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Abstract

We develop a novel theory of real estate foreclosure auctions, which have the special feature that the lender acts as a seller for low and as a buyer for high prices. The theory yields several empirically testable predictions concerning the strategic behavior of the agents, both under symmetric and asymmetric information. Using novel data from Palm Beach County (FL, US), we find evidence of both strategic behavior and asymmetric information, with the lender being the informed party. Moreover, the data are consistent with moral hazard in mortgage securitization: banks collect less information about the value of the mortgage collateral.

Keywords: foreclosure auctions, asymmetric information, bunching, discontinuous strategies, securitization

JEL Codes: C72, D44, D82, G21

1 Introduction

Foreclosures of real estate have a substantial economic impact. In 2013, 609,000 residential sales in the U.S. were foreclosure related (foreclosure auctions or sales of real estate owned by

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a lender), amounting to 10.3% of total residential sales.[1] Foreclosures played an even larger role during the 2007 and 2008 financial crisis, when "four million families [...] lost their homes to foreclosure and another four and a half million [...] slipped into the foreclosure process or [were] seriously behind on the mortgage payments" (Financial Crisis Inquiry Report, Angelides et al., 2011, p. xv). Preceding the foreclosure wave, the securitization of mortgages increased from 30 per cent in 1995 to 80 per cent in 2006 (Dewatripont et al., 2010, p. 19). It is widely believed that the securitization of mortgages led to moral hazard for originating banks and that "collapsing mortgage-lending standards and the mortgage securitization pipeline lit and spread the flame of contagion and crisis" (Angelides et al., 2011, p. xxiii).[2]

We develop a novel theory of foreclosure auctions, incorporating the institutional details of foreclosures. Our theory both serves a better understanding of the foreclosure process and provides insight about the moral hazard involved in securitization.

A foreclosure auction is run by a government agency after a mortgagee stopped making payments to the lender. The lender (typically a bank) and third-party bidders (typically real estate brokers) participate in this auction. An important feature of foreclosure auctions is that only payments up to the *judgment amount* (which is basically the amount owed) are paid to the lender.[3] Payments above the judgment amount, if any, are paid to the owner of the property. The owner typically does not participate. In such an auction, the bank essentially acts as a seller below the judgment amount, and as a buyer above the judgment amount.

Another important feature of foreclosure auctions is that the bank is likely to have superior information about the quality of the property. There are several reasons for this. First, lenders typically spend large resources on appraisals to get a more precise value of the collateral before granting mortgages. Second, banks also typically put significant effort in assessing the probability of default of potential borrowers, which is well known to be negatively correlated with the value of the collateral (see Qi and Yang, 2009, and the references therein).

Unlike in a regular real estate sale, this information is not readily available to the buyers. The owner is neither obligated nor may have an incentive to cooperate with the foreclosing

[1] See `http://www.realtytrac.com/Content/foreclosure-market-report/december-and-year-end-2013-us-residential-and-foreclosure-sales-report-7967`.

[2] See Brunnermeier (2009); Mian and Sufi (2009); Keys et al. (2010); Tirole (2011) for the academic literature.

[3] The judgment amount chosen by the court is by and large the amount owed to the bank. It can additionally include unpaid utilities fees, legal fees, etc.

lender in providing access to the property to the potential buyers. Moreover, the properties in foreclosure are sometimes abandoned by their owners, making buyer access and physical inspection problematic.

In contrast to the bank, which is mostly interested in reselling the property and may have superior information about quality, brokers typically participate with the intent of renovating the property.[4] The costs of renovation are to a much larger degree driven by brokers' idiosyncratic components, which, at least to a first approximations, may be independent from the bank's information about the quality.[5] In our model, the broker's valuation generally depends both on the quality and its own idiosyncratic component.

We derive participants' equilibrium bidding strategies in a foreclosure auction with a common value component on the bank's side. However, as a benchmark, we first analyze a symmetric information environment, where the bank's information is known to the brokers. In this environment brokers bid their own type as in a standard English auction. The bank bids its valuation plus a markup for valuations far below the judgment amount. The bank's bid is equal to the reserve price a seller would set. The bank's bid takes into account the classical trade-off of a seller: a higher bid lowers the bank's probability of selling the house, but increases the bank's revenue in case of sale. However, one side of this trade-off disappears for banks that bid exactly the judgment amount: a higher bid still lowers the probability of sale, but does not lead to higher revenues for the bank, since the additional revenue goes to the original owner. This change of the trade-off causes a bunching of banks' bids at the judgment amount. If the bank's valuation is sufficiently high – higher than the judgment amount – the bank does not want to sell the property, but would want to keep it. Hence, for high valuations, the bank bids its own type, just as a buyer would do in an English auction.

A bank intent on selling the property will bid more in an asymmetric information environment than in a symmetric information environment. The reason is that there is another effect in the bank's trade-off: the bank's bid also serves as a costly signal about the quality of the

[4]When visiting websites of several brokers participating in the foreclosure auctions in our data set, we could see that they often specialize in renovating foreclosed properties.

[5]Assuming private values also helps us to focus on the common value component on the seller's side, which has been ignored so far in the empirical literature. Our theoretical results are likely to extend to a setup in which buyers also have information about the common value component. However, such a setup would add an additional layer of complexity to our model. Further, for a better understanding of moral hazard in the securitization process, what matters is the common value component on the *seller's side*.

property. Intuitively, the bank is willing to incur the cost of an excessively low probability of sale (given the price conditional on sale), in order to prove that the quality of the house is high. This cost has to be sufficiently high, so that banks with lower qualities do not have an incentive to imitate banks with higher qualities. This *signalling premium* means that the bank charges more than what it would have charged in a symmetric information environment.

The bunching result from the symmetric information setup holds also with asymmetric information. Now, however, bunching has a surprising implication: there cannot be an equilibrium where the bank's bidding strategy is continuous around the bunch. The reason for this is that brokers who are supposed to drop slightly below the judgment amount will instead prefer to wait for the price to rise above the judgment amount, as they would then be certain to obtain a significantly higher quality house at a slightly higher price. Since brokers will not have an incentive to drop out at prices slightly below the judgment amount, neither will the bank: increasing the bid to the judgment amount will lead to a higher revenue for the bank, while not changing the probability of sale. This logic implies that there has to be a discontinuity in the bank's bidding strategy below the judgment amount. We show in this paper that the bank's equilibrium bidding strategy is strictly increasing and continuous for low valuations, then exhibits a discontinuous jump to the judgment amount and bunching at the judgment amount, and is equal to the bidding strategy of a buyer for valuations above the judgment amount.

Our model has two striking empirically testable implications. First, there is bunching of banks' bids at the judgment amount. Second, in the presence of both auctions with symmetric and asymmetric information, one should expect to observe the following. For values slightly below the judgment amount, the probability of sale (the probability of the bank selling the house to a third-party bidder) first increases with the bank's bid and then drops down discontinuously at the judgment amount. The explanation for this consists of two pieces. First, with asymmetric information there is a gap in the bank's bid distribution just below the judgment amount (corresponding to the discontinuous bidding strategy), whereas for symmetric information there is no gap. Hence, bids observed just below the judgment amount will be from symmetric information auctions. For other bids, one will observe a mixture of symmetric and asymmetric information. Second, under asymmetric information, the signalling premium leads to higher bids by the bank and hence a lower probability of sale. Putting the two pieces together reveals that the probability of sale will be lower in the gap than just below and just above the gap. This implies a probability of sale which is non-monotonic with the bank's bid at the lower bound of the gap and decreases discontinuously at the upper bound of the gap (which is the judgment amount). This *non-monotonicity* (which appears like a violation of the law of demand, but is really a consequence of sample selection) and *demand discontinuity* are empirically testable implications of our theory.

To bring our theory to the data, we have collected a novel data set with foreclosure auctions from Palm Beach County in Florida. We have data on 12,788 auctions from 2010 to 2013 with a total judgment amount of $4.2bn. The data reveal that bunching indeed occurs at the judgment amount. We also observe that the probability of sale increases with the bank's bid just below the judgment amount. Further, demand exhibits a downward discontinuity at the judgment amount. This points, as we have argued, to the presence of auctions with both symmetric and asymmetric information in our data set.

Our theory and our data allow us to investigate how securitization affects the information about the mortgage collateral. Moral hazard in the securitization of mortgages was a major concern during the financial crisis. Originating banks securitized their mortgages and sold the mortgage backed securities on the capital market. This reduced banks' "skin in the game" and thus led to moral hazard: banks did not exert sufficient effort in gathering information before granting mortgages and hence excessively granted mortgages (see Mian and Sufi, 2009; Dewatripont et al., 2010; Keys et al., 2010).[6]

The connection between moral hazard at the stage of granting a mortgage and asymmetric information at the time of the foreclosure is the following. In the presence of moral hazard for securitized mortgages, a bank will exert less effort in collecting information both about the value of the collateral and the financial situation of the borrower. One effect of this is that the bank will be less informed about the probability of default – this is indeed what the literature has focused on so far. However, there is also another effect: once a mortgage defaults, the loss given default (and hence the value of the collateral) matters. If the originating bank spent less

[6]Besides moral hazard, informational asymmetries can also lead to adverse selection. However, as argued in the literature (see Keys et al., 2010), the institutional details of the securitization process suggest that adverse selection is much less important than moral hazard.

effort gathering the information about the value of the collateral when granting the mortgage, it will also have less of an informational advantage during the foreclosure auction.[7]

One would therefore expect the bank to have less private information about the quality of the house for securitized mortgages than for non-securitized mortgages. This implies that the non-monotonicity and the discontinuity of the probability of sale as a function of the bank's bid should be present for non-securitized mortgages, but not for securitized mortgages (or at least only present to a smaller extent). An analysis of the data reveals that this is indeed the case.

We provide further evidence for banks having less information for securitized than for non-securitized mortgages by considering bank's bidding strategies. Because of the signalling premium, a bank bids higher under asymmetric information given the same opportunity cost of selling. In order to estimate banks' bidding strategies for securitized and non-securitized mortgages and to correct for possible selection effects, we collected additional data on the resale prices and the tax assessment values of foreclosed properties.[8] These two additional variables allow us to deal with the selection effect while simultaneously controlling for unobserved heterogeneity. We then estimate banks' bidding functions for securitized and non-securitized mortgages. We show that for a given opportunity cost, a bank holding a non-securitized mortgage bids more than a bank holding a securitized mortgage. This provides additional evidence for different levels of asymmetric information at the foreclosure stage and is consistent with moral hazard at the mortgage underwriting stage.

Related Literature. To the best of our knowledge, this is the first paper providing an economic analysis of the bidding behavior in foreclosure auctions. So far, the details of foreclosure auctions have been mainly analyzed in the law literature (see e.g. Nelson and Whitman (2004)).

For our theoretical analysis, we build on two strands of literature. The first analyzes bidding behavior of buyers with information about a common value component, starting with Milgrom and Weber (1982). The second, more recent, literature analyzes auctions in which the seller

[7]The originating bank typically also acts as the servicer of the mortgage, i.e. it bids in the foreclosure auction. This makes an alternative explanation – that information is collected by originating banks, but lost when the mortgage backed securities are sold – less plausible.

[8]A different distribution of banks' bids for securitized and non-securitized mortgages could be either due to the signalling premium or due to banks having a different distribution of opportunity costs for securitized and non-securitized mortgages.

has superior information, see Jullien and Mariotti (2006), Cai et al. (2007), and Lamy (2010). For bids above the judgment amount, we can use results from the former; for bids sufficiently below the judgment amount, we can use results from the latter literature. Our theoretical contribution is to show that there are surprising predictions for the intermediate range of bids, which are between the informed seller and the informed buyer regions.

This is also the first paper to provide empirical evidence for a common value component on the *seller's side* in an auction. In this sense, it complements the existing empirical auction literature which has considered testable predictions for models with a common value component on the *buyer's side*, see Hendricks and Porter (1988) and Hendricks et al. (2003) for offshore oil auctions, and Nyborg et al. (2002) and Hortaçsu and Kastl (2012) for treasury auctions.

Foreclosure auctions are particularly well suited to analyze the presence of a common value component on the seller's side. This is because (i) participation in the sales is required by law and (ii) the seller is a private organization. Without (i) (i.e. with voluntary participation) submarkets with particularly strong adverse selection would operate at a low scale or might even completely break down by an Akerlof (1970) type of argument. Hence, if one cannot perfectly separate submarkets with symmetric and asymmetric information, there is a selection effect working against finding asymmetric information.[9] Point (ii) matters, because governments are less likely to have superior information. Much of the literature has analyzed markets in which either (i) or (ii) was not satisfied, hence even if one had looked for a common value component on the seller's side rather than on the buyer's side, it would have been difficult to find it.

The application of our theory to securitization relates to the literature dealing with moral hazard in the securitization process, such as Mian and Sufi (2009), Keys et al. (2010), Tirole (2011). Our analysis is consistent with the view that securitization led to moral hazard. Our analysis is complementary to the analysis in the existing literature that provides evidence for moral hazard with respect to the probability of default. Our analysis highlights the importance of moral hazard with respect to the other determinant of the expected shortfall of a mortgage: the loss given default (which is determined by the value of the mortgage collateral).

In a wider sense, our article relates to the growing literature that uses insights from auctions

[9] It is plausible that one cannot perfectly separate different submarkets. At least in our data set, we find evidence consistent with both symmetric and asymmetric information auctions being present.

to analyze important questions in financial markets, such as Heller and Lengwiler (1998), Hortacsu and McAdams (2010), Cassola et al. (2013), Zulehner et al. (2013), where our novelty is to apply it to mortgages.

2 Foreclosure Process

Property foreclosure is a remedy allowed by law to the lender if the borrower defaults on the mortgage. While it generally transfers the ownership of the property to the lender, the process may be a lengthy one and the details depend on the jurisdiction. In the United States, roughly 40% of the states adopt what is called a *judicial* foreclosure, while the rest is comprised of the *nonjudicial* foreclosure states. In the following, we will restrict our attention to judicial foreclosures, since we are using a data set from a state with this foreclosure system (Florida).[10]

When the borrower defaults on a mortgage, the court grants the lender a judgment of foreclosure. The judgment will specify the date of the foreclosure sale. The judgment holder is often required to advertise the sale in a local newspaper. The title is transferred to the highest bidder, and certain liens and encumbrances may survive the foreclosure sale. Finding out this information is the sole responsibility of the prospective buyers.

Unlike in a regular real estate sale, the bank is not allowed to run an open house, or provide individual access to prospective buyers in a foreclosure sale. The reason for this is that until the sale is complete, any access to the property usually needs owner permission. The owner, however, may have little incentive to cooperate with the foreclosing lender, or may even have abandoned the property.

The sale is conducted through a public auction. The bank is an active participant and is allowed to bid with credit up to the amount owed (also called the judgment amount), while other participants (usually real estate brokers) are required to submit bids in cash or cash equivalent.[11] The auction is usually conducted in an open format. The property title is awarded to the highest bidder, and the auction price is equal to the highest bid. The proceedings of

[10] The main legal difference between judicial and nonjudicial foreclosure is that in the former, the process takes place in the court system, while in the latter, it takes place outside the courts. Judicial foreclosures always result in the property being sold at a public auction, *the foreclosure auction*. This and other information concerning judicial and nonjudicial foreclosures can be found in Nelson and Whitman (2004).

[11] The bidders are often required to make an advance deposit as a certain percentage of the judgment amount. This deposit will be credited towards the bid, should the bidder win the auction.

the auction are collected by the court. The court transfers any payments up to the judgment amount to the lender. In case the auction price exceeds the judgment amount, the additional proceedings are paid to junior liens holders and to the owner of the property.[12]

We use foreclosure auction data from Palm Beach County (FL). Palm Beach County moved to an online foreclosure auction system in 2010. Bidders bid in an Internet English auction through an electronic bidding proxy. The lender can submit their bid one week prior to the auction, while third-party bidders can start bidding on the day of the auction. The web interface shows the leading bid and the identity of the leading bidder (in particular also whether the leading bidder is the lender's representative).

In the next section, we present a stylized model of the foreclosure auction that captures the main institutional details. First, the payment to the lender in the auction is capped at the judgment amount. As we show, this will result in a differential role of the lender, effectively making it a seller of the property if the bid is lower than the judgment amount, and a buyer otherwise. Second, the bank will have private information about the quality of the house. This information is also relevant to the brokers, but their valuations will also depend on other idiosyncratic factors.

