The Season Package (by Ekkehart Schlicht)

Revision 3 of February 2017. Adapted to *Mathematica* 6 and *Mathematica* 8.

The package implements Schlicht's (1984) seasonal adjustment method. It decomposes a time series $x_t,\ t \in \{1, ..., T\}$ into a trend $y_t,\ t \in \{1, ..., T\}$, a seasonal component $z_t,\ t \in \{1, ..., T\}$, and an irregular component $u_t,\ t \in \{1, ..., T\}$ such that $x_t = y_t + z_t + u_t$ for all t. The method uses non-parametric splines. It combines the trend filter proposed by Leser (1961), known as the HP-Filter filter, the seasonal filter proposed by Schlicht and Pauly (1983) and the orthogonal parametrization proposed by Schlicht (1984, 2005). For the treatment of missing observations and structural breaks, see Schlicht(2009). In contrast to prevailing methods, the method used here is based on an explicit statistical model (state-space).

Season is appropriate for smoothing time series with higher than annual frequency. Season is inappropriate for smoothing annual data or deseasonalized time series; for this, use the HPFilter package by Johannes Ludsteck.

This package provides the following functions for seasonal adjustment:

Season[x,s] splits the series x into trend, seasonal component, and irregular component, using estimated optimum smoothing parameters.
Season[x,s,α,γ] splits the series x into trend, seasonal component, and irregular component, using the smoothing parameters α and γ.

Both versions of Season[ ] return a list $\{\{x,y,z,u\},\{\alpha,\gamma\},\sigma^2\}$ *where x is the input series, y is the trend, z is the seasonal component, u is the irregular component,* α *and* γ *are the smoothing parameters used and* σ^2 *is the variance of the irregular component..*

LL[x,s,α,γ] give the log likelihood of the constellation $\{x,s,\alpha,\gamma\}$.

LLPlot[x,s, $\{\alpha_{min}, \alpha_{max}\}$,$\{\gamma_{min}, \gamma_{max}\}$] plots the log likelihood for the range $\{\alpha,\gamma\} \in [\alpha_{min}, \alpha_{max}] \times [\gamma_{min}, \gamma_{max}]$.

The method needs the time series x and the length of the season s as an input (*e.g.* $s=4$ for quarterly data or $s=12$ for monthly data). The function Season[x,s] computes smoothing parameters α and γ by a maximum likelihood method and splits the time series, using these weights. Alternatively, the parameters can be provided in Season[x,s,α,γ]. The parameter α controls the smoothness of the trend - the larger α, the smoother the trend. The parameter γ controls the rigidity of the seasonal pattern - the larger γ, the more time-invariant is the seasonal pattern. Both parameters interact: Increasing α while keeping γ constant will shift some variabilty from the trend to the seasonal component, for example (and the rest to the irregular component).

The method assumes that trend, season and irregular component are *additive* components of the time series. For some time series, such as trended time series, a *multiplicative* formulation is preferable. In this case, take the logarithms of the original data as the input.

Instructions: Place the package season6.m in $UserBaseDirectory\Applications\ (usually C : \Users \< username > \AppData\Roaming\Mathematica\ Applications\) and successively evaluate the cells.

To obtain information of the available functions, such as `Season[],` evaluate it with a question mark, like `?Season.` The list of of functions in the package season6.m can be obtained by opening it oin *Mathematica* and click on “Functions”.

