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Nuclear Physics B 897 (2015) 500–554

NUCLEAR PHYSICS B

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A flux-scaling scenario for high-scale moduli stabilization in string theory

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Received 28 April 2015; accepted 2 June 2015

Available online 4 June 2015

Editor: Stephan Stieberger

Abstract

Tree-level moduli stabilization via geometric and non-geometric fluxes in type IIB orientifolds on Calabi–Yau manifolds is investigated. The focus is on stable non-supersymmetric minima, where all moduli are fixed except for some massless axions. The scenario includes the purely axionic orientifold-odd moduli. A set of vacua allowing for parametric control over the moduli vacuum expectation values and their masses is presented, featuring a specific scaling with the fluxes. Uplift mechanisms and supersymmetry breaking soft masses on MSSM-like D7-branes are discussed as well. This scenario provides a complete effective framework for realizing the idea of F-term axion monodromy inflation in string theory. It is argued that, with all masses close to the Planck and GUT scales, one is confronted with working at the threshold of controlling all mass hierarchies.

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http://dx.doi.org/10.1016/j.nuclphysb.2015.06.003
0550-3213/

1. Introduction

The central element in relating string theory to the real world is moduli stabilization, i.e. a dynamical mechanism that gives a mass to the ubiquitous massless scalar fields. Most of the more detailed questions about string phenomenology and string cosmology can only be answered in a framework of moduli stabilization. Of course, it would be a big advance to isolate generic but specific predictions of string theory derived models, but so far there are only very general predictions, such as the existence of supersymmetry at a high scale, the existence of axions, gauge interactions with chiral fermions and the existence of inflaton candidates.

The usual approach to moduli stabilization [1–6] is to start with an $\mathcal{N}=1$ supersymmetric compactification to four dimensions of one of the ten-dimensional superstring theories, and then generate a scalar potential for the many moduli by taking into account additional ingredients. These are tree-level background fluxes as well as perturbative and non-perturbative corrections to the Kähler potential and the superpotential. Once these data are specified, one can compute the resulting scalar potential for the moduli and search for minima, which can either preserve supersymmetry or break it spontaneously. A *scenario* of moduli stabilization is a restricted set-up, where a certain type of minima is guaranteed to exist and where one has parametric control over the emerging scales of the vacuum expectation values and masses of the moduli. It is fair to say that in view of the vast landscape, so far there only exist few such scenarios. The most studied ones are the racetrack, the KKLT [7] the large volume scenario (LVS) [8,9], and variations thereof.

The aim of this paper is to propose a scenario of moduli stabilization, which is entirely based on the tree-level flux induced scalar potential. The motivation for this study is two-fold, and to appreciate our approach and its historical embedding let us elucidate this point further.

In the first run of LHC no direct indication of supersymmetry has been found, so that naturalness as a guiding principle is under pressure, and fine-tuning of the Higgs mass (in the string landscape) might eventually be something we have to face. In most approaches to string phenomenology, a supersymmetry breaking scale of the order $M_{\rm susy}\sim 1$ TeV was used as an input to fix the stringy scales. Due to $M_{\rm Pl}/M_{\rm susy}\sim 10^{15}$, a moduli stabilization scenario, dynamically generating exponential hierarchies, seemed very natural. This is precisely what the LVS achieves. However, if $M_{\rm susy}$ is indeed much larger or even close to the GUT scale, then scenarios generating only polynomial hierarchies might also be interesting to consider.

Furthermore, the BICEP2 claim [10] to have measured primordial B-modes with a large tensor-to-scalar ratio of $r\sim 0.2$ has triggered quite some activity in realizing large-field inflation models in string theory. Although by now there is agreement between the PLANCK and the BICEP2 Collaborations that the main component of the B-modes is due to dust in the foreground [11–13], BICEP2's initial results have led to a number of developments in string cosmology. Invoking string theory is motivated because inflation is UV sensitive. For instance, for chaotic inflation with a quadratic potential, the mass scale of inflation is at $M_{\rm inf}\sim 10^{16}$ GeV, the Hubble scale of inflation at $H_{\rm inf}\sim 10^{14}$ GeV and the mass of the inflaton is $m_\theta\sim 10^{13}$ GeV. Therefore, a mechanism such as the shift symmetry of an axion is necessary to gain control over higher-order Planck-suppressed operators.[2] Various scenarios for axion inflation have been proposed, such as natural inflation [15], N-flation [16], or aligned inflation [17]. During the last year, it was analyzed how these scenarios can be embedded into string theory [18–32].

[2] For other symmetry-based mechanisms to suppress higher-order corrections, see [14].

A promising string theory approach, still allowing for some control over the higher-order corrections, is axion monodromy inflation [33,34], for which a field-theory version has been proposed in [35,36] (for a review see for instance [37]). More recently, axion monodromy inflation has been realized via the F-term scalar potential induced by background fluxes [38–40]. This has the advantage that supersymmetry is broken spontaneously by the very same effect by which usually moduli are stabilized. Various scenarios have been studied, mostly in the type IIB context, which differ by what kind of axion is identified as the inflaton. The latter can be the universal axion [39], a geometric axion like a complex structure modulus or a D7-brane deformation developing a shift symmetry in the large complex structure regime [40–42], a Higgs-like open string modulus [43], or the Kalb–Ramond or Ramond–Ramond (R–R) two-form field [38,44]. Moreover, in [45] non-geometric fluxes were employed and the inflaton was given by a Kähler modulus.

In view of single-field large-field inflation, a challenge for string theory is to find a scheme of moduli stabilization such that a single axion θ is the lightest state, beyond maybe some lighter axions providing candidates for the QCD axion or dark radiation [46]. In fact, the challenge is to fix the moduli such that during inflation the following sensitive hierarchy of scales is guaranteed

$$M_{\rm Pl} > M_{\rm s} > M_{\rm KK} > M_{\rm inf} \sim M_{\rm mod} > H_{\rm inf} > |M_\theta| \,, \tag{1.1}$$

where neighboring scales differ by (only) a factor of $O(10)$. An argument for the second-last relation was presented in [47]. Such a hierarchy can either appear just by numerical coincidence, or by having a parameter that controls the quotient of two scales. On the formal side such a string theory realization is constrained by the no-go theorem of [48]. It states that once an axion is completely unstabilized in a supersymmetry-preserving minimum, its saxionic partner is tachyonic. Therefore, non-supersymmetric minima are a better-suited starting point.

In [49] it was analyzed whether the no-scale scalar potential for the complex structure and axio-dilaton moduli in type IIB orientifolds with NS–NS and R–R three-form fluxes admits non-supersymmetric minima, where a single axion can be parametrically lighter than the rest of the moduli (see also [50] for an alternative approach invoking tunings in the string landscape). The procedure, that we will also follow in this paper, is to first turn on large fluxes such that all moduli except one axion are frozen. In a second step, we turn on order-one fluxes freezing also the last axion, which was parametrically lighter than the rest. The main shortcoming of the original approach was to neglect the Kähler moduli. If they could be stabilized by some subleading effect (as in KKLT or LVS) their masses would be parametrically lighter than the tree-level induced inflaton mass. Since the F-term monodromy potential for the latter is a tree-level effect, one should better stabilize all heavy moduli by a flux induced tree-level potential.

Working in type IIB superstring theory, a superpotential for the Kähler moduli is generated (at tree-level) by turning on non-geometric fluxes. In this paper, following [51,52], we study type IIB orientifolds on Calabi–Yau three-folds and their flux induced scalar potential for the Kähler, the complex structure and the axio-dilaton moduli. Therefore, the induced scalar potential is the one of (orientifolded) $\mathcal{N}=2$ gauged supergravity [53], where the fluxes are considered as small perturbations around the Calabi–Yau geometry. Vacua of this potential have been discussed in toroidal backgrounds in [54–70] and in more general CY three-folds [71,72]. Here, we examine this framework for the existence of a scheme (a subset of fluxes) such that the following aspects are realized:

- There exist non-supersymmetric minima stabilizing the saxions in their perturbative regime.
- All mass eigenvalues are positive semi-definite, where the massless states are only axions.

- For both the values of the moduli in the minima and the mass of the heavy moduli one has parametric control in terms of ratios of fluxes.
- One has either parametric or at least numerical control over the mass of the lightest (massive) axion, i.e. the inflaton candidate.[3]
- The moduli masses are smaller than the string and the Kaluza–Klein scale.

Since, as in the LVS, we deal with a non-supersymmetric minimum, once this is determined we can continue to study many string-phenomenological and cosmological questions. For instance, we can compute the effect of this supersymmetry breaking on the MSSM-like theory on stacks of D7-branes, wrapping a four-cycle not forbidden by Freed–Witten anomalies.

Since we introduce non-geometric fluxes, let us mention some of the open questions and limitations of our approach:

- We work in the effective four-dimensional supergravity theory. The uplift of new minima of the scalar potential to genuine solutions of the ten-dimensional string equations of motion is a subtle problem.
- It is often questioned whether there exists a clear separation of Kaluza–Klein and moduli mass scales that allow to only consider a finite number of modes in the effective theory.
- Here we are mostly interested in the scalar potential and its mathematical structure for a small treatable number of moduli. Therefore, we are not carefully specifying global Calabi–Yau geometries and orientifold projections that concretely realize our supergravity models.

Concerning the first item, we will confirm that for non-geometric fluxes a proper dilute-flux limit does not exist, so that backreaction is expected. Our point of view is that not having an explicit uplift does not mean that orientifolded $\mathcal{N}=2$ gauged supergravity cannot be a consistent truncation for the dynamics of the string modes kept in the model.

Having expressed our concerns, let us summarize how this paper is organized. In Section 2 we describe the string theory framework that will be used, i.e. type IIB orientifolds on Calabi–Yau three-folds with various kinds of geometric and non-geometric fluxes. These induce a scalar potential which is the one of (orientifolded) $\mathcal{N}=2$ gauged supergravity. This section partially reviews known results from generalized geometry, but also adds some new aspects, like the couplings of orientifold-odd moduli to geometric fluxes and the inclusion of non-geometric R–R P-flux. We also derive the tadpole cancellation conditions and the generalized Freed–Witten anomaly conditions. Generically, the fluxes fix all closed string moduli at tree-level.

In Section 3 we first present some simple examples, with a small number of Kähler moduli and no complex structure moduli, that show a peculiar pattern. Namely, for n moduli the superpotential contains $n+1$ terms so that the requirement that all terms scale in the same way with the fluxes uniquely fixes the scaling of the frozen moduli with the fluxes. By minimizing the scalar potential we find that this intended scaling behavior indeed shows up in the minima. Generically, both supersymmetric AdS and stable non-supersymmetric AdS minima appear, where the latter can be tachyon-free and are our main interest throughout this paper. The scaling behavior allows us to gain parametric control over the physics in these minima, in particular to fix the dilaton and the Kähler moduli in their perturbative regime, or to adjust the relative sizes of the string,

[3] Axions staying massless at tree-level can only receive tiny masses from non-perturbative effects and are expected not to interfere with the moduli dynamics in the early universe.

Kaluza–Klein and moduli-mass scale. We also consider models which cannot be realized on tori, having for instance a Kähler potential of swiss-cheese or K3-fibration type. Thus, the flux scaling behavior is the tool to design certain properties in the minimum.

In the second part of Section 3 we generalize the construction to models also having complex structure moduli. The more moduli we add, the more likely it becomes that one encounters tachyonic directions. We also discuss more models with odd Kähler moduli and non-geometric P-flux. At the end, we describe how we can systematically search for this kind of scaling minima.

In Section 4 we investigate whether there exists a general mechanism to uplift tachyons, in particular those arising when more than one Kähler modulus is involved. Indeed, we find that the D-term of an appropriate stack of D7-branes, subject to the generalized Freed–Witten anomalies, precisely adds a positive contribution to the mass-square of this type of tachyons, while not affecting the masses of the other moduli. We believe that this is a fairly non-trivial result. In this section we also discuss the uplift of the cosmological constant by adding a simple term of the form $\varepsilon/\mathcal{V}^\alpha$ with $\alpha > 0$. We find that the uplift by anti D3-branes does not work for these tree-level models in the sense that α has to be smaller than $4/3$.

Section 5 is devoted to discussing aspects related to string phenomenology. First, we comment on the generic issue of justifying the existence of a string theory uplift of these flux vacua to the full ten-dimensional string theory. Then, we concretely evaluate the various resulting mass scales, which generically are only a few orders below the Planck-scale. For that purpose we introduce a well-defined notion of parametric equality or inequality, respectively. We also estimate the tunneling amplitude between different branches of the flux landscape. Having a source of supersymmetry breaking in the bulk, we compute the gravity-mediated soft-masses on stacks of D7-branes, both for a bulk and a sequestered set-up. For the latter case anomaly-mediation is the dominant contribution. Such a sequestered scenario is important for lowering the supersymmetry breaking scale down to the intermediate or even the TeV regime.

In Section 6 we analyze the models with respect to the presence of axions capable of realizing F-term axion monodromy inflation. We mostly consider the scenario where an axion can gain a parametrically small mass via turning on additional fluxes. In this case we follow the ideas put forward in [49] in the context of no-scale models. Generically, a tension between this kind of parametric control and the Kaluza–Klein scale shows up. In a separate article [73] we discuss a toy model for this kind of scenario, in which the backreaction [74] of the heavy moduli onto the flow of the inflaton can be taken into account analytically.

2. Fluxes and branes in type IIB orientifolds

In this section, we describe the set-up we will be employing in the following: This is type IIB orientifolds with geometric and non-geometric fluxes, where the latter are used to stabilize all moduli at string tree-level. We also derive conditions arising when fluxes and D-branes are present simultaneously, which can be considered as generalized Freed–Witten anomaly cancellation conditions.