3 Model

Consider the bank (also called the seller, S) and n real estate brokers (or buyers, B) who participate in the foreclosure auction. The original owner of the property is not modeled as a strategic player, since he typically lacks the resources to participate in the auction.[13] The judgment amount (i.e. the balance of the mortgage) is denoted as v_J. The foreclosure auction is modeled as variant of the (button) English auction as in Milgrom and Weber (1982). The auction website expedites bidding by allowing the participants to employ automatic bidding agents. The bidders provide their agents the maximum price they are willing to pay (their *dropout* price), and the agent then bids on their behalf. These proxy bids can be updated at

[12]If the broker wins the auction but the sale price falls short of the judgment amount, the lender can then ask the court for a deficiency judgment, to be applied against the borrower's other assets. Deficiency judgments are relatively rare, as they are only pursued when the borrower owns other significant assets. But in that case, absent a sudden crash of the real estate market, the borrower is better off selling these assets and avoiding a costly foreclosure that is likely to reduce the borrower's credit rating.

[13]In legal documents, the bank (or lender) is referred to as the plaintiff. The original owner is referred to as the defendant.

any time.[14] The brokers are able to observe if the bank's maximum bid has been exceeded, while a broker's dropout price is unobservable to the bank. The broker may therefore change its maximum bid upon observing the bank's dropout price. As for the bank, we assume that it commits to its dropout price.[15]

The key difference from a standard auction is that the proceeding of the foreclosure auction up to the judgment amount goes to the bank. Anything above the judgment amount goes to the original owner.[16] This effectively turns the bank into a seller for prices below and into a buyer for prices above the judgment amount.

The winner pays the auction price p. If the price exceeds the judgment amount, $p \geq v_J$, then the bank gets v_J and the owner pockets the difference $p - v_J$. If the price is below the judgment amount, $p < v_J$, then the bank gets p and the owner gets nothing. The property is transferred to a broker only if a broker wins; otherwise, the bank keeps the property. It follows that if the bank wins the auction, it effectively pays the auction price to itself. So in reality no money changes hands in this case. But if a broker wins, then money is actually transferred, from the broker to the bank and possibly the owner as well (if the auction price exceeds the judgment amount).

As usual, we model the foreclosure sale (auction) as a game of incomplete information. As explained in the empirical section of the paper, banks and brokers buy houses for different purposes. While banks mostly simply sell the houses, brokers typically renovate the property before reselling. Motivated by this, we make the following assumption concerning the information of the bank and the brokers. First, we assume that the ith broker's idiosyncratic signal, denoted as $X_B^i \in \mathbb{R}_+$, only concerns its renovation value added to the house. Second, we assume that the bank's signal $X_S \in \mathbb{R}_+$ concerns the baseline resale value of the house. The signals X_B^i and X_S will be sometimes referred to as buyers' and seller's *types*. Their realizations will be denoted as x_B^i and x_S, respectively.

Since the bank has typically collected information about the value of the collateral before

[14]Such proxy bidding makes the button model even more applicable here, as it alleviates the need to model difficult features such as e.g. jump bidding that may be present in the traditional open auctions.

[15]As it will become apparent later, the bank does not have an incentive to change its dropout price during the auction, so that the assumption of commitment is irrelevant in equilibrium.

[16]Or, junior liens holders. The distinction between payments to the original owner and to junior liens holders does not matter for our purposes.

granting the mortgage, it is assumed to be the informed party.[17] Its signal X_S is normalized to equal the expected value of the house in the market, so the bank's valuation is

$$u_S(x_S) = x_S.$$

The brokers do not observe X_S; they only privately observe their own signals X_B^i. Broker i's expected value of the house, given its own signal x_B^i and the bank's signal x_S, is denoted as $u_B(x_B^i, x_S)$.

We make the following assumptions concerning the expected valuations of the brokers.

Assumption 1 (Broker valuations)**.** *A broker's expected valuation is differentiable and strictly increasing in its own signal x_B, and nondecreasing in the bank's signal x_S,*

$$\frac{\partial u_B(x_B, x_S)}{\partial x_B} \geq \theta, \quad \frac{\partial u_B(x_B, x_S)}{\partial x_S} \geq 0,$$

for some constant $\theta > 0$. Moreover, the derivatives satisfy the single crossing *condition*

$$\frac{\partial u_B(x_B, x_S)}{\partial x_S} < \frac{du_S(x_S)}{dx_S}.$$

This single crossing condition is sufficient to ensure that the broker's dropout strategy in the open (button) auction is increasing in the broker's signal. If u_B does not depend on x_S, we have a special case of *private values*. Otherwise, the valuations are interdependent. For reasons that will be clear in the sequel, we normalize the broker signals so that the value conditional on winning the auction is equal to the signal,

$$u_B(x_B, x_B) = x_B. \tag{1}$$

This normalization is without loss of generality because Assumption 1 ensures that $u_B(x_B, x_B)$ is continuous and strictly increasing in x_B.[18]

We make the following assumption regarding the distribution of the signals.

[17] The bank also typically collects information about the financial situation of the borrower, which is correlated with the value of the collateral.

[18] To see that the normalization is indeed without loss of generality, consider the general case $u_B(x_B, x_B) = v(x_B)$ for some function v. We can define the transformed signal $\tilde{x}_B = v(x_B)$ and the transformed utility function $\tilde{u}_B(\tilde{x}_B, x_S) = u_B(v^{-1}(\tilde{x}_B), v^{-1}(x_S))$. Then, for the transformed utility function and signal, the normalization holds: $\tilde{u}_B(\tilde{x}_B, \tilde{x}_B) = \tilde{x}_B$.

Assumption 2 (Signals)**.** *The bank's signal X_S is drawn from a distribution F_S supported on $\mathbb{R}_+$, with density f_S continuous and positive on the support. The broker signals X_B^i, $i = 1, ..., n$, are identically and independently distributed, and drawn from the distribution F_B, supported on $\mathbb{R}_+$, with density f_B continuous and positive on the support.*

The independence assumption is made to simplify the analysis of the game, by eliminating the need to consider adjustments that brokers would otherwise make to their proxy bids following dropouts by other brokers. Under independence, we shall see that the information in the auction will be transmitted only from the bank to the brokers, following the bank's dropout from the auction. After that, the brokers essentially have independent private values. They simply enter those values as their (updated) proxy bids, and there will be no updating from brokers' dropout prices.[19]

The *Myerson virtual value* is defined in the present setting as

$$J_B(x_B, x_S) = u_B(x_B, x_S) - \frac{\partial u_B(x_B, x_S)}{\partial x_B} \frac{1 - F_B(x_B)}{f_B(x_B)}. \tag{2}$$

We make the standard monotonicity assumption concerning $J_B(x_B, x_S)$, a variant of the Myerson regularity condition. This assumption ensures quasi-concavity of the profit functions, so that the second order conditions are fulfilled.

Assumption 3 (Virtual value monotonicity)**.** *The function $J_B(x_B, x_S)$ is strictly increasing in x_B.*

In the following, we will use $x_{(1)}$ and $x_{(2)}$ for the highest and second highest order statistics among n brokers' signals. Further, we will use $F_{(1)}$ and $F_{(2)}$ for the corresponding distributions. All technical proofs are banned to the Appendix.

We have two reasons to focus on a model in which the seller's superior information about the quality of the property being auctioned is at the center of interest. First, the bank is likely to have better information due to having gathered information about the property when granting the mortgage. Information about the mortgagee can also serve as an indicator of how well the mortgagee maintains the house. Second, we will later consider securitized and non-securitized

[19] Independence is also assumed in Jullien and Mariotti (2006), while Cai et al. (2007) and Lamy (2010) allow the buyer signals to be correlated, but still independent of the seller's signal.

mortgages. As it will become clear later, it is reasonable to believe that the informativeness of the bank's signal about the quality of the house is different for securitized and non-securitized mortgages.

4 Symmetric Information

It is useful to start with the case of *symmetric information*, where the bank's information is known to the brokers. This independent private values (IPV) setup is particularly useful to highlight the role of the judgment amount, below which the bank acts as a seller and above which the bank acts as a buyer.

Standard arguments imply that it is a weakly dominant strategy for the broker to choose its valuation as the drop-out price, so

$$p_B(x_B) = u_B(x_B, x_S).$$

The bank's bidding behavior can be best derived by first considering two hypothetical cases. First, nothing is owed to the bank ($v_J = 0$) and hence the bank always acts as a buyer. Second, an infinite amount is owed to the bank ($v_J = \infty$) and hence the bank always acts as a seller. After deriving these two hypothetical cases, we can put the two pieces together and additionally derive the bank's bidding behavior for the transitional region in which the bank turns from a seller to a buyer.

First, consider the case in which the bank always acts as a buyer ($v_J = 0$). By standard arguments for English auctions, the bank's optimal strategy is to bid its own type, i.e. $p_S(x_S) = x_S$.

Next, consider the case when $v_J = \infty$, i.e. the standard auction where the bank acts as the seller. The brokers will drop out at prices $u_B(x_B^i, x_S)$. If the bank decides to drop out at price p, its auction revenue will be equal to p if there is one, and only one broker that is active in the auction at that price. If there are multiple brokers active at p, the bank's revenue will be equal to the second-highest dropout broker price, i.e. $u_B(x_{(2)}, x_S)$. Denote by $\hat{x}_B$ the broker's type indifferent between staying in or dropping out at p, so that $p = u_B(\hat{x}_B, x_S)$. Then the bank's expected profit is the usual expression from auction theory

$$\Pi_S(x_S, \hat{x}_B) = u_B(\hat{x}_B, x_S) n F_B(\hat{x}_B)^{n-1}(1 - F_B(\hat{x}_B)) + \int_{\hat{x}_B}^{\infty} u_B(y, x_S) dF_{(2)}(y) + x_S F_B(\hat{x}_B)^n,$$

where the first term in the sum is the revenue if the property is sold at the bank's bid. Observe that the bank bid can be viewed as a *reserve price* in this context. The second term is the revenue if the property is sold above the reserve, and the third is the bank's utility if it retains the property. (If there is only one broker, $n = 1$, then the integral term should be excluded from the above formula.)

The bank will choose the price, or, equivalently, the broker's marginal cutoff $\hat{x}_B$, optimally. The optimal marginal cutoff is given by the first order condition

$$J_B(x_B^*(x_S), x_S) = x_S, \tag{3}$$

which can be derived by setting $\partial \Pi_S / \partial \hat{x}_B = 0$. It can be verified that the second order condition is satisfied because of Assumption 3.

This is, of course, a well-known result in auction theory concerning the optimal reserve price, adapted to the setting where the buyer's valuations depend on the seller's information x_S, observable to the buyers.[20] A sufficient condition for the bank's optimal strategy to be monotone is

$$\frac{\partial J_B(x_B, x_S)}{\partial x_S} < 1, \tag{4}$$

which can be obtained by totally differentiating (3) with respect to x_S. It can be checked that (4) is implied by the stronger but more intuitive conditions

$$\frac{\partial u_B(x_B, x_S)}{\partial x_S} < 1, \qquad \frac{\partial^2 u_B(x_B, x_S)}{\partial x_B \partial x_S} \geq 0. \tag{5}$$

Now consider the case of our primary interest, $0 < v_J < \infty$. If the bank has valuation $x_S \leq v_J$, it will not drop out at the price above v_J: the bank is not entitled to any revenue in excess of v_J, so staying in the auction beyond v_J will only reduce the probability of sale. So effectively, the bank's problem is to maximize its profit with the additional constraint $p \leq v_J$, or, equivalently, $u_B(\hat{x}_B, x_S) \leq v_J$. As we have seen, the bank's profit is quasiconcave in $\hat{x}_B$, so the optimal price $p_S^0(x_S)$ is equal to $u_B(x_B^*(x_S), x_S)$ if $x_S < \underline{x}_S$ and v_J otherwise, where the border between the separating and the bunching region $\underline{x}_S$ is implicitly defined by $u_B(x_B^*(\underline{x}_S), \underline{x}_S) = v_J$.

[20]See Myerson (1981).

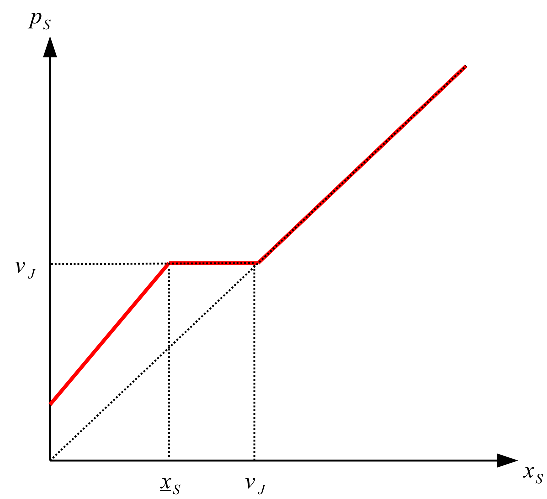


Figure 1: The bank's bidding strategy $p_S(x_S)$ under independent private values. For $x_S < \underline{x}_S$ the bank bids its valuation x_S plus a markup, for $x_S > v_J$ the bank bids its valuation. The bids of banks with $x_S \in [\underline{x}_S, v_J]$ are bunched at v_J.

If, on the other hand, $x_S > v_J$, then the bank is not willing to sell the house at a price below x_S, but also will not benefit from a price above x_S as the excess $p - v_J$ will go to the owner. It follows that the bank's optimal strategy in this case is to drop out at x_S, $p_S(x_S) = x_S$.

We summarize these findings in the proposition below.

Proposition 1. *The bank's optimal strategy is given by*

$$p_S^0(x_S) = \begin{cases} u_B(x_B^*(x_S), x_S) & \text{if } x_S < \underline{x}_S, \\ v_J & \text{if } x_S \in [\underline{x}_S, v_J] \\ x_S, & \text{if } x_S > v_J \end{cases}$$

where $x_B^*(x_S)$ *is uniquely determined from* $J_B(x_B^*(x_S), x_S) = x_S$ *and* $\underline{x}_S$ *is implicitly given by* $u_B(x_B^*(\underline{x}_S), \underline{x}_S) = v_J$*. The optimal strategy is strictly increasing in* x_S *for* $x_S < \underline{x}_S$ *provided* (4) *or* (5) *hold, in which case the bank's prices are pooled over an interval* $[\underline{x}_S, v_J]$.

The bank's equilibrium behavior is different depending on whether the bank's valuation is below or above the judgment amount. The bank acts in a seller role in the former case, and in a buyer role in the latter. The most interesting feature of the equilibrium is the bunching region. In this region, the bank's dropout prices are pooled at the judgment amount. This is illustrated in Figure 1.

The following intuition can be given for the bunching of banks' bids at the judgment amount. Consider comparative statics with respect to the bank's valuation x_S. For low valuations x_S, the bank bids its valuation plus the monopoly markup. The optimal bid of the bank balances two effects in a trade-off: a higher bid increases the bank's expected profits conditional on sale, but also lowers the probability of selling the house to a third party. As we increase x_S, this trade-off changes in a way that makes higher bids more attractive to the bank. Hence, the bank's bid increases with x_S. However, at the point where the bank's bid reaches v_J, one part of the trade-off disappears for price increases (but not decreases): a higher bid by the bank does not increase the bank's profit conditional on selling, since any additional revenues go to the original owner. Therefore, as x_S increases, the bank's optimal bid stays at v_J. Once x_S surpasses v_J, the bank actually prefers retaining the property, since it is worth more than the amount owed. Therefore, the bank's bid will again increase with x_S for x_S above v_J. The flat piece in the bank's bidding function corresponds to a mass point in banks' bid distributions.

5 Asymmetric Information

We now consider the general environment with common values, where the bank will be the informed party. As we shall see, there are some novel features in this setting compared to the symmetric information setup. The main new feature is that the equilibrium will involve a *gap* below the judgment amount.

As with symmetric information, the bank will act as a seller for lower types, and as a buyer for higher types. Again, we begin by considering the extreme cases $v_J = 0$, where the bank always acts as a buyer, and $v_J = \infty$, when the bank always acts as a seller.

