Reduction of couplings in the MSSM

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A B S T R A C T

In this Letter, we first demonstrate the existence of renormalization group invariant relations among the top, bottom Yukawa and the gauge colour couplings in the minimal supersymmetric SM. Based on this observation and assuming furthermore the existence of a renormalization group invariant relation among the trilinear couplings in the superpotential and the soft supersymmetry breaking sector, we obtain predictions for the Higgs masses and the supersymmetric spectrum.

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1. Introduction

With the recent discovery of the Higgs-like boson at the LHC [1], the new bounds on supersymmetric particles which place supersymmetry at least at the TeV scale [2], and the new data on B physics [3], the search for theoretical scenarios beyond the Standard Model in which all these experimental facts can be accommodated becomes more pressing.

Frameworks such as Superstrings and Noncommutative Theories were developed aiming to provide a unified description of all interactions, including gravity. However, the main goal from a unified description of interactions should be the understanding of the present day free parameters of the Standard Model (SM) in terms of a few fundamental ones, or in other words to achieve *reduction of couplings* at a more fundamental level. Unfortunately, the above theoretical frameworks have not provided yet an understanding of the free parameters of the SM.

We have developed a complementary strategy in searching for a more fundamental theory, possibly realized near the Planck scale, whose basic ingredients are Grand Unified Theories (GUTs) and supersymmetry (SUSY), but its consequences certainly go beyond the known ones [4–6]. The method consists in searching for renormalization group invariant (RGI) relations holding below the Planck scale, which in turn are preserved down to the GUT scale. An impressive aspect of the RGI relations is that one can guarantee

their validity to all-orders in perturbation theory by studying the uniqueness of the resulting relations at one-loop, as was proven in the early days of the programme of *reduction of couplings* [7]. Even more remarkable is the fact that it is possible to find RGI relations among couplings that guarantee finiteness to all-orders in perturbation theory [8]. This programme, called Gauge–Yukawa unification (GYU) scheme, has been applied to the dimensionless couplings of supersymmetric GUTs, such as gauge and Yukawa couplings, with remarkable successes since it predicted correctly the top quark and the Higgs masses in finite $N = 1$ supersymmetric SU(5) GUTs [4–6,9].

Supersymmetry seems to be an essential feature of the GYU programme and understanding its breaking becomes crucial, since the programme has the ambition to supply the SM with predictions for several of its free parameters. Indeed, the search for RGI relations was extended to the soft supersymmetry breaking (SSB) sector of these theories [6,10], which involves parameters of dimension one and two. Based conceptually and technically on the work of Ref. [11], considerable progress was made concerning the renormalization properties of the SSB parameters [12–17]. In Ref. [11] the powerful supergraph method [18,19] was applied to softly broken SUSY theories using the "spurion" external space–time independent superfields [20,21].

In the spurion method, a softly broken supersymmetric gauge theory is considered as a supersymmetric one in which the various parameters such as couplings and masses have been promoted to external superfields that acquire "vacuum expectation values". Thus, the β-functions of the parameters of the softly broken theory are expressed in terms of partial differential operators involving the dimensionless parameters of the unbroken theory. By transforming the partial differential operators involved into total derivative operators it is possible to express all parameters in a RGI way

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[16,17], and in particular on the RGI surface which is defined by the solution of the reduction equations. Crucial to the success of this programme is that the soft scalar masses obey a sum rule [22,23], which is RGI to all orders in perturbation theory, both for the general GYU as for the particular finite case [17]. Based on the above tools and results we would like to apply the above programme in the case of MSSM.

2. The reduction of couplings method

In this section we will briefly outline the reduction of couplings method. Any RGI relation among couplings (i.e. which does not depend on the renormalization scale μ explicitly) can be expressed, in the implicit form $\Phi(g_1, \ldots, g_A) = $ const., which has to satisfy the partial differential equation (PDE)

$$\frac{d\Phi}{dt} = \sum_{a=1}^{A} \frac{\partial \Phi}{\partial g_a} \frac{dg_a}{dt} = \sum_{a=1}^{A} \frac{\partial \Phi}{\partial g_a} \beta_a = \vec{\nabla}\Phi \cdot \vec{\beta} = 0, \tag{1}$$

where $t = \ln\mu$ (μ being the renormalization scale) and β_a is the β-function of g_a. This PDE is equivalent to a set of ordinary differential equations, the so-called reduction equations (REs) [7,24],