2.1. Orientifold compactifications

The framework we are considering is that of type IIB string theory compactified on orientifolds of Calabi–Yau manifolds $\mathcal{M}$. The orientifold projection $\Omega_{\mathrm{P}}(-1)^{F_{\mathrm{L}}}\sigma$ contains, besides the world-sheet parity operator Ω_{P} and the left-moving fermion number F_{L}, a holomorphic involution $\sigma : \mathcal{M} \to \mathcal{M}$. We choose the latter to act on the Kähler form J and the holomorphic $(3,0)$-form Ω_3 of the Calabi–Yau three-fold $\mathcal{M}$ as

$$\sigma^* : J \to +J\,, \qquad \sigma^* : \Omega_3 \to -\Omega_3\,. \tag{2.1}$$

The fixed loci of this involution correspond to O7- and O3-planes, which in general require the presence of D7- and D3-branes to satisfy the tadpole cancellation conditions.

2.1.1. Cohomology

In order to establish the conventions for our subsequent discussion, let us note the following. We denote a symplectic basis for the third cohomology of the Calabi–Yau manifold $\mathcal{M}$ by

$$\{\alpha_\Lambda, \beta^\Lambda\} \in H^3(\mathcal{M})\,, \qquad \Lambda = 0, \ldots, h^{2,1}\,, \tag{2.2}$$

which can be chosen such that the only non-vanishing pairings satisfy

$$\int_{\mathcal{M}} \alpha_\Lambda \wedge \beta^\Sigma = \delta_\Lambda{}^\Sigma\,. \tag{2.3}$$

For the $(1,1)$- and $(2,2)$-cohomology of $\mathcal{M}$ we introduce bases of the form

$$\begin{aligned} \{\omega_{\mathsf{A}}\} &\in H^{1,1}(\mathcal{M})\,, \\ \{\tilde\omega^{\mathsf{A}}\} &\in H^{2,2}(\mathcal{M})\,, \end{aligned} \qquad \mathsf{A} = 1, \ldots, h^{1,1}\,, \tag{2.4}$$

and for later convenience we also define $\{\omega_A\} = \{1, \omega_{\mathsf{A}}\}$ and $\{\tilde\omega^A\} = \{d\mathrm{vol}_6, \tilde\omega^{\mathsf{A}}\}$, with $A = 0, \ldots, h^{1,1}$. The latter two bases are chosen as

$$\int_{\mathcal{M}} \omega_A \wedge \tilde\omega^B = \delta_A{}^B\,. \tag{2.5}$$

Turning to the orientifold projection, we have to take into account that the holomorphic involution σ shown in equation (2.1) splits the cohomology into even and odd parts. This means in particular that

$$H^{p,q}(\mathcal{M}) = H^{p,q}_+(\mathcal{M}) \oplus H^{p,q}_-(\mathcal{M})\,, \qquad h^{p,q} = h^{p,q}_+ + h^{p,q}_-\,. \tag{2.6}$$

We also note that constants as well as the volume form $d\mathrm{vol}_6$ on $\mathcal{M}$ are always even under the involution. For the other bases introduced above, we employ the following notation

$$\begin{array}{llll} \{\omega_\alpha\} \in H^{1,1}_+(\mathcal{M}) & \alpha = 1, \ldots, h^{1,1}_+, & \{\omega_a\} \in H^{1,1}_-(\mathcal{M}) & a = 1, \ldots, h^{1,1}_-, \\ \{\tilde\omega^\alpha\} \in H^{2,2}_+(\mathcal{M}) & \alpha = 1, \ldots, h^{1,1}_+, & \{\tilde\omega^a\} \in H^{2,2}_-(\mathcal{M}) & a = 1, \ldots, h^{1,1}_-, \\ \{\alpha_{\hat\lambda}, \beta^{\hat\lambda}\} \in H^3_+(\mathcal{M}) & \hat\lambda = 1, \ldots, h^{2,1}_+, & \{\alpha_\lambda, \beta^\lambda\} \in H^3_-(\mathcal{M}) & \lambda = 0, \ldots, h^{2,1}_-. \end{array} \tag{2.7}$$

2.1.2. Moduli fields

Compactifications of type IIB string theory on Calabi–Yau orientifolds with O7- and O3-planes are well-studied. Here, we recall only some results which are needed below; for more details we would like to refer the reader to the original papers [75–77], and for a broader overview for instance to [4,78].

We first note that under the combined world-sheet parity and left-moving fermion number operator $\Omega_{\mathrm{P}}(-1)^{F_{\mathrm{L}}}$ the ten-dimensional bosonic fields in type IIB string theory transform as

Table 1
Moduli in type IIB orientifold compactifications.

Number	Modulus	Name
1	$S = e^{-\phi} - i\, C_0$	axio-dilaton
$h^{2,1}_-$	$U^i = v^i + i\, u^i$	complex structure
$h^{1,1}_+$	$T_\alpha = \tau_\alpha + i\, \rho_\alpha + \cdots$	Kähler
$h^{1,1}_-$	$G^a = S\, b^a + i\, c^a$	axionic odd

$$\Omega_{\rm P}(-1)^{F_{\rm L}} = \begin{cases} g,\, \phi,\, C_0,\, C_4 & \text{even}\,, \\ B_2,\, C_2 & \text{odd}\,, \end{cases} \tag{2.8}$$

where g, ϕ, B_2 are the metric, dilaton and Kalb–Ramond field, and C_p denote the Ramond–Ramond potentials. The components of the ten-dimensional form fields which are purely in the six-dimensional space $\mathcal{M}$ can then be expanded as

$$e^{-\phi/2} J = t^\alpha \omega_\alpha\,, \qquad B_2 = b^a \omega_a\,, \qquad C_2 = c^a \omega_a\,, \qquad C_4 = \rho_\alpha\, \tilde{\omega}^\alpha\,, \tag{2.9}$$

where the factor of $e^{-\phi/2}$ for J has been included for later convenience. It implies that $\{t^\alpha\}$ is expressed in Einstein frame. We also note that the potential C_4 appears in the five-form field strength as $\widetilde{F}_5 = dC_4 - C_2 \wedge dB_2$. The moduli fields of the effective four-dimensional theory after compactification are summarized in Table 1 (see [76] for more details). The full definition of the Kähler moduli T_α is given by

$$T_\alpha = \frac{1}{2}\kappa_{\alpha\beta\gamma} t^\beta t^\gamma + i\left(\rho_\alpha - \frac{1}{2}\kappa_{\alpha ab} c^a b^b\right) - \frac{1}{4} e^\phi \kappa_{\alpha ab} G^a (G + \overline{G})^b\,, \tag{2.10}$$

where the triple intersection numbers are defined as $\kappa_{\sf ABC} = \int_{\mathcal{M}} \omega_{\sf A} \wedge \omega_{\sf B} \wedge \omega_{\sf C}$. Note that since the holomorphic involution σ has to leave the constants $\kappa_{\sf ABC}$ invariant, it follows that the components κ_{abc} and $\kappa_{a\beta\gamma}$ are vanishing.

The complex structure moduli U^i are contained in the holomorphic three-form Ω_3. The latter can be expanded in the basis of odd three-forms shown in (2.7) as follows

$$\Omega_3 = X^\lambda \alpha_\lambda - F_\lambda\, \beta^\lambda\,. \tag{2.11}$$

Usually, the periods F_λ can be expressed as derivatives $F_\lambda = \partial F/\partial X^\lambda$ of a prepotential F. In the large complex structure limit Re $U^i \gg 1$, the prepotential takes the form

$$F = \frac{d_{ijk}\, X^i X^j X^k}{X^0}\,, \qquad i = 1, \ldots, h^{2,1}_-\,, \tag{2.12}$$

where the constants d_{ijk} are symmetric in their indices. In terms of the periods X^λ, the complex structure moduli are given by

$$U^i = v^i + i\, u^i = -i\, \frac{X^i}{X^0}\,. \tag{2.13}$$

Note that X^0 does not contain any physical information and can be chosen as $X^0 = 1$. For later reference, we also note the following relation

$$0 < -i \int_{\mathcal{M}} \Omega_3 \wedge \overline{\Omega}_3 = 8\left|X^0\right|^2 d_{ijk} v^i v^j v^k \quad \text{for } h^{2,1}_- \neq 0\,. \tag{2.14}$$

For $h^{2,1}_{-}=0$ the right-hand side in (2.14) is replaced by $2\left|X^0\right|^2 \operatorname{Im}\left(F_0/X^0\right)$ and we usually take $F_0=i$.

2.1.3. Scalar potential

After compactifying the ten-dimensional theory on a Calabi–Yau orientifold, the F-term potential of the four-dimensional theory is given by the standard supergravity formula

$$V_F=\frac{M_{\rm Pl}^4}{4\pi}\, e^K\left(K^{I\bar{J}} D_I W D_{\bar{J}} \overline{W}-3\left|W\right|^2\right), \tag{2.15}$$

expressed in terms of a Kähler potential K, the corresponding Kähler metric $K_{I\bar{J}}=\partial_I\partial_{\bar{J}} K$ and a superpotential W. The Kähler-covariant derivative is given by $D_I W=\partial_I W+(\partial_I K)\, W$, and the sum runs over all holomorphic and anti-holomorphic fields in the theory. At tree-level and in the large-volume regime, the Kähler potential reads

$$K=-\log\left(-i\int_{\mathcal{M}}\Omega\wedge\overline{\Omega}\right)-\log\big(S+\overline{S}\big)-2\log\mathcal{V}, \tag{2.16}$$

where $\mathcal{V}=\frac{1}{6}\,\kappa_{\alpha\beta\gamma}t^\alpha t^\beta t^\gamma$ denotes the volume of the Calabi–Yau three-fold $\mathcal{M}$ in Einstein frame. It is expressed in terms of the two-cycle volumes t^α introduced in (2.9). Note that in order to write $\mathcal{V}$ in terms of the moduli fields T_α, G^a, S, one has to invert the relation (2.10). We furthermore observe that the Kähler potential (2.16) satisfies a no-scale relation [76]

$$K^{I\bar{J}}(\partial_I K)(\partial_{\bar{J}} K)=4, \tag{2.17}$$

where the sum runs over the axio-dilaton S, and the even and odd moduli T_α and G^a. However, perturbative corrections to the Kähler potential will spoil this no-scale structure.

2.2. Geometric and non-geometric fluxes

The moduli fields shown in Table 1 are a priori massless, and therefore are in conflict with experimental observations. However, by turning on fluxes on the Calabi–Yau manifold $\mathcal{M}$, a mass term for the moduli can be generated. Usually, one considers the three-form flux

$$G_3=\mathfrak{F}-i\,S\,H, \tag{2.18}$$

where $\mathfrak{F}=\langle dC_2\rangle$ and $H=\langle dB_2\rangle$ are fluxes for the two-form potentials C_2 and B_2. These fluxes can be expanded in the basis of three-forms as

$$\mathfrak{F}=-\tilde{\mathfrak{f}}^\Lambda\,\alpha_\Lambda+\mathfrak{f}_\Lambda\,\beta^\Lambda, \qquad H=-\tilde{h}^\Lambda\,\alpha_\Lambda+h_\Lambda\,\beta^\Lambda. \tag{2.19}$$

In addition to the $\mathfrak{F}$- and H-flux, in this work we also take into account geometric and non-geometric fluxes $F^I{}_{JK}$, $Q_I{}^{JK}$ and R^{IJK}. In the context of type IIB orientifolds, these fluxes have been studied for instance in [52,54,57,71,79]; here we will not repeat this analysis but recall only those expressions needed in our discussion.

2.2.1. Twisted differential and Bianchi identities

Following the approach of [52,54,57,71], let us introduce a twisted differential acting on p-forms. This differential contains the constant fluxes H, F, Q and R, and is given by

$$\mathcal{D}=d-H\wedge-F\circ-Q\bullet-R\llcorner, \tag{2.20}$$

where the operators appearing in (2.20) implement the mapping

$$\begin{aligned} H\wedge: &\quad p\text{-form} \to (p+3)\text{-form}\,,\\ F\circ: &\quad p\text{-form} \to (p+1)\text{-form}\,,\\ Q\bullet: &\quad p\text{-form} \to (p-1)\text{-form}\,,\\ R\llcorner: &\quad p\text{-form} \to (p-3)\text{-form}\,. \end{aligned} \tag{2.21}$$

For the present example of a Calabi–Yau three-fold, we can be more specific about the action of $\mathcal{D}$. Recalling our notation (2.7) and following [52], we introduce the geometric and non-geometric fluxes as

$$\begin{aligned} \mathcal{D}\alpha_\Lambda &= q_\Lambda{}^A\omega_A + f_{\Lambda A}\tilde{\omega}^A\,, &\qquad \mathcal{D}\beta^\Lambda &= \tilde{q}^{\Lambda A}\omega_A + \tilde{f}^\Lambda{}_A\tilde{\omega}^A\,,\\ \mathcal{D}\omega_A &= \tilde{f}^\Lambda{}_A\alpha_\Lambda - f_{\Lambda A}\beta^\Lambda\,, &\qquad \mathcal{D}\tilde{\omega}^A &= -\tilde{q}^{\Lambda A}\alpha_\Lambda + q_\Lambda{}^A\beta^\Lambda\,. \end{aligned} \tag{2.22}$$

Here, $f_{\Lambda A}$ and $\tilde{f}^\Lambda{}_A$ denote the geometric fluxes, while $q_\Lambda{}^A$ and $\tilde{q}^{\Lambda A}$ are the non-geometric ones. Moreover, we use the following convention for the H- and R-flux

$$\begin{aligned} f_{\Lambda 0} &= h_\Lambda\,, &\qquad \tilde{f}^\Lambda{}_0 &= \tilde{h}^\Lambda\,,\\ q_\Lambda{}^0 &= r_\Lambda\,, &\qquad \tilde{q}^{\Lambda 0} &= \tilde{r}^\Lambda\,. \end{aligned} \tag{2.23}$$