5.1 Bank-buyer Equilibrium ($v_J = 0$)

If $v_J = 0$, both the bank and the brokers act as buyers bidding in a standard English auction. The solution can be found by adapting a Milgrom and Weber (1982) type of argument to our setup. The bank knows its valuation x_S, so will drop out at the price equal to x_S. In a symmetric equilibrium, a broker's dropout strategy $p_B(x_B)$ is found through the inverse bidding strategy $X_B(p)$, which in turn is found through the following standard indifference condition.[21]

$$u_B(X_B(p), p) = p \tag{6}$$

The intuition is that, given the auction has reached price p, if a broker decided to drop out at price p instead of $p + \epsilon$, this decision will change the broker's expected payoff only if the bank (and the other brokers) dropped out at prices $p \in (p, p + \epsilon)$. If the bank drops out at $p = x_S$, then in the limit as $\epsilon \to 0$, the expected value of the house to the broker will be $u_B(x_B, p)$. In equilibrium, the broker will drop out at a price such that he is indifferent between dropping out and continuing, while single crossing $\partial u_B/\partial x_S < \partial u_S/\partial x_S = 1$ implies that the broker will not benefit from waiting and dropping out at higher prices. Also, the same single crossing condition implies that brokers with higher types will prefer to continue. Thus, a broker's type $X_B(p)$ dropping out at price p is (uniquely) found from the condition (6). Given our normalization $u_B(x, x) = x$, we see that $X_B(p) = p$, so the broker, even though uninformed, in equilibrium will also drop out at the price equal to its type.

If the bank drops out while there are at least two brokers remaining in the auction, the auction becomes essentially an independent private values auction, since all the information about the common value component is public. In this case, each remaining broker i revises his drop-out price to $\tilde{p}_B^i = u_B(x_B^i, \tilde{p})$, where $\tilde{p}$ is the drop-out price of bank.

This equilibrium is described in the proposition below.

Proposition 2 (Bank-buyer equilibrium)**.** *In a broker-symmetric equilibrium, both the bank and each broker i will drop out at the prices equal to their signals,*

$$p_S(x_S) = x_S, \qquad p_B^i(x_B^i) = x_B^i \text{ for all } i.$$

If the bank drops out at price $\tilde{p}$ and at least two brokers remain in the auction, then the remaining brokers' drop-out prices are $\tilde{p}_B^i = u_B(x_B^i, \tilde{p})$ for each broker i remaining in the auction.

5.2 Bank-seller Equilibrium ($v_J = \infty$)

Auctions in which the seller has information about the common value component have been considered in Jullien and Mariotti (2006), Cai et al. (2007) and Lamy (2010). These papers

[21] By symmetry here, we mean that the *brokers* adopt the same strategy.

consider the case of a public reserve price and characterize a separating equilibrium in strictly increasing, continuous and differentiable strategies. We begin by adapting the equilibrium characterization results in the aforementioned papers to our setting.

We restrict attention to equilibria where the bank adopts an increasing and continuous equilibrium dropout strategy $p^*_S(x_S)$, with a differentiable inverse $X^*_S(p)$. Given our assumption that broker signals are independent, only the bank's dropout price is relevant for information updating. Denote a broker's dropout strategy as $p^*_B(x_B)$, with the inverse denoted as $X^*_B(p)$. As in Milgrom and Weber (1982), $X^*_B(p)$ is found by equating the object's expected value to the broker assuming the bank drops out at p, to the price p:

$$u_B(X^*_B(p), X^*_S(p)) = p \tag{7}$$

After a drop-out of the bank at a price $\tilde{p}$, a broker's dropout strategy is simply $u_B(x_B, X_S(\tilde{p}))$ as the brokers will then have independent private values.

The following proposition describes the separating equilibrium in our model.

Proposition 3 (Bank-seller equilibrium)**.** *There is a unique equilibrium in monotone differentiable strategies. The bank's inverse bidding strategy $X^*_S(p)$ and the brokers' inverse bidding strategies $X^*_B(p)$ before a drop-out of the bank are given by the (unique) solutions to the differential equation system*

$$\frac{dX^*_S(p)}{dp} = \frac{(J_B(X^*_B, X^*_S) - X^*_S)]f_{(1)}(X^*_B)}{\frac{\partial u_B(X^*_B,X^*_S)}{\partial x_S}(u_B(X^*_B, X^*_S) - X^*_S)f_{(1)}(X^*_B) + \frac{\partial u_B(X^*_B,X^*_S)}{\partial x_B}\int_{X^*_B}^{\infty}\frac{\partial u_B(x,X^*_S)}{\partial x_S}f_{(2)}(x)dx}, \tag{8}$$

$$\frac{dX^*_B(p)}{dp} = \frac{\frac{\partial u_B(X^*_B,X^*_S)}{\partial x_S}(F_{(2)}(X^*_B) - F_{(1)}(X^*_B)) + \int_{X^*_B}^{\infty}\frac{\partial u_B(x,X^*_S)}{\partial x_S}f_{(2)}(x)dx}{\frac{\partial u_B(X^*_B,X^*_S)}{\partial x_S}(u_B(X^*_B, X^*_S) - X^*_S)f_{(1)}(X^*_B) + \frac{\partial u_B(X^*_B,X^*_S)}{\partial x_B}\int_{X^*_B}^{\infty}\frac{\partial u_B(x,X^*_S)}{\partial x_S}f_{(2)}(x)dx}, \tag{9}$$

subject to the initial conditions $X^*_S(\underline{p}) = 0$ and $X^*_B(\underline{p}) = \underline{p}$. The lowest price offered by the bank $\underline{p}$ is given by $\underline{p} = p_S(0) = u_B(\underline{x}_B, 0)$, where $\underline{x}_B$ is the lowest broker type that purchases with positive probability, given by the unique solution to $J_B(\underline{x}_B, 0) = 0$. For an out-of-equilibrium reserve price $p < \underline{p}$, brokers believe that the bank's type is the lowest possible.

If the bank drops out of the auction, the remaining bidders bid up to $u_B(x_B, X_S(\tilde{p}))$, where $\tilde{p}$ is the bank's drop-out price.

The proof adapts Cai et al. (2007) to our setting where the bank is an active bidder in the open auction. We also show that the full-information price is the only price that can be offered by the lowest-type bank in such a separating equilibrium. Thus the equilibrium outcome is unique (assuming that the strategies are differentiable and monotone). The out-of-equilibrium beliefs for prices lower than $\underline{p}$ are indeterminate, but must be sufficiently pessimistic so as to provide the bank with an incentive not to drop out at lower prices. The most pessimistic beliefs (i.e. believing that the bank's type is the lowest possible) are reasonable, and they indeed support this unique equilibrium outcome.

We note in the following Corollary a noteworthy property of this separating equilibrium. In comparison with the symmetric information setup considered in Section 4, there is *signalling premium* in the asymmetric information equilibrium.

Corollary 1 (Signalling premium). *Under asymmetric information, the bank bids higher, $p_S^*(x_S) > p_S^0(x_S)$ for $x_S > 0$ and $p_S^*(0) = p_S^0(0)$; brokers' bids are lower, $p_B^*(x_B) < p_B^0(x_B)$. The probability of sale to the broker as a function of the seller's price is* lower *under asymmetric information than under symmetric information.*

The intuition for the bank bidding more under asymmetric information is that bidding more is a costly signal about quality, since it decreases the bank's probability of sale: a lower probability of selling hurts a bank with a low opportunity cost of selling more than a bank with a high opportunity cost of selling. The intuition for brokers bidding less under asymmetric information is a variant of the standard *winner's curse*: a broker is more likely to win the auction if the bank knows that the quality is low. Observe that the signalling premium and the winner's curse lead to welfare losses, since less gains from trade are exploited than under symmetric information.[22] This is illustrated in Figure 2.

Motivated by our empirical application, we now investigate how the bank's strategy is affected as its information concerning the common value component becomes less precise, and hence less relevant to the brokers. Following Cai et al. (2007), we consider a linear specification in the form

$$u_B(x_B, x_S) = x_B + \alpha x_S, \qquad \alpha \in [0, 1).$$

[22]This is on top of an existing distortion due to the seller's market power.

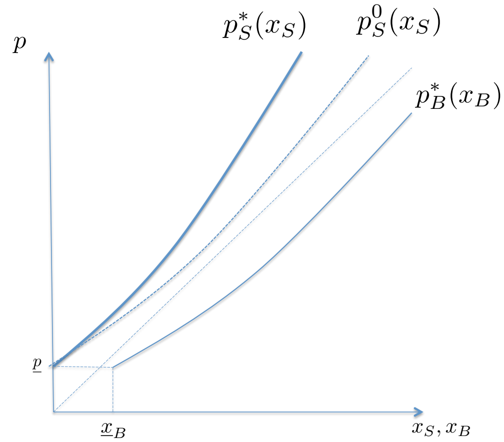


Figure 2: Bank-seller equilibrium ($v_J = \infty$) under asymmetric information. The thick line represents the bank's bidding strategy $p_S^*(x_S)$, the thin line represents a broker's bidding strategy $p_B^*(x_B)$. The dashed line represents the bank's strategy $p_S^0(x_S)$ under symmetric information.

Here, α reflects the relevance of the bank's information for the broker.[23] As α decreases, the bank's information becomes progressively less relevant to the broker. The case $\alpha = 0$ corresponds to independent private values. Note that this is different from the symmetric information case considered above since now the bank's information becomes irrelevant rather than being revealed to the brokers. Denoting the bank's strategy as $p_S(x_S;\alpha)$, we can write the following Proposition.

Proposition 4 (Equilibrium of the linear model)**.** *The bank's strategy in the linear model is given by*

$$p_S^*(x_S;\alpha) = s^{-1}(x_S) + \alpha x_S,$$

where

$$s(x_B) := \frac{1}{\alpha}(1 - F_{(1)}(x_B))^{\frac{1}{\alpha}-1} \int_{\underline{x}_B}^{x_B} (1 - F_{(1)}(t))^{-\frac{1}{\alpha}} f_{(1)}(t) \left(t - \frac{1 - F_B(t)}{f_B(t)} \right) dt$$

is the maximal bank's type that trades with a broker of type x_B, and $\underline{x}_B$, the lowest broker participating type, is the same as under symmetric information, i.e. given implicitly by

$$\underline{x}_B - \frac{1 - F_B(\underline{x}_B)}{f_B(\underline{x}_B)} = 0.$$

Moreover, $p_S^*(x_S;\alpha)$ is increasing in α.

[23] This specification does not satisfy our normalization $u_B(x_B, x_B) = x_B$, but it would if we change the broker's signal to $\tilde{x}_B = \frac{x_B}{1-\alpha}$.

This result shows that, as the bank's information becomes more relevant to the broker, the bank bids higher. In this model, α affects the bank's bidding incentives through two channels. First, there is a direct channel since the bank's information directly affects the broker's payoff. Second, there is an indirect channel due to the signalling premium, as we have discussed previously.

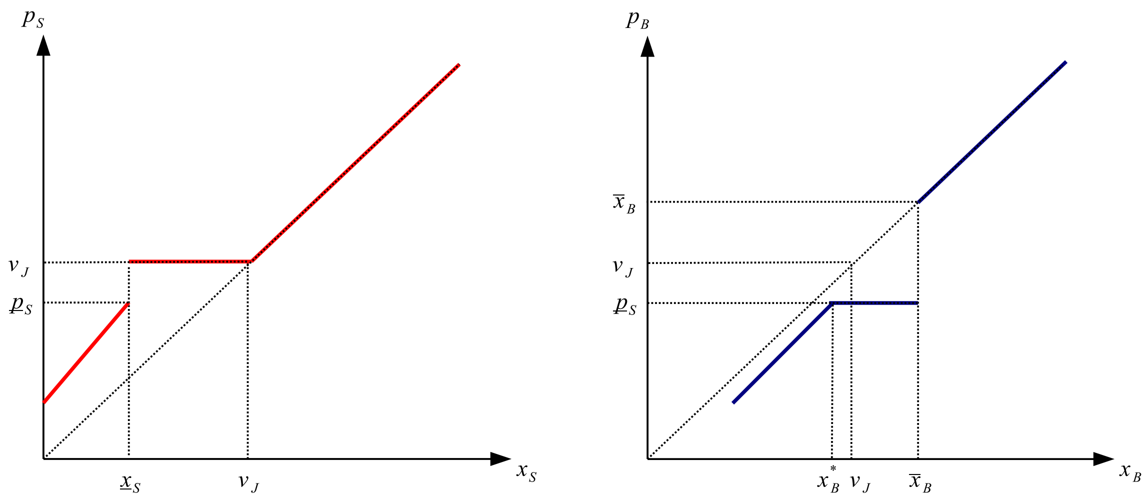
5.3 General Case: $v_J \in (0, \infty)$

In a foreclosure auction with $v_J \in (0, \infty)$, the equilibrium will combine the features of the bank-seller equilibrium for lower x_S, where the bank will drop out at $p_S^*(x_S)$ as described for the $v_J = \infty$ case, and bank-buyer equilibrium for higher x_S, where the bank will drop out at $p_S^*(x_S) = x_S$. However, "stitching" these two equilibria under common values is by no means a simple matter. To see the difficulties that arise, suppose we have a bunching equilibrium with common values where the bank types over a certain interval bid v_J:

$$p_S(x_S) = v_J, \qquad x_S \in [\underline{x}_S, \overline{x}_S].$$

To begin, we observe that there does not exist an equilibrium in continuous strategies that involves bunching. If the bank's dropout strategy $p_S(x_S)$ were continuous and involved bunching, then the brokers' dropout strategy $p_B(x_B)$ would involve a gap below v_J. Otherwise, the brokers who would contemplate bidding slightly below v_J, would prefer to wait and drop out at a price slightly above v_J, to take advantage of a dramatically higher quality of the house they would be able to get from banks in the bunching region. Such a gap in broker bids, however, creates an incentive for the bank types that bid somewhat below v_J to deviate to the bid v_J, since this deviation leads to a higher expected price, but does not change the probability of sale.

In order to prevent such deviations, an equilibrium with bunching must involve a *gap* below v_J, with $\underline{x}_S$, the seller at the lower bound of support of the bunching region, being indifferent between bidding v_J and bidding $\underline{p}_S < v_J$. Sellers with valuations slightly below $\underline{x}_S$ will strictly prefer bidding slightly below $\underline{p}_S$ to bidding v_J. The bank's strategy in our (semi-)separating foreclosure equilibrium coincides with $p_S^*(x_S)$ derived previously for $v_J = \infty$ when $x_S < \underline{x}_S$, involves a jump at $\underline{x}_S$ to v_J, bunching at v_J, and truthful bidding for $x_S > v_J$.



(a) Bank's bid (b) Broker's bid

Figure 3: Equilibrium under asymmetric information. The left panel shows the bank's bidding strategy $p_S(x_S)$. The right panel shows a broker's bidding strategy $p_B(x_B)$ when the bank has not dropped out.

The broker's dropout strategy will be a best response to the bank's strategy. It will coincide with $p_B^*(x_B)$ derived for the $v_J = \infty$ case for $x_B < x_B^*$, where x_B^* is implicitly defined by $p_B^*(x_B^*) = \underline{p}_S$. Then, the types $x_B \in (x_B^*, \overline{x}_B)$ will drop out as soon as the price has surpassed $\underline{p}_S$. These brokers realize that they cannot beat the bank profitably at any higher price. The types $x_B \geq \overline{x}_B$ will continue, and bid up to their values. We must have $\overline{x}_B > v_J$, since otherwise the broker bidding slightly above v_J would prefer a deviation to a bid slightly above $\underline{p}$ in order to avoid a loss due to a lower average quality over the bunch.

So the bank's dropout strategy $p_S(x_S)$ and the broker's dropout strategy when the bank has not dropped out, $p_B(x_B)$, are given respectively by

$$p_S(x_S) = \begin{cases} p_S^*(x_S), & x_S \in [0, \underline{x}_S], \\ v_J, & x_S \in [\underline{x}_S, v_J], \\ x_S, & x_S > v_J, \end{cases} \qquad p_B(x_B) = \begin{cases} p_B^*(x_B), & x_B \in [\underline{x}_B, x_B^*), \\ \underline{p}_S, & x_B \in [x_B^*, \overline{x}_B], \\ x_B, & x_B > \overline{x}_B. \end{cases} \tag{10}$$

See Figure 3, showing the bank's and a broker's bidding strategies.