$$\beta_g \frac{dg_a}{dg} = \beta_a, \quad a = 1, \ldots, A, \tag{2}$$

where g and β_g are the primary coupling and its β-function, and the counting on a does not include g. Since maximally $(A-1)$ independent RGI "constraints" in the A-dimensional space of couplings can be imposed by the Φ_a's, one could in principle express all the couplings in terms of a single coupling g. The strongest requirement is to demand power series solutions to the REs,

$$g_a = \sum_{n=0} \rho_a^{(n)} g^{2n+1}, \tag{3}$$

which formally preserve perturbative renormalizability. Remarkably, the uniqueness of such power series solutions can be decided already at the one-loop level [7,24]. To illustrate this, let us assume that the β-functions have the form

$$\beta_a = \frac{1}{16\pi^2}\left[\sum_{b,c,d \neq g} \beta_a^{(1)\,bcd} g_b g_c g_d + \sum_{b \neq g} \beta_a^{(1)\,b} g_b g^2\right] + \cdots,$$

$$\beta_g = \frac{1}{16\pi^2}\beta_g^{(1)} g^3 + \cdots, \tag{4}$$

where $\cdots$ stands for higher order terms, and $\beta_a^{(1)\,bcd}$'s are symmetric in b, c, d. We then assume that the $\rho_a^{(n)}$'s with $n \leqslant r$ have been uniquely determined. To obtain $\rho_a^{(r+1)}$'s, we insert the power series (3) into the REs (2) and collect terms of $O(g^{2r+3})$ and find

$$\sum_{d \neq g} M(r)_a^d \rho_d^{(r+1)} = \text{lower order quantities}, \tag{5}$$

where the r.h.s. is known by assumption, and

$$M(r)_a^d = 3\sum_{b,c \neq g} \beta_a^{(1)\,bcd} \rho_b^{(1)} \rho_c^{(1)} + \beta_a^{(1)\,d} - (2r+1)\beta_g^{(1)}\delta_a^d, \tag{6}$$

$$0 = \sum_{b,c,d \neq g} \beta_a^{(1)\,bcd} \rho_b^{(1)} \rho_c^{(1)} \rho_d^{(1)} + \sum_{d \neq g} \beta_a^{(1)\,d} \rho_d^{(1)} - \beta_g^{(1)} \rho_a^{(1)}. \tag{7}$$

Therefore, the $\rho_a^{(n)}$'s for all $n > 1$ for a given set of $\rho_a^{(1)}$'s can be uniquely determined if $\det M(n)_a^d \neq 0$ for all $n \geqslant 0$.

Our experience examining specific examples has taught us that the various couplings in supersymmetric theories could have the same asymptotic behaviour. Therefore, searching for a power series

solution of the form (3) to the REs (2) is justified and moreover, one can rely that keeping only the first terms a good approximation is obtained in realistic applications.

3. Sum rule for soft breaking terms

The method of reducing the dimensionless couplings has been extended [6,10], as we have discussed in the Introduction, to the soft supersymmetry breaking (SSB) dimensionful parameters of $N = 1$ supersymmetric theories. In addition it was found [22,23] that RGI SSB scalar masses in Gauge–Yukawa unified models satisfy a universal sum rule.

Consider the superpotential given by

$$W = \frac{1}{2}\mu^{ij}\Phi_i\Phi_j + \frac{1}{6}C^{ijk}\Phi_i\Phi_j\Phi_k, \tag{8}$$

along with the Lagrangian for SSB terms

$$-\mathcal{L}_{\text{SSB}} = \frac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + \frac{1}{2}(m^2)_i^j \phi^{*\,i}\phi_j$$
$$+ \frac{1}{2}M\lambda\lambda + \text{H.c.}, \tag{9}$$

where the ϕ_i are the scalar parts of the chiral superfields Φ_i, λ are the gauginos and M their unified mass.

Let us recall that the one-loop β-function of the gauge coupling g is given by [25]

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i T(R_i) - 3C_2(G)\right], \tag{10}$$

where $C_2(G)$ is the quadratic Casimir of the adjoint representation of the associated gauge group G. $T(R)$ is given by the relation $\text{Tr}[T^a T^b] = T(R)\delta^{ab}$ where T^a is the generators of the group in the appropriate representation. Similarly the β-functions of C_{ijk}, by virtue of the non-renormalization theorem, are related to the anomalous dimension matrix γ_j^i of the chiral superfields as:

$$\beta_C^{ijk} = \frac{dC_{ijk}}{dt} = C_{ijl}\gamma_k^l + C_{ikl}\gamma_j^l + C_{jkl}\gamma_i^l. \tag{11}$$

