, when to invest. Myopically, we could consider the following simple rules: (i) invest (i.e., $x^i = 1$) if and only if $E[\pi^i_t] > 0$; (iia) if an investment is to be made, then make it at t = 1 if and only if $E[\pi^i_1] > E[\pi^i_2]$, otherwise wait. In these rules the pro¯t function explicitly includes discounting the short run time horizon. This might seem a sensible rule to adopt, but while we are capturing a notion of the cost of delay since we have an implicit $\delta < 1$ in the second period payo®, we are not capturing the bene¯t of delay, namely the option value of waiting. This option value comes about because of the possibility that for some reason agent i may have invested at time 1 even though doing so was foolish given the information available to him at time 2. We will consider the cost and bene¯t of delay in turn, but ¯rst we will de¯ne a symmetric signal value τ such that $\theta^i > \tau > 0$ ( ) $x^i = 1$. We have not yet

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[8]By the same argumentation as above, we have now $\delta > \delta^n > \delta$.

said anything about what to do at $t = 2$, but we have already defined an alternative decision rule for $t = 1$: (iib) invest at $t = 1$ (i.e. set $x_1^i = 1$) if and only if $\mu_1^i > \bar\tau > 0$.

Proposition 1. There is some symmetric $\bar\tau$ such that it is optimal for agent i to invest at time $t = 1$ if and only if $\mu_1^i > \bar\tau > 0$.

    Proof. See Appendix. ∎

Proposition 2. (a) The game will end at $t = 2$, i.e. if agent i did not invest at time $t = 1$ he will either invest when $t = 2$ or never invest. (b) Agent i will only invest at $t = 2$ if agent j invested at $t = 1$.

    Proof. See Appendix. ∎

We have now specified the basic ingredients of the investment herding model. The next subsection will briefly summarize some modifications and extensions undertaken in the literature on herding with endogenous timing. However, before going on it is worth noting a number of interesting results which have come about so far. From now on we will refer to the value $\bar\tau$ established above as $\bar\tau\,(\tfrac{1}{2})$ since it is a function of $\tfrac{1}{2}$ only. The features that characterize the above model as a herding model are: information is not fully revealed, there is no direct mapping from signal to action which can be inverted to reveal agents' signals; errors are made and private information may be ignored, in particular even if $\mu_1^i > 0$ for $i = 1; 2$ neither of the players will invest unless $\mu_1^i > \bar\tau\,(\tfrac{1}{2})$ for at least some player i. There will be short-term delay in this model. This scenario clearly describes what Bikhchandi, Hirshleifer and Welch (1992) entitled an informational cascade: An individual, having observed the actions of those ahead of him in a sequence, who follows the behavior of the preceding individual, without regard of his own information, is said to be in a cascade. It embodies also a herd externality in the definition of Banerjee (1992); accordingly, a herd externality is the loss of information contained in later agents' private signals that comes about when individuals in a sequence ignore their own private information and join a herd. This has also been shown since in this basic setting the game will effectively end at $t = 2$, beyond this point agents have either invested in the project or will never do so. The addition of further agents would allow the game to continue beyond two periods of interest, but at least one agent is needed to invest in a myopic time interval or investment will stop, as in the two agents case. This is formally shown to be true in the statement and proof of the following proposition 3 which extends proposition 2 to the multi-agent case.

Proposition 3. Investment at $t = 2$ takes place if some or at least one agent invested at $t = 1$. A single period of no investment will end the prospect of any further investment in a model with $N \, 2 \, N_{++}$ agents: A Lemmings-Effect is triggered.

Proof. See Appendix. ∎

A summary of the propositions derived in this section is given in Table 2. It displays for the myopic point in time $t = 1$ all the potential constellations of aggregate and individual investments, where > 0 denotes investment and $= 0$ no investment. Furthermore it reflects the reaction at $t = 2$ and $t = 3$ of the individual investor for every constellation at $t = 1$. It can be noted that the first column, i.e. the two parallel actions $\sum I_j > 0$ (aggregate investment)/ $I_i > 0$ (individual investment)[9] and $\sum I_j = 0$ (no aggregate investment)/ $I_i > 0$ (individual investment) are clearly situations where the individual i does not observe the aggregate behavior - otherwise he would have waited up to $t = 2$. In the following, we will call such agents Type II Agents. The second part of the $t = 1$-column of Table 2 represents Type I Agents who only observe at this point in time. Therefore the first part of the $t = 2$-column contains no actions since type II agent i already invested at $t = 1$. In the second part of this column the action depends on the signal transported by the aggregate behavior at $t = 1$. Now it should be obvious that in this game only the delay strategy: $t = 1 : \sum I_j = 0 = I_i = 0;\ t = 2 : I_i = 0;\ t = 3 : I_i = 0;\ ...$ etc., survives the myopic time space and has a longer reaching impact also on medium-term investment (see proposition 3 above).

