Ludwig-Maximilians-University

Department of Statistics

Asymmetric Correlations in Financial Returns

Bachelor Thesis

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Munich, 12th May 2015

Abstract

Modeling dependence in finance is a challenging task, which is important for return and risk estimation of portfolios. Some recent studies provide evidence of an increase in correlations in a market crash eroding benefits of portfolio diversification. In this thesis dependence between financial returns will be carefully examined using Local Gaussian Correlation, which is obtained by approximating an arbitrary bivariate return distribution by a family of bivariate Gaussian distributions. The correlation parameter in a local neighbourhood of a value is taken as a local measure of dependence. Furthermore we make use of the copula class, which provides flexible modeling of dependence, and run a goodness-of-fit test for these copulas.

Contents

1 Introduction

Modeling dependence between financial returns is essential for all portfolios with at least two assets. Choosing low correlated assets offers benefits from portfolio diversification because a fall in one market could be offset by a rise in another market.

Unfortunately some recent studies provide evidence that correlations increase in bear markets, that are periods of low and volatile returns. In a financial crisis all markets move closely together eroding the benefits of portfolio diversification when these benefits are needed most. Studies providing evidence are e.g. Longin and Solnik [1995, 2001], Ang and Chen [2002], Campbell et al. [2002, 2008], Butler and Joaquin [2002], Okimoto [2008], Nikoloulopoulos et al. [2012] and Støve et al. [2012, 2014].

Longin and Solnik [1995] found an increase in correlations between international markets studying monthly index returns of seven major markets in Europe, America and Japan from 1960 to 1990. They also found a rise in correlations in volatile market periods. In their study in 2001, see Longin and Solnik [2001], they could reject normality in the lower tail (negative returns), but not in the upper tail (positive returns), indicating that high volatility does not necessarily affect correlations, because the change in correlations depends on the market trend. In bear markets there was an increase in correlations, but not so in bull markets, which are periods of positive returns.

However there seems to be no agreement of how such asymmetries should be measured. Standard dependence measures like the Pearson correlation coefficient fail to capture nonlinear dependence, e.g. asymmetric correlations in the tails of the return distribution. Recent studies use quite different models for measuring tail asymmetries (e.g. conditional correlations, copulas, Local Gaussian Correlation and regime-switching models). Some of these measures like the conditional correlation may lead to biased results [Støve et al., 2012].

In this thesis correlations in financial returns will be carefully investigated using Local Gaussian Correlation, which enables us to detect local asymmetries. A short introduction to the models will be given in the first sections. Thereafter several financial returns (daily, weekly and monthly) from stocks and stock markets will be thoroughly studied to look for asymmetric dependence. We therefore use Local Gaussian Correlation and combine this with standard copula theory to model the dependence in bivariate return distributions. Furthermore a Likelihood based comparison of several copulas is given and a goodness of fit test carried out to find adequate models for the observed dependence structures.

2 Local Gaussian Correlation

Local Gaussian Correlation is a new measure of dependence introduced by Tjøstheim and Hufthammer [2013]. An arbitrary bivariate distribution can be locally approximated by a family of bivariate Gaussian distributions with correlation parameter ρ, which can be interpreted as a local measure of dependence.

The density f of a bivariate random variable $X = [X_1, X_2]$ can be approximated in the neighbourhood of any point $x = (x_1, x_2)$ by a Gaussian bivariate density defined by

$$\begin{aligned}
&\psi(u, v, \mu_1(x), \mu_2(x), \sigma_1(x), \sigma_2(x), \rho(x)) \\
&= \frac{1}{2\pi\,\sigma_1(x)\,\sigma_2(x)\,\sqrt{1-\rho^2(x)}} \exp\Bigg\{-\frac{1}{2(1-\rho^2(x))} \\
&* \left[\left(\frac{u-\mu_1(x)}{\sigma_1(x)}\right)^2 - 2\rho(x)\,\frac{(u-\mu_1(x))\,(v-\mu_2(x))}{\sigma_1(x)\,\sigma_2(x)} + \left(\frac{v-\mu_2(x)}{\sigma_2(x)}\right)^2\right]\Bigg\}
\end{aligned} \tag{1}$$

with local means μ, local standard deviations σ and correlation parameter ρ.

The parameters $(\mu_1(x), \mu_2(x), \sigma_1(x), \sigma_2(x), \rho(x))$ of the bivariate Gaussian density will be obtained by maximising the local log-likelihood function defined by

$$\begin{aligned}
l =& \frac{1}{n}\sum_{i=1}^{n} K_{b_1}(X_{1i}-x_1)\,K_{b_2}(X_{2i}-x_2)\log[\psi(X_{1i}, X_{2i})] \\
&- \int K_{b_1}(u-x_1)\,K_{b_2}(v-x_2)\,\psi(u,v)dudv
\end{aligned} \tag{2}$$

for observations $X = [(X_{11}, X_{21}), ..., (X_{1n}, X_{2n})]$.
The localisation of the estimation is achieved by defining a neighbourhood around x by

$$K_{b_j}(X_{ji}-x_j) = \frac{1}{b_j}K\left(\frac{1}{b_j}(X_{ji}-x_j)\right); \quad i \in \{1, ..., n\} \quad \text{and} \quad j \in \{1, 2\} \tag{3}$$

with kernel function K and bandwidth parameters b_1 and b_2. K is a weight function giving observations close to the point x a larger weight than more distant observations. For large values of b all observations are included in the neighbourhood, so the local parameter estimates converge to the global estimates of the parameters. Therefore the Local Gaussian Correlation will be the same as the ordinary Pearson correlation. Properties and interpretation of the Local Gaussian Correlation are the same as for the Pearson correlation.

In the following analysis the R-package "localgauss" will be used for estimating the Local Gaussian Correlation. An introduction to this package is given in Berentsen, Kleppe and Tjøstheim [2014].

3 Copulas and canonical Local Gaussian Correlation

A copula is a multivariate cumulative distribution function of random variables, which are marginally standard uniformly distributed [Ruppert, 2010]. The copula models the dependence between the random variables. It contains a few parameters (often only one) which describe the dependence structure. Because these parameters are difficult to interpret, they are often transformed to a value of a rank based correlation coefficient like Kendalls τ or Spearmans ρ.

In this thesis only bivariate copulas will be considered (Normal (=Gaussian), t, BB1, BB7). Formulas and details on these copulas are given in Joe [2015]. Berentsen, Støve, Tjøstheim and Nordbø [2014] derive a new tool to visualise the dependence structure of a given copula: the canonical Local Gaussian Correlation. First a bivariate variable $X = (X_1, X_2)$ must be transformed to

$$Z = (Z_1, Z_2) = (\Phi^{-1}(F_1(X_1)), \Phi^{-1}(F_2(X_2))) \tag{4}$$

with Φ denoting the standard normal distribution function. Since X_1 and X_2 are both marginally uniformly distributed, Z_1 and Z_2 are both marginally standard normal distributed.

The canonical Local Gaussian Correlation is then defined by

$$\rho_\theta(z_1, z_2) = \frac{-C_{11}(\Phi(z_1), \Phi(z_2))\, \phi(z_1)}{\sqrt{\phi^2(\Phi^{-1}(C_1(\Phi(z_1), \Phi(z_2)))) + C_{11}^2(\Phi(z_1), \Phi(z_2))\, \phi^2(z_1)}} \tag{5}$$

with $C(u,v)$ the copula formula, $C_1(u,v) = \frac{\partial\, C(u,v)}{\partial\, u}$, $C_{11}(u,v) = \frac{\partial^2\, C(u,v)}{\partial\, u^2}$, ϕ denoting the density function of the standard normal distribution and θ the copula parameter(s). This formula is valid on the diagonal $z_1 = z_2$ and holds for all copulas that are exchangeable.

Normal copula

For a Normal copula there exists a one-to-one correspondence between the correlation matrix of a bivariate Normal distribution and the copula. The formula for the canonical Local Gaussian Correlation simplifies to $\rho_\theta(z_1, z_2) = \theta$ and is therefore constant for all points on the diagonal.

t copula

The t copula is similar to the Normal copula but shows more tail dependence. The derivatives of the copula with ν degrees of freedom and ρ as the correlation coefficient are given in Berentsen, Støve, Tjøstheim and Nordbø [2014]. In Figure 1 the canonical Local Gaussian Correlation is given for the t copula with 2.5 and 6 degrees of freedom and different levels of Kendalls Tau τ, which has a one-to-one relation to copula parameter ρ valid for Normal and t copula given by $\rho = \sin(\frac{\pi}{2}\,\tau)$. With increasing degrees of freedom the t copula resembles more and more the Normal copula.

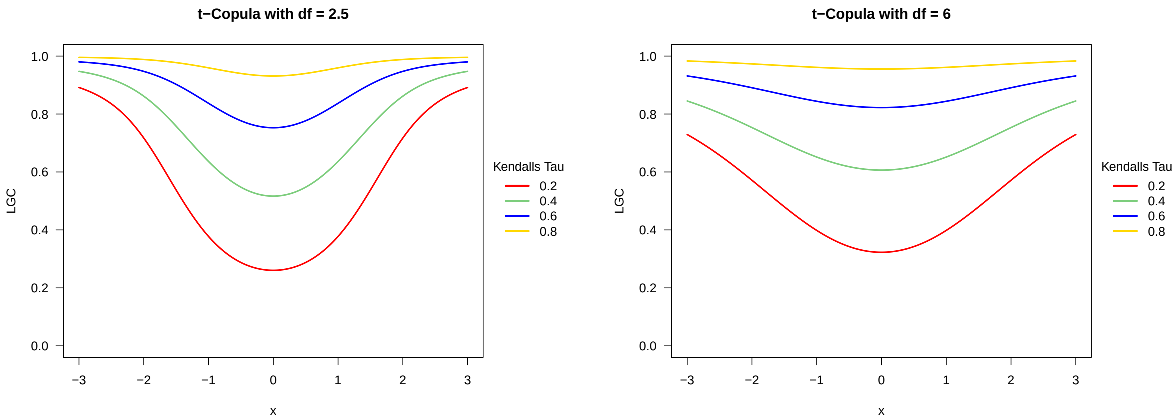

Figure 1: Canonical Local Gaussian Correlation for the t copula along the diagonal z1=z2

While Normal and t copula are symmetric, BB copulas allow asymmetric lower and upper tail dependence and may be therefore useful to model dependencies of financial returns.

BB1 copula

The BB1 copula is given by

$$C(u,v;\theta,\delta) = \left(1 + \left[(u^{-\theta}-1)^{\delta} + (v^{-\theta}-1)^{\delta}\right]^{\frac{1}{\delta}}\right)^{-\frac{1}{\theta}} \tag{6}$$

with $\theta \in (0,\infty)$ and $\delta \in [1,\infty)$. It can model dependence in the range [0,1].