At one-loop level the anomalous dimension, $\gamma^{(1)\,i}_j$ of the chiral superfield is [25]

$$\gamma^{(1)\,i}_j = \frac{1}{32\pi^2}\left[C^{ikl}C_{jkl} - 2g^2 C_2(R_i)\delta_{ij}\right], \tag{12}$$

where $C_2(R_i)$ is the quadratic Casimir of the representation R_i, and $C^{ijk} = C^*_{ijk}$. Then, the $N = 1$ non-renormalization theorem [19,26] ensures there are no extra mass and cubic-interaction-term renormalizations, implying that the β-functions of C_{ijk} can be expressed as linear combinations of the anomalous dimensions γ_j^i.

Here we assume that the reduction equations admit power series solutions of the form

$$C^{ijk} = g\sum_{n=0} \rho_{(n)}^{ijk} g^{2n}. \tag{13}$$

In order to obtain higher-loop results instead of knowledge of explicit β-functions, which anyway are known only up to two-loops, relations among β-functions are required.

The progress made using the spurion technique [18–20] leads to the following all-loop relations among SSB β-functions (in an obvious notation) [12–14,16]

$$\beta_M = 2\mathcal{O}\left(\frac{\beta_g}{g}\right), \tag{14}$$

$$\beta_h^{ijk} = \gamma^i_l h^{ljk} + \gamma^j_l h^{ilk} + \gamma^k_l h^{ijl}$$

$$-2\gamma_{1\,l}^{i}C^{ljk}-2\gamma_{1\,l}^{j}C^{ilk}-2\gamma_{1\,l}^{k}C^{ijl}, \tag{15}$$

$$(\beta_{m^2})^i{}_j = \left[\Delta + X\frac{\partial}{\partial g}\right]\gamma^i{}_j, \tag{16}$$

where

$$\mathcal{O} = \left(Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial C^{lmn}}\right), \tag{17}$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2\frac{\partial}{\partial g^2} + \tilde{C}_{lmn}\frac{\partial}{\partial C_{lmn}} + \tilde{C}^{lmn}\frac{\partial}{\partial C^{lmn}}, \tag{18}$$

$$(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j, \tag{19}$$

$$\tilde{C}^{ijk} = (m^2)^i{}_l C^{ljk} + (m^2)^j{}_l C^{ilk} + (m^2)^k{}_l C^{ijl}. \tag{20}$$

The assumption, following [13], that the relation among couplings

$$h^{ijk} = -M(C^{ijk})' \equiv -M\frac{dC^{ijk}(g)}{d\ln g}, \tag{21}$$

is RGI and furthermore, the use of the all-loop gauge β-function of Novikov et al. [27] given by

$$\beta_g^{\mathrm{NSVZ}} = \frac{g^3}{16\pi^2}\left[\frac{\sum_l T(R_l)(1-\gamma_l/2) - 3C_2(G)}{1 - g^2 C_2(G)/8\pi^2}\right], \tag{22}$$

lead to the all-loop RGI sum rule [17] (assuming $(m^2)^i{}_j = m_j^2\delta_j^i$),

$$m_i^2 + m_j^2 + m_k^2$$
$$= |M|^2\left\{\frac{1}{1 - g^2 C_2(G)/(8\pi^2)}\frac{d\ln C^{ijk}}{d\ln g} + \frac{1}{2}\frac{d^2\ln C^{ijk}}{d(\ln g)^2}\right\}$$
$$+ \sum_l \frac{m_l^2 T(R_l)}{C_2(G) - 8\pi^2/g^2}\frac{d\ln C^{ijk}}{d\ln g}. \tag{23}$$

Surprisingly enough, the all-loop result of Eq. (23) coincides with the superstring result for the finite case in a certain class of orbifold models [23,28] if

$$\frac{d\ln C^{ijk}}{d\ln g} = 1,$$

as discussed in Ref. [5].

4. All-loop RGI relations in the SSB sector

Let us now see how the all-loop results on the SSB β-functions, Eqs. (14)–(20), lead to all-loop RGI relations. We assume:

(a) the existence of a RGI surfaces on which $C = C(g)$, or equivalently that

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g} \tag{24}$$

holds, i.e. reduction of couplings is possible, and

(b) the existence of a RGI surface on which

$$h^{ijk} = -M\frac{dC(g)^{ijk}}{d\ln g} \tag{25}$$

holds too in all-orders.