Table 2.

t = 1 (investment opportunity)		t = 2		t = 3			t = T	
$\sum I_j > 0$	$\sum I_j > 0$	i	i	i	i		i	i
$I_i > 0$	$I_i = 0$	i	$I_i > 0$	i	i	....	i	i
$\sum I_j = 0$	$\sum I_j = 0$	i	i	i	i		i	i
$I_i > 0$	$I_i = 0$	i	$I_i = 0$	i	$I_i = 0$		i	$I_i = 0$

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[9]Note $\sum I_j$ denotes sum over all respectively observed other industries, i.e. the respectively observed aggregate, where $j = 1; ...; N$ n f i g:

The Information Revelation Extension. Recently, Sgroi (1999) noted that the numerous contributions on herding and social learning did not consider the possibility of introducing a stochastic environment and a third party, as e.g. a governmental or commercial research institute, capable of revealing the true state, through which the value of information due to be revealed at a pre-determined date might be evaluated. In his paper, Sgroi (1999) undertakes a re-consideration of a \standard problem" or suggestive idea in modelling investment: \With common value multi-agent investment under uncertainty it is easy to foresee a failure of investment even if the true value of the state is strictly positive. This might lead to the suggested solution that a third party, such as the government, a regulator or even a joint body established by the agents to gather information, should attempt to evaluate the true state and correct such an investment breakdown, by revealing positive value investments to all agents." In contrast, his ¯ndings in the framework of a modi¯ed version of the above outlined basic herding model point in a di®erent direction: \A far more damaging point is that in the great majority of cases complete revelation provides no bene¯ts." In his analysis the only setting that bears a breakdown of the herding cascade is a third party's surprise revelation of the true state at zero costs for the agents. We will return to this point when it comes to highlight political implications and draw conclusions. Another remarkable extension undertaken by Sgroi (1999) is to let the state of the world w follow a Markow process. He ¯nds that all qualitative results derived in the preceding paragraphs also hold true under such a setting.

3.3. The Integrated Model. It should be noted that propositions 1, 2 and 3 of the preceding section could be extended to the case of disinvestment, i.e. we can simply reformulate them including the case of agent i facing a disinvestment decision. By disinvestment, not disinvestment in the narrow sense of a ¯rm's accounting but rather decisions like closing a speci¯c production plant, shutting down machinery equipment or ¯ring sta®, i.e. observable disinvesting activities, are meant. This inclusion is just the other side of the same coin and it should be evident that the informational herding results derived and outlined in 3.2 hold analogously in an investment or disinvestment scenario. This extension although quite obvious is crucial in making our model symmetric in contrast to the ones of Gale (1996b), González (1997) and Chalkley and Lee (1998) who challenge the explanation of potential assymetry in investment cycles. There is no argument in the derivation of section 3.2 that should prevent the herding results of propositions 1, 2 and 3 from occuring also in the disinvestment case. Propositions 1, 2 and especially proposition 3 then have a direct implication on the individual acceleration rate of capital a_i in equation (11) of

section 3.1, in making it a function of the other agents' investment activity:

$$a_i = a_i \left(@^{\sum_j} I_j^0 A^1 \right) \quad I_i^0 = f \left(@^{\sum_j} I_j^0 A^1 \right), \text{ where } i \neq j : \tag{13}$$
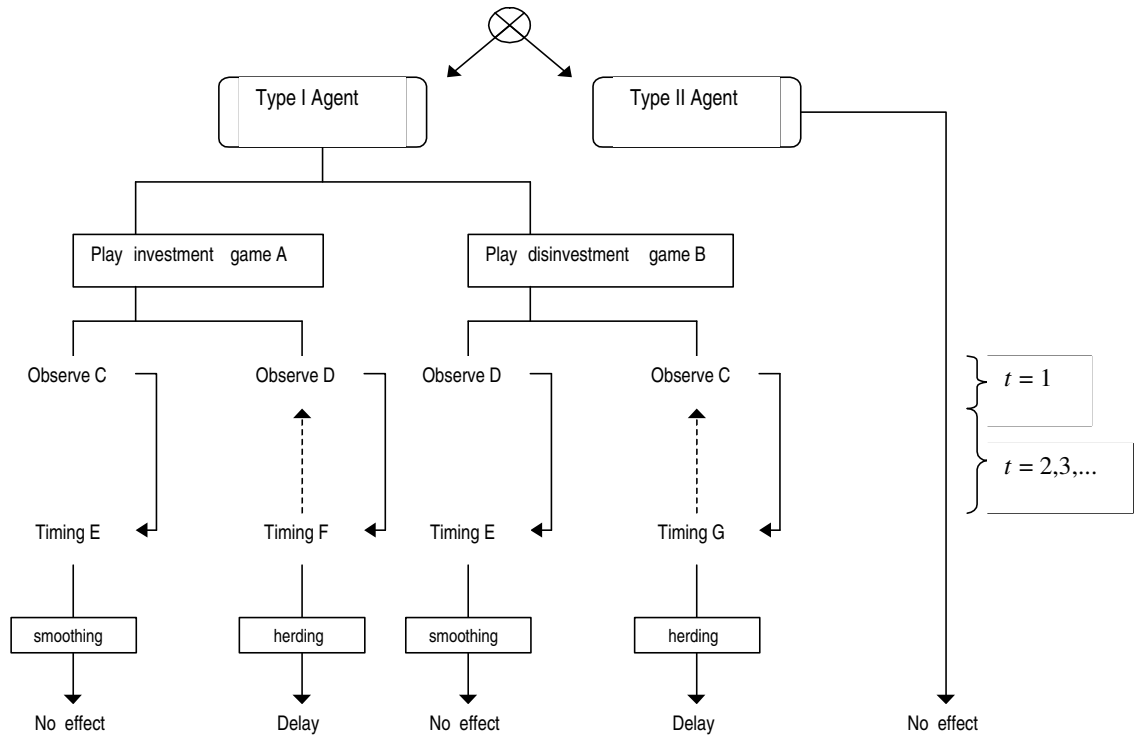
This in°uence could be quite heterogeneous between di®erent ¯rms, industries or sectors i and j and, as we have argued above, even needs not to hold true at all for some i (type II agents), depending on short-term characteristics (especially the short-term discount factor ½²). According to proposition 1 to 3 the functional relationship (13) applies for all agents i whose short-term characteristics are such that:

$$1^i <^\top (\tfrac{1}{2}) < 0 =) \quad E \; \tfrac{1}{4}_1^{h \; i} < E \; \tfrac{1}{4}_2^{h \; i}, \tag{14}$$

where ¼¹ is the respective payo® in the investment or disinvestment (in terms of avoided costs of production factors) case for the short-run instance t. Therefore we can discriminate two types of agents: Firms who, due to their short-term characteristics as re°ected in the inequality relations (14), tend to be in°uenced by the aggregate investment behavior, i.e. more precisely their probability to strategically delay and to get locked into a herding cascade is a priori positive

$$\Pr \tfrac{2}{4} I_i^0 = f \left(@^{\sum_j} I_j^0 A^1 \right) \tfrac{3}{5} > 0, \tag{15}$$

and those who do not. The logic of our synthesized model can best be illustrated by a game tree-like scheme as given in Figure 1. It consists in ¯ve stages of possible alternatives and gives a chronological order of the events: Type of agent: in°uenced by aggregate behavior vs. not in°uenced by aggregate behavior, type of game: investment vs. disinvestment, observation: aggregate activity vs. aggregate inactivity or destructivity, timing: follow vs. delay and ¯nally action: smoothing vs. herding.

type of game	observation	timing
A: $K_i^{\alpha} - K_i > 0 \rightarrow$	C: $\sum_j K_j^0 = \sum_j I_j > 0 \rightarrow$	E: $\frac{3 \partial \phi_i^0}{\partial \sum_j I_j} = 0$
B: $K_i^{\alpha} - K_i < 0 \rightarrow$	D: $\sum_j K_j^0 = \sum_j I_j < 0 \rightarrow$	E: $\frac{3 \partial \phi_i^0}{\partial \sum_j I_j} = 0$
A: $K_i^{\alpha} - K_i > 0 \rightarrow$	D: $\sum_j K_j^0 = \sum_j I_j < 0 \rightarrow$	F: $\frac{3 \partial \phi_i^0}{\partial \sum_j I_j} < 0$
B: $K_i^{\alpha} - K_i < 0 \rightarrow$	C: $\sum_j K_j^0 = \sum_j I_j > 0 \rightarrow$	G: $\frac{3 \partial \phi_i^0}{\partial \sum_j I_j} > 0$

Figure 1: Order of events

Consider ¯rst the leftmost branch of the game tree. Here, we face a type I agent's decision ful¯lling (14) and (15) who is confronted with an individual capital de¯cit $(K_i^{\pi} ¡ K_i) > 0$, i.e. who is about to \play the investment game". On the next stage, he observes at an arbitrary point in time $t = 1$ that the other industries $j = 1; ::::; J$ of his sector constituting \the aggregate" invest, i.e. take the opportunity in $t = 1$ and make an investment in the project at stake. So does agent (subsector, industry or ¯rm) i and he does it immediately at the myopic point in time $t = 1$. Therefore there is no additional e®ect from i's observation of the aggregate behavior. The investor smoothes his capital stock by running the investment project.[10] This is what he would have done anyway according to the objective of section 3.1 expressed in equation (11) and therefore

$$\text{for aggregate activity:} \quad \sum_{j} I_j > 0) \quad \left. \tilde{A} \frac{@\phi_i^0}{@ \sum_{j} I_j} !\right|_{(K_i^{\pi} ¡ K_i) > 0} = 0:$$