Details and formulas C_1 for the BB1 and BB7 copula are given in Joe [2015], pages 190 and 202. The formula C_{11} for the BB1 copula is given by

$$\begin{aligned} C_{11}(u_1,u_2) = &-\frac{1}{x^{\theta}-1} x^{-\theta-2} a^{\frac{\delta-1}{\delta}} \left((a+y)^{\frac{1}{\delta}-2} (a+y)^{\frac{1}{\delta}} + 1\right)^{-\frac{1}{\theta}-2} * \\ &\left((\theta+1)\, a \left(x^{\theta}\left((a+y)^{\frac{1}{\delta}}+1\right)-1\right) + y\left(\theta\left(x^{\theta}-\delta\right) + x^{\theta} - 1\left((a+y)^{\frac{1}{\delta}}+1\right)\right)\right). \end{aligned} \tag{7}$$

with $a = \left(x^{-\theta}-1\right)^{\delta}$, $x = \left(u_1^{-\theta-1}\right)^{\delta}$ and $y = \left(u_2^{-\theta-1}\right)^{\delta}$.

The canonical Local Gaussian Correlation for the BB1 copula is shown in Figure 2 for different parameter combinations.

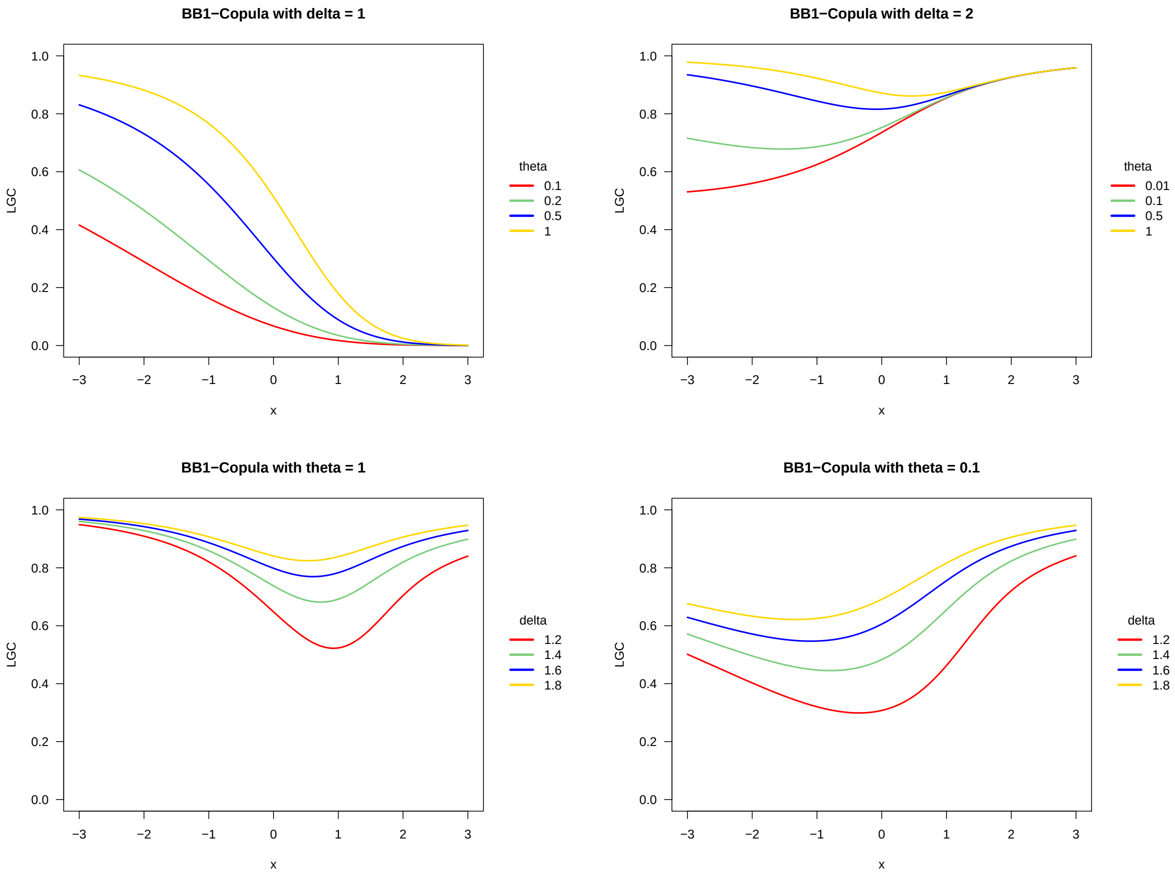

Figure 2: Canonical Local Gaussian Correlation for the BB1 copula along the diagonal z1=z2

BB7 copula

The BB7 copula is given by

$$C(u,v;\theta,\delta) = 1 - \left(1 - \left[(1-u^{-\theta})^{-\delta} + (1-v^{-\theta})^{-\delta} - 1\right]^{-\frac{1}{\delta}}\right)^{\frac{1}{\theta}} \tag{8}$$

with $\theta \in [1,\infty)$ and $\delta \in (0,\infty)$. It can model dependencies in the range [0,1]. The formula C_{11} for the BB7 copula is given in Equation 9.

$$\begin{aligned} C_{11}(u_1,u_2) = &\Bigg((1-x)^{\theta-2} \left(a^{-\delta}\right)^{1/\delta} \left(1-\left(a^{-\delta}+y\right)^{-1/\delta}\right)^{1/\theta} \left(ya^{\delta}\left(\theta+\delta\,\theta\,(1-x)^{\theta}\right)(1-x)^{\theta}-1\right) * \\ &\left(\left(a^{-\delta}+y\right)^{1/\delta}-1\right) - (\theta-1)\left((1-x)^{\theta}\left(a^{-\delta}+y\right)^{1/\delta} - \left(a^{-\delta}+y\right)^{1/\delta}+1\right)\Bigg) / \\ &\left(\left((1-x)^{\theta}-1\right)\left(ya^{\delta}+1\right)^{2}\left(\left(a^{-\delta}+y\right)^{1/\delta}-1\right)^{2}\right) \end{aligned} \tag{9}$$

with $a = 1 - (1-x)^\theta$, $x = (1-(1-u_1)^\theta)^{-\delta-1}$ and $y = (1-(1-u_2)^\theta)^{-\delta-1}$.

The canonical Local Gaussian Correlation for the BB7 copula is shown in Figure 3 for different parameter combinations.

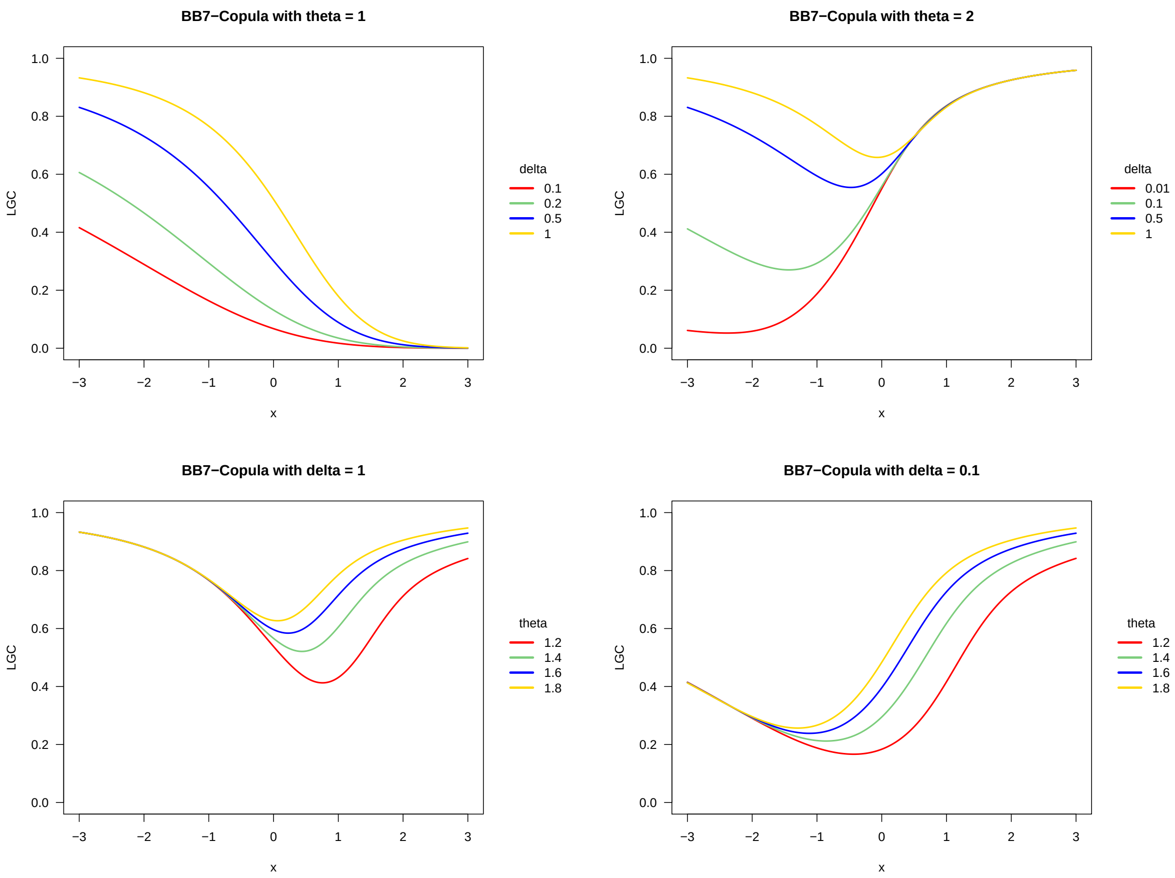

Figure 3: Canonical Local Gaussian Correlation for the BB7 copula along the diagonal z1=z2

The BB1 copula with $\delta = 1$ is equal to the BB7 copula with $\theta = 1$. Both copulas allow flexible modeling of lower and upper tail dependence.

4 Goodness-of-fit test for copulas

Berentsen, Støve, Tjøstheim and Nordbø [2014] introduce a goodness-of-fit test for copulas based on the canonical Local Gaussian Correlation. Thereby the difference between the empirical estimate for the Local Gaussian Correlation and the canonical Local Gaussian Correlation for a copula $\mathcal{C}$ will be computed. The test null hypothesis is

$$H_0 : \mathrm{C} \in \mathcal{C} = \{\mathrm{C}_\theta : \theta \in \Theta\} \tag{10}$$

given iid observations $X_1, ..., X_n$ from $F(x_1, x_2) = C(F_1(x_1), F_2(x_1))$ and copula parameter(s) θ.

