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Kurzbeiträge / *Short Articles*

Reference Group Behaviour and Economic Incentives: A Further Remark*

by

EKKEHART SCHLICHT

The argument given in SCHLICHT [1981] can be generalized to the case of heterogeneous individuals.

Assume n workers forming a reference group. Denote actual productivity of a given worker by π_i and his reference productivity by π_i^*. In presence of an economic incentive e, the i-th worker determines his productivity according to

$$(1) \qquad \pi_i = f^i(\pi_i^*, e) \, , \qquad 0 < f_1^i < 1, \quad f_2^i > 0 \, ,$$

where f_1^i denotes the partial derivative of $f^i(\cdot)$ with respect to the first argument, etc.

Reference productivity π_i^* follows the observed average productivity of the other workers which is perceived as

$$(2) \qquad \frac{1}{n-1} \sum_{j \neq i} \pi_j \, .$$

Hence the process (4) in SCHLICHT [1981] is replaced by the system of differential equations[1]

$$(3) \qquad \dot{\pi}_i^* = \mu_i \left\{ \frac{1}{n-1} \cdot \sum_{j \neq i} f^j(\pi_j^*, e) - \pi_i^* \right\} , \qquad i = 1, 2, \dots n \, .$$

The equilibrium values of $\pi^* = (\pi_1^*, \dots, \pi_n^*)$ for given e are defined by the conditions

$$(4) \qquad \bar{\pi}_i^* = \frac{1}{n-1} \sum_{j \neq i} f^j(\bar{\pi}_j^*, e) \, , \qquad i = 1, 2, \dots n \, .$$

This gives rise to the following proposition:
If there exists a set of equilibrium aspiration levels $\bar{\pi}^* = (\bar{\pi}_1^*, \dots \bar{\pi}_n^*)$ *satisfying (4), it is unique and globally stable.*

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* This note has been written in response to one of several points of criticism of SCHLICHT [1981] by R. Cremer.

[1] Due to a printing error, the dot over $\dot{\pi}^*$ on the left-hand side of (4) in SCHLICHT [1981] has been omitted.

Proof:

1. (Uniqueness) It is easy to check that the Jacobian of system (3) has a dominant diagonal. This establishes uniqueness, see NIKAIDO [1968], theorems 21.1 and 20.4.

2. (Global stability) Define

$$A_i = \mu_i \left\{ \frac{1}{n-1} \sum_{j \neq i} f^j(\pi_j^*, e) - \pi_i^* \right\} \tag{5}$$

as functions of π^*, and consider the Ljapunov function[2]

$$V = \max_i |A_i| \ . \tag{6}$$

It is to be shown that this function is decreasing over time. For a given state π^*, let $\nu \varepsilon \{1, 2, \ldots, n\}$ denote the index which maximizes the expression in (6) during the next instant (ν need not be unique). Hence we have $V = |A_\nu|$ and the time derivative

$$\dot{V} = \dot{A}_\nu \cdot \operatorname{sign} A_\nu \ . \tag{7}$$

Since

$$\dot{A}_\nu = \mu_\nu \left[\frac{1}{n-1} \sum_{j \neq \nu} f_1^j A_j - A_\nu \right] \tag{8}$$

and since $|A_\nu| \geq |A_j|, f_1^j < 1$ for all j, this implies $\dot{A}_\nu < 0$ for $A_\nu > 0$ and $\dot{A}_\nu > 0$ for $A_\nu < 0$.

Hence $\dot{V}$ is negative whenever it is defined.

Since V is a continuous function which is piecewise continuously differentiable, the above argument is sufficient to establish the proposition that V is strictly decreasing over time. This proves stability.

The impact of a change of the economic incentive e on equilibrium productivity can be calculated by noting that (4) implies

$$\frac{1}{n} \sum_i \bar{\pi}_i^* = \frac{1}{n} \sum_i f^i(\bar{\pi}_i^*, e) \ , \tag{9}$$

i.e. that the average aspiration level and the average productivity are equal in equilibrium. Hence average productivity $\bar{\pi}$ can be identified with the average aspiration level

$$\bar{\pi} = \frac{1}{n} \sum_i \bar{\pi}_i^* \ . \tag{10}$$

Formula (4) can be re-written now as

$$\bar{\pi}_i^* = \frac{n}{n-1} \cdot \bar{\pi} - \frac{1}{n-1} \cdot f^i(\bar{\pi}_i^*, e) \ . \tag{11}$$

[2] On the Ljapunov technique, see BHATIA and SZEGÖ [1970], theorem 2.2, p. 66.

This can be differentiated and solved for $d\bar{\pi}_i^*/de$. Summation with respect to i yields

$$(12)\qquad n \cdot \frac{d\bar{\pi}}{de} = \sum_i \frac{n}{n-1+f_1^i} \cdot \frac{d\bar{\pi}}{de} - \sum_i \frac{f_2^i}{n-1+f_1^i} \,.$$

Solving for $d\bar{\pi}/de$ gives the desired expression describing the impact of the economic incentive on productivity

$$(13)\qquad \frac{d\bar{\pi}}{de} = \sum_i a_i \cdot f_2^i$$

where

$$(14)\qquad a_i = \frac{1}{n} \cdot \left(\frac{1}{n+f_1^i-1}\right) \Big/ \left(\sum_j \frac{1}{n+f_1^j-1} - 1\right) > \frac{1}{n} \,.$$

It is easy to see that a_i is close to $1/n$ if social interdependence is weak (f_1^j close to zero for all j) and that it tends to infinity if social interdependence is strong (f_1^j close to unity for all j). Furthermore, each a_i is strictly increasing if f_1^j increases for one j. Hence, if one individual changes in such a way that his reference productivity becomes more important for him, this will increase the social productivity effect for all other individuals.

Formula (13) is the analogue to (6) in SCHLICHT [1981]. The associated "social multiplier" can be defined as

$$(15)\qquad s = \frac{\sum_i a_i f_2^i}{\frac{1}{n} \sum_i f_2^i} \,.$$

It gives the productivity increase induced by a "direct" productivity increase of unity. This term will be particularily large if reference group behaviour is important, i.e. if f_1^i is close to unity for many individuals. In this respect, the proposition of the earlier paper remains valid.

The same can be said with regard to the relationship between the speed of the approach to equilibrium and the strength of the social multiplier: If all f_1^j were equal to unity, the eigenvalues of the Jacobian of system (3) would be all equal to zero. Hence the speed of adjustment in any direction would be zero. Since the eigenvalues are continuous functions of the matrix elements, it can be concluded that the speed of adjustment will be slow, and the social multiplier will be large, if reference group behaviour is important.

References

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Professor Dr. Ekkehard Schlicht
Technische Hochschule Darmstadt
Fachgebiet Wirtschaftstheorie
Residenzschloß
D-6100 Darmstadt
Bundesrepublik Deutschland