Imposing then a nilpotency condition of the form $\mathcal{D}^2 = 0$ leads to the well-known Bianchi identities for the fluxes [54]

$$\begin{aligned} 0 &= \tilde{q}^{\Lambda A}\tilde{f}^\Sigma{}_A - \tilde{f}^\Lambda{}_A\tilde{q}^{\Sigma A}\,, &\qquad 0 &= q_\Lambda{}^A f_{\Sigma A} - f_{\Lambda A}q_\Sigma{}^A\,,\\ 0 &= q_\Lambda{}^A\tilde{f}^\Sigma{}_A - f_{\Lambda A}\tilde{q}^{\Sigma A}\,, &\qquad 0 &= \tilde{f}^\Lambda{}_A q_\Lambda{}^B - f_{\Lambda A}\tilde{q}^{\Lambda B}\,,\\ 0 &= \tilde{f}^\Lambda{}_A f_{\Lambda B} - f_{\Lambda A}\tilde{f}^\Lambda{}_B\,, &\qquad 0 &= \tilde{q}^{\Lambda A}q_\Lambda{}^B - q_\Lambda{}^A\tilde{q}^{\Lambda B}\,. \end{aligned} \tag{2.24}$$

We now want to take the orientifold projection into account. To do so, we first note that under the combined world-sheet parity and left-moving fermion-number transformation, the five types of fluxes behave as

$$\Omega_{\mathrm{P}}(-1)^{F_{\mathrm{L}}}: \begin{cases} \mathfrak{F} \to -\mathfrak{F}\,,\\ H \to -H\,,\\ F \to F\,,\\ Q \to -Q\,,\\ R \to R\,. \end{cases} \tag{2.25}$$

Thus, under $\Omega_{\mathrm{P}}(-1)^{F_{\mathrm{L}}}$ only the fluxes F and R are even. Including the holomorphic involution σ defined in (2.1) and recalling (2.7), we can deduce the non-vanishing flux components as follows

$$\begin{array}{lllll} \mathfrak{F}: & & & \mathfrak{f}_\lambda\,, & \tilde{\mathfrak{f}}^\lambda\,,\\ H: & & & h_\lambda\,, & \tilde{h}^\lambda\,,\\ F: & f_{\hat{\lambda}\alpha}\,, & \tilde{f}^{\hat{\lambda}}{}_\alpha\,, & f_{\lambda a}\,, & \tilde{f}^\lambda{}_a\,,\\ Q: & q_{\hat{\lambda}}{}^a\,, & \tilde{q}^{\hat{\lambda}a}\,, & q_\lambda{}^\alpha\,, & \tilde{q}^{\lambda\alpha}\,,\\ R: & r_{\hat{\lambda}}\,, & \tilde{r}^{\hat{\lambda}}\,. & & \end{array} \tag{2.26}$$

2.2.2. Superpotential

Let us turn to the scalar F-term potential (2.15). The Kähler potential appearing in V_F is shown in equation (2.16), whereas the superpotential W induced by the background fluxes will be determined in the following. For non-trivial fluxes $\mathfrak{F}$ and H, it has been shown in [80] that the superpotential takes the form

$$W^{(1)} = \int_{\mathcal{M}} \left[\mathfrak{F} + d_H \Phi_c^{\rm ev} \right]_3 \wedge \Omega_3 \,, \tag{2.27}$$

where, in the present conventions, the complex multi-form of even degree $\Phi_c^{\rm ev}$ is defined as follows

$$\Phi_c^{\rm ev} = i\,S - i\,G^a \omega_a - i\,T_\alpha\, \tilde{\omega}^\alpha \,. \tag{2.28}$$

The subscript on the parentheses in (2.27) means that the three-form part of a multi-form should be selected, and the operator d_H is defined as $d_H = d - H\wedge$. Evaluating then (2.27) leads to the familiar Gukov–Vafa–Witten superpotential [81].

However, in order to account for other geometric as well as non-geometric fluxes, the authors in [54] (see also [57,71,79]) proposed to replace the operator d_H in (2.27) by $\mathcal{D}$ defined in (2.20), that is

$$d_H \to \mathcal{D} \,. \tag{2.29}$$

We mention that in [71], the case $h_-^{1,1} = 0$ was studied, which we generalize here to $h_-^{1,1} \neq 0$. The superpotential we are therefore considering is expressed as

$$\begin{aligned} W^{(2)} &= \int_{\mathcal{M}} \left[\mathfrak{F} + \mathcal{D}\, \Phi_c^{\rm ev} \right]_3 \wedge \Omega_3 \\ &= \int_{\mathcal{M}} \left[\mathfrak{F} - i\,S\,H + i\,G^a\,(F \circ \omega_a) + i\,T_\alpha \left(Q \bullet \tilde{\omega}^\alpha \right) \right]_3 \wedge \Omega_3 \,. \end{aligned} \tag{2.30}$$

Employing then the expansions (2.11) and (2.19) together with (2.3), and using the action of $\mathcal{D}$ on the cohomology defined in (2.22), we find the following expression for the superpotential

$$\begin{aligned} W^{(2)} = &-\left(\mathfrak{f}_\lambda X^\lambda - \tilde{\mathfrak{f}}^\lambda F_\lambda \right) \\ &+ i\,S \left(h_\lambda X^\lambda - \tilde{h}^\lambda F_\lambda \right) \\ &- i\,G^a \left(f_{\lambda a} X^\lambda - \tilde{f}^\lambda{}_a F_\lambda \right) \\ &+ i\,T_\alpha \left(q_\lambda{}^\alpha X^\lambda - \tilde{q}^{\lambda\alpha} F_\lambda \right) . \end{aligned} \tag{2.31}$$

Note that the fluxes are subject to the Bianchi identities (2.24). We observe that the R-flux does not appear in $W^{(2)}$ and that the geometric flux couples to the odd moduli G^a. Furthermore, the peculiar feature of this superpotential is that it only depends linearly on the three kinds of moduli S, G^a, T_α.

2.2.3. Contribution to the tadpoles

The fluxes appearing in the superpotential (2.30) contribute to the tadpole cancellation conditions. In general, for a Dp-brane charge the tadpole cancellation conditions take the form

$$N_{{\rm D}p}^{\rm flux} + \sum_{\substack{\text{D-branes} \\ \text{O-planes } i}} Q_{{\rm D}p}^{(i)} = 0 \,, \tag{2.32}$$

where the sum runs over all D-branes and orientifold planes present in the setting. The flux part can be derived by varying the type IIB action with respect to the R–R potentials C_p (in the democratic formulation [82]). We find that [83]

$$\delta_{C_p} \mathcal{S}_{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \int_{\mathbb{R}^{3,1}\times\mathcal{M}} \frac{(-1)^{\frac{p}{2}}}{2}\, \delta C_p \wedge \Big[(d - H_3\wedge)\, \widetilde{\mathfrak{F}} \Big]_{10-p} , \tag{2.33}$$

where $\widetilde{\mathfrak{F}}_p = d\,C_{p-1} - H_3 \wedge C_{p-3}$ denotes the generalized R–R field strength. To obtain the contribution of the non-geometric fluxes, we then perform the replacement shown in (2.29) (see also [71]).

The D3-brane tadpole originates from the variation with respect to C_4, and leads to the familiar $H \wedge \mathfrak{F}$ expression. For the D5- and D7-brane tadpole we consider the variation with respect to C_6 and C_8, to which non-geometric fluxes contribute. After a short computation, we obtain

$$\begin{aligned} N^{\text{flux}}_{\text{D3}} &= -\mathfrak{f}_\lambda\, \tilde{h}^\lambda + \tilde{\mathfrak{f}}^\lambda\, h_\lambda\,, \\ \big[N^{\text{flux}}_{\text{D5}}\big]_a &= +\mathfrak{f}_\lambda\, \tilde{f}^\lambda{}_a - \tilde{\mathfrak{f}}^\lambda\, f_{\lambda\, a}\,, \\ \big[N^{\text{flux}}_{\text{D7}}\big]^\alpha &= -\mathfrak{f}_\lambda\, \tilde{q}^{\lambda\,\alpha} + \tilde{\mathfrak{f}}^\lambda\, q_\lambda{}^\alpha\,. \end{aligned} \tag{2.34}$$

Below we discuss the contribution to $Q_{\text{D}p}$ due to magnetized D7-branes.

2.3. D7*-branes, tadpoles and Freed–Witten anomalies*

When constructing models of particle physics in type IIB string theory, we are required to introduce D-branes which, in the present setting, are D3- and magnetized D7-branes. Note that these D-branes contribute to the tadpole cancellation conditions (2.32), and that the mutual presence of fluxes and D-branes gives rise to a number of consistency conditions. The most famous one is the Freed–Witten anomaly condition [84], following from the relation $d\mathcal{F} = H$ for the two-form gauge field

$$\mathcal{F} = F_2 + B_2 \tag{2.35}$$

on the D-brane. This implies that $\int_{\Gamma_3} H = 0$ for every three-cycle Γ_3 in the D-brane world-volume. In this section, we discuss the contribution of D-branes to the tadpole cancellation conditions, and we derive generalized Freed–Witten anomalies for D-branes in non-geometric flux backgrounds.

2.3.1. D-branes and tadpole contributions

We begin by recalling some fact about D7-branes in type IIB orientifolds. In order to keep our discussion general, it is useful to work in the upstairs picture and carry out the orientifold projection later. For that purpose, upstairs, we introduce all objects such that the orientifold projection leaves the whole configuration invariant.

The generic single-brane configuration we are considering consists of a D7-brane wrapping a homological four-cycle Σ in $\mathcal{M}$, and carries an abelian gauge flux $\mathcal{E} = \langle \mathcal{F} \rangle$ along the four-cycle. From the upstairs point of view, the orientifold projection $\Omega_{\text{P}}(-1)^{F_{\text{L}}}\sigma$ maps such a brane to an image brane, wrapping an image four-cycle Σ' with an image gauge flux $\mathcal{E}'$. Neither item of the data has to be orientifold invariant by itself. Instead, for a generic $U(1)$ brane configuration we have

$$\begin{pmatrix} \Sigma = \Sigma_+ + \Sigma_- \\ \mathcal{E} = \mathcal{E}_+ + \mathcal{E}_- \end{pmatrix} \quad \xrightarrow{\Omega_{\text{P}}(-1)^{F_{\text{L}}}\sigma} \quad \begin{pmatrix} \Sigma' = \Sigma_+ - \Sigma_- \\ \mathcal{E}' = -\mathcal{E}_+ + \mathcal{E}_- \end{pmatrix}, \tag{2.36}$$

where the overall minus sign for $\mathcal{E}'$ comes from the fact that the gauge field is odd under $\Omega_{\text{P}}(-1)^{F_{\text{L}}}$. The fluxes as well as the Poincaré duals of the four-cycles are two-forms, that we can expand in the basis (2.7) as

$$\begin{aligned} [\Sigma]_+ &= m^\alpha \omega_\alpha \,, & [\Sigma]_- &= m^a \omega_a \,, \\ \mathcal{E}_+ &= e^\alpha \omega_\alpha \,, & \mathcal{E}_- &= e^a \omega_a \,. \end{aligned} \tag{2.37}$$

A stack of N such brane–image-brane pairs contributes to the D7-, D5- and D3-brane tadpole equations (2.32). For the present setting, we can refer for instance to [83,85] (see also [78] for a more extended discussion) for the contribution to the tadpoles. With $\chi(\Sigma)$ the Euler number of the four-cycle Σ wrapped by the D7-brane, we find

$$\begin{aligned} Q_{\rm D3} &= -N\, e^{\mathsf{A}} e^{\mathsf{B}} m^{\mathsf{C}}\, \kappa_{\mathsf{ABC}} - \tfrac{N}{24}\, \chi(\Sigma)\,, \\ \big[Q_{\rm D5}\big]_a &= +2\, N \left(m^\alpha e^b + e^\alpha m^b\right) \kappa_{\alpha a b}\,, \\ \big[Q_{\rm D7}\big]^\alpha &= +2\, N\, m^\alpha\,. \end{aligned} \tag{2.38}$$

2.3.2. Freed–Witten anomalies

Let us now turn to the Freed–Witten anomaly cancellation conditions. This issue has been discussed in similar configurations of D-branes and fluxes in [56,65,86–88].

In the presence of geometric flux, the Freed–Witten conditions guarantee that the cycle wrapped by the D-brane is still closed in the deformed geometry. For the previously introduced D7-brane wrapping $\Sigma = \Sigma_+ + \Sigma_-$ this means that

$$\mathcal{D}[\Sigma] = m^a \left(\tilde{f}^\lambda{}_a \alpha_\lambda - f_{\lambda\, a} \beta^\lambda\right) + m^\alpha \left(\tilde{f}^{\hat\lambda}{}_\alpha \alpha_{\hat\lambda} - f_{\hat\lambda\, \alpha} \beta^{\hat\lambda}\right) = 0\,. \tag{2.39}$$

In components, this relation can be expressed in the following way

$$\begin{aligned} 0 &= m^a\, \tilde{f}^\lambda{}_a\,, & 0 &= m^\alpha\, \tilde{f}^{\hat\lambda}{}_\alpha\,, \\ 0 &= m^a\, f_{\lambda\, a}\,, & 0 &= m^\alpha\, f_{\hat\lambda\, \alpha}\,. \end{aligned} \tag{2.40}$$

In order to generalize (2.39) to include non-geometric Q- and R-fluxes, let us note that a $U(1)$ brane can result in a gauging of axionic shift symmetries. This leads to the so-called generalized Green–Schwarz mechanism, which plays an important role for canceling possible chiral gauge anomalies in four dimensions. The couplings relevant for the Green–Schwarz mechanism originate from the Chern–Simons action of a D7-brane and read

$$\begin{aligned} \mathcal{S}_{\rm CS} \sim \int\limits_{\mathbb{R}^{3,1}\times\Sigma} C_6 \wedge F_2 - \int\limits_{\mathbb{R}^{3,1}\times\Sigma'} C_6 \wedge F_2 \\ + \int\limits_{\mathbb{R}^{3,1}\times\Sigma} C_4 \wedge \mathcal{E} \wedge F_2 - \int\limits_{\mathbb{R}^{3,1}\times\Sigma'} C_4 \wedge \mathcal{E}' \wedge F_2 + \cdots\,, \end{aligned} \tag{2.41}$$

where F_2 denotes the four-dimensional abelian gauge field and $\mathcal{E}$, as before, denotes the internal background gauge field supported on the D7-brane. The ellipsis indicate that there are additional terms in the Chern–Simons action, which are however not of importance here.