As before, if the bank drops out and at least two brokers remain in the auction, the auction becomes essentially an independent private values auction. The only difference to the previous sections is that if the bank drops out at v_J, the brokers do not know the exact value of x_S, but only that it is in the interval $[\underline{x}_S, v_J]$. The remaining brokers' drop-out prices are hence

$$\tilde{p}_B(x_B^i) = \begin{cases} u_B(x_B^i, X_S^*(\tilde{p})), & \tilde{p} \in [p_S^*(0), p_S^*(\underline{x}_S), \\ \int_{\underline{x}_S}^{v_J} u_B(x_B^i, x_S) \frac{dF_S(x_S)}{F_S(v_J) - F_S(\underline{x}_S)}, & \tilde{p} = v_J, \\ u_B(x_B^i, \tilde{p}), & \tilde{p} > v_J, \end{cases}$$

where $\tilde{p}$ is the bank's drop-out price.

We now introduce the two key conditions that will uniquely pin down the cutoff types $\underline{x}_S$ and $\overline{x}_B$ and therefore an equilibrium. First, the $\overline{x}_B$-type broker must be indifferent between dropping out at a price slightly above $\underline{p}$ or staying in up to $\overline{x}_B$. By deviating to a higher bid $\overline{x}_B$, the broker would gain the property value $u_B(\overline{x}_B, x_S)$ at a price equal to v_J if $x_S \in [\underline{x}_S, v_J]$, and x_S if $x_S \in [v_J, \overline{x}_B]$. So the broker's indifference condition takes the form

$$0 = \int_{\underline{x}_S}^{\overline{x}_B} \Big(u_B(\overline{x}_B, x_S) - \max\{v_J, x_S\} \Big) \frac{dF_S(x_S)}{F_S(\overline{x}_B) - F_S(\underline{x}_S)}, \tag{11}$$

where $f_S(x_S)/(F_S(\overline{x}_B) - F_S(\underline{x}_S))$ is the density of the bank's types conditional on $x_S \in [\underline{x}_S, \overline{x}_B]$.

The second condition specifies that the $\underline{x}_S$-type bank is either indifferent between bidding $\underline{p}_S = p_S(\underline{x}_S)$ or $p = v_J$ (if $\underline{x}_S > 0$), or weakly prefers the bid at v_J to $\underline{p}_S$ (if $\underline{x}_S = 0$):

$$\Pi_S(\underline{x}_S, \underline{p}_S) = \Pi_S(\underline{x}_S, v_J), \qquad (\underline{x}_S > 0) \tag{12}$$

$$\Pi_S(0, \underline{p}_S) \leq \Pi_S(0, v_J), \qquad (\underline{x}_S = 0) \tag{13}$$

where $\Pi_S(x_S, p)$ is the bank's expected equilibrium profit if its type is x_S and it bids p.[24]

Our first result is a technical lemma below that shows existence of cutoffs $\underline{x}_S, \overline{x}_B$ that solve the indifference conditions (11) and (12). Note that it could happen that $\underline{x}_S = 0$, in which case the bunch extends all the way to the left.

Lemma 1 (Existence and uniqueness of the cutoffs)**.** *There exists a unique solution* $(\underline{x}_S, \overline{x}_B)$ *to the broker's and bank's indifference conditions* (11)*,* (12)*, and* (13)*.*

Given the existence of the cutoffs, we now establish existence (and uniqueness) of a semi-pooling equilibrium under asymmetric information of the kind described above.

Proposition 5 (Equilibrium existence)**.** *There exists a unique equilibrium in the class of strategies given by* (10)*.*

[24]See the Appendix for the explicit formula for $\Pi_S(x_S, p)$.

6 Testable Hypotheses

Our theory predicts bunching of banks' bids at v_J for both symmetric and asymmetric information. Denoting the distribution of bank's bids as $G_S(\cdot)$, we therefore have the following testable hypothesis.

Hypothesis 1 (Bunching at v_J)**.** *Under both symmetric and asymmetric information, the distribution $G_S(p)$ has an atom at $p = v_J$, formally,*

$$\lim_{p \uparrow v_J} G_S(p) < G_S(v_J).$$

In a hypothetical world, the econometricians could perfectly observe all forms of heterogeneity. It would then be straightforward to test for asymmetric information: our theory predicts a gap in banks' bid distributions just below v_J for asymmetric information auctions, but not for symmetric information auctions. Hence, the presence of a gap could serve as a test for asymmetric information.

However, one should not expect a gap if there is unobserved heterogeneity with respect to informational asymmetry, as one certainly would expect in reality. To what extent informational asymmetries play a role typically depends on characteristics of the property. For example, empirical studies show that there is little uncertainty about the quality of condominiums and hence symmetric information is likely to be a good approximation. For single family houses (especially in poor neighborhoods), on the other hand, there is more uncertainty about quality and asymmetric information is likely to play more of a role.[25] Another reason for differences in asymmetric information can be that for some properties, the owner cooperates with the lender by allowing access to the property, thereby making the information essentially symmetric.

If the sales of some of the houses are characterized by asymmetric information (ASI), whereas for others the auction is essentially under symmetric information (SI), then the SI auction will fill out the gap. Here, by SI we mean the houses where the bank's information is also available to the broker; in our model, this implies that x_S is observable to the broker.

[25] For example, Allen et al. (1995) find that for condominiums observable characteristics explain a large part of the rent (adjusted $R^2 = 0.911$ in a regression with the rent as the dependent and different property characteristics, such as the number of square feet and bathrooms, and central air conditioning, as independent variables). For single family houses observable characteristics explain a much smaller fraction of the rent (adjusted $R^2 = 0.685$).

But heterogeneity with respect to informational asymmetry does have empirically testable implications. The testable implication can be derived from Corollary 1 (signaling premium), by observing that for ASI houses the probability of sale is lower than for SI houses for a given bid by the bank. The empirically testable implication stems from a selection due to the gap for ASI auctions as described below.

For SI houses, the bank and the brokers are symmetrically informed. In our framework, this means that the bank's information x_S is observable to the broker and that there is no gap below v_J. For ASI houses, x_S is not observable to the broker and there is a gap in banks' bids in the interval $(\underline{p}_S, v_J)$ for $\underline{p}_S = p_S(\underline{x}_S)$. For bank's bids below the gap for ASI houses ($p_S < \underline{p}_S$), we will observe a mixture of SI and of ASI houses. The probability of sale for such prices is the average of the high SI and the low ASI probability of sale. In the gap ($p_S \in (\underline{p}, v_J)$), we only observe high probability of sale SI houses. This selection effect leads to an increase of the probability of sale at $\underline{p}_S$. For $p_S = v_J$ and above, we again observe both SI and ASI houses, hence a downward jump in the probability of sale at $p_S = v_J$. This is illustrated in Figure 4.

In reality, one would expect that there are more than two types of properties, that there are different degrees of informational asymmetry and hence different lower bounds $\underline{p}_S$ for the gaps $(\underline{p}_S, v_J)$. This will smooth out the discontinuity at $\underline{p}_S$ in Figure 4, but we should still expect that the probability of sale to increase in the bank's maximum bid in an interval below v_J, and jump downwards at v_J. Note that the upper bound of the gap v_J is the same irrespective of the degree of asymmetric information, hence the discontinuity at v_J will not be smoothed out. We thus arrive at the following testable hypothesis concerning the probability of sale to the broker, denoted as $\rho(p_S)$. We assume that the prices are normalized by v_J, so that all houses have the same effective v_J.

Hypothesis 2 (Probability of sale)**.** *If all auctions in the data are under symmetric information, the probability of sale $\rho(p_S)$ is a continuous, decreasing function of p_S. If, on the other hand, the data exhibit a mixture of auctions with a varying degree of informational asymmetry, including SI auctions with a positive probability, then we should expect the probability of sale to the broker $\rho(p_S)$ to exhibit the following pattern. Initially, $\rho(p_S)$ decreases in p_S. Then, over a certain interval $p_S \in [\tilde{p}, v_J)$, where $\tilde{p} < v_J$, $\rho(p_S)$ increases in p_S. At $p_S = v_J$, $\rho(p_S)$ drops discontinuously to a lower value, $\rho(v_J) < \lim_{p_S \uparrow v_J} \rho(p_S)$, and from that point on, decreases in*

Figure 4: Probability of sale to the broker as a function of the bank's maximum bid for symmetric information houses (dotted, blue), asymmetric information houses (red, dashed), and a mixture of both types of houses (black, solid).

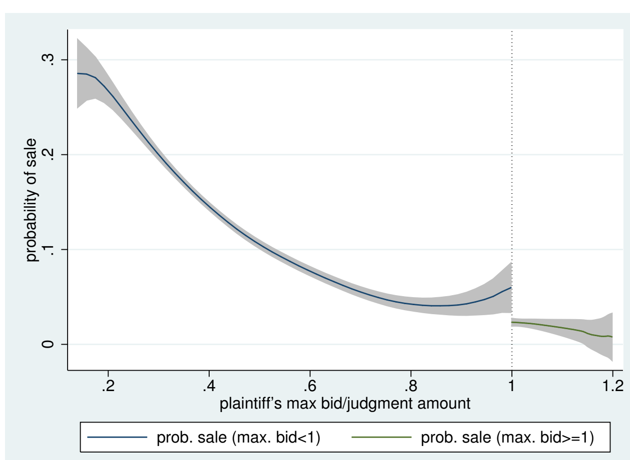


Figure 5: Monte Carlo simulation for the probability of sale to the broker with linear payoffs and lognormal distributions of signals.

a continuous fashion.

We further illustrate the predicted pattern in Hypothesis 2 through a Monte Carlo simulation. In this example, we assume that there is one broker in each auction. Her payoff is linear in the signals, $u_B(x_B, x_S) = (1-\beta)x_B + \beta x_S$. The signals are lognormally distributed. β, which is the weight on the bank's information x_S, represents the degree of asymmetric information. It is assumed to be uniformly distributed on $[0, 1/2]$, with $\beta = 0$ corresponding to symmetric information.

To detect the discontinuity at v_J, we split the sample at v_J in two subsets. The first subset contains the observations with the bank's bid below v_J, while in the second one contains all observations with the bank's bid above v_J. Then we apply a local linear regression estimator to each subset by following Fan and Gijbels (1996). The well-known advantage of this method is that it provides automatic correction of the boundary bias.[26] It allows us to consistently estimate the regression curve $\rho(p)$ not only for $p < v_J$ and $p > v_J$, but also their limits at the boundary, i.e., $\lim_{p\uparrow v_J} \rho(p)$ and $\lim_{p\downarrow v_J} \rho(p)$.

We choose the Epanechnikov kernel function, and follow the rule-of-thumb bandwidth selection by Fan and Gijbels (1996), which offers the asymptotically optimal constant bandwidth by minimizing the conditional weighted mean integrated squared error.[27] The estimated probability of sale functions are plotted in Figure 5. We took overall 50,000 random draws. The shaded region is the 95% confidence interval. The figure shows both a discontinuity at v_J and an upward sloping probability of sale function in the left neighborhood of v_J.

7 Data

Palm Beach County (FL) switched to an electronic system for foreclosure auctions following Administrative Policy 10-01, effective January 6, 2010.[28] The foreclosure auction website `https://mypalmbeachclerk.clerkauction.com/` allows plaintiffs (typically banks) and third-party bidders (typically brokers) to bid for property being foreclosed. The ClerkAuction online platform conducts foreclosure sales on all business days, which provides a large amount of data.

[26] See, e.g., the discussion in Fan and Gijbels (1996), pp. 69-70.

[27] See section 4.2 on pp. 110-113 in Fan and Gijbels (1996) for details.

[28] Administrative Policy 10-01 was later superceded by Administrative Policy 11-01, effective June 17, 2011.

Furthermore, the auction data are electronically available.

We collected data from the website for foreclosure sales between January 21, 2010 and November 27, 2013. Our data record all transaction details on these sales, including winning bid, winner identities, and judgment amounts. Our data set contains 12,788 auctions with a total judgment amount of \$4.2bn.[29] Table 1 reports the summary statistics for the main variables. The variable *bank winning* indicates that 81% of auctions under study ended up having properties transferred to bank's ownership.

Banks are required to submit a bid for the foreclosed property. We have data on the maximum bid the bank entered in the bidding proxy. Closing times for auctions conducted on the same day differ by 1-2 minutes, so that bidding is not simultaneous.[30] Further, the data suggest that bidding by the original owner plays a negligible role if any.[31]

Table 1: Summary statistics

Variable	**Mean**	**Std. Dev.**	**Min**	**Max**
bank winning	0.813	0.390	0	1
number of brokers	1.065	1.362	0	14

Variable	**Mean**	**Std. Dev.**	**1st Percentile**	**99th Percentile**
Variables with original scales				
bank's bid	210,996	1,294,181	4,200	1,476,989
judgment amount	329,951	1,953,172	4,985	2,380,127
Variables normalized by judgment amount				
bank's bid	0.766	1.363	0.096	1.167

We now turn to empirical tests of our hypotheses. Bunching at the judgment amount (Hypothesis 1) can be easily checked by plotting the cumulative distribution function of banks' bids. Figure 6 shows the distribution of banks' maximum bids. Bunching shows up very clearly: there are roughly 4,000 observations bunched at the judgment amount.

[29] 25,725 auctions in this period were canceled. For 30,976 auctions, information on the bank's maximum bid is not available.

[30] If the leading bid in the auction increases in the last minute, then the closing time of the auction and all subsequent auctions is extended by a minute.

[31] We have run a comparison between the names of the defendant and the winning bidder in the auction. Out of 12,788 auctions, the defendant's last name only shows up in the name of the winning bidder for four auctions. For three out of these four auctions, the first names do not match (which could mean either relatives of the defendant bidding or coincidentally matching last names).

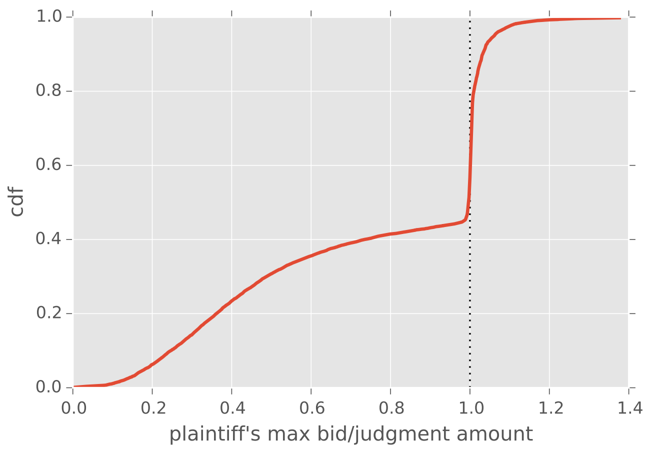


Figure 6: Distribution of the bank's maximum bids (normalized by the judgment amounts). Number of observations: 12,788.

Next, we turn to the non-monotonicity and the discontinuity in the probability of sale as a function of the bank's maximum bid. This is plotted in Figure 7. Figure 7 shows the same kernel regression as the one shown for the Monte Carlo simulation in Figure 5. It is a local linear kernel regression, the sample being split into observations below ($p_S < v_J$) and weakly above ($p_S \geq v_J$) the judgment amount. The most striking feature of the graph is that the probability of sale increases with the bank's bid just before the judgment amount and drops down discontinuously at v_J. This is exactly what theory predicts in case that there is heterogeneity in terms of asymmetric information (see Figures 4, 5, and Hypothesis 2).

The data suggest that there is indeed asymmetric information involved in foreclosure auctions: banks seem to have indeed an informational advantage. This appears plausible: banks collect information about the value of the collateral before granting mortgages.

Asymmetric information has been recognized to be highly relevant for the understanding of how markets work, on policy implications, and on welfare analyses, at least since Akerlof's seminal contribution on the lemon's market (Akerlof, 1970). In the following section, we will describe one important application in which asymmetric information plays a crucial role.