This loads the package

```
In[1]:= << Season6`
```

Here is a series with 128 elements - the time series of monthly Gereman unemployment rates for 1992:01 through 2002:08.

```
In[2]:= x = {8.3, 8.2, 7.7, 7.6, 7.4, 7.3, 7.8, 7.7, 7.5, 7.6, 7.7, 8.1, 9, 9, 8.7, 8.6, 8.4, 8.5,
   9.1, 9.1, 8.9, 9.1, 9.2, 9.6, 10.5, 10.5, 10.1, 9.9, 9.5, 9.3, 9.6, 9.4, 9.1, 8.9,
   8.9, 9.2, 10, 10, 9.6, 9.4, 9, 9, 9.4, 9.3, 9.2, 9.2, 9.3, 9.9, 10.8, 11.1, 10.8,
   10.4, 10, 9.9, 10.2, 10.2, 10.1, 10.1, 10.3, 10.8, 12.2, 12.2, 11.7, 11.3, 11.1,
   11, 11.4, 11.4, 11.2, 11.2, 11.3, 11.8, 12.6, 12.6, 12.1, 11.4, 10.9, 10.5, 10.7,
   10.6, 10.3, 10.1, 10.2, 10.9, 11.5, 11.6, 11.1, 10.7, 10.2, 10.1, 10.3, 10.3, 10.1,
   9.9, 10, 10.3, 11, 10.9, 10.6, 9.8, 9.3, 9.1, 9.3, 9.3, 9, 8.9, 8.9, 9.3, 10, 10.1,
   9.8, 9.5, 9, 8.9, 9.2, 9.2, 9, 9, 9.2, 9.6, 10.4, 10.4, 10, 9.7, 9.5, 9.5, 9.7, 9.6};
```

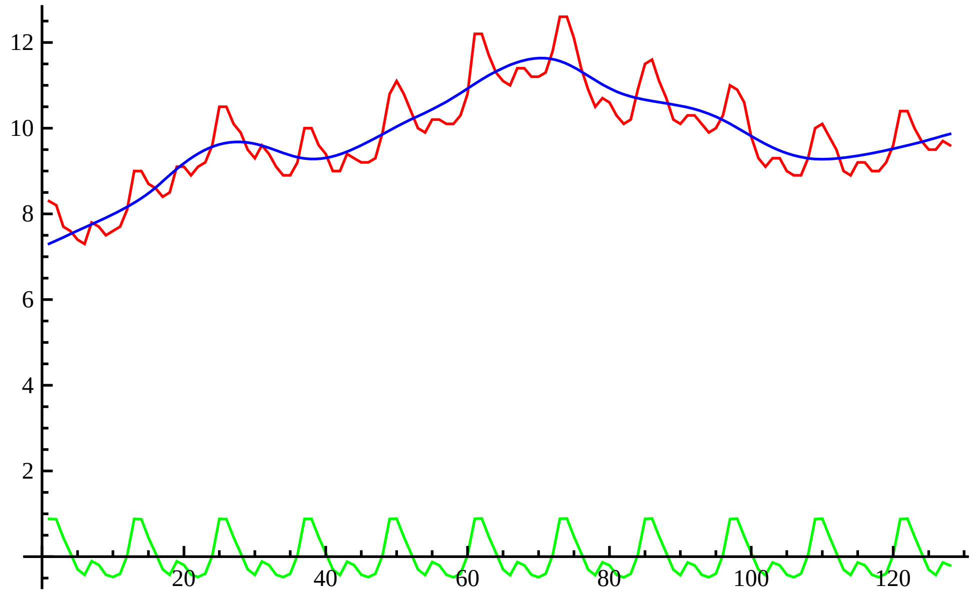
The following code calls the decomposition routine for monthly data with smoothing parameter values α=100 and γ=100 and returns the list $\{\{x,y,z,u\},\{\alpha,\gamma\}\}$.

```
In[3]:= {{x, y, z, u}, {α, γ}, varU} = Season[x, 12, 100, 100];
```

Here is a plot of the original time series *x* (red), the trend *y* (blue), and the seasonal component *z* (green).

```
In[4]:= ListPlot[{x, y, z }, Joined → True,
   PlotStyle → {{RGBColor[1, 0, 0]}, {RGBColor[0, 0, 1]}, {RGBColor[0, 1, 0]}}]
```

