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Masterarbeit, Sommersemester 2016

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https://doi.org/10.5282/ubm/epub.31967

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Alternative Approaches to Regulatory Risk Calibrations in Solvency II

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MASTER THESIS

July 2016

Declaration of Authorship

I hereby confirm that I have authored this master thesis independently and without use of others than the indicated resources.

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Abstract

The directive Solvency II harmonizes the insurance regulation in the European Union. A central aspect of the regulatory framework is the calibration of the Solvency Capital Requirement (SCR) as a capital buffer insurers and reinsurers are required to hold. The SCR can be calculated based on a regulatory model. The so called standard formula follows a modular approach where capital requirements for sub-risk elements are calculated and subsequently aggregated. Thus, the standard formula requires two input parameters – capital requirements of risk elements and dependence measures between them. This thesis examines the scenario-based approach to calibrate those input parameters for property risk and equity risk, two sub-modules of the risk module market risk. The regulatory scenario-based approach follows calibrations based on historical market data and includes the application of rolling-window annualization. Unfortunately, this methodology causes severe distortions in calibration results for both input parameters. In this thesis alternative approaches to regulatory risk calibrations are examined. This includes the application of Filtered Historical Simulation (FHS) to gather a sufficient amount of historical data. Outcomes of empirical calculations based on FHS annual returns imply, that the regulatory calibration approach is exposed to overestimate equity as well as property risk while diversification effects might be underestimated. Additionally, alternative approaches to determine the correlation between risk elements are examined. This also includes the calibration of correlation coefficients on a more granular level. Empirical investigations show, that the assumption of perfect correlation within the equity category "other equity" cannot be verified. Diversification effects might mistakenly be neglected. For property risk, the decision of no further breakdown into sub-categories can be supported. Lastly, the choice of Value-at-Risk (VaR) as appropriate risk measure for Solvency II risk calibrations is discussed since authorities of other regulatory frameworks like Solvency II's Swiss equivalent Swiss Solvency Test (SST) decided differently.

Acknowledgements

First and foremost I would like to thank my supervisor Prof. Stefan Mittnik for his exceptional support and his continuous advice throughout the course of this thesis. His expertise and his ideas were of great value for my research.

Secondly, I would like to thank Joseph Reiter who patiently answered my questions and supplied me with valuable literature and data.

I am thankful to Stefan Pilz for his valuable input. I also would like to thank my colleagues at Munich Re for giving me the time and space to finish this thesis.

Finally, I would like to thank my parents who not only supported me during the creation of this thesis but also throughout the whole of my studies.

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Abbreviations

AIC	**A**kaike **I**nformation **C**riteria
ARCH	**A**uto **R**egressive **C**onditional **H**eteroskedasticity
ARMA	**A**uto **R**egressive **M**oving **A**verage
BIC	**B**ayesian **I**nformation **C**riteria
BIS	**B**ank for **I**nternational **S**ettlements
BSCR	**B**asic **S**olvency **C**apital **R**equirement
CI	**C**onfidence **I**nterval
CEIOPS	**C**ommittee of **E**uropean **I**nsurance and **O**ccupational **P**ensions **S**upervisors
CVaR	**C**onditional **V**alue **a**t **R**isk
DM	**D**eveloped **M**arket
EC	**E**uropean **C**ommission
EEA	**E**uropean **E**conomic **A**rea
EIOPA	**E**uropean **I**nsurance **O**ccupational **P**ensions **A**uthority
EIOPC	**I**nsurance and **O**ccupational **P**ensions **C**ommittee
ES	**E**xpected **S**hortfall
EU	**E**uropean **U**nion
EC	**E**uropean **C**ommission
FHS	**F**iltered **H**istorical **S**imulation
FOPI	**F**ederal **O**ffice of **P**rivate **I**nsurance
GARCH	**G**eneralized **A**uto **R**egressive **C**onditional **H**eteroscedasticity
HFS	**H**edge **F**und re**S**earch
HS	**H**istorical **S**imulation

iGARCH	**i**ntegrated **G**eneralized **A**uto **R**egressive **C**onditional **H**eteroscedasticity
IPD	**I**nvestment **P**roperty **D**atabank
MCR	**M**inimum **C**apital **R**equirement
MSCI	**M**orgen **S**tanlay **C**apital **I**nternational
NRI	**N**et **R**eturn **I**ndex
OECD	**O**rganisation for **E**conomic **C**ooperation and **D**evelopment
QIS	**Q**uantitative **I**mpact **S**tudy
PI	**P**rice **I**ndex
SCR	**S**olvency **C**apital **R**equirement
SST	**S**wiss **S**olvency **T**est
TRI	**T**otal **R**eturn **I**ndex
TVaR	**T**ail **V**alue **a**t **R**isk
UK	**U**nited **K**ingdom
VaR	**V**alue **at** **R**isk
FINMA	**FIN**ancial **M**arket **A**uthority

Chapter 1

Introduction

1.1 Motivation

Solvency II is a supervisory framework for the insurance and reinsurance sector with the purpose to reform and to harmonize insurance regulation throughout the European Union (EU). The new directive is a project of the European Commission which was initiated decades ago and became applicable as of the 1st of January 2016.
One of Solvency II's main objectives is to ensure insurance and reinsurance companies are holding sufficient economic capital to fulfill their responsibilities towards policyholders and beneficiaries. This capital buffer is referred to as Solvency Capital Requirement (SCR). A fundamental innovation of Solvency II in comparison to former legacies, often referred to as Solvency I, is the risk-based calculation of capital requirements. This implies that the calibration of the SCR is tailored to an insurer's or reinsurer's individual risk structure [European Commission, 2007]. By definition, the SCR covers unexpected losses which will not be exceeded with a probability of 99.5% and an assumed holding period of 12 month. For the calibration of the SCR, insurance and reinsurance companies can either make use of a formula provided by the regulator, develop their own internal model, or use a mixture of both. The regulatory *standard formula* has a modular structure. Risks insurance or reinsurance undertakings might be exposed to, are categorized into risk modules which themselves comprise sub-risk elements. The overall SCR is calculated by the calibration of capital requirements for those sub-risks and the subsequent step-by-step aggregation to the highest level.
In 2015 more than three quarters of all insurance and reinsurance undertakings under Solvency II indicated their intention to base SCR calculations on the standard formula – either completely or at least partially [KPMG, 2015]. Hence, a

flawless calibration of the standard formula's input parameters is of great importance to ensure sound risk management in the insurance and reinsurance sector throughout the European Union.
During the development of Solvency II, a lot of time, effort and knowledge was put into the design of the standard formula and the calibration of its input parameters. Even though the final implementation of the Solvency II framework already took place, this process is not concluded yet. The concept of Solvency II is designed as a lively framework open for development and improvement rather than a rigid legislation which is supposed to be applicable in its original form for the following decades. To ensure resilient risk management, regulatory frameworks in general need to be able to respond to economic developments in the concerning market. Moreover, the Solvency II framework and specifically the standard formula might still contain inconsistencies which require particular attention.
In the course of Solvency II's go-live in January 2016, Gabriel Bernardino, Chairman of the insurance regulation authority (EIOPA) stated the following on this matter:

> *"Now with Solvency II a modern, robust and proportionate supervisory regime will be implemented. This is a huge step forward for enhanced policyholder protection and the single European insurance market. However, this is not a time for complacency" [EIOPA, 2016b].*

In fact, the insurance regulation authority EIOPA already scheduled additional work on Solvency II for the time after its implementation. From regulatory side it is declared that future work on Solvency II will specifically involve the review of the SCR standard formula [EIOPA, 2016a].
The developments in the emergence of Solvency II have opened up discussions in the academic world, too. A wide range of academic publications is concerned with a variety of challenges and drawbacks concerning the regulatory framework its calibrations. Some of them even contributed to the decisions and developments on the regulatory side. On behalf of the role of academic research in the development of regulatory frameworks Paul Embrechts stated that

> *"[...] academia has a crucial role to play in commenting officially on proposed changes in the regulatory landscape. Second, when well-documented, properly researched and effectively communicated, we may have an influence on regulatory and industry practice"*[Embrechts et al., 2014, p. 2].

1.2 Subjects and Aims

This thesis is concerned with calibration approaches, limitations and possible alternatives regarding regulatory risk calibrations in Solvency II. Here, the focus is on two specific risk elements – *equity risk* and *property risk*. Both are sub-risks of the risk module market risk. Basically, the intentions behind this thesis can be divided into three blocks.

1. Review of regulatory standard formula input parameter calibrations.
2. Outline of limitations regarding regulatory risk calibrations.
3. Introduction and analysis of alternative approaches.

The calculation of capital requirements occupies an important part of Solvency II. The calibration of standard formula parameters is challenging for most of the risks, an insurance or reinsurance company may face. The purpose of the first block is to better understand the regulator's approach to calibrate standard formula input parameters specifically for the sub-risk modules equity and property. Methodological descriptions and empirical reproductions aim to bring clarity about obstacles the regulator had to overcome and decisions it had to take in consequence.
The second block comprises the outline of problems regarding SCR calibrations based on the standard formula. This part aims to highlight severe consequences of regulatory calibration procedures. This part of the thesis aims to reveal, how regulatory decisions and simplifications throughout the calibration process of standard formula input parameters alter resulting capital requirements.
The third purpose of this thesis is to introduce alternative calibration approaches which aim to avoid distortions the regulatory approach might cause. Empirical analysis shows differences in calibration outcomes between the regulatory approach and its alternatives.

1.3 Structure

In the second chapter basic statistical definitions are introduced. The focus is on terms and concepts in the field of financial risk management. This contains the introduction of certain risk measures, the concept of dependency and specific aspects of (financial) time series analysis. Also, the random sampling methodology bootstrapping is explained.
The third chapter addresses the long-term project Solvency II. First, a general description of the Solvency II directive and its additional components is given. In

the further course, the standard formula and is structure as well as the calibration of its input parameters is described. The last part of the chapter focuses on structure and calibration of the market risk module and specifically its sub-risk modules equity and property.
Chapter 4 is concerned with the stability of regulatory risk calibrations and possible alternatives to the regulatory approach. It provides the basis for the empirical analysis comprised by this thesis. First, general concerns are discussed. Subsequently, specific parts of regulatory risk calibrations are questioned and alternative approaches are introduced.
Chapter 5 contains the empirical part of this thesis. The first section introduces the data used for the analysis. The following sections comprise main findings of empirical investigations.
The final chapter summarizes the results and draws conclusions.
The appendix comprises additional concepts and proofs for the methodological parts of this thesis as well as relevant plots and tables complementing the empirical analyses.

Chapter 2

Basic Definitions

Chapter 2 introduces basic terms and concepts used in this thesis. The first section deals with the quantification of risk in general and introduces two important risk measures. The second part is concerned with the concept of dependence. Together, the two sections build the basis for the calibration of standard formula input parameters. The third part of this chapter discusses selected areas of (financial) time series analysis. The last section introduces the random sampling methodology bootstrapping. Section 3 and Section 4 are important for the understanding of specific problems concerning regulatory calibration approaches as well as the methodology of alternative approaches discussed in this paper.

2.1 Risk Measures

A central aspect of risk management is given by the quantification of risk. This is accomplished by risk measures which have, among others, the purpose to determine the amount of economic capital a financial institution is required to hold against unexpected losses.
In a mathematical context, a future value of any financial position is represented by the random variable X. Comparably, $L = -X$ refers to the random variable representing the financial position's future loss. F_L denotes the corresponding cumulative distribution function, also referred to as loss distribution. A risk measure is then given by the mapping ρ(L) of the random variable L[1] to a real number.
Based on [McNeil et al., 2005], two important and well known risk measures are introduced in the following.

[1]Note that the formal definitions given in this chapter are based on loss distributions. For the empirical part of this thesis, risk measures are largely calibrated based on the historical distribution of actual returns instead of losses. This results in a change of signs but does not effect the understanding and interpretation of the risk measure in general.

Value-at-Risk
The Value-at-Risk (VaR) describes the loss which will not be exceeded given a certain probability $(1-\alpha)$. α is also referred to as confidence level. Formally, VaR is given by

$$\begin{aligned}\rho(L) = VaR_\alpha(L) = inf\{l \in \mathbb{R}: &\quad P(L > l) \leq 1-\alpha\} \\ = inf\{l \in \mathbb{R}: &\quad F_L(l) \geq \alpha\}.\end{aligned} \tag{2.1}$$

VaR_α is therefore represented by the α-quantil of the loss distribution F_L.

Expected Shortfall
For a given confidence level α, the Expected Shortfall (ES) describes the expected value of the loss given the loss is higher than the corresponding value of VaR_α. That is,

$$\rho(L) = ES_\alpha = \mathbb{E}(L|L > VaR_\alpha). \tag{2.2}$$

2.2 Correlation and Dependency

In a statistical environment, dependency describes the relationship between two random variables. Several approaches are conceivable to determine the dependence – but not all of them are suitable in any situation. In this section, two different concepts will be introduced. Both are discussed in [Embrechts et al., 2003].

Linear Correlation
A well known method to measure the correlation between two random variables X and Y is the *(linear) correlation coefficient*, also known as *Pearson's* ρ. It is given by

$$\rho_{X,Y} = \frac{\mathbb{C}ov(X,Y)}{\sqrt{\mathbb{V}ar(X)\mathbb{V}ar(Y)}}. \tag{2.3}$$

$\rho_{X,Y}$ can equal values between [-1,1]. If $\rho_{X,Y}$ equals 0, the random variables X and Y are independent. Nonetheless, the converse does not hold in general. This is caused by a special property of linear correlation which solely covers the linear part of dependency. Another pitfall is that linear correlation coefficients are only defined for finite variances.

Tail Dependence
Tail dependence coefficients measure the strength of dependence in the tails of the bivariate distribution of a pair of random variables. They give an estimation about extremal dependence, meaning the likelihood of a coincidental appearance of extreme events. This is especially interesting for heavy-tailed distributions. Other than linear correlation, tail dependence does not depend on the marginal distribution of the two random variables but is rather based on their copula[2].
It can be distinguished between the *upper and the lower tail dependence coefficient.* With a given pair of continuous random variables X_1 and X_2 and their respective marginal distributions F_{X_1} and F_{X_2} the latter is given by

$$\begin{aligned}\lambda_l(X_1, X_2) &= \lim_{q \to 0^+} P(X_2 \leq F_{X_2}^{-1}(q) | X_1 \leq F_{X_1}^{-1}(q)) \\ &= \lim_{q \to 0^+} \frac{P(X_2 \leq F_{X_2}^{-1}(q), X_1 \leq F_{X_1}^{-1}(q))}{P(X_1 \leq F_{X_1}^{-1}(q))} \\ &= \lim_{q \to 0^+} \frac{C(q,q)}{q} \end{aligned} \tag{2.4}$$

with $C(q, q)$ denoting the copula of the bivariate distribution of the random variables X_1 and X_2 and provided the limit $\lambda_l \in [0, 1]$ exists. X_1 and X_2 exhibit lower tail dependence if $\lambda_l \in (0, 1]$. If $\lambda_l = 0$, X_1 and X_2 are said to be asymptotically independent.

2.3 Financial Time Series Analysis

Financial time series analysis is concerned with the development of assets over time. In this section, selected concepts of this area will be introduced.

Returns
In most cases, it is more convenient to investigate price changes instead of the prices itself. Those changes in price are referred to as returns. In general, it is distinguished between discrete and continuous returns. Discrete returns can be further divided into net returns and gross returns. In this thesis, we deal with discrete net returns. Given an asset price P_t at time t, the *k-period net return* is

[2]For a short introduction to copulas refer to Annex A.

given by

$$r_t^k = \frac{P_t - P_{t-k}}{P_{t-k}}. \tag{2.5}$$

k refers to the length of period for example one day, one week or one year [Tsay, 2005].

Stationarity

The analysis of time series is often based on certain assumptions. The concept of stationarity is basic in this context. It can be distinguished between strict and weak stationarity. Strict stationarity is a very strong condition and hard to verify whereas weak stationarity is a common and most often required assumption in time series modeling. In order to be weakly stationary, a time series r_t must fulfill

(a) $\mathbb{E}(r_t) = \mu \qquad \forall t$ and

(b) $\mathbb{C}ov(r_t, r_{t-l}) = \gamma_l$.

In other words, a time series r_t is weakly stationary if its mean μ is constant over time and the covariance between r_t and r_{t+l} only depends on the lag l between them rather then the values themselves.
Time series failing this property are often referred to as unit-root nonstationary time series. A well known example for such a time series is the *Random Walk*, that is

$$r_t = r_{t-1} + a_t \tag{2.6}$$

where a_t be a sequence of *i.i.d.* random variables with constant mean and variance, further referred to as *White Noise* [Tsay, 2005].

ARMA-GARCH Models

The collection of tools for the analysis of financial time series includes a large number of econometric models with different properties[3]. Examples are AutoRegressive Moving Average (ARMA) models and Generalized AutoRegressive Conditional Heteroscedastic (GARCH) models. The latter allows for volatile variances over time. Both modes can be combined to *ARMA(p,q)-GARCH(m,n)* models.

[3]A detailed explanation of various time series models can for example be found in [Tsay, 2005]

A time series r_t follows an *ARMA(p,q)-GARCH(m,n)* model if it satisfies

$$\begin{aligned} r_t &= \mu_t + a_t \\ &= \phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i} + \sum_{j=1}^{q} \psi_j a_{t-j} + a_t \end{aligned} \tag{2.7}$$

with

$$a_t = \sigma_t \epsilon_t, \qquad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2. \tag{2.8}$$

z_t is *i.i.d.* with constant variance 1. For the paramterts it holds $\alpha_0 > 0$, $\alpha_i \geq 1$ $\forall i = 1, \ldots m$, $\beta_j \geq 0$ $\forall j = 0, \ldots s$, $\phi_i \geq 0$ $\forall i = 0, \ldots p$, $\psi_j \geq 0$ $\forall j = 0, \ldots q$ and $\sum_{i=1}^{max(m,s)} (\alpha_i + \beta_i) < 1$. The last constraint ensures that the unconditional variance of the innovation a_t is finite while the conditional variance evolves over time [Tsay, 2005].

Information Criteria

Information criteria are used to estimate the order of an appropriate time series model. Given a set of time series models with different orders, the information criteria help to select one of them as an appropriate fit of the analyzed data. Two well known criteria will be briefly introduced in the following. Both are discussed in [Burnham and Anderson, 2004].

The *Akaike Information Criterion (AIC)* is a likelihood based criterion. The AIC aims to minimize the distance between the model and the true value while using as few parameters as possible on the same time. The criteria is given by

$$\text{AIC} = -2 \times \log \Big(L(\hat{\theta} | x_1, \ldots, x_T) \Big) + 2 \times K \tag{2.9}$$

with K denoting the number of parameters related to the order of the model. With $x_1, \ldots, x_T$ denoting the observed data and $\hat{\theta}$ denoting the vector of estimated parameters, $L(\hat{\theta}|x_1, \ldots, x_T)$ represents the likelihood function of $\hat{\theta}$. The first part of Equation 2.9 evaluates the distance between the model and the truth and therefore the goodness of fit of the model. The second part is referred to as the penalty function of the criterion since it penalizes the model for each additional parameter is uses.

The *Bayesian Information Criterion (BIC)* is closely related to the AIC. It can be derived by

$$\text{BIC} = -2 \times \ln \Big(L(\hat{\theta} | x_1, \ldots, x_T) \Big) + \ln(T) \times K \tag{2.10}$$

Equation 2.9 and equation2.10 show, that the definitions of AIC and BIC are similar to each other. However, in comparison to Akaike's criterion, the BIC considered the sample size T of the observed data $x_1, \ldots, x_T$ in the penalty term of the function.

2.4 Random Sampling: Bootstrapping

The resampling technique bootstrapping was introduced by [Efron, 1979]. Given a random sample, bootstrapping aims to estimate the unknown distribution of a prespecified random variable based on observed data. In other words, bootstrapping can be used to determine parameters without making any parametric assumptions about the distribution of the corresponding random variables.
Let $X = (X_1, X_2, \ldots, X_n)$ denote a random sample of size n with realizations $x = (x_1, x_2, \ldots x_n)$. Let the random variables follow an unspecified distribution F, such that

$$X_i = x_i, \qquad X_i \overset{i.i.d}{\sim} F \qquad i = 1, 2, \ldots, n.$$

Let $R(F, X)$ denote some prespecified random variable depending on X and F. Traditionally, $R(F, X)$ represents a parameter of interest, e.g. the mean or the Value-at-Risk of F. The goal of bootstrapping is to estimate the sampling distribution of the random variable $R(F, X)$. The bootstrap methodology works as follows

1. Construct a sample probability distribution $\hat{F}$ by assigning a probability of $\frac{1}{n}$ for each observed data point $x_1, x_2 \ldots x_n$.
2. Out of $\hat{F}$ draw a random sample with replacement, such that

   $$X_i^* = x_i^*, \qquad X_i^* \overset{i.i.d}{\sim} \hat{F} \qquad i = 1, 2, \ldots, n.$$

   The resulting sample is called bootstrap sample $X^* = (X_1^*, X_2^*, \ldots X_n^*)$ with realizations $x^* = (x_1^*, x_2^*, \ldots x_n^*)$.
3. Use the bootstrap sample approximate $R(F, X)$ by the bootstrap estimate $\theta = R(X^*, (\hat{F}))$.
4. Repeat step 2 and 3 T times to obtain T bootstrap samples $x^{*^1}, \ldots, x^{*^T}$ and T bootstrap estimates $\theta^1, \ldots \theta^T$.

Bootstrap confidence intervals

Step four results in T bootstrap estimates. Based on those, a bootstrap confidence interval can be derived as follows

1. Order $\theta^1, \ldots \theta^T$. from the smallest value to the largest: $\theta^{(1)}, \ldots \theta^{(T)}$.
2. Choose a confidence level α and find the $T(1-\frac{\alpha}{2})$ and $T(\frac{\alpha}{2})$ estimate. Those are referred to as $\theta_{C_{1-\frac{\alpha}{2}}}$ and $\theta_{C_{\frac{\alpha}{2}}}$.
3. $[\theta_{C_{1-\frac{\alpha}{2}}}, \theta_{C_{\frac{\alpha}{2}}}]$ is an $(1-\alpha)$ bootstrap confidence interval.

Chapter 3

Solvency II

3.1 Solvency II Directive

Solvency II is a project of the European Union (EU) with the purpose to reform the previous insurance supervision law, often referred to as Solvency I, and to enforce a harmonized EU insurance regulation. The legislation of Solvency II was implemented in several stages. The basis of the regulation forms the EU Directive 2009/138/EC adopted in November 2009. This Solvency II directive is assigned to Level 1. The development ot the Solvency II framework was technically advised by the European Insurance and Occupational Pensions Authority (EIOPA), former CEIOPS (Committee of European Insurance and Occupational Pensions Supervisions)[1]. Between 2005 and 2009, CEIOPS/EIOPA conducted five field tests known as Quantitative Impact Studies (QIS). Based on the outcomes of the five impact studies and as requested by the European Commission (EC), CEIOPS/EIOPA provided technical advise on implementing measures, known as Level 2 Advices. On a third level, CEIOPS/EIOPA provided supervisory guidelines and recommendations. The Solvency II directive came into force as of the 1st of January 2016. The structure of Solvency II follows a three-pillar approach. Each pillar covers a specific sector to assure sound risk management.

- **Pillar I** focuses on the quantitative topics. It specifies a Solvency Capital Requirement (SCR) and a Minimum Capital Requirement (MCR) to ensure the solvency of an insurance or reinsurance undertaking.
- **Pillar II** is concerned with qualitative requirements in insurance and reinsurance companies. This part of Solvency II aims to assure the application,

[1]In the following, we will not distinguish between CEIOPS and EIOPA and will refer to both as CEIOPS/EIOPA

maintenance, and regulation of an efficient risk management system within the companies.

- **Pillar III** deals with the transparency of companies in the insurance and reinsurance sector. It contains requirements concerning the disclosure of risk management towards the supervising regulator as well as the market.

Figure 3.1 visualizes the three pillars of Solvency II and shows their specific area of risk management.

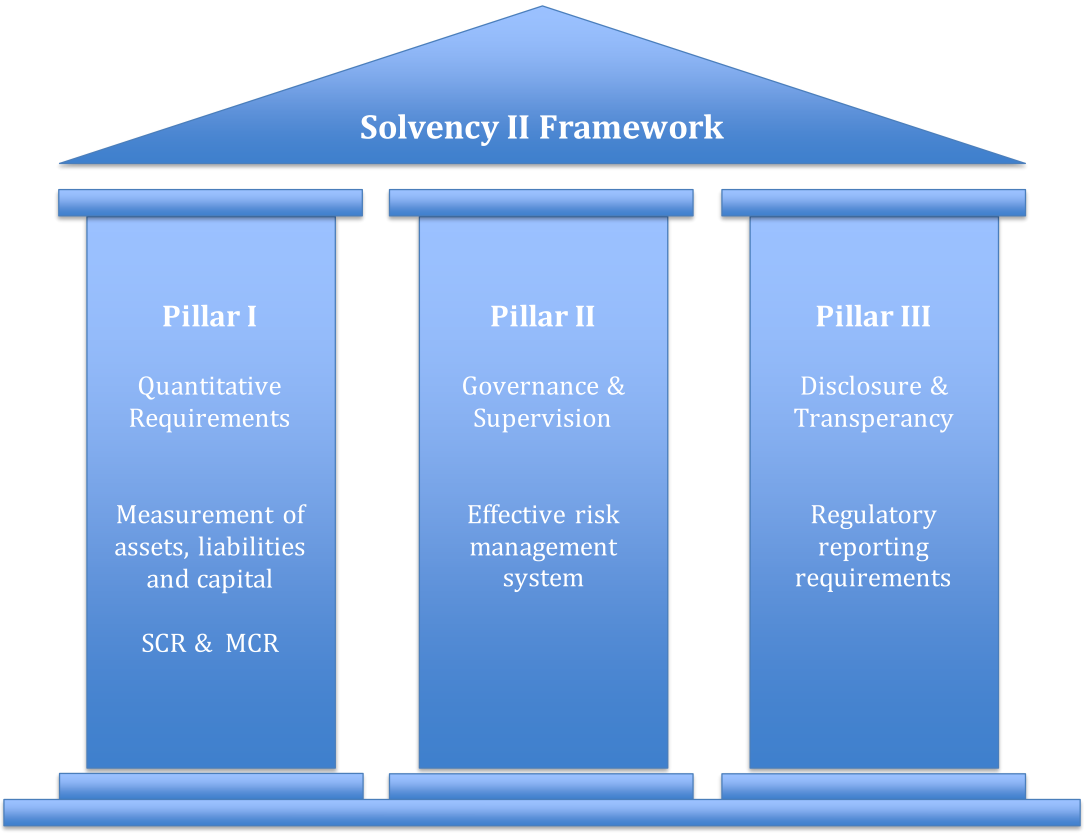


FIGURE 3.1: Structure of Solvency II: Three-pillar approach[2].

[BaFin, 2016].

Pillar I includes one of the main aspects of the Solvency II framework – the calculation of the risk-based Solvency Capital Requirement (SCR). The purpose of the SCR is to

[2]Figure 3.1 based on [Lloyds, 2010].

> *"... reflect a level of eligible own funds that enables insurance and reinsurance undertakings to absorb significant losses and that gives reasonable assurance to policy holders and beneficiaries that payments will be made as they fall due.'* [European Commission, 2009, p.13, Article (62)].

The SCR calculations can be based either on an internally developed model, in accordance to the standard formula provided by the regulator or by mixture of both. This thesis will specifically focus on features and idiosyncrasies of the standard formula.

3.2 SCR Standard Formula

Based on the standard formula, the SCR is calculated by the sum of three parts, the Basic Solvency Capital Requirement (Basic SCR), the capital requirement covering operational risk and adjustments. The Basic SCR can be considered as the main part of the overall Solvency Capital Requirement. It covers five types of risks identified as risk modules:

1. non-life underwriting risk,
2. life underwriting risk,
3. health underwriting risk,
4. market risk,
5. credit risk.

In most cases, the five individual risk modules can be further divided into sub-modules, which could possibly comprise sub-modules themselves. According to [EIOPA, 2014], the overall structure of the standard formula can be summarized as follows

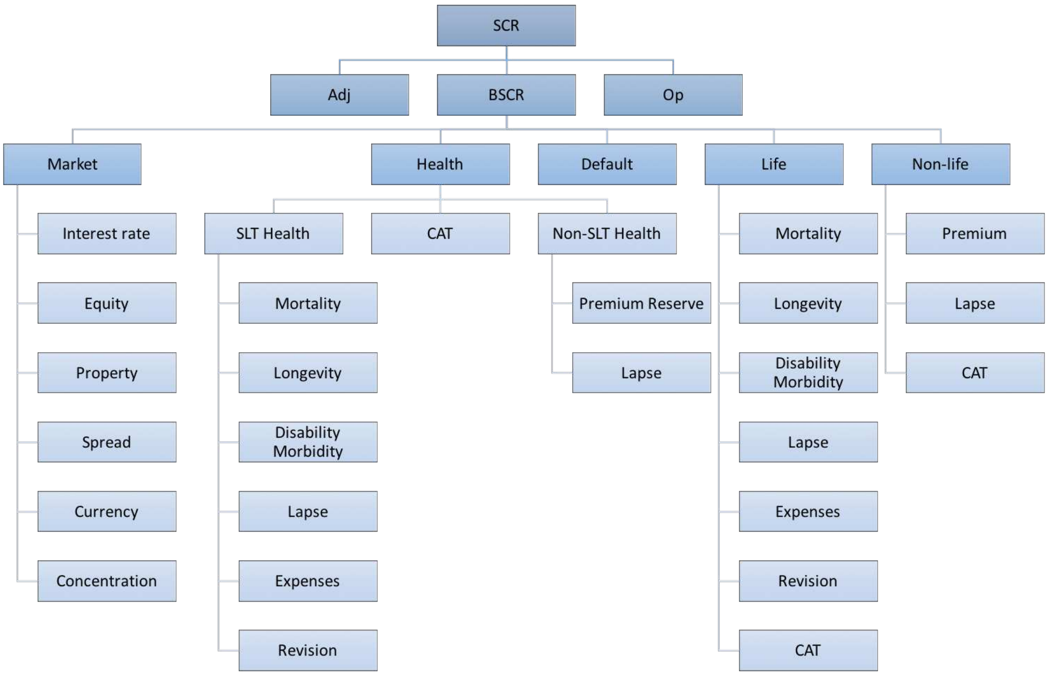


FIGURE 3.2: Segmentation of the standard formula in its risk modules and sub-modules. [EIOPA, 2014].

