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Licensing Innovations: The Case of the Inside Patent Holder

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Abstract
The present paper reconsiders the inside innovators’ licensing problem under incomplete information. Employing an optimal mechanism design approach, we show that, contrary to what is claimed in the literature, the optimal mechanism may prescribe fixed fees, royalty rates lower than the cost reduction, and even negative royalty rates.

KEYWORDS: Innovation, licensing, industrial organization.
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1 Introduction
The literature on patent licensing in oligopoly markets draws a sharp distinction between licensing by an outside innovator and an inside innovator holder who is a competitor of potential licensees. Whereas outside innovators are advised to use fixed fee contracts or auction patent licenses, inside innovators are advised to employ pure output based royalty contracts without fixed fees.

The present paper reconsiders the inside innovators’ licensing problem. Employing an optimal mechanism design approach, we show that pure royalty contracts are generally not optimal. The optimal mechanism may prescribe fixed fees, royalty rates lower than the cost reduction, and even negative royalty rates.

2 The model
Consider a dynamic licensing game played between an inside innovator of a non-drastic process innovation (firm 1) and a potential licensee (firm 2). In the first stage the innovator offers a license contract which the licensee either accepts or rejects. In the second stage firms play a Cournot game.

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Prior to licensing, firms’ unit costs are \((c_1, c_2) = (d, c), c > d\). Licensing reduces the unit cost of firm 2 to \(x\), which is that firm’s private information. From the perspective of the innovator \(x\) is a random variable drawn from the c.d.f. \(F\) with support \([x, \bar{x}]\).

The payoff functions of the duopoly games are: \(\pi_1(q_1, q_2; c_1) = (P(Q) - c_1)q_1, \pi_2(q_1, q_2; c_2) = (P(Q) - c_2)q_2, Q := q_1 + q_2\).

The inverse demand, \(P\), is strictly decreasing, twice differentiable, and concave, which assures that duopoly subgames have a unique solution.

The innovator employs a direct mechanism, \(M := (t(x, q_2), \gamma(x))\), which consists of an allocation rule \(\gamma(x)\) and a transfer rule \(t(x, q_2)\). The transfer rule prescribes the licensee to pay a royalty rate \(r(x)\) per output unit plus a state independent fixed fee, \(f\). The licensee reports his cost, and the innovator adopts the allocation and transfers prescribed by the mechanism for the reported cost.

We consider incentive compatible mechanisms with a deterministic allocation rule, \(\gamma(x) \in \{0, 1\}\), that prescribes a threshold level, \(\hat{x}\) above which no license is awarded.

Due to antitrust concerns, royalty rates cannot exceed the licensee’s cost reduction.

3 Optimal license mechanism

The innovator maximizes his expected payoff (which is the sum of his own expected profit and expected license revenue), subject to incentive compatibility, participation, and “antitrust” constraints.

In order to solve the set of incentive compatible mechanisms, we first compute the payoffs of the licensee for all combinations of his true cost \(x\) and reported cost \(z\). This requires solving all oligopoly subgames that may occur on and off the equilibrium path.

Oligopoly subgames on and off the equilibrium path Suppose the licensee with cost \(x\) deviates from truth-telling and reports the cost \(z \leq \hat{x}\). In that case firm 2 is awarded the license and firm 1 believes to play a duopoly game with the cost profile \((c_1, c_2) = (d, z + r(z))\). Denote the equilibrium of that game by \((q_1(z), \bar{q}_2(z))\), defined as \(q_1(z) := \arg\max_q \pi_1(q, \bar{q}_2(z); d), \bar{q}_2(z) := \arg\max_{q^2} \pi_2(q_1(z), q; z + r(z))\). However, firm 2 privately knows that the cost profile is \((c_1, c_2) = (d, x + r(z))\) and therefore plays its best reply to \(q_1(z): q_2(x, z) = \arg\max_q \pi_2(q_1(z), q; x + r(z))\).

A special case are the “on the equilibrium path” strategies and profits that apply when firm 2 reports truthfully: \(q^*_1(x) := q_1(x), q^*_2(x) := q_2(x, x), \pi^*_1(x) := \pi_1(q^*_1(x), q^*_2(x); d), \pi^*_2(x) := \pi_2(q^*_1(x), q^*_2(x); x + r(x))\).