For the estimation of the copula parameter(s) pseudo observations are needed. They are defined by

$$\hat{U}_i = \frac{1}{n+1} \sum_{i=1}^{n} \mathbf{I}_{X_i \leq x}. \tag{11}$$

The empirical Local Gaussian Correlation is then estimated for Gaussian pseudo observations

$$\hat{Z}_i = \Phi^{-1}(\hat{U}_i), \tag{12}$$

The test is carried out using a parametric bootstrap with following steps:

1. estimate copula parameter(s) θ under H_0 using the pseudo observations $\hat{U}_i$
2. generate a random sample from the copula C_θ with the parameters estimated from the original observations
3. convert the sampled observations into pseudo observations
4. estimate copula parameter(s) θ_n from these pseudo observations
5. obtain canonical Local Gaussian Correlation ρ_{θ_n}
6. convert pseudo observations $\hat{U}_i$ into Gaussian pseudo observations $\hat{Z}_i$
7. estimate the empirical Local Gaussian Correlation ρ_n using $\hat{Z}_i$
8. compute the test statistic $T_n = \int_{z_{\alpha/2}}^{z_{1-\alpha/2}} P_n(t,t)^2 dt$ with $P_n(\cdot) = \rho_n(\cdot) - \rho_{\theta_n}(\cdot)$ along the diagonal with $z_{\alpha/2}$ to $z_{1-\alpha/2}$ quantiles of the standard normal distribution

Repeat steps 2-8 for some large number R. The integral can be approximated by a sum. The p-value can be approximated by

$$\frac{1}{R} \sum_{k=1}^{R} \mathbf{I}(T_{n,k} > T_n). \tag{13}$$

with T_n the test statistic for the original sample in the same interval.

5 Empirical study

In this section the methods of the previous sections will be used to empirical investigate the dependence in financial returns.

5.1 Stock markets

First we look at stock market index data from four major markets (DAX 30 - Germany, CAC 40 - France, S&P 500 - USA, FTSE 100 - Great Britain) from 3rd January 1995 to 5th February 2015 (a total of 5027 daily observations). These markets have been studied earlier by Longin and Solnik [2001], Campbell et al. [2002], Campbell et al. [2008] and Støve et al. [2012] in different time periods with daily or monthly returns.

Figure 4 shows the market trend normalised on the first observation (3rd January 1995).

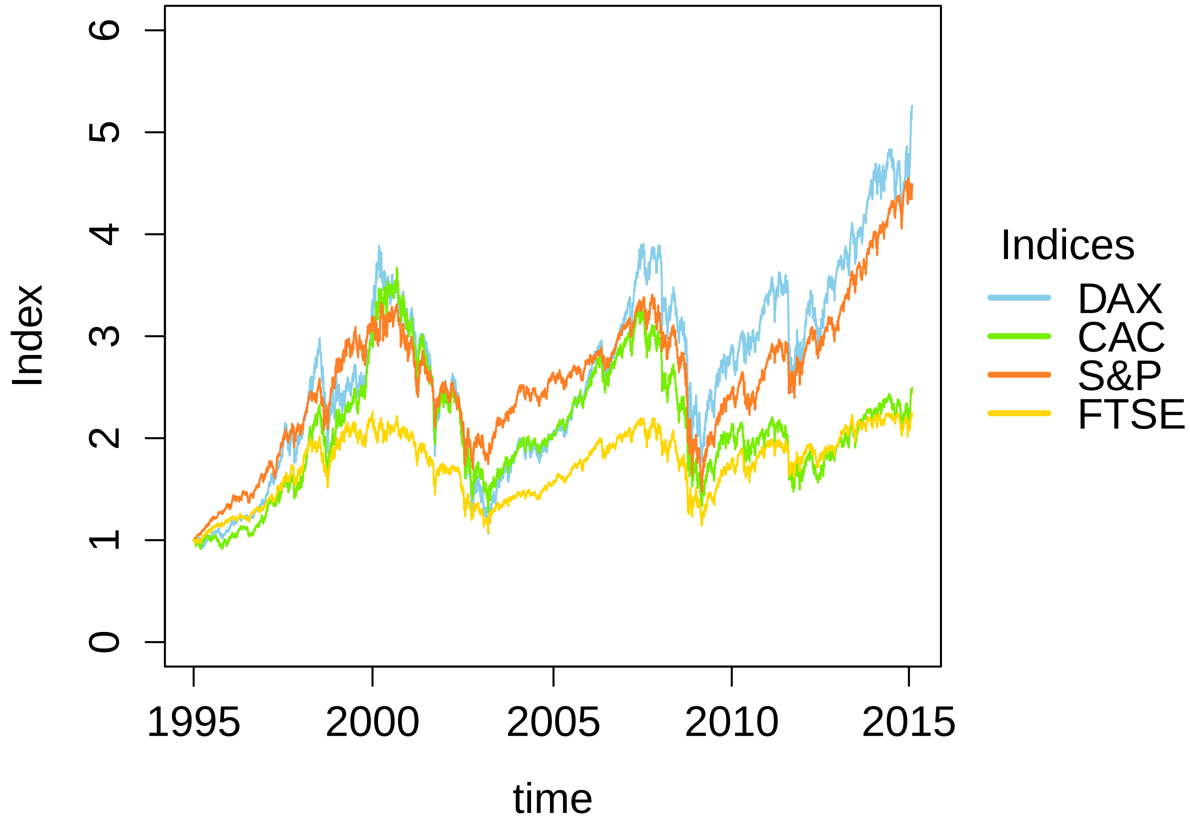

Figure 4: Market trend of stock market index data from 3rd January 1995 to 5th February 2015, normalised on the first observation

The S&P, DAX and CAC move closely together over the chosen time period, whilst the FTSE develops rather poor. A number of bear and bull markets can be identified in the 20 years. From 1995 to 2000 markets rise. Then in the early 2000s a decline in markets can be observed. From 2003 till the beginning of the financial crisis in late 2007 there is another bull market, until in 2008 and 2009 a sharp downturn occurs. In the recent past the market development diverges. In Germany and USA markets strongly increase, while in Great Britain and France indices develop rather poor.

In the following analysis we will be looking at the log returns defined by

$$r_t = 100 * (\log(p_t) - \log(p_{t-1})) \tag{14}$$

with the stock value p_t at a time point t.

Table 1 shows a summary of the index return data.

	Minimum	Mean	Maximum	Variance	Skewness	Kurtosis
DAX	-8.88%	0.033%	10.8%	2.29%	-0.08	4.23
CAC	-9.47%	0.018%	10.59%	2.14%	-0.015	4.36
FTSE	-9.27%	0.016%	9.38%	1.39%	-0.14	5.92
S&P	-9.47%	0.03%	10.96%	1.50%	-0.24	8.05

Table 1: Summary statistics for index returns

The daily returns reach from -9.47% to 10.96%. The daily mean is positive for all indices, largest for the DAX (0.033%). For a year (about 250 observations) this results in an annual mean return of 8.25%. Variances are lower for the FTSE and S&P in comparison to DAX and CAC, which was already observed by Campbell et al. [2008] in a different time period.
Skewness is defined as

$$s = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{r_i - \bar{r}}{\sigma}\right)^3 \tag{15}$$

with mean $\bar{r}$ and standard deviation σ and describes the asymmetry of a distribution. The observed negative skewness tells us that the mass of the distribution is concentrated in the right tail. For a skewness of 0 the distribution is symmetric (like e.g. the Normal distribution). All indices have a negative skewness (lowest value -0.24 for S&P).

The excess kurtosis of a sample is defined by

$$w = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{r_i - \bar{r}}{\sigma}\right)^4 - 3. \tag{16}$$

This equation implies that a normal distribution has a kurtosis of 0. For the index data the kurtosis assumes large values indicating that a normal distribution might not be a good fit for the data and so the ordinary Pearson correlation not a good measure for dependence. Definitions for skewness and kurtosis are taken from Ruppert [2010].

Figure 5 contains time series of the returns.

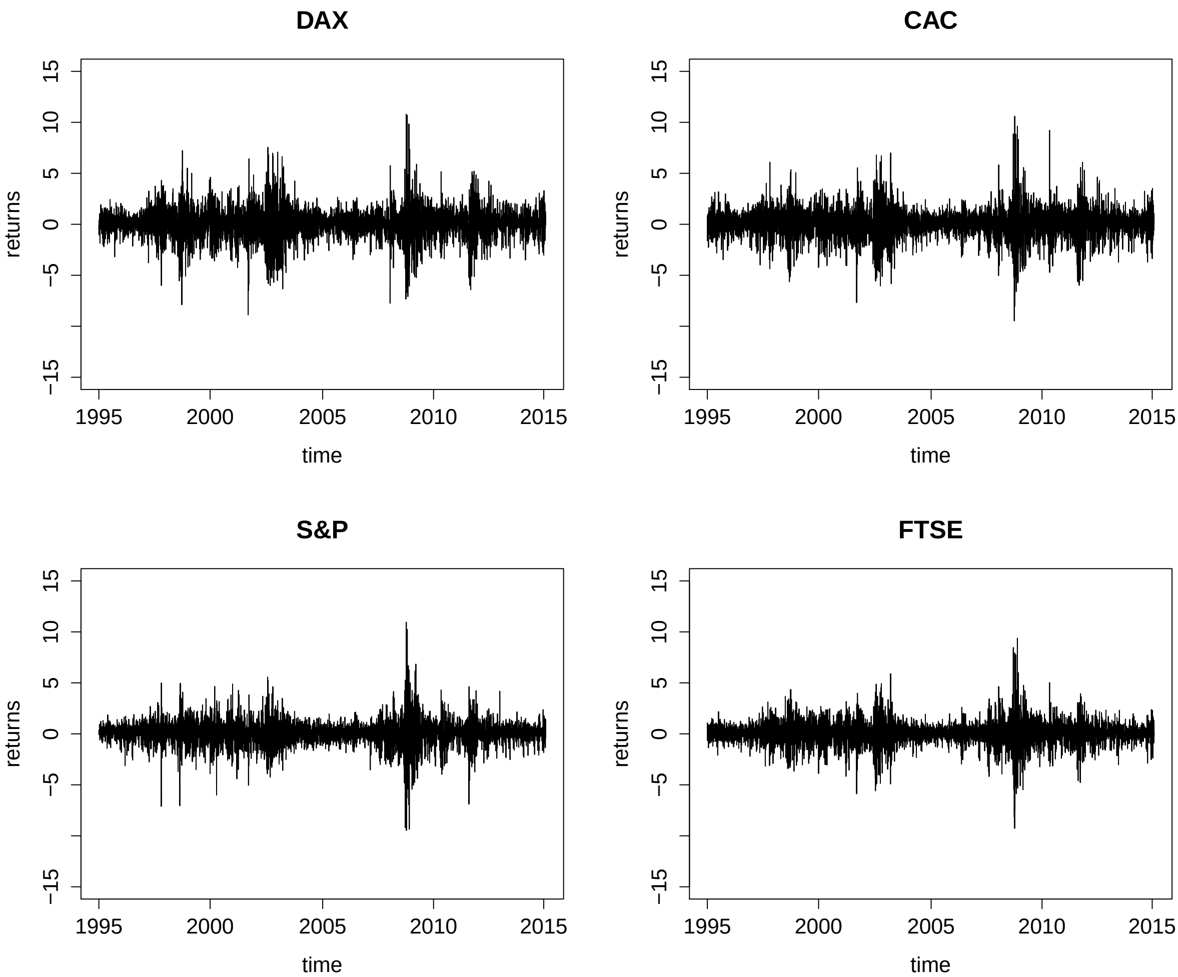

Figure 5: Time series of the index returns

Figure 5 shows that there are periods of low variance (e.g. 2004-2007) and periods of high variance (e.g. 2008 and 2009), so the volatility of the stock markets is not constant. Large returns appear in clusters. Financial returns are known to be fat tailed (e.g. Ruppert [2010]). In the further analysis we will need independently and identically distributed data, so we apply a time series filter to standardise the variance. Table 2 compares the fit of several GARCH(1,1) models similar to the comparison in Campbell et al. [2008].