The structure of the standard formula already implies its application. It follows a modular approach and is applied in a stepwise, bottom-up manner. First capital requirements are calculated for (sub-) modules of the lowest level. This is followed by the stepwise aggregation of capital requirements. Consequently, on the highest level, the standard formula is given by

$$\mathrm{SCR}_{\mathrm{Basic}} = \sqrt{\sum_{i=1}^{5}\sum_{j=1}^{5} \rho_{i,j} \cdot \mathrm{SCR}_i \cdot \mathrm{SCR}_j}, \tag{3.1}$$

where SCR_i and SCR_j represent the capital charges for the ith and jth risk module and $\rho_{i,j}$ the correlation between them. Since the Basic SCR comprises five risk modules i and j can attain values between 1 and 5 on this level.
In case the risk module can be further divided, capital charges for sub-risks can similarly be obtained by

$$\mathrm{SCR}_i = \sqrt{\sum_{k=1}^{n_i}\sum_{l=1}^{n_i} \rho_{k,l} \cdot \mathrm{SCR}_k \cdot \mathrm{SCR}_l}, \tag{3.2}$$

where SCR_i and SCR_j respectively represent the capital charges for sub-module i's risk elements and $\rho_{k,l}$ the correlation between them. k and l can attain values between 1 and n_i where n_i denotes the number of risk elements comprised by risk module i. Independent from its aggregation level, the application of the standard formula requires two input parameters:

- Capital charges for each sub-risk comprised by the considered risk module.
- Correlation coefficients describing the dependence of each possible combination of sub-risks comprised by the considered risk module.

3.2.1 Sub-risk Capital Requirements

Capital charges for sub-risks arise from the aggregation of their risk elements according to equation 3.2. On the lowest level, meaning in case the concerning risk is not further divided into sub-risks, the capital requirements are directly linked to the 99.5% VaR associated with the concerning risk module. This is constituted on the Solvency II Level 1 guidelines which demands that insurance and reinsurance undertakings

> "*will still be in a position with a probability of at least* 99.5%*, to meet their obligations to policyholders and beneficiaries over the following twelve month.*" [European Commission, 2009, p.13, Article (64)].

The regulatory approach to calibrate VaR depends on the structure and characteristics of the individual risk modules. For many of them scenario-based approaches are applied, meaning risk calibrations are directly based on historical data of representing financial intstruments.

3.2.2 Correlation Coefficients

To aggregate capital charges of risk elements, a parameter estimating the correlation between those elements is required. The most common and probably easiest approach to measure dependence is the linear correlation coefficient also known as Pearson correlation. However, linear correlation does not fully reflect the overall dependence structure for each and every class of probability distributions. In those cases, the use of linear correlation could lead to spurious aggregation results. Unfortunately, risks insurance and reinsurance undertakings are exposed to exhibit characteristics distorting the aggregation results when using linear correlation as a dependence measure. These include skewed probability distributions and the existence of tail dependencies. CEIOPS/EIOPA is fully aware of this problem and thus proposes the use tail correlation coefficients instead of Pearson correlation to measure the dependence between risks [CEIOPS, 2010c]. To obtain tail correlation coefficients, CEIOPS/EIOPA discusses two different approaches.

VaR-implied correlation

The *VaR-implied correlation approach*[3] is based on the inversion of the standard formula. Let r_1 and r_2 denote the returns of two different assets[4]. Let r_p denote the portfolio return of an equally weighted[5] combination of r_1 and r_2, meaning $r_p = r_1 + r_2$. Based on an inversion of the standard formula, the portfolio VaR is given by

$$\mathrm{VaR}(r_p) = \sqrt{\mathrm{VaR}(r_1)^2 + \mathrm{VaR}(r_2)^2 + 2\rho_{r_1,r_2}\mathrm{VaR}(r_1)\mathrm{VaR}(r_2)}$$

and therefore

$$\mathrm{VaR}(r_p)^2 = \mathrm{VaR}(r_1)^2 + \mathrm{VaR}(r_2)^2 + 2\rho_{r_1,r_2}\mathrm{VaR}(r_1)\mathrm{VaR}(r_2) \tag{3.3}$$

with ρ_{r_1,r_2} denoting the correlation for the risk components r_1 and r_2. Based on CEIOPS/EIOPA's Level 2 Advice concerning Correlations, ρ_{r_1,r_2} should be chosen

> "*in such way as to achieve the best approximation of the* 99.5% *VaR for the aggregated capital requirement.*" [CEIOPS, 2010c, p. 9, Article 3.15]

Thus, ρ_{X_1,X_2} must be chosen in so as to minimize the so called aggregation error given by

$$|\mathrm{VaR}(r_p)^2 - \mathrm{VaR}(r_1)^2 + \mathrm{VaR}(r_2)^2 + 2\rho_{r_1,r_2}\mathrm{VaR}(r_1)\mathrm{VaR}(r_2)| \tag{3.4}$$

[CEIOPS, 2010c]. The minimization of aggregation error 3.4 is given by a transformation of equation 3.3 such that

$$\rho^{\mathrm{VaR}}_{r_1,r_{2\alpha}} = \frac{\mathrm{VaR}(r_p)^2 - \mathrm{VaR}(r_1)^2 + \mathrm{VaR}(r_2)^2}{2\mathrm{VaR}(r_1)\mathrm{VaR}(r_2)}. \tag{3.5}$$

[3] CEIOPS/EIOPA does not explicitly talk about VaR-implied correlation. However, in the literature, the methodology described in its Level 2 Advice is often referred to this term. See for example in [Mittnik, 2013].

[4] For the sake of simplicity, we assume that r_1 and r_2 are elliptically distribution with zero expectation.

[5] Without loss of generality, we assume $r_p = w_1 r_1 + w_2 r_2$ with $w_1, w_2 = 0.5$

As a coefficient of correlation, $\rho^{\mathrm{VaR}}_{r_1,r_{2\alpha}}$ is bounded in the interval $[-1,1]$. This constraint is not given in general. Since

$$\begin{aligned}
\rho^{\mathrm{VaR}}_{r_1,r_{2\alpha}} \geq 1 &\Leftrightarrow \mathrm{VaR}(r_p)^2 - \mathrm{VaR}(r_1)^2 - \mathrm{VaR}(r_2)^2 \geq 2\mathrm{VaR}(r_1)\mathrm{VaR}(r_2) \\
&\Leftrightarrow \mathrm{VaR}(r_1)^2 \geq 2\mathrm{VaR}(r_1)\mathrm{VaR}(r_2) + \mathrm{VaR}(r_1)^2 + \mathrm{VaR}(r_2)^2 \\
&\Leftrightarrow \mathrm{VaR}(r_1)^2 \geq \big(\mathrm{VaR}(r_1) + \mathrm{VaR}(r_2)\big)^2 \\
&\Leftrightarrow \mathrm{VaR}(r_p) \geq \mathrm{VaR}(r_1) + \mathrm{VaR}(r_2)
\end{aligned}$$

and

$$\begin{aligned}
\rho^{\mathrm{VaR}}_{r_1,r_{2\alpha}} \leq -1 &\Leftrightarrow \mathrm{VaR}(r_p)^2 - \mathrm{VaR}(r_1)^2 - \mathrm{VaR}(r_2)^2 \leq -2\mathrm{VaR}(r_1)\mathrm{VaR}(r_2) \\
&\Leftrightarrow \mathrm{VaR}(r_1)^2 \leq -2\mathrm{VaR}(r_1)\mathrm{VaR}(r_2) + \mathrm{VaR}(r_1)^2 + \mathrm{VaR}(r_2)^2 \\
&\Leftrightarrow \mathrm{VaR}(r_1)^2 \geq (\mathrm{VaR}(r_1) - \mathrm{VaR}(r_2))^2 \\
&\Leftrightarrow \mathrm{VaR}(r_p) \leq |\mathrm{VaR}(r_1) + \mathrm{VaR}(r_2)|
\end{aligned}$$

a truncated version of equation 3.4 is given by

$$\rho^{\mathrm{VaR}}_{r_1,r_{2\alpha}} = \begin{cases} +1, & \text{if } \mathrm{VaR}(r_p) \geq \mathrm{VaR}(r_1) + \mathrm{VaR}(r_2) \\ -1, & \text{if } \mathrm{VaR}(r_p) \leq |\mathrm{VaR}(r_1) + \mathrm{VaR}(r_2)| \\ \frac{\mathrm{VaR}(r_p)^2 - \mathrm{VaR}(r_1)^2 + \mathrm{VaR}(r_2)^2}{2*\mathrm{VaR}(r_1)\mathrm{VaR}(r_2)} & \text{otherwise.} \end{cases} \tag{3.6}$$

Data-cutting correlation

Another approach is proposed by CEIOPS/EIOPA under the name of *data-cutting method*. Associated with a given $(1-\alpha)\%$ VaR or respectively a given $\alpha - quantile$ cthe idea is to compute common Pearson correlations from joint tail observations which comprise all those pairs of observations simultaneously falling below their respective α quantile.

Let r_1, r_2 again denote risk components evoked by two different asset price changes. Then the data cutting correlation coefficient denoted by $\rho^{DC}_{r_1,r_{2\alpha}}$ is given by

$$\rho^{DC}_{r_1,r_{2\alpha}} = Corr(r_1, r_2 | X_1 < \mathrm{VaR}_\alpha(r_1), r_2 < \mathrm{VaR}_\alpha(r_2)) \tag{3.7}$$

[Mittnik, 2011]. Like capital requirements, tail correlation coefficients are calculated on the basis of historical market data of financial instruments for the majority of risk categories.

3.3 Market Risk

The Basic SCR is obtained by the aggregation of five risk modules. Figure 3.3 shows their average proportion. By far the largest of the five components of the standard formula is represented by the risk module market risk.

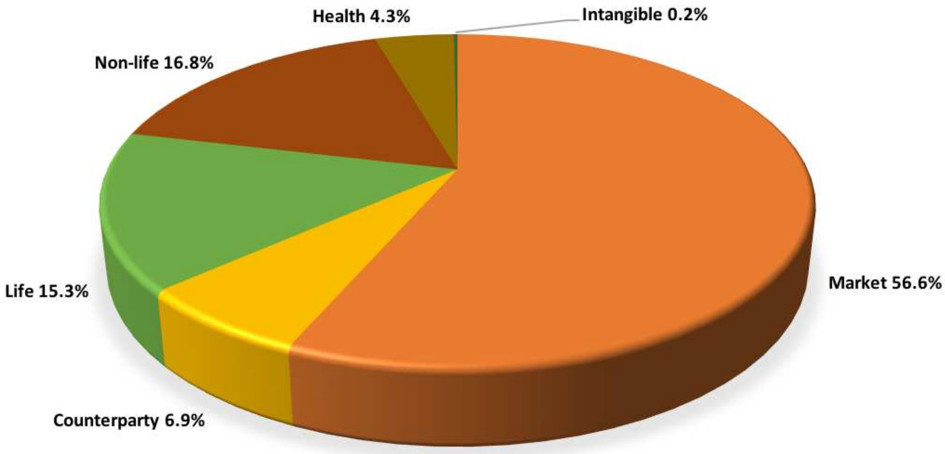


FIGURE 3.3: Decomposition of the BSCR (diversified) [EIOPA, 2011].

Market risk itself comprises seven sub-modules. In the following, we will mainly focus on two of those components - equity and property risk.

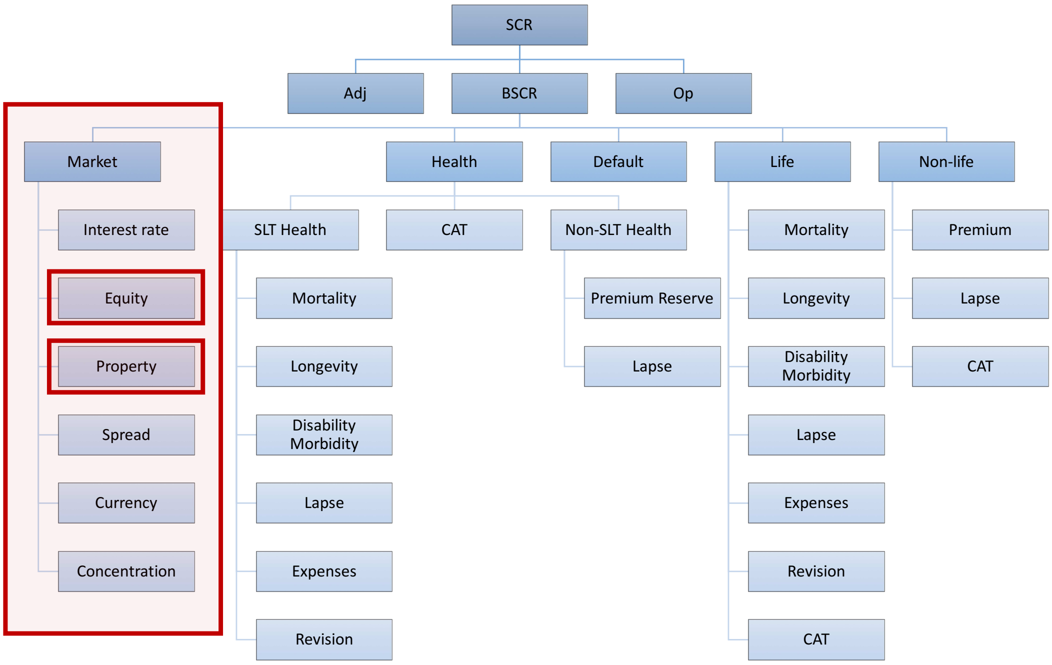


FIGURE 3.4: The segmentation of the standard formula in its risk modules and their sub-modules with a focus on the sub-modules of interest for this thesis[6].

3.3.1 Equity Risk

Equity risk is divided into two categories – "global equity" and "other equity". The category "global equity" covers equities listed in European Economic Area (EEA) countries or countries belonging to the Organization for Economic Cooperation and Development (OECD). The equity category "others" is more diverse and covers several equity types. This includes non-listed equities or equities listed in countries which are not EEA/OECD, private equities, hedge funds, commodities ant other alternative financial instruments. In accordance with the standard formula, capital charges for equity risk are derived by

$$\mathrm{SCR}_{\mathrm{equity}} = \mathrm{SCR}^2_{\mathrm{global}} + \mathrm{SCR}^2_{\mathrm{other}} + \rho_{\mathrm{global,other}} \cdot \mathrm{SCR}^2_{\mathrm{global}} \cdot \mathrm{SCR}^2_{\mathrm{other}}, \tag{3.8}$$

where $\rho_{\mathrm{global,other}}$ denotes the correlation between the two equity categories and $\mathrm{SCR}^2_{\mathrm{global}}$, $\mathrm{SCR}^2_{\mathrm{other}}$ the categories' capital charges.
CEIOPS/EIOPA's calibrations concerning the input parameters of equation 3.8 are based on the so called scenario-based approach. For the category "global equity" the regulator concludes a capital charge of -45%, also referred to as stress factor or shock scenario. For the category "other equities" a stress factor of -55% is proposed. For the correlation between the two equity categories, CEIOPS/EIOPA proposes a correlation coefficient of 0.75 [CEIOPS, 2010a].

3.3.2 Property Risk

Like for the sub-risk module equity, standard formula input parameters for property risk are derived on the basis of a scenario based approach. However, unlike in the first case, the sub-risk module property is not further divided into subcategories. Thus, CEIOPS/EIOPA only derives a single shock scenario for the sub-risk module property. The regulator's analysis results in a stress factor of -25%. The relinquishment of a breakdown into different property classes also results in the assumption of perfect correlation within the sub-risk module[CEIOPS, 2010b].

Chapter 4

Stability of Regulatory Risk Calibrations and its Alternatives

The project of Solvency II not only comprises the development of the legal framework itself but also numerous additional publications, studies, advices and discussions. A crucial facet is given by five field tests where insurance and reinsurance companies throughout the European Union were invited to test the quantitative aspects of the Solvency II framework. Findings of those exercises were used to develop advices on a second level, so called Level 2 Implementing Measures that complete and implement the Solvency II Level 1 Framework Directive. Therewith, the development of the Solvency II framework incorporated lessons learned. Nonetheless, parts of the final Solvency II Directive are controversially discussed since the beginning of the project.

Criticism and comments are raised concerning various parts of the Solvency II framework.[Pfeifer and Strassburger, 2008] as well as [Sandström, 2007] deal with properties of individual risk distributions and their effect on the stability of the standard formula. The underlying probability distributions of risks an insurance or reinsurance undertaking is exposed to are not normal but rather skewed. Both authors discuss the effect of neglecting those characteristics. They stress the need to calibrate for skewness of risk distributions in order to maintain the standard formula's recommended level of confidence. However, it should be noted that the findings of both authors attracted CEIOPS/EIOPA's attention. Two years after the two publications, CEIOPS/EIOPA revealed their Level 2 Advice which addresses in depth the calibration of correlation parameters. Based on the authors' research, the regulator discusses certain characteristics shared by many risks reinsurance and insurance undertakings are exposed to and proposes alternative correlation approaches to calibrate for them [CEIOPS, 2010c].

[Aria et al., 2010] point their criticism in a more specified direction. For the scenario-based calibration approaches of certain risk-modules, the authors scrutinize the suitability of the corresponding representing market data. Similar concerns are raised concerning the choice of representative indexes for the risk module property. CEIOPS/EIOPA bases its analysis on historical data for the United Kingdom (UK) property market. This choice has been argued from various sides. UK property market are not considered to be able to cover the complexity and diversity of property risk throughout the European market.

In the following we will mainly focus on two rather fundamental problems concerning the regulatory calibration of standard formula input parameters – specifically concerning equity and property risk. The first part is concerned with the handling of historical data used for scenario-based calibration approaches. Subsequently, several aspects concerning the aggregation of sub-risk module capital requirements will be assessed. As an excursus, we will discuss the appropriate choice of risk measures in regulatory frameworks.

4.1 Risk Calibrations based on Historical Data

The calibration of standard formula input parameters concerning equity and property risk is subject to a scenario-based approach which includes analysis carried out on historical market data. Risk measure calibrations on the basis of historical data are referred to as Historical Simulation (HS). The determination of the historical VaR follows a simple concept. First, the return series is ordered from the lowest to the highest value. The $(1-\alpha)n$ smallest observation then denotes the $(1-\alpha)\%$ VaR, where n denotes the length of the historical time series [Li et al., n.d.].

4.1.1 Rolling-Window Annualization

Solvency II demands capital requirement calibrations subject to a 99.5% probability of remaining solvent within a one year horizon. Therewith, VaR calibrations are associated with a "one in 200 years event", meaning the likelihood of an insurer being ruined must not exceed one in 200 cases. Historical VaR calibrations therefore require at least 200 years of historical data for each representing index. However, in the majority of cases, annual data is not available in a sufficient amount. To overcome this problem, CEIOPS/EIOPA proposes a methodology to compute annual data out of data on a daily basis using 12-month rolling windows.

A return computed by this methodology is further referred to as *annualized return* and is given by

$$r_{annualized_i} = \frac{P_t - P_{t-259}}{P_{t-259}}, \qquad t = 260, \ldots, n. \tag{4.1}$$

where P_t denotes the daily closing price of an asset at day t and n the number of days where data is available. Similarly annualized returns can be calculated on the basis of monthly data where

$$r_{annualized_i} = \frac{P_t - P_{t-12}}{P_{t-12}}, \qquad t = 13, \ldots, n. \tag{4.2}$$

Here, P_t refers to monthly closing prices whereas n denotes the number of available monthly data points. The rolling-window annualization results in almost as many annualized returns as daily returns are available.

However, the annualization procedure implicates severe problems. The resulting returns overlap to a large extent and hence share a lot of information. [Mittnik, 2011] analyzes consequences of this procedure in general and shows severe contortions affecting the dependence structure of the return series over time as well as across different assets. Thus, with regard to standard formula calibrations, both input parameters are affected. The rolling-window annualization can imply highly unstable VaR calibration results and simultaneously, it might severely distort the correlation structures between risk modules.

CEIOPS/EIOPA was well aware of the problems arising from one-year rolling-window returns. On this matter the regulator stated:

> "*There is a balance to be struck between an analysis based on the richest possible set of relevant data and the possibility of distortion resulting from autocorrelation*" [CEIOPS, 2010a, p. 8, Article 3.12].

Nonetheless, CEIOPS/EIOPA decided to hazard the consequences in favor of the method's advantages.

> "*In this case, we have chosen to take a rolling one-year window in order to make use of the greatest possible quantity of relevant data*" [CEIOPS, 2010a, p. 9, Article 3.12].

In the following, we will introduce a possible alternative to simulate a sufficient amount of annualized data for scenario-based approaches to calibrate standard formula input parameters.

4.1.2 Filtered Historical Simulation

Rolling-window annualization is used to overcome the problem of too less data history for VaR calibrations based on Historical Simulation (HS). Alternatively, VaR calibrations could also carried out on the basis of an underlying model. However, this approach imposes assumptions about an underlying loss distribution and is thus often referred to as parametric approach. Filtered Historical Simulation (FHS) aims to overcome the limitations of both approaches. The methodology was introduced by Giovanni Barone-Adessi in 1997 and is based on the combination of parametric GARCH models and non-parametric historical simulation. Historical data is filtered and subsequently used as basis for the simulation of future return pathways.

The first step of the FHS is the removal of serial correlation and volatility clusters from the data. This can be done by an ARMA(p,q)-GARCH(m,n) filter. A time series r_t following an ARMA(1,1)-GARCH(1,1) model[1] is given by

$$r_t = \phi_1 r_{t-1} + \psi_1 \epsilon_{t-1} + \epsilon_t \tag{4.3}$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1} + \beta \sigma_{t-1}^2. \tag{4.4}$$

where σ_t represents the non constant variance of ϵ_t. Let s denote the available amount of daily returns[2]. Estimating the parameters in equations 4.3 and 4.4 leads to

- a set of estimated returns of length s: $\{\hat{r_1}, \hat{r_2}, \ldots, \hat{r_s}\}$,
- a set of estimated volatility of length s: $\{\hat{\sigma_1}, \hat{\sigma_2}, \ldots, \hat{\sigma_s}\}$,
- a set of estimated residuals of length s: $\{\hat{\epsilon_1}, \hat{\epsilon_2}, \ldots, \hat{\epsilon_s}\}$.

The model residuals are standardized by the corresponding volatility with

$$\hat{z_t} = \frac{\hat{\epsilon_t}}{\hat{\sigma_t}}, \tag{4.5}$$

Leaving us with

- a set of standardized residuals of length s: $\{\hat{z_1}, \hat{z_2}, \ldots, \hat{z_s}\}$.

[1] Without loss of generality we use an ARMA(1,1)-GARCH(1,1) with zero mean for exemplary reasons.

[2] For simplicity, we assume the data to be available as a series of returns on a daily basis. As we will see in the following chapter, another pattern is easily conceivable too.

For the simulation of future returns, initial values for equation 4.3 and equation 4.4 need to be determined. It is reasonable, that the most recent data forecasts the future better than data lying further in the past. Calling in mind that the length of the available time series is denoted by s, initial values are determined as follows

- initial volatility: $\sigma_s^* = \hat{\sigma}_s$
- initial residual: $\epsilon_s^* = \hat{\epsilon}_s$

Let T denote the length of the future return path. Out of the set of standardized residuals $\{\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_s\}$, T returns are drawn randomly with replacement, resulting in

- a set of random standardized returns of length T: $\{z_{s+1}^*, z_{s+2}^*, \ldots, z_{s+T}^*\}$.

To obtain the innovation forecasts for periods $t = (s+1, s+2, \ldots s+T)$, the random standardized returns are scaled by the corresponding current volatility. For each period $t = s+1, s+2, \ldots s+T$ we iteratively calculate

$$\sigma_t^{2*} = \hat{\alpha}_0 + \hat{\alpha}_1 \epsilon_{t-1}^* + \hat{\beta}_1 {\sigma_{t-1}^2}^* \tag{4.6}$$

$$\epsilon_t^* = z_t^* \sigma_t^* \tag{4.7}$$

Based on the results above, the pathway of future returns can be generated by

$$r_t^* = r_{s+1} = \hat{\phi}_1 r_s + \hat{\psi}_1 \epsilon_s^* + \epsilon_{s+1}^*. \tag{4.8}$$

for $t = s+1$ and

$$r_t^* = \hat{\phi}_1 r_{t-1} + \hat{\psi}_1 \epsilon_{t-1}^* + \epsilon_t^*. \tag{4.9}$$

for the following periods $t = s+2, \ldots s+T$.
To obtain a distribution of future returns, the proceeding described above is now replicated K times[3] resulting in a (KxT)-matrix including K pathways of T future returns.

$$\begin{pmatrix} r_{1,1}^* & r_{1,2}^* & \cdots & r_{1_T}^* \\ r_{2,1}^* & r_{2,2}^* & \cdots & r_{2_T}^* \\ \vdots & \vdots & \ddots & \vdots \\ r_{K,1}^* & r_{K,2}^* & \cdots & r_{K_T}^* \end{pmatrix} \tag{4.10}$$

[Barone Adesi et al., 1999].

[3]To obtain reliable results, K should be chosen as a large number.

4.2 Aggregation of Capital Requirements

4.2.1 Choice of Correlation Coefficient

As described in Chapter 3, CEIOPS/EIOPA recognizes the insufficiency of the common linear Pearson correlation to cover the correlation structure between financial instruments. In its Level 2 Advice [CEIOPS, 2010c], the regulator proposes the use of tail correlation coefficients instead of linear correlation concepts and explains two methodologies to calibrate them – the VaR-implied correlation and the data-cutting correlation. However, CEIOPS/EIOPA does not further specify which of the two proposed methodologies is used in practice. Nonetheless, in the calibration report belonging to the fifth Quantitative Impact Study (QIS5), CEIOPS/EIOPSA states on this matter that

> *"[...] in view of the assumed tail dependence of market risks in stressed situation the correlation analysis was based on 'cutting out' adequate subsets of data pairs in order to obtain a measure of the tail correlation [...]"* [CEIOPS, 2010d, p. 346, Article 3.1285].

This statement suggests, that CEIOPS/EIOPA decides upon tail correlation coefficients obtained through the data cutting methodology concerning the risk module market risk. However correlation calibration approaches concerning market risk's sub-module are not explicitly specified.
Assuming the calibration of data-cutting correlation coefficients CEIOPS/EIOPA does not make a clear statement about the quantile used either. On this matter it is solely revealed that

> *"[...] the overall correlation matrix should produce a level of stress equivalent to a 99.5% VaR event, so each individual pair can be equivalent to significantly less than a 99.5th percentile stress, but still should be firmly in the tail. The analysis must be subject to sensitivities for different percentiles, and should be taken as providing an indication of the correct correlation."* [CEIOPS, 2010d, p. 367, Article 3.1385]

In general, the use of data cutting correlation coefficients contains problems. Per definition, data-cutting correlation measures the dependency for the specific set of joint tail observations meaning all those pairs of observations which simultaneously fall below a certain predefined quantile. Even for large data histories this specific set of joint tail observations can be extremely small. The correlation coefficient solely depends on a minority of data points and could lead to rather unstable estimations of dependency between the two risks. Figure 4.1 illustrates the problem. It shows a scatter plot of annualized returns extracted from two financial market indexes[4]. VaRs for three different confidence levels are marked by red lines. The respective lines for both indexes construct a box separating the observations the data-cutting correlation coefficient calibration is based on. The plot shows that the number of observations falling into the box of the 99.5% confidence level box is particularly small.