Next consider a type I agent playing the investment game and observing the aggregate not investing or disinvesting[11] in $t = 1$: Agent i does not start an investment project and waits like the other agents j do. A vicious circle or informational cascade is triggered for $t = 2; 3; :::$ as suggested in proposition 3 above and indicated by the backward loop in Figure 1. This strategic delay induced by herding behavior has now a longer lasting e®ect[12] that even impacts medium-term behavior by slowing down the pace of the medium-term investment °ow:

$$\text{for aggregate inactivity:} \quad \sum_{j} I_j \cdot 0) \quad \left. \tilde{A} \frac{@\phi_i^0}{@ \sum_{j} I_j} !\right|_{(K_i^{\pi} ¡ K_i) > 0} < 0:$$

The argumentation for a ¯rm i playing the disinvestment game would proceed analogously to the investment decision case. The only and crucial di®erence is the fact that the consequence of strategic delay of disinvestment decisions implies in turn

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[10]Haltiwanger (1997) and Doms and Dunne (1998) o®er some empirical evidence of the crucial role and the impact of single plant-level investment projects a®ecting in form of \investment episodes" the medium-term investment performance of US ¯rms.

[11]Since we are analyzing net equipment investment we can abstract from the weak inequality case.

[12]Actually the e®ect lasts until a new upcoming investment opportunity has to be decided upon.

of course a positive e®ect on actual medium-run capital acceleration or velocity of investment:

$$\text{for aggregate inactivity: } \left(\sum_j I_j \cdot 0\right) \quad \left. \tilde{A} \frac{@ \Phi_i^0}{@ \sum_j I_j} \right|_{(K_i^\pi \, ¡ \, K_i) < 0} = 0;$$

$$\text{for aggregate activity: } \left(\sum_j I_j > 0\right) \quad \left. \tilde{A} \frac{@ \Phi_i^0}{@ \sum_j I_j} \right|_{(K_i^\pi \, ¡ \, K_i) < 0} > 0:$$

Now we turn to combine the arguments brought forward in section 3.2 with the medium-term objective of section 3.1 meeting the argumentation of the last paragraphs and in Figure 1. We achieve the synthesis by modifying our model of equation (11) in the following way:

$$I_i^0 = K_i^\infty = a_i \cdot \left[1 + \hat{A}_i \, ¢ \, \Phi^a \, 4(K_i^\pi \, ¡ \, K_i) \sum_j I_j 5 \right] \cdot (K_i^\pi \, ¡ \, K_i) \, ¡ \, b_i K_i^0, \text{ where} \quad (16)$$

Φ^a is de¯ned as a transform function for example of the simple form:

$$\Phi^a[x] = \begin{cases} 0 \text{ for } x , 0 \\ ¡ x \text{ for } x < 0 \end{cases} \text{ and} \quad (17)$$

parameter $\hat{A}_i 2 (0; 1]$ is assumed to be strictly positive, allowing it being interpreted as the strength of individual interaction with the aggregate behavior. To see that the requirements stated in this section so far are met consider ¯rst the case of strategic delay in the investment case. This case implies $(K_i^\pi \, ¡ \, K_i) > 0$ and $\sum_j I_j < 0$ so that the product of both expressions will be negative and

$$\left. \tilde{A} \frac{@ \Phi_i^0}{@ \sum_j I_j} \right|_{(K_i^\pi \, ¡ \, K_i) > 0} = \left[\frac{@ \Phi_i^0}{@ (K_i^\pi \, ¡ \, K_i) \sum_j I_j} \right] = ¡ \, a_i \, ¢ \hat{A}_i \, (K_i^\pi \, ¡ \, K_i) < 0:$$

is achieved. On the contrary according to (16) and (17) a strategic delay in the

disinvestment case characterized by $(K_i^\pi - K_i) < 0$ and $\sum_j I_j > 0$ yields:

$$\left. \tilde{A} \frac{@ \vartheta_i^0}{@ \sum_j I_j} \right|_{(K_i^\pi - K_i) < 0} = \left[\frac{@ \vartheta_i^0}{@ (K_i^\pi - K_i) \sum_j I_j} \right] = -a_i \, \cancel{A}_i \, (K_i^\pi - K_i) > 0;$$

i.e. a positive e®ect on the pace of investment, as we stated above. The integrated model as represented in reduced form by equations (16) and (17) therefore parallely captures the qualitative result of the myopic herding model and the medium run investment objective of smoothing the capital stock adjustment process. Besides it is a macroeconomic model that bears a complete disaggregative structure.