Let us now focus on the first line in (2.41) and expand the R–R form C_6 in the basis of even and odd four-forms shown in (2.7). Ignoring terms of different degree in the compact space, we find

$$C_6 = C_{2,\alpha}\, \tilde{\omega}^\alpha + C_{2,a}\, \tilde{\omega}^a + \cdots \tag{2.42}$$

Performing a dimensional reduction of the first line in (2.41) to four dimensions, we obtain the Stückelberg mass terms

$$\mathcal{S}^{(1)}_{\rm CS} \sim \int\limits_{\mathbb{R}^{3,1}} 2\, m^a\, C_{2,a} \wedge F_2\,. \tag{2.43}$$

As usual, such a term implies a gauging of shift symmetries of the zero-forms dual to $C_{2,a}$ in four dimensions. In the present setting, we have the relation (see for instance [78])

$$C_{2,a} \;\longleftrightarrow\; -c^a\,, \tag{2.44}$$

where the four-dimensional scalars c^a have been defined in (2.9). The gauging of the shift symmetry means that under a $U(1)$ gauge transformation $A \to A + d\lambda$ the scalars transform as

$$c^a \to c^a + m^a \lambda\,, \tag{2.45}$$

where A is the open string gauge field on the D-brane with field strength F_2. Since gauge invariance is a fundamental property of all interactions, we have to require that the flux induced superpotential (2.31) is gauge invariant. This imposes extra constraints on the fluxes, which are the generalized Freed–Witten anomalies. More concretely, in order for the superpotential (2.57) to be invariant under (infinitesimal) transformations (2.45), we have to impose

$$\begin{aligned} &0 = m^a\, \tilde{f}^{\lambda}{}_{a}\,, && 0 = m^a\, f_{\lambda\, a}\,, \\ &0 = \kappa_{\alpha bc}\, m^b\, q_{\lambda}{}^{\alpha}\,, && 0 = \kappa_{\alpha bc}\, m^b\, \tilde{q}^{\lambda\,\alpha}\,. \end{aligned} \tag{2.46}$$

Note that the first column corresponds to half of the conditions shown in (2.39).

We next turn to the second line in (2.41) and perform a similar analysis. We expand the R–R four-form C_4 as

$$C_4 = C_2^{\alpha}\, \omega_\alpha + C_2^a\, \omega_a + \cdots\,, \tag{2.47}$$

and dimensionally reduce the Chern–Simons terms. The resulting Stückelberg mass terms take the form

$$\mathcal{S}^{(2)}_{\rm CS} \sim \int\limits_{\mathbb{R}^{3,1}} 2\left(\kappa_{\alpha\beta\gamma}\, m^\beta e^\gamma + \kappa_{\alpha bc}\, m^b e^c\right) C_2^\alpha \wedge F_2\,. \tag{2.48}$$

The four-dimensional two-forms C_2^α are dual to the four-dimensional scalars ρ_α appearing in the Kähler moduli T_α, whose shift symmetry is again gauged. In particular, under open string $U(1)$ gauge transformations $A \to A + d\lambda$, we have

$$\rho_\alpha \to \rho_\alpha + \left(\kappa_{\alpha\beta\gamma}\, m^\beta e^\gamma + \kappa_{\alpha bc}\, m^b e^c\right)\lambda\,. \tag{2.49}$$

The gauge invariance of the superpotential (2.57) together with the relations (2.46) then leads to the following Freed–Witten conditions

$$0 = \kappa_{\alpha\beta\gamma}\, m^\beta\, e^\gamma\, q_\lambda{}^\alpha\,, \qquad 0 = \kappa_{\alpha\beta\gamma}\, m^\beta\, e^\gamma\, \tilde{q}^{\lambda\,\alpha}\,. \tag{2.50}$$

Let us conclude this section with the following two remarks on the generalized Freed–Witten constraints derived above:

- The relations shown in (2.46) and (2.50) imply that a pure D7-brane placed on an orientifold-even four-cycle with vanishing gauge flux satisfies the generalized Freed–Witten constraints. Such a brane would carry SO/SP gauge group.

- Once we deform it to a $U(1)$ brane, either geometrically or by gauge flux, we find extra non-trivial constraints. Note that this generically also leads to a chiral matter spectrum on the D7-branes. Thus, we expect that the Kähler moduli governing the size of the four-cycle wrapped by a (chiral) D7-brane cannot be stabilized by turning on (non)-geometric fluxes. We investigate this effect for certain examples later. This situation is reminiscent of the freezing of Kähler moduli via brane instanton effects [89].

2.4. S-dual completion of non-geometric fluxes

With the additional geometric and non-geometric fluxes F, Q and R, the superpotential (2.30) does not transform covariantly under S-duality. In particular, a non-vanishing Q-flux spoils the invariance of the scalar potential (2.15) under $SL(2,\mathbb{Z})$; it has therefore been suggested to introduce a so-called P-flux in order to restore the covariance of W [55]. Further aspects of P-fluxes have been considered in [61,65,88,90,91]. Here we are mostly interested in extending the superpotential to include G-moduli.

To address this point, let us begin by recalling that the ten-dimensional type IIB action (in Einstein frame) is invariant under the following transformation[4]

$$S \to \frac{a\,S - i\,b}{i\,c\,S + d}\,, \qquad \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} C_2 \\ B_2 \end{pmatrix}, \tag{2.51}$$

where the matrix with components a, b, c, d is an element of $SL(2,\mathbb{Z})$. The Kähler potential (2.16) transforms under $SL(2,\mathbb{Z})$, and thus the superpotential W has to transform as well for the scalar F-term potential (2.15) to be invariant. More concretely, we have

$$K \to K + \log\left(\left|i\,c\,S + d\right|^2\right) \quad \Longrightarrow \quad W \to \frac{1}{i\,c\,S + d}\,W\,. \tag{2.52}$$

Turning now to the moduli shown in Table 1, from (2.51) we can determine the following transformations

$$G^a \to \frac{1}{i\,c\,S + d}\,G^a\,, \qquad T_\alpha \to T_\alpha + \frac{i}{2}\,\frac{c}{i\,c\,S + d}\,\kappa_{\alpha bc}G^b G^c\,. \tag{2.53}$$

Let us note that despite these somewhat involved transformations rules, the term $-2\log\mathcal{V}$ in the Kähler potential (2.16) stays invariant. This is to be expected since $\mathcal{V} = \frac{1}{6}\kappa_{\alpha\beta\gamma}t^\alpha t^\beta t^\gamma$ depends only on the two-cycle volumina $\{t^\alpha\}$ (in Einstein frame), which do not transform under $SL(2,\mathbb{Z})$.

Given the above results, we can now construct an $SL(2,\mathbb{Z})$-covariant extension of the superpotential (2.30). In particular, in analogy to the Q-flux we introduce a so-called P-flux [55] as a map

$$P\,\bullet\;: \quad p\text{-form} \to (p-1)\text{-form}\,, \tag{2.54}$$

which transforms in combination with Q as a $SL(2,\mathbb{Z})$ doublet

$$\begin{pmatrix} Q \\ P \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} Q \\ P \end{pmatrix}. \tag{2.55}$$

Note that the action of the P-flux (2.54) is analogous to that of the Q-flux. In particular, extending (2.22) we have

[4] In the conventions employed here, the combination $C_4 - \frac{1}{2}C_2 \wedge B_2$ is invariant under $SL(2,\mathbb{Z})$, and the unusual factors of i in the transformation of S are due to $S = e^{-\phi} - i\,C_0$.

$$
\begin{aligned}
-P \bullet \alpha_\Lambda &= p_\Lambda{}^A \omega_A\,, & -P \bullet \beta^\Lambda &= \tilde{p}^{\Lambda\, A} \omega_A, \\
-P \bullet \omega_A &= 0\,, & -P \bullet \tilde{\omega}^A &= -\tilde{p}^{\Lambda\, A} \alpha_\Lambda + p_\Lambda{}^A \beta^\Lambda\,.
\end{aligned}
\tag{2.56}
$$

For the superpotential, we then require the transformation behavior shown in (2.52), which leads us to the following expression

$$
\begin{aligned}
W^{(3)} = \int_{\mathcal{M}} \Big[&\mathfrak{F} - i\, S\, H \\
&+ i\, G^a\, (F \circ \omega_a) \\
&+ i\, T_\alpha \left([Q - i\, S\, P] \bullet \tilde{\omega}^\alpha \right) + \frac{1}{2} \kappa_{\alpha bc}\, G^b G^c \left(P \bullet \tilde{\omega}^\alpha \right) \Big]_3 \wedge \Omega_3\,.
\end{aligned}
\tag{2.57}
$$

Evaluating this expression leads to

$$
W^{(3)} = W^{(2)} + \left(S\, T_\alpha + \frac{1}{2} \kappa_{\alpha bc}\, G^b G^c \right) (\, p_\lambda{}^\alpha X^\lambda - \tilde{p}^{\lambda\, \alpha} F_\lambda)\,. \tag{2.58}
$$

Note that this superpotential contains new terms $S\,T_\alpha$, as well as terms quadratic in the G^a fields. Furthermore, the term involving the geometric flux F transforms covariantly under $SL(2, \mathbb{Z})$, and therefore no additional flux parameters have to be introduced. This observation is particularly interesting, because it contradicts the common expectation that for every known flux one has to introduce a dual flux, when constructing a duality-invariant theory. One explanation might be that the geometric flux involves solely the metric which does not transform under S-duality.

3. Non-supersymmetric flux vacua

In this section, we analyze the flux induced scalar potential in several examples with increasing complexity. We will not be completely specific in the sense that we do not present a concrete Calabi–Yau three-fold with a fixed orientifold projection. Instead, we only specify the data at the level of supergravity by taking a set of moduli together with their Kähler potential and superpotential. We believe that this approach gives a representative picture of the structure of mathematically well-defined models of background geometries. The intention is to learn about the structure of the flux landscape, in particular about the space of stable non-supersymmetric minima of the scalar potential. Since in later sections we apply our findings to string phenomenology and string cosmology, in this section we focus on the following properties:

- Vacua should be non-supersymmetric and tachyon-free, so that after uplifting they can lead to stable de Sitter vacua.
- The moduli should be stabilized in the perturbative regime, i.e. at weak string coupling and large radius.
- All saxionic moduli should be stabilized with axions providing candidates for the inflaton and possibly dark radiation.

The aim of this section is to gain some insight into a subset of generic and well-treatable models, which show a particular scaling behavior with the fluxes. The ratios of the latter provide relations among the various mass scales of the background, such as the string scale, the Kaluza–Klein scale, the moduli-mass scale, the inflaton-mass scale and the soft-term scale. After briefly describing some generalities, we consider particular examples. First, we discuss models without complex structure moduli, in which Kähler-moduli stabilization can occur due to non-geometric fluxes. Then, we study cases with all types of moduli. Readers who are primarily interested in the phenomenology can skip parts of this section and come back later when a certain model is used later.

3.1. Generalities

Our starting point is the $\mathcal{N}=1$ supergravity scalar potential (2.15), which we recall for convenience

$$V=\frac{M_{\rm Pl}^4}{4\pi}\,e^K\Big(K^{I\bar J}D_IW\,D_{\bar J}\overline{W}-3\big|W\big|^2\Big)\,, \tag{3.1}$$

where $K_{I\bar J}=\partial_I\partial_{\bar J}K$, and $D_IW=\partial_IW+(\partial_IK)\,W$. The indices run over the moduli fields displayed in Table 1, and for ease of notation we set

$$S:=s+i\,c\,,\qquad G^a:=\psi^a+i\,\eta^a\,. \tag{3.2}$$

The Kähler potential is determined from (2.16), and turning on fluxes induces superpotentials of the form shown in (2.31). When S-dual P-flux is included, there are additional terms derived from (2.57).

The Planck mass in (3.1) is $M_{\rm Pl}=(8\pi G)^{-1/2}\approx 2.435\cdot 10^{18}\,$GeV in our conventions. As usual the string mass is $M_{\rm s}=(\alpha')^{-\frac{1}{2}}$, and in terms of $M_{\rm Pl}$ the string and Kaluza–Klein scales can be expressed as

$$M_{\rm s}=\frac{\sqrt{\pi}\,M_{\rm Pl}}{s^{\frac14}\,\mathcal{V}^{\frac12}}\,,\qquad M_{\rm KK}=\frac{M_{\rm Pl}}{\sqrt{4\pi}\,\mathcal{V}^{\frac23}}\,, \tag{3.3}$$

where $s=e^{-\phi}$ (see e.g. [92]). Recall that $\mathcal{V}$ is the volume of the Calabi–Yau manifold in Einstein frame measured in string units, namely $\mathcal{V}={\rm Vol}/\ell_s^6$ with $\ell_s=2\pi\sqrt{\alpha'}$.

Supersymmetric extrema of the potential (3.1) are generically AdS. Such extrema are stable even if the Hessian of the potential has negative eigenvalues. Indeed, as it is well known, for AdS vacua tachyonic fluctuations are stable provided they satisfy the Breitenlohner–Freedman bound [93]

$$m^2\,M_{\rm Pl}^2\geq\frac{3}{4}\,V_0\,, \tag{3.4}$$

where V_0 is the value of the potential at the extremum and m^2 is the physical mass. For supersymmetric extrema the bound is always verified. At a given extremum of a potential V the squared physical masses for the canonically normalized fields can be computed as the eigenvalues of the matrix

$$(M^2)^i_{\;j}=K^{ik}V_{kj}\,, \tag{3.5}$$

with $V_{kj}=\frac12\partial_k\partial_jV$ [94]. It is also useful to introduce the gravitino mass term

$$M^2_{\frac32}=e^{K_0}\,|W_0|^2\,\frac{M_{\rm Pl}^2}{4\pi}\,, \tag{3.6}$$

with K_0 and W_0 denoting the value of the Kähler and superpotential in the minimum. The scale of supersymmetry breaking is determined by the non-vanishing F-terms $F^I=e^{\frac{K}{2}}K^{I\bar J}D_{\bar J}\overline{W}$.