Then one can prove [29,30], that the following relations are RGI to all-loops (note that in both (a) and (b) assumptions above we do not rely on specific solutions of these equations)

$$M = M_0\frac{\beta_g}{g}, \tag{26}$$

$$h^{ijk} = -M_0\beta_C^{ijk}, \tag{27}$$

$$b^{ij} = -M_0\beta_\mu^{ij}, \tag{28}$$

$$(m^2)^i{}_j = \frac{1}{2}|M_0|^2\mu\frac{d\gamma^i{}_j}{d\mu}, \tag{29}$$

where M_0 is an arbitrary reference mass scale to be specified shortly. The assumption that

$$C_a\frac{\partial}{\partial C_a} = C_a^*\frac{\partial}{\partial C_a^*} \tag{30}$$

for a RGI surface $F(g, C^{ijk}, C^{*ijk})$ leads to

$$\frac{d}{dg} = \left(\frac{\partial}{\partial g} + 2\frac{\partial}{\partial C}\frac{dC}{dg}\right) = \left(\frac{\partial}{\partial g} + 2\frac{\beta_C}{\beta_g}\frac{\partial}{\partial C}\right) \tag{31}$$

where Eq. (24) has been used. Now let us consider the partial differential operator $\mathcal{O}$ in Eq. (17) which, assuming Eq. (21), becomes

$$\mathcal{O} = \frac{1}{2}M\frac{d}{d\ln g}. \tag{32}$$

In turn, β_M given in Eq. (14), becomes

$$\beta_M = M\frac{d}{d\ln g}\left(\frac{\beta_g}{g}\right), \tag{33}$$

which by integration provides us [29,31] with the generalized, i.e. including Yukawa couplings, all-loop RGI Hisano–Shifman relation [12]

$$M = \frac{\beta_g}{g}M_0, \tag{34}$$

where M_0 is the integration constant and can be associated to the unification scale M_U in GUTs or to the gravitino mass $m_{3/2}$ in a supergravity framework. Therefore, Eq. (34) becomes the all-loop RGI Eq. (26). Note that β_M using Eqs. (33) and (34) can be written as

$$\beta_M = M_0\frac{d}{dt}(\beta_g/g). \tag{35}$$

Similarly

$$(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j = \frac{1}{2}M_0\frac{d\gamma^i{}_j}{dt}. \tag{36}$$

Next, from Eq. (21) and Eq. (34) we obtain

$$h^{ijk} = -M_0\beta_C^{ijk}, \tag{37}$$

while β_h^{ijk}, given in Eq. (15) and using Eq. (36), becomes [29]

$$\beta_h^{ijk} = -M_0\frac{d}{dt}\beta_C^{ijk}, \tag{38}$$

which shows that Eq. (37) is all-loop RGI. In a similar way Eq. (28) can be shown to be all-loop RGI.

Finally we would like to emphasize that under the same assumptions (a) and (b) the sum rule given in Eq. (23) has been proven [17] to be all-loop RGI, which (using Eq. (34)) gives us a generalization of Eq. (29) to be applied in considerations of non-universal soft scalar masses, which are necessary in many cases including the MSSM.

Having obtained Eqs. (26)–(29) from Eqs. (14)–(20) with the assumptions (a) and (b), we would like to conclude the present section with some remarks. First it is worth noting the difference, say in first order in g, among the possibilities to consider specific

solution of the reduction equations or just assume the existence of a RGI surface, which is a weaker assumption. So in the case we consider the reduction equation (24) without relying on a specific solution, the sum rule (23) reads

$$m_i^2 + m_j^2 + m_k^2 = |M|^2 \frac{d \ln C^{ijk}}{d \ln g}, \qquad (39)$$

and we find that

$$\frac{d \ln C^{ijk}}{d \ln g} = \frac{g}{C^{ijk}} \frac{dC^{ijk}}{dg} = \frac{g}{C^{ijk}} \frac{\beta_C^{ijk}}{\beta_g}, \qquad (40)$$

which is clearly model dependent. However assuming a specific power series solution of the reduction equation, as in Eq. (3), which in first order in g is just a linear relation among C^{ijk} and g, we obtain that

$$\frac{d \ln C^{ijk}}{d \ln g} = 1 \qquad (41)$$

and therefore the sum rule (39) becomes model independent. We should also emphasize that in order to show [13] that the relation

$$(m^2)^i{}_j = \frac{1}{2} \frac{g^2}{\beta_g} |M|^2 \frac{d\gamma^i{}_j}{dg}, \qquad (42)$$

which using Eq. (34) becomes Eq. (29), is RGI to all-loops a specific solution of the reduction equations has to be required. As it has already been pointed out above such a requirement is not necessary in order to obtain the all-loop RG invariance of the sum rule (23).