	GARCH:Normal	GARCH:Skewed Normal	GARCH:t	GARCH:Skewed t
DAX	3.311	3.304	3.293	3.289
CAC	3.302	3.297	3.287	3.284
FTSE	2.781	2.774	2.769	2.764
S & P	2.831	2.820	2.791	2.787

Table 2: Standardised AIC (=AIC/number of observations) of univariate GARCH models for the daily index return series. Results are given for GARCH(1,1) models with underlying Normal or t-distribution.

Based on the maximised log likelihoods the univariate GARCH(1,1) model with skewed Student-t distributed errors provides the best fit for all return time series. These findings are consistent with Campbell et al. [2008]. The GARCH equations are given by

$$r_t = \mu + a_t, \tag{17}$$

$$a_t = \sigma_t \, \epsilon_t, \tag{18}$$

$$\sigma_t^2 = \omega + \alpha a_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{19}$$

with r_t denoting the log returns and GARCH parameters μ, σ, α, β and ω.

All coefficients are significant on the 5%-level and the Ljung-Box tests carried out on the standardized residuals and squared standardized residuals reject the null hypothesis of auto correlation on the 5%-level. These results are not reported here but can be found in the R-code. In the following analysis the standardised residuals defined by

$$\hat{\epsilon}_t = \frac{r_t - \hat{\mu}}{\hat{\sigma}_t} \tag{20}$$

will be used.

Table 3 reports the Pearson correlations between the index returns.

	DAX	CAC	FTSE	S&P
DAX	1	0.87 [0.87]	0.77 [0.80]	0.54 [0.57]
CAC		1	0.81 [0.86]	0.52 [0.54]
FTSE			1	0.50 [0.52]
S&P				1

Table 3: Pearson correlations for the GARCH filtered index returns. Correlations for the unfiltered returns are given in square brackets.

The highest correlation is observed between DAX and CAC with 0..87, the lowest correlation between S&P and FTSE with 0.5. The correlations between European markets are higher than the correlations with the American S&P. GARCH-filtered returns exhibit lower correlations than unfiltered returns due to removed volatility effects. These findings are consistent with Campbell et al. [2008], who studied the same markets in the time period 1990 to 2005. In comparison to their results all correlations have increased by 0.1 to 0.2, showing that correlations between international markets have increased in the recent past. This could be due to a growing political and economic integration between markets worldwide. Most companies have expanded their foreign activities (exports and investments), so the influence of global factors on a domestic economy has increased [see also Longin and Solnik, 1995].

In the following part the Local Gaussian Correlation will be used to find asymmetries in the dependence structure of index pairs. We will restrict ourselves to looking at the diagonal from -3 to 3, because outside of this area there are only a few observations (e.g. 34 of 5026 standardised DAX returns with absolute value larger than 3). Restricting ourselves to the diagonal enables us to present Local Gaussian Correlation estimates and confidence bands in one plot. The bandwidths are chosen as 1, similar to the standard deviation of the standardised returns, which was recommended by Støve et al. [2014]. For an more advanced choice see Tjøstheim and Hufthammer [2013] and Støve et al. [2014]. Because of the GARCH filtering the standardised returns are now independently and identically distributed. With that assumption pointwise confidence intervals for the Local Gaussian Correlation can be constructed. Therefore 1000 bootstrap replicas will be used. The confidence intervals will contain 95% of the estimated Local Gaussian Correlation values at each point on the diagonal.The resulting plots for every index pair are presented in Figure 6.

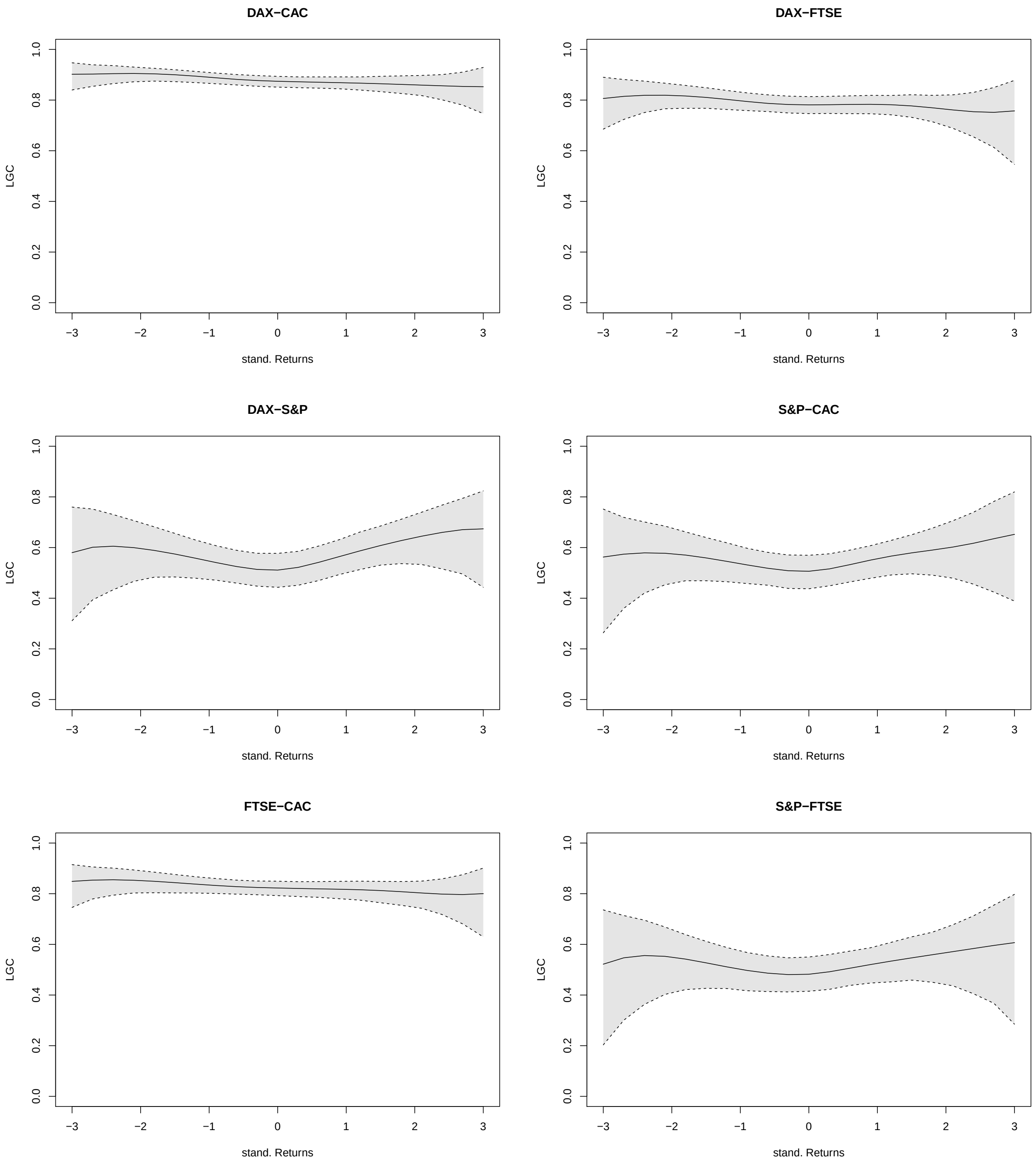

Figure 6: Local Gaussian Correlation on the diagonal with confidence intervals for standardised index returns.

The Local Gaussian Correlation curves between the European markets show a negative slope from the left tail to the right tail. So there is higher correlation for negative returns confirming that there is some asymmetry in dependence between bull and bear markets. The correlations are generally high and the bootstrapped confidence intervals are thin showing that there is not much uncertainty about the estimation. The combinations of the S&P 500 and one of the European markets show a differing LGC curve. The uncertainty shown in the confidence intervals is higher, especially in the tails. There seems to be higher tail dependence without much asymmetry between lower and upper tail. All curves are similar to the ones shown in Støve et al. [2012], who studied the same markets in the time span from 1987 to 2011.

A moving window is used to analyse the time variation in the Local Gaussian Correlation for the daily returns. Every window contains 500 observations, which correspond roughly to two years. Every ten days new standardised returns are obtained via a GARCH filter, for which the Local Gaussian Correlation and the Pearson correlation are calculated. This enables us to find time periods with differing correlation structure, e.g. particular high correlation in one of the tails in a certain time interval.

Figure 7 shows the time variation of the Local Gaussian Correlation for the S&P 500 and the DAX 30.

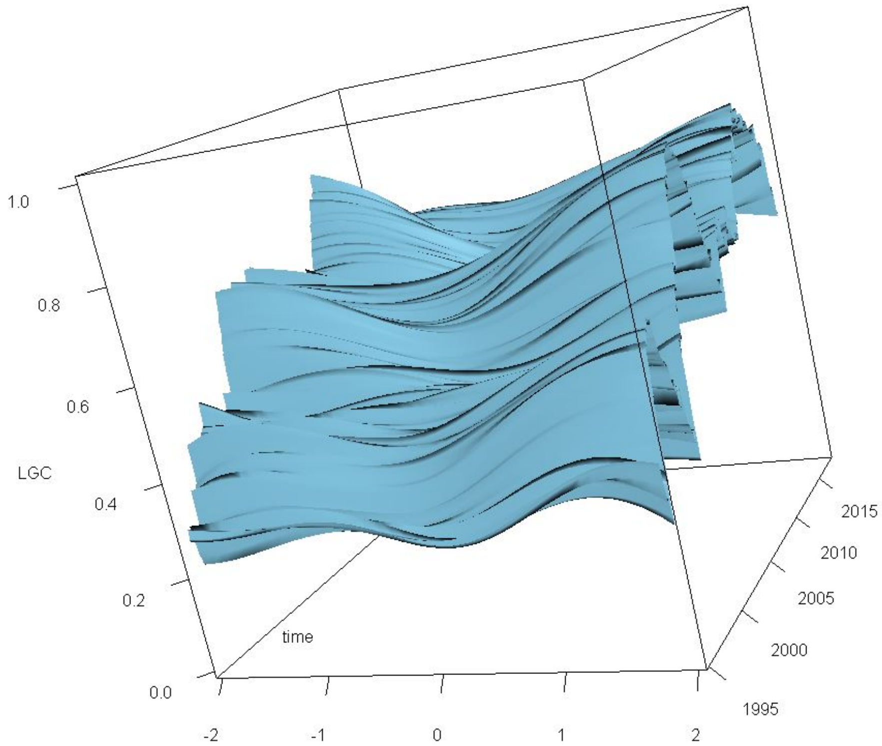

Figure 7: Time variation of the Local Gaussian Correlation between S&P and DAX using a moving window of 500 daily observations.