In general there are numerical factors coming from e^K that are fixed by the geometry which we do not specify here.

We use conventions in which the flux parameters entering in the superpotential are quantized. These fluxes are constrained by the Bianchi identities (2.24), and they induce tadpoles (2.34) for the R–R p-form potentials that can be cancelled by couplings to appropriate sources. In particular, magnetized D7-branes give a specific contribution to the R–R tadpoles as indicated in (2.38). In turn, including D-branes leads to additional Freed–Witten cancellation conditions on the fluxes, cf. (2.46) and (2.50).

Non-geometric fluxes were originally considered in [54]. Subsequently many authors have looked into the question of moduli stabilization due to a scalar potential induced by non-geometric fluxes [55–57,59–72]. Similar vacua have also been constructed and analyzed in detail in the T-dual type IIA language [86,95–109]. In this paper we direct our search to non-supersymmetric and tachyon-free vacua, that can give rise to de Sitter after uplifting. Moreover, we will look for vacua in which all saxionic moduli are stabilized whereas axions furnish viable candidates for the inflaton.

3.2. Models without complex structure moduli

In this section we describe examples in which the fields are the axio-dilaton and up to two Kähler moduli, while complex structure moduli are absent.

3.2.1. Model A

Let us consider the simple case of a CY manifold with $h^{2,1}_{-} = 0$ and $h^{1,1}_{+} = 1$. One can consider this model as the isotropic six-torus with frozen complex structure modulus. In this situation the Kähler potential is given by

$$K = -3\log(T+\overline{T}) - \log(S+\overline{S})\,. \tag{3.7}$$

We also consider NS–NS flux $h_0 = h$, the non-geometric flux ${q_0}^1 = q$, and the R–R three-form flux $\tilde{\mathfrak{f}}^0 = \tilde{\mathfrak{f}}$. These fluxes satisfy the Bianchi identities (2.24), and are subject to the quantization condition $\tilde{\mathfrak{f}}, h, q \in \mathbb{Z}$. From (2.31) we determine the corresponding superpotential as

$$W = i\,\tilde{\mathfrak{f}} + i\,h\,S + i\,q\,T\,, \tag{3.8}$$

where we have set $X^0 = 1$ and $F_0 = i$. The resulting scalar potential takes the very simple form

$$V = \frac{M_{\rm Pl}^4}{4\pi\cdot 2^4}\left[\frac{(hs-\tilde{\mathfrak{f}})^2}{s\tau^3} - \frac{6hqs+2q\tilde{\mathfrak{f}}}{s\tau^2} - \frac{5q^2}{3s\tau} + \frac{1}{s\tau^3}\,(hc+q\rho)^2\right], \tag{3.9}$$

which only depends on the following linear combination of axions

$$\theta = h\,c + q\,\rho\,. \tag{3.10}$$

Hence, the orthogonal linear combination of axions is not stabilized by the potential (3.9).

The extremal points of (3.9) are obtained by solving for $\partial_s V = \partial_\tau V = \partial_\theta V = 0$, and we find the three solutions shown in Table 2. Note that the fluxes must be chosen so that the values of s and τ are inside the physical domain $s, \tau > 0$.

- The first solution in Table 2 is the supersymmetric one, since here $D_T W = D_S W = 0$. As W does not depend on one axionic direction, the no-go theorem of [48] implies that the

Table 2
Extrema of the scalar potential (3.9) for Model A.

Solution	(s, τ, θ)	Susy	Tachyons	Λ
1	$(-\frac{\tilde{\mathfrak{f}}}{2h}, -\frac{3\tilde{\mathfrak{f}}}{2q}, 0)$	yes	yes	AdS
2	$(\frac{\tilde{\mathfrak{f}}}{8h}, \frac{3\tilde{\mathfrak{f}}}{8q}, 0)$	no	yes	AdS
3	$(-\frac{\tilde{\mathfrak{f}}}{h}, -\frac{6\tilde{\mathfrak{f}}}{5q}, 0)$	no	no	AdS

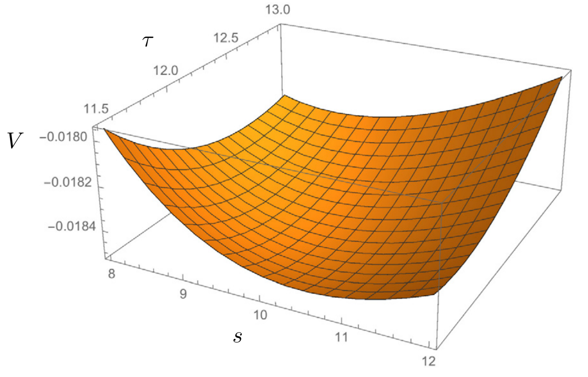

Fig. 1. The scalar potential V in units of $\frac{M_{\rm Pl}^4}{4\pi\cdot 2^4}$ for $h=q=1$, $\tilde{\mathfrak{f}}=10$, showing the expected stable minimum at $s_0=10$ and $\tau_0=12$.

saxionic partner is tachyonic, which can indeed be confirmed for this minimum. However, as expected in the supersymmetric case, the Breitenlohner–Freedman bound (3.4) is satisfied and the minimum is stable.

- Solution three of Table 2 is non-supersymmetric and has no tachyonic directions. Such AdS vacua we call strictly stable, as they can be uplifted to stable de Sitter vacua. Moreover, for $|\tilde{\mathfrak{f}}/h| \gg 1$ and $|\tilde{\mathfrak{f}}/q| \gg 1$ we obtain weak string coupling and large radius, so that it is justified to ignore higher-order corrections to the scalar potential. Note that the scaling of the stabilized moduli with the fluxes implies that all terms in the superpotential are of the same order. For $\theta=0$, the potential has the shape shown in Fig. 1.

So far, one linear combination of axions remains unstabilized. Let us analyze whether this modulus can be given a parametrically-light mass by turning on more general fluxes. If that can be achieved, this axion is a good candidate for realizing F-term axion monodromy inflation. From (2.31) we determine the most general superpotential (without P-flux) as follows

$$W = -\mathfrak{f} + i\,\tilde{\mathfrak{f}} + i(h - i\,\tilde{h})\,S + i(q - i\,\tilde{q})\,T\,, \tag{3.11}$$

with $\mathfrak{f}=\mathfrak{f}_0$, $\tilde{h}=\tilde{h}^0$ and $\tilde{q}=\tilde{q}^{0\,1}$. The only non-trivial Bianchi identity following from (2.24) reads

$$\tilde{h}\,q - h\,\tilde{q} = 0\,, \tag{3.12}$$

so that the superpotential (3.11) reduces to

$$W = -\mathfrak{f} + i\,\tilde{\mathfrak{f}} + \left(1 - i\,\frac{\tilde{q}}{q}\right) i\,(hS + qT)\,. \tag{3.13}$$

Therefore, W still depends only on the linear combination of axions (3.10), so that the orthogonal direction remains unfixed. In fact, the vacua of the superpotential (3.13) can be determined analytically and share the same qualitative structure of the three minima shown in Table 2.

Let us now turn to the contribution of the fluxes to the tadpoles. From the expressions shown in (2.34) we find

$$N_{\rm D3}^{\rm flux} = \tilde{\mathfrak{f}} h\,, \qquad N_{\rm D5}^{\rm flux} = 0\,, \qquad N_{\rm D7}^{\rm flux} = \tilde{\mathfrak{f}} q\,. \tag{3.14}$$

For the extrema 1 and 3 in Table 2, we have $N_{\rm D3}^{\rm flux} < 0$ and $N_{\rm D7}^{\rm flux} < 0$, whereas it is the opposite in extremum 2. Note that since in this model the Q-flux ${q_0}^1$ has been turned on, the FW-condition forbids to wrap a magnetized D7-brane on the single homological four-cycle $\Sigma_1 = [\omega_1]$. Therefore, for non-trivial flux, the D7-brane tadpole can only be cancelled by a number of un-magnetized D7-branes wrapping Σ_1 and giving SO/SP gauge symmetry. A chiral gauge/matter sector is not possible.

We finally compute the mass eigenvalues and eigenstates for the canonically normalized fields and compare them to the string and Kaluza–Klein scales. Evaluating the physical mass matrix (3.5) for the non-supersymmetric tachyon free minimum (solution 3 of Table 2) gives

$$M^2 = \frac{M_{\rm Pl}^2}{4\pi \cdot 2^4}\,\frac{5\,q^2}{54\,\tilde{\mathfrak{f}}^2}\begin{pmatrix} 60hq & 12h^2 & 0 & 0 \\ 25q^2 & 25hq & 0 & 0 \\ 0 & 0 & 12hq & 12h^2 \\ 0 & 0 & 25q^2 & 25hq \end{pmatrix}. \tag{3.15}$$

The mass eigenvalues can be written as

$$M_{{\rm mod},i}^2 = \mu_i\,\frac{hq^3}{\tilde{\mathfrak{f}}^2}\,\frac{M_{\rm Pl}^2}{4\pi \cdot 2^4}\,, \tag{3.16}$$

with the numerical values

$$\mu_i = \left(\frac{25(17+\sqrt{97})}{108}, \frac{25(17-\sqrt{97})}{108}\,;\, \frac{185}{54}, 0\right) \approx (6.2, 1.7\,;\, 3.4, 0)\,. \tag{3.17}$$

The eigenvectors of the first (last) two masses are combinations of saxions (axions). The massless state is the axionic combination $(q\,c - h\,\rho)$, and the three massive states are parametrically of the same mass. The gravitino shares the same flux dependence as the moduli (3.16) with the numerical prefactor $\mu_{\frac{3}{2}} = \frac{5}{6} \approx 0.833$. This model should be considered as our simplest prototype example and we will come back to it throughout this paper.

3.2.2. *Model B: inclusion of $H_-^{1,1}$ moduli*

Let us consider a generalization of the previous model and add a G-modulus as well as a geometric flux. The underlying CY manifold therefore has to have Hodge numbers $h_-^{2,1} = 0$ and $h_+^{1,1} = h_-^{1,1} = 1$. Using (2.10), the Kähler potential is

$$K = -3\log\Big((T+\overline{T}) + \frac{\kappa}{4\,(S+\overline{S})}\,(G+\overline{G})^2\Big) - \log(S+\overline{S})\,, \tag{3.18}$$

where for later convenience we have set $\kappa := 2\,\kappa_{\alpha ab}$ for $\alpha = a = b = 1$. We turn on fluxes such that the superpotential (2.31) becomes

$$W = i\,\tilde{\mathfrak{f}} + i\,h\,S + i\,q\,T - i\,f\,G\,, \tag{3.19}$$

with $\tilde{\mathfrak{f}} = \tilde{\mathfrak{f}}^0$, $h = h_0$, $q = q_0{}^1$, and $f = f_{01}$. For this set of fluxes, the contribution to the tadpoles (2.34) is given by

$$N_{\rm D3}^{\rm flux} = \tilde{\mathfrak{f}}\, h\,, \qquad N_{\rm D5}^{\rm flux} = -\tilde{\mathfrak{f}}\, f\,, \qquad N_{\rm D7}^{\rm flux} = \tilde{\mathfrak{f}}\, q\,. \tag{3.20}$$

The signs of these tadpoles depend on the signs of the fluxes, which are in turn fixed by the condition that the saxions have positive vevs. In the most interesting vacuum discussed below we must demand $q, h < 0 < f, \tilde{\mathfrak{f}}$ for which all tadpole contributions in (3.20) are negative.

The scalar potential can be computed from (3.18) and (3.19), for which we find three AdS extrema. One of them is supersymmetric and tachyonic, another one non-supersymmetric and tachyonic and the other one non-supersymmetric and non-tachyonic. In the following, we focus on the non-supersymmetric non-tachyonic extremum which is characterized by

$$\begin{aligned} &\tau = -\frac{(6+x)}{5(1+x)}\,\frac{\tilde{\mathfrak{f}}}{q}\,, \qquad s = -\frac{1}{x+1}\,\frac{\tilde{\mathfrak{f}}}{h}\,, \qquad \psi = \frac{2x}{x+1}\,\frac{\tilde{\mathfrak{f}}}{f}\,, \\ &0 = q\,\rho - f\,\eta + h\,c\,, \end{aligned} \tag{3.21}$$

where we remind the reader that our conventions for the modulus G are shown in (3.2). In (3.21), we have simplified the formulas by introducing the parameter

$$x = \frac{f^2}{\kappa\, h\, q}\,. \tag{3.22}$$

As expected, the superpotential (3.19) fixes only one linear combination of axions. Furthermore, notice that all the saxion vevs in (3.21) scale with $\tilde{\mathfrak{f}}$, which has to be large to be in the perturbative regime. In the minimum specified by (3.21), the superpotential becomes x-independent and we are left with

$$W_0 = -\frac{6\,i}{5}\,\tilde{\mathfrak{f}}\,. \tag{3.23}$$

We also note that for the other extrema in this model, we find a similar scaling with the flux, namely $W_0 \sim \tilde{\mathfrak{f}}$. The scalar potential at the above-mentioned minimum is given by

$$V_0 = -\frac{2^2 \cdot M_{\rm Pl}^4}{4\pi}\,\frac{25}{864}\,\frac{h\,q^3}{\tilde{\mathfrak{f}}^2}\,(1+x)\,, \tag{3.24}$$

where the dependence on x originates from the e^K factor. The masses are given by the following expression

$$M_{{\rm mod},i}^2 = \mu_i\,\frac{2^2 \cdot M_{\rm Pl}^2}{4\pi}\,\frac{h\,q^3}{\tilde{\mathfrak{f}}^2}\,(1+x)\,, \tag{3.25}$$

with the numerical coefficients

$$\mu_i \approx \big(0.097,\ 0.026,\ 0;\ 0.054,\ 0,\ 0\big)\,. \tag{3.26}$$

The first three entries correspond to (linear combinations of) saxionic moduli, while the last three entries are axionic combinations. A novel feature is the appearance of a massless saxion in the direction of $(f\,\tau + q\,\psi)$. The gravitino mass has the same flux dependence (3.25), with the numerical factor given by $\mu_{\frac{3}{2}} = \frac{5}{384} \approx 0.013$.