As it was emphasized in Ref. [29] the set of the all-loop RGI relations (26)–(29) is the one obtained in the *Anomaly Mediated SB Scenario* [32], by fixing the M_0 to be $m_{3/2}$, which is the natural scale in the supergravity framework.

A final remark concerns the resolution of the fatal problem of the anomaly induced scenario in the supergravity framework, which is here solved thanks to the sum rule (23), as it will become clear in the next section. Other solutions have been provided by introducing Fayet–Iliopoulos terms [33].

5. MSSM and RGI relations

We would like now to apply the RGI relations to the SSB sector of the MSSM, assuming power series solutions of the reduction equations at the unification scale. According to the analysis presented in Section 4 the RGI relations in the SSB sector hold, assuming the existence of RGI surfaces where Eqs. (24) and (25) hold. We show first that Eq. (24) indeed holds in the MSSM, then we assume the validity of Eq. (25) and examine the consequences in the MSSM phenomenology.

Using a perturbative ansatz concerning the solutions of Eqs. (24) and (25), the set of Eqs. (26)–(28) and Eq. (39) together with Eq. (41), clearly hold. Then one easily finds that Eq. (25) with (the first order) perturbative ansatz at the unification scale leads to the condition

$$h^{ijk} = -M_U C^{ijk}, \qquad (43)$$

where M_U is the gaugino mass and C^{ijk} are the Yukawa couplings, both at the unification scale. Therefore, this assumption leads to Eqs. (43) as boundary conditions at the unification scale.

In a similar way, starting from Eq. (28) and assuming that μ^{ij} are reduced in favour of g, i.e. that the reduction equation holds

$$\beta_\mu^{ij} = \beta_g d\mu^{ij}/dg \qquad (44)$$

and moreover has power series type solutions, we obtain

$$b^{ij} = -M_U \mu^{ij} \qquad (45)$$

as boundary conditions at the unification scale.

Finally the sum rule (39) also holds at the unification scale in the form,

$$m_i^2 + m_j^2 + m_k^2 = M_U^2. \qquad (46)$$

Therefore, the above Eqs. (43), (45) and (46) have to be imposed as boundary conditions at the unification scale in the renormalization group equations that govern the evolution of the SSB parameters.

Let us now consider more specifically the MSSM, which is defined by the superpotential,

$$W = Y_t H_2 Q t^c + Y_b H_1 Q b^c + Y_\tau H_1 L \tau^c + \mu H_1 H_2, \qquad (47)$$

with soft breaking terms,

$$-\mathcal{L}_{SSB} = \sum_\phi m_\phi^2 \phi^* \phi + \left[m_3^2 H_1 H_2 + \sum_{i=1}^3 \frac{1}{2} M_i \lambda_i \lambda_i + \text{h.c.} \right] + \left[h_t H_2 Q t^c + h_b H_1 Q b^c + h_\tau H_1 L \tau^c + \text{h.c.} \right], \qquad (48)$$

where the last line refers to the scalar components of the corresponding superfield. In general $Y_{t,b,\tau}$ and $h_{t,b,\tau}$ are 3×3 matrices, but we work throughout in the approximation that the matrices are diagonal, and neglect the couplings of the first two generations.

5.1. Reduction of couplings

Assuming perturbative expansion of all three Yukawa couplings in favour of α_3 satisfying the reduction equations

$$\beta_{Y_{t,b,\tau}} = \beta_{g_3} \frac{dY_{t,b,\tau}}{dg_3}, \qquad (49)$$

we run into trouble since the coefficients of the Y_τ coupling turn imaginary. Therefore, we take Y_τ at the GUT scale to be an independent variable. In that case, the coefficients of the expansions (again at the GUT scale)

$$\frac{Y_t^2}{4\pi} = c_1 \frac{g_3^2}{4\pi} + c_2 \left(\frac{g_3^2}{4\pi} \right)^2, \qquad (50)$$

$$\frac{Y_b^2}{4\pi} = p_1 \frac{g_3^2}{4\pi} + p_2 \left(\frac{g_3^2}{4\pi} \right)^2 \qquad (51)$$

are given by

$$c_1 = \frac{157}{175} + \frac{1}{35} K_\tau = 0.897 + 0.029 K_\tau,$$

$$p_1 = \frac{143}{175} - \frac{6}{35} K_\tau = 0.817 - 0.171 K_\tau,$$

$$c_2 = \frac{1}{4\pi} \frac{1457.55 - 84.491 K_\tau - 9.66181 K_\tau^2 - 0.174927 K_\tau^3}{818.943 - 89.2143 K_\tau - 2.14286 K_\tau^2},$$