As already shown in Figure 6 high tail dependence appear in both tails and the level of correlation increases over time. Periods with particular high correlation can now be identified. From circa 2002 to 2007 high correlations appear for large positive returns, in the same period a bull market can be identified in Figure 4. The financial crisis of 2008 and 2009 can be recognised by high correlation in the lower tail for large negative returns. In the past five years another peak in the upper tail can be observed, but in the last two years correlations are once more decreasing.

Figure 8 shows the development of the correlation in the past 20 years by comparison of the ordinary Pearson correlation coefficient (referred to as global correlation) and the Local Gaussian Correlation in the lower tail (point (-2,-2) = large losses) and upper tail (point (2,2) = large gains) for all index pairs. The restriction to these two points enables us to directly compare the development of lower and upper tail dependence as well as the global correlation over time in one plot. As the Local Gaussian Correlation curves are mostly linear between -2 and 2 not much information is lost by the restriction to these two points.

Figure 8: Time variation of the Pearson correlation in comparison to the Local Gaussian Correlation for points (-2,-2) and (2,2) of the standardised returns.

Figure 8 confirms that correlations between markets are unstable over the last 20 years. This holds not only for global correlations but also for tail correlations. Hence predictability of correlations and thus risk estimations are affected. A positive time trend can be observed though there are some ups and downs. Correlation curves are very similar for the combinations of one of the European stock markets and the S&P 500 and among the European markets themselves. For the German and French market there are higher correlations in the lower tail than in the upper tail from 1995 to 2005, thereafter the asymmetry is smaller with all correlations approaching a very high level. Similar results are found for CAC-FTSE and DAX-FTSE. For all combinations with the S&P the volatility of the correlation is higher and there seems to be high tail dependence, especially in the upper tail for positive returns. With the beginning of the financial crisis in late 2007 correlations increase not only in the lower tail but also in the upper tail and globally. A possible explanation is given in Støve et al. [2012]. Even when the market trend is falling there are some days with high positive returns (compare with Figure 5). This "bear market rally effect" should be vanishing when looking at monthly returns. With the ending of the financial crisis in 2010 and 2011 correlations between markets are falling.

Furthermore we will analyse weekly (1049 observations) and monthly (242 observations) returns for the DAX 30 with S&P 500 or CAC 40 to analyse if there are any differences to the correlation in daily data for the same time span.

Table 4 shows summary statistics of the weekly and monthly index return data.

		Minimum	Mean	Maximum	Variance	Skewness	Kurtosis
DAX	weekly	-24.35%	0.18%	14.94%	11.56%	-0.92	6.22
CAC	weekly	-25.05%	0.09%	12.43%	9.37%	-0.77	5.42
S&P	weekly	-20.08%	0.16%	11.36%	6.88%	-0.90	6.37
DAX	monthly	-25.27%	0.77%	19.37%	43.63%	-0.84	2.05
CAC	monthly	-19.23%	0.42%	19.79%	31.57%	-0.45	0.97
S&P	monthly	-28.08%	0.68%	10.23%	23.02%	-1.46	5.78

Table 4: Summary statistics for index returns

As one would expect the variation in weekly and especially monthly returns is higher than in daily returns. Variances, means and absolute values of minimum and maximum returns are much higher than for daily returns (compare to Table 1). The skewness of the data assumes larger negative values indicating that the distribution of the returns is not symmetric.

The Local Gaussian Correlation on the diagonal for weekly and monthly returns is presented in Figure 9 with bootstrapped 95% confidence intervals (n=1000).

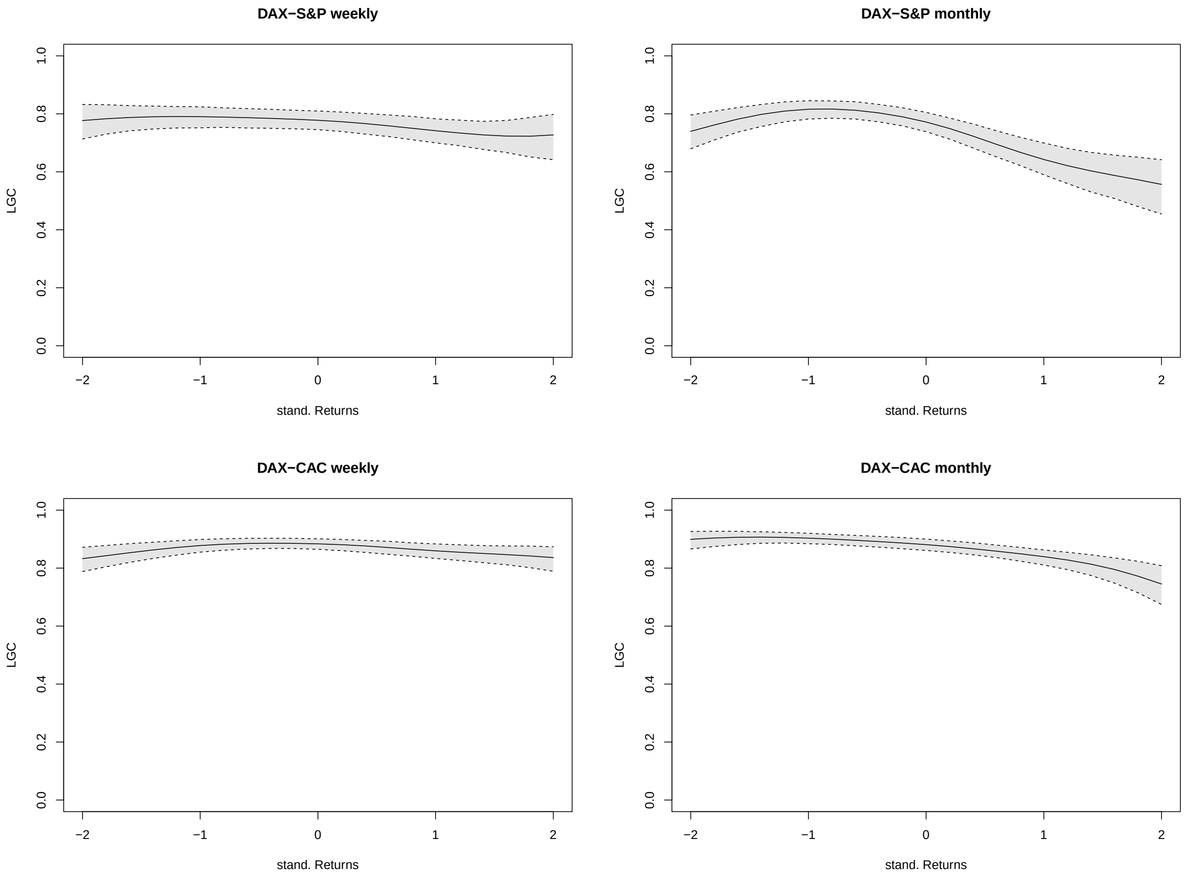

Figure 9: Local Gaussian Correlation for weekly (left plot) and monthly (right plot) index returns.

The correlations for weekly and monthly returns between DAX and S&P are higher than for daily returns (0.76 for monthly returns compared to 0.54 for daily returns) showing that the overall market trend is more similar between international markets than the daily movements. A possible reason for that could be the different opening hours of markets in Europe and USA due to the time shift, which may affect daily correlations. For monthly returns an asymmetry between the levels of correlation in the tails can be observed. The correlations are much higher in the lower tail than in the upper tail. These results are consistent with Støve et al. [2012] and confirm that the "bear market rally" effect has vanished.

In the following analysis we will use copulas to model the dependence structure. First the GARCH residuals will be transformed to the [0,1] interval by building pseudo observations. The definition is given in Equation 11.

For all index combinations the copula parameter(s) will be estimated. The goodness of fit based on maximised log likelihood estimates is presented in Table 5.

index-pair	observations	Normal	t	BB1	BB7
DAX-S&P	daily	-1744	-1827	-1820	-1796
	weekly	-745	-799	-789	-762
	monthly	-150	-162	-170	-167
DAX-CAC	daily	-7141	-7552	-7474	-7198
	weekly	-1386	-1422	-1397	-1319
	monthly	-332	-337	-340	-328
DAX-FTSE	daily	-4583	-4772	-4708	-4539
CAC-FTSE	daily	-5308	-5650	-5563	-5361
CAC-S&P	daily	-1555	-1665	-1646	-1620
FTSE-S&P	daily	-1417	-1487	-1477	-1453

Table 5: AIC of Normal, t, BB1 and BB7 copulas for estimated parameters using the full sample (1995-2015) for different index pairs with daily, weekly or monthly observations.

The t copula fits the data best for all daily and weekly index return combinations. The BB1 copula comes second, the Normal and BB7 copula provide a worse fit. A possible explanation for the superior fit of the t copula in comparison to the Normal copula is the more frequent occurrence of extreme values in financial return data (even then filtered for volatility clustering) than expected by a Normal distribution. This can be better modeled by the t distribution, which allows more tail dependence. For monthly returns the BB1 copula fits best. Moreover this is the only combination, where the correlation in the lower tail is much higher than in the upper tail. The BB1 copula can model asymmetric dependence and might be therefore an appropriate model for the monthly data. To visualise the fit of the copulas we will use the canonical Local Gaussian Correlation (see Section 3) for some examples, see Figure 10.

The empirical Local Gaussian Correlation is estimated on Gaussian pseudo observations

$$\hat{Z}_i = \Phi^{-1}(\hat{U}_i), \tag{21}$$

so that $\hat{Z}$ can be interpreted as a sample from the copula $C(\Phi(z_1), \Phi(z_2))$, for which the canonical Local Gaussian Correlation was defined.