Table 3
Extrema of the scalar potential in the K3-fibration model.

Solution	$(s, \tau_1, \tau_2, \theta)$	Susy	Tachyons	Λ
1	$(-\frac{\tilde{\mathfrak{f}}}{2h}, -\frac{\tilde{\mathfrak{f}}}{q_1}, -\frac{\tilde{\mathfrak{f}}}{2q_2}, 0)$	yes	2	AdS
2	$(\frac{\tilde{\mathfrak{f}}}{8h}, \frac{\tilde{\mathfrak{f}}}{4q_1}, \frac{\tilde{\mathfrak{f}}}{8q_2}, 0)$	no	2	AdS
3	$(-\frac{\tilde{\mathfrak{f}}}{h}, -\frac{4\tilde{\mathfrak{f}}}{5q_1}, -\frac{2\tilde{\mathfrak{f}}}{5q_2}, 0)$	no	1	AdS
4	$(-\frac{2\tilde{\mathfrak{f}}}{5h}, -\frac{4\tilde{\mathfrak{f}}}{5q_1}, -\frac{\tilde{\mathfrak{f}}}{q_2}, 0)$	no	1	AdS

3.2.3. Models with two Kähler moduli, $h^{1,1}_+ = 2$

Next, we investigate the effect of having several Kähler moduli. To this end, we consider models, whose Kähler moduli sectors can be thought to be based on the K3-fibration $\mathbb{P}_{1,1,2,2,2}[8]$, and the swiss-cheese manifold $\mathbb{P}_{1,1,1,6,9}[18]$. It turns out that the vacuum solutions of the $h^{1,1}_+ = 1$ example generalize, but the spectrum contains additional tachyons.

K3-fibration In the Kähler sector of $\mathbb{P}_{1,1,2,2,2}[8]$ the intersection numbers are such that the Kähler potential splits into sums and is given by

$$K = -2\log(T_1 + \overline{T}_1) - \log(T_2 + \overline{T}_2) - \log(S + \overline{S}), \tag{3.27}$$

where for simplicity we have set $h^{2,1}_- = 0$. Fluxes are chosen such that the superpotential (2.31) takes the form

$$W = i\,\tilde{\mathfrak{f}} + i\,h\,S + i\,q_1\,T_1 + i\,q_2\,T_2\,, \tag{3.28}$$

with $\tilde{\mathfrak{f}} = \tilde{\mathfrak{f}}^0$, $h = h_0$ and – for ease of notation – with $q_i = q_0{}^i$. The resulting scalar potential has four AdS vacua summarized in Table 3, three of which are generalizations of those in Table 2. The stabilized axion is $\theta = q_1\rho_1 + q_2\rho_2 + hc$, and the potential does not depend on the two orthogonal axion combinations which thus remain unstabilized.

The physical masses of the fields scale with the fluxes in the following way

$$M^2_{\mathrm{mod},i} = \mu_i \frac{h\,q_1^2\,q_2}{\tilde{\mathfrak{f}}^2}\frac{M^2_{pl}}{4\pi \cdot 2^4}\,, \tag{3.29}$$

where the numerical factors μ_i depend on the specific solution. The cosmological constant is negative and has the same relation to the fluxes as the physical masses. The supersymmetric case contains, as expected, two tachyons above the Breitenlohner–Freedman bound; for the non-supersymmetric vacua, tachyons are below the bound. In vacua 1, 2 and 3 there is a tachyon given by the combination of saxions $\tau_{\mathrm{tac}} = q_2\,\tau_1 - q_1\,\tau_2$. In Section 4.1 we will see that this tachyon can be lifted by adding a D-term to the F-term potential.

Turning to the tadpole conditions, according to (2.34) in this model the flux contributions are given by

$$N^{\mathrm{flux}}_{\mathrm{D3}} = \tilde{\mathfrak{f}}\,h\,, \qquad \left[N^{\mathrm{flux}}_{\mathrm{D7}}\right]^1 = \tilde{\mathfrak{f}}\,q_1\,, \qquad \left[N^{\mathrm{flux}}_{\mathrm{D7}}\right]^2 = \tilde{\mathfrak{f}}\,q_2\,. \tag{3.30}$$

For the vacua 1, 2 and 3 to have positive vevs for the saxions, we take for concreteness $\tilde{\mathfrak{f}} < 0$ and the remaining fluxes positive. The contributions (3.30) to the flux tadpoles are then all negative.

Table 4
Extrema of the 'swiss-cheese' scalar potential.

Solution	(s,τ_1,τ_2,θ)	Susy	Tachyons	Λ
1	$(-\frac{\tilde{\mathfrak{f}}}{2h},-\frac{3\tilde{\mathfrak{f}}q_1^2}{2(q_1^3+q_2^3)},-\frac{3\tilde{\mathfrak{f}}q_2^2}{2(q_1^3+q_2^3)},0)$	yes	2	AdS
2	$(\frac{\tilde{\mathfrak{f}}}{8h},\frac{3\tilde{\mathfrak{f}}q_1^2}{8(q_1^3+q_2^3)},\frac{3\tilde{\mathfrak{f}}q_2^2}{8(q_1^3+q_2^3)},0)$	no	2	AdS
3	$(-\frac{\tilde{\mathfrak{f}}}{h},-\frac{6\tilde{\mathfrak{f}}q_1^2}{5(q_1^3+q_2^3)},-\frac{6\tilde{\mathfrak{f}}q_2^2}{5(q_1^3+q_2^3)},0)$	no	1	AdS

Swiss cheese As a second example we discuss the Kähler sector of the swiss-cheese Calabi–Yau $\mathbb{P}_{1,1,1,6,9}[18]$ with Kähler potential

$$K=-\log(S+\overline{S})-2\log\Big((T_1+\overline{T}_1)^{3/2}-(T_2+\overline{T}_2)^{3/2}\Big)\,. \tag{3.31}$$

As in the K3-fibration example, for the superpotential we choose

$$W=i\,\tilde{\mathfrak{f}}+i\,h\,S+i\,q_1\,T_1+i\,q_2\,T_2\,, \tag{3.32}$$

and the complex structure sector of $\mathbb{P}_{1,1,1,6,9}[18]$ is again set to zero for simplification.

In case that the non-geometric fluxes satisfy $q_1\,q_2<0$, we find four extrema of the potential with data summarized in Table 4. The linear combination of axions $\theta=q_1\,\rho_1+q_2\,\rho_2+h\,c$ is stabilized, but two orthogonal combinations remain unstabilized.

The physical mass eigenvalues exhibit the following scaling with the flux parameters

$$M^2_{\mathrm{mod},i}=\mu_i\,\frac{h\,(q_1^3+q_2^3)}{\tilde{\mathfrak{f}}^2}\,\frac{M^2_{pl}}{4\pi}\,, \tag{3.33}$$

with vacuum-dependent numerical coefficients μ_i. The vacua are analogous to those in the examples discussed so far. In particular, as in the two previous models with $h^{1,1}_+=2$, there is an additional tachyonic state given by a combination of the saxions of the two Kähler moduli, either T and G, or T_1 and T_2. In Section 4.1 we show that this tachyon can be uplifted adding a D-term to the potential.

To have positive saxion vevs, for definiteness we take $h>0$, $(q_1^3+q_2^3)>0$ and $\tilde{\mathfrak{f}}<0$ in vacua 1 and 3, but $\tilde{\mathfrak{f}}>0$ in vacuum 2. The flux tadpoles are again given by (3.30). Then, in vacua 1 and 3 the D3-brane tadpole is $N^{\mathrm{flux}}_{\mathrm{D3}}<0$. Satisfying $q_1q_2<0$ via $q_1>0$ and $q_2<0$, the two D7-brane tadpoles have signs $\big[N^{\mathrm{flux}}_{\mathrm{D7}}\big]^1<0$, and $\big[N^{\mathrm{flux}}_{\mathrm{D7}}\big]^2>0$.

3.3. Flux scaling minima with complex structure moduli

Let us now consider models which have complex structure moduli, and analyze whether they admit strictly stable, non-supersymmetric minima of the flux-scaling type encountered before.

3.3.1. Model C

To begin, we analyze a model with $h^{2,1}_-=1$ and $h^{1,1}_+=1$, for which the Kähler potential in the large complex structure limit can be written as

$$K=-3\log(T+\overline{T})-\log(S+\overline{S})-3\log\big(U+\overline{U}\big)\,. \tag{3.34}$$

One can view this model as the isotropic six-torus. In the superpotential we have now more fluxes available, which of course have to satisfy the Bianchi identities. For the flux superpotential (2.30) we choose

$$W = -\mathfrak{f}_0 - 3\,\tilde{\mathfrak{f}}^1 U^2 - h\,U\,S - q\,U\,T\,, \tag{3.35}$$

where we note that U^2 denotes the square of the modulus U, and where $h := h_1$ and $q := q_1$. Note that this superpotential only depends on the linear combination of axions $(h\,c + q\,\rho)$, and thus leaves its orthogonal combination unstabilized. The latter is a possible inflaton candidate, and we analyze below whether by turning on additional fluxes a (parametrically small) mass can be generated.

Analyzing the scalar potential following from (3.34) and (3.35), we find two interesting extrema. The first one is the supersymmetric AdS minimum with values of the moduli

$$\begin{aligned} &\tau = -18\,v\,\frac{\tilde{\mathfrak{f}}^1}{q}\,, \qquad s = -6\,v\,\frac{\tilde{\mathfrak{f}}^1}{h}\,, \qquad v^2 = \frac{1}{9}\,\frac{\mathfrak{f}_0}{\tilde{\mathfrak{f}}^1}\,, \\ &0 = h\,c + q\,\rho\,, \qquad u = 0\,. \end{aligned} \tag{3.36}$$

Since one axion is unstabilized, this extremum contains tachyons which are above the Breitenlohner–Freedman bound. The second extremum is a non-supersymmetric tachyon-free AdS minimum with frozen moduli

$$\begin{aligned} &\tau = -15\,v\,\frac{\tilde{\mathfrak{f}}^1}{q}\,, \qquad s = -12\,v\,\frac{\tilde{\mathfrak{f}}^1}{h}\,, \qquad v^2 = \frac{1}{3\cdot 10^{\frac{1}{2}}}\,\frac{\mathfrak{f}_0}{\tilde{\mathfrak{f}}^1}\,, \\ &0 = h\,c + q\,\rho\,, \qquad u = 0\,. \end{aligned} \tag{3.37}$$

For $h, q < 0 < \mathfrak{f}_0, \tilde{\mathfrak{f}}^1$, all moduli are in the physical regime. The scaling of the moduli with the fluxes can already be detected from the superpotential, which is also the case for the other extrema we found. For this scaling, all terms in W are of the same order $\mathfrak{f}_0$. The contribution of the fluxes to the tadpoles are

$$N_{\mathrm{D3}}^{\mathrm{flux}} = \tilde{\mathfrak{f}}^1\,h\,, \qquad N_{\mathrm{D5}}^{\mathrm{flux}} = 0\,, \qquad N_{\mathrm{D7}}^{\mathrm{flux}} = \tilde{\mathfrak{f}}^1\,q\,. \tag{3.38}$$

Note that the flux $\mathfrak{f}_0$ does not contribute to any of the tadpoles. Therefore, by scaling $\mathfrak{f}_0 \gg \tilde{\mathfrak{f}}^1, h, q \sim O(1)$, we can ensure that all moduli are fixed in the perturbative regime.

Let us now analyze the non-tachyonic model (3.37) in more detail. The moduli masses in the canonically normalized basis are

$$M^2_{\mathrm{mod},i} = \mu_i\,\frac{hq^3}{(\mathfrak{f}_0)^{\frac{3}{2}}(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}\,\frac{M^2_{\mathrm{Pl}}}{4\pi\cdot 2^7}\,, \tag{3.39}$$

with numerical values

$$\mu \approx \big(2.1,\ 0.37,\ 0.25\,;\ 1.3,\ 0.013,\ 0\big)\,. \tag{3.40}$$

The first three eigenstates are saxions and the last three are axions. The massless mode is the axionic combination $(q\,c - h\,\rho)$. Note that the lightest massive mode is axionic, and although not parametrically light, its mass is numerically light. In fact, it is by a factor of $1/5$ smaller than the second-lightest massive state, which is purely saxionic. For the gravitino mass the flux dependence is the same as for the moduli masses, with the numerical prefactor given by $\mu_{\frac{3}{2}} \approx 0.152$.

3.3.2. Model D

We next consider a variation of Model C, with the same Kähler potential

$$K = -3\log(T+\overline{T}) - \log(S+\overline{S}) - 3\log\left(U+\overline{U}\right), \tag{3.41}$$

but with a different superpotential

$$W = i\hat{\mathfrak{f}}_1\, U + i\,\tilde{\mathfrak{f}}^0 U^3 + 3\,i\,\tilde{h}^1\, U^2\, S + 3\,i\,\tilde{q}^1\, U^2\, T\,. \tag{3.42}$$

Note that here we introduced $\hat{\mathfrak{f}}_1 = -\mathfrak{f}_1$ for notational convenience.