$$p_2 = \frac{1}{4\pi} \frac{1402.52 - 223.777 K_\tau - 13.9475 K_\tau^2 - 0.174927 K_\tau^3}{818.943 - 89.2143 K_\tau - 2.14286 K_\tau^2} \qquad (52)$$

where

$$K_\tau = Y_\tau^2 / g_3^2. \qquad (53)$$

The important new observation is that the couplings Y_t, Y_b and g_3 are not only reduced, but they provide predictions consistent with the observed experimental values (as it will be explained later in the discussion of Fig. 3).

Given the above solutions of the reduction equations

$$\beta_{Y_{t,b}} = \beta_{g_3} \frac{dY_{t,b}}{dg_3}, \qquad (54)$$

and assuming the validity of Eq. (25) then, according to our earlier discussion, the following relations are RGI

$$M = \frac{\beta_{g_3}}{g_3} M_U,$$ (55)

$$h_{t,b} = -M g_3 \frac{dY_{t,b}}{dg_3},$$ (56)

$$m_3^2 = -M g_3 \frac{d\mu}{dg_3},$$ (57)

$$m_i^2 + m_j^2 + m_k^2 = M^2,$$ (58)

where i, j, k refer to the superfields appearing in the trilinear terms in the superpotential (47).[2]

Note that in the application of the reduction of couplings in the MSSM that we examine here, in the first stage we neglect the Yukawa couplings of the first two generations, while we keep Y_τ and the gauge couplings g_2 and g_1, which cannot be reduced consistently, as corrections. Therefore, strictly speaking, when we say above that Eqs. (55)–(58) are RGI we refer to the case that not only the first two generations but also the Y_τ, g_2 and g_1 are switched off.

In turn, since all gauge couplings in the MSSM meet at the unification point, we are led to the following boundary conditions at the GUT scale:

$$Y_t^2 = c_1 g_U^2 + c_2 g_U^4/(4\pi) \quad \text{and} \quad Y_b^2 = p_1 g_U^2 + p_2 g_U^4/(4\pi), \quad (59)$$

$$h_{t,b} = -M_U Y_{t,b},$$ (60)

$$m_3^2 = -M_U \mu,$$ (61)

where $c_{1,2}$ and $p_{1,2}$ are the solutions of the algebraic system of the two reduction equations (49) taken at the GUT scale (while keeping only the first term[3] of the perturbative expansion of the Yukawas in favour of g_3 for Eqs. (60) and (61)), and a set of equations resulting from the application of the sum rule (46)

$$m_{H_2}^2 + m_Q^2 + m_{t^c}^2 = M_U^2,$$ (62)

$$m_{H_1}^2 + m_Q^2 + m_{b^c}^2 = M_U^2,$$ (63)

noting that the sum rule introduces four free parameters.

6. Discussion and conclusions

In the present Letter we have made a new important observation, that the Y_t, Y_b and α_3 obey RGI relations within the MSSM. Therefore, they can be reduced and can be considered as parameters dependent among themselves. This "reduced" system holds at all scales, and thus serve as boundary conditions of the RGEs of the MSSM at the unification scale, where we assume that the gauge couplings meet. With these boundary conditions we run the MSSM RGEs down to the SUSY scale, which we take to be the geometrical average of the stop masses, and then run the SM RGEs down to the electroweak scale (M_Z), where we compare with the experimental values of the third generation quark masses. The RGEs are taken at two-loops for the gauge and Yukawa couplings and at one-loop for the soft breaking parameters. We let M_U and $|\mu|$ at the unification scale to vary between ~ 1 TeV ~ 11 TeV, for the two possible signs of μ. In evaluating the τ and bottom masses we have taken

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[2] There is another RGI term in the form of the b-parameter that could be included in Eq. (28) as was suggested in Ref. [33]. This term would turn m_3^2 in Eqs. (57) in a free parameter to be determined by the minimization of the electroweak potential. Although we omit this term here, following other treatments in the literature, we plan to include this possibility in a future examination.