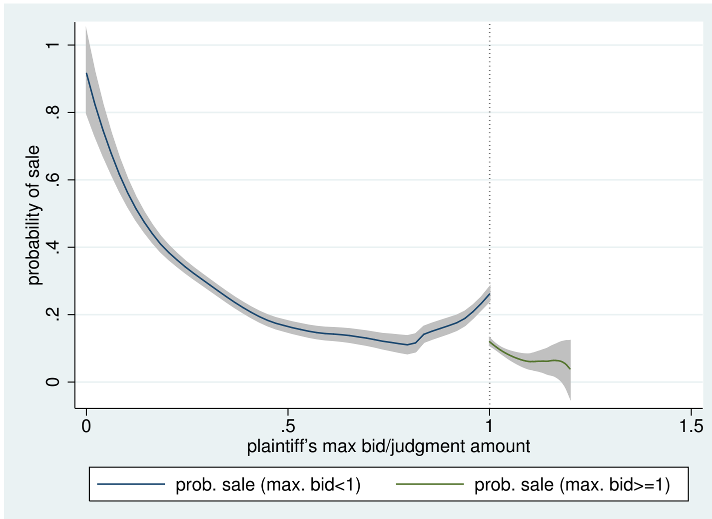


Figure 7: Probability of sale as a function of the bank's maximum bid. Local linear kernel regression with the data split at $p/v_J = 1$ (confidence interval: 95%, Epanechnikov kernel and rule-of-thumb (ROT) bandwidth selection, see Fan and Gijbels (1996)).

8 Securitization

The securitization of mortgages massively increased before and moved to the forefront of attention during the financial crisis. While only 30% of mortgages were securitized in 1995, this fraction increased to 80% in 2006 (Dewatripont et al., 2010, p. 19). Angelides et al. (2011) considered "collapsing mortgage-lending standards and the mortgage securitization pipeline [to have] lit and spread the flame of contagion and crisis" (see p. xxiii). The academic literature has confirmed this view and found that mortgage securitization led to moral hazard and to banks excessively granting mortgages, which later caused a collapse of the mortgage backed securities market (Mian and Sufi, 2009; Dewatripont et al., 2010; Keys et al., 2010). See Gorton and Metrick (forthcoming) for an overview of the theory and of the empirical evidence for moral hazard.

Securitization can take many forms. We provide a description of the typical procedure here that captures the basic aspects of securitization.

The relevant actors in the securitization process are the originator, the issuer, the special purpose vehicle, the mortgage trust, and the servicer.[32] Securitization typically involves the

[32]Additional actors are the underwriter, the rating agency, and the trustee. However, they are not relevant

following steps. First, the originating bank grants a mortgage to the home owner. Second, the issuer pools mortgages and sells the pool to a special purpose vehicle. Third, the special purpose vehicle transfers the assets to a trust that is specially created for a particular securitization deal. Fourth, the trust issues security shares backed by the assets in the pool, which are then bought by investors. Interest payments by mortgagees are collected by the servicer and transferred to the investors through the trust. Fifth, if a mortgagee defaults, the servicer sues for foreclosure on behalf of the trust.[33] Proceedings from the foreclosure are collected by the servicer and transferred to investors through the trust. For our purposes, the only actors that are relevant are the originating bank and the servicer, which is typically the same bank: the originating bank collects information about the value of the collateral when granting the mortgage and uses this information when acting as a servicer in the foreclosure auction. For more details on the securitization process and the different variations of securitization, see Cetorelli and Peristiani (2012); Gorton and Metrick (forthcoming).

To avoid adverse selection, several measures are usually taken. First, only mortgages can be securitized for which the credit score (the FICO score) of the borrower is above a threshold.[34] Second, the originating bank has to offer all properties with similar observable characteristics eligible for securitization and the special purpose vehicles randomly pick mortgages to be securitized (Keys et al., 2010).

However, these measures do not prevent moral hazard, since the credit score is not a perfect signal about the expected loss of a mortgage – which is what the originating bank or the investors buying the mortgage backed securities ultimately care about. One reason is that the expected loss is the product of the probability of default and the loss given default. The credit score is only a proxy for the probability of default, but not the loss given default. Before the crisis, banks were known to grant mortgages with a high loan-to-value ratio – the mortgage sometimes even exceeding the value of the house. Since this means that only a small fraction of the mortgage can be recovered in case of a foreclosure, this led to high losses given default.

for our analysis and hence left out.

[33] Court documents show the trustee of the mortgage trust as the plaintiff in the foreclosure auction. However, the servicer acts as plaintiff on behalf of the trust.

[34] This is a somewhat simplified account, since securitization agencies chose two cutoffs for FICO scores: above a FICO score of 620, banks can securitize *low documentation mortgages*, below a score of 600, it is very difficult to securitize mortgages. Between 600 and 620, banks can securitize mortgages, but have to provide full documentation.

Another reason is that the credit score is not a perfect signal about the probability of default of a borrower. Banks typically need to collect additional (soft) information to assess the probability of default.

The securitization of mortgages reduces a bank's incentive to collect more information on additional components of the expected loss (i.e. the loss given default and soft information about the probability of default), which in turn leads to moral hazard. Keys et al. (2010) have shown that moral hazard led to banks collecting insufficient soft information about the probability of default.

While the literature so far has mainly focused on the probability of default, it is reasonable to suspect that moral hazard also led to banks collecting insufficient information about the other component of the expected loss, the *loss given default*. This is, first, because a high loan-to-value ratio is known to be positively correlated both with the probability of default and the loss given default (see Qi and Yang, 2009, and the references therein).[35] Second, a more careful investigation of a potential borrower should also directly reveal information about the value of the property.

Our theory and our data allow us to look more carefully at the question of moral hazard with respect to the loss given default: if a bank collected information about the value of the collateral (and hence the loss given default) when it granted the mortgage, it is likely to have an informational advantage over third-party bidders during the foreclosure auction.

In particular, we can use information on whether a foreclosed mortgage was securitized or not. If securitization leads to moral hazard, we should expect the bank to have no (or only a minor) informational advantage over third-party bidders for the case of securitized mortgages. This means an independent private value auction (or only a small common value component). For non-securitized mortgages we should expect the bank to have an informational advantage. This means an auction with a common value component on the bank's side. This leads us to the empirically testable implication that is an adaptation of Hypothesis 2:

Hypothesis 3 (Probability of sale for securitized and non-securitized mortgages)**.** *For se-*

[35]In which direction the causality goes is not important for our analysis. It could (and mostly likely does) go both ways: a high loan-to-value ratio makes strategic default more likely. At the same time, borrowers with financial difficulties are less likely to invest in the maintenance of the property, which decreases its value and increases the loss given default.

curitized mortgages, the probability of sale $\rho(p_S)$ is a continuous, decreasing function of p_S. For non-securitized mortgages, we should expect the probability of sale to the broker $\rho(p_S)$ to exhibit the following pattern. Initially, $\rho(p_S)$ decreases in p_S. Then, over a certain interval $p_S \in [\tilde{p}, v_J)$, where $\tilde{p} < v_J$, $\rho(p_S)$ increases in p_S. At $p_S = v_J$, $\rho(p_S)$ drops discontinuously to a lower value, $\rho(v_J) < \lim_{p_S \uparrow v_J} \rho(p_S)$, and from that point on, decreases in a continuous fashion.

The hypothesis that the bank's information concerning the common value component is *less precise* for the securitized mortgages can also be tested using banks' estimated bidding strategies. We do so assuming the linear model

$$u_B(x_B, x_S) = x_B + \alpha x_S.$$

Proposition 4 shows that the bank's strategy $p_S(x_S; \alpha)$ is increasing in α. If banks holding securitized mortgages are less informed than banks holding non-securitized mortgages, then α is expected to be smaller for the former. This leads to the following testable hypothesis.

Hypothesis 4. *For bids below the judgment amount v_J, a bank holding a non-securitized mortgage chooses a higher bid p^{nonsec} than a bank holding a securitized mortgage p^{sec} given the same opportunity cost x_S. Formally,*

$$p_S^{nonsec}(x_S) > p_S^{sec}(x_S), \qquad x_S > 0.$$

In the following, we test these hypotheses with our data.

8.1 Data

Since we have the name of the plaintiff in each foreclosure auction, we can categorize mortgages as securitized vs non-securitized. We use a simple classification rule. We classify a mortgage as securitized if the name of the plaintiff contains at least one of the following keywords: "trust", "asset backed", "asset-backed", "certificate", "security", "securities", "holder".[36] This simple

[36] Two examples of plaintiff's names that are classified as securitized are "US BANK NATIONAL ASSOCIATION AS TRUSTEE ON BEHALF OF THE HOLDERS OF THE ASSET BACKED SECURITIES CORPORATION HOME EQUITY LOAN TRUST SERIES AEG 2006-HE1 ASSET BACKED PASS-THROUGH CERTIFICATES SERIES AEG 2006-HE1 HSBC MORTGAGE SERVCIES INC" and "DEUTSCHE BANK NATIONAL TRUST COMPANY AS TRUSTEE FOR ARGENT SECURITIES INC ASSET-BACKED PASS-THROUGH CERTIFICATES SERIES 2006-W2". It should be noted that on legal documents the name of the trustee of the mortgage backed securities fund shows up. However, trustees typically do not bid themselves, but are rather represented by the servicer of the mortgage (typically the originating bank) in the foreclosure process.

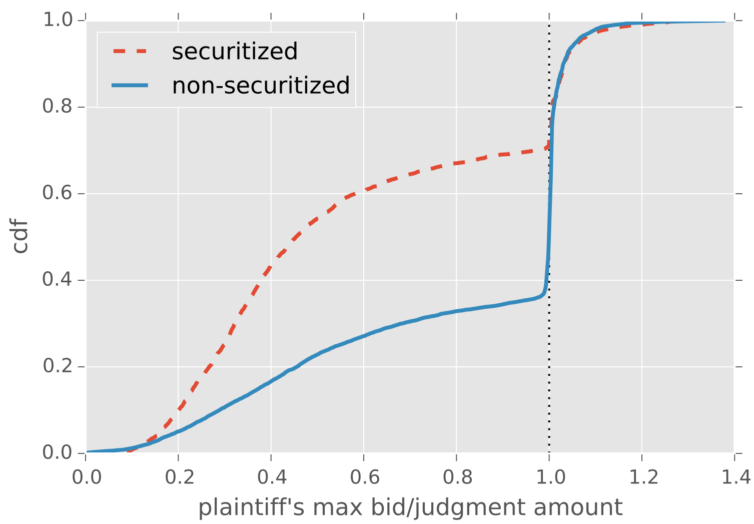


Figure 8: Distribution of the bank's maximum bid as a fraction of the judgment amount p_S/v_J for securitized (dashed line) and non-securitized (solid line) mortgages.

categorization does give false negatives (for some securitized mortgages none of the keywords shows up in the name of the plaintiff), but almost no false positives.

We have classified 3,249 mortgages as securitized and the remaining 9,539 as non-securitized.[37] Figure 8 plots the distributions of banks' bids for securitized and non-securitized mortgages. The average difference of the bank's bid as a fraction of the judgment amount is much lower for securitized than for non-securitized mortgages, roughly 40 percentage points. This could be explained by two effects. First, a selection effect: houses used as collateral for securitized mortgages may be of lower quality than houses used as collateral for non-securitized mortgages. Second, the signaling premium leads to banks bidding higher for non-securitized than for securitized mortgages, since for non-securitized mortgages the bank may have more of an informational advantage than for securitized mortgages.

One cannot directly disentangle these two effects, hence it cannot be seen immediately whether there is more asymmetric information for non-securitized than for securitized mortgages. However, we can use our theory as a tool to uncover evidence of asymmetric information.

[37]We only considered private label securitization, since only 218 of the mortgages in our sample were securitized by Government Sponsored Enterprises. We identified mortgages as securitized by a Government Sponsored Enterprise as those for which the name of the plaintiff contains one of the following: "Federal National Mortgage Assoc[iation]", "Fannie Mae", "Federal Home Loan Mortgage Corp[oration]", or "Freddie Mac".

We will take two approaches for this purpose. The first relies on our theoretical results on the discontinuity of the probability of sale at the judgment amount. For the second approach, we use additional data that we hand-collected for a subsample of the data set to correct for the selection effect.

8.2 Discontinuity

Our theoretical results on the judgment amount and the effect of the common value component help us gain more insight on asymmetric information. Recall that in the presence of asymmetric information, we should expect an increase and then a discontinuity in the probability of sale as a function of the bank's bid (Hypothesis 3).

Figures 9 and 10 show the probability of sale as a function of the bank's bid for securitized and non-securitized mortgages. For securitized mortgages, there is no increase in the probability of sale below the judgment amount. Further, the discontinuity at v_J is much less pronounced and indeed not statistically significant. For non-securitized mortgages, we still see the same pattern as in Figure 7. This is consistent with the theory that asymmetric information plays less of a role for securitized mortgages, since the servicer of a securitized mortgage pool is less likely to have an informational advantage over brokers than a bank bank servicing its own mortgage.

8.3 Bidding Strategy

Figures 9 and 10 provide indirect evidence for asymmetric information for non-securitized mortgages, while there is no such evidence for the securitized mortgages. In this section, we develop and implement a direct test of banks bidding more for houses used as collateral for non-securitized rather than securitized mortgages, formally $p_S^{nonsec}(x_S) > p_S^{sec}(x_S)$ (Hypothesis 4).

While the theoretical predictions are clear, the empirical strategy is more involved. It is not sufficient to observe that banks' bids are higher for non-securitized than for securitized mortgages as depicted in Figure 8. The reason for a different distribution of bids may be either the signaling premium (higher $p_S(x_S)$ for a given x_S), a selection effect (different distributions of x_S for securitized and non-securitized mortgages), or a combination of both. We have to

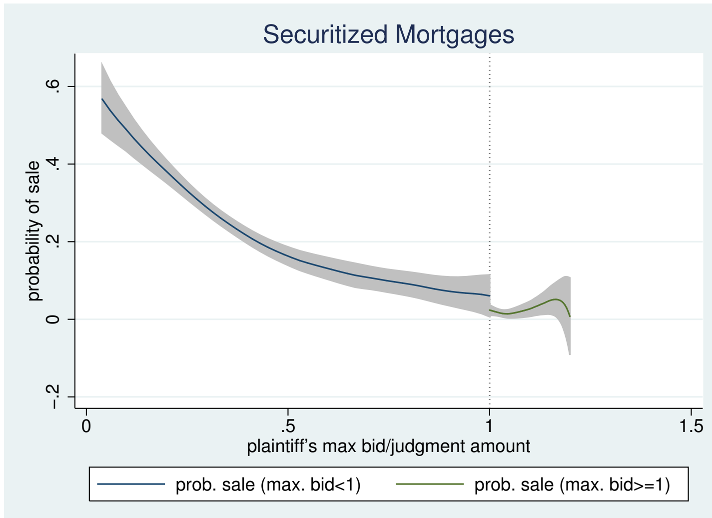


Figure 9: Probability of sale as a function of the bank's maximum bid for securitized mortgages. Local linear kernel regression with the data split at $p/v_J = 1$ (confidence interval: 95%, Epanechnikov kernel and rule-of-thumb (ROT) bandwidth selection, see Fan and Gijbels (1996)).

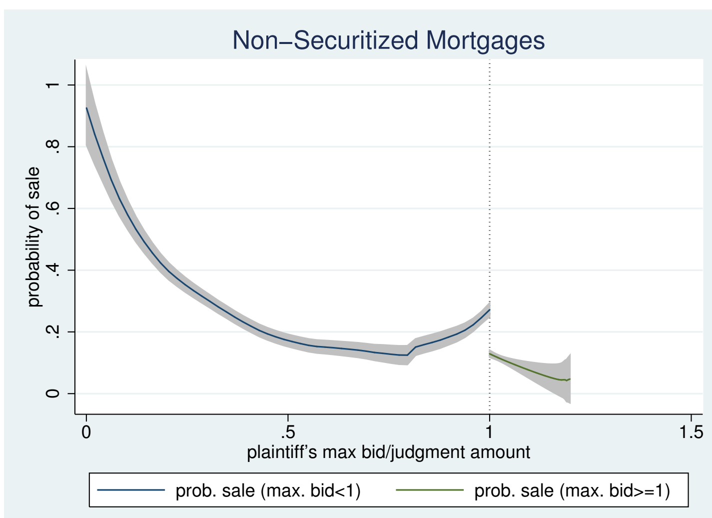


Figure 10: Probability of sale as a function of the bank's maximum bid for non-securitized mortgages. Local linear kernel regression with the data split at $p/v_J = 1$ (confidence interval: 95%, Epanechnikov kernel and rule-of-thumb (ROT) bandwidth selection, see Fan and Gijbels (1996))

control for x_S in order to identify the signaling premium.