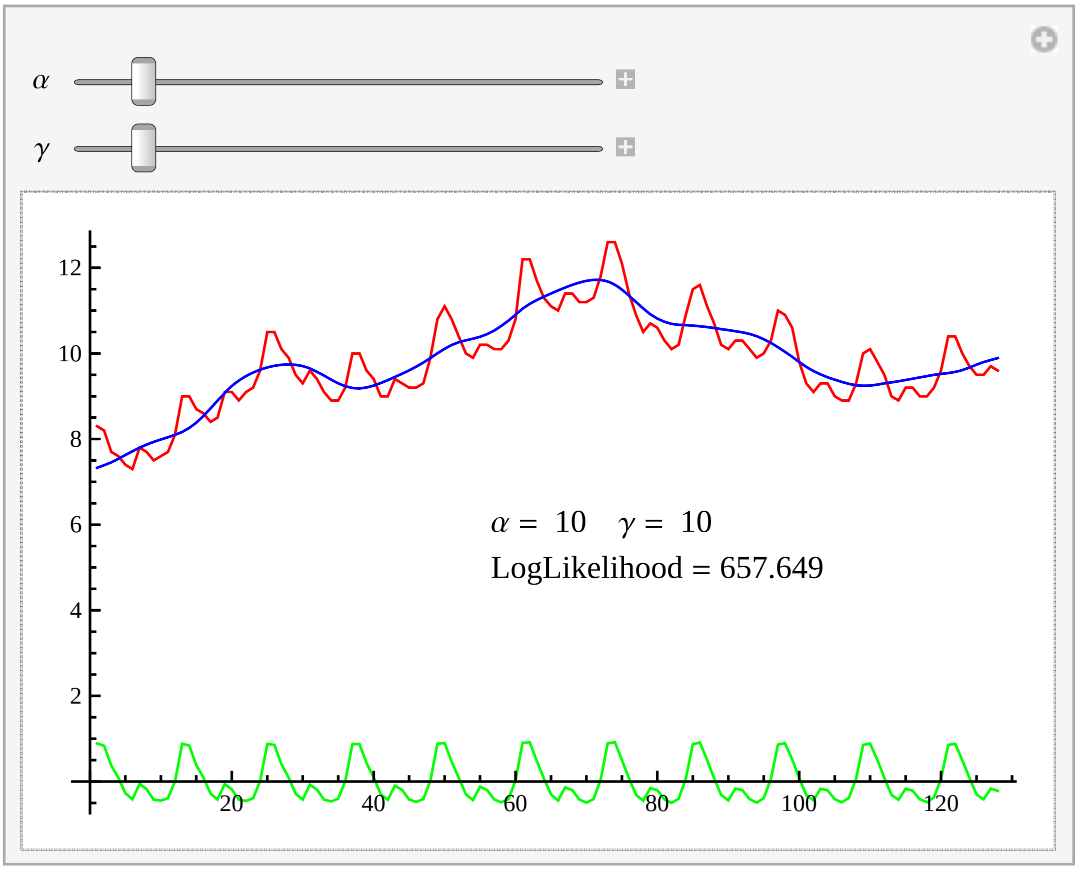
Out[4]=



To view the effect of the choice of the smoothing parameters α and γ over the ranges (α_{min}, α_{max}) and (γ_{min}, γ_{max}) evaluate the following. It uses α_{init} = 10, α_{min} = .1, α_{max} = 100, γ_{init} = 10, γ_{min} = .1, γ_{max} = 100.

In[5]:=
```
Manipulate[{{x, y, z, u}, {α, γ}, varU} = Season[x, 12, α, γ];
  Show[ListPlot[{x, y, z }, Joined → True,
    PlotStyle → {{RGBColor[1, 0, 0]}, {RGBColor[0, 0, 1]}, {RGBColor[0, 1, 0]}}],
   Graphics[Text[Style["α =  " <> ToString[ α ] <> "     γ =  " <>
       ToString[ γ ] <> "\nLogLikelihood = " <> ToString[LL[x, 12, α, γ]],
      12, TextAlignment → Left],  {80, 5.5}]]],
  {{α, 10}, .1, 100}, {{γ, 10}, .1, 100}]
```