[3] The second term can be determined once the first term is known.
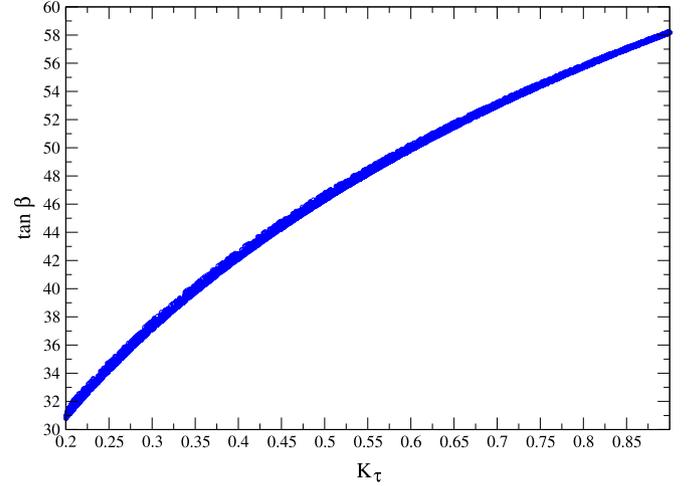

Fig. 1. Required values of $\tan\beta$ as a function of $K_\tau = Y_\tau^2/g_3^2$ in order to get the experimentally accepted tau mass.

into account the one-loop radiative corrections that come from the SUSY breaking [34]. These corrections have a dependence on the soft breaking parameters, in particular for large $\tan\beta$ they can give sizeable contributions to the bottom quark mass.

The observation that Y_t, Y_b and α_3 are a reduced system is best demonstrated in Fig. 3, where we plot the predictions for the top quark mass, M_t, and the bottom quark mass, M_b, as they result from Eqs. (50) and (51) with $c_{1,2}$ and $p_{1,2}$ given in Eq. (52), for sign(μ) = −. As one can see the predicted values agree comfortably with the corresponding experimental values within 1σ. Recall that Y_τ is not reduced and is a free parameter in this analysis. In Fig. 1 we present a plot relating the values of $\tan\beta$ and $K_\tau = Y_\tau^2/g_3^2$ which are compatible with the observed experimental value of the tau mass M_τ (fixed at its experimental central value). In the case that sign(μ) = +, there is no value for K_τ where both the top and the bottom quark masses agree simultaneously with their experimental value, therefore we only consider the negative sign of μ from now on.

The parameter K_τ is further constrained by allowing only the values that are also compatible with the top and bottom quark masses within 1 and 2σ of their central experimental value. We use the experimental value of the top quark pole mass as [35]

$$M_t^{\text{exp}} = (173.2 \pm 0.9) \text{ GeV}.$$ (64)

The bottom mass is calculated at M_Z to avoid uncertainties that come from running down to the pole mass and, as previously mentioned, the SUSY radiative corrections both to the tau and the bottom quark masses have been taken into account [36]

$$M_b(M_Z) = (2.83 \pm 0.10) \text{ GeV}.$$ (65)

In Fig. 2, we show these constrained K_τ values plotted against M_t (its central value corresponds to the purple dashed line), within 1σ (orange dashed lines), and 2σ (upper border of the graph), where also M_b is constrained to be within 1 and 2σ of its experimental value. We can do the same for M_b but we prefer to present in Fig. 3 the values of M_t vs M_b for the constrained K_τ values. From Fig. 3 it can be clearly seen that there is a set of values for the parameter K_τ where both M_t and M_b agree simultaneously within 1σ of their experimental values, for the boundary conditions given by the reduced system Y_t, Y_b and α_3.