In terms of canonical Local Gaussian Correlation all copulas fail to reproduce the empirical Local Gaussian Correlation of the data. A possible reason for that could be the estimation of the copula parameters using the full sample. But with the canonical Local Gaussian Correlation we restrict ourselves on the diagonal. So we can not make a point about the fit of the copulas outside of the diagonal. For the U-shaped correlation pattern between daily DAX and S&P returns the t copula, the BB1 and especially the BB7 copula assign to large correlation in the outer tails of the distribution, while the fit from [-1,-1] to [1,1] seems to be good. On the other hand the Normal copula is not flexible enough to capture the tail dependence and estimate too small values for the correlation in the tails. This could be a problem, because for large losses the correlation between financial returns is underestimated, so the diversification effect is

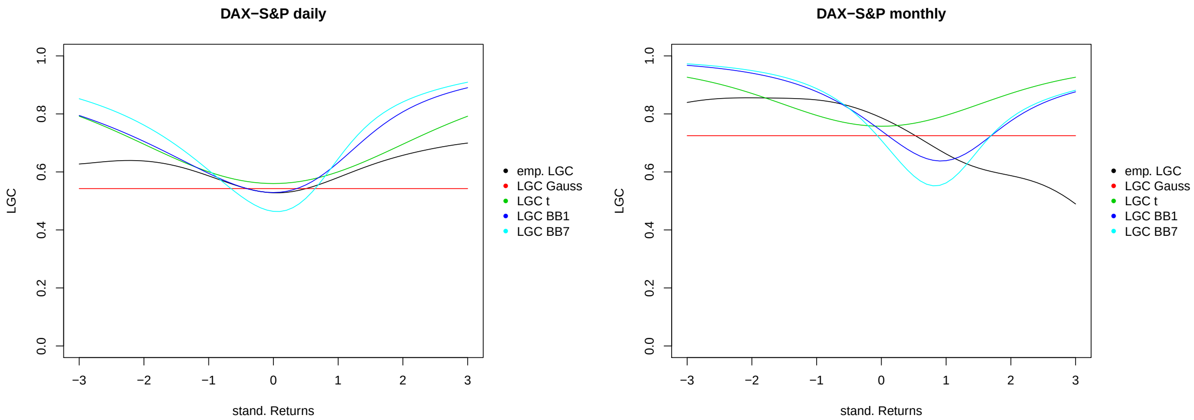

Figure 10: Local Gaussian Correlation and canonical Local Gaussian Correlation estimates

overestimated.

For the monthly data the t copula provides the best fit in the lower tail, but it overestimates the correlation in the upper tail. Similar results for the BB copulas. The Normal copula underestimates the correlation in the lower tail and overestimates the correlation in the upper tail and is therefore a poor choice to model the dependence in the data because it underestimates the risk. Hence one of the other copulas may be a better choice for risk averse investors.

The copula goodness-of-fit test presented in Section 4 confirms that for daily returns of DAX and S&P the t copula provides a good statistical fit. For 500 bootstrap iterations the t, BB1 and BB7 copula could not be rejected on a 5% significance level. The obtained p-value for the t copula is 0.948, the p-value for the BB1 copula is 0.78 and for the BB7 copula 0.552. In contrast the Normal copula is rejected (p-value=0).
For monthly returns between DAX and S&P, where there is high asymmetry in Local Gaussian Correlations, test results based on 500 iterations are a bit different. The Normal copula is still strongly rejected (p=0.008), so is the t copula (p=0.002), whereas BB1 (p=0.52) and BB7 (p=0.238) could not be rejected on a 5% level. The results seem to be reasonable, as the BB copulas were superior in terms of the AIC to Normal and t copula (compare Table 5). To test the reliabilty of the test procedure the test was repeated with another 500 bootstrap iterations. The obtained p values are: Normal copula: 0.01, t copula: 0.004, BB1 copula: 0.496, BB7 copula: 0.238. Because the p-values do not differ much, the test seems to be reliable.

5.2 DAX and its components

Next the Local Gaussian Correlation between stocks and the stock market will be analysed. Figure 11, Figure 12, Figure 13 and Figure 14 show the Local Gaussian Correlation between GARCH filtered daily returns of 12 single German stocks and the DAX 30 in the time period 2000-2015 (a total of 3842 observations). On the right side the Local Gaussian Correlation is given for monthly returns in the same time period (161 observations). 95% confidence bands are estimated via bootstrap (n=1000).

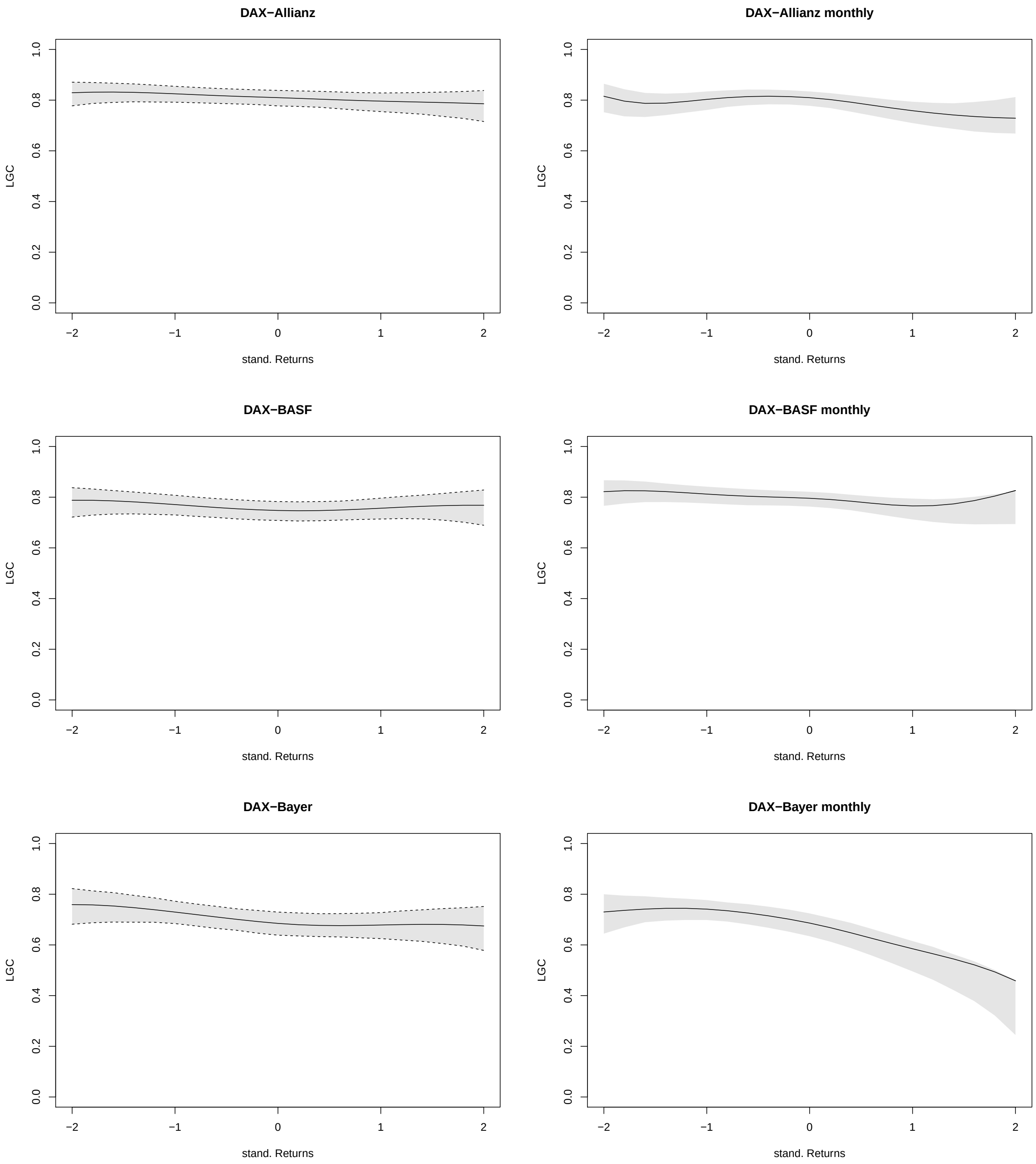

Figure 11: Local Gaussian Correlation for German stocks to index from 2000-2015. Left: Daily Returns, right: monthly returns

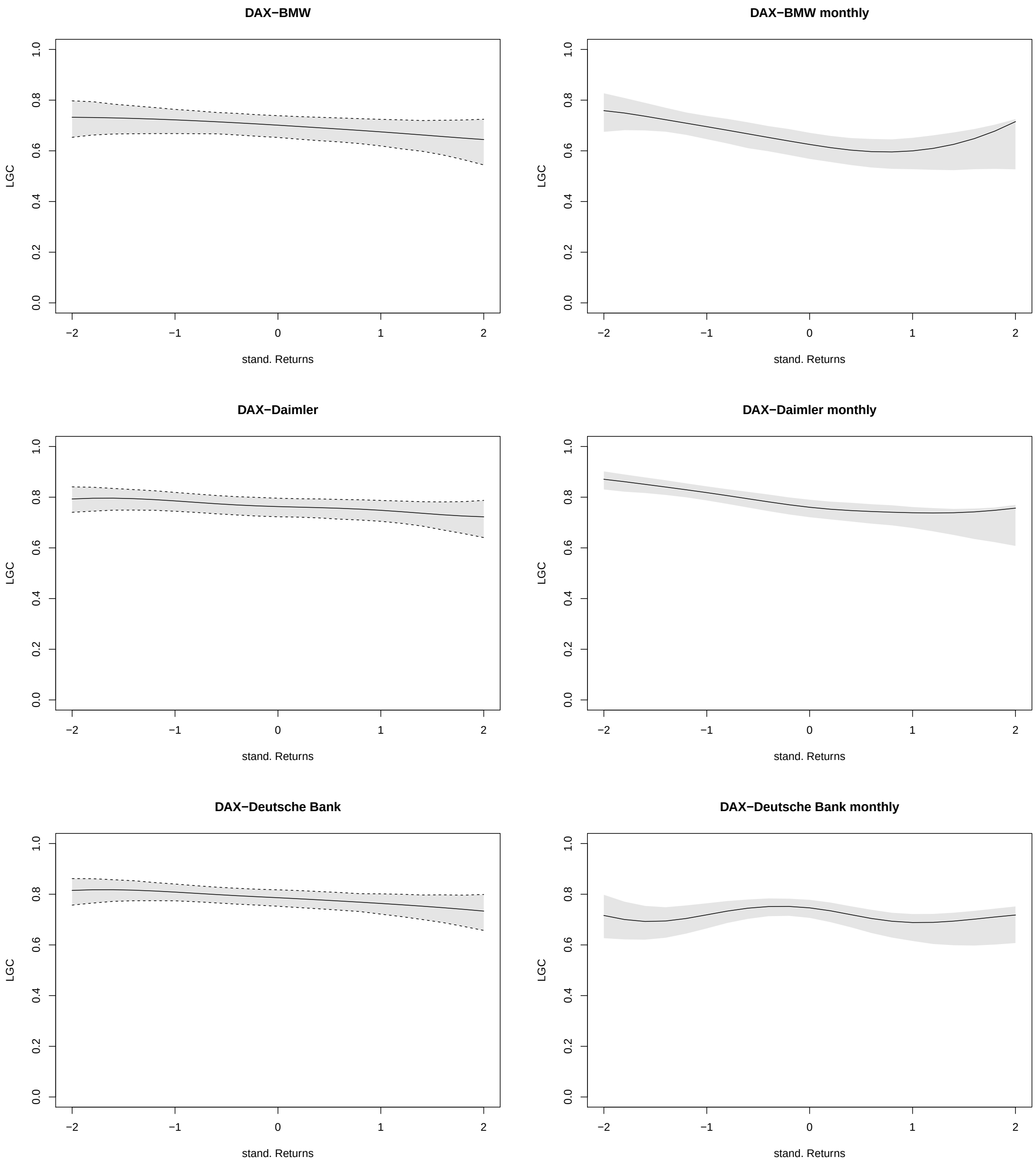

Figure 12: Local Gaussian Correlation for German stocks to index from 2000-2015. Left: Daily Returns, right: monthly returns