In order to be able to control for x_S, we have hand-collected additional information for a subsample of our data set, namely for properties foreclosed in September 2011. In particular, we collected the information on property transactions after foreclosure sales from a different data source, a website powered by the Palm Beach County Property Appraiser (a government agent). The information regarding the property transactions are for ad valorem tax assessment purposes, as stated in the disclaimer of the website. This implies the appraiser exercises auditing procedures strictly to ensure the validity of any transaction received and posted by that office.

For each foreclosure case, we first traced back the original legal documents, from which we found the property address. We then used the address to search in the database for the detailed features regarding the property and its transaction history. The property appraiser database provides information regarding the type of property (single family, townhouse, zero lot line, etc.), its appraisal values for the most recent three years, the next sale date, the next sale price,and information on the next owner. Using this information, we were able to recover some of the next sale prices of the properties at foreclosure.

A comparison of the data on foreclosed properties and the data from the Property Appraiser's database revealed that a perfect matching of observations in the two data sets is not possible, since the address listed in the legal documents is not always of the same format (or containing the same details) as the information recorded in the appraisal database. In order to err on the side of caution, we only used observations for which we can be certain about the matching.

We identified 332 properties among 644 foreclosed cases in the month.[38] The next sale of the foreclosed properties happened mostly in the year of 2012, though a few of the properties were not resold until early 2013. This leaves us with 250 observations for which the resale price is available. Since our aim is to get an estimate of the bank's valuation distribution, we only used the data from the foreclosure auctions in which the bank won. This restricts our sample to 199 observations, with 77 of them in the category of securitized mortgages and 122 cases of non-securitized mortgages.

[38]There were 983 properties listed for foreclosure for September 2011, but 339 of them were canceled prior to the auction dates.

Table 2 provides descriptive statistics of the data collected. The table suggests that part of the difference in sales prices between securitized and non-securitized mortgages is explained by a selection effect: the bank's resale price in case it wins the auction is higher for non-securitized than for securitized mortgages. In the following, we will disentangle the selection and the signaling premium effects.

Table 2: Descriptive statistics for securitized and non-securitized mortgages.

	Resale Price/Judgment Amount Mean	#
All auctions	0.391	199
Securitized	0.349	77
Non-securitized:	0.417	122

The bank's bidding behavior is determined by its opportunity cost of selling x_S, which is the expected resale price.[39] We identify the distribution of the unobservable x_S by specifying the following correlation structure between x_S, the observable resale price r and the (also observable) tax assessment value a:

$$\tilde{r} = \tilde{x}_S + \epsilon_r, \qquad \tilde{a} = \tilde{x}_S + \epsilon_a$$

where $\tilde{x}_S := \ln x_S - E[\ln x_S]$ is the de-meaned log-opportunity cost of the bank, and $\tilde{r}$, $\tilde{a}$ are the de-meaned log-resale price and tax assessment, respectively. In this specification, the noise terms ϵ_r and ϵ_S have zero mean, are mutually independent, and are also independent of $\tilde{x}_S$. Following Li and Vuong (1998) and Krasnokutskaya (2011), the underlying distribution of x_S is non-parametrically identifiable under this assumption. One could use standard non-parametric deconvolution techniques based on Fourier transforms to obtain the distribution of x_S.

However, because of sample size issues, we take a semi-parametric approach and parametrically deconvolute the noise in the following way. We can identify the variance of $\tilde{x}_S$ by considering the variance of different linear combinations of r and a. A simple example for this is the following set of variances and the corresponding equations:

$$\mathrm{Var}\left[w\tilde{r} + (1-w)\tilde{a}\right] = w^2\sigma_r^2 + (1-w)^2\sigma_a^2 + 2w(1-w)\sigma_S^2, \qquad w \in \{0, \frac{1}{2}, 1\},$$

[39] To be more precise, the bank's opportunity cost is the difference between the expected price and the admistrative costs of conducting a sale. However, for our purposes, this distinction is irrelevant.

where σ_S^2, σ_r^2, and σ_a^2 are the variances of $\tilde{x}_S$, ϵ_r, and ϵ_a, respectively. The above is a system of three linear equations with three unknowns σ_i^2, $i \in \{S, r, a\}$, and has a unique solution.

The empirical variances of $\tilde{r}$, $\tilde{a}$, and $\tilde{r}/2 + \tilde{a}/2$ will thus lead to a consistent estimate of σ_S. Further, we use the empirical mean of r as an estimate for the mean of x_S, which is also consistent. We further make the parametric assumption that x_S, r, and a are log-normally distributed. Estimates of the distributions of x_S are reported in Table 3. The difference in means reveals that there is indeed a selection effect: securitized mortgages have a lower resale price to judgment amount ratio x_S/v_J than non-securitized mortgages. In the following, we will show that this selection effect cannot explain all the difference between securitized and non-securitized mortgages.

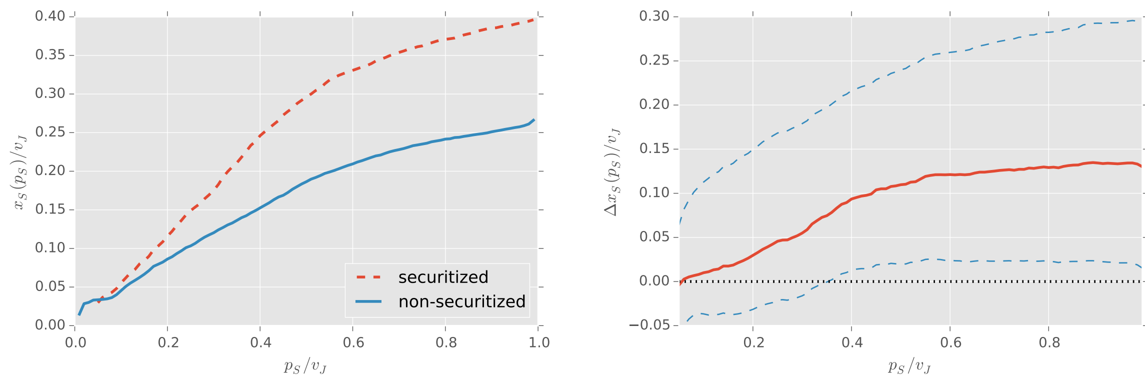
Table 3: Distribution of x_S/v_J for securitized and non-securitized mortgages. The estimate is based on a log-normal distribution $x_S/v_J \sim \ln N(\mu_S, \sigma_S)$.

	μ_S	σ_S	mean	std
securitized	-1.17236	0.429416	0.33954	0.152791
non-securitized	-1.01577	0.588605	0.430615	0.277086

Next, observe that the distribution of banks' bids satisfies $G_S(p_S(x_S)) = F_S(x_S)$ for $p_S(x_S) < v_J$ if there is homogeneity with respect to asymmetric information.[40] Since we only observe the distribution of the bank's resale prices if the bank wins the auction, it is useful to define $\tilde{F}_S(x_S)$ and $\tilde{G}_S(p_S)$ the distributions of x_S and p_S conditional on winning the auction. Because $\tilde{F}_S(x_S) = \tilde{G}_S(p_S(x_S))$, we can write $p_S(x_S) = \tilde{G}_S^{-1}(\tilde{F}_S(x_S))$. Given that we have estimates of $\tilde{G}_S$ and $\tilde{F}_S$ both for securitized and for non-securitized mortgages, we can estimate p_S for both types of mortgages.

Note that comparing the bidding functions $p_S(x_S)$ for bids below v_J is difficult, since the bidding function below v_J has a different support for securitized and non-securitized mortgages, $[0, x_S^{\text{sec}}(v_J)]$ and $[0, x_S^{\text{non-sec}}(v_J)]$, respectively. It is more practical to compare the inverse bidding function $x_S(p_S)$ for securitized and non-securitized mortgages, as the support is $[0, v_J]$ in both cases. The inverse bidding functions are shown in Figure 11a. Computing the 90% confidence interval of the difference of the inverse bidding functions for securitized and non-securitized

[40] We state results under the assumption of homogeneity of asymmetric information for the sake of expositional clarity. These results trivially extend to mixtures of symmetric and asymmetric auctions.



(a) Inverse bidding function

(b) Difference of inverse bidding functions

Figure 11: Inverse bidding function $x_S(p_S)$ relating the bank's opportunity cost of selling x_S and the bank's bid p_S for securitized (dashed line) and non-securitized (solid line) mortgages (left panel). Difference between banks' inverse bidding functions (i.e. the bank's opportunity cost of selling x_S as a function of its bid p_S) for securitized and non-securitized mortgages (solid line) and the 90% confidence interval of the estimates (dashed lines) (right panel). The estimates are constructed by matching quantiles, the confidence intervals are computed using a bootstrap estimate.

mortgages reveals that the difference is significant for high values of p_S, see Figure 11b. The estimates were computed using matching quantiles, the confidence interval was computed using bootstrap estimation.

8.4 Implications

The above analysis provides evidence for the existence of asymmetric information in mortgage markets. The analysis uncovers a difference between securitized and non-securitized mortgages and is consistent with the hypothesis that banks holding securitized mortgages are indeed less informed about the quality of the foreclosed property than banks holding non-securitized mortgages.

This is consistent with the hypothesis of moral hazard in the securitization process: if a mortgage is expected to be securitized, then the originating bank has less incentives to collect information about the quality of the mortgage and hence will be less informed. While the literature so far has focused on moral hazard with respect to acquiring information about the probability of default, our analysis has uncovered another dimension of moral hazard: the bank exerts insufficient effort to collect information about the collateral value, which directly affects the loss given default. Our results suggest that the effect of securitization on the loss given default is equally important for the expected loss as the effect on the probability of default. The bank's resale price as a fraction of the judgment amount is 21% lower for securitized than for non-securitized mortgages (0.340 vs 0.431).[41] This is comparable to the 10% to 20% increase of the probability of default for securitized versus non-securitized mortgages (5.5% to 6% versus 5%) estimated by Keys et al. (2010).

The alternative explanation of adverse selection rather than moral hazard is implausible because of the institutional details of the securitization described at the beginning of this section.[42] In particular, the random selection rules according to which mortgages are typically chosen to be transferred to the special purpose vehicles prevent originating banks from cherry picking the mortgages they want to securitize. See Keys et al. (2010) for more details.

Another alternative explanation is that originating banks do collect information about the loss given default, but this information is lost at a later stage. However, this is also implausible, because the originating bank typically serves as the servicer of the mortgage and hence deals with the foreclosure process (see e.g. Cetorelli and Peristiani, 2012). Since the originating bank is required to hold tranches of the securitized mortgage pool, they have an incentive to make use of their information about quality of the collateral.

Another possible concern is a selection effect. Our first approach relies on patterns showing up in the data (the non-monotonicity and discontinuity of the probability of default) and does not rely on the assumption that there is no selection effect. Our second approach – the estimation of the bidding function – explicitly corrects for a selection effect in the quality of securitized and non-securitized mortgages. However, there is a selection effect we cannot control for with our foreclosure data: we only observe mortgages that defaulted. An alternative hypothesis to moral hazard is that securitized mortgages were less likely to default and hence it was less important to collect information about the value of the collateral. However, the

[41]We provide results for the resale prices rather than losses given default, since there is more reliable data on this. A rough estimate of the difference in terms of the loss given default can be obtained by assuming that the loss given default is $v_J - x_S$. This would mean a 16% higher loss given default for securitized than for non-securitized mortgages (1-0.340 versus 1-0.431). The loss given default is typically higher than $v_J - x_S$, because of administrative costs.

[42]The candidate for an alternative explanation would be adverse selection with a pooling equilibrium, which leads to loss of information. For adverse selection and a separating equilibrium, no information is lost.

existing literature actually finds the very opposite of this: securitized mortgages are *more* likely to default.[43] Hence more effort should be exerted when investigating the collateral of securitized mortgages.

Yet another possible concern is that the bidding behavior of the servicing bank in the foreclosure auction might be different depending on whether the mortgage is securitized or not. The idea is that if the originating bank (which typically acts as the servicer) holds on to fraction $\mu < 1$, then its payoff is $\mu\Pi_S$ rather than Π_S, which might affect its bidding behavior. However, this is not the case, since $\mu\Pi_S$ is a linear transformation of Π_S, represents hence the same von Neumann-Morgenstern preferences, and leads to the same bidding behavior by the bank.[44]

In the following, we will state the additional insight about policy questions that can be gained from our analysis of foreclosure auctions. The usual caveats apply: we are using data from Palm Beach County (FL) from 2010 to 2013. While it appears reasonable that data from other regions should reveal similar patterns, more research is needed. Our theoretical results and empirical techniques should facilitate such future research. Data from other periods of time is likely to reveal other patterns, since we are picking up foreclosures in the aftermath of the financial crisis in our data set. Such data should provide additional insight about the evolution of asymmetric information over time.

Another caveat is that policy should be guided by the bigger picture rather than individual pieces of the picture, such as foreclosure auctions. Therefore, we will mostly discuss policy recommendations made in the previous literature based on the bigger picture and state how our insights contribute to these.

[43] Keys et al. (2010) show that mortgages for which the borrower had a credit score just sufficient for the mortgage to be securitized were more likely to default than borrowers whose credit score was just below the securiitzation threshold. Also see Elul (2011) for evidence that securitized mortgages were more likely to default on average than non-securitized mortgages.

[44] The originating bank often does not hold the same fraction of the junior, the senior, and the mezzanine tranches of the mortgage pool. This means that the payoff of the originating bank is a piecewise linear function of the proceedings from the foreclosure auctions and interest rate payments from the mortgage pool. The kinks in this piecewise linear function would lead to distortions in the bank's bidding behavior. However, one should expect the effect of the kinks to be negligible in practice, since a mortgage pool typically contains mortgages worth several billion dollars, whereas the value of an individual mortgage is much smaller (roughly $300,000 in our data). It is very unlikely that a single foreclosure auction would move a bank to the other side of a kink. To put it differently: a non-linear (here piecewise linear) von Neumann-Morgenstern utility function of the bank does mean non-risk-neutrality, but given that the money at stake is very small in relative terms for the bank, risk neutrality is a good approximation.

Our results provide further support for the concern about moral hazard in the securitization process. Our findings hence speak in favor of policies seeking to reduce moral hazard, such as requiring better documentation for securitized mortgages, reducing tax benefits for securitization, or holding the originating bank liable in case a mortgagee defaults (as is the case for European covered bonds), see e.g. Keys et al. (2010), Dewatripont et al. (2010), Tirole (2011), Campbell (2013).

Further, our results emphasize a new aspect of moral hazard in the securitization process: the value of the collateral, which directly affects the loss given default. This suggests that documentation not just of the characteristics of the borrower, but also of the property is important. At the same time, our findings also highlight some of the difficulties in new regulations: new regulatory requirements set criteria for the documented loan-to-value ratio of mortgages, e.g. the new Solvency II Directive of the European Union requires mortgages in the securitization pool to meet certain requirements on the loan-to-value ratio.[45] However, our results suggest that there is asymmetric information about the value of properties and hence also about the loan-to-value ratio, which reduces the effectivity of such regulation.

9 Conclusions

We develop a novel theory of foreclosure auctions and test some of its predictions with data from Palm Beach county (Florida, US). We find evidence for strategic bidding and asymmetric information, with the bank being the informed party. First, the data reveal bunching in bids at the judgment amount as the theory predicts both under symmetric and asymmetric information. Second, there is a non-monotonicity and discontinuity in the probability of sale to the brokers, as the theory predicts under asymmetric information.

Our theory and empirical techniques have practical relevance: the presence of asymmetric information is crucial for mortgage securitization. We look separately at securitized and non-securitized mortgages and find that for securitized mortgages, the bank has less of an informational advantage, for two reasons. First, non-monotonicity and discontinuity are not present for securitized mortgages. Second, banks bid lower for securitized mortgages.

This is consistent with the hypothesis that moral hazard is involved in the securitization

[45] See the Commission Delegated Regulation (EU) 2015/35, Article 177(1)(h).

process: the banks may exert less effort collecting information about the value of the property used as collateral for a mortgage. These findings complement the findings in Keys et al. (2010) that banks collect less information about the probability of default for securitized than for non-securitized mortgages.

Our results can also be used for other policy relevant questions. One such question is a welfare comparison of judicial and non-judicial foreclosures. Roughly 40% of the states in the U.S. (including Florida) only allow judicial foreclosures, i.e. the foreclosure auction has to be run by a court with rules as the ones described in this article (see Nelson and Whitman (2004)). The other states allow for both judicial and non-judicial foreclosures. Mortgage contracts with a so-called "power of sale" clause allow the bank to choose a non-judicial foreclosure if the borrower defaults, i.e. the bank can directly seize the property and sell it without going through a court.