For this model, we find extrema of the scalar potential in which the flux dependence of the moduli is governed by the following overall scaling of the superpotential

$$W_0 \sim \frac{(\hat{\mathfrak{f}}_1)^{\frac{3}{2}}}{(\tilde{\mathfrak{f}}^0)^{\frac{1}{2}}}\,. \tag{3.43}$$

For instance, there exists a supersymmetric AdS minimum with values of the moduli

$$\begin{aligned} &\tau = -\frac{2}{3}\,\frac{\tilde{\mathfrak{f}}^0}{\tilde{q}^1}\,v\,, \qquad s = -\frac{2}{9}\,\frac{\tilde{\mathfrak{f}}^0}{\tilde{h}^1}\,v\,, \qquad v^2 = 3\,\frac{\hat{\mathfrak{f}}_1}{\tilde{\mathfrak{f}}^0}\,,\\ &0 = \tilde{h}^1\, c + \tilde{q}^1\,\rho\,, \qquad u = 0\,. \end{aligned} \tag{3.44}$$

For this solution there are tachyons in the spectrum, that however fulfill the Breitenlohner–Freedman bound. The value of the potential at the minimum is given by

$$V_0 = -27\cdot 3^{\frac{1}{2}}\,\frac{\tilde{h}^1\,(\tilde{q}^1)^3}{(\hat{\mathfrak{f}}_1)^{\frac{1}{2}}(\tilde{\mathfrak{f}}^0)^{\frac{3}{2}}}\,\frac{M_{\rm Pl}^4}{4\pi\cdot 2^7}\,. \tag{3.45}$$

We also find a non-supersymmetric strictly stable AdS minimum with frozen moduli

$$\begin{aligned} &\tau = -\frac{5^{\frac{1}{2}}}{3\cdot 2^{\frac{1}{2}}}\,\frac{\tilde{\mathfrak{f}}^0}{\tilde{q}^1}\,v\,, \qquad s = -\frac{2^{\frac{3}{2}}}{3\cdot 2^{\frac{1}{2}}}\,\frac{\tilde{\mathfrak{f}}^0}{\tilde{h}^1}\,v\,, \qquad v^2 = 10^{\frac{1}{2}}\,\frac{\hat{\mathfrak{f}}_1}{\tilde{\mathfrak{f}}^0}\,,\\ &0 = \tilde{h}^1\, c + \tilde{q}^1\,\rho\,, \qquad u = 0\,. \end{aligned} \tag{3.46}$$

Note that for $\tilde{q}^1, \tilde{h}^1 < 0 < \hat{\mathfrak{f}}_1, \tilde{\mathfrak{f}}^0$, the moduli are in their physical regime. The non-trivial contribution of the fluxes to the tadpoles given by

$$N_{\rm D3}^{\rm flux} = \tilde{h}^1\,\hat{\mathfrak{f}}_1\,, \qquad N_{\rm D7}^{\rm flux} = \tilde{q}^1\hat{\mathfrak{f}}_1\,, \tag{3.47}$$

which in the physical regime are negative. We furthermore note that the flux $\hat{\mathfrak{f}}_0$ does not enter into the tadpoles. Thus, to guarantee weak coupling and large radius we can take large $\hat{\mathfrak{f}}_0$, together with $\hat{\mathfrak{f}}_1, \tilde{h}^1, \tilde{q}^1 \sim O(1)$. The value of the potential of the non-tachyonic model (3.46) in the minimum is found to be

$$V_0 = -\frac{216\cdot 2^{\frac{3}{4}}}{5^{\frac{5}{4}}}\,\frac{\tilde{h}^1\,(\tilde{q}^1)^3}{(\hat{\mathfrak{f}}_1)^{\frac{1}{2}}(\tilde{\mathfrak{f}}^0)^{\frac{3}{2}}}\,\frac{M_{\rm Pl}^4}{4\pi\cdot 2^7}\,, \tag{3.48}$$

which is smaller than the potential (3.45) in the supersymmetric extremum. The moduli masses in the canonically normalized basis are

$$M^2_{{\rm mod},i} = \mu_i\,\frac{\tilde{h}^1(\tilde{q}^1)^3}{(\hat{\mathfrak{f}}_1)^{\frac{1}{2}}(\tilde{\mathfrak{f}}^0)^{\frac{3}{2}}}\,\frac{M_{\rm Pl}^2}{4\pi\cdot 2^7}\,, \tag{3.49}$$

with

$$\mu_i = \big(291,\, 52,\, 35\, ;\, 210,\, 1.8,\, 0\big)\,. \tag{3.50}$$

The first three eigenstates are saxions and the last three are axions. Note that again the lightest massive mode is an axion, whose mass is smaller than the lightest saxion by a factor $1/5$. The gravitino mass is again of the same order as the fluxes with numerical prefactor $\mu_{\frac{3}{2}} \approx 114$.

It is interesting to notice that the isotropic torus model D is related to Model C by the transformation $U \to 1/U$ under which

$$W \to -\frac{i}{U^3}\Big[-\hat{\mathfrak{f}}_1 U^2 - \tilde{\mathfrak{f}}^0 - 3\,\tilde{h}^1 U\,S - 3\,\tilde{q}^1 U\,T\Big] = -\frac{i}{U^3} W' \tag{3.51}$$

Hence, $e^K |W|^2 = e^K |W'|^2$ and the resulting scalar potential is basically the same as in Model C because W' above has the same form as the superpotential in (3.35). Indeed, notice that the vevs in both models, for instance (3.37) and (3.46), match under $U \to 1/U$ and appropriate redefinition of the fluxes involved. This kind of transformation was exploited in [60,62] to classify the allowed superpotentials induced by non-geometric fluxes. Moreover, duality symmetries in moduli space allow to fix the moduli vevs, thereby simplifying the search for vacua [66,110]. In non-toroidal models the transformation $U \to 1/U$ is not expected to be a duality, but still it can be used as a solution-generating technique.

3.3.3. Freezing axionic $H^{1,1}_-$ moduli

Let us consider the case of a CY manifold with $h^{2,1}_- = 1$ and $h^{1,1}_+ = h^{1,1}_- = 1$, for which the Kähler potential reads

$$K = -3\log\Big((T+\overline{T}) + \frac{\kappa}{4\,(S+\overline{S})}\,(G+\overline{G})^2\Big) - \log(S+\overline{S}) - 3\log\big(U+\overline{U}\big). \tag{3.52}$$

Although the resulting Kähler metric is off diagonal, we can still find extrema by extending the superpotentials of models C and D to include a term depending on the G modulus. Here, we just present the generalization of model D. Turning on an additional geometric flux $\tilde{f}^1$ leads to

$$W = i\,\hat{\mathfrak{f}}_1\,U - i\,\tilde{\mathfrak{f}}^0\,U^3 + 3\,i\,\tilde{h}^1\,U^2\,S + 3\,i\,\tilde{q}^1\,U^2\,T - 3\,i\,\tilde{f}^1\,U^2\,G\,, \tag{3.53}$$

where we again introduced $\hat{\mathfrak{f}}_1 = -\mathfrak{f}_1$. Similarly to model D, we obtain extrema with the flux scaling of W_0 shown in (3.43). In particular, we find a strictly stable non-supersymmetric AdS minimum characterized by

$$\begin{aligned}
\tau &= -\frac{1}{3\cdot 10^{\frac{1}{2}}}\,\frac{x+5}{x+1}\,\frac{\tilde{\mathfrak{f}}^0}{\tilde{q}^1}\,\upsilon\,, & s &= -\frac{4}{3\cdot 10^{\frac{1}{2}}}\,\frac{1}{1+x}\,\frac{\tilde{\mathfrak{f}}^0}{\tilde{h}^1}\,\upsilon\,,\\
\upsilon^2 &= 10^{\frac{1}{2}}\,\frac{\hat{\mathfrak{f}}_1}{\tilde{\mathfrak{f}}^0}\,, & \psi &= -\frac{8}{3\cdot 10^{\frac{1}{2}}}\,\frac{x}{1+x}\,\frac{\tilde{\mathfrak{f}}^0}{\tilde{f}^1}\,\upsilon\,,\\
0 &= \tilde{h}^1\,c + \tilde{q}^1\,\rho - \tilde{f}^1\,\eta\,, & u &= 0\,,
\end{aligned} \tag{3.54}$$

where

$$x = \frac{(\tilde{f}^1)^2}{\kappa\,\tilde{h}^1\,\tilde{q}^1}\,. \tag{3.55}$$

To ensure that the moduli are in the physical regime, we require $\tilde{q}^1, \tilde{h}^1 < 0 < \hat{\mathfrak{f}}_1, \tilde{\mathfrak{f}}^0$. The flux induced tadpoles, which are again given by (3.47), are negative and do not depend on $\hat{\mathfrak{f}}_0$. We can then achieve weak coupling as well as large radius taking $\hat{\mathfrak{f}}_0 \gg 1$, and $\hat{\mathfrak{f}}_1, \tilde{h}^1, \tilde{q}^1 \sim O(1)$. The value of the scalar potential at the minimum is

$$V_0 = -\frac{27}{20 \cdot 10^{\frac{1}{4}}} \, \frac{\tilde{h}^1 \, (\tilde{q}^1)^3 \, (1+x)}{(\hat{\mathfrak{f}}_1)^{\frac{1}{2}} \, (\tilde{\mathfrak{f}}^0)^{\frac{3}{2}}} \, \frac{M_{\rm Pl}^4}{4\pi \cdot 2} , \tag{3.56}$$

and the mass eigenvalues of the fields are

$$M^2_{{\rm mod},i} = \mu_i \, \frac{\tilde{h}^1 \, (\tilde{q}^1)^3 \, (1+x)}{(\hat{\mathfrak{f}}_1)^{\frac{1}{2}} \, (\tilde{\mathfrak{f}}^0)^{\frac{3}{2}}} \, \frac{M_{\rm Pl}^2}{4\pi \cdot 2} , \tag{3.57}$$

with the numerical prefactors given by

$$\mu_i = \big(4.6,\ 0.82,\ 0.55,\ 0;\ 3.3,\ 0.028,\ 0,\ 0\big) , \qquad \mu_{\frac{3}{2}} = 0.333 . \tag{3.58}$$

The lightest eigenstate is still axionic, but the mass gap to the second-lightest state has decreased as compared to model D. As in Model B, a saxionic combination of τ and ψ has become massless. It would be interesting to know whether this is a generic feature of non-tachyonic, non-supersymmetric minima for models with odd-moduli.

3.3.4. Stabilization with non-geometric P-fluxes

We now want to present models including the S-dual P-fluxes discussed in Section 2.4. The allowed P-fluxes are in general constrained by Bianchi identities. The models we analyze fulfill the constraints derived for instance in [55]. We will consider examples with and without odd Kähler moduli G_a.

$\mathbf{h^{1,1}_- = 0}$ We come back to Model C with $h^{1,2}_- = 1$ and $h^{1,1}_+ = 1$, for which the Kähler potential is given by (3.34). The new ingredient is an additional P-flux. As an illustrative example we consider the superpotential

$$W = \hat{\mathfrak{f}} - 3\tilde{\mathfrak{f}}\, U^2 - h\, S\, U + p\, S\, T , \tag{3.59}$$

with $\hat{\mathfrak{f}} := -\mathfrak{f}_0$, $\tilde{\mathfrak{f}} := \tilde{\mathfrak{f}}^1$, $h := h_1$ and $p := p_0$. We find the same structure of minima as in the other examples with the same Hodge numbers. Note that the superpotential is chosen in such a way that every modulus is stabilized. For h negative and other fluxes positive, we find a tachyon-free *supersymmetric* AdS vacuum at

$$\begin{aligned} &\tau = \frac{3}{2}\, \frac{h}{p}\, v , \qquad s = -\frac{12}{5}\, \frac{\tilde{\mathfrak{f}}}{h}\, v , \qquad v^2 = \frac{5}{9}\, \frac{\hat{\mathfrak{f}}}{\tilde{\mathfrak{f}}} , \\ &\rho = 0 , \qquad c = 0 , \qquad u = 0 . \end{aligned} \tag{3.60}$$

The scalar potential and the superpotential at the minimum read

$$V_0 = \frac{288}{5^{\frac{5}{2}}} \, \frac{p^3 \, (\tilde{\mathfrak{f}})^{\frac{5}{2}}}{h^2 \, (\hat{\mathfrak{f}})^{\frac{3}{2}}} \, \frac{M_{\rm Pl}^4}{4\pi \cdot 2^7} , \qquad W_0 = -\frac{4}{3}\, \hat{\mathfrak{f}} , \tag{3.61}$$

and the masses are given by

$$M^2_{{\rm mod},i} = -\mu_i \, \frac{p^3 \, (\tilde{\mathfrak{f}})^{\frac{5}{2}}}{h^2 \, (\hat{\mathfrak{f}})^{\frac{3}{2}}} \, \frac{M_{\rm Pl}^2}{4\pi \cdot 2^7} , \tag{3.62}$$

where the numerical prefactors take the values

$$\mu_i = \big(52,\, 31,\, 15\,;\, 73,\, 17,\, 5.7\big)\,, \qquad \mu_{\frac{3}{2}} = 1.72\,. \tag{3.63}$$

To be in the physical regime we demand $h, p < 0$ such that the masses are positive and the cosmological constant is negative.

The non-supersymmetric minimum is analogous to the above, except that in (3.60) a minus appears in front of $\hat{\mathfrak{f}}$ in v^2. Thus $\hat{\mathfrak{f}}$ should now be taken negative. In this case $W_0 = 10\hat{\mathfrak{f}}/3$, while the scalar potential and the flux dependence of the masses are the same as in the supersymmetric vacuum, except for the extra minus sign. The numerical prefactors of the masses are now $\mu_{\frac{3}{2}} = 10.7$ for the gravitino and $\mu_i = (52,\, 31,\, 15\,;\, 45,\, -3.4,\, 19)$ for the moduli, signaling the appearance of one tachyon above the Breitenlohner–Freedman bound.

$\mathbf{h^{1,1}_{-} = 1}$ It is interesting to extend the previous model by adding one odd Kähler modulus $G = \psi + i\eta$. In this case the Kähler potential is given by (3.18). According to the general superpotential (2.57) the same p flux in (3.59) then leads to an additional term quadratic in G, namely

$$W = \hat{\mathfrak{f}} - 3\tilde{\mathfrak{f}}\,U^2 - h\,S\,U + p\left(S\,T + \frac{\kappa}{4}\,G^2\right)\,, \tag{3.64}$$

where as in the Kähler potential $\kappa = 2\kappa_{\alpha ab}$ with $\alpha = a = b = 1$. It is important to notice that the G^2 and ST terms are generated by the same P-flux. Since the Kähler potential and the superpotential differ from the previous ones only by terms depending on G there are supersymmetric and non-supersymmetric minima with the axionic odd moduli stabilized at $\psi = \eta = 0$. The remaining moduli still take the values (3.60) in the supersymmetric minimum, while in the non-supersymmetric counterpart the difference is again a minus in front of $\hat{\mathfrak{f}}_0$. The potential and the superpotential at the minima are still given by (3.61).