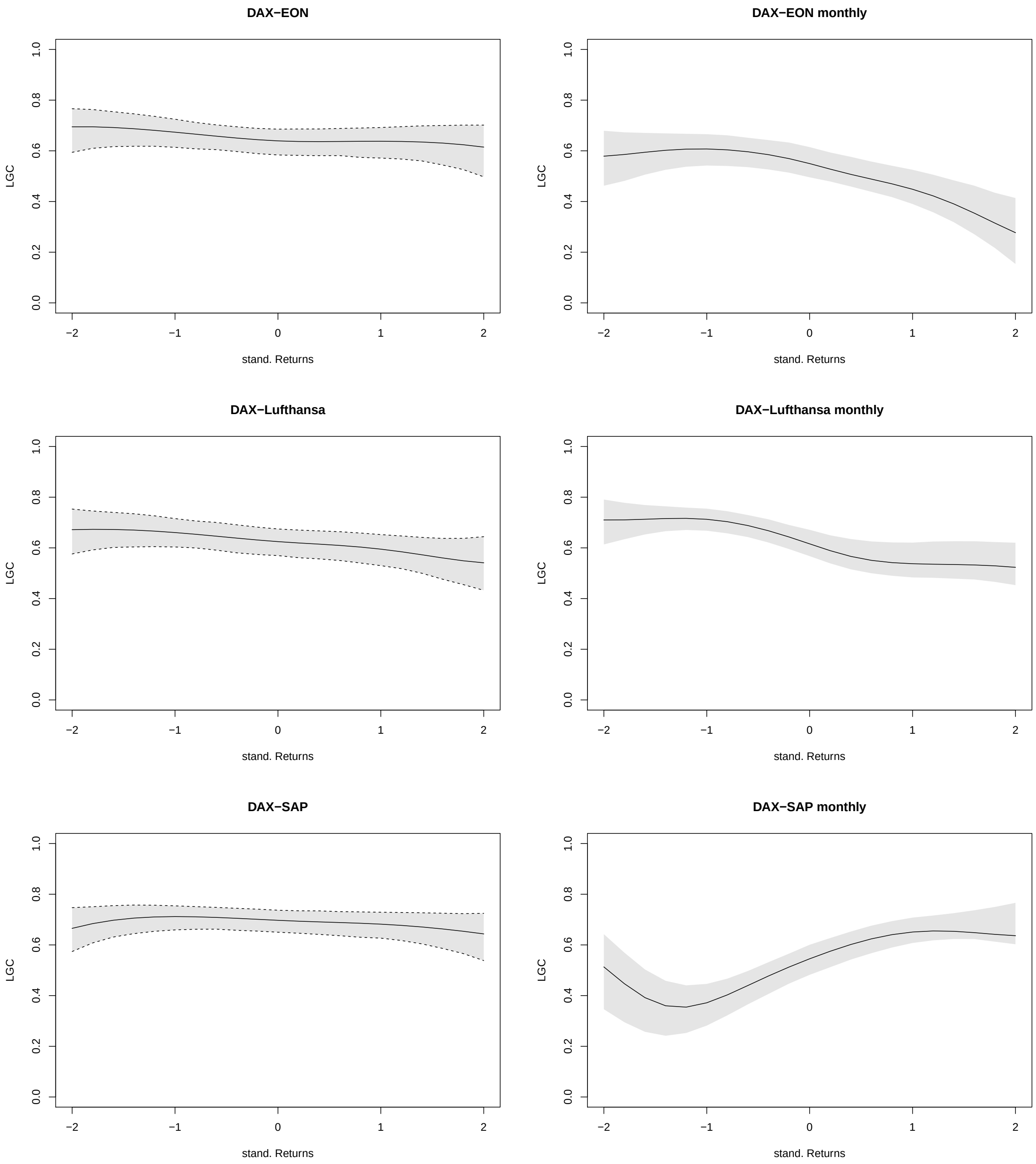

Figure 13: Local Gaussian Correlation for German stocks to index from 2000-2015. Left: Daily Returns, right: monthly returns

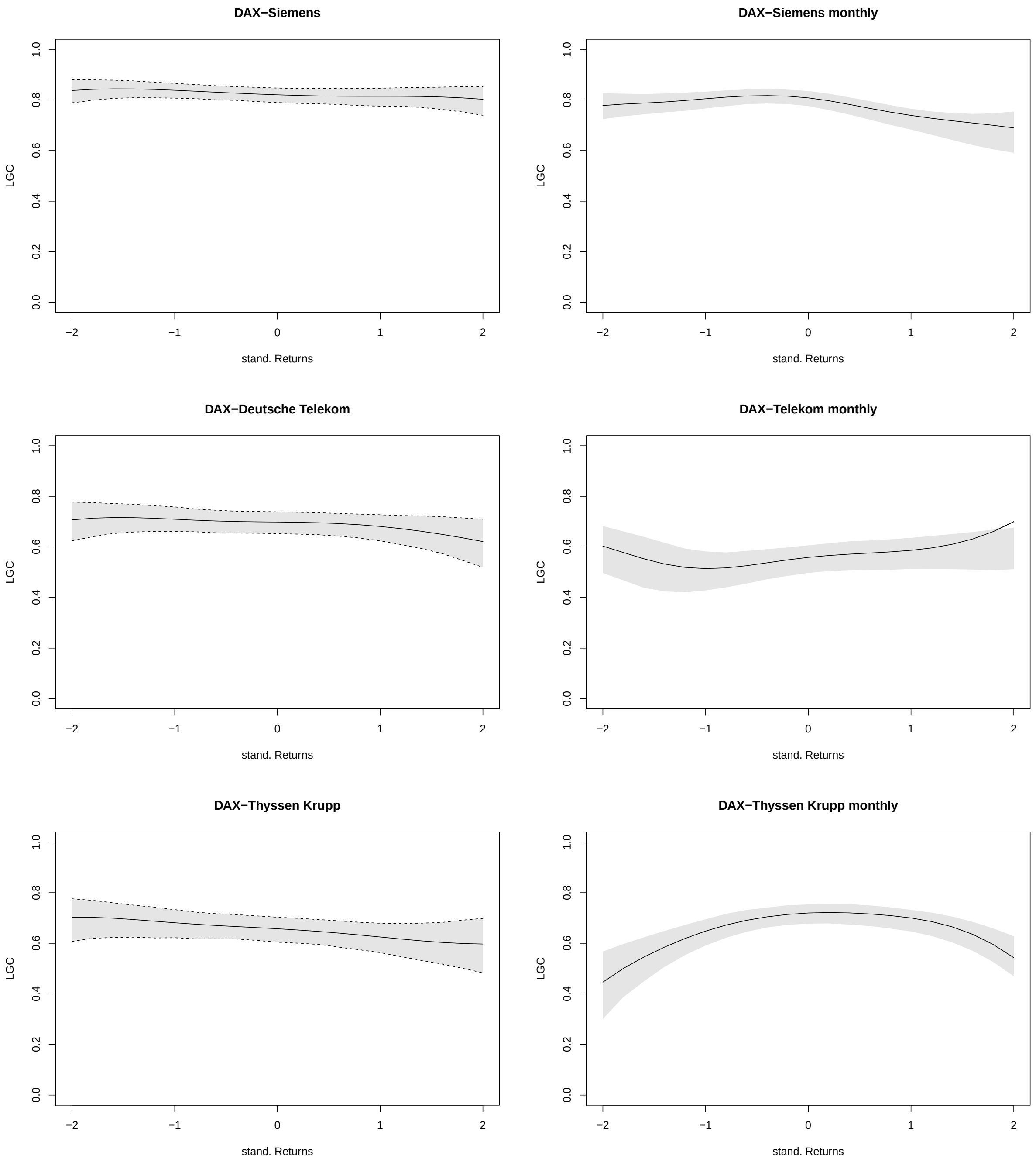

Figure 14: Local Gaussian Correlation for German stocks to index from 2000-2015. Left: Daily Returns, right: monthly returns

Correlations between daily DAX 30 and stocks returns vary between 0.5 and 0.8, the highest correlation is observed in DAX-Allianz with 0.78, the lowest correlation in DAX-EON with 0.52. The uncertainty expressed in bootstrapped (1000 replicas) 95% confidence intervals is low. For all pairs the Local Gaussian Correlation curves for daily returns are similar. They are nearly linear with negative slope, so there is higher correlation in the lower tail (= negative returns). An exception is the combination DAX-SAP with nearly constant local correlations. For monthly returns the local correlations differ. For DAX-Allianz and DAX-BASF there is no great difference to the correlations in daily returns, but many of the other pairs exhibit noticeable differences to the correlations in daily returns. Some pairs show higher correlation in the lower tail (large losses), so the stocks follow the index movement especially strong in a crisis period with declining monthly prices. Examples for that are BMW, Daimler and Lufthansa. EON, Bayer and Siemens show no increase in lower tail correlations but much smaller upper tail correlations compared to the daily data, hence these stocks are not anticipating bull markets as much as expected when looking at daily data. SAP and Thyssen Krupp behave relatively exceptional. Thyssen Krupp exhibits low tail correlations, while SAP show a minimum near (-1,-1) and before and after higher correlations. These correlation patterns can not be observed for daily returns.