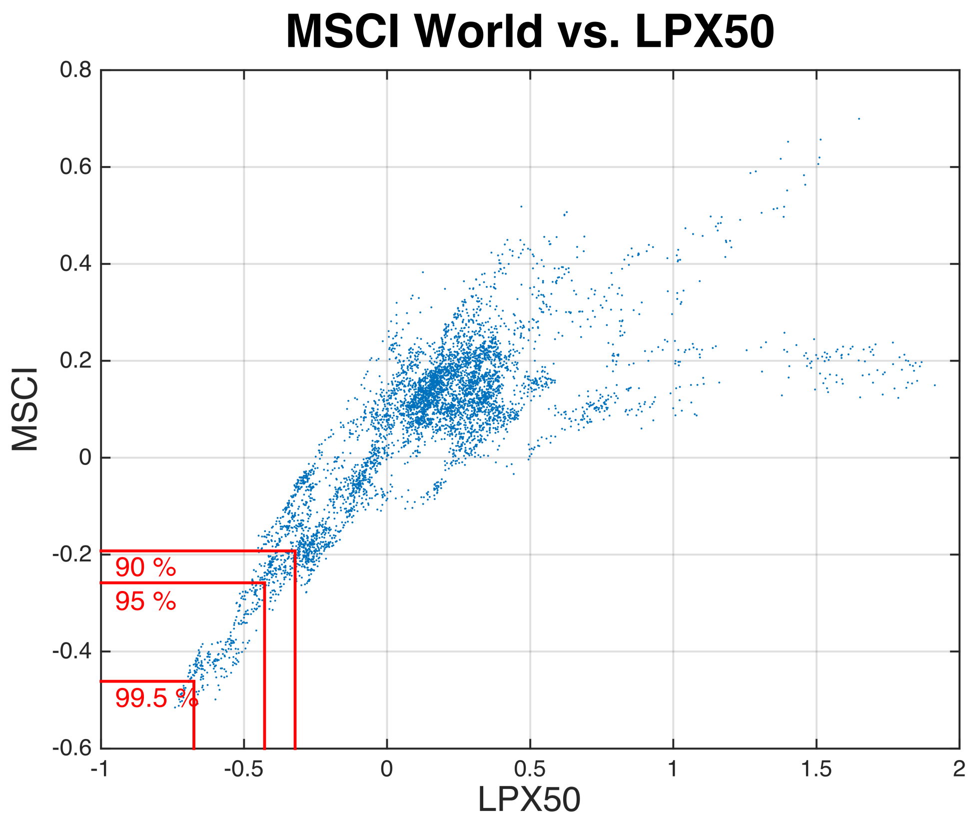


FIGURE 4.1: Principle of the data-cutting approach to calibrate the correlation coefficient between two risks.

A possible alternative to the data-cutting correlation approach is given by the calibration of VaR-implied correlation coefficients. CEIOPS/EIOPA's Level 2 Advice concerning correlation calibration shows that the regulator certainly considers VaR-implied correlation estimates as possible alternative. However, there is no sign for its actual application.

[4]In the further course of this thesis we will see, that these two indexes are used by CEIOPS/EIOPA to represent the sub-risk module equity.

4.2.2 Granularity

The risk module market risk is subdivided into seven sub-risk modules where some of them are further divided into several types or categories. However, in most cases, CEIOPS/EIOPA relinquishes the use of a more granular analysis. The sub-risk module equity is divided into two categories which are investigated separately. The category "other equity" comprises four different types of equities. CEIOPS/EIOPA indeed analyzes all five representing indexes of equity risk empirically and derives stress factors for all of them. Correlations between the category "global equity" and the category "other equity" are calibrated for each of the four possible index combinations. However, zhe regulator only yields one stress factor per equity category and one correlation parameter between "global equity" and "other equity". Any diversification effects within "other equity" are thereby neglected. CEIOPS/EIOPA again is well aware of this matter but is reluctant towards a more granular analysis. It is stated that

> *"CEIOPS notes a potential diversification benefit between the other equity types, but considers it to be low and difficult to calibrate, so proposes that the standard formula contains no diversification benefit within the other equity sub-module (an implicit correlation of 1)."* [CEIOPS, 2010a, p. 24, Article 3.66].

A similar attitude is adopted concerning the sub-risk module property. Analysis is carried out for five types of property risk. Notwithstanding, concerning the influence of property risk to the Basic Solvency Capital Requirement calculated according to the standard formula, CEIOPS/EIOPA decides that

> *"No breakdown in different property classes is needed as the historical values at risk for the different classes do not diverge too much."* [CEIOPS, 2010a, p. 24, Article 3.66].

The regulators decisions regarding the granularity of market risk's sub-risk modules are arguable. The negligence of diversification effects at several points could cause a distort picture of an insurance or reinsurance situation of risk exposure.

4.3 Excursus: Choice of Risk Measure

Solvency II risk calibrations are subject to the 99.5% Value-at-Risk risk measure. In financial risk management, the VaR is a popular and widely used risk measure.

Nonetheless its application is controversial and has been conceptually criticized from various directions. In 2008, David Einhorn, manager of the Greenlight Capital Hedgefund described VaR as

> "*An airbag that works all the time, except when you have a car accident.*" [Einhorn, 2008]

Among other professionals and risk managers, Einhorn accuses VaR to cause a false sense of security. A popular alternative to the VaR is given by the Expected Shortfall (ES) which gives an estimation about the size of loss, in case the VaR is exceeded. On that account, ES is also referred to as Tail VaR (TVaR).

Arguments in favor of ES
The ES entails two popular advantages towards the VaR – ES is a coherent risk measure and it is not only a measure of location.
The concept of coherence was introduced by [Artzner et al., 1999] and summarizes four desirable properties of risk measures[5].

Translation invariance: For all losses L and all constants $a \in \mathbb{R}$ it holds that

$$\rho(L + a) = \rho(L) + l,$$

meaning adding or subtracting a deterministic amount l to a financial position leading to loss L alters the capital required to buffer this loss by exactly the same amount.

Positive homogeneity: For all losses L and every $\lambda \geq 0$ it holds that

$$\rho(\lambda L) = \lambda \rho(L),$$

meaning the scale of financial positions do not have an influence on the risk measure. For example the choice of currency has no influence on positive homogeneous risk measures.

Monotonicity: For all losses L_1, L_2 with $P(L_1 \leq L_2) = 1$ it holds that

$$\rho(L_1) \leq \rho(L_2),$$

meaning financial positions alway leading to higher losses must result in higher capital requirements.

[5]The interpretations of the properties described below are based on [Kriele and Wolf, 2012].

Subadditivity: For all losses L_1, L_2, it holds that

$$\rho(L_1 + L_2) \leq \rho(L_1) + \rho(L_2),$$

meaning the combination of financial positions lead to diversification effects. In the financial world, diversification means the reduction of the overall risk of a portfolio by investing in a variety of assets which are not perfectly correlated. A loss of a financial position can be neutralized by a positive return or at least a less severe loss of another [Kriele and Wolf, 2012].
The ES fulfills all four axiomes and can thus be considered as coherent risk measure. However, Appendix A shows that the VaR does fulfill the first three axioms. However, a simple counterexample implies that VaR is not a subadditiv risk measure in general. Thus, VaR possibly neglects diversification effects.

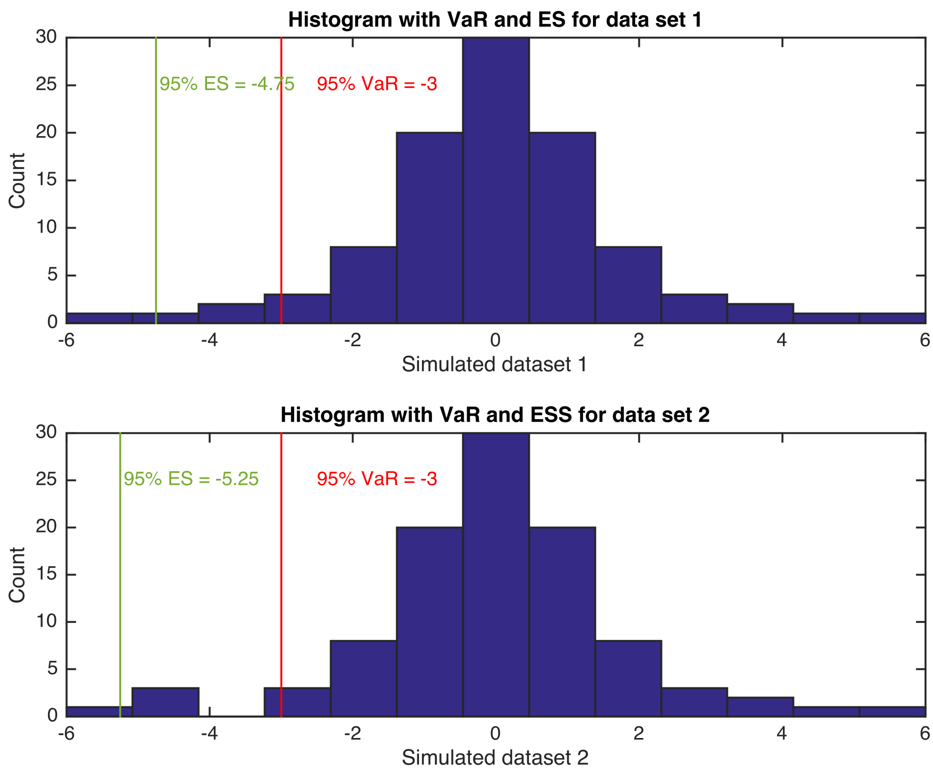


FIGURE 4.2: Differences between the risk measures VaR and ES.

The second advantage of the ES towards the VaR is implied by its definition. Theoretically, VaR is a quantile of a loss distribution, meaning VaR determines the loss which will not be exceeded within a pre-specified probability and time horizon. hus, VaR is a measure of location and does not consider the situation beyond this level whereas the ES gives an estimation about the size of this loss [Acerbi, 2002]. This is illustrated by figure 4.2. The figure shows histograms of two similar but not identical data sets[6]. They differ in their behavior in the left tail. Nonetheless, the VaR is identical for both data sets. By contrast, ES considers the tail behavior behind the VaR and is therefore more negative for the second data set.

Arguments in favor of VaR

In the course of the scientific discourse regarding the suitability of certain risk measures as well as their advantages towards others, the four desirable properties of risk measures described above have been complemented by the so called axiom of elicitabilty. As described in section 2.1, risk measures map random variables or data vectors to a real number. Thus, risk measures are functionals of the underlying data. In general, a functional is called elicitable if

> *"it can be defined as the minimizer of a suitable, strictly convex scoring function."* [Embrechts et al., 2014, p. 17].

Those scoring functions can be used to compare competing forecasts and thus, to backtest certain risk measures. It can be shown that VaR is an elicitable risk measure whereas ES is not. The failure of the elicitability axiom is considered as the reason for the difficulties on finding a suitable backtesting procedure for ES[7]. On this behalf [Gneiting, 2011] stated that the lack of elicitability

> *"may provide a partial explanation for the lack of literature on the evaluation of CVaR [ES] forecasts, as opposed to quantile or VaR forecasts[...]"* [Gneiting, 2011, p. 11].

The absence of methodologies for the validation of ES forecast is a crucial disadvantage of ES compared to VaR and probably one of the main reasons to prefer the latter.

A second problem concerning ES is its dependence on the existence of a finite first moment. According to its definition, ES is based on the expected size of loss in case VaR is exceeded. Thus, ES does only exist for probability distributions with finite mean. However, this is not necessarily the case for probability distributions in general. Especially particularly heavy-tailed distributions tend to an infinite mean [Ghosh, 2012]. An example could be a Pareto distribution with $\alpha = 1$.

[6]Data sets are simulated and comprise 100 observations each

[7]There is a controversial discussion about the dependence between a lack elicitability and difficulties in the construction of backtesting procedures. Some articles consider the topic of elicitability as irrelevant for the choice of risk measure (refer for example to [Acerbi and Szekely, 2014]).

Regulatory practice

The advantages and disadvantages of both risk measures make room for discussions concerning their suitability for risk calibrations as part of regulatory frameworks. While Solvency II sticks to the use of VaR as a risk measure, its Swiss counterpart Swiss Solvency Test (SST) introduced the use of ES instead [FOPI, 2004]. A similar decision was taken for the International regulatory framework for banks (Basel III) developed by the Basel Commitee on Banking Supervision [BIS, 2010]. During the development of the Solvency II framework, the topic concerning the choice of risk measure was addressed at various points. In several official publications and protocols CEIOPS/EIOPA's discussed the advantages and disadvantages of ES and VaR and examined their suitability for the specific needs of Solvency II[8]. In 2007 CEIOPS/EIOPA acknowledged the superiority of VaR in comparison to ES in terms of the Solvency II framework and stated on this matter

> *However, CEIOPS recognizes that the Commission's Amended Framework for Consultation continues to support VaR as the risk measure for the SCR, and that the decision on the appropriate risk measure should not only be based on theoretical considerations, but also on practical issues. Therefore, CEIOPS believes that the SCR, at least for an initial implementation of Solvency II, should be based on VaR [...]. [CEIOPS, 2007, p. 16, paragraph 2.31].*

[8]Most of them were published in 2006, refer for example to a publication of the European Insurance and Occupational Pensions Commitee (EIOPC) [EIOPC, 2006]. EIOPC advises the European Commission in terms of directives on insurance, reinsurance and occupational pensions.

Chapter 5

Empirical Analysis

The previous chapters paid close attention to the standard formula in general and were specifically concerned with the structure and calibration of the sub-risk modules equity and property. Chapter 3 described the modular structure of the aggregation formula, explained regulatory approaches for the calibration of input parameters and outlined CEIOPS/EIOPA's results for parameters of equity and property risk. Chapter 4 addressed stability problems of the standard formula and discussed possible alternative approaches for its calibration.
In the empirical analysis discussed in this chapter we replicate CEIOPS/EIOPA's calibration approach aiming to reproduce the regulator's results and decisions. In a second step, we empirically investigate alternative risk calibration approaches discussed in Chapter 4. The empirical analysis specifically refers to the sub-risk modules equity risk and property risk and is carried out on authentic data, meaning financial market indexes chosen by CEIOPS/EIOPA to represent the individual risk modules[1]. In most cases, the empirical analysis described in this chapter is carried out on two different data sets, both built upon the representing indexes. The first data set includes approximately the same length of historical data as it is used by CEIOPS/EIOPA. We will further refer to this set of observations as "original data set". The second data set simply includes additional, more up to date data and is thus further referred to as "recent data set".
The empirical analysis discussed in this chapter is conducted with the numerical computing environment [MATLAB, 2015].

[1] The majority of data is extracted from Bloomberg.

5.1 Data

For scenario-based approaches like it is applied for equity and property risk calibrations, CEIOPS/EIOPA carries out analysis on historical market data of financial instruments. In the following, the representing indexes of both sub-risk modules will be explained in more detail.

5.1.1 Equity Risk

The sub-risk module equity is divided into two categories. The category "global equity" is represented by a single index while "other equity" covers four equity types, each represented by an individual index. Table 5.1 summarizes CEIOPS/EIOPA's choice of indexes representing equity risk. The coverage, initiation and affiliation of each index will be described below.

Category	**Equity type**	**Index**
Global	Global	MSCI World Developed Price Equity Index
Other	Private Equity	LPX50 Total Return Index
	Commodities	S&P GSCI Total Return Index
	Hedge Funds	HFRX Global Hedge Fund Index
	Emerging Markets	MSCI Emerging Markets BRIC

TABLE 5.1: Indexes representing the two categories of the sub-risk module equity.

Global equity

- The **MSCI World Developed Price Equity Index** is traded in US Dollars and covers instruments across 23 Developed Market (DM) countries[2]. The MSCI World was constructed by the American financial services provider Morgan Stanley Capital International (MSCI). To ensure up-to-dateness it is reviewed quarterly and thus reflects changes in the underlying equity markets to all times. The index was launched on March 31, 1986. Data prior to the launch date is back-tested data [MSCI, 2016b]. We will further refer to this index as *MSCI World* or sometimes simply as *MSCI*.

[2]Countries involved: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, UK, U.S.

Other equity

- The **LPX50 Total Return Index** represents the equity type private equity and comprises the 50 largest listed private equity companies given they fulfill certain liquidity constraints. The index is constructed by the LPX Group, a Swiss financial services provider and is designed with the purpose to ensure tradability, transparency and investability [Group, 2016]. We will further refer to this index as *LPX50*.

- The **S&P GSCI Total Return Index** represents the equity type commodities. It is designed to provide a benchmark for investment performance on the commodity markets. It is calculated on a world production weighted basis and comprises the most liquid physical commodity futures. The index involves five sectors – agriculture, energy, livestock, industrial metals and precious metals reflecting 24 commodity futures in total. The weight of each commodity indicates its significance in the world economy given the index preserves to be tradable. The index was initiated by Goldman Sachs and has been calculated since 1969. Standard&Poors acquired the index from Goldman Sachs in 2007 [Standard&Poors, 2012]. We will further refer to this index as *S&P GSCI* or sometimes simply as *S&P*.

- The **HFRX Global Hedge Fund Index**[3] represents the equity type hedge funds and aims to reflect the overall composition of the hedge fund segment. It comprises a combintation of all eligible hedge fund strategies weighted according to their asset distribution in the segment. The index is constructed by the Hedge Fund Research Inc. (HFR) [Hedge Fund Research, 2014]. We will further refer to this index as *HFRX*.

- The **MSCI Emerging Market BRIC Index** represents the equity type emerging markets. It is is traded in US Dollars and is designed to measure the performance of the equity market in four countries – Brazil, Russia, India, China. Like the MSCI World, the MSCI BRIC is constructed by the American financial services provider MSCI. It is based on the same quality standards to ensure global views across all market capitalization segments of sector, style and size a well as to assure actuality. The MSCI BRIC was launched on Dec 06, 2005. Data prior to the launch date is back-tested data [MSCI., 2016a]. We will further refer to this index as *MSCI BRIC* or sometimes simply as *BRIC*.

[3] In comparison to the other indexes, the HFRX is not extracted from Bloomberg but, after registration, directly downloaded from the HFR website: https://www.hedgefundresearch.com/family-indices/hfrx.

Figure 5.1 shows daily historical prices of the five indexes described above starting at their individual initiation[4].

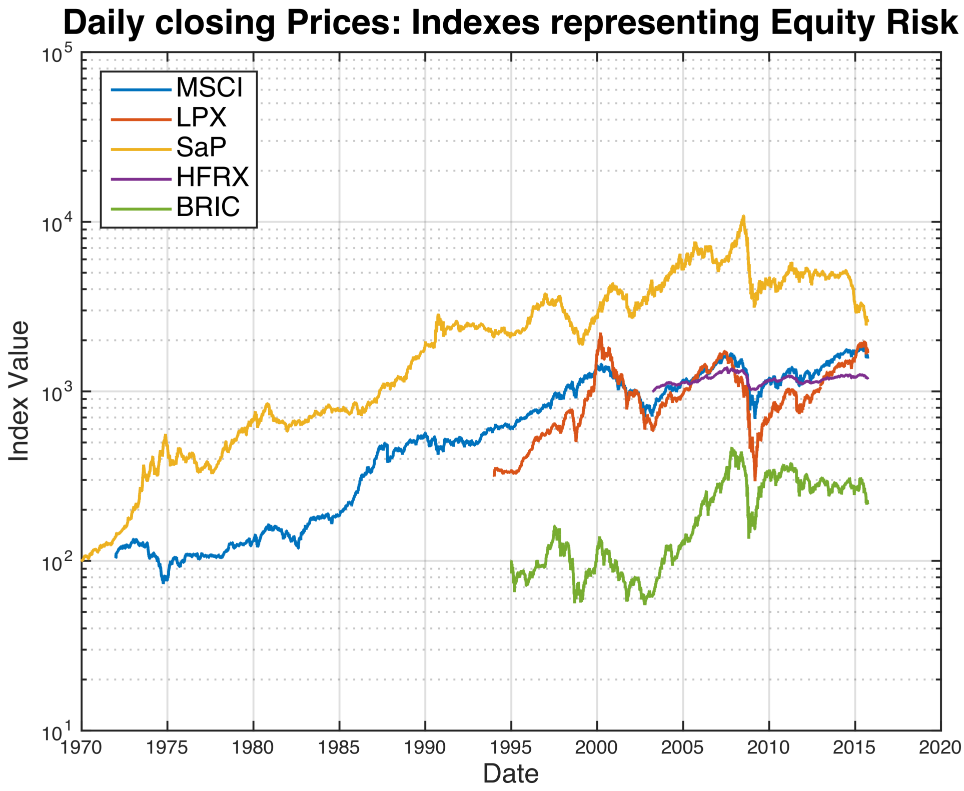


FIGURE 5.1: Monthly prices for representing indexes of equity risk.

For equity risk, CEIOPS/EIOPA bases its analysis on index data ending in 2009. The regulator does not indicate an exact date. We therefore choose the year-end of 2009 as end date for the "original data set". The more recent analysis is carried out on data ending in September 2015. Table 5.2 summarizes the initiation date of each index as well as the composition of both data sets.

		Original Data		Recent Data	
Index	**Initiation**	**End**	**Length**	**End**	**Length**
MSCI World	31.12.1971		9914		11412
LPX50	31.12.1993		4174		5672
S&P GSCI	01.01.1970	31.12.2009	10436	28.09.2015	11933
HFRX[5]	31.03.2003		1763		3260
MSCI BRIC	30.12.1994		3915		5412

TABLE 5.2: Data history of indexes representing equity risk.

[4]Note that the y-axis is log scaled in order to ensure a better visualization.

5.1.2 Property Risk

Property risk is represented by index data extracted from the Investment Property Databank (IPD). The index is provided by IPD Ltd. which was acquired by MSCI in 2012. According to [CEIOPS, 2010b], it is the most widely used commercial property index. The IPD indexes are produced for 17 European countries and 7 countries outside Europe in total. They measure total returns for directly held real estate assets as well as for the four main market sectors retail, office, industrial and residential. Depending on the country and its conditions, the results can be more granular [IPD, 2012].
For most countries data are provided annually or quarterly. However, for the UK market sector, monthly closing prices are available and therewith provide the greatest pool of available data within the databank. CEIOPS/EIOPA therefore decided to carry out analysis on the **Monthly IPD UK Total Return Index**. The UK property market sector is represented by six individual indexes. However, CEIOPS/EIOPA only defines five categories to carry out individual analysis. For the category 'commercial' it is not automatically perceptible which UK IPD index is used [CEIOPS, 2010b]. Table 5.3 summarized the five defined property categories together with the six available UK IPD indexes.

Categories	**Index**
All properties UK	IPD UK Total Return 'All Property'
Offices UK excl. London city	IPD UK Total Return 'Offices'
Offices London city	IPD UK Total Return 'City Offices'
Retail UK	IPD UK Total Return 'Retail'
Commercial UK	IPD UK Total Return 'Industry' IPD UK Total Return 'Warehouse'

Table 5.3: Indexes representing the sub-risk module property.

All six indexes representing property risk were initiated simultaneously in December 1986. For regulatory analysis carried out on those indexes, CEIOPS/EIOPA makes clear statements about the length as well as the end date of the data set. Those specifications are however inconclusive. Two possible original data sets are conceivable. Table 5.4 summarizes initiation date, end date and length of time series for those two possible original data sets as well as the recent data.

[5] HFRX includes several missing values (66 for original data and 11 for recent data).

		31.07.2008		31.12.2008		Recent Data Set	
Index	**Initiation**	**End**	**Length**	**End**	**Length**	**End**	**Length**
UK IPD	31.12.1986	31.12.2008	264	31.07.2008	259	29.02.2016	351

TABLE 5.4: Data history of indexes representing property risk.

Figure 5.2 shows the historical development of the six UK IPD indexes between 1986 and the beginning of 2016.

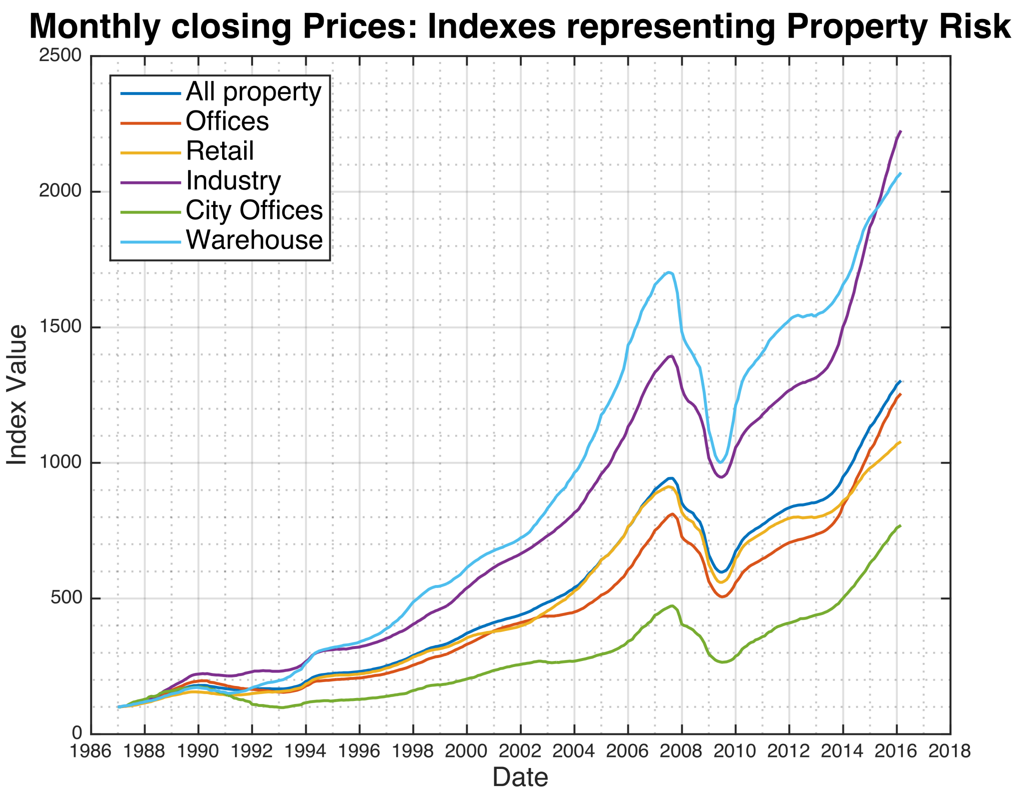


FIGURE 5.2: Monthly prices for representing indexes of property risk.

5.2 Replication of CEIOPS/EIOPA's Approach

The first step of the empirical analysis described in this chapter is the replication of CEIOPS/EIOPA's approach to calibrate standard formula input parameters for equity and property risk. To overcome the lack of sufficient annual returns, the regulator applies one-year rolling-windows to transform daily prices into annualized returns. For the first part of the analysis we base our calibrations on returns generated by the same technique. Histograms of the resulting rolling-window annualized returns can be found in Appendix B.

5.2.1 Stress Scenarios

Equity risk

Although reporting only one stress factor per equity category in their Level 2 Advice, CEIOPS/EIOPA carries out analysis on each of the five indexes described above. Table 5.5 summarizes the regulator's results for VaR calibrations together with the final proposed stress factors for the two categories and compares them to the outcomes of our replication study carried out on both data sets described above.

	Regulator		**Replication**	
	Empirical	**Final**	**Original Data**	**Recent Data**
MSCI World	−44.25%	−45.00%	−44.27%	−43.88%
LPX50	−68.67%	−55.00%	−68.95%	−67.50%
S&P GSCI	−59.45%		−59.66%	−59.21%
HFRX	−23.11%		−23.18%	−22.80%
MSCI BRIC	−63.83%		−63.90%	−61.43%

TABLE 5.5: Replicative VaR calibrations for indexes representing equity risk.

The outcomes of the replicative analysis are very similar to the regulator's results. Minor differences between CEIOPS/EIOPA's results and the replicative analysis carried out on the original data set could result from small differences in the length of data history since the regulator does not state the exact end date of the used time series.

Property risk

For the sub-risk module property, CEIOPS/EIOPA intially carries out analysis on 6 indexes. As stated above, the regulator's statements concerning the number of observations and the end date are inconclusive. The replication study reveals, that both conceivable data sets do not fit to the regulator's empirical VaR calibrations results. Via trial and error we find a data set where our outcomes of the replication study fit well to CEIOPS/EIOPA's results. Table 5.5 summarizes the calibration results based on all three original data sets as well as recent data and compares them to regulatory outcomes.

	Regulator		Replication			
	Empirical	**Final**	**31.07.2008**	**31.12.2008**	**30.03.2009**	**Recent**
All property	−25.74%	−25%	−15.18%	−2158%	−25.72%	−26.23%
Offices	−25.93%		−15.11%	−22.50%	−26.06%	−26.56%
City Offices	−30.03%		−24.41%	−27.27%	−30.73%	−30.68%
Retail	−27.47%		−16.20%	−22.11%	−27.30%	−27.95%
Industry	−27.67%		−13.58%	−19.17%	−21.55%	−21.86%
Warehouse			−19.02%	−23.84%	−28.07%	−28.30%

Table 5.6: Replicative VaR calibrations for indexes representing property risk.

According to the results in table 5.5 we assume that the category commercial is represented by the index for warehouse. Replicative VaR calibrations fit significantly better for this index in comparison to the index for industry.

5.2.2 Correlation

For the sub-risk module equity, CEIOPS/EIOPA proposes final stress factors for two equity categories. The aggregation of those two categories requires a parameter measuring the correlation between them. Similar to the stress calibration, regulatory analysis is carried out on all five indexes meaning CEIOPS/EIOPA calibrates a correlation parameter between the MSCI World representing "global equity" and each index representing "other equity". However, only an "average" correlation coefficient is considered in its final Level 2 Advice. Table 5.7 shows the results.

		Global	
		Empirical	**Final**
Other	LPX50	0.8359	0.75
	S&P GSCI	0.4472	
	HFRX	0.7731	
	MSCI BRIC	−0.5282	

Table 5.7: Regulatory tail correlation coefficients between the MSCI World representing "global equity" and each of the representative indexes of the category "other equity".

In the following we replicate CEIOPS/EIOPA's analysis on correlation within the risk-module equity[6].

Data-cutting correlation

As described in Chapter 3, CEIOPS/EIOPA reveals the use of the data-cutting approach for the risk module market risk [CEIOPS, 2010d]. Figure 5.3 illustrates the functionality of this approach. It shows scatter plots of each of the four possible combinations of "global equity" and "other equity". The red boxes in the diagrams are composed of the VaR for the MSCI World and the VaR of the respective representative for "other equity" for a certain confidence interval, here $1 - \alpha_1 = 0.995$, $1 - \alpha_2 = 0.95$ and $1 - \alpha_3 = 0.9$. The data points located in the particular box illustrate the data 'cutted out', meaning the data points considered for the calibration of the data-cutting correlation coefficient estimate.