For convenience of exposition, define

\[
\Pi^*_2(x, z) := \pi_2(q_1(z), q_2(x, z); x + r(z))
\]

\[
\Pi^*_2 := \pi_2(q^*_1, q^*_2; c), \quad \Pi^*_1 := \pi_1(q^*_1, q^*_2; d)
\]

\[
\Pi_2(x, z) := \gamma(z) (\Pi^*_2(x, z) - f_L) + (1 - \gamma(z)) (\Pi^*_2 - f_N).
\]

There, \(L\) and \(N\) are mnemonic for licensing and no-licensing.

Incentive compatibility The mechanism is incentive compatible if \(x = \arg\max_x \Pi_2(x, z), \forall x\).

Lemma 1. The mechanism is incentive compatible if and only if,

\[
\gamma(x) = \begin{cases} 
1 & \text{if } x \leq \hat{x} \\
0 & \text{otherwise}
\end{cases}
\]
\begin{equation}
\begin{aligned}
\ell(x, q_2) &= \begin{cases} 
  f_L + r(x)q_2 & \text{if } x \leq \hat{\gamma} \\
  f_N & \text{otherwise}
\end{cases} \\
-1 < r'(x) - P'(\cdot)q_1'(x) < 0, & \forall x \leq \hat{\gamma} \\
f_L - f_N = \Pi_L^q(\hat{x}, \hat{\gamma}) - \Pi_N^q.
\end{aligned}
\end{equation}

**Proof.** Necessity: To prove (4), suppose the mechanism prescribes \( \gamma(x') = 1, \gamma(x) = 0 \) for some \( x', x \) with \( x' > x \). Then, the licensee with cost \( x \) has an incentive to report \( z = x' \), because in that case he obtains the license and earns an even higher profit than type \( x' \).

Using the envelope theorem, the first order conditions for incentive compatibility can be written as

\[ \partial_2 \Pi_L^q |_{z=x} = (P'(\cdot)q_1'(x) - r'(x)) q_2(x, x) = 0, \quad \forall x \leq \hat{\gamma}. \tag{8} \]

Rearranging yields \( r'(x) = P'(\cdot)q_1'(x) \), as asserted in (6).

To prove that \( r'(x) < 0 \), it is sufficient to show that \( q_1'(x) > 0 \). Suppose it is not positive. Then, with slight abuse of notation, \( q_1'(x) = \partial_2 q_1 \cdot (1 + r'(x)) = \partial_2 q_1 \cdot (1 + P'(\cdot)q_1'(x)) > 0 \), which is a contradiction. Therefore, \( q_1'(x) = \partial_2 q_1 \cdot (1 + r'(x)) > 0 \) which implies \( r'(x) > -1 \).

To prove (7), suppose \( f_L - f_N < \Pi_L^q(\hat{x}, \hat{\gamma}) - \Pi_N^q \). Then, all types \( x \) slightly above \( \hat{x} \) have an incentive to report \( z = \hat{\gamma} \). Similarly, if \( f_L - f_N > \Pi_L^q(\hat{x}, \hat{\gamma}) - \Pi_N^q \), type \( x = \hat{x} \) has an incentive to report \( z > \hat{\gamma} \).

Sufficiency: Assuming a mechanism that has the properties stated in Lemma 1 one can show that, for all \( x, z < \hat{x} \), the function \( \Pi_L(x, z) \) is pseudo-concave in \( z \), i.e., it is non-decreasing in \( z \) for \( z < x \) and non-increasing for \( z > x \). The proof for the case \( z < x \) is (using the concavity of inverse demand, (8)), and the fact that \( q_2(x, z) \) is decreasing in \( x \),

\[ \partial_1 \Pi_L = (P'(\cdot)q_1(z) + q_2(x, z)q_1'(z) - r'(z)) q_2(x, z) \geq (P'(\cdot)q_1(z) + q_2(x, z)q_1'(z) - r'(z)) q_2(x, z) = 0 \quad \text{(by (6)).} \]

The proof for the case \( z > x \) is similar and omitted.

Using this property, incentive compatibility confirms as follows:

\[ z < x \leq \hat{x} \Rightarrow \Pi_L(x, z) \equiv \Pi_L(x, x) - \int_z^x \partial_1 \Pi_L(x, y) dy \leq \Pi_L(x, x) \]

\[ \hat{x} \geq z > x \Rightarrow \Pi_L(x, z) \equiv \Pi_L(x, x) + \int_x^z \partial_1 \Pi_L(x, y) dy \leq \Pi_L(x, x) \]

\[ z > \hat{x} \geq x \Rightarrow \Pi_L(x, z) \equiv \Pi_L^N - f_N = \Pi_L(\hat{x}, \hat{x}) < \Pi_L(x, \hat{x}) \leq \Pi_L(x, x) \]

\[ x \geq \hat{x} > z \Rightarrow \Pi_L(x, z) \equiv \Pi_L(\hat{x}, \hat{x}) = \Pi_N^N - f_N = \Pi_L(x, x) \]

\[ z, x > \hat{x} \Rightarrow \Pi_L(x, z) = \Pi_N^N - f_N = \Pi_L(x, x). \]

\[ \square \]