Next we are interested in the goodness of fit for several copulas. An AIC-based comparison is given in Table 6.

index	stock		Normal	t	BB1	BB7
DAX	Allianz	daily	-3740	-4008	-3910	-3734
		monthly	-156.9	-164.3	-165.5	-160.1
	BASF	daily	-2967	-3236	-3152	-3047
		monthly	-161.7	-162.4	-167.3	-167.3
	Bayer	daily	-2220	-2458	-2386	-2315
		monthly	-99.1	-97.7	-110.2	-109.7
	BMW	daily	-2282	-2444	-2392	-2307
		monthly	-101.7	-100.7	-106.8	-108.5
	Daimler	daily	-3086	-3255	-3194	-3067
		monthly	-152.6	-153.3	-158.0	-156.7
	Deutsche Bank	daily	-3390	-3573	-3506	-3359
		monthly	-110.0	-112.4	-112.9	-110.9
	EON	daily	-1741	-1922	-1856	-1791
		monthly	-36.2	-46.8	-45.2	-44.9
	Lufthansa	daily	-1639	-1764	-1714	-1654
		monthly	-75.5	-78.3	-86.5	-86.3
	SAP	daily	-2205	-2327	-2259	-2153
		monthly	-59.6	-64.1	-62.9	-62.7
	Siemens	daily	-3938	-4276	-4155	-3969
		monthly	-143.3	-156.8	-155.2	-150.2
	Deutsche Telekom	daily	-2251	-2344	-2272	-2166
		monthly	-56.0	-69.8	-65.6	-66.9
	Thyssen Krupp	daily	-2006	-2071	-2031	-1957
		monthly	-100.9	-99.4	-94.9	-89.9

Table 6: AIC of Normal, t, BB1 and BB7 copulas on daily and monthly index to stock returns.

In terms of AIC the t copula fits best for all pairs with daily returns, which might be explained by the small decrease in local correlations from the left to the right tail. The BB1 copula fits second best, Normal and BB7 copulas fit worse.

For monthly data results differ. The BB1 copula fits best for pairs DAX with one of Allianz, BASF, Bayer, Daimler, Deutsche Bank and Lufthansa and the t copula fits best for EON, SAP, Siemens and Deutsche Telekom. An explanation for this is the stronger asymmetry in monthly return correlations. The BB7 copula fits best for DAX-BMW and the Normal copula only for DAX-Thyssen Krupp. For DAX-Thyssen Krupp we had observed a rather untypical Local Gaussian Correlation curve in Figure 14 with low tail correlations.

Furthermore we apply the goodness of fit test to the stock-index combinations. Table 7 shows the approximated p-values for 500 test iterations for the monthly return data from 2000 to 2015.

index	stock		Normal	t	BB1	BB7
DAX	Allianz	daily	0	0	0.27	0
		monthly	0.194	0.12	0.412	0.248
	BASF	daily	0	0.43	0.8	0.06
		monthly	0.328	0.248	0.48	0.226
	Bayer	daily	0	0.08	0.73	0.11
		monthly	0.016	0.004	0.766	0.53
	BMW	daily	0	0	0.16	0
		monthly	0.094	0.118	0.642	0.492
	Daimler	daily	0	0.01	0.21	0
		monthly	0.218	0.212	0.668	0.386
	Deutsche Bank	daily	0	0	0.18	0
		monthly	0.344	0.172	0.482	0.242
	EON	daily	0	0.09	0.48	0
		monthly	0.07	0.124	0.822	0.66
	Lufthansa	daily	0	0	0.07	0
		monthly	0.062	0.078	0.944	0.84
	SAP	daily	0	0.03	0.09	0
		monthly	0.346	0.546	0.68	0.596
	Siemens	daily	0	0.04	0.46	0
		monthly	0.12	0.114	0.524	0.258
	Telekom	daily	0	0.01	0.04	0
		monthly	0.19	0.762	0.848	0.766
	Thyssen Krupp	daily	0	0	0.1	0
		monthly	0.874	0.768	0.398	0.152

Table 7: p values of a Goodness-of-fit test reported for Normal, t, BB1 and BB7 copulas using the daily or monthly stock returns. For monthly data 500 iterations were used, for daily data only 100 because of higher computation time.

Results for daily and monthly returns differ widely. While for daily data most copulas are rejected on a a 5% significance level, with monthly data none of the copulas can be rejected. The BB1 copula is not rejected for daily and monthly data except the combination DAX-Telekom and may be therefore useful to model asymmetric dependence in financial return data. To test the reliability the test procedure was repeated with 5000 iterations for the pair DAX-EON using monthly data. The obtained p values are very similar (Normal: 0.0732, t: 0.1252, BB1: 0.8176, BB7: 0.6498), so it can be concluded that results are reliable.

Finally we want to investigate the time variation of the Local Gaussian Correlation by building sub periods. These sub periods are obtained by a rolling window of 500 observations. The time variation of the Local Gaussian Correlation for the lower tail (-2,-2) and the upper tail (2,2) with global Pearson correlation is shown in Figure 15 for some stocks.

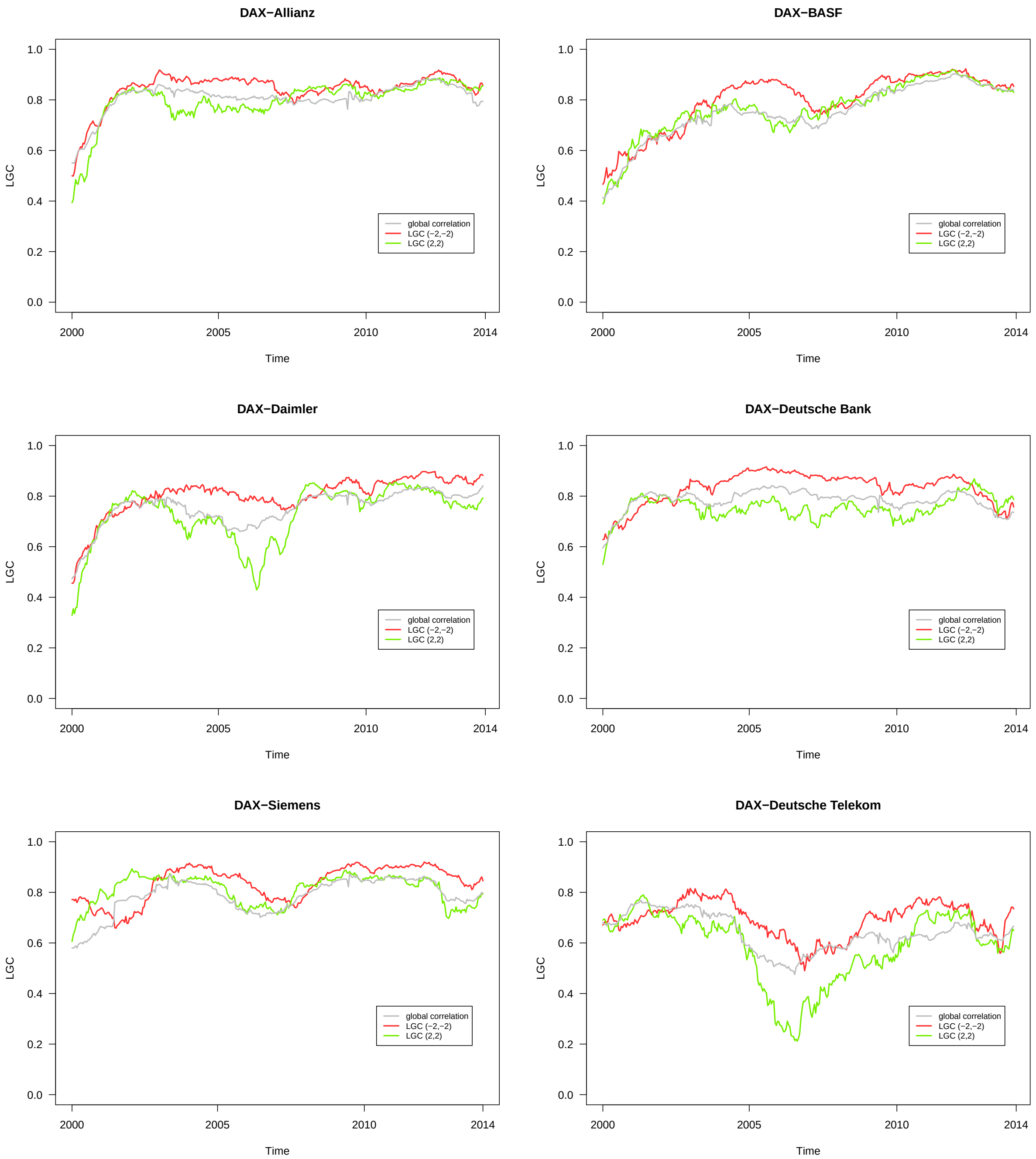

Figure 15: Time Variation of the Local Gaussian Correlation in the lower and upper tail and Pearson correlation for German stocks to index from 2000-2015 estimated by a rolling window of 500 daily observations.

Periods with high correlation in the lower tail can be easily identified in Figure 15. In the early 2000s the global correlation increases for most combinations. Thereafter it remains more or less constant on a high level. A high correlation in the upper tail can be identified from 2000-2003 for some of the stocks (e.g. Siemens). From 2004 to 2009 a period of mostly high correlations in the lower tail and low correlations in the upper tail can be observed. In the past five years correlations in the lower and upper tail are similar to the global correlation (e.g. BASF and Allianz) or there is still higher correlation in the lower tail (e.g. Siemens and Daimler).

6 Conclusions

In this Bachelor thesis dependence between financial returns has been investigated using GARCH models, Local Gaussian Correlation and copulas. When looking at a moving window of time intervals correlations are found to be unstable with a positive time trend, so financial returns have become more interlinked in the last 20 years.

Furthermore this thesis confirms that there is usually higher correlation for negative returns, which may affect the risks of portfolios. The typical Local Gaussian Correlation curve for daily return data is nearly linear with a slight negative slope. Between European stock markets we observe particularly high correlations, which have increased in the last 20 years. Local correlations for negative returns are higher than correlations for returns close to 0 or positive returns showing that there is some asymmetry in dependence. For a combination of a European market and the American S&P index high correlations appear in both lower and upper tail when looking at daily returns. For monthly returns however the local correlations for negative returns are higher whilst local correlations for positive returns are constant or lower. Also for most combinations of German stocks to the DAX index there were higher correlations in the lower tail and lower correlations in the upper tail for monthly returns in comparison to daily returns. These results show that the change in prices of stock markets is more similar in a market crash than in a calm market period with rising prices. The stocks follow the monthly market trend stronger than daily movements in prices.

In terms of models, which are appropriate for the observed dependence structures, different copulas were tested with the financial data. For daily returns the t copula fits best, while the Normal copula can be rejected. For monthly data with stronger lower tail dependence in addition to the t copula the BB1 copula should be considered because it is more flexible as it allows asymmetric lower and upper tail dependence. The canonical Local Gaussian Correlation can be used to find and visualise differences between the local correlations implied by a copula and the empirical correlation of the data.

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Erklärung der Urheberschaft

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