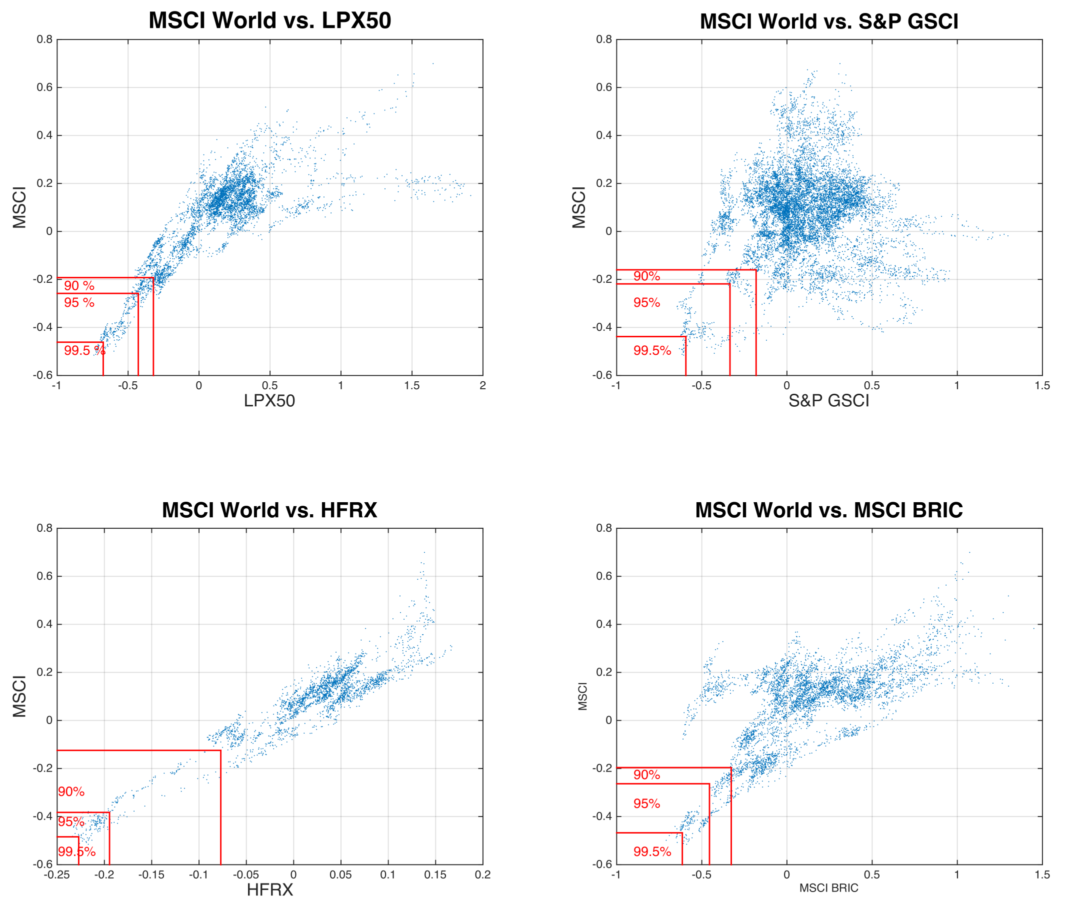


FIGURE 5.3: Relationship between the MSCI World and each representative of "other equity" including data-cutting boxes for exemplary percentiles.

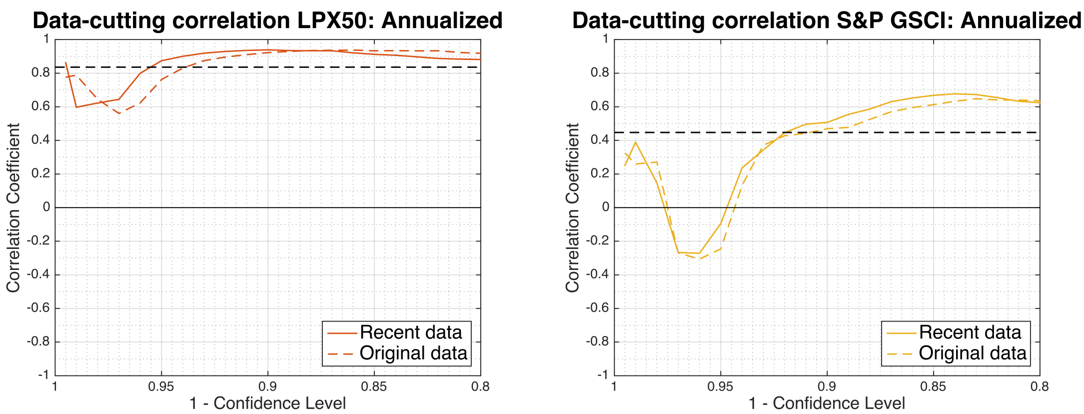
[6] Note that at this point of the study we focus on equity risk. For the sub-risk module property, there is no further breakdown into sub-categories. Hence, CEIOPS/ EIOPA does not carry out any analysis concerning correlation in this risk module.

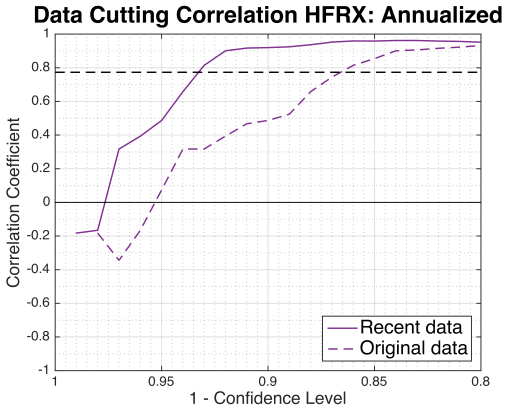
Assuming data-cutting correlation calibrations within equity risk, the regulator does not disclose the confidence level, the calibrations are based on. We therefore calibrate data-cutting correlation coefficients between the MSCI World and the four indexes representing "other equity" for various confidence levels. A range of results for both data sets is summarized in table 5.8.

	LPX50		S&P GSCI		HFRX		MSCI BRIC	
$1-\alpha$	**Original**	**Recent**	**Original**	**Recent**	**Original**	**Recent**	**Original**	**Recent**
0.995	0.7758	0.8624	0.3253	0.2506	NaN	NaN	0.0225	0.2722
0.99	0.7881	0.5974	0.2594	0.3894	NaN	−0.1829	0.3437	0.34
0.98	0.6445	0.6225	0.2708	0.1475	−0.1829	−0.1655	0.1511	0.2352
0.97	0.5606	0.6445	−0.2696	−0.2675	−0.3448	0.3171	0.3473	0.5089
0.96	0.624	0.7999	−0.306	−0.2716	−0.1655	0.3934	0.5091	0.4088
0.95	0.7623	0.8739	−0.2457	−0.0946	0.0734	0.4864	0.4467	0.4395
0.90	0.9222	0.9386	0.4695	0.5071	0.4864	0.9199	0.7495	0.871
0.85	0.9334	0.9123	0.6126	0.668	0.8544	0.9587	0.8849	0.9122
0.80	0.9183	0.8815	0.6361	0.6245	0.9322	0.9514	0.9045	0.8798

TABLE 5.8: Data-cutting correlation coefficients for varying confidence levels based on rolling-window annualized returns of indexes representing equity risk.

For most indexes the data-cutting correlation coefficients subject to the different confidence levels vary widely. Table 5.8 also reveals, that we cannot calibrate a data-cutting correlation coefficient for the HFRX when the confidence level is particularly small. For those confidence levels we do not find enough observations falling simultaneously below the confidence level. This could be reasoned by the relatively short data history of the index.
Figure 5.4 graphically shows the behavior of the data-cutting correlation coefficients over varying confidence levels for each of the four indexes. The black dashed line implies the correlation coefficient calibrated by CEIOPS/EIOPA.





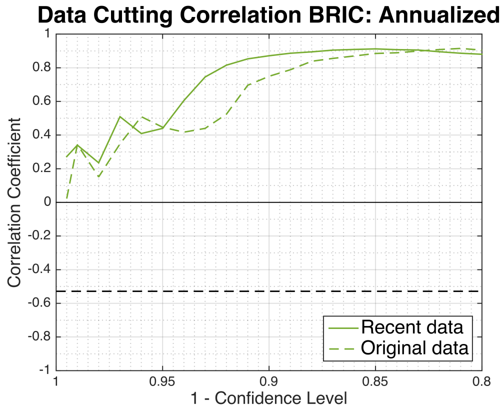


FIGURE 5.4: Behavior of data-cutting correlation estimates over varying confidence levels based on annualized returns of indexes representing equity risk.

Except for the HFRX, there are only minor differences between original and recent. The non-accordance for this index could again be explained by its short data history and the resulting relative difference between the original and the recent data set. For the MSCI BRIC we cannot find a match with CEIOPS/EIOPA's calibration result. For the remaining indexes there are matches. Nonetheless we cannot identify a coincident confidence level for them.

VaR-implied correlation

In its Level 2 Advice, CEIOPS/EIOPA also takes the VaR-implied correlation calibration methodology into consideration. Similar to the data-cutting analysis we calibrate VaR-implied correlation coefficients over varying confidence levels and for both data sets. Table 5.9 summarizes the results.

	LPX50		**S&P GSCI**		**HFRX**		**MSCI BRIC**	
α	**Original**	**Recent**	**Original**	**Recent**	**Original**	**Recent**	**Original**	**Recent**
0.995	1	1	0.831	0.8249	1	1	1	1
0.99	1	1	0.7759	0.8259	1	1	1	1
0.98	1	1	0.6221	0.5426	1	1	1	1
0.97	1	1	0.2509	0.1024	1	1	1	1
0.96	1	1	−0.0985	−0.2024	1	1	1	0.7551
0.95	1	1	−0.1275	−0.293	1	1	0.9487	0.6311
0.90	1	1	−0.5226	−0.3491	1	−0.0452	0.5485	0.355
0.85	1	1	−0.7641	−0.6205	1	0.2679	0.5491	1
0.80	1	1	−0.9559	−0.9645	1	1	1	1

TABLE 5.9: VaR-implied correlation coefficients for varying confidence levels based on rolling-window annualized returns of indexes representing equity risk.

For the LPX50, the VaR-implied correlation is constantly 1. The HFRX and the MSCI BRIC exhibit perfect correlations for numerous confidence levels, too. The results for the S&P GSCI vary extremely.
Figure 5.5 illustrates the result in comparison to CEIOPS/EIOPA's calibration result, again visualized by a dashed black line.

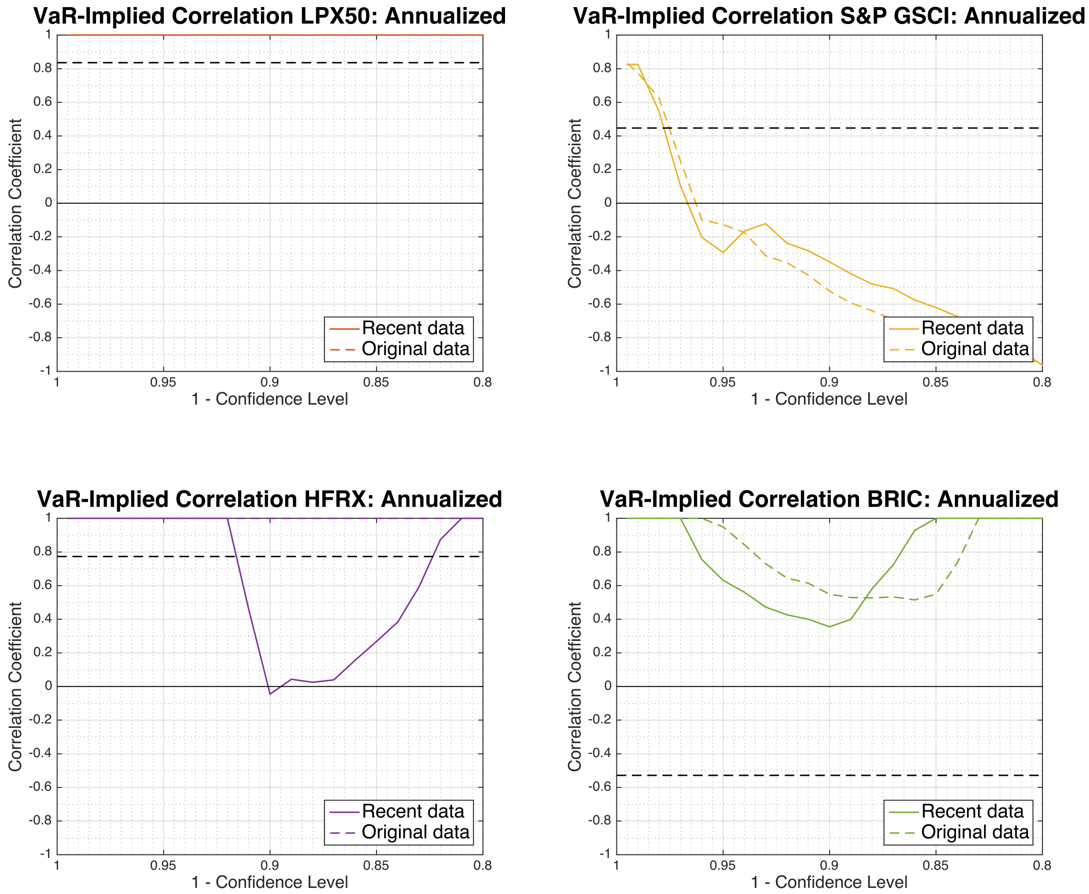


FIGURE 5.5: Behavior of VaR-implied correlation estimates over varying confidence levels based on annualized returns of indexes representing equity risk.

Figure 5.5 reveals that for the data used by the regulator, only the S&P GSCI exhibits a match with CEIOPS/EIOPA's empirical correlation coefficient.

Pearson Correlation

As discussed at several points, CEIOPS/EIOPA acknowledges the insufficiency of common linear correlation coefficients to fully reflect the dependence structure between risks. However, for the sake of completeness, we additionally calibrate the Pearson correlation coefficient between the MSCI World and the representative indexes of "other equity". Table 5.10 shows the results for both data sets in comparison to regulatory (tail) correlation calibration results.

	Regulator		Replication	
	Empirical	Adjusted	"Original Data"	"Recent Data"
LPX50	0.8359	0.75	0.7502	0.7680
S&P GSCI	0.4472		0.0653	0.0825
HFRX	0.7731		0.9664	0.9433
MSCI BRIC	−0.5282		0.6950	0.6660

Table 5.10: Linear Pearson correlation coefficients between "global equity" and each representing index of the equity category other.

The Pearson correlation coefficient between the LPX50 and the MSCI world is close to the result CEIOPS/EIOPA obtained. The others however differ considerably.

5.3 VaR based on FHS Annual Returns

Chapter 4 explained the possibility of severe distortions caused by the use of rolling-window annualized returns as basis for historical risk calibrations. The chapter also introduced Filtered Historical Simulation (FHS) as an alternative simulation approach. In this part of the empirical study we examine the application of FHS to gather a sufficient amount of annual data as basis for risk calibrations concerning the sub-risk modules equity and property. Our analysis is carried out according to a variation of the FHS approach described by [Barone Adesi et al., 1999]. The algorithm is given by

1. Transform daily prices of indexes representing equity risk into weekly[7] returns by

$$r_t = \frac{P_{w_{t+1}} - P_{w_t}}{P_{w_t}}, \qquad t = 1, \ldots, s-1 \tag{5.1}$$

   where P_{wt} denotes the closing price of the last trading day in week t and s refers to the amount of weeks available in the data. Respectively, for property risk transform monthly prices into monthly returns by

$$r_t = \frac{P_{t+1} - P_t}{P_t}, \qquad t = 1, \ldots, s-1 \tag{5.2}$$

   Here, P_t denotes the closing price of month t while s represents the length of the concerning price index.

[7]Weekly returns are chosen since those returns ca be aggregated to yearly data more easily as it would be the case with weekly returns.

2. Fit ARMA(p,q)-GARCH(1,1) models with p,q $\in \{0, 1, 2\}$ to each return series. Choose the best model via AIC/BIC criteria[8].

3. On the basis of the standardized model residuals randomly with replacement draw 100,000 subsets of length 52 and use them to generate pathways of future returns.

4. For each path aggregate pathways to future non-overlapping annual returns via

$$r^*_{ann\,j} = \sum_{i=1}^{p} (r^*_{j,i} + 1) - 1 \qquad j = 1, \ldots 100000$$

   with $p = 52$ for indexes representing equity risk and $p = 12$ for representatives of property risk.

5. On the basis of the set of future non-overlapping annual returns build bootstrap samples and calibrate 95% bootstrap confidence intervals of estimates for the 99.5% VaR.

5.3.1 Calibration Results

According to the algorithm described above we carried out analysis on all indexes representing equity risk and property risk. For equity risk, we analyze the original as well as th recent data. For property risk, we limit the analysis to recent data[9]. For each index, ARMA-GARCH models are fitted with two different assumptions concerning the distribution of ϵ_t.

Equity risk

For each of the two data sets we choose the best models via AIC/BIC criteria, one of them referring to the best model assuming normal distribution for ϵ_t while the other represents the best model assuming student-t distributed ϵ_t. VaR calibration results based on FHS annual returns obtained from those models are summarized in table 5.11.

[8] In case AIC/BIC criteria are inconclusive, choose in favor of the model with lower order.

[9] Results for the original data set appear to be highly unstable. Adding or excluding a minor number of observations to the investigated data set changed the result to a high degree.

Index	Data	ϵ_t	Model	95% CI for $\text{VaR}_{99.5}$	AIC	BIC
MSCI	Original	$\sim N$	$(0,0)$	$[-41.28\%, -39.32\%]$	-1.0248	-1.0225
		$\sim t$	$(0,0)$	$[-40.33\%, -38.85\%]$	-1.0322	-1.0294
	Recent	$\sim N$	$(0,0)$	$[-43.38\%, -41.48\%]$	-1.1755	-1.1732
		$\sim t$	$(0,0)$	$[-40.82\%, -39.26\%]$	-1.1842	-1.1813
LPX50	Original	$\sim N$	$(1,2)$	$[-81.91\%, -79.40\%]$	-3.7951	-3.7668
		$\sim t$	$(1,2)$	$[-80.71\%, -78.04\%]$	-3.8354	-3.8023
	Recent	$\sim N$	$(0,0)$	$[-74.51\%, -71.09\%]$	-5.1414	-5.1212
		$\sim t$	$(2,0)$	$[-67.96\%, -65.65\%]$	-5.2209	-5.1857
S&P	Original	$\sim N$	$(0,0)$	$[-48.68\%, -47.14\%]$	-9.8338	-9.8112
		$\sim t$	$(0,0)$	$[-49.87\%, -48.38\%]$	-9.866	-9.8378
	Recent	$\sim N$	$(0,0)$	$[-44.01\%, -42.19\%]$	-1.1194	-1.1171
		$\sim t$	$(0,0)$	$[-45.05\%, -43.19\%]$	-1.1244	-1.1215
HFRX	Original	$\sim N$	$(0,0)$	$[-14.97\%, -13.89\%]$	-2.5596	-2.5442
		$\sim t$	$(1,1)$	$[-25.76\%, -24.39\%]$	-2.597	-2.5699
	Recent	$\sim N$	$(1,1)$	$[-20.80\%, -20.01\%]$	-4.8823	-4.8554
		$\sim t$	$(1,1)$	$[-22,00\%, -21.43\%]$	-4.9375	-4.9062
BRIC	Original	$\sim N$	$(0,0)$	$[-64.18\%, -61.93\%]$	-2.9551	-2.9365
		$\sim t$	$(0,0)$	$[-68.33\%, -65.76\%]$	-2.987	-2.9645
	Recent	$\sim N$	$(2,0)$	$[-61.34\%, -59.64\%]$	-4.248	-4.2281
		$\sim t$	$(0,0)$	$[-68.04\%, -65.86\%]$	-4.2985	-4.2636

Table 5.11: Results for VaR calibrations based on annual returns obtained through filtered historical simulation for sub-risk module equity.

Again based on AIC/BIC criteria, we decide between the model assuming a normal distribution for ϵ_t and the model based on the assumption of student-t distributed ϵ_t. For all indexes representing equity risk, models assuming student-t distribution for ϵ_t appear to be the preferred models. In table 5.12 we compare the final model with CEIOPS/EIOPA's results for empirical VaR calibrations as well as our replication results.

		FHS	Rolling Window	
Index	**Data**	**95% CI for $\mathbf{VaR_{99.5}}$**	**Replication**	**Regulator**
MSCI	Original	[−40.05%, −38.57%]	−44.27%	−44.25%
	Recent	[−40.42%, −38.74%]	−43.81%	
LPX50	Original	[−80.71%, −78.04%]	−68.96%	−68.67%
	Recent	[−67.96%, −65.65%]	−67.50%	
S&P GSCI	Original	[−49.87%, −48.38%]	−59.66%	−59.45%
	Recent	[−45.05%, −43.19%]	−59.21%	
HFRX	Original	[−25.76%, −24.39%]	−23.10%	−23.11%
	Recent	[−22.00%, −21.43%]	−22.72%	
BRIC	Original	[−68.33%, −65.76%]	−61.96%	−63.83%
	Recent	[−68.04%, −65.86%]	−61.43%	

Table 5.12: VaR calibration results based on FHS annual returns vs. original and replicative results for calculations on the basis of rolling-window annual returns for indexes representing equity risk.

For the MSCI World and the S&P GSCI, the calibration results unequivocally imply an overestimation of risk by CEIOPS/EIOPA's approach. The same applies to the recent data sets of the LPX50 and the HFRX. However, the MSCI BRIC as well as the original data of the LPX50 exhibit more negative results for calibrations based on FHS annual returns.

Property risk

Table 5.13 summarizes VaR calibration results for the best models fitted to indexes representing property risk.

Index	**Data**	ϵ_t	**Model**	**95% CI for VaR$_{99.5}$**	**AIC**	**BIC**
All prop.	Recent	$\sim N$	$(2,2)$	$[-6.72\%, -6.06\%]$	-2.9621	-2.9312
		$\sim t$	$(1,1)$	$[-9.16\%, -8.11\%]$	-2.9963	-2.9693
Offices	Recent	$\sim N$	$(1,1)$	$[-10.58\%, -9.51\%]$	-2.8384	-2.8153
		$\sim t$	$(1,1)$	$[-10.53\%, -9.25\%]$	-2.8733	-2.8463
City off.	Recent	$\sim N$	$(2,2)$	$[-9.29\%, -8.51\%]$	-2.3912	-2.3603
		$\sim t$	$(1,1)$	$[-7.41\%, -6.67\%]$	-2.4555	-2.4285
Retail	Recent	$\sim N$	$(1,1)$	$[-6.95\%, -6.36\%]$	-2.9046	-2.8815
		$\sim t$	$(1,1)$	$[-6.50\%, -5.93\%]$	-2.9285	-2.9015
Warehouse	Recent	$\sim N$	$(1,1)$	$[-5.17\%, -4.54\%]$	-2.6579	-2.6347
		$\sim t$	$(1,1)$	$[-7.31\%, -6.49\%]$	-2.7194	-2.6924

Table 5.13: Results for VaR calibrations based on annual returns obtained through filtered historical simulation for sub-risk module property

Here, too, the assumption of student-t distribution is superior to models assuming normal distribution for ϵ_t. Table 5.14 compares the VaR calibration results based on FHS annual returns obtained through the preferred models to CEIOPS/EIOPA's empirical calibration results.

		FHS	**Rolling Window**	
Index	**Data**	**95% CI for VaR$_{99.5}$**	**Replication**	**Regulator**
All prop.	Recent	$[-9.16\%, -8.11\%]$	-25.74%	-26.23%
Offices	Recent	$[-10.53\%, -9.25\%]$	-25.93%	26.56%
City off.	Recent	$[-7.41\%, -6.67\%]$	-30.03%	-30.68%
Retail	Recent	$[-6.50\%, -5.93\%]$	-27.47%	-27.95%
Warehouse	Recent	$[-7.31\%, -6.49\%]$	-27.67%	-28.30%

Table 5.14: VaR calibration results based on FHS annual returns vs. original and replicative results for calculations on the basis of rolling-window annual returns for indexes representing property risk.

VaR calibrations based on FHS annual returns clearly reveal an overestimation of risk by the regulatory rolling-window approach. We obtain deviations from CEIOPS/EIOPA's empirical results accounting to 15 to 20 percent.

5.3.2 Diagnostics and Limitations

For property risk, we renounced the reporting of outcomes obtained from analysis carried out on the original data sets. Calibration results reacted extremely volatile to minor changes in the length of time series. The recent data behaves more stable. However, since the UK IPD Property Index is only provided on a monthly basis, both data sets have a relatively short data history. Indexes representing equity risk are daily available and therewith exhibit a longer history. However, in comparison to the other four representatives of equity risk, the time series for HFRX is significantly shorter.
Besides problems concerning the amount of historical observations, the indexes analyzed in this study exhibit some characteristics which complicate profound analyses and conclusions. In this section we will look into the sample moments of weekly as well as FHS annual returns for each index. Further, we will discuss two difficulties encountered in the course of fitting ARMA(p,q)-GARCH(1,1) models.

Moments
The first four sample moments are given by *mean*, *standard deviation*, *skewness* and *kurtosis*. Skewness measures the asymmetry of the data around the sample mean. Kurtosis looks into the tails of the distribution and measures the sample's outlier-sensibility. The skewness of a normal distribution is zero while its kurtosis is 3. Table 5.15 summarizes the four described moments of weekly returns in comparison to FHS annual returns for indexes representing equity risk.

		Equity: Original Data			
Index	**Return**	**Mean**	**Standard Deviation**	**Skewness**	**Kurtosis**
MSCI	weekly	0.0014	0.0204	−0.7268	11.368
	FHS	0.0693	0.1613	0.0765	4.4537
LPX	weekly	0.0015	0.0326	−1.0481	12.7299
	FHS	0.0792	0.3068	0.6517	7.249
S&P	weekly	0.0022	0.026	−0.4489	6.7476
	FHS	0.1227	0.2492	0.9405	8.3531
HFRX	weekly	0.0004	0.0075	−2.3527	15.52
	FHS	0.0422	0.0861	−0.786	6.0586
BRIC	weekly	0.0023	0.0403	−0.3957	7.1007
	FHS	0.1523	0.3189	0.3685	4.5555

TABLE 5.15: Mean, standard deviation, skewness and kurtosis for for weekly as well as FHS annual returns of indexes representing equity risk: Original data.

Index	Return	Equity: Recent Data Mean	Standard Deviation	Skewness	Kurtosis
MSCI	weekly	0.0014	0.0204	−0.6929	10.4743
	FHS	0.0658	0.1618	0.074	4.3494
LPX	weekly	0.002	0.031	−1.0058	12.2782
	FHS	0.0792	0.3068	0.6517	7.249
S&P	weekly	0.0017	0.0259	−0.4588	6.4825
	FHS	0.0926	0.2103	0.6276	6.6992
HFRX	weekly	0.0003	0.0065	−2.1576	15.455
	FHS	0.0056	0.0704	−0.4629	4.131
BRIC	weekly	0.0014	0.0375	−0.338	7.3336
	FHS	0.0742	0.2772	0.4489	4.5882

Table 5.16: Mean, standard deviation, skewness and kurtosis for for weekly as well as FHS annual returns of indexes representing equity risk: Recent data.

Table 5.15 and table 5.16 imply that FHS annual returns are ”more normal” than weekly returns. For the majority of indexes, the skewness of FHS annual returns is closer to zero than it is for weekly data. Also, the kurtosis reduces for almost all indexes when moving from weekly to FHS annual returns. An exception is solely given by the S&P GSCI. Here, skewness and kurtosis are slightly higher for FHS annual returns.

Moments of monthly returns in comparison FHS annual returns for indexes representing property risk are shown in table 5.17. The moments summarized in table 5.17 do not generally imply a ”normalization” of data when moving from weekly to FHS annual returns. The results for skewness are diverse. Some indexes exhibit a decrease in skewness while others seem to become more skew. The kurtosis increases for all five indexes and literally explodes for warehouse and all property. Remembering the course of the time series illustrated in figure 5.2, a possible explanation could be the short data history in combination with the extreme variations of prices within a short time period.

		Property: Recent Data			
Index	**Return**	**Mean**	**Standard Deviation**	**Skewness**	**Kurtosis**
All property	weekly	0.0074	0.0107	−1.7173	10.1969
	FHS	0.0474	0.1058	4.6851	152.086
Offices	weekly	0.0073	0.0119	−1.3619	7.8432
	FHS	0.0536	0.0982	1.2397	22.681
City Offices	weekly	0.006	0.0157	−1.5011	8.7655
	FHS	0.104	0.0616	0.8828	10.5438
Retail	weekly	0.0069	0.0109	−1.7716	11.5282
	FHS	0.0354	0.089	2.4166	46.4059
Warehouse	weekly	0.0088	0.013	−1.3066	9.3049
	FHS	0.0883	0.0562	10.4302	906.1114

Table 5.17: The first four moments mean, standard deviation, skewness and kurtosis for weekly as well as FHS annual returns of indexes property risk.

iGARCH effects

Per model definition, the parameters of ARMA-GARCH models are required to meet specific conditions. One of them requires the sum of the ARCH and GARCH parameters to be less than 1, that is $\sum_{i=1}^{max(m,s)}(\alpha_i + \beta_i) < 1$. If the condition is not fulfilled, the unconditional variance of the innovation a_t approaches infinity. Weak stationarity is not longer given. The phenomenon of the GARCH parameters summing up to 1 is called integrated GARCH (iGARCH) effect. Those iGARCH effects are likely to have spuriously impact on volatility forecasts [Franke et al., 2015].