Out[5]=



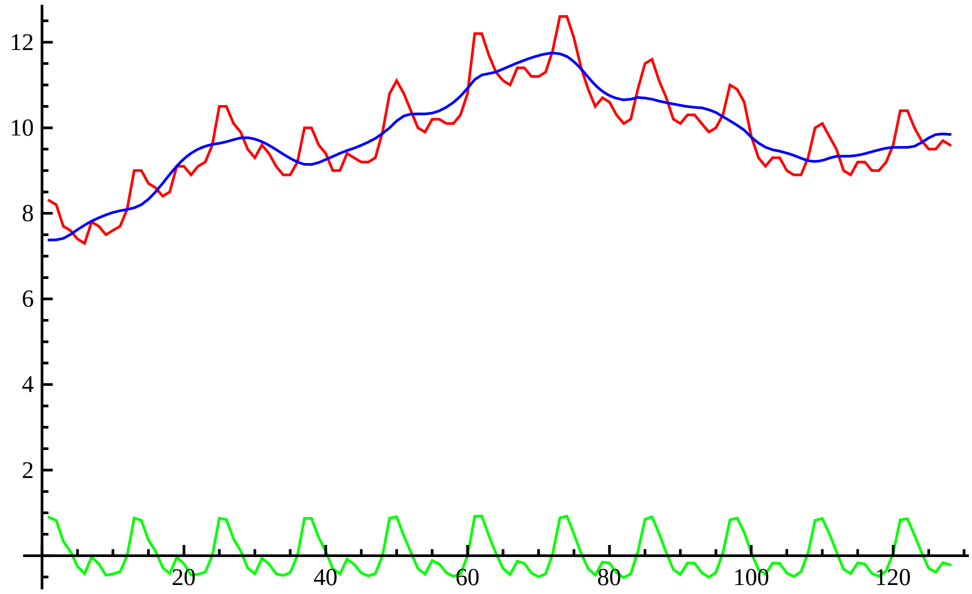
If Season is called without smoothing parameters, it computes the optimum smoothing parameters by a maximum-likelihood method and returns the list $\{\{x,y,z,u\},\{\alpha,\gamma\},\sigma^2\}$.

In[6]:=
```
{{x, y, z, u}, {a, g}, var} = Season[x, 12];
```

Here is a plot of the original time series x (red), the trend y (blue), and the seasonal component z (green). It can be seen that the smoothing parameters are estimated smaller than the parameters used in the previous example. In particular, the parameter α for the trend is estimated as being quite small, entailing a rougher trend.

```
In[7]:= ListPlot[{x, y, z }, Joined → True,
         PlotStyle → {{RGBColor[1, 0, 0]}, {RGBColor[0, 0, 1]}, {RGBColor[0, 1, 0]}}]
        Print["Smoothing parameters: α = ", a, ",   γ = ", g];
        Print["Estimated variances: σu² = ",
          varU, ", σv² = ", varU / a, ", σw² = ", varU / g];
```

Out[7]=



Smoothing parameters: α = 0.923708, γ = 1.88736

Estimated variances: σ_u^2 = 0.00719997, σ_v^2 = 0.00779464, σ_w^2 = 0.00381484

The option "SearchRange" → $\{\{\alpha_{min}, \alpha_{max}\}, \{\gamma_{min}, \gamma_{max}\}\}$ permits to select a range over which Season searches for a solution, where α is restricted to the interval $[\alpha_{min}, \alpha_{max}]$ and γ is restricted to the interval $[\gamma_{min}, \gamma_{max}]$. If a parameter is estimated close to the boundary of the relevant interval, a warning is issued.

The standard option for the search range is: "SearchRange" → {{0.5, 1000}, {0.5, 1000}}.

Here is an example:

```
In[10]:= {{x, y, z, u}, {α, γ}, var} = Season[x, 12, "SearchRange" → {{1, 10}, {1, 10}}];
```

Season::cornera : Estimate 1.0000000783988976` of alpha appears to be a corner solution

Season calls NMaximize to determine the optimum smoothing parameter. NMaximize options can be given to Season to cope with convergency problems and other issues. They are handed over to NMaximize. If NMaximize does not find a solution, a warning is issued.