Finally, assuming the validity of Eq. (25) for the corresponding couplings to those that have been reduced before, we calculate the Higgs mass as well as the whole Higgs and sparticle spectrum using Eqs. (59)–(63), and we present them in Figs. 4 and 5. The Higgs
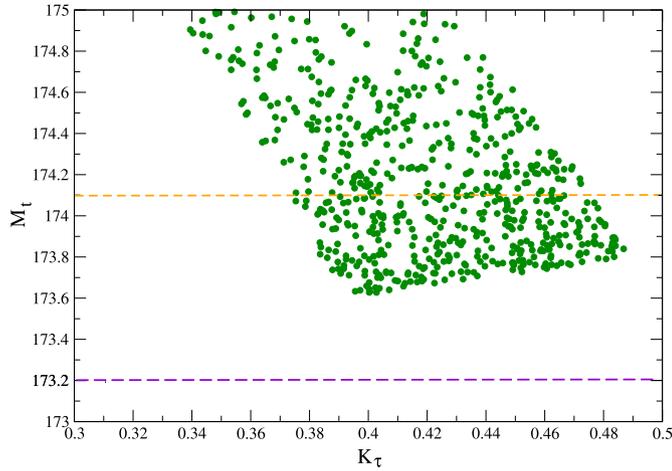
Fig. 2. The top mass as a function of $K_\tau = Y_\tau^2/g_3^2$, the purple dashed line is the experimental central value and the orange one is the 1σ value. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
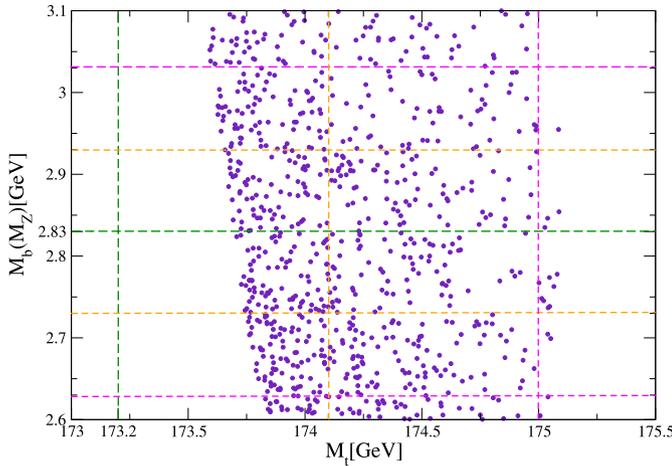

Fig. 4. The Higgs mass as a function of $K_\tau = Y_\tau^2/g_3^2$.

Fig. 3. Using the regions of values for $K_\tau = Y_\tau^2/g_3^2$ and $\tan\beta$ which give experimentally accepted tau mass, this figure shows the resulted points in the (M_t, M_b) phase space. The central value (green dashed lines), as well as the 1 and 2σ deviation (orange and magenta lines respectively), for the top and bottom masses is also drawn. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 5. The Higgs mass and s-spectrum for values of $M_U \sim 1.3$ TeV to ~ 11 TeV.

mass was calculated using a "mixed-scale" one-loop RG approach, which is known to be a very good approximation to the full diagrammatic calculation [37].

From Fig. 4 we notice that the lightest Higgs mass is in the range 123.7–126.3 GeV, where the uncertainty is due to the variation of K_τ, the gaugino mass M_U and the variation of the scalar soft masses, which are however constrained by the sum rules (62) and (63). The gaugino mass M_U is in the range ~ 1.3 TeV ~ 11 TeV, the lower values having been discarded since they do not allow for radiative electroweak symmetry breaking. The variation of K_τ is in the range $\sim 0.37 \sim 0.49$ in order to agree with the experimental values of the bottom and top masses at 1σ, and $\sim 0.34 \sim 0.49$ if the agreement is at the 2σ level. To the lightest Higgs mass value one has to add at least ± 2 GeV coming from unknown higher order corrections [38]. Therefore it is in excellent agreement with the experimental results of ATLAS and CMS [1].

From Fig. 5 we find that the masses of the heavier Higgses have relatively high values, above the TeV scale. In addition we
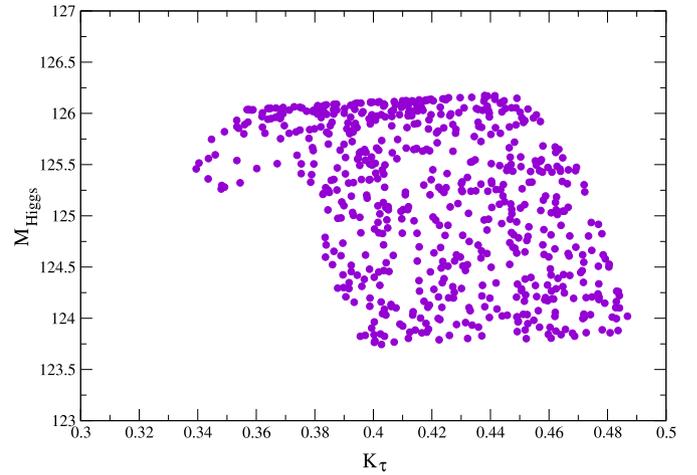
find a generally heavy supersymmetric spectrum starting with a neutralino as LSP at ~ 500 GeV and comfortable agreement with the LHC bounds due to the non-observation of coloured supersymmetric particles [2]. Finally note that although the $\mu < 0$ found in our analysis would disfavour the model in connection with the anomalous magnetic moment of the muon, such a heavy spectrum gives only a negligible correction to the SM prediction. We plan to extend our analysis by examining the restrictions that will be imposed in the spectrum by the B-physics and CDM constraints.

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