Table 5.18 gives an overview of iGARCH effects detected[10] for ARMA-GARCH fits of data representing equity risk. The table shows, that models fitted for the LPX50 cumulatively exhibit iGARCH effects when assuming normal distribution for ϵ_t. The remaining models seem to be free of those effects.

[10] Affected models identified by [MATLAB, 2015].

	Equity: Original Data										Equity: Recent Data									
	MSCI		LPX		S&P		HFRX		BRIC		MSCI		LPX		S&P		HFRX		BRIC	
	nv	*t*	*nv*	*t*	*nv*	*t*	*nv*	*t*	*nv*	*t*	*nv*	*t*	*nv*	*t*	*nv*	*t*	*nv*	*t*	*nv*	*t*
(0,0)	-	-	x	-	-	-	-	-	-	-	-	-	x	-	-	-	-	-	-	-
(0,1)	-	-	x	-	-	-	-	-	-	-	-	-	x	-	-	-	-	-	-	-
(0,2)	-	-	x	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(1,0)	-	-	x	-	-	-	-	-	-	-	-	-	x	-	-	-	-	-	-	-
(1,1)	-	-	x	-	-	-	-	-	-	-	-	-	x	-	-	-	-	-	-	-
(1,2)	-	-	x	-	-	-	-	-	-	-	-	-	x	-	-	-	-	-	-	-
(2,0)	-	-	-	-	-	-	-	-	-	-	-	-	x	-	-	-	-	-	-	-
(2,1)	-	-	x	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2,2)	-	-	x	-	-	-	-	-	-	-	-	-	x	-	-	-	-	-	-	-

TABLE 5.18: Present iGARCH effects for ARMA(p,q)-GARCH(1,1) models fitted for indexes representing the sub-risk module equity.

Table 5.19 shows the same reports for ARMA(p,q)-GARCH(1,1) fitted to indexes representing the sub-risk module property.

	Property: Recent Data									
	All prop.		Offices		City off.		Retail		Wareh.	
	nv	*t*	*nv*	*t*	*nv*	*t*	*nv*	*t*	*nv*	*t*
$(0,0)$	x	x	x	x	x	x	x	x	x	x
$(0,1)$	x	-	x	x	-	x	x	x	x	-
$(0,2)$	x	x	x	x	-	x	x	x	x	x
$(1,0)$	x	x	x	x	-	x	-	-	x	x
$(1,1)$	x	x	-	x	-	-	-	x	x	-
$(1,2)$	x	-	x	x	x	-	-	x	-	x
$(2,0)$	x	x	-	-	-	-	-	x	-	x
$(2,1)$	-	-	x	-	x	x	-	x	-	x
$(2,2)$	x	x	x	x	x	x	-	x	x	-

TABLE 5.19: Present iGARCH effects for ARMA(p,q)-GARCH(1,1) models fitted for indexes representing the sub-risk module property.

According to table 5.19 the majority of models is affected by iGARCH effects. This includes most of the models chosen as best fits.

Pole-zero cancellation

Both models chosen for the original data of the LPX50 exhibit so called pole-zero cancellation. To understand this phenomenon, consider an ARMA(1,1) model[11] given by

$$y_t = \phi_1 y_{t-1} + \psi_1 \epsilon_{t-1} + \epsilon_t. \tag{5.3}$$

With the help of a lag operator L defined by $Ly_t = y_{t-1}$, y_t can be written as

$$y_t = \frac{1 + \phi_1 y_t}{1 + \psi_1 \epsilon_t} \epsilon_t. \tag{5.4}$$

If $\phi_1 = -\psi_1$, y_t reduces to a white noise given by

$$y_t = \epsilon_t. \tag{5.5}$$

Thus, the ARMA(1,2) model reduces to an ARMA(0,0) [Verbeek, 2001]. This effect is referred to as pole-zero cancellation.
For the original data of the LPX50 we chose ARMA(1,1) models for both assumptions concerning the distribution of ϵ_t. With $\phi_1 = 0.768521$ and $\psi_1 = -0762193$ assuming normal distribution for ϵ_t and $\phi_1 = 0.781838$ and $\psi_1 = -0.780076$ assuming student-t distributed ϵ_t, both best models exhibit pole-zero cancellation. Allowing for this effect, the models reduce to ARMA(0,1) models.
Table 5.20 compares VaR calibration results for the final model with the outcomes for the reduced model. For the sake of completeness, the results for the unaffected recent data set is included.

			FHS	**Rolling Window**	
Index	**Data**	**Model**	95% CI for $\text{VaR}_{99.5}$	Replication	Regulator
LPX50	Original	$(1,1)$	$[-80.71\%, -78.04\%]$	-68.96%	-68.67%
		$(0,0)$	$[-69.13\%, -66.22\%]$		
	Recent	$(1,2)$	$[-67.96\%, -65.65\%]$	-67.50%	

TABLE 5.20: Comparison between VaR calibration results of final model and calibration outcomes of the reduced model when allowing for pole-zero canellation.

Allowing for pole-zero cancellation, VaR calibrations based on FHS anual returns would rather imply an overestimation of risk by the regulatory approach.

[11] For exemplary purposes, we assume the constant parameter ϕ_0 equals zero.

5.4 Correlation based on FHS Annual Returns

[Mittnik, 2011] clearly showed, that the application of rolling-windows not only affects results of VaR calibrations. It also distorts the dependence structure between risks. In this section, we analyze the correlation between the two equity categories "global equity" and "other equity" based on annual returns obtained via FHS. To simulate FHS annual returns suitable for correlation calibrations, we use a slightly modified FHS algorithm[12].

1. Transform daily prices into weekly returns.
2. Fit ARMA(p,q)-GARCH(1,1) models with p,q $\in \{0, 1, 2\}$ to each return series. Choose the best model via AIC/BIC criteria[13].
3. Simultaneously, randomly with replacement draw pairs of residuals composed of a residual of time t from the best model for the MSCI World and a residual of the same time t from the best model for one of the indexes representing "other equity". Based on the resulting $100,000$ subsets of pairs with length 52 generate 100,000 pathways of pairs of future returns.
4. Aggregate the pathways to non-overlapping pairs of annual returns.
5. Build bootstrap samples and calibrate the 95% bootstrap confidence interval for the correlation coefficient bootstrap estimate.

Based on annual FHS data obtained via the algorithm above, we calibrate 95% bootstrap confidence intervals for correlation coefficients based on the data-cutting approach, the VaR-implied correlation approach and Pearson correlation.

Data-cutting correlation

Table 5.21 and table 5.22 show the results for bootstrap confidence intervals of correlation coefficients obtained via the data-cutting approach. Again, analysis is carried out on original data as well as the recent data set.

[12]For familiar reasons, we again focus on equity risk. The algorithm is accordingly tailored to characteristics of data representing this risk module.

[13]The best models are obviously the same as in the previous FHS study for VaR calibrations, since we use the same data.

	Original Data			
$1-\alpha$	**LPX50**	**S&P GSCI**	**HFRX**	**BRIC**
0.995	$[-0.0017, 0.3232]$	$[-0.9735, 0.9990]$	$[-0.1132, 0.2266]$	$[-0.0792, 0.2925]$
0.99	$[0.1870, 0.3648]$	$[-0.3867, 0.3983]$	$[0.0551, 0.2652]$	$[0.0398, 0.2852]$
0.98	$[0.2631, 0.3938]$	$[-0.1389, 0.2194]$	$[0.1063, 0.2732]$	$[0.1411, 0.2911]$
0.97	$[0.3026, 0.4047]$	$[-0.2088, 0.0943]$	$[0.1728, 0.2969]$	$[0.1804, 0.3068]$
0.96	$[0.3084, 0.4035]$	$[-0.1295, 0.1100]$	$[0.2371, 0.3437]$	$[0.1862, 0.2900]$
0.95	$[0.3307, 0.4150]$	$[-0.0696, 0.1165]$	$[0.2612, 0.3458]$	$[0.2034, 0.2991]$
0.90	$[0.4198, 0.4757]$	$[-0.0181, 0.0869]$	$[0.3408, 0.4025]$	$[0.2839, 0.3398]$
0.85	$[0.4549, 0.4935]$	$[0.0191, 0.0879]$	$[0.3863, 0.4333]$	$[0.3061, 0.3526]$
0.80	$[0.4647, 0.5003]$	$[0.0420, 0.1073]$	$[0.4134, 0.4573]$	$[0.3190, 0.3651]$

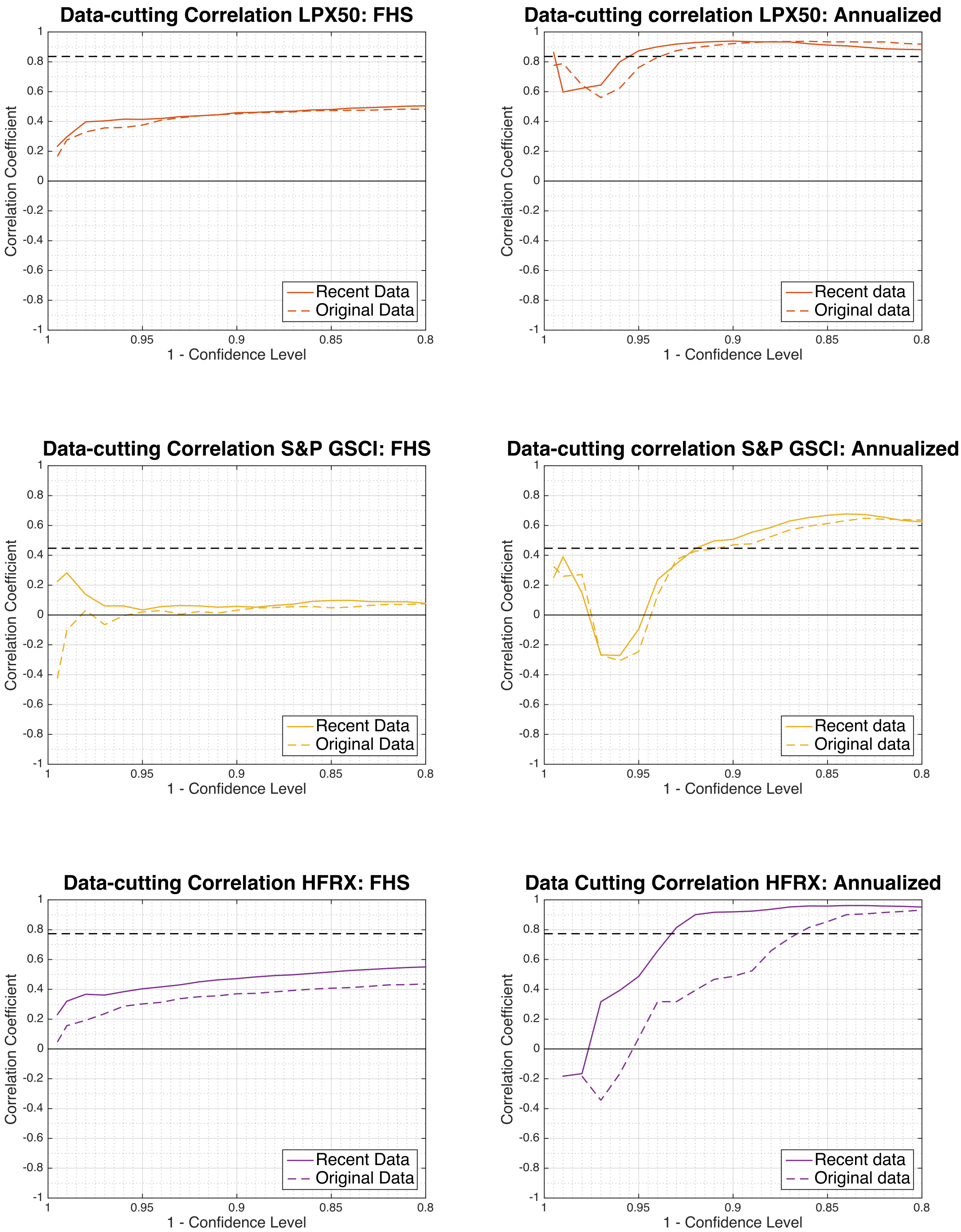
Table 5.21: 95% bootstrap confidence intervals for data-cutting correlation coefficients between "global equity" and "other equity" based on FHS annual returns of original data.

	Recent Data			
$1-\alpha$	**LPX50**	**S&P GSCI**	**HFRX**	**BRIC**
0.995	$[0.0536, 0.3828]$	$[-0.4511, 0.9416]$	$[0.0618, 0.3486]$	$[0.1037, 0.5131]$
0.99	$[0.1929, 0.3976]$	$[-0.1413, 0.6725]$	$[0.2038, 0.4298]$	$[0.1709, 0.4383]$
0.98	$[0.3288, 0.4666]$	$[-0.1127, 0.4167]$	$[0.2913, 0.4328]$	$[0.2395, 0.4031]$
0.97	$[0.3406, 0.4552]$	$[-0.1024, 0.2268]$	$[0.3094, 0.4191]$	$[0.2925, 0.4047]$
0.96	$[0.3585, 0.4594]$	$[-0.0652, 0.2073]$	$[0.3360, 0.4277]$	$[0.2715, 0.3736]$
0.95	$[0.3598, 0.4539]$	$[-0.0669, 0.1574]$	$[0.3571, 0.4420]$	$[0.2754, 0.3683]$
0.90	$[0.4237, 0.4902]$	$[0.0062, 0.1125]$	$[0.4433, 0.4981]$	$[0.3272, 0.3933]$
0.85	$[0.4543, 0.5024]$	$[0.0590, 0.1396]$	$[0.4964, 0.5374]$	$[0.3604, 0.4143]$
0.80	$[0.4823, 0.5238]$	$[0.0480, 0.1085]$	$[0.5324, 0.5658]$	$[0.3703, 0.4147]$

Table 5.22: 95% bootstrap confidence intervals for data-cutting correlation coefficients between "global equity" and "other equity" based on FHS annual returns of recent data.

For small confidence levels α the 95% bootstrap confidence interval are conspicuously wide, implying rather unstable data-cutting correlation coefficients for the bootstrap samples. This is specifically noticeable for original data of the S&P GSCI and $(1-\alpha) = 0.995$. The 95% bootstrap confidence interval almost spans the interval of possible values for correlation coefficients.

Figure 5.6 illustrates the course of the data-cutting correlation coefficients with varying confidence levels and compares them to the corresponding data-cutting correlation coefficients calibrated on the basis of rolling-window annualized returns. For the visualization in figure 5.6, we used the median data-cutting correlation coefficient bootstrap estimate. CEIOPS/EIOPA's calibration results are indicated by a dashed black line.



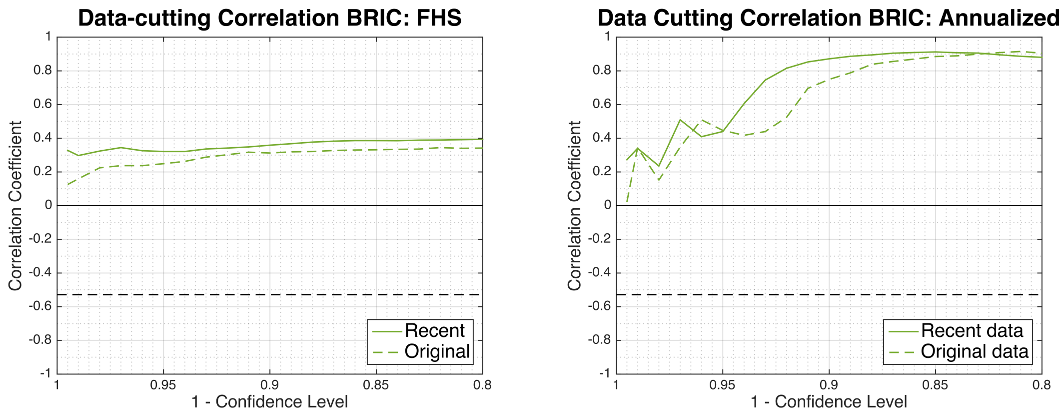


FIGURE 5.6: Data-cutting correlation based on FHS annual returns in comparison to results of calibrations based on rolling-window annual returns.

Except for the MSCI BRIC, the median correlation coefficient estimates are clearly below the regulator's correlation calibration outcome. Also, the course of the median data-cutting correlation coefficients based on FHS annual data seems to be more stable in comparison to coefficients calibrated on the basis of rolling-window annualized returns. Nonetheless, the width of the bootstrap confidence intervals should be kept in mind.

VaR-implied correlation

Table 5.23 contains the 95% bootstrap confidence intervals for VaR-implied correlation analysis carried out on FHS annual returns.

	Original Data			
$\mathbf{1-\alpha}$	**LPX50**	**S&P GSCI**	**HFRX**	**BRIC**
0.995	$[0.6484, 0.7272]$	$[-0.1354, -0.0750]$	$[0.6058, 0.7197]$	$[0.4884, 0.5672]$
0.99	$[0.6825, 0.7435]$	$[-0.1154, -0.0628]$	$[0.6209, 0.7126]$	$[0.5483, 0.6093]$
0.98	$[0.6201, 0.6781]$	$[-0.1119, -0.0746]$	$[0.6586, 0.7308]$	$[0.5217, 0.5809]$
0.97	$[0.6145, 0.6617]$	$[-0.1256, -0.0912]$	$[0.6483, 0.7088]$	$[0.5238, 0.5709]$
0.96	$[0.6179, 0.6575]$	$[-0.1387, -0.0992]$	$[0.6899, 0.7429]$	$[0.5239, 0.5693]$
0.95	$[0.6254, 0.6653]$	$[-0.1552, -0.1158]$	$[0.7054, 0.7623]$	$[0.5240, 0.5709]$
0.90	$[0.6025, 0.6449]$	$[-0.2513, -0.2111]$	$[0.6907, 0.7544]$	$[0.5142, 0.5614]$
0.85	$[0.5491, 0.5955]$	$[-0.4120, -0.3665]$	$[0.6564, 0.7233]$	$[0.4814, 0.5355]$
0.80	$[0.4467, 0.5202]$	$[-0.6598, -0.6064]$	$[0.5023, 0.6380]$	$[0.3783, 0.4485]$

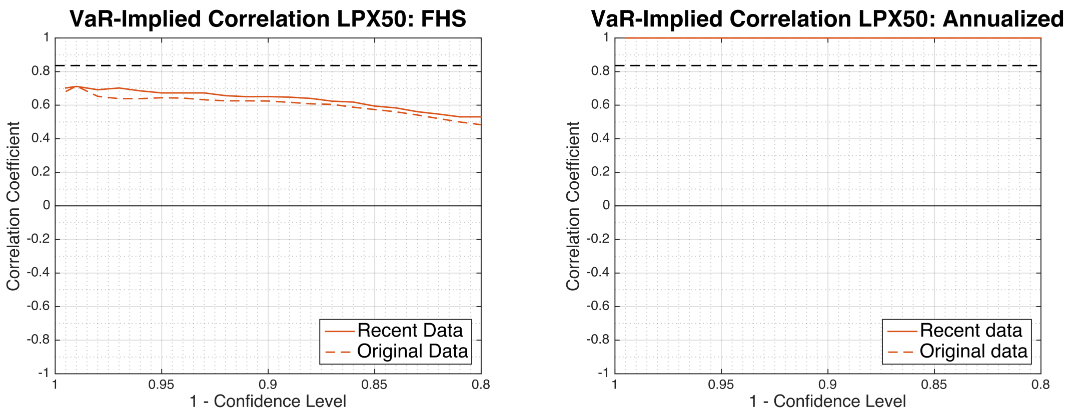
TABLE 5.23: 95% bootstrap confidence intervals for VaR-implied correlation coefficients between "global equity" and "other equity" based on FHS annual returns of original data.

	Recent Data			
$1-\alpha$	**LPX50**	**S&P GSCI**	**HFRX**	**BRIC**
0.995	$[0.6549, 0.7448]$	$[-0.0387, 0.0193]$	$[0.6623, 0.7366]$	$[0.599, 0.6723]$
0.99	$[0.6804, 0.7461]$	$[-0.0070, 0.0551]$	$[0.7605, 0.8370]$	$[0.5985, 0.6580]$
0.98	$[0.6664, 0.7214]$	$[-0.0034, 0.0392]$	$[0.8045, 0.8577]$	$[0.6035, 0.6583]$
0.97	$[0.6767, 0.7307]$	$[-0.0121, 0.0345]$	$[0.7981, 0.8498]$	$[0.5944, 0.6406]$
0.96	$[0.6640, 0.7099]$	$[-0.0244, 0.0166]$	$[0.7971, 0.8513]$	$[0.5930, 0.6428]$
0.95	$[0.6489, 0.7018]$	$[-0.0526, -0.0103]$	$[0.7637, 0.8127]$	$[0.581, 0.6199]$
0.90	$[0.6294, 0.6723]$	$[-0.1386, -0.1010]$	$[0.8052, 0.8500]$	$[0.5807, 0.6207]$
0.85	$[0.5680, 0.6225]$	$[-0.2591, -0.2108]$	$[0.7900, 0.8398]$	$[0.5626, 0.6093]$
0.80	$[0.5045, 0.5645]$	$[-0.4876, -0.4281]$	$[0.7693, 0.8228]$	$[0.4755, 0.5338]$

TABLE 5.24: 95% bootstrap confidence intervals for VaR-implied correlation coefficients between "global equity" and "other equity" based on FHS annual returns of recent data.

In comparison to the bootstrap confidence intervals for the data-cutting approach, the span of bootstrap confidence intervals for VaR-implied correlation coefficient estimates are considerably lower. The size of the bootstrap confidence interval seems to be approximately constant over varying confidence levels.

Again, we visualize the course of the median VaR-implied correlation coefficient estimates with varying confidence levels and compare them to the regulator's results as well as VaR-implied correlation calibration outcomes based on rolling-window annualized returns. Figure 5.7 shows the results.



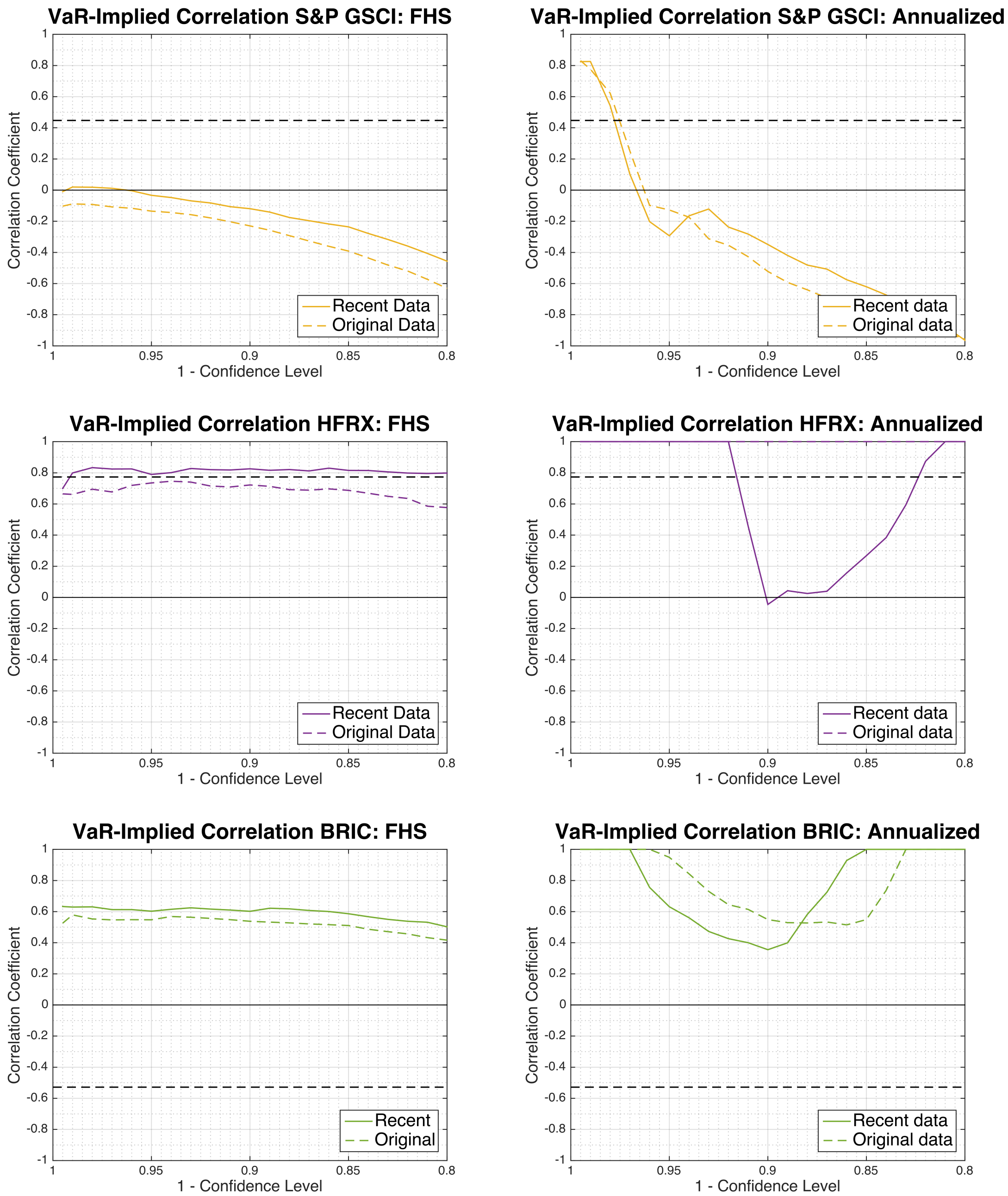


FIGURE 5.7: VaR-implied correlation based on FHS annual returns in comparison to results of calibrations based on rolling-window annual returns.

For the VaR-implied correlation approach too, the course of the median correlation coefficient estimate seems to be more stable than results for calibrations based on rolling-window annualized returns. Correlations for the LPX50 as well as the S&P GSCI are considerably lower than the regulator's result.

Pearson correlation

For the sake of completeness, table 5.25 compares linear Pearson correlation coefficient estimates based on FHS annual returns with calibration results on the basis of rolling-window annual returns as well as CEIOPS/EIOPA's empirical outcomes.

Index	Data	FHS annual	Annualized	Regulator
LPX	Original	[0.6113, 0.6240]	0.7502	0.8359
	Recent	[0.6767, 0.6852]	0.7680	
S&P GSCI	Original	[0.0951, 0.1075]	0.0653	0.4472
	Recent	[0.1446, 0.1568]	0.0825	
HFRX	Original	[0.7611, 0.7670]	0.9664	0.7731
	Recent	[0.8071, 0.8114]	0.9433	
BRIC	Original	[0.6166, 0.6247]	0.6950	−52.82
	Recent	[0.6433, 0.6504]	0.6660	

Table 5.25: 95% bootstrap confidence intervals for linear Perason correlation coefficient estimates in comparison to regulatory outcomes as well as results for calibrations based on rolling-window annual returns.

For most indexes, the results using FHS annual returns seem to be lower than the corresponding outcomes of calibrations based on rolling-window annualized returns. Repeatedly, we cannot find a match with regulatory results.

In summary, the VaR-implied correlation approach as well as the data-cutting correlation approach based on FHS annual returns exhibits results considerably different form CEIOPS/EIOPA's outcomes. In fact, our results imply, that CEIOPS/EIOPA's proposed correlation of 0.75 between "global equity" and "other equity" might be too close to "1".

The course of the median correlation coefficient estimates seems to be more stable for calibrations based on FHS annual returns than rolling-window annualized returns. However, for the data-cutting approach, the width of the 95% bootstrap confidence intervals for particularly low confidence intervals should be kept in mind. Since per definition data-cutting correlation subject to small confidence intervals rely on a particularly small set of observations, the large variation of calibration results for the different bootstrap samples is not astonishing. This again implies the superiority of VaR-implied correlation towards data-cutting correlation estimates.

We also carried out correlation analysis on the basis of daily returns. Similar to calibrations based on FHS annual returns, the course of VaR-implied and data-cutting correlation coefficients seems to be more stable over varying confidence levels than the corresponding coefficients based on rolling-window annualized returns. The values of the correlation coefficients based on daily returns are similar to the median outcomes obtained on the basis of FHS annual returns. However, they tend to be slightly closer to 1. For plots and tables of calibration results based on daily returns refer to Annex B.

5.5 Diversification within sub-risk modules

CEIOPS/EIOPA analyzes the correlation between the MSCI World as representative of the equity category "global equity" and each of the four indexes representing the category "other equity". However, the regulator proposes a single "averaged" correlation coefficient of 0.75 between the two categories. Correlation within the category "other equity" is not analyzed at all. Diversification benefits within this category are thereby neglected. For property risk, there is no further breakdown into sub-categories. Analyses concerning correlation within this risk module are therefor relinquished.

In this part of our study, we analyze the regulator's decisions concerning correlations within property risk and within the equity category "other equity". We calibrate Pearson correlations as well as VaR-implied correlations based on daily returns[14].

5.5.1 Equity Risk

Pearson correlation

Table 5.26 contains linear Pearson correlation coefficients based on daily returns for all possible combinations of the four indexes representing the equity category "other equity". Table 5.26 and table 5.27 show the results for calibrations carried on the original and recent data set.

[14]The use of annual FHS results for this part of analysis is conceivable, too

	Original data: Daily			
Index	**LPX**	**S&P GSCI**	**HFRX**	**BRIC**
LPX	1	0.1857	0.5734	0.5114
S&P GSCI		1	0.3384	0.2264
HFRX			1	0.6606
BRIC				1

Table 5.26: Linear perason correlation coefficients within "other equity" based on the original data set.