Standard options for NMaximize in Season are: Method → {"NelderMead", PostProcess → False }. To speed up calculation, use AccuracyGoal → 3.

Here is an example:

```
In[11]:= {{x, y, z, u}, {α, γ}, var} = Season[x, 12, AccuracyGoal → 8, MaxIterations → 5];
```

NMaximize::cvmit :
Failed to converge to the requested accuracy or precision within 5 iterations. ≫

Season::maxfail : Aborting due to problems in numerical maximization procedure.
Consider fine−tuning of NMaximize options.

```
Out[11]= $Aborted
```

The function LL[x,s,α,γ] returns the log likelihood of the constellation {x,s,α,γ}.

Here is an example:

```
In[12]:= LL[x, 12, 2, 3]

Out[12]= 681.781
```

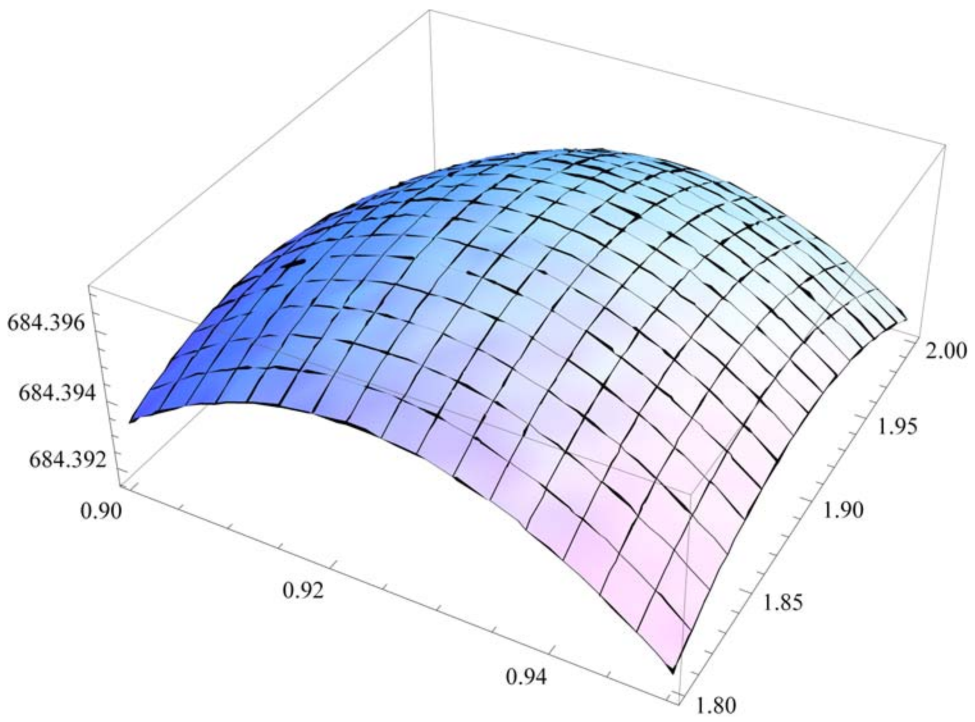
The function LLPlot[x,s,{α_{min}, α_{max}}, {γ_{min}, γ_{max}}] returns a plot of the log likelihood over the range $\{\alpha,\gamma\}\in[\alpha_{min}, \alpha_{max}]\times[\gamma_{min}, \gamma_{max}]$.

LLPlot calls Plot3D and hands any options over to Plot3D. Be aware that the system matrix becomes near-singular for very large or very small values of the smoothing parameters, and that irregularities in these regions may be computational artifacts.

Here is an example:

```
In[13]:= LLPlot[x, 12, {0.9, 0.95}, {1.8, 2}, PlotPoints → 20]
```

Out[13]=



References

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Acknowledgement

I thank Johannes Ludsteck for helpimg me with expert advice in preparing this package.

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October 2005
revised July 2006
revised February 2017