The Markow process (see the last paragraph of subsection 3.2 above) or in a continuous time world the Wiener process that the state of the world follows would ultimately in°uence the evolution of K_i^π over time, depending on the demand of goods in a certain sector i. This connection to the demand-side of the economy makes our model essentially di®erent from multi-sector models with an origin in the supply-sided RBC-tradition, see e.g. the impressive work on sectoral cyclicality by Horvath (1998a, 1998b). An in some respects similar story underlies the models of Caplin and Leahy (1993) and Zeira (1994): In their models agents try to reveal information on the \potential demand" - in our notation a (linear) transform of K_i^π - by observing other agents' actions. The similarity can be found in the ¯rst and second part of our model's derivation: The ¯rst part in section 3.1, e.g. , is cogenial to the argumentation in Zeira (1994): \Since investment is costly, output expands gradually, as ¯rms try to reduce the expected costs of over-investment"; the second part, i.e. section 3.2, relates more or less to Caplin and Leahy (1993), where sectors interact with the aggregate whereby \the process of investment itself helps to reveal information concerning the pro¯tability of further investment". On the other hand, the cyclicality in the model presented here is not generated by changes in investors' information on demand and certain transmission mechanisms of demand shocks. The informational interplay just in°uences the pace of an endogenously modeled capital adjustment process.

4.   The Synchronization Mechanism

This section brie°y outlines the potential of the model basically described by equations (16) and (17) to synchronize heterogeneous microeconomic °uctuations resulting in a robust macroeconomic cyclic pattern of the aggregate investment series. A strong criticism that multi-sector models or generally speaking business cycle models with disaggregative structure have been confronted with is the reasoning that \as de¯nition

of sectors became more disaggregate, aggregate volatility converged to zero at the rate implied by the law of large numbers," see Horvath (1998a). The reason of why it does not apply in our model is the synchronization of heterogeneous individual cyclic investment behavior. This synchronizing is the result of a phenomenon underlying our model of investment for which Krugman (1996) gives an intuitive description and a list of synonyma: \Phase locking, otherwise known as mode locking, frequency pulling or simple synchronization of two oscillators, is one of those phenomena that occur in wildly di®erent contexts and at a very di®erent scale." This list of synonyma could be extended by the expressions: phase coordination, nonlinear or periodic entrainment, resonant stimulation or feedback e®ects of interacting oscillations that are also frequently used to describe the same phenomenon. In the context of international business cycles' interplay and synchronization Krugman (1996) suggested it as an adequate track of explanation to follow. Other authors like Mosekilde et al. (1992), Larsen et al. (1993) and Sterman and Mosekilde (1994)[13] applied it in a simple model of the interplay between medium-term and long-term economic cycles. In the context of inventories phase locking leading to resonant stimulation and self-organization is analyzed in Focardi and Marchesi (1998). A model the most closely related to the one suggested here, though not based on myopic herding behavior but rather on ad hoc plausibility, was already sketched in a quite brief manner by Hillinger and Weser (1988) and simulated on the basis of empirical data by Sässmuth (1998) one decade later.

But how does this synchronization actually come about in our model? Where do we ¯nd the interaction of oscillators leading to resonant stimulation? To illustrate this, let us reconsider equation (16) above. It is an equation of the so called Hill's class of equations, since it assigns the second temporal derivative of a variable K_i^{∞} to a function including a temporally variable coe±cient $a_i [K_i(t); I_j(t)]$) $a_i(t)$, see Arnol'd (1983). Consider next the case of quasi-cyclic investing behavior on the sectoral or individual level, such that I_i idealistically °uctuates according to a sinusoidal function depending on microeconomic characteristics as outlined in section 3.1 and contained in a_i, e.g. $\cos(a_i t)$. For a su±ciently strong prominence of this cyclic component in the individual investment activity, i.e. for a relatively small damping corresponding to relatively small values of b_i, we can write

$$K_i^{\infty} = I_i^0 = f[(K_i^{\pi} ¡ K_i)] ! \cos(a_i t).$$

Suppose now that the aggregate investment series is constituted by many such quasi-

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[13]Sterman and Mosekilde (1994) contributed their work to the Business Cycles compilation edited by Semmler (1994) which can be seen as the counterbalance of the Frontiers of Business Cycle Research compilation of the RBC research agenda edited by Cooley (1995).

cycles with roughly equal periodicities, see Figure 2,[14] so that the sum of its first-order term follows approximately the same cyclical dynamics than I_i^0, i.e. $\cos(a_i t)$. This implies that

$$\sum_j I_j \frac{1}{4} \int_t^T I_i^0 dt = \int_t^T f\left[(K_i^\pi \mathop{;} K_i)\right] dt \; ! \; \sin(a_i t) :$$