	Recent Data: Daily			
Index	**LPX**	**S&P GSCI**	**HFRX**	**BRIC**
LPX	1	0.2311	0.6277	0.5253
S&P GSCI		1	0.3695	0.2648
HFRX			1	0.6467
BRIC				1

Table 5.27: Linear perason correlation coefficients within "other equity" based on the recent data set.

The results shown in the tables above imply a dependence pattern contrary to CEIOPS/EIOPA's assumption of perfect correlation within the category "other equity". For almost all pairs of indexes and both data sets, linear correlation is below 65%. According to linear correlation coefficients, the presence of diversification effects is likely.

VaR-implied correlation

Next, we consider VaR-implied tail correlations between each possible pair of the four indexes[15]. Table 5.28 summarizes the results.

[15]Note that at this point, we carry out analysis solely for the recent data sets. Analysis regarding Pearson correlations within "other equity" as well as previous analysis concerning VaR-implied correlations based on non-annualized data showed that the difference between those two data sets is negligible for this part of the study.

	LPX				S&P			HFRX
$1-\alpha$	S&P	HFRX	BRIC	$1-\alpha$	HFRX	BRIC	$1-\alpha$	BRIC
0.995	0.6363	1	0.5775	**0.995**	1	0.5817	**0.995**	0.93875
0.99	0.4819	1	0.7166	**0.99**	1	0.4364	**0.99**	1
0.98	0.3553	1	0.6032	**0.98**	1	0.4404	**0.98**	0.53965
0.97	0.3766	1	0.4894	**0.97**	1	0.4238	**0.97**	0.42351
0.96	0.3586	1	0.5819	**0.96**	1	0.4561	**0.96**	0.24361
0.95	0.348	1	0.5238	**0.95**	1	0.5078	**0.95**	0.17306
0.9	0.4272	1	0.5137	**0.9**	1	0.4489	**0.9**	0.19029
0.85	0.5191	0.9556	0.4664	**0.85**	1	0.4715	**0.85**	0.30163
0.8	0.5487	0.9982	0.5222	**0.8**	1	0.599	**0.8**	0.46722

Table 5.28: VaR-implied tail correlation coefficients within "other equity" for varying confidence levels based on the recent data set.

The results for VaR-implied correlations within "other-equities" confirm the impression implied by results for Pearson correlation calibrations. Except for pairs involving the HFRX, correlations are clearly different from 1.

In summary, assuming perfect correlation within the sub-risk module equity seems inexplicable. Our empirical study implies that diversification effects are likely to exist within this module. With the relinquishment of correlation calibrations within the equity category, those benefits are neglected.

5.5.2 Property

Pearson correlation

Table 5.29 and table 5.30 show linear Pearson correlation calibration results for dependencies within the sub-risk module property. For this part of the study, we carry out additional analysis on the original data set of indexes representing property risk.

	Original data: Monthly				
Index	**All prop.**	**Offices**	**City Off.**	**Retail**	**Warehouse**
All prop.	1	0.9619	0.7927	0.9673	0.9000
Offices		1	0.8550	0.8689	0.8035
City Off.			1	0.7103	0.6683
Retail				1	0.9315
Warehouse					1

Table 5.29: Linear pearson correlation coefficients within property risk based on the original data set.

	Recent data: Monthly				
Index	**All prop.**	**Offices**	**City off.**	**Retail**	**Warehouse**
All prop.	1	0.9590	0.7887	0.9643	0.8946
Offices		1	0.8572	0.8575	0.7822
City Off.			1	0.6965	0.6393
Retail				1	0.9368
Warehouse					1

Table 5.30: Linear Pearson correlation coefficients within property risk based on the recent data set.

For indexes representing property risk, linear correlation coefficients are close to 1 for almost all of the index combinations. Differences between the two data sets are small and thus negligible.

VaR-implied correlation

Table 5.31 shows the results for VaR-implied tail correlation coefficient calibrations carried out on recent data for property risk.

	All property			
α	**Offices**	**City off**	**Retail**	**Warehouse**
0.995	1	0.8397	1	1
0.99	1	0.5349	0.9049	0.9106
0.98	0.9756	0.2796	1	0.939
0.97	1	1	0.8471	0.8361
0.96	0.9588	0.7944	1	0.5843
0.95	0.7713	0.9555	1	1
0.9	0.8746	1	0.6266	−0.2759
0.85	1	0.7872	1	1
0.8	1	−1	0.9354	1

	Offices		
α	**City off.**	**Retail**	**Wareh.**
0.995	0.8976	1	1
0.99	0.5644	1	0.9483
0.98	0.4067	0.9638	1
0.97	1	1	0.7593
0.96	1	1	0.78
0.95	1	0.7457	0.9651
0.9	0.8765	0.3689	−1
0.85	1	1	−0.9886
0.8	1	0.3014	1

	City offices	
α	**Retail**	**Wareh.**
0.995	0.8849	0.8169
0.99	0.6144	0.6427
0.98	0.3144	0.393
0.97	0.3961	1
0.96	1	0.6029
0.95	1	0.9461
0.9	1	−1
0.85	1	1
0.8	−1	−1

	Retail
α	**Wareh.**
0.995	0.8169
0.99	0.6427
0.98	0.393
0.97	1
0.96	0.6029
0.95	0.9461
0.9	−1
0.85	1
0.8	−1

Table 5.31: VaR-implied tail correlation coefficients within property risk for varying confidence levels based on the recent data set.

The results for VaR-implied correlation coefficients within property risk are diverse. Overall, the results again imply a correlation of 1 ore close to 1. However, for some index combinations and specific confidence levels there are a couple of outliers reaching up to −1. Nonetheless, those outliers do not necessarily imply a "wrong" regulatory decision concerning the allowance of diversification effects within the sub-risk module equity but rather leave room for doubts concerning the choice of historical data representing property risk or respectively their length of history.

Chapter 6

Conclusion and Outlook

The Solvency II framework was fully implemented with the beginning of 2016. However, its development is not completed yet. Additional regulatory work on Solvency II is already scheduled. This includes to a large extent the review of Solvency Capital Requirement calculations based on the standard formula.

This thesis discussed the regulatory approach to calibrate standard formula parameters for the sub-risk modules equity and property risk. The main focus was on the consequences of CEIOPS/EIOPA's unfortunate decision to confront the problem of too short data histories with the use of rolling-window annual returns. [Mittnik, 2011] clearly showed, that this calibration methodology has severe falsifying impact on the calibration results for both standard formula input parameters. In the empirical part of this thesis, CEIOPS/EIOPA's calibrations concerning standard formula input parameters for equity risk and property risk were replicated. The stress factors obtained by the regulator's empirical analysis could be unequivocally confirmed for both risk modules. However, regarding the correlation coefficients between representatives of the equity category "other equity" and the index representing "global equity" we could not obtain a clear match with CEIOPS/EIOPA's empirical result.
In a second step, we used a slightly modified version of the Filtered Historical Simulation methodology to circumvent the need of rolling-windows to transform daily or respectively monthly prices into annual returns. We analyzed representing indexes of both sub-risk modules. In general, the analysis implied an overestimation of risk for calibrations foll regulatory approach, meaning VaR calibrations based on rolling-window annualized returns. Especially for property risk, we obtained substantially less negative results for VaR calibrations based on FHS annual returns. For equity risk, the results were more diverse. For most of the indexes calibrations annual returns obtained through FHS resulted in a similar or lower

risk than proposed by the regulator. However, the MSCI BRIC as representative of the equity type emerging markets revealed rather higher risk than it is implied by CEIOPS/EIOPA.
Analysis concerning the aggregation of the categories "global equity" and "other equity" implied that the proposed correlation of 0.75 might be too close to 1. Although CEIOPS/EIOPA's approach concerning calibration approaches has not been fully determined, our analysis showed, that the VaR-implied correlation approach is superior to the data-cutting method. Also, analysis carried out on rolling-window annualized returns possibly underestimates diversification benefits.
Lastly, the presence of diversification effects within the category "other equity" as well as the sub-risk module property was investigated. For property risk, CEIOPS/EIOPA's decision to neglect diversification within the sub-risk module could be supported. However, indexes representing "other equity" exhibit correlations which are unequivocally different from 1. According to our analysis, diversification benefits within the category are indeed present.
At several points, results of the empirical analysis implied that short data histories might cause unstable calibration results. Time series representing property risk are particularly short. In order to obtain more reliable calibration results, their replacement might be considered.

Solvency II pursues the maintenance of sound risk management within the insurance an reinsurance sector – in the present as well as in future. Along with many others, this thesis indicated, that the calibration of the standard formula input parameters still exhibits room for improvements. Parts of regulatory calibration approaches might overestimate certain risks while present diversification effects are neglected. This clearly impacts the amount of capital and insurer is required to hold under Solvency II to a large degree. In order to prevent disadvantages against insurance or reinsurance companies being reliant on the regulatory model, those aspects should be taken into consideration when reviewing the SCR standard formula.

Appendix A

Methodical Appendix

A.1 Copulas

The concept of copulas allows us to extend the measure of dependence beyond the concept of correlation. With a given set of random variables $(X_1, \dots X_n)$, the idea behind the approach is extract every information about their dependence structure out of their joint distribution function F.
Let $U = (U_1 \dots U_d)$ be a d-dimensional random vector and U_j its standard-uniformly distributed margins on the interval $[0, 1]$. Then the d-dimensional distribution function $C_U : [0, 1]^d \rightarrow [0, 1]$ is called copula. The importance and of copulas and their meaning concerning the measurement of dependence is summarized in (Sklar's Theorem).

Theorem A.1. Let $X_1, \dots X_d$ be random variables with respective probability distribution function $F_{X_1}, \dots F_{X_d}$. Let F_X denote the joint probability distribution function. Then there is a Copula $C : [0, 1]^d \rightarrow [0, 1]$ such that $\forall x_1, \dots x_n \in [-\infty, \infty]$

$$F(x_1 \dots x_n) = C(F_{X_1}(x_1), \dots F_{X_d}(x_d)) \tag{A.1}$$

If the marginal distributions $F_{X_1}, \dots F_{X_d}$ are all continuous, the copula is unique [McNeil et al., 2005]. In this case it also holds

$$C(u_1, \dots, u_d) = (F^{-1}_{X_1}(u_1), \dots F^{-1}_{X_d}(u_d)), \qquad u_1, \dots, u_n \in [0, 1] \tag{A.2}$$

where $F^{-1}_{X_1} \dots F^{-1}_{X_n}$ denote the inverse distribution functions of $F_{X_1} \dots F_{X_n}$ [Schmidt and Stadtmüller, 2006].

A.2 Properties of VaR

VaR is translation invariant: $\forall a \in \mathbb{R}$ it holds that $VaR_\alpha(X+a) = VaR_\alpha(L)+a$.

Proof.

$$\begin{aligned} VaR_\alpha(L+a) &= inf\{l \in \mathbb{R}: \quad F_{L+a}(l) \geq \alpha\} \\ &= inf\{l \in \mathbb{R}: \quad P(L+a \leq l) \geq \alpha\} \\ &= inf\{l \in \mathbb{R}: \quad P(L \leq \underbrace{l-a}_{:=y}) \geq \alpha\} \\ &= inf\{y+a \in \mathbb{R}: \quad P(L \leq y) \geq \alpha\} \\ &= a + inf\{y \in \mathbb{R}: \quad P(L \leq y) \geq \alpha\} \\ &= a + inf\{y \in \mathbb{R}: \quad F_L(y) \geq \alpha\} \\ &= a + VaR_\alpha(L) \end{aligned}$$

□

VaR is positive homogen: $\forall \lambda \geq 0$ it holds that $\rho(\lambda L) = \lambda \rho(L)$.

Proof. First consider $\lambda = 0:$

$$\begin{aligned} VaR_\alpha(\lambda L) &= VaR_\alpha(0) \\ &= inf\{l \in \mathbb{R}: \quad F_0(l) \geq \alpha\} \\ &= inf\{l \in \mathbb{R}: \quad P(O \leq l) \geq \alpha\} \\ &= 0 = 0 * VaR_\alpha(L). \end{aligned}$$

Now consider $\lambda > 0:$

$$\begin{aligned} VaR_\alpha(\lambda L) &= inf\{l \in \mathbb{R}: \quad F_{\lambda L}(l) \geq \alpha\} \\ &= inf\{l \in \mathbb{R}: \quad P(\lambda L \leq l) \geq \alpha\} \\ &= inf\{l \in \mathbb{R}: \quad P(L \leq \underbrace{\frac{1}{\lambda} l}_{=:y}) \geq \alpha\} \\ &= inf\{\lambda y \in \mathbb{R}: \quad P(L \leq y) \geq \alpha\} \\ &= \lambda inf\{y \in \mathbb{R}: \quad F_L(y) \alpha\} \\ &= \lambda VaR_\alpha(L) \end{aligned}$$

□

VaR is monoton: $\forall L_1, L_2$ with $P(L_1 \leq L_2) = 1$ it holds that $VaR_\alpha(L_1) \leq VaR_\alpha(L_2)$.

Proof. We know that $F_l(L) \leq F_Y(l)$ since it holds $L \leq Y$ with probability 1.

$$\begin{aligned} VaR_\alpha(L) = inf\{l \in \mathbb{R}: \quad F_L(l) \geq \alpha\} \\ \leq inf\{l \in \mathbb{R}: \quad F_Y(l) \geq \alpha\} = VaR_\alpha(Y) \end{aligned}$$

□

VaR is not sub-additiv: $VaR(L_1 + L_2) \leq VaR(L_1) + VaR(L_2)$ does not hold in general.

Proof. Counterelample: Let L_1, L_2 be two identically distributed rvs with

$$L_i = \begin{cases} 1, & \text{with} \quad P(L_i = 1) = 0.5 \\ 0, & \text{with} \quad P(L_i = 0) = 0.5 \end{cases} \qquad i = 1, 2.$$

Now, choose $\alpha = 0.3$. Then

$$VaR_\alpha(L_i) = inf\{l \in \mathbb{R}: \quad F_{L_i}(l) \geq 0.3\} = 0.$$

Now consider the sum of $L_1 + L_2$. It holds

$$L_1 + L_2 = \begin{cases} 2, & \text{with} \quad P(L1 + L_2 = 2) = \frac{1}{4} \\ 1, & \text{with} \quad P(L1 + L_2 = 1) = \frac{1}{2} \\ 0, & \text{with} \quad P(L1 + L_2 = 0) = \frac{1}{4} \end{cases} \qquad i = 1, 2.$$

$$VaR_\alpha(L_1 + L_2) = inf\{l \in \mathbb{R}: \quad F_{L_1+L_2}(l) \geq 0.3\} = 1$$

since

$$P(L_1 + L_2 < 0) = \frac{1}{4} \leq 0.3 \quad \text{and} \quad (L_1 + L_2 \leq 1) = \frac{3}{4} > 0.3.$$

Thus

$$VaR_\alpha(L_1 + L_2) \neq VaR_\alpha(L_1) + VaR_\alpha(L_1) = 0 + 0 = 0,$$

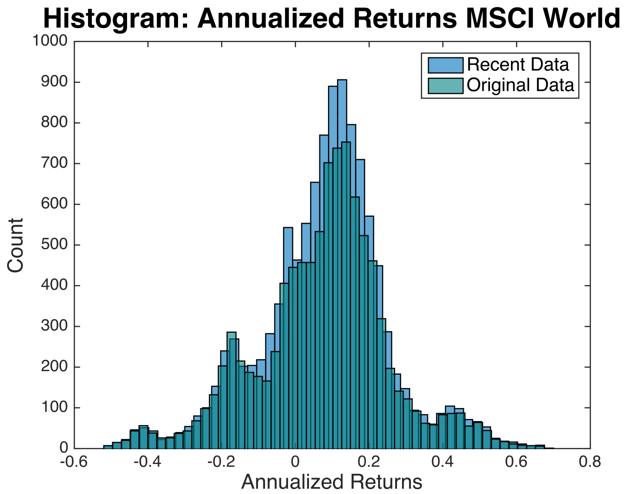
and therefore VaR is not subadditiv in general. □

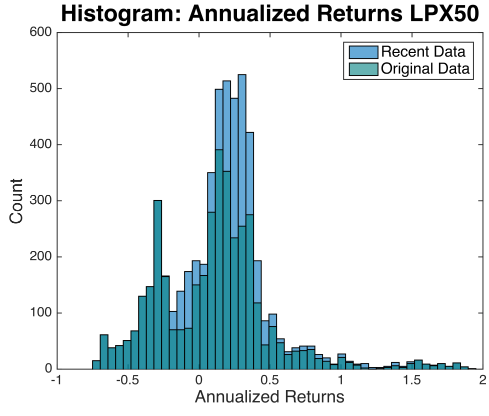
Appendix B

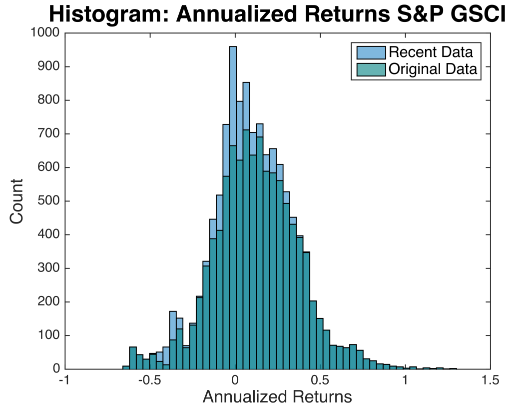
Empirical Appendix

B.1 Histograms of annualized returns

Equity







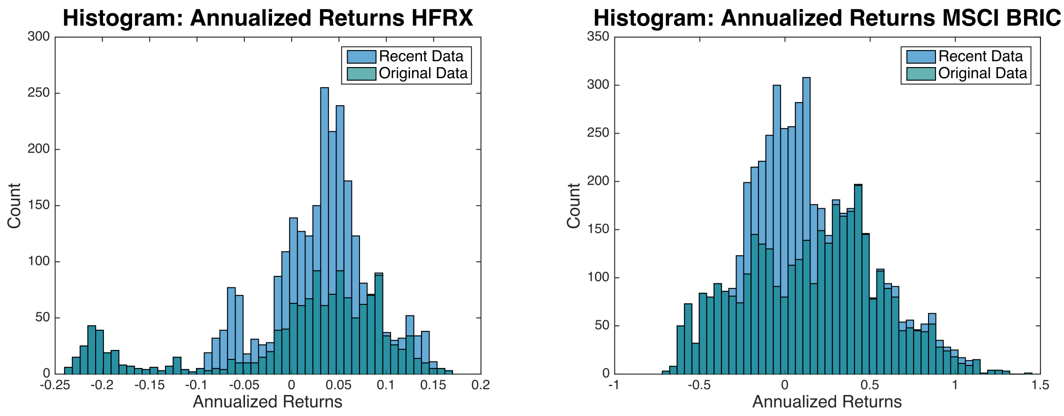
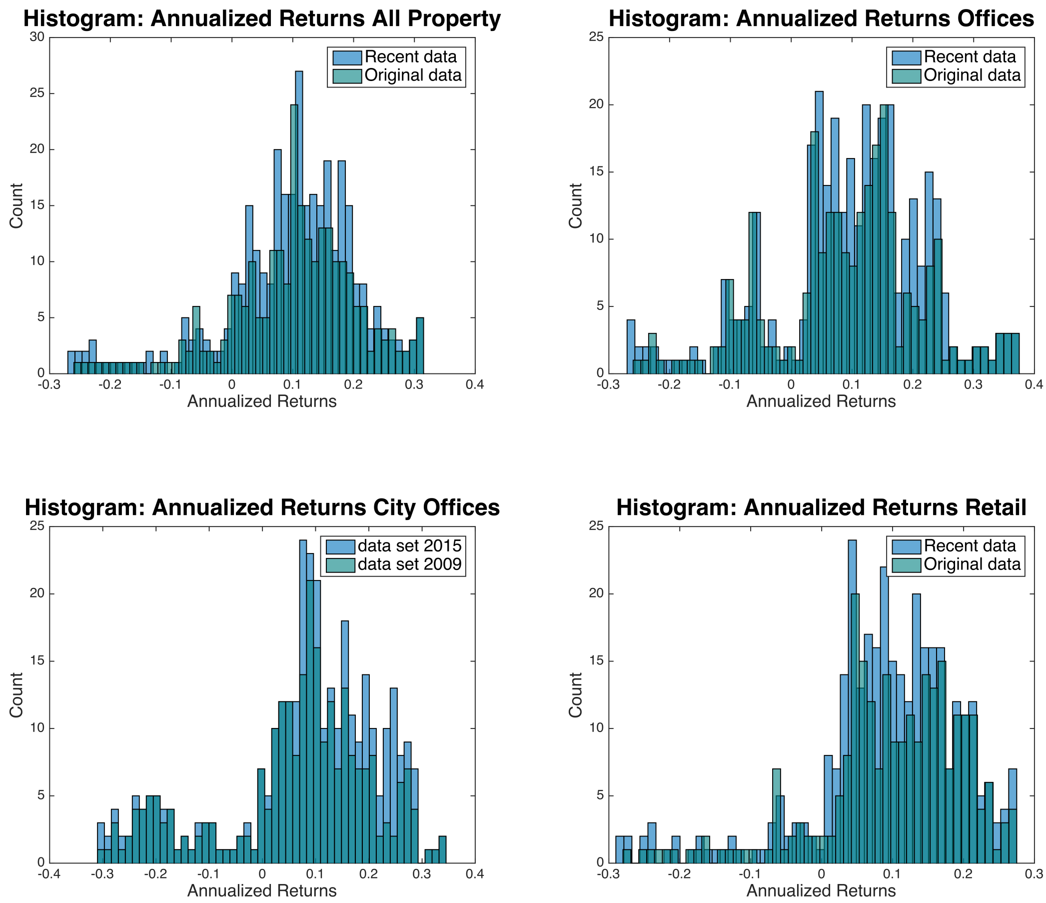


Figure B.1: Histograms for indexes representing equity risk.

Property



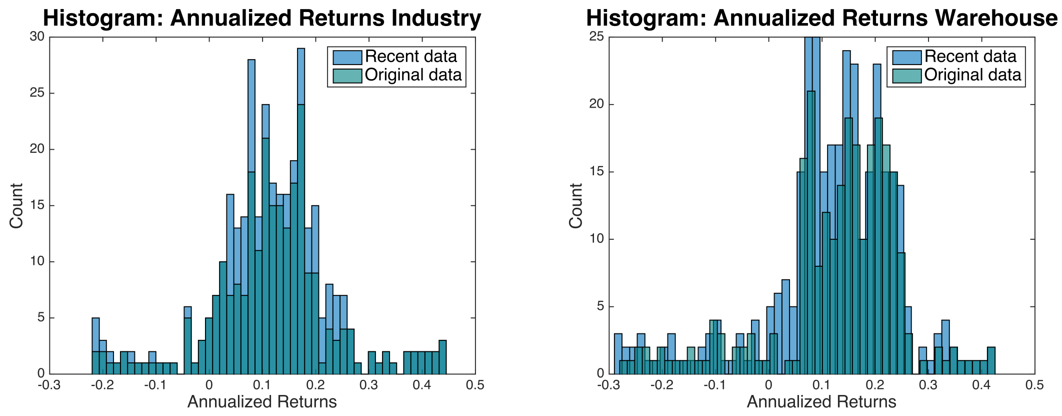
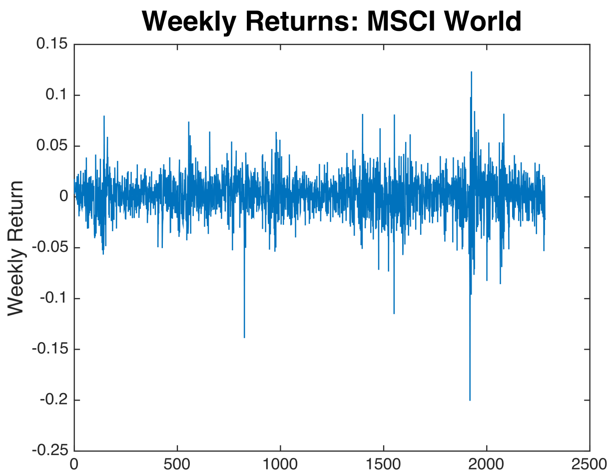
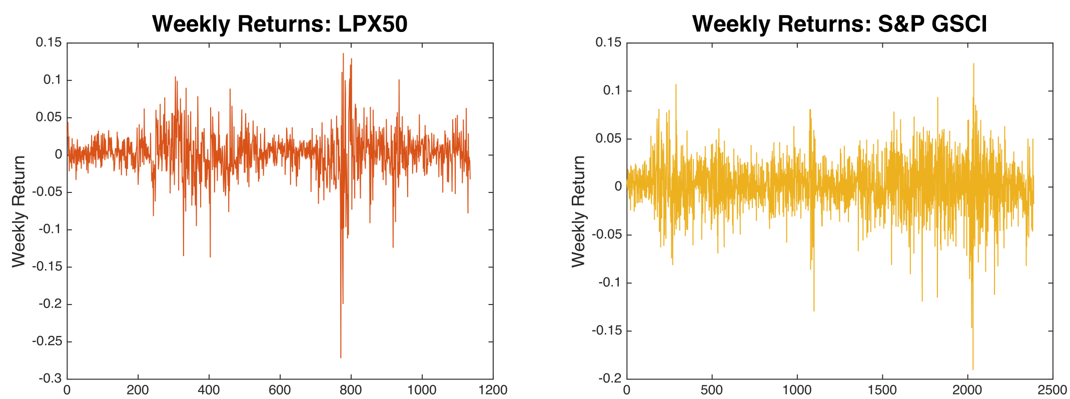


FIGURE B.2: Histograms for indexes representing property risk.

B.2 Returns

Equity





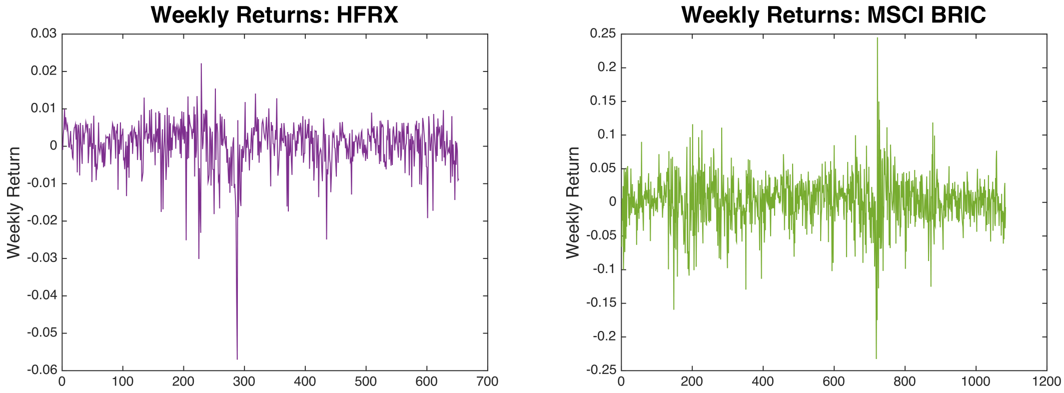
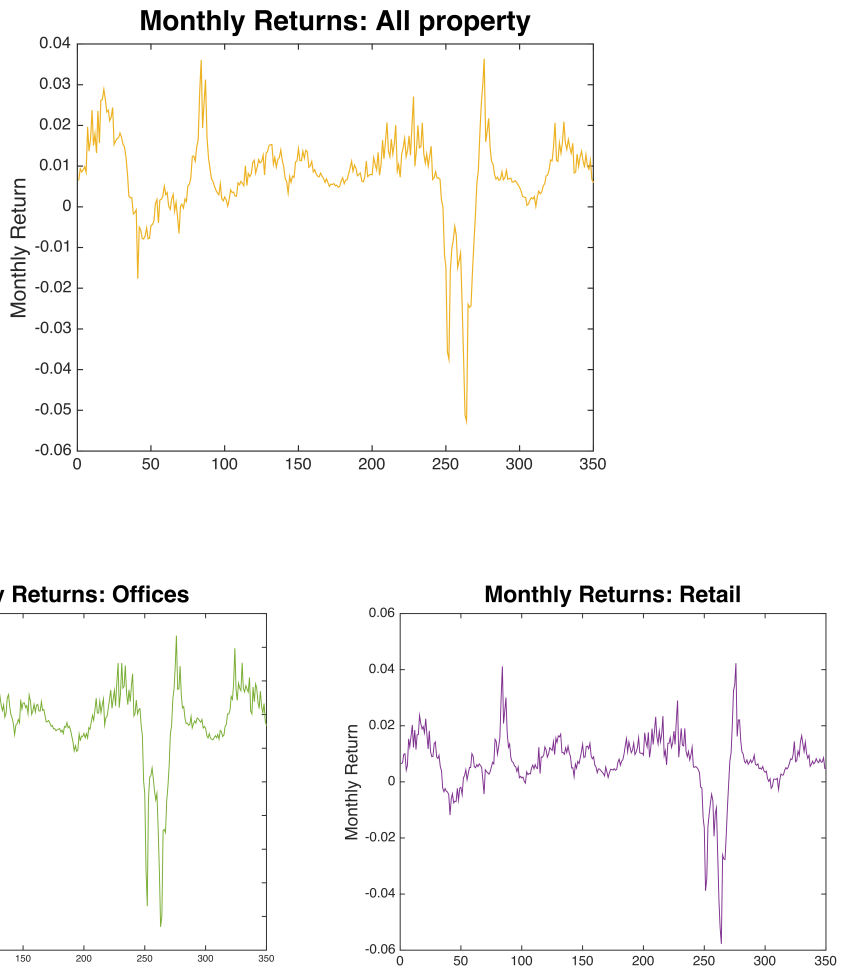


FIGURE B.3: Weekly Returns of indexes representing equity risk.