The power of sale clause allows the bank to market the property, and to verifiably disclose, through inspections, some of the information regarding its condition. This reduces the signalling premium, but introduces another distortion. The bank, acting as a de facto owner of the property, is no longer obligated to pay the owner back any auction proceedings above the judgment amount. Thus the monopoly price distortion now extends to prices above the judgment amount. So the overall welfare effect of the power of sale is ambiguous. We plan to estimate this effect in future work.

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Online Appendix

A Omitted Proofs

Proof of Proposition 3. For the exposition, we assume $n \geq 2$. The proof for $n = 1$ is parallel. It turns out convenient to restate the problem a bit differently. Consider the bank of type x_S that contemplates a dropout price p, and assume that the brokers hold the belief $\hat{x}_S$ concerning the bank's type following the bank's dropout. For now, this belief is not necessarily the equilibrium belief.

We restrict attention to equilibria where the broker's dropout strategy (if the bank has not dropped out) is a continuous, strictly increasing function, with a differentiable inverse $X_B^*(p)$. Then the bank's expected profit, as a function of own type x_S, the perceived type $\hat{x}_S$, and the price p is given by

$$\begin{aligned}\hat{\Pi}(x_S, \hat{x}_S, p) = & \, p(F_{(2)}(X_B^*(p)) - F_{(1)}(X_B^*(p))) \\ & + \int_{X_B^*(p)}^{\infty} u_B(x, \hat{x}_S) f_{(2)}(x) dx + x_S F_{(1)}(X_B^*(p)).\end{aligned} \tag{14}$$

A direct calculation shows that

$$\begin{aligned}\frac{\partial \hat{\Pi}_S}{\partial p} = & \, (p - u_B(X_B^*(p), \hat{x}_S)) f_{(2)}(X_B^*(p)) X_B^{*'}(p) \\ & + (x_S - p) f_{(1)}(X_B^*(p)) X_B^{*'}(p) \\ & + F_{(2)}(X_B^*(p)) - F_{(1)}(X_B^*(p))\end{aligned} \tag{15}$$

and

$$\frac{\partial \hat{\Pi}_S}{\partial \hat{x}_S} = \int_{\hat{X}_B(p)}^{\infty} \frac{\partial u_B(x, \hat{x}_S)}{\partial \hat{x}_S} f_{(2)}(x) dx. \tag{16}$$

The bank's expected profit following a deviation from the equilibrium price to some other price $\hat{p}$ is equal to $\hat{\Pi}_S(x_S, X_S(\hat{p}), \hat{p})$. In equilibrium, such a deviation should not be profitable,so that the following first-order condition (FOC) must hold for $p \in (\underline{p}, \infty)$:

$$\frac{\partial \hat{\Pi}_S(x_S, X_S(p), p)}{\partial p} = 0.$$

Substituting into this FOC the bank's type that bids p, $x_S = X_S^*(p)$, we obtain the following differential equation:

$$\frac{dX_S^*(p)}{dp} = -\frac{\partial \hat{\Pi}_S(X_S^*(p), X_S^*(p), p)/\partial p}{\partial \Pi_S(X_S^*(p), X_S^*(p), p)/\partial \hat{x}_S} \tag{17}$$

We now turn to the broker's equilibrium dropout strategy if the bank has not yet dropped out. We claim that the broker's strategy $X_B^*(p)$ defined as the solution (which will be shown unique later in the proof) to

$$u_B(X_B^*(p), X_S^*(p)) = p, \tag{18}$$

is a best response. Consider first the scenario when it is known to the broker that the bank will drop out at price p. Then it is optimal for the broker to drop out at the price $u_B(x_B, X_S^*(p))$, and the brokers with $x_B < X_B^*(p)$ will drop out at prices lower than p, while the brokers with $x_B > X_B^*(p)$ will drop out at higher prices. If $X_B^*(p)$ defined by (18) is increasing, it follows that it is optimal to drop out at price p' for a broker of type $X_B^*(p')$. Since this best response does not depend on p, we see that $X_B^*(p')$ is a best response also when the broker does not know the bank's dropout price p.

Totally differentiating (18) yields another differential equation linking $X_S^*(p)$ and $X_B^*(p)$:

$$\frac{\partial u_B}{\partial x_B}\frac{dX_B^*(p)}{dp} + \frac{\partial u_B}{\partial x_S}\frac{dX_S^*(p)}{dp} = 1. \tag{19}$$

Equations (17) and (19) form a linear system for $\frac{dX_S^*(p)}{dp}$ and $\frac{dX_B^*(p)}{dp}$; solving this system yields (8) and (9) in Proposition 3:

$$\frac{dX_S^*(p)}{dp} = \frac{(J_B(X_B^*, X_S^*) - X_S^*)]f_{(1)}(X_B^*)}{\frac{\partial u_B}{\partial x_S}(u_B(X_B^*, X_S^*) - X_S^*)f_{(1)}(X_B^*) + \frac{\partial u_B}{\partial x_B}\int_{X_B^*}^{\infty}\frac{\partial u_B}{\partial x_S}f_{(2)}(x)dx}, \tag{20}$$

$$\frac{dX_B^*(p)}{dp} = \frac{F_{(2)}(X_B^*) - F_{(1)}(X_B^*) + \int_{X_B^*}^{\infty}\frac{\partial u_B}{\partial x_S}f_{(2)}(x)dx}{\frac{\partial u_B}{\partial x_S}(u_B(X_B^*, X_S^*) - X_S^*)f_{(1)}(X_B^*) + \frac{\partial u_B}{\partial x_B}\int_{X_B^*}^{\infty}\frac{\partial u_B}{\partial x_S}f_{(2)}(x)dx}, \tag{21}$$

subject to the initial conditions $X_S^*(\underline{p}) = 0$ and $X_B^*(\underline{p}) = \underline{x}$, where the cutoff $\underline{x}$ is uniquely determined from $u_B(\underline{x}, 0) = \underline{p}$. With a given value for $\underline{x}$, the solution to the system (20) and (21) is unique by standard results in the theory of differential equations.

We now show that the broker who is indifferent between staying in and dropping out at p will not have an incentive to wait for a price higher than p. It will be sufficient to demonstrate that the rate of increase in the expected utility from trading with higher quality seller types is less than the rate of increase in the price, i.e. less than 1. By differentiating

$$u_B(x_B, X_S^*(p)) - p$$

with respect to p, we indeed get the slope less than 1,

$$\frac{\partial u_B}{\partial x_S}\frac{dX_S^*(p)}{dp} < \frac{J_B - X_S^*}{u_B - X_S^*} < 1,$$

where the first inequality can be obtained by substituting in the left-hand-side of (20) and observing that the integral in (20) is positive. The second inequality follows from the definition of J_B in (2).

Next, we claim that the only value of $\underline{x}$ compatible with equilibrium is the one given in the proposition, namely with $\underline{x} = \underline{x}_B$, uniquely determined from $J_B(\underline{x}_B, 0) = 0$. This value corresponds to the full information outcome.

Since $\hat{X}_B'(p) > 0$, it follows that the expected profit function $\hat{\Pi}(x_S, \hat{x}_S, p)$ satisfies the following *single-crossing* condition: the ratio of the slopes

$$\frac{\partial \hat{\Pi}_S(x_S, \hat{x}_S, p)/\partial p}{\partial \hat{\Pi}_S(x_S, \hat{x}_S, p)/\partial \hat{x}_S} \tag{22}$$

is increasing in x_S. This single-crossing condition implies that there are no profitable within-equilibrium deviations. Indeed, if $\hat{p} \geq \underline{p}$ is such a deviation, then differntial equation (17) implies

$$\frac{dX_S^*(\hat{p})}{dp} = -\frac{\partial \hat{\Pi}_S(X_S^*(\hat{p}), X_S^*(\hat{p}), \hat{p})/\partial p}{\partial \hat{\Pi}_S(X_S^*(\hat{p}), X_S^*(p), p)/\partial \hat{x}_S}$$

This, however, contradicts the single-crossing condition, according to which $X_S^*(\hat{p}) < x_S$ implies

$$\frac{\partial \Pi_S(X_S^*(\hat{p}), X_S^*(\hat{p}), \hat{p})/\partial p}{\partial \Pi_S(X_S^*(\hat{p}), X_S^*(\hat{p}), \hat{p})/\partial \hat{x}_S} < \frac{\partial \Pi_S(x_S, X_S^*(\hat{p}), \hat{p})/\partial p}{\partial \Pi_S(x_S, X_S^*(\hat{p}), \hat{p})/\partial \hat{x}_S}$$

Similarly, we can rule out an within-equilibrium deviation to a price $\hat{p} \in (p, \infty)$.

We now claim that only the full information cutoff $\underline{x} = \underline{x}_B$ is compatible with the separating equilibrium. First, observe that any value $\underline{x} < \underline{x}_B$ corresponds to the solution of the system that is not monotonically increasing and therefore cannot correspond to a separating equilibrium. Next, if $\underline{x} > \underline{x}_B$, then it is profitable for the type $x_S = 0$ to deviate to $p < \underline{p}$ even when the brokers' beliefs are the most pessimistic, $\hat{x}_S = 0$. Indeed, if $\underline{x} > \underline{x}_B$, the slope of the expected profit (15) at $p = \underline{p}$ is of the same sign as

$$-J_B(\underline{x}, 0) < -J_B(\underline{x}_B, 0) = 0.$$

The cutoff $\underline{x} = \underline{x}_B$ does indeed yield the solution $(X_B^*(p), X_S^*(p))$ in which each function is monotonically increasing in p. First, note that equations (20) and (21) form an autonomous system of differential equations. Every solution curve that is entirely contained in the region

$$\mathcal{M} := \{(x_B, x_S) : x_S \geq 0, J_B(x_B, x_S) - x_S \geq 0$$

corresponds to a monotone increasing solution because the r.h.s. of (20) and (21) are non-negative in (positive in the interior of) $\mathcal{M}$. Next, observe that the solution curve that starts at the point $(x_B, x_S) = (\underline{x}_B, 0)$, i.e. in the south-west corner of $\mathcal{M}$ will never leave $\mathcal{M}$. The left boundary of $\mathcal{M}$ is given by the full information outcome, $\{(x_B, x_S) : J_B(x_B, x_S) - x_S = 0\}$. This boundary is an increasing locus because $J_B(x_B.x_S)$ is assumed increasing in x_B. The vector field of the system points inside $\mathcal{M}$, with $dX_S^*(p)/dp = 0$ and $dX_B^*(p)/dp \geq 0$. Therefore, any solution with the initial condition in $\mathcal{M}$ will stay in $\mathcal{M}$. Since the initial condition $(\underline{x}_B, 0) \in \mathcal{M}$, we conclude that $X_B^*(p)$ and $X_S^*(p)$ are increasing in p.

Finally, in order to ensure that the bank with $x_S = 0$ indeed prefers to choose $\underline{x}_B$, it is sufficient to assume that for any lower (out of equilibrium) price, the brokers believe that the bank's type that deviated is the lowest one, $\hat{x}_S = 0$. Then (15) implies that the slope of the expected profit is *positive* for $p < \underline{p}$.

□

Proof of Corollary 1. Our previous analysis in that section implies that the bank with valuation x_S will set the price so that the marginal broker type willing to purchase at this price, $X_B^0(p)$, is found from the "marginal revenue equals cost" equation $J_B(X_B^0(p), x_S) - x_S = 0$. The price strategy itself is given by $p_S^0(x_S) = u_B(X_B^0(p), x_S)$. How does this price compare to the one with asymmetric information, $p_S^*(x_S)$? Proposition 3 shows that there is no distortion at the bottom, so that the two price are equal: $p_S^*(0) = p_S^0(0)$.

For $x_S > 0$, we have $dX_S^*(p)/dp > 0$. Going back the differential equation (8), this means that for $p = p_S^*(x_S)$, we must have $J_B(X_B^*(p), x_S) > x_S$, which, by the monotonicity of $J_B(x_B, x_S)$ in x_B, implies $X_B^*(p) > X_B^0(p)$. Since $u_B(X_B^0(p), X_S^0(p)) = p$ and $u_B(X_B^*(p), X_S^*(p)) = p$, with $u_B(x_B, x_S)$ being monotone increasing in both arguments (under common values), we must have

$$X_S^*(p) < X_S^0(p) \implies p_S^*(x_S) > p_S^0(x_S)$$

for $x_S > 0$. Finally, to show that the probability of sale is lower under asymmetric information, note that

$$\begin{aligned} u_B(X_B^*(p), X_S^*(p)) = p, \quad u_B(X_B^0(p), X_S^0(p)) = p, \\ X_S^*(p) < X_S^0(p) \implies X_B^*(p) > X_B^0(p). \end{aligned} \tag{23}$$

Since the probabilities of sale under asymmetric and symmetric information are given by $1 - F_{(1)}(X_B^*(p))$ and $1 - F_{(1)}(X_B^0(p))$ respectively, we see that the latter is higher than the former. Further, (23) implies $p_B^*(x_B) < p_B^0(X_B)$. $\square$

Proof of Proposition 4. Instead of the bank's bid strategy directly, in this proof it turns out convenient to work with the function $s(x_B)$, which gives the the bank's type s as a function of the broker's type x_B that would drop out at the same price as the bank. Then the bank's bidding strategy is given by

$$\begin{aligned} p_S(x_S) &= u_B(s^{-1}(x_S), x_S) && (24) \\ &= s^{-1}(x_S) + \alpha x_S. && (25) \end{aligned}$$

Dividing equation (8) by (9), we get the following differential equation for $s(x_B)$,

$$\frac{ds}{dx_B} = \frac{(\bar{J}(x_B) - (1-\alpha)s) f_{(1)}(x_B)}{\alpha \bar{F}_{(1)}(x_B)}$$

where

$$\bar{J}(t) = t - \frac{1 - F_B(t)}{f_B(t)}$$

and $\bar{F}_{(1)}(x_B) = 1 - F_{(1)}(x_B)$.

This equation can be integrated explicitly[46]

$$s(x_B) = \gamma \bar{F}_{(1)}(x_B)^{\gamma-1} \int_{\underline{x}_B}^{x_B} \bar{F}_{(1)}(t)^{-\gamma} f_{(1)}(t) \bar{J}(t) dt$$

where

$$\gamma = \frac{1}{\alpha}$$

and $\underline{x}_B$ is the lowest broker participating type, the same as under symmetric information,

$$\bar{J}(\underline{x}_B) = 0.$$

The slope of $s(x_B)$ is given by

$$s'(x_B) = \gamma(\gamma-1) f_{(1)}(x_B) \bar{F}_{(1)}(x_B)^{\gamma-2} \int_{\underline{x}_B}^{x_B} \bar{F}_{(1)}(t)^{-\gamma} f_{(1)}(t) \bar{J}(t) dt + \gamma \bar{F}_{(1)}(x_B)^{-1} f_{(1)}(x_B) \bar{J}(x_B) \tag{26}$$

[46] The integration method is essentially the same as in the proof of Theorem 3 in Cai et al. (2007). However, our linear specification is different and is not a special case of the linear specification in Cai et al. (2007).