To summarize this idealistic reference case, let us assign an inner (in) and outer (out) frequency to the system described by equation (16):

$$I_i^0 = K_i^\infty = a_i \left[1 + \hat{A}_i \, \mathbb{c}^{\mathrm{a}} \, (K_i^\pi \mathop{;} K_i) \, \mathbb{c} \, I_j \, (K_i^\pi \mathop{;} K_i) \mathop{;} b_i K_i^0 : \right.$$

$$\underbrace{\underbrace{}_{\cos(a_i t)}}_{\frac{1}{2}\sin(2a_i t)} \underbrace{}_{\sin(a_i t)} \quad \text{in} = 2a_i$$

$$\underbrace{}_{\cos(a_i t)} \quad \text{out} = a_i$$

Arnol'd (1983) notes: "Equations or systems with constant coefficients in the leading term and with coefficients in the form of trigonometric polynomials in the lower-order terms have special properties, and can be called equations of Mathieu type." Obviously, equation (16), in its above idealistic interpretation, represents an equation of the Mathieu type. The special property of such an equation or system lies in its potential to resonantly stimulate different phases (parametric resonance phenomenon) and periodically entrain them, see Arnol'd (1983), Hillinger and Weser (1988) and Norris (1992). This effect depends on what is usually called forcing amplitude or depth of modulation, corresponding to the strength of interaction with the aggregate, in our notation $\hat{A}_i$. The general restriction is $\hat{A}_i < 1$. Since we defined $\hat{A}_i$ as being $2\ [0;1]$ our model meets this requirement. The other conditio sine qua non is a certain constellation of outer to inner frequency (in is sometimes also called perturbation or frequency of driving signal): If many such constellations for different values of $\hat{A}_i$ exist, these zones of parametric resonance or periodic entrainment are usually visualized in so called Arnol'd tongues, see Arnol'd (1983) and Norris (1992) in general and Mosekilde et al. (1992) or Sterman and Mosekilde (1994) for applied examples of the Arnol'd tongue concept. For systems of Mathieu type the by far

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[14] As can be seen in Figure 2, about 90% of Standard Industrial Classification (SIC) 4-digit industrial investment series of the U.S. manufacturing sector, for the period 1958-1994, show periodicities corresponding to quasi-cycle lengths of 4-7 years. The corresponding empirical result, i.e. the estimated deterministic cyclic component, of the aggregate series lies at 6.5 years.

largest zone of periodic entrainment lies for a depth of modulation $\hat{A}_i$ 2 [0; 1] at the constellation of

$$\frac{out}{in} \frac{1}{4} \frac{1}{2};$$

see Arnol'd (1983) or Hillinger and Weser (1988). The approximative equality sign comes about, since this large area of parametric resonance also captures many constellations of out to in that only roughly equal 0:5. As outlined above the system characterized by equation (16), idealistically interpreted, exactly matches this condition, since

$$\frac{out}{in} = \frac{a_i}{2a_i} = \frac{1}{2}:$$

The expression \idealistic" now gets a more meaningful interpretation, since for all de¯ned values of $\hat{A}_i$ periodic entrainment is guaranteed. It should be noted that even deviations (as long as they are not major ones) from this ideal case are capable to produce resonant stimulation and phase locking.
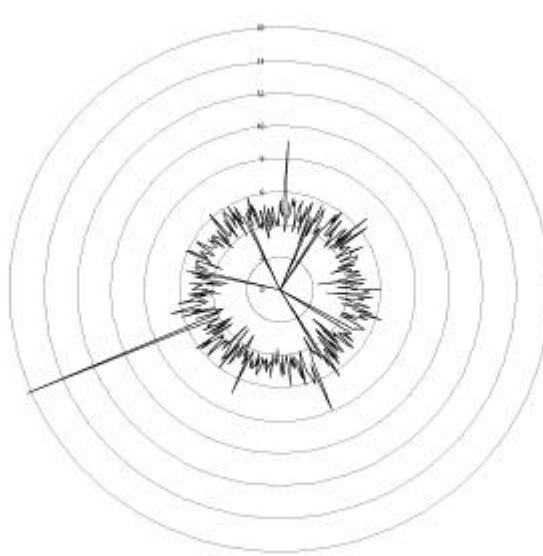

Figure 2: Univariate spectral analytic results - estimated cycle lengths (years)[15]

Data: 450 SIC4-sectoral investment series, 1987 prices, annual: 1958-1994, detrending: see Baxter and King (1995)

Source: NBER Manufacturing Productivity Database, see Bartelsman and Gray (1996)

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[15]Note: the relatively few rays pointing into the center of the polar diagram mark series that did not show any periodicity or were distorted due to missing values.

5. Implications and Conclusion

The model of this paper suggested an interplay of medium- and short-term investing behavior leading to an endogenous and deterministic synchronization of microeconomic cyclic investment activities. This synchronization prevents di®erent microphases to cancel out according to the law of large numbers or similar laws as suggested in Horvath (1998a, 1998b). The model implies that less herding behavior reduces this inherent synchronizing process, as long as there are no signi¯cant aggregate or partial aggregate[16] shocks that would exogenously set the microeconomic quasi-cycles into a similar phase constellation. Recent ¯ndings of the literature on endogenous timing and information revelation in herding models state that an unexpected revelation of the pro¯tability of major investment opportunities, at zero costs for ¯rms or industries, by a regulatory institution (government, research institute etc.) would prevent herding behavior and - in the framework of this paper's model - thereby synchronization from happen. Finally, as a direct political implication, the establishment of such an institution providing information of the true state with a reasonable degree of accuracy, could ultimately lead to a stabilization of macroeconomic investment. This is especially remarkable with regard to the fact that the inherent cyclicality of the investment series is a widely agreed on empirical fact and seen by many authors as the major cause of business cycles.