Property



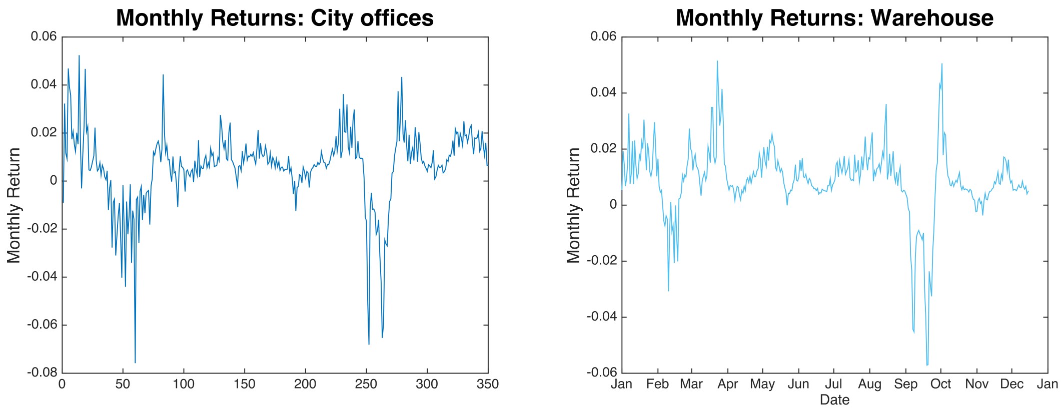


FIGURE B.4: Monthly Returns of indexes representing property risk.

B.3 FHS Results

Equity

	MSCI World Original Data		MSCI World Recent Data	
(p,q)[1]	$\epsilon_t \sim N$	$\epsilon_t \sim t$	$\epsilon_t \sim N$	$\epsilon_t \sim t$
$(0,0)$	$[-41.28\%, -39.32\%]$	$[-40.33\%, -38.85\%]$	$[-42.33\%, -40.59\%]$	$[-40.42\%, -38.74\%]$
$(0,1)$	$[-41.6\%, -39.6\%]$	$[-40.7\%, -39.2\%]$	$[-42.75\%, -40.92\%]$	$[-40.91\%, -39.26\%]$
$(0,2)$	$[-42.66\%, -40.92\%]$	$[-42.17\%, -40.57\%]$	$[-43.27\%, -41.62\%]$	$[-41.49\%, -39.8\%]$
$(1,0)$	$[-41.61\%, -39.61\%]$	$[-40.75\%, -39.27\%]$	$[-42.77\%, -40.94\%]$	$[-40.93\%, -39.29\%]$
$(1,1)$	$[-41.29\%, -39.27\%]$	$[-40.35\%, -38.87\%]$	$[-43.37\%, -41.58\%]$	$[-42.06\%, -40.4\%]$
$(1,2)$	$[-43.4\%, -41.39\%]$	$[-43.37\%, -41.75\%]$	$[-42.78\%, -40.96\%]$	$[-42.17\%, -40.34\%]$
$(2,0)$	$[-42.76\%, -40.96\%]$	$[-42.38\%, -40.74\%]$	$[-43.3\%, -41.68\%]$	$[-41.65\%, -39.9\%]$
$(2,1)$	$[-43.38\%, -41.35\%]$	$[-43.32\%, -41.75\%]$	$[-43.46\%, -41.78\%]$	$[-42.17\%, -40.36\%]$
$(2,2)$	$[-42.97\%, -40.84\%]$	$[-43.08\%, -41.55\%]$	$[-43.38\%, -41.73\%]$	$[-40.68\%, -39.23\%]$

TABLE B.1: 95% bootstrap confidence intervals for 99.5% VaR of MSCI World Data after FHS.

	LPX50 Original Data		LPX50 Recent Data	
(p,q)	$\epsilon_t \sim N$	$\epsilon_t \sim t$	$\epsilon_t \sim N$	$\epsilon_t \sim N$
$(0,0)$	$[-72.86\%, -69.36\%]$	$[-68.12\%, -65.02\%]$	$[-74.51\%, -71.09\%]$	$[-65.25\%, -62.95\%]$
$(0,1)$	$[-73.67\%, -70.42\%]$	$[-69.13\%, -66.22\%]$	$[-74.99\%, -71.66\%]$	$[-65.44\%, -62.98\%]$
$(0,2)$	$[-77.35\%, -74.19\%]$	$[-73.17\%, -70.22\%]$	$[-75.58\%, -72.45\%]$	$[-67.18\%, -64.94\%]$
$(1,0)$	$[-74.14\%, -70.89\%]$	$[-69.65\%, -66.68\%]$	$[-75.09\%, -71.78\%]$	$[-65.46\%, -63.02\%]$
$(1,1)$	$[-81.71\%, -79.08\%]$	$[-80.54\%, -77.59\%]$	$[-79.55\%, -76.92\%]$	$[-63.27\%, -61.03\%]$
$(1,2)$	$[-81.91\%, -79.4\%]$	$[-80.71\%, -78.04\%]$	$[-79.14\%, -76.43\%]$	$[-73.77\%, -71.58\%]$
$(2,0)$	$[-78.46\%, -75.63\%]$	$[-74.97\%, -71.93\%]$	$[-76.14\%, -72.88\%]$	$[-67.96\%, -65.65\%]$
$(2,1)$	$[-82.02\%, -79.66\%]$	$[-80.95\%, -78.18\%]$	$[-79.13\%, -76.39\%]$	$[-73.6\%, -71.38\%]$
$(2,2)$	$[-81.83\%, -79.14\%]$	$[-80.69\%, -78.01\%]$	$[-78.63\%, -76.04\%]$	$[-73.23\%, -71.01\%]$

TABLE B.2: 95% bootstrap confidence intervals for 99.5% VaR of LPX50 Data after FHS.

	S&P GSCI Original Data		**S&P GSCI Recent Data**	
(p,q)	$\epsilon_t \sim N$	$\epsilon_t \sim t$	$\epsilon_t \sim N$	$\epsilon_t \sim t$
$(0,0)$	[−48.68%, −47.14%]	[−49.87%, −48.38%]	[−44.01%, −42.19%]	[−45.05%, −43.19%]
$(0,1)$	[−48.67%, −47.13%]	[−50.08%, −48.65%]	[−44.21%, −42.43%]	[−45.83%, −44%]
$(0,2)$	[−50.31%, −48.66%]	[−51.9%, −50.34%]	[−44.92%, −43.14%]	[−46.74%, −44.64%]
$(1,0)$	[−48.66%, −47.12%]	[−50.12%, −48.65%]	[−44.22%, −42.46%]	[−45.9%, −44.15%]
$(1,1)$	[−52.41%, −50.82%]	[−58.54%, −56.97%]	[−48%, −46.1%]	[−57.07%, −55.8%]
$(1,2)$	[−52.23%, −50.84%]	[−54.45%, −53.07%]	[−46.15%, −44.59%]	[−57.11%, −55.81%]
$(2,0)$	[−50.45%, −48.77%]	[−52.12%, −50.61%]	[−45.12%, −43.3%]	[−46.93%, −44.92%]
$(2,1)$	[−52.33%, −50.87%]	[−54.65%, −53.37%]	[−46.38%, −44.81%]	[−57.11%, −55.81%]
$(2,2)$	[−51.25%, −49.62%]	[−53.31%, −51.78%]	[−44.63%, −42.87%]	[−56.85%, −55.6%]

TABLE B.3: 95% bootstrap confidence intervals for 99.5% VaR of S&P GSCI Data after FHS.

	HFRX Original Data		**HFRX Recent Data**	
(p,q)	$\epsilon_t \sim N$	$\epsilon_t \sim t$	$\epsilon_t \sim N$	$\epsilon_t \sim N$
$(0,0)$	[−14.97%, −13.89%]	[−16.41%, −15.31%]	[−14.37%, −13.73%]	[−13.98%, −13.37%]
$(0,1)$	[−16.28%, −15.38%]	[−17.66%, −16.56%]	[−15.23%, −14.58%]	[−15.38%, −14.78%]
$(0,2)$	[−17.66%, −16.64%]	[−19.58%, −18.3%]	[−17.84%, −17.16%]	[−17.44%, −16.7%]
$(1,0)$	[−16.85%, −16.12%]	[−18.6%, −17.33%]	[−16.99%, −16.25%]	[−16.08%, −15.48%]
$(1,1)$	[−21.06%, −20%]	[−25.76%, −24.39%]	[−20.8%, −20.01%]	[−22%, −21.43%]
$(1,2)$	[−20.98%, −19.77%]	[−25.15%, −23.91%]	[−20.53%, −19.74%]	[−21.95%, −21.29%]
$(2,0)$	[−19.03%, −17.99%]	[−21.53%, −20.36%]	[−19.24%, −18.44%]	[−18.64%, −17.94%]
$(2,1)$	[−21.03%, −19.85%]	[−25.39%, −24.14%]	[−20.45%, −19.69%]	[−21.88%, −21.19%]
$(2,2)$	[−21.54%, −20.33%]	[−26.59%, −25.08%]	[−21.2%, −20.49%]	[−22.73%, −22.04%]

TABLE B.4: 95% bootstrap confidence intervals for 99.5% VaR of HFRX Data after FHS.

	MSCI BRIC Original Data		**MSCI BRIC Recent Data**	
(p,q)	$\epsilon_t \sim N$	$\epsilon_t \sim t$	$\epsilon_t \sim N$	$\epsilon_t \sim t$
$(0,0)$	[−64.18%, −61.93%]	[−68.33%, −65.76%]	[−61.34%, −59.64%]	[−62.44%, −60.74%]
$(0,1)$	[−65.47%, −63.33%]	[−69.78%, −67.64%]	[−62.66%, −61.06%]	[−64%, −62.07%]
$(0,2)$	[−67.6%, −65.65%]	[−73.67%, −72.01%]	[−65.25%, −63.37%]	[−66.87%, −64.72%]
$(1,0)$	[−65.79%, −63.67%]	[−70.24%, −68.32%]	[−63.1%, −61.34%]	[−64.48%, −62.49%]
$(1,1)$	[−70.79%, −69.01%]	[−78.33%, −76.44%]	[−68.61%, −66.89%]	[−70.63%, −68.77%]
$(1,2)$	[−66.31%, −64.19%]	[−71.72%, −69.59%]	[−68.66%, −66.96%]	[−64.77%, −62.87%]
$(2,0)$	[−68.46%, −66.53%]	[−75.82%, −73.61%]	[−66.11%, −64.22%]	[−68.04%, −65.86%]
$(2,1)$	[−67.02%, −64.81%]	[−72.88%, −70.68%]	[−64.14%, −62.33%]	[−65.71%, −63.83%]
$(2,2)$	[−71.51%, −69.69%]	[−79.08%, −77.45%]	[−68.02%, −66.29%]	[−70.46%, −68.86%]

TABLE B.5: 95% bootstrap confidence intervals for 99.5% VaR of MSCI BRIC Data after FHS.

Property

	(p,q)	$\epsilon_t \sim N$	$\epsilon_t \sim t$
All Property	$(0,0)$	$[4.58\%, 4.95\%]$	$[4.58\%, 4.91\%]$
	$(0,1)$	$[2.54\%, 2.89\%]$	$[1.79\%, 2.20\%]$
	$(0,2)$	$[-0.62\%, -0.01\%]$	$[-1.53\%, -0.71\%]$
	$(1,0)$	$[-10.65\%, -9.52\%]$	$[-13.98\%, -12.93\%]$
	$(1,1)$	$[-6.20\%, -5.52\%]$	$[-9.16\%, -8.11\%]$
	$(1,2)$	$[-6.44\%, -5.63\%]$	$[-9.21\%, -8.11\%]$
	$(2,0)$	$[-7.91\%, -7.04\%]$	$[-10.15\%, -9.24\%]$
	$(2,1)$	$[-9.84\%, -8.87\%]$	$[-9.13\%, -8.04\%]$
	$(2,2)$	$[-6.72\%, -6.06\%]$	$[-7.90\%, -7.09\%]$

Table B.6: 95% bootstrap confidence intervals for 99.5% VaR of All Property Data after FHS.

	(p,q)	$\epsilon_t \sim N$	$\epsilon_t \sim t$
Offices	$(0,0)$	$[2.21\%, 2.69\%]$	$[2.08\%, 2.58\%]$
	$(0,1)$	$[-0.02\%, 0.65\%]$	$[-0.14\%, 0.55\%]$
	$(0,2)$	$[-3.74\%, -3.00\%]$	$[-3.46\%, -2.70\%]$
	$(1,0)$	$[-14.78\%, -13.59\%]$	$[-13.77\%, -12.45\%]$
	$(1,1)$	$[-10.58\%, -9.51\%]$	$[-10.53\%, -9.25\%]$
	$(1,2)$	$[-10.35\%, -9.24\%]$	$[-10.55\%, -9.23\%]$
	$(2,0)$	$[-13.22\%, -11.90\%]$	$[-12.15\%, -10.65\%]$
	$(2,1)$	$[-10.55\%, -9.35\%]$	$[-10.52\%, -9.25\%]$
	$(2,2)$	$[-9.87\%, -8.75\%]$	$[-9.74\%, -8.53\%]$

Table B.7: 95% bootstrap confidence intervals for 99.5% VaR of Offices Data after FHS.

	(p,q)	$\epsilon_t \sim N$	$\epsilon_t \sim t$
City offices	$(0,0)$	$[-00.04\%, +00.48\%]$	$[-01.23\%, -00.50\%]$
	$(0,1)$	$[-02.95\%, -02.38\%]$	$[-03.51\%, -02.95\%]$
	$(0,2)$	$[-04.42\%, -03.78\%]$	$[-05.15\%, -04.42\%]$
	$(1,0)$	$[-10.82\%, -10.12\%]$	$[-10.78\%, -09.99\%]$
	$(1,1)$	$[-10.10\%, -09.25\%]$	$[-07.41\%, -06.67\%]$
	$(1,2)$	$[-09.88\%, -09.04\%]$	$[-07.33\%, -06.61\%]$
	$(2,0)$	$[-11.88\%, -11.09\%]$	$[-09.83\%, -09.08\%]$
	$(2,1)$	$[-10.73\%, -09.87\%]$	$[-07.32\%, -06.61\%]$
	$(2,2)$	$[-09.29\%, -08.51\%]$	$[-07.20\%, -06.39\%]$

Table B.8: 95% bootstrap confidence intervals for 99.5% VaR of City Offices Data after FHS.

	(p,q)	$\epsilon_t \sim N$	$\epsilon_t \sim t$
Retail	$(0,0)$	$[2.56\%, 2.99\%]$	$[2.57\%, 2.99\%]$
	$(0,1)$	$[2.38\%, 2.68\%]$	$[1.44\%, 1.86\%]$
	$(0,2)$	$[-0.03\%, 0.37\%]$	$[-0.36\%, 0.17\%]$
	$(1,0)$	$[-8.70\%, -7.97\%]$	$[-8.94\%, -8.17\%]$
	$(1,1)$	$[-6.95\%, -6.36\%]$	$[-6.50\%, -5.93\%]$
	$(1,2)$	$[-4.22\%, -3.84\%]$	$[-6.59\%, -6.00\%]$
	$(2,0)$	$[-7.49\%, -6.87\%]$	$[-6.86\%, -6.29\%]$
	$(2,1)$	$[-6.99\%, -6.37\%]$	$[-6.53\%, -5.91\%]$
	$(2,2)$	$[-6.29\%, -5.72\%]$	$[-6.30\%, -5.74\%]$

TABLE B.9: 95% bootstrap confidence intervals for 99.5% VaR of Retail Data after FHS.

	(p,q)	$\epsilon_t \sim N$	$\epsilon_t \sim t$
Warehouse	$(0,0)$	$[04.06\%, 04.44\%]$	$[03.90\%, 04.23\%]$
	$(0,1)$	$[02.62\%, 03.11\%]$	$[02.13\%, 02.64\%]$
	$(0,2)$	$[01.42\%, 01.94\%]$	$[00.29\%, 00.84\%]$
	$(1,0)$	$[-06.10\%, -05.38\%]$	$[-08.99\%, -08.05\%]$
	$(1,1)$	$[-05.17\%, -04.54\%]$	$[-07.31\%, -06.49\%]$
	$(1,2)$	$[-05.24\%, -04.64\%]$	$[-07.32\%, -06.36\%]$
	$(2,0)$	$[-05.51\%, -04.87\%]$	$[-08.03\%, -07.15\%]$
	$(2,1)$	$[-05.24\%, -04.61\%]$	$[-07.22\%, -06.44\%]$
	$(2,2)$	$[-05.29\%, -04.70\%]$	$[-05.95\%, -05.14\%]$

TABLE B.10: 95% bootstrap confidence intervals for 99.5% VaR of Industrial Data after FHS.

B.4 AIC and BIC Criteria

Equity

	MSCI Original Data				MSCI Recent Data			
	$\epsilon_t \sim N$		$\epsilon_t \sim t$		$\epsilon_t \sim N$		$\epsilon_t \sim t$	
(p,q)	**AIC**	**BIC**	**AIC**	**BIC**	**AIC**	**BIC**	**AIC**	**BIC**
$(0,0)$	-1.0248	-1.0225	-1.0322	-1.0294	-1.1755	-1.1732	-1.18420	-1.1813
$(0,1)$	-1.0246	-1.0218	-1.0321	-1.0287	-1.1753	-1.1725	-1.1840	-1.1805
$(0,2)$	-1.0245	-1.0212	-1.0321	-1.0282	-1.1753	-1.1718	-1.1840	-1.18
$(1,0)$	-1.0246	-1.0218	-1.0321	-1.02870	-1.1753	-1.1725	-1.184	-1.1805
$(1,1)$	-1.0244	-1.0211	-1.0319	-1.0279	-1.1753	-1.1719	-1.1838	-1.1798
$(1,2)$	-1.0244	-1.0205	-1.0321	-1.0276	-1.1751	-1.1711	-1.1839	-1.1793
$(2,0)$	-1.0245	-1.0212	-1.0321	-1.0282	-1.1753	-1.1718	-1.1840	-1.18
$(2,1)$	-1.0244	-1.0205	-1.0321	-1.0276	-1.1751	-1.1711	-1.1839	-1.1793
$(2,2)$	-1.0243	-1.0198	-1.0318	-1.0268	-1.1749	-1.1703	-1.1837	-1.1785
	$\times 1.0e+04$		for all values					

TABLE B.11: AIC/BIC for the MSCI World.

	LPX50 Original Data				LPX50 Recent Data			
	$\epsilon_t \sim N$		$\epsilon_t \sim t$		$\epsilon_t \sim N$		$\epsilon_t \sim t$	
(p,q)	**AIC**	**BIC**	**AIC**	**BIC**	**AIC**	**BIC**	**AIC**	**BIC**
$(0,0)$	-3.7832	-3.7643	-3.8199	-3.7963	-5.1414	-5.1212	-5.2105	-5.1853
$(0,1)$	-3.7827	-3.7591	-3.8197	-3.7913	-5.1396	-5.1144	-5.2085	-5.1783
$(0,2)$	-3.7872	-3.7589	-3.8259	-3.7928	-5.1467	-5.1165	-5.2202	-5.185
$(1,0)$	-3.783	-3.7594	-3.8201	-3.7917	-5.1396	-5.1144	-5.2085	-5.1783
$(1,1)$	-3.7951	-3.7633	-3.8372	-3.8023	-5.1485	-5.1183	-5.2117	-5.1765
$(1,2)$	-3.7964	-3.7668	-3.8354	-3.7994	-5.1509	-5.1157	-5.2239	-5.1836
$(2,0)$	-3.7888	-3.7605	-3.8279	-3.7948	-5.1476	-5.1174	-5.2209	-5.1857
$(2,1)$	-3.7963	-3.7632	-3.8371	-3.7993	-5.1512	-5.116	-5.2246	-5.1843
$(2,2)$	-3.7944	-3.7566	-3.8352	-3.7927	-5.15	-5.1097	-5.2241	-5.1788
	$\times 1.0e+03$		for all values					

TABLE B.12: AIC/BIC for the LPX50.

	S&P GSCI Original Data				S&P GSCI Recent Data			
	$\epsilon_t \sim N$		$\epsilon_t \sim t$		$\epsilon_t \sim N$		$\epsilon_t \sim t$	
(p,q)	**AIC**	**BIC**	**AIC**	**BIC**	**AIC**	**BIC**	**AIC**	**BIC**
$(0,0)$	-9.8338	-9.8112	-9.8660	-9.8378	-1.1194	-1.1171	-1.1244	-1.1215
$(0,1)$	-9.8318	-9.8036	-9.8642	-9.8304	-1.1193	-1.1164	-1.1243	-1.1208
$(0,2)$	-9.8325	-9.7987	-9.866	-9.8261	-1.1194	-1.1159	-1.1245	-1.1204
$(1,0)$	-9.8318	-9.8036	-9.8643	-9.8304	-1.1193	-1.1164	-1.1243	-1.1208
$(1,1)$	-9.8323	-9.7984	-9.8683	-9.8288	-1.1195	-1.1160	-1.125	-1.121
$(1,2)$	-9.8330	-9.7935	-9.8666	-9.8215	-1.1195	-1.1154	-1.1248	-1.1202
$(2,0)$	-9.8325	-9.7986	-9.8658	-9.8263	-1.1194	-1.1160	-1.1245	-1.1205
$(2,1)$	-9.8328	-9.7933	-9.8665	-9.8213	-1.1195	-1.1154	-1.1248	-1.1202
$(2,2)$	-9.8330	-9.7879	-9.8658	-9.815	-1.1196	-1.115	-1.1249	-1.1197
	$\times 1.0e+03$	for original data						
	$\times 1.0e+04$	for recent data						

Table B.13: AIC/BIC for the S&P GSCI.

	HFRX Original Data				HFRX Recent Data			
	$\epsilon_t \sim N$		$\epsilon_t \sim t$		$\epsilon_t \sim N$		$\epsilon_t \sim t$	
(p,q)	**AIC**	**BIC**	**AIC**	**BIC**	**AIC**	**BIC**	**AIC**	**BIC**
$(0,0)$	-2.5596	-2.5442	-2.5857	-2.5664	-4.8662	-4.8483	-4.9181	-4.8957
$(0,1)$	-2.5606	-2.5413	-2.5871	-2.5640	-4.8490	-4.8266	-4.9252	-4.8983
$(0,2)$	-2.5623	-2.5392	-2.5903	-2.5633	-4.8801	-4.8532	-4.9329	-4.9015
$(1,0)$	-2.536	-2.5167	-2.5883	-2.5651	-4.8771	-4.8547	-4.9275	-4.9006
$(1,1)$	-2.5651	-2.5420	-2.597	-2.5699	-4.8823	-4.8554	-4.9375	-4.9062
$(1,2)$	-2.5637	-2.5367	-2.5960	-2.5651	-4.8804	-4.8490	-4.9355	-4.8997
$(2,0)$	-2.5639	-2.5407	-2.5931	-2.566	-4.8818	-4.8549	-4.9348	-4.9035
$(2,1)$	-2.5637	-2.5367	-2.5961	-2.5652	-4.8804	-4.8491	-4.9356	-4.8997
$(2,2)$	-2.5632	-2.5323	-2.5962	-2.5614	-4.8803	-4.8444	-4.9375	-4.8971
	$\times 1.0e+03$	for all values						

Table B.14: AIC/BIC for the HFRX.

	MSCI BRIC Original Data				MSCI BRIC Recent Data			
	$\epsilon_t \sim N$		$\epsilon_t \sim t$		$\epsilon_t \sim N$		$\epsilon_t \sim t$	
(p,q)	**AIC**	**BIC**	**AIC**	**BIC**	**AIC**	**BIC**	**AIC**	**BIC**
$(0,0)$	-2.9551	-2.9365	-2.9870	-2.9645	-4.2480	-4.2281	-4.2864	-4.2615
$(0,1)$	-2.9552	-2.9319	-2.9874	-2.9594	-4.2493	-4.2244	-4.2878	-4.2578
$(0,2)$	-2.9572	-2.9292	-2.9947	-2.9620	-4.2552	-4.2252	-4.2975	-4.2626
$(1,0)$	-2.9555	-2.9322	-2.988	-2.96	-4.25	-4.225	-4.2885	-4.2586
$(1,1)$	-2.9608	-2.9328	-2.9972	-2.9645	-4.2566	-4.2267	-4.2981	-4.2632
$(1,2)$	-2.9571	-2.9245	-2.9949	-2.9576	-4.2547	-4.2197	-4.2954	-4.2555
$(2,0)$	-2.9577	-2.9298	-2.9962	-2.9636	-4.2557	-4.2258	-4.2985	-4.2636
$(2,1)$	-2.9574	-2.9248	-2.9959	-2.9587	-4.2538	-4.2189	-4.2971	-4.2572
$(2,2)$	-2.9574	-2.9201	-2.9992	-2.9573	-4.2547	-4.2148	-4.3010	-4.2561
	$\times 1.0e+03$	for all values						

TABLE B.15: AIC/BIC for the MSCI BRIC.

Property

		$\epsilon_t \sim N$		$\epsilon_t \sim t$	
	(p,q)	**AIC**	**BIC**	**AIC**	**BIC**
All Property	$(0,0)$	-2.581	-2.5656	-2.5804	-2.5611
	$(0,1)$	-2.7351	-2.7158	-2.7309	-2.7078
	$(0,2)$	-2.7809	-2.7578	-2.8032	-2.7762
	$(1,0)$	-2.9274	-2.9081	-2.9823	-2.9591
	$(1,1)$	-2.9539	-2.9307	-2.9963	-2.9693
	$(1,2)$	-2.953	-2.9260	-2.9951	-2.9642
	$(2,0)$	-2.9493	-2.9262	-2.9940	-2.9670
	$(2,1)$	-2.9467	-2.9197	-2.9945	-2.9636
	$(2,2)$	-2.9621	-2.9312	-2.9931	-2.9584
	$\times 1.0e+03$	for all values			

TABLE B.16: AIC/BIC for all property.

	(p,q)	$\epsilon_t \sim N$ AIC	BIC	$\epsilon_t \sim t$ AIC	BIC
Offices	$(0,0)$	-2.4430	-2.4276	-2.4443	-2.425
	$(0,1)$	-2.6148	-2.5955	-2.6356	-2.6125
	$(0,2)$	-2.6754	-2.6522	-2.6952	-2.6682
	$(1,0)$	-2.8239	-2.8046	-2.8599	-2.8367
	$(1,1)$	-2.8411	-2.8179	-2.8762	-2.8492
	$(1,2)$	-2.8394	-2.8123	-2.8742	-2.8433
	$(2,0)$	-2.8338	-2.8107	-2.8718	-2.8448
	$(2,1)$	-2.8391	-2.8121	-2.8742	-2.8433
	$(2,2)$	-2.8420	-2.8111	-2.8783	-2.8436
$\times 1.0e+03$		for all values			

Table B.17: AIC/BIC for offices.

	(p,q)	$\epsilon_t \sim N$ AIC	BIC	$\epsilon_t \sim t$ AIC	BIC
City offices	$(0,0)$	-2.183	-2.1676	-2.2259	-2.2066
	$(0,1)$	-2.2848	-2.2656	-2.3354	-2.3122
	$(0,2)$	-2.2888	-2.2657	-2.3548	-2.3278
	$(1,0)$	-2.3443	-2.3250	-2.4157	-2.3926
	$(1,1)$	-2.3743	-2.3512	-2.4555	-2.4285
	$(1,2)$	-2.3725	-2.3455	-2.4535	-2.4227
	$(2,0)$	-2.3610	-2.3379	-2.4376	-2.4105
	$(2,1)$	-2.3728	-2.3458	-2.4536	-2.4227
	$(2,2)$	-2.3912	-2.3603	-2.4556	-2.4209
$\times 1.0e+03$		for all values			

Table B.18: AIC/BIC for city offices.

	(p,q)	$\epsilon_t \sim N$ AIC	BIC	$\epsilon_t \sim t$ AIC	BIC
Retail	$(0,0)$	-2.5838	-2.5684	-2.5816	-2.5624
	$(0,1)$	-2.7048	-2.6855	-2.7221	-2.699
	$(0,2)$	-2.7613	-2.7381	-2.7722	-2.7452
	$(1,0)$	-2.8887	-2.8694	-2.9145	-2.8914
	$(1,1)$	-2.9046	-2.8815	-2.9285	-2.9015
	$(1,2)$	-2.8632	-2.8362	-2.9268	-2.8959
	$(2,0)$	-2.9034	-2.8803	-2.9258	-2.8988
	$(2,1)$	-2.9032	-2.8762	-2.9266	-2.8957
	$(2,2)$	-2.8994	-2.8685	-2.9264	-2.8917
$\times 1.0e+03$		for all values			

Table B.19: AIC/BIC for retail.

	(p,q)	$\epsilon_t \sim N$ AIC	BIC	$\epsilon_t \sim t$ AIC	BIC
All Property	$(0,0)$	-2.4166	-2.4011	-2.4221	-2.4028
	$(0,1)$	-2.5304	-2.5111	-2.5683	-2.5452
	$(0,2)$	-2.5576	-2.5345	-2.6063	-2.5793
	$(1,0)$	-2.6435	-2.6242	-2.7131	-2.6899
	$(1,1)$	-2.6579	-2.6347	-2.7194	-2.6924
	$(1,2)$	-2.6562	-2.6292	-2.7191	-2.6883
	$(2,0)$	-2.6564	-2.6332	-2.7169	-2.6899
	$(2,1)$	-2.6562	-2.6292	-2.7177	-2.6869
	$(2,2)$	-2.6554	-2.6246	-2.7201	-2.6854
$\times 1.0e+03$		for all values			

Table B.20: AIC/BIC for warehouse.