With a change of variable

$$y = \log \frac{\bar{F}_{(1)}(x_B)}{\bar{F}_{(1)}(t)},$$

we have

$$s'(x_B) = f_{(1)}(x_B) \int_{\underline{y}}^{0} \gamma(\gamma-1)e^{(\gamma-1)y}\tilde{J}(y)dy + \gamma \bar{F}_{(1)}(x_B)^{-1} f_{(1)}(x_B)\bar{J}(x_B)$$

where $\tilde{J}(y) = \bar{J}(t(y))$,

$$\underline{y} < 0, \qquad \tilde{J}(\underline{y}) = 0.$$

Taking the derivative of the slope $s'(x_B)$ with respect to γ, and using the estimate

$$\begin{aligned} \frac{d}{d\gamma}\Big(\gamma(1-\gamma)e^{(\gamma-1)y}\Big) &= \Big(2\gamma - 1 + \gamma(\gamma-1)\Big)e^{(\gamma-1)y} \\ &\geq \Big(1 + (\gamma-1)y\Big)e^{(\gamma-1)y} \end{aligned}$$

we obtain the estimate

$$\frac{ds'(x_B)}{d\gamma} \geq f_{(1)}(x_B) \int_{\underline{y}}^{0} \Big(1 + (\gamma-1)y\Big)e^{(\gamma-1)y}\tilde{J}(y)dy + \bar{F}_{(1)}(x_B)^{-1} f_{(1)}(x_B)\bar{J}(x_B).$$

The second term above is positive. As far as the first term, the extended mean-value theorem for integrals implies for some $a \in [\underline{y}, 0]$[47]

$$\begin{aligned} \int_{\underline{y}}^{0} \Big(1 + (\gamma-1)y\Big)e^{(\gamma-1)y}\tilde{J}(y)dy &= \tilde{J}(0) \int_{\underline{y}}^{0} \Big(1 + (\gamma-1)y\Big)e^{(\gamma-1)y}dy \\ &= J(x_B) \int_{\underline{y}}^{0} d\Big(ye^{(\gamma-1)y}\Big) \\ &= -J(x_B)ae^{(\gamma-1)\underline{a}} \geq 0 \end{aligned}$$

where the last line follows from $a < 0$. So we conclude

$$\frac{ds'(x_B)}{d\gamma} > 0.$$

Since $\gamma = 1/\alpha$, this implies $\frac{ds'(x_B)}{d\alpha} < 0$, which in turn implies that the slope of the inverse $s^{-1}(x_S)$ is increasing in α. Since $p_S(x_S; \alpha) = s^{-1}(x_S) + \alpha x_S$, we conclude that the slope of

[47]The second mean-value theorem for integrals states that $\int_a^b f(t)g(t)dt = g(a)\int_a^c f(t)dt + g(b)\int_c^b f(t)dt$ whenever f, g are continuous functions on $[a, b]$. See Theorem 2.12.17 on p.150 in Bogachev (2007). Here, this theorem is applied with $g(t) = \tilde{J}(t)$, taking into account $\tilde{J}(\underline{y}) = 0$.

$p_S(x_S;\alpha)$ is increasing in α. Since $p_S(0) = \underline{x}_B$ is independent of α, this implies $p_S(x_S;\alpha)$ increases in α.

□

Proof of Lemma 1. Define

$$H(\underline{x}_S, \overline{x}_B) := \int_{\underline{x}_S}^{\overline{x}_B} \Big(u_B(\overline{x}_B, x_S) - \max\{v_J, x_S\} \Big) f_S^*(x_S) dx_S,$$

and

$$f_S^*(x_S) = \frac{f_S(x_S)}{F_S(\overline{x}_B) - F_S(\underline{x}_S)}.$$

Indifference condition (12) can be equivalently stated as

$$(\hat{p}(\underline{p}_S) - \underline{x}_S)(1 - F_{(1)}(X_B^*(p_S^*(\underline{x}_S)))) = (\hat{p}(v_J) - \underline{x}_S)(1 - F_{(1)}(\overline{x}_B)), \tag{27}$$

where $\hat{p}(p)$ denotes the equilibrium price received by the bank *conditional* on winning the auction with a reserve p. The bank's indifference condition (27) defines $\overline{x}_B$ as an implicit function of $\underline{x}_S$. This function is denoted as $y_B(\cdot)$. The indifference condition (27) can be re-written as

$$F_{(1)}(\overline{x}_B) = 1 - \frac{\Pi_S(\underline{x}_S)}{\hat{p}(v_J) - \underline{x}_S}$$

where we denoted the broker's type that corresponds to $\underline{x}_S$ as $\tilde{x}_B(\underline{x}_S) = X_B(p_S^*(\underline{x}_S)))$, and

$$\Pi_S(\underline{x}_S) = (\hat{p}(p_S^*(\underline{x}_S)) - x_S)(1 - F_{(1)}(\tilde{x}_B(\underline{x}_S)).$$

Next, we show that $\Pi_S(\underline{x}_S)/(\hat{p}(v_J) - \underline{x}_S)$ is increasing in $\underline{x}_S$, which implies that $\overline{x}_B$ is decreasing in $\underline{x}_S$. The derivative of this function is

$$\frac{d}{d\underline{x}_S} \frac{\Pi_S(\underline{x}_S)}{\hat{p}(v_J) - \underline{x}_S} = \frac{\Pi_S'(\underline{x}_S)(\hat{p}(v_J) - \underline{x}_S) + \Pi_S(\underline{x}_S)}{(\hat{p}(v_J) - \underline{x}_S)^2}$$

The envelope theorem implies $\Pi_S'(\underline{x}_S) = -(1 - F_{(1)}(\tilde{x}_B(\underline{x}_S)))$. So the numerator is equal to

$$\begin{aligned}
&\Pi_S(\underline{x}_S) - (\hat{p}(v_J) - \underline{x}_S)(1 - F_{(1)}(\tilde{x}_B(\underline{x}_S))) \\
&= \Pi_S(\underline{x}_S) - (\hat{p}(p_1) - \underline{x}_S)(1 - F_{(1)}(\tilde{x}_B(\underline{x}_S))) - (\hat{p}(v_J) - \hat{p}(p_1))(1 - F_B(\tilde{x}_B(\underline{x}_S))) \\
&= -(\hat{p}(v_J) - p_1)(1 - F_{(1)}(\tilde{x}_B(\underline{x}_S))) < 0
\end{aligned}$$

where the inequality follows since $p_1 < v_J$ and $\hat{p}(\cdot)$ is a an increasing function. Hence, $y_B(\underline{x}_S)$ is a *decreasing* function.

The broker's indifference condition (11), $H(\underline{x}_S, \overline{x}_B) = v_J$, defines $\overline{x}_B$ as an implicit function of $\underline{x}_S$,

$$\overline{x}_B = z(\underline{x}_S).$$

Indeed, we have $H(\underline{x}_S, v_J) < 0$ and, for $\overline{x}_B \geq v_J$,

$$\begin{aligned}
\frac{\partial H(\underline{x}_S, \overline{x}_B)}{\partial \overline{x}_B} &= \Big(u_B(\overline{x}_B, \overline{x}_B) - \overline{x}_B\Big) f_S(\overline{x}_B) + \int_{\underline{x}_S}^{\overline{x}_B} \frac{\partial u_B(\overline{x}_B, x_S)}{\partial \overline{x}_B} f_S(x_S) dx_S \\
&= \int_{\underline{x}_S}^{\overline{x}_B} \frac{\partial u_B(\overline{x}_B, x_S)}{\partial \overline{x}_B} f_S(x_S) dx_S \\
&\geq \theta(F_S(\overline{x}_B) - F_S(v_J)),
\end{aligned}$$

where we have used the assumption that $\partial u_B / \partial x_B \geq \theta > 0$. By integration, it then follows that for $\overline{x}_B \geq v_J$,

$$H(\underline{x}_S, \overline{x}_B) \geq H(\underline{x}_S, v_J) + \int_{v_J}^{\overline{x}_B} (F_S(y) - F_S(v_J)) dy \to \infty$$

as $\overline{x}_B \to \infty$. So for a given $\underline{x}_S < v_J$, $H(\underline{x}_S, \overline{x}_B)$ is an increasing function of $\overline{x}_B$, tending to ∞, with $H(\underline{x}_S, v_J) < 0$. This implies that the equation $H(\underline{x}_S, \overline{x}_B) = 0$ defines $\overline{x}_B$ as an implicit function of $\underline{x}_S$. This function will be denoted as $z(\underline{x}_S)$,

$$H(\underline{x}_S, z_B(\underline{x}_S)) = 0.$$

We now show that $z_B(\cdot)$ is an *increasing* function. This will follow from the fact that $H(\underline{x}_S, \overline{x}_B)$ defined in (11) is an increasing function in first two arguments. We have already shown that $H(\underline{x}_S, \overline{x}_B)$ is increasing in $\overline{x}_B$ for $\overline{x}_B \geq v_J$. Now

$$\frac{\partial H(\underline{x}_S, \overline{x}_B)}{\partial \underline{x}_S} = (v_J - u_B(\overline{x}_B, \underline{x}_S)) f_S(\underline{x}_S).$$

We claim that $u_B(\overline{x}_B, \underline{x}_S) < v_J$, so that $H(\underline{x}_S, \overline{x}_B)$ is indeed increasing in $\underline{x}_S$. We argue by contradiction. If not, we would have

$$H(\underline{x}_S, \overline{x}_B) \geq \int_{v_J}^{\overline{x}_B} \Big(u_B(\overline{x}_B, x_S) - x_S\Big) f_S(x_S) dx_S, \tag{28}$$

Given our assumption $\partial u_B/\partial x_S < 1$, we have for $x_S < \overline{x}_B$,

$$\begin{aligned} u_B(\overline{x}_B, x_S) &= u_B(\overline{x}_B, \overline{x}_B) - \int_{x_S}^{\overline{x}_B} \frac{\partial u_B(\overline{x}_B, x_S)}{\partial x_S} dx_S \\ &> \overline{x}_B - \int x_S \overline{x}_B dx_S = x_S. \end{aligned}$$

Substituting this bound into (28), we get

$$\begin{aligned} H(\underline{x}_S, \overline{x}_B) &> \int_{v_J}^{\overline{x}_B} \Big(u_B(\overline{x}_B, x_S) - x_S \Big) f_S(x_S) dx_S \\ &> 0, \end{aligned}$$

a contradiction to $H(\underline{x}_S, \overline{x}_B) = 0$. So we conclude

$$\frac{\partial H(\underline{x}_S, \overline{x}_B)}{\partial \underline{x}_S} > 0.$$

The Implicit Function Theorem now implies that $z_B(\cdot)$ is increasing:

$$z_B'(\underline{x}_S) = -\frac{\partial H/\partial \underline{x}_S}{\partial H/\partial \overline{x}_B} > 0.$$

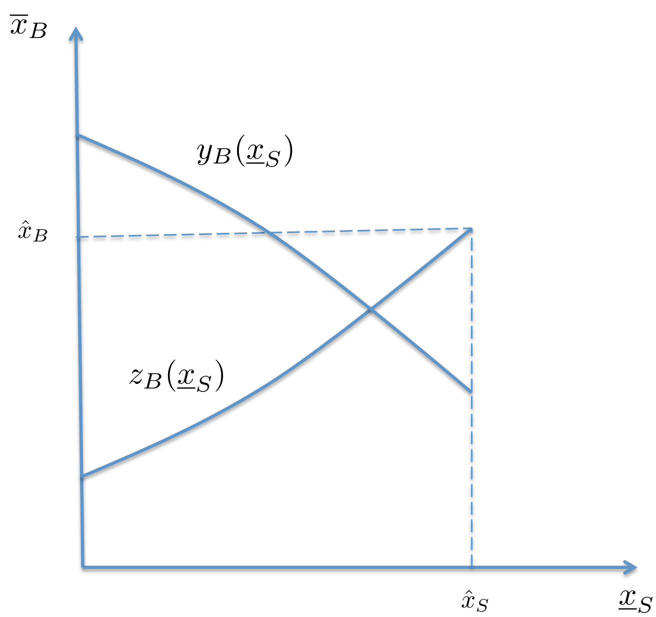

Figure 12: Functions $y_B(\cdot)$ and $z_B(\cdot)$.

Thus $y_B(\underline{x}_S)$ and $z_B(\underline{x}_S)$ are both continuous, and are, respectively, decreasing and increasing functions. Continuing with the proof, let $\hat{x}_S = X_S^*(v_J)$ and $\hat{x}_B = X_B^*(v_J)$ be the bank's and broker's types in the bank-seller equilibrium ($v_J = 0$). The lower cutoff values are restricted between the lowest possible value 0, and $\hat{x}_S$. Both mapping $y_B(\cdot)$ and $z_B(\cdot)$ are defined on the domain $[0, \hat{x}_S]$.

Since the $\hat{x}_B$ type breaks even if the bank drops out at v_J in the bank-seller equilibrium ($v_J = 0$), we must have $H(\hat{x}_S, \hat{x}_B) > 0$. The monotonicity of $H(\underline{x}_S, \overline{x}_B)$ in $\overline{x}_B$ implies $z_B(\hat{x}_S) < \hat{x}_B$. At the same time, the definition of $y_B(\cdot)$ implies $\hat{x}_B = y_B(\hat{x}_S)$, and it follows that $z_B(\hat{x}_S) < y_B(\hat{x}_S)$. Refer to Figure 12. In view of the monotonicity of $y_B(\cdot)$ and $z_B(\cdot)$, there are two possibilities. First, if $z_B(0) \geq y_B(0)$, then type-0 bank prefers to bid $\underline{p}$ rather than v_J. In this case, the curves $y_B(\underline{x}_S)$ and $z_B(\underline{x}_S)$ have a unique intersection given by

$$\overline{x}_B = y_B(\underline{x}_S) = z_B(\underline{x}_S), \tag{29}$$

with $\overline{x}_S \in (0, \hat{x}_S)$. If, on the other hand, $z_B(0) < y_B(0)$, then type-0 bank prefers to bid v_J, so that the equilibrium involves $\overline{x}_B = 0$ and the bunching extends all the way to $x_S = 0$.

□

Proof of Proposition 5. We begin with the bank's equilibrium strategy $p_S(x_S)$. The arguments from the section preceding the Proposition imply that $p_S(x_S)$ is an equilibrium best response for $x_S \leq \underline{x}_S$ and $x_S \geq \overline{x}_B$, so in this proof we only consider $x_S \in (\underline{x}_S, \overline{x}_B)$. For $x_S \in (v_J, \overline{x}_B)$, the bank acts as a buyer, and it is a best response for it to bid its value, x_S. So it only remains to consider $x_S \in (\underline{x}_S, v_J]$. These types will prefer to bid v_J over any bid in $(\underline{p}_S, v_J)$. The reason is that brokers with values $x_B < \overline{x}_B$ drop out immediately once the price has gone over $\underline{p}_S$. The brokers who remain will bid up to their values $x_B \geq \overline{x}_S$. The bank will prefer to bid v_J over any $(\underline{p}_S, v_J)$ since doing so will not reduce the probability of selling to a broker, but will at least weakly increase the price conditional on sale.[48]

Having shown equilibrium incentives for the bank, we now turn to the broker. The results in the previous section imply that, for $x_B \leq x_B^*$, $p_B(x_B)$ dominates any other bid $p \leq \underline{p}_S = p_B(x_B^*)$. Since bidding $p \in (\underline{p}_S, v_J)$ will not affect the broker's expected profit due to the fact

[48]The price conditional on sale will be strictly higher $v_J > \underline{p}$ if there is only one broker who is active, i.e. has $x_B \geq \overline{x}_S$. If there is more than one broker, the expected price conditional on sale will be unchanged.

that no other participant bids there, it remains to be shown that a broker with $x_B < \overline{x}_B$ will not have an incentive to deviate to a $p \geq v_J$. It is clear that such a broker will not have an incentive to deviate to a bid $p > \overline{x}_B$, since this would lead the broker to buy at prices that are too high and would result in a loss. Indeed, by single crossing, $u_B(x_B, p) < p$ for $p > x_B$, which implies $u_B(x_B, p) < p$ for $p > \overline{x}_B$.

The incremental expected profit from deviation to a price $p \in [v_J, \overline{x}_B]$ is

$$\begin{aligned}
\Delta\Pi_B &= \int_{\underline{x}_S}^{p} (u_B(x_B, x_S) - \max\{v_J, x_S\}) f_S(x_S) dx_S \\
&< \int_{\underline{x}_S}^{p} (u_B(\overline{x}_B, x_S) - \max\{v_J, x_S\}) f_S(x_S) dx_S \\
&\leq \int_{\underline{x}_S}^{\overline{x}_B} (u_B(\overline{x}_B, x_S) - \max\{v_J, x_S\}) f_S(x_S) dx_S = 0
\end{aligned}$$

where the first inequality follows from the fact that $u_B(x_B, x_S)$ is increasing in x_B, while the second inequality follows from the definition of $\overline{x}_B$ as the type that is indifferent This shows that the broker with $x_B < \overline{x}_B$ will not have an incentive to deviate to a $p \geq v_J$, and it also follows that the broker types $x_B \in [x_B^*, \overline{x}_B]$ will bunch at $\underline{p}_S$.

The equilibrium uniqueness follows because there are unique cutoff types $\underline{x}_S$ and $\overline{x}_S$ according to Lemma 1. $\square$