APPENDIX

Proof of Proposition 1:

Consider ¯rst the cost of delay that can be seen intuitively as $(1 \text{¡} \frac{1}{2})^{1 \text{¡} i}$. This is simply the expected payo® at the myopic point in time 1 minus the expected payo® at 2. The di®erence displays in some sense, the cost of delay. Since the unconditional expectation $E[^{1 \text{¡} j}] = 0$ which is true for any signal distribution symmetric around zero, such as the uniform $[\text{¡} 1; 1]$. Consider now the bene¯t in delay: the option value. Here we need to take into account the possibility of regret, where an investment made at $t = 1$ actually seems less sensible when information made available at $t = 2$ is revealed. Information of this sort comes about if it is observed that agent j did not invest at $t = 1$, therefore revealing that $^{1 \text{¡} j} <^{\top}$ which provides some evidence that the state of the world is less likely to merit investment. This can be avoided if agent i waits and so provides the option value of waiting which occurs with probability $\Pr(^{1 \text{¡} j} <^{\top})$. The option value can therefore be de¯ned as

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the expected loss avoided by agent i by not investing at t = 1 in the event that agent j does not invest at t = 1:

$$-\tfrac{1}{2}\Pr\left[h_{1j} < \tau^{in}_{1i}\right] + E\left[h_{1j}\,|\,h_{1j} < \tau^{io}\right]: \tag{18}$$

Let us now consider the condition which leaves the marginal decision-maker indi®erent when deciding to invest at t = 1: indi®erence occurs when the option value exactly o®sets the delay costs; this is none other than the standard value matching condition for a dynamic planning problem. This condition implicitly de¯nes the value τ using the properties of the uniform distribution:

$$(1 - \tfrac{1}{2})\,\tau = -\tfrac{1}{2}\Pr\left[h_{1j} < \tau^{in}_{1i}\right] + E\left[h_{1j}\,|\,h_{1j} < \tau^{io}\right] \tag{19}$$

$$) \quad \tau = \frac{-(4 - 2\tfrac{1}{2}) \pm \left[(4 - 2\tfrac{1}{2})^2 + 12(\tfrac{1}{2})^2\right]^{\tfrac{1}{2}}}{6\tfrac{1}{2}}:$$

For $\tfrac{1}{2} \in (0;1)$ and $\tau \in [-1;1]$ we can rule out one of this two results, eliminating:

$$\tau = \frac{1}{6}(\tfrac{1}{2})^{-1}\left[-(4 - 2\tfrac{1}{2}) - \left[(4 - 2\tfrac{1}{2})^2 + 12(\tfrac{1}{2})^2\right]^{\tfrac{1}{2}}\right]^{\tfrac{3}{4}} \notin [-1;1] \text{ for } \tfrac{1}{2} \in (0;1): \tag{20}$$

This leaves the value of τ uniquely given as:

$$\tau = \frac{1}{3} + \frac{2}{3}(\tfrac{1}{2})^{-1}\left[(\tfrac{1}{2})^2 - \tfrac{1}{2} + 1\right]^{\tfrac{1}{2}} - 1: \tag{21}$$

Equation (21) is well de¯ned for $\tfrac{1}{2} \in (0;1)$ and gives a range of values for τ of $\tau \in \left[0;\tfrac{1}{3}\right]$, that can be roughly approximated by the linear function $\tau = \tfrac{1}{3}\tfrac{1}{2}$ over the relevant range of values of $\tfrac{1}{2}$. It has been shown that there exists a unique value of τ given in equation (21) such that if $1_i > \tau$ the cost of delay is strictly o®set by the option value of waiting. We face the > relation since the cost of delay is rising in 1_i (and falling in $\tfrac{1}{2}$) which therefore de¯nes the optimal decision rule for agent i at t = 1. The assumption of a positive option value to delay immediately implies that $\tau > 0$. So far for the su± ciency-part of proposition 1. For $1_i > \tau > 0$ to be also a necessary condition of investment, consider the value 1_i must take on if agent i has optimally decided to invest at t = 1. Optimally deciding to invest implies that the delay cost is strictly o®set by the option value, hence we have:

$$(1 - \tfrac{1}{2})\,1^{\alpha} < -\tfrac{1}{2}\Pr\left[h_{1j} < \tau^{in}_{1i}\right] + E\left[h_{1j}\,|\,h_{1j} < 1^{\alpha io}\right]; \tag{22}$$

where 1^{α} implicitly de¯nes the value of 1 required for this inequality relation to hold. But this is exactly the value τ we de¯ned above. This completes the proof of proposition 1.