B.5 Param. Estimates

Equity

Data	ϵ_t	**(p,q)**	**Parameter**	**Estimate**	**Stand. Error**	**t-statistic**
MSCI World Original	$\sim N$	(0,0)	ϕ_0	0.00216979	0.000365323	5.93938
			α_0	1.8045e-05	3.14972e-06	5.72906
			α_1	0.142444	0.00973617	14.6304
			β_1	0.817434	0.015557	52.5443
	$\sim t$	(0,0)	ϕ_0	0.00212008	0.00035961	5.89549
			α_0	1.1314e-05	3.29802e-06	3.43054
			α_1	0.0922449	0.0160336	5.75324
			β_1	0.879075	0.0209436	41.9734
			DoF	8.29272	1.13508	7.30581
MSCI World Recent	$\sim N$	(0,0)	ϕ_0	0.00215796	0.000347232	6.21474
			α_0	1.98181e-05	3.31083e-06	5.98583
			α_1	0.142324	0.00955968	14.8879
			β_1	0.813466	0.0154083	52.7942
	$\sim t$	(0,0)	ϕ_0	0.00211991	0.000339589	6.24259
			α_0	1.2692e-05	3.39447e-06	3.73903
			α_1	0.0953547	0.0153441	6.21442
			β_1	0.872878	0.0201109	43.4032
			DoF	8.29278	1.05563	7.85576

Table B.21: Parameter estimates for chosen ARMA-GARCH models: MSCI World.

Data	ϵ_t	(p,q)	Param.	Estimate	Stand. Error	t-statistic
LPX50 Original	$\sim N$	(1,2)	ϕ_0	0.000763745	0.000459514	1.66207
			ϕ_1	0.768521	0.120179	6.39479
			ψ_1	-0.762193	0.126291	-6.0352
			ψ_2	0.0703712	0.037389	1.88214
			α_0	1.04693e-05	3.32891e-06	3.14496
			α_1	0.13804	0.0119963	11.5069
			β_1	0.861959	0.0134639	64.0202
	$\sim t$	(1,2)	ϕ_0	0.000750974	0.000386505	1.94299
			ϕ_1	0.781838	0.0965395	8.09864
			ψ_1	-0.780076	0.101069	-7.71824
			ψ_2	0.0766913	0.0340933	2.24945
			α_0	9.07685e-06	3.91286e-06	2.31975
			α_1	0.103634	0.0213897	4.84506
			β_1	0.890795	0.0201646	44.1761
			DoF	6.25678	0.986726	6.34096
LPX50 Recent	$\sim N$	(0,0)	ϕ_0	0.00346488	0.00061159	5.665361
			α_0	1.17509e-05	3.76704e-06	3.11939
			α_1	0.149019	0.012309	12.1065
			β_1	0.850981	0.0144427	58.921
	$\sim t$	(2,0)	ϕ_0	0.00309297	0.000601204	5.14462
			ϕ_1	0.00708075	0.0312474	0.226603
			ϕ_2	0.114293	0.0303428	3.76671
			α_0	9.15048e-06	3.92017e-06	2.3342
			α_1	0.10053	0.0209365	4.80169
			β_1	0.893057	0.0200618	44.5153
			DoF	6.28284	0.986663	6.36777

Table B.22: Parameter estimates for chosen ARMA-GARCH models: LPX50.

Data	ϵ_t	(p,q)	Param.	Estimate	Stand. Error	t-statistic
S&P GSCI Original	$\sim N$	(0,0)	ϕ_0	0.00219891	0.000418299	5.25679
			α_0	1.01743e-05	2.329e-06	4.36853
			α_1	0.100473	0.0114386	8.78369
			β_1	0.886929	0.0120913	73.3526
	$\sim t$	(0,0)	ϕ_0	0.00238073	0.000433009	5.49811
			α_0	9.82857e-06	3.08145e-06	3.18959
			α_1	0.100828	0.0148571	6.78655
			β_1	0.887844	0.0155529	57.0853
			DoF	9.75914	2.1304	4.5809
S&P GSCI Recent	$\sim N$	(0,0)	ϕ_0	0.00178212	0.00040193	4.4339
			α_0	8.79903e-06	1.85943e-06	4.73212
			α_1	0.0878656	0.00931001	9.43775
			β_1	0.901318	0.009459	95.2868
	$\sim t$	(0,0)	ϕ_0	0.00203403	0.000413185	4.92282
			α_0	9.18197e-06	2.75048e-06	3.33831
			α_1	0.0908625	0.0129594	7.01131
			β_1	0.898363	0.0133348	67.3697
			DoF	8.82261	1.56374	5.64198

Table B.23: Parameter estimates for chosen ARMA-GARCH models: GSCI.

Data	ϵ_t	(p,q)	Parameter	Estimate	Stand. Error	t-statistic
HFRX Original	$\sim N$	(0,0)	ϕ_0	0.00128108	0.000303174	4.22558
			α_0	7.11971e-06	1.42674e-06	4.99019
			α_1	0.309917	0.067815	4.57003
			β_1	0.563896	0.0805339	7.00197
	$\sim t$	(1,1)	ϕ_0	0.000362522	0.000179108	2.02404
			ϕ_1	0.823612	0.0902572	9.12517
			ψ_1	-0.696608	0.118892	-5.85916
			α_0	8.39716e-06	3.11567e-06	2.69514
			α_1	0.274593	0.103007	2.66577
			β_1	0.567122	0.124238	4.56479
			DoF	4.36598	1.13111	3.85991
HFRX Recent	$\sim N$	(1,1)	ϕ_0	0.00022922	0.000153631	1.49202
			ϕ_1	0.683861	0.12722	5.37544
			ψ_1	-0.531457	0.15707	-3.38356
			α_0	5.47725e-06	2.15146e-06	2.54583
			α_1	0.200921	0.0314088	6.39697
			β_1	0.658891	0.0631905	10.4271
	$\sim t$	(1,1)	ϕ_0	0.000261955	0.000120519	2.17355
			ϕ_1	0.811242	0.0837947	9.6813
			ψ_1	-0.695121	0.105417	-6.59401
			α_0	5.65734e-06	2.67864e-06	2.11202
			α_1	0.149581	0.0447186	3.34494
			β_1	0.697307	0.0855676	8.14919
			DoF	4.69863	0.906382	5.18394

Table B.24: Parameter estimates for chosen ARMA-GARCH models: HFRX.

Data	ϵ_t	(p,q)	Parameter	Estimate	Stand. Error	t-statistic
MSCI BRIC Original	$\sim N$	(0,0)	ϕ_0	0.00469493	0.00118844	3.95051
			α_0	8.24773e-05	2.09489e-05	3.93707
			α_1	0.138927	0.0216343	6.42161
			β_1	0.810021	0.0290884	27.8468
	$\sim t$	(0,0)	ϕ_0	0.00537562	0.00112216	4.79043
			α_0	6.57746e-05	2.67399e-05	2.45979
			α_1	0.138913	0.0340882	4.07512
			β_1	0.825952	0.0387672	21.3054
			DoF	6.02959	1.46622	4.11233
MSCI BRIC Recent	$\sim N$	(0,0)	ϕ_0	0.00276169	0.000972906	2.8386
			α_0	6.64198e-05	1.6203e-05	4.09923
			α_1	0.124209	0.017512	7.09276
			β_1	0.827093	0.0249185	33.1918
	$\sim t$	(2,0)	ϕ_0	0.0026862	0.000903528	2.97302
			ϕ_1	0.0579067	0.0314365	1.84202
			ϕ_2	0.108501	0.0319006	3.40122
			α_0	5.95883e-05	2.11898e-05	2.81213
			α_1	0.115328	0.0257362	4.48118
			β_1	0.841532	0.0329732	25.5217
			DoF	6.45653	1.34318	4.80689

Table B.25: Parameter estimates for chosen ARMA-GARCH models: MSCI BRIC.

Property

Data	ϵ_t	(p,q)	Parameter	Estimate	Stand. Error	t-statistic
All Prop Recent	$\sim N$	(2,2)	ϕ_0	0.00102886	0.000392135	2.62375
			ϕ_1	-0.0591907	0.0224054	-2.6418
			ϕ_2	0.919202	0.0220665	41.6559
			ψ_1	0.707915	0.0649291	10.9029
			ψ_2	-0.274542	0.0628853	-4.36576
			α_0	5.18075e-07	6.28978e-07	0.823677
			α_1	0.677037	0.0380464	17.795
			β_1	0.322963	0.0545632	5.91906
	$\sim t$	(1,1)	ϕ_0	0.000493899	0.000171297	2.88328
			ϕ_1	0.922869	0.0188978	48.8348
			ψ_1	-0.247396	0.061299	-4.03589
			α_0	8.21526e-07	7.85982e-07	1.04522
			α_1	0.384214	0.0997638	3.85124
			β_1	0.615786	0.0622294	9.89542
			DoF	4.61414	0.966867	4.77226

Table B.26: Parameter estimates for chosen ARMA-GARCH models: IPD UK All Property.

Data	ϵ_t	(p,q)	Parameter	Estimate	Stand. Error	t-statistic
Offices Recent	$\sim N$	(1,1)	ϕ_0	0.000370845	0.000173782	2.13396
			ϕ_1	0.947684	0.0183722	51.5825
			ψ_1	-0.364651	0.0553234	-6.59126
			α_0	5.24782e-07	6.85012e-07	0.766091
			α_1	0.303069	0.0483977	6.26206
			β_1	0.696931	0.0382543	18.2184
	$\sim t$	(1,1)	ϕ_0	0.000463707	0.000174777	2.65313
			ϕ_1	0.924637	0.0203226	45.4981
			ψ_1	-0.292208	0.0644576	-4.53333
			α_0	5.16142e-07	7.30348e-07	0.706707
			α_1	0.312918	0.0768141	4.07371
			β_1	0.687081	0.0556175	12.3537
			DoF	5.95557	1.35268	4.4028

Table B.27: Parameter estimates for chosen ARMA-GARCH models: IPD UK Office.

Data	ϵ_t	(p,q)	Parameter	Estimate	Stand. Error	t-statistic
City offices Recent	$\sim N$	(2,2)	ϕ_0	0.000898617	0.000523684	1.71595
			ϕ_1	0.330214	0.0887795	3.71948
			ϕ_2	0.586614	0.0763998	7.67822
			ψ_1	0.229703	0.108091	2.1251
			ψ_2	-0.453643	0.0682958	-6.64233
			α_0	1.0805e-06	7.96288e-07	1.35692
			α_1	0.168969	0.0239039	7.0687
			β_1	0.831031	0.0158745	52.3499
	$\sim t$	(1,1)	ϕ_0	0.00100725	0.000276742	3.63965
			ϕ_1	0.86812	0.0285575	30.3991
			ψ_1	-0.476912	0.0591585	-8.0616
			α_0	2.42071e-06	1.90292e-06	1.2721
			α_1	0.18787	0.0769987	2.43991
			β_1	0.81213	0.04257	19.0775
			DoF	2.93884	0.516924	5.68524

Table B.28: Parameter estimates for chosen ARMA-GARCH models: IPD UK City Offices.

Data	ϵ_t	(p,q)	Parameter	Estimate	Stand. Error	t-statistic
Retail Recent	$\sim N$	(1,1)	ϕ_0	0.000515259	0.00024487	2.10422
			ϕ_1	0.925479	0.0281024	32.9324
			ψ_1	-0.284466	0.0748298	-3.80151
			α_0	1.54438e-06	9.64595e-07	1.60106
			α_1	0.360844	0.056004	6.44317
			β_1	0.593749	0.0546824	10.8581
	$\sim t$	(1,1)	ϕ_0	0.000562495	0.000200358	2.80745
			ϕ_1	0.907769	0.0244622	37.109
			ψ_1	-0.257638	0.0689975	-3.73403
			α_0	1.44203e-06	1.04655e-06	1.37789
			α_1	0.425464	0.123124	3.45556
			β_1	0.574536	0.077588	7.40496
			DoF	4.65734	1.32627	3.5116

Table B.29: Parameter estimates for chosen ARMA-GARCH models: IPD UK Retail.

Data	ϵ_t	(p,q)	Parameter	Estimate	Stand. Error	t-statistic
Warehouse Recent	$\sim N$	(1,1)	ϕ_0	0.000804766	0.00035475	2.26854
			ϕ_1	0.908565	0.0351814	25.8252
			ψ_1	-0.304846	0.077401	-3.93853
			α_0	8.59639e-07	7.58427e-07	1.13345
			α_1	0.259074	0.0410113	6.31714
			β_1	0.740926	0.0284269	26.0643
	$\sim t$	(1,1)	ϕ_0	0.000948878	0.000294503	3.22197
			ϕ_1	0.863781	0.0309399	27.918
			ψ_1	-0.20804	0.0703957	-2.95529
			α_0	1.3596e-06	1.06883e-06	1.27205
			α_1	0.358192	0.100956	3.54799
			β_1	0.641808	0.0598325	10.7268
			DoF	4.05647	0.736296	5.50929

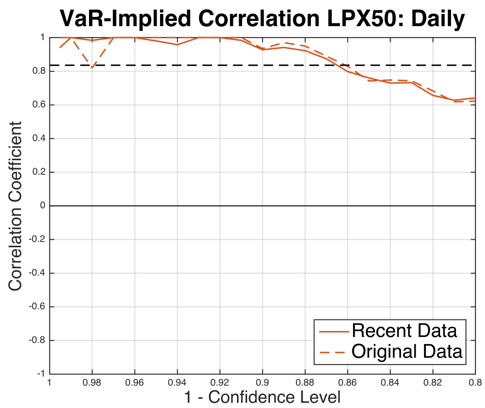
Table B.30: Parameter estimates for chosen ARMA-GARCH models: IPD UK Warehouse.

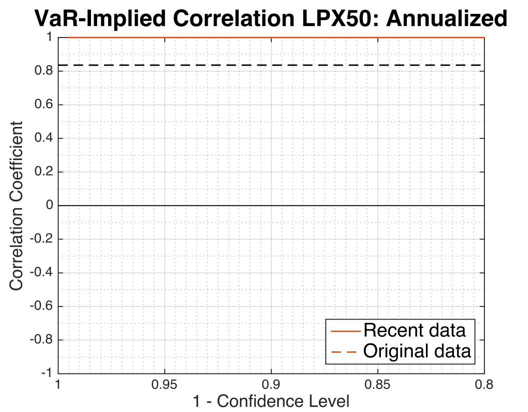
B.6 Correlation based on Daily Returns

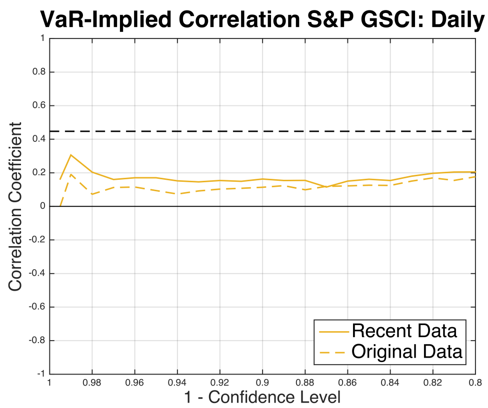
VaR-Implied

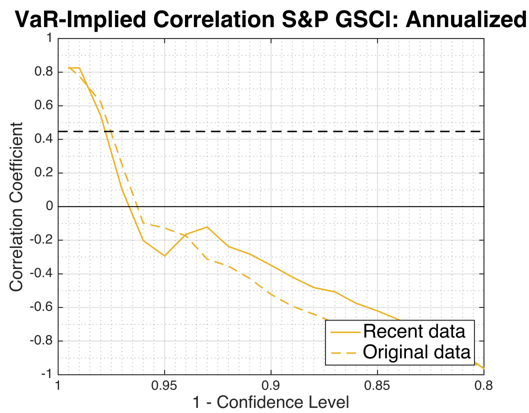
	LPX50		S&P GSCI		HFRX		BRIC	
α	Original	Recent	Original	Recent	Original	Recent	Original	Recent
0.995	1	0.9427	0.0015	0.1625	1	1	1	0.8094
0.99	1	1	0.1908	0.306	1	1	1	0.8903
0.98	0.8203	0.9839	0.0723	0.2037	1	1	0.908	0.8732
0.97	1	1	0.1124	0.1598	1	1	0.8835	0.828
0.96	1	1	0.1154	0.1707	1	1	0.7994	0.9052
0.95	1	0.98	0.094	0.17	1	1	0.7421	0.7966
0.9	0.9349	0.9278	0.1139	0.1627	1	1	0.8011	0.7947
0.85	0.7427	0.76	0.1258	0.1617	1	1	0.805	0.8088
0.8	0.6224	0.6414	0.1768	0.205	1	1	0.8083	0.7649

TABLE B.31: 95% bootstrap confidence intervals for VaR-implied correlation coefficients between ”global equity” and ”other equity” based on daily returns of recent data.









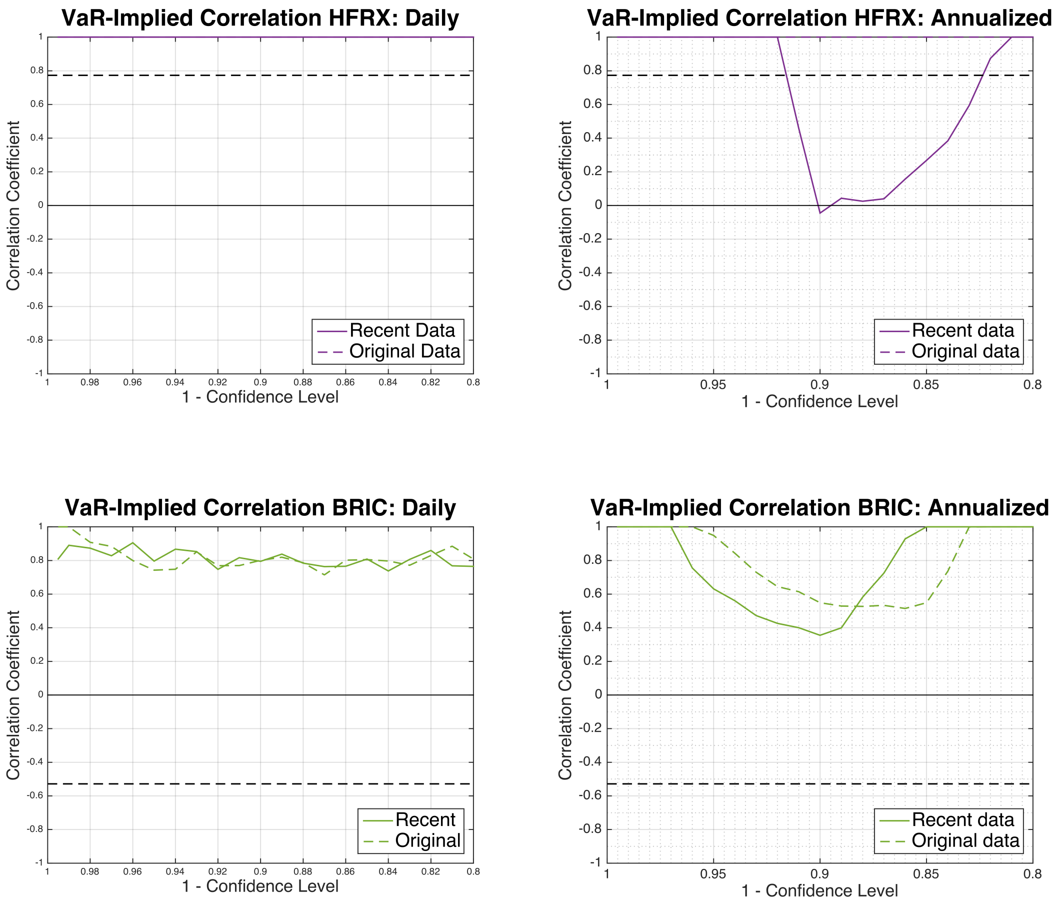


FIGURE B.5: based on daily returns in comparison to results of calibrations based on rolling-window annual returns.

Data Cutting

	LPX50		**S&P GSCI**		**HFRX**		**BRIC**	
α	**Original**	**Recent**	**Original**	**Recent**	**Original**	**Recent**	**Original**	**Recent**
0.995	0.2123	0.2927	0.4152	0.4606	0.4541	0.4677	0.544	0.6533
0.99	0.5822	0.5405	0.6236	0.6396	0.5076	0.7078	0.4858	0.491
0.98	0.6661	0.7001	0.7292	0.6955	0.6212	0.5823	0.5905	0.6068
0.97	0.6724	0.6944	0.7487	0.7287	0.5467	0.3909	0.5407	0.5591
0.96	0.7168	0.7189	0.7644	0.7338	0.4065	0.5169	0.5197	0.5315
0.95	0.747	0.7306	0.605	0.596	0.4864	0.5501	0.5394	0.5476
0.9	0.6943	0.7023	0.5639	0.5406	0.4917	0.5316	0.5633	0.5485
0.85	0.6873	0.701	0.5163	0.4915	0.5092	0.5353	0.5695	0.5492
0.8	0.6844	0.6996	0.4581	0.4439	0.5555	0.5722	0.5787	0.5624

TABLE B.32: 95% bootstrap confidence intervals for data-cutting correlation coefficients between "global equity" and "other equity" based on daily returns of recent data.

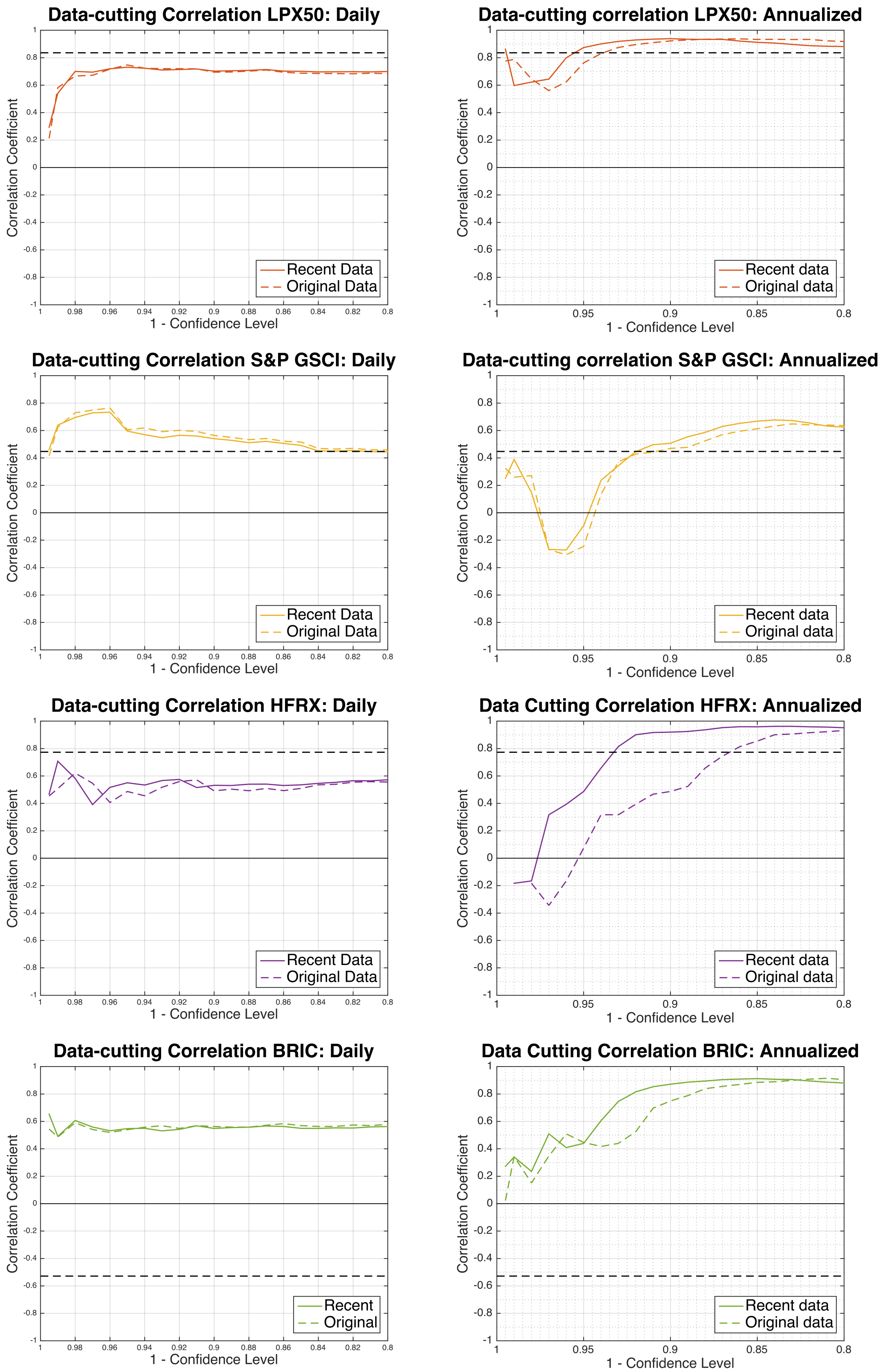


FIGURE B.6: Data-cutting correlation based on daily returns in comparison to results of calibrations based on rolling-window annual returns.

Pearson Correlation

	Annualized Returns		Daily Returns		Regulator
	Original Data	Recent Data	Original Data	Recent Data	
LPX50	0.7502	0.7680	0.6706	0.6971	0.8359
S&P GSCI	0.0653	0.0825	0.1290	0.1848	0.4472
HFRX	0.9664	0.9433	0.6900	0.7110	0.7731
MSCI BRIC	0.6950	0.6660	0.5921	0.6072	−0.5282

Table B.33: Linear Pearson correlation coefficients between "global equity" and "other equity" based on daily and annualized returns in comparison.

Appendix C

Electronic Appendix

The Electronic Appendix in form of a CD-Rom comprises

1. Data
2. Matlab Code
3. Thesis

Data
The indexes are extracted from Bloomberg[1]. Data representing equity risk are available in form of daily closing prices. Indexes representing property risk are provided on a monthly price basis. For each of the two investigated risk modules, indexes are combined in one excel file. For each module, there are two data sets – the original and the recent data set. Thus, data for this thesis is provided in four excel files,

- Daily_Prices_Equity_Recent.xlsx
- Daily_Prices_Equity_Original.xlsx
- Monthly_Prices_Property_Recent.xlsx
- Monthly_Prices_Property_Original.xlsx

[1]Except the HFRX Global Hedge Fund Index which is directly extracted from the HFR website: https://www.hedgefundresearch.com/family-indices/hfrx.

MATLAB Code

The MATLAB code required for the empirical part of this thesis is divided in several functions and programs. Those functions and programs are topically stored in several sub-folders. Figure C.1 gives an overview.

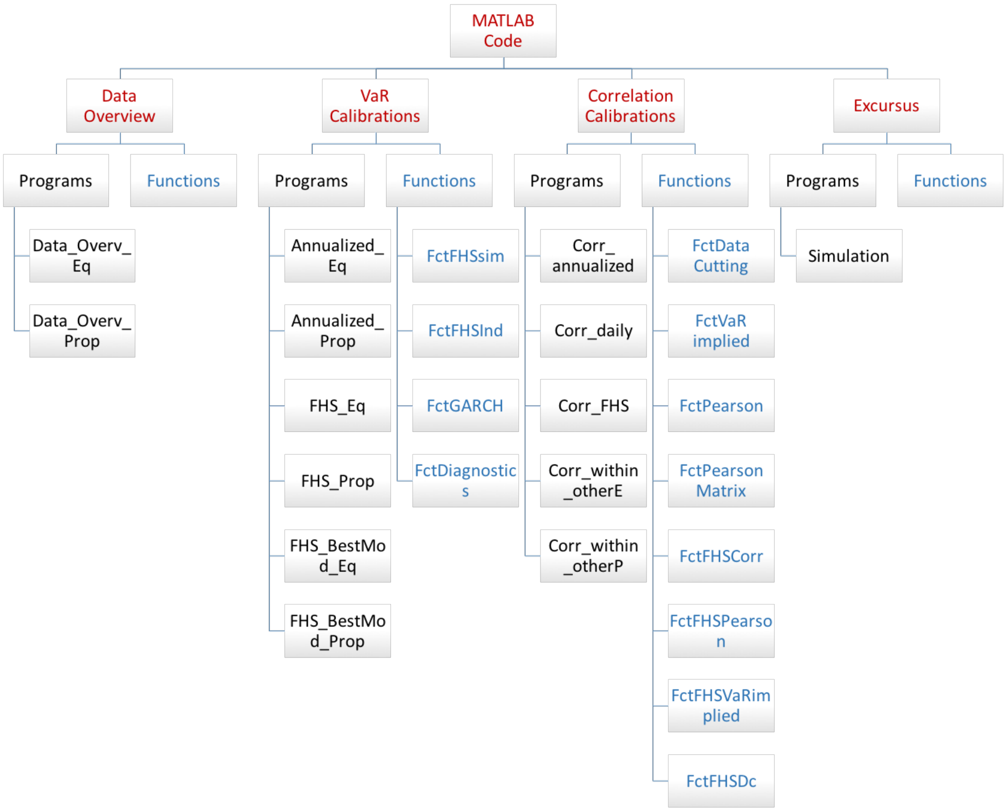


FIGURE C.1: Summary of the written programs and functions for the empirical analysis.

Thesis

This folder contains a pdf version of the master thesis.

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