Integrating (6) one obtains the family of incentive compatible \( r \) functions:

\[ r(x) = r_0 + \int_{\hat{x}}^x P'(\cdot)q_1'(y) dy; \quad r_0 = r(\hat{x}). \tag{9} \]

Specifically, if demand is linear, \( P(Q) = 1 - Q \), one has \( q_1^*(x) = (1 - 2d + x + r(x))/3 \), and, together with (6) one finds: \( r'(x) = P'(\cdot)q_1^*(x) = -(1 + r(x))/3 \), which implies \( r(x) = r_0 - (x-\hat{x})/4 \).
Participation and “antitrust” constraints The contract has to assure voluntary participation:

$$\Pi_2(x,x) \geq \Pi_2^N, \quad \forall x \quad \text{(participation constraints).}$$ (10)

Note that $x' > x \Rightarrow \Pi_2(x,x) \geq \Pi_2(x,x') \geq \Pi_2(x',x')$. Therefore, if the participation constraint is satisfied for some $x'$, it is also satisfied for all $x < x'$. It follows that participation constraints are satisfied if and only if $f_N \leq 0$.

Moreover, due to antitrust concerns, royalty rates cannot exceed the cost reduction, $r(x) \leq c - x$. This is assured if and only if

$$f_L - f_N \geq 0 \quad \text{(antitrust constraint).}$$ (11)

Without this constraint, the optimal mechanism may raise the marginal cost of the licensee above the prior cost $c$, while compensating him with negative fixed fee $f_L$. In the extreme this could implement the monopoly outcome.

Optimal mechanism The optimal mechanism maximizes the innovator’s expected payoff,

$$\Pi_1 = \int_\xi \left( \pi^*_1(x) + r(x)q^*_2(x) + f_L \right) dF(x) + \left( \Pi_1^N + f_N \right) \left( 1 - F(\hat{x}) \right)$$ (12)

subject to the incentive compatibility constraints (4)-(7), the participation constraint : $f_N \leq 0$, and the antitrust constraint (11).

Obviously, it is optimal to set $f_N = 0$. Also, $r(x)$ is uniquely determined by incentive compatibility except for the choice of $r_0$, and $f_L$ is determined by $r_0$ and $\hat{x}$, because $f_L = \Pi_2^L(\hat{x},\hat{x}) - \Pi_2^N$. Therefore, the optimization problem simplifies to:

$$\max_{\{r_0, \hat{x}\}} \Pi_1, \quad \text{s.t.} \quad \Pi_2^L(\hat{x},\hat{x}) \geq \Pi_2^N, \quad \hat{x} \leq \hat{x} \leq \hat{x}.$$ (13)

The optimal contract depends on the efficiency of the licensee relative to that of the innovator. If the innovator is unambiguously more efficient, the pure royalty contract tends to be optimal and exclusion, i.e., $\hat{x} < \hat{x}$, tends to occur. Whereas if the licensee is more efficient with positive probability, the optimal contract tends to employ fixed fees and exclusion is less attractive.

Proposition 1. Suppose the innovator is more efficient, i.e., $\hat{x} \geq d$, and $\frac{d}{dx} \pi^*_2(x)$ is non-decreasing in $x$. Then, the pure royalty contract (with zero fixed fee) is optimal. The optimal contract extracts the full surplus from the marginal type, $\hat{x}$, and leaves a positive surplus to all lower types.

Proof. Suppose the optimal contract exhibits $f_L > 0$. Then, one can increase the innovator’s payoff $\Pi_1$ by reducing $f_L$ while increasing $r_0$ in such a way that $\hat{x}$ remains unchanged, as shown below. By definition of $\hat{x}$ such a change in $r_0$ requires:

$$df_L = \frac{d}{dr_0} \pi^*_2(\hat{x}) dr_0,$$ (14)

and one finds:

$$\frac{d\Pi_1}{dr_0} = \int_\xi \frac{d}{dr_0} \left( \pi^*_1(x) + r(x)q^*_2(x) + f_L \right) dF(x)$$