Proof of Proposition 2:

We are given that agent i did not invest at the myopic datum $t = 1$. If this was so we know from proposition 1 that $\theta^i < \bar{\tau}$. Investment will bene¯t agent i if $E[\mu_2] > 0$. Thereby two rationales for delay at $t = 1$ are possible and will be considered in turn:

(a) If $\theta^i \in (\_1; 0]$ and therefore $E[\mu_2] < 0$, only if new information suggested a rise in $E[\mu_2]$ would it be rational to decide to invest. Agent i must have observed one of two possible histories: $x_1^j = 1$ or $x_1^j = 0$. Only if he observed $x_1^j = 1$ would he raise his expectation of μ_2 as follows:

$$E\left[\mu_2^h \mid x_1^j = 1\right]^i = \theta^i + E\left[\theta^{1j} \mid \theta^{1j} > \bar{\tau}\right]^i = \theta^i + \frac{1+\bar{\tau}}{2} > \theta^i = E\left[\mu_1^h\right]^i \qquad (23)$$

$$E\left[\mu_2^h \mid x_1^j = 0\right]^i = \theta^i + E\left[\theta^{1j} \mid \theta^{1j} < \bar{\tau}\right]^i = \theta^i \_ \frac{1\_\bar{\tau}}{2} < \theta^i = E\left[\mu_1^h\right]^i : \qquad (24)$$

Since this is a symmetric problem the same holds true for agent j if $\theta^{1j} \in (\_1; 0]$, therefore if agent j did not invest at $t = 1$ then he too would only raise his expectation if $x_1^i = 1$. If neither of the two agents invests then no increase in expectations occurs at $t = 2$ and so no investment at all is undertaken at $t = 2$, and hence no rise in expectation occurs at $t = 3$, etc. Therefore we have shown that if one agent does not observe investment from the other he will not invest and the next period after a small additional incremental change in time will look much like the second one, so the decision not to invest becomes permanent. If either agent invested the other would increase his expectation, but only once (since the other player may never move again) and will therefore raise his expectation, i.e. $E\left[\mu_2^h \mid x_1^j = 1\right]^i > 0$, and invest at $t = 2$ or despite the increase it will be the case that $E\left[\mu_2^h \mid x_1^j = 1\right]^i < 0$ because his signal was so low, and no investment will take place at $t = 2$ or ever.

(b) If $\theta^i \in 0; \bar{\tau}$ and $E[\mu_2] > 0$ then player i was delaying despite expecting positive pro¯t because of the positive option value to delay. This option value has however been expended. If $x_1^j = 1$ then he would have been better o® investing at $t = 1$ and would have done so had he realized that agent j would de¯nitely invest. He will invest at $t = 2$ since there will be no further revelations as agent j has de facto left the game. Now if $x_1^j = 0$ agent i will lower his payo® expectation as will agent j therefore if it was optimal for them to delay at $t = 1$ it is optimal to delay at $t = 2$ a fortiori and so it will be optimal not to invest at $t = 2; 3; 4; ::::$ etc. We have shown that in all cases, agent i will either invest at $t = 1$, invest at $t = 2$, or never invest and thereby given the proof of the ¯rst part of proposition 2. Furthermore in all cases examined it is only optimal for player i to invest at $t = 2$ if the other agent j invested at $t = 1$ and vice versus. Therefore also the second part of proposition 2 is proven.

Proof of Proposition 3:

To proof this statement, one has to show that if there is no investment at an arbitrary point of the time continuum, $t = \varsigma$, then there will be no investment at time $t = \varsigma + 1; \varsigma + 2; \ldots$ etc.[17] We know from proposition 2 above that if there is no investment at time $t = \varsigma$ then agent i will not alter his optimal decision not to invest, and by symmetry this will be the case for all players i. The only additional information revealed at time $t = \varsigma + 1$ lowers expected payoffs, so as in proposition 2 agents will either go from a position where $\varsigma^{1 \, i} \, 2 \, (\mathstrut_i 1; 0]$ $=) \, E \, \frac{1}{4}_{t+\varsigma} < 0$ and will then certainly not invest at $t = \varsigma + 2$, or $^{1 \, i} \, 2 \quad 0;^{\top} \quad =) \quad E \, \frac{1}{4}_{t+\varsigma} > 0$ and they will have decided optimally to delay because of a positive option value, and it will remain optimal to delay a fortiori just as in the two-agent case. At $t = \varsigma + 3$ agent i is in an identical position to the position at $t = \varsigma + 2$, since no agents have invested once more, so there is no additional information at all being revealed, and this will clearly be the case for $t = \varsigma + 4; \varsigma + 5; \ldots$ etc. Therefore there will be no reason for any agent to change his optimal decision not to invest.

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[17]Obviously, this would link the short- to the medium-term time horizon.

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