Proof of necessity: suppose $r(x) \leq c - x$. Then, $f_L - f_N = \Pi_2^L(\hat{x},\hat{x}) - \Pi_2^N \geq \Pi_2^L(\hat{x},\hat{x})|_{r(x)=c-x} - \Pi_2^N = 0$. The proof of sufficiency is similar and hence omitted.
\[ = \int_{x}^{\hat{x}} \frac{d}{dr_0} \left( \pi^1_1(x) + r(x)q^2_1(x) + \pi^2_{2}(\hat{x}) \right) dF(x) \quad \text{(by (14))} \]
\[ \geq \int_{x}^{\hat{x}} \frac{d}{dr_0} \left( \pi^1_1(x) + r(x)q^2_1(x) + \pi^2_{2}(x) \right) dF(x) > 0. \]

The second last inequality follows from the assumed monotonicity of \( \frac{d}{dr_0} \pi^2_{2}(x) \). The last inequality follows from the fact that \( \pi^1_1(x) + r(x)q^2_1(x) + \pi^2_{2}(x) \) is equal to the sum of duopoly profits which is increasing in \( r_0 \). The latter holds because increasing \( r_0 \) moves the sum of profits closer to the more efficient firm’s monopoly profit. This completes the proof that \( f_L \) must be equal to zero.

The assertion that \( r(\hat{x}) = c - \hat{x} \) follows immediately from (7) and \( f_L = f_N = 0 \), and \( r(x) < c - x, \forall x < \hat{x} \) follows from the fact that \( -1 < r'(x) < 0 \) (see Lemma). Finally, we mention that exclusion may or may not occur, depending on the probability distribution.

Proposition generalizes Heywood et al. (2014) who showed that it is not optimal to employ fixed fees, assuming that the licensee’s cost is either equal or greater than that of the innovator, in a binary model with linear demand.

However, if one permits that the licensee is more efficient, it is no longer the case that the optimal contract does not employ fixed fees.

**Proposition 2.** Suppose the licensee is more efficient with positive probability, i.e., \( x < d \). Then, the optimal mechanism may prescribe fixed fees, \( f_L > 0 \), royalty rates lower than the cost reduction, \( r(x) < c - x \), and even negative royalty rates, \( r(x) < 0 \).

**Proof.** The proof is by counterexample. Assume linear demand and the uniform distribution with support \([0, d]\) (firm 2 is more efficient), and \( c = 0.4 \). We compute the optimal contract for various values of \( d \), summarized in the table in Figure. Evidently, the optimal contract prescribes positive fixed fees and royalty rates lower than the cost reduction, \( r(x) < c - x \), whenever the relative efficiency of the licensee is sufficiently high, i.e., for all \( d \geq 0.31 \). Pure royalty contracts are optimal only if the relative efficiency of the licensee is sufficiently low, i.e., if \( d \leq 0.27 \). The optimal contract even prescribes negative royalty rates if \( d \) is sufficiently high, i.e., if the relative efficiency of firm 1 is sufficiently low.

If the licensee is sufficiently more efficient, the innovator benefits from shifting output to the licensee by subsidizing the marginal cost of the licensee.

**4 Discussion**

The literature on patent licensing by an inside innovator claimed that the optimal contract does not employ fixed fees and typically adopts a pure royalty contract. While it is generally true that the innovator may wish to charge a high royalty rate in order to maintain a strategic cost advantage, it

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4 This assumption is satisfied, for example, if demand is linear, because in that case \( q^2_1(x) = (1 - 2(x + r(x)) + d)/3 \), \( \pi^2_{2}(x) = q^2_1(x)^2 \). Using the incentive compatible \( r \) function, it follows immediately that \( \frac{d}{dx} \pi^2_{2}(x) = 2/3 > 0 \).

5 Sketch of a formal proof: 1) \( d_0 > 0 \) reduces aggregate output, \( Q \), reduces \( q_2 \), and increases \( q_1 \). 2) Suppose, for the moment, that the reduction of \( Q \) is exclusively borne by firm 2; then, total surplus increases because \( Q \) moves towards the monopoly output of firm 1. 3) However, firm 2 reduces its output by more than \( dQ \) while firm 1 increases its output by that extra amount, i.e., output is shifted to the more efficient firm 1. Therefore, the surplus increases even more.

Heywood et al. (2014) also consider ad valorem royalties and show that for most parameter values output based royalties are superior.
can actually be more profitable to reduce that cost advantage by subsidizing the marginal cost of the licensee. The pivotal issue is whether the licensee or the innovator is able to make better use of the innovation.

Our analysis indicates that the optimality of pure royalty contracts asserted in the literature is driven by the assumption that the inside innovator is more efficient than the licensee, rather than by the assumption that the innovator is an inside patent holder.

**References**