The G modulus decouples from the S, T, U moduli in the canonically normalized mass matrix, thereby leading to the same masses in the S, T, U sector as before. On the other hand, η and ψ turn out to be eigenstates of the canonically normalized mass matrix. The corresponding mass eigenvalues are of the form (3.62) with numerical prefactors $(\mu_\psi, \mu_\eta) = (0, -3.4)$ and $(0, 17)$, for the supersymmetric and non-supersymmetric extrema, respectively. Therefore both cases are now plagued with a tachyon and a massless saxion. There exist additional extrema with unstabilized $\psi \neq 0$ showing the same qualitative behavior.

3.3.5. A simple example with $h^{2,1}_{-} = 2$

As the final example, let us discuss the case of more than one complex structure modulus. We choose $h^{2,1}_{-} = 2$, $h^{1,1}_{+} = 1$ and $h^{1,1}_{-} = 0$ and work with a particularly simple prepotential such that Kähler potential reads

$$K = -2\log(U_1 + \overline{U}_1) - \log(U_2 + \overline{U}_2) - \log(S + \overline{S}) - 3\log(T + \overline{T})\,. \tag{3.65}$$

This corresponds to the mirror dual of the Kähler sector of $\mathbb{P}_{1,1,2,2,2}[8]$, which was discussed in Section 3.2.3. For the superpotential we take

$$W = -\mathfrak{f}_0 - \Big(h\,S + q\,T + \tilde{\mathfrak{f}}_2\,U_1 + 2\,\tilde{\mathfrak{f}}_1\,U_2\Big)\,U_1\,, \tag{3.66}$$

where we have set $h_1 = h$ and $q_1 = q$. In the extrema we obtain $u_1 = 0$, thus the superpotential depends effectively on the axionic combination $\theta = h\,c + q\,\rho + 2\,\tilde{\mathfrak{f}}_1\,u_2$, and the two orthogonal axions are unstabilized. In addition, all extrema contain at least one additional tachyon. As expected from the no-go theorem of [48], the supersymmetric minimum has in fact two tachyonic states, although above the Breitenlohner–Freedman bound. A representative non-supersymmetric AdS extremum is given by

$$\begin{aligned}
&\tau = 2\,\frac{\tilde{\mathfrak{f}}_2}{q}\,v_1\,, \qquad s = \frac{2}{3}\,\frac{\tilde{\mathfrak{f}}_2}{h}\,v_1\,, \qquad v_1^2 = \frac{\mathfrak{f}_0}{\tilde{\mathfrak{f}}_2}\,, \qquad v_2^2 = \frac{1}{3}\,\frac{\tilde{\mathfrak{f}}_2}{\tilde{\mathfrak{f}}_1}\,v_1\,,\\
&0 = h\,c + q\,\rho + 2\,\tilde{\mathfrak{f}}_1\,u_2\,, \qquad u_1 = 0\,,
\end{aligned} \tag{3.67}$$

with

$$V_0 = -3\,\frac{\tilde{\mathfrak{f}}_1 h\,q^3}{\mathfrak{f}_0^{\frac{3}{2}}\,\tilde{\mathfrak{f}}_2^{\frac{3}{2}}}\,\frac{M_{\rm Pl}^4}{4\pi\cdot 2^7}\,, \qquad W_0 = -\frac{16}{3}\,\mathfrak{f}_0\,. \tag{3.68}$$

The eigenstates of the normalized mass matrix can be computed to be

$$M^2_{{\rm mod},i} = \mu_i\,\frac{\tilde{\mathfrak{f}}_1\,h\,q^3}{\mathfrak{f}_0^{\frac{3}{2}}\,\tilde{\mathfrak{f}}_2^{\frac{3}{2}}}\,\frac{M_{\rm Pl}^2}{4\pi\cdot 2^7}\,, \tag{3.69}$$

where the numerical prefactor is $\mu_{\frac{3}{2}} \approx 16$ for the gravitino mass and $\mu_i = (18, 18, -2, -2; 10, 10, 0, 0)$ for the moduli masses. The massless eigenstates refer to the axionic combinations $(2\,\tilde{\mathfrak{f}}_1\,c - h\,u_2)$ and $(h\rho - qc)$, respectively, and the tachyonic directions are $(h\,\tau - q\,s)$ and $(2\,s\,\tilde{\mathfrak{f}}_1 - h\,v_2)$. One can verify that the tachyons lie below the Breitenlohner–Freedman bound.

Let us mention one new feature arising in this model. Computing the values of the auxiliary fields $F^i = e^{\frac{K}{2}}\,K^{i\bar{j}}\,F_{\bar{j}}$ in the minimum (3.67), we obtain

$$\begin{aligned}
&F^S = e^{\frac{K}{2}}\,\frac{16\,(\mathfrak{f}_0)^{\frac{3}{2}}\,(\tilde{\mathfrak{f}}^2)^{\frac{1}{2}}}{3h}\,, \qquad && F^T = e^{\frac{K}{2}}\,\frac{16(\mathfrak{f}_0)^{\frac{3}{2}}\,(\tilde{\mathfrak{f}}^2)^{\frac{1}{2}}}{q}\,,\\
&F^{U_1} = 0, && F^{U_2} = e^{\frac{K}{2}}\,\frac{8(\mathfrak{f}_0)^{\frac{3}{2}}\,(\tilde{\mathfrak{f}}^2)^{\frac{1}{2}}}{3\tilde{\mathfrak{f}}^1}\,.
\end{aligned} \tag{3.70}$$

This exemplifies that accidentally it can happen that an F-term vanishes in a certain minimum. As we discuss in Section 5, such a result is essential for realizing sequestered supersymmetry breaking on the Standard Model branes.

3.4. *General properties of the flux scaling minima*

In this section we summarize the salient aspects of the models constructed previously. We first explain the systematics behind our search of vacua and then address more specific features.

The defining property of the models is a common flux scaling of W which in turn implies a common flux scaling of V and the moduli masses. This is potentially powerful to achieve parametric control over the hierarchies among the relevant scales $M_{\rm s}$, $M_{\rm KK}$ and the moduli masses. On the other hand, parametrically controlled hierarchies among the different moduli masses are then excluded. Later in Section 6, we will try to circumvent this problem by introducing additional fluxes in W that break the scaling.

The strategy is to choose a superpotential dictated by a particular scaling. In practice this means that only a subset of the allowed fluxes is turned on to ensure that the moduli vevs scale in a simple way. A typical example is model C in Section 3.3.1. In this case, at the extrema $W_0 \sim \mathfrak{f}_0$, whereas $s\,v \sim \mathfrak{f}_0/h$, $\tau\,v \sim \mathfrak{f}_0/q$ and $v^2 \sim \mathfrak{f}_0/\tilde{\mathfrak{f}}^1$, as we can easily read from (3.35).

Generically, for n complex moduli it suffices to switch on $n+1$ flux parameters. For instance, to stabilize T^α we include one flux of type q_λ^α or one of type $\tilde{q}_\lambda^\alpha$. Similarly, for S we take one h_λ or one $\tilde{h}_\lambda$. For the complex structure moduli we need one R–R flux of type $\mathfrak{f}_\lambda$ and one $\tilde{\mathfrak{f}}_\lambda$. Of course, we have to be careful that the chosen NS–NS, R–R and non-geometric fluxes satisfy the Bianchi identities. We observe that in the studied examples an off diagonal Kähler metric did not spoil the scaling of the potential and the masses. We want to stress that the scaling strategy can be used to efficiently engineer models with the desired pattern of masses. This will become apparent when we discuss the moduli spectroscopy in Section 5.2 and axion monodromy inflation in Section 6.

All models include NS–NS and R–R three-form fluxes that lead to stabilization of the real parts of the axio-dilaton and the complex structure moduli. Further addition of non-geometric fluxes, as well as of geometric ones when $h_-^{1,1} \neq 0$, enables the stabilization of the Kähler moduli saxions. In general, the fluxes always allow the existence of supersymmetric AdS vacua which often have tachyons above the Breitenlohner–Freedman bound. One of the main and maybe unexpected results of this paper is that we have also found non-supersymmetric, non-tachyonic AdS vacua. We have seen that, once we introduce more Kähler or more complex structure moduli, new tachyonic modes appear. A natural question is whether one can identify an uplift mechanism for these tachyons, hence enlarging the space of good models. In Section 4, we will see that a D-term uplift exists for tachyons appearing for multiple Kähler moduli. For the tachyons appearing for multiple complex structure moduli, such an uplift is still an open question.

Equipped with a class of tachyon-free, non-supersymmetric AdS minima, as in the LVS scenario, one can proceed to perform string phenomenology studies in order to explore what particle physics predictions can be made. This will be analyzed in more detail in Section 5. Recall that in these models (without P-flux) typically only one axionic combination is fixed. In Section 6 we examine how the axion sector can give rise to large field inflation.

A common feature of the models is the existence of R–R tadpoles due to the fluxes. Interestingly, in most examples of AdS vacua we find that $N_{\rm D3}^{\rm flux}$ and $N_{\rm D7}^{\rm flux}$ are negative, as it happens in related T-dual type IIA models [86,100]. Thus, the flux tadpoles can be compensated by introducing D3- and D7-branes instead of O3- and O7-planes. Magnetized D7-branes that induce D3-charge are in principle allowed but they are constrained by cancellation of Freed–Witten anomalies.

The R–R fluxes play a special role in the models since they determine the vevs of the moduli. This is indeed the case for the complex structure moduli. On the other hand, the scale of the dilaton and the Kähler moduli is also set by the R–R fluxes upon taking the NS–NS, non-geometric and geometric fluxes to be $O(1)$. Moreover, as we have seen, in models with complex structure moduli there are R–R fluxes that do not enter the flux tadpoles at all. It turns out that such fluxes can be chosen large enough to attain moduli vevs leading to large volume and small string coupling. In Section 5 we will discuss to what extent these fluxes are diluted in order to guarantee a reliable supergravity approximation.

4. Uplift mechanisms

The models studied in the last section face two problems: all of them have a negative cosmological constant, and some of them have tachyonic mass eigenstates. To make an uplift to de Sitter possible, we therefore discuss a mechanism to uplift tachyonic Kähler moduli to a positive mass. Afterwards, we identify possible terms that uplift the cosmological constant and discuss their behavior.

4.1. D-term uplifting of the tachyon

To uplift tachyons one could think that taking perturbative and non-perturbative corrections to K and W into account might help. However, since we have taken care of freezing the moduli in the perturbative regime, these corrections are generically suppressed against the tree-level values. Of course, this also holds for the tachyonic mass. The second and more natural option is to have an additional positive-definite contribution such as a D-term potential. Thus, in the following we study how a D-term of a stack of D7-branes contributes to moduli stabilization and the mass terms. An analogous mechanism to uplift tachyons via D-terms from D-branes was proposed in [56].

K3-fibration To show how the D-term uplift works, we perform our analysis in a concrete model. In particular, we consider the K3-fibration with $h^{1,1}_{+} = 2$ and $h^{2,1}_{-} = 0$ studied in Section 3.2.3. The Kähler potential is given by

$$K = -2\log(T_1 + \overline{T}_1) - \log(T_2 + \overline{T}_2) - \log(S + \overline{S})\,, \tag{4.1}$$

whereas the superpotential is taken to be

$$W = i\,\tilde{\mathfrak{f}} + i\,h\,S + i\,q_1\,T_1 + i\,q_2\,T_2\,. \tag{4.2}$$

Recall that in this model the supersymmetric AdS minimum is at

$$\tau_1 = -\frac{\tilde{\mathfrak{f}}}{q_1}\,, \quad \tau_2 = -\frac{\tilde{\mathfrak{f}}}{2q_2}\,, \quad s = -\frac{\tilde{\mathfrak{f}}}{2h}\,, \quad hc + q_1\rho_1 + q_2\rho_2 = 0\,, \tag{4.3}$$

and that there also exists a non-supersymmetric AdS minimum at

$$\tau_1 = -\frac{4\tilde{\mathfrak{f}}}{5q_1}\,, \quad \tau_2 = -\frac{2\tilde{\mathfrak{f}}}{5q_2}\,, \quad s = -\frac{\tilde{\mathfrak{f}}}{h}\,, \quad hc + q_1\rho_1 + q_2\rho_2 = 0\,, \tag{4.4}$$

which has mass eigenvalues

$$M^2_{\mathrm{mod},i} = \mu_i\,\frac{h\,q_1^2\,q_2}{\tilde{\mathfrak{f}}^2}\,\frac{M^2_{\mathrm{Pl}}}{4\pi\cdot 2^4}\,, \tag{4.5}$$

with $\mu_i = (-15,\ 11,\ 42\,;23,\ 0,\ 0)$. The tachyonic mode corresponds to a linear combination of Kähler saxions given by $\tau_{\mathrm{tac}} = q_2\tau_1 - q_1\tau_2$. To obtain positive vevs for the saxions we take $\tilde{\mathfrak{f}} < 0$, $h > 0$, $q_1 > 0$ and $q_2 > 0$.

We now introduce a stack of N D7-branes equipped with a $U(1)$ gauge flux with

$$[c_1(L)] = [\mathcal{E}] = l_1 D_1 + l_2 D_2\,, \tag{4.6}$$

where $D_{1,2}$ are two (effective) divisors in $\mathbb{P}_{1,1,2,2,2}[8]$ and $l_{1,2} \in \mathbb{Z}$. The D7-branes are wrapping a four-cycle defined by

$$\Sigma = m_1 D_1 + m_2 D_2\,, \tag{4.7}$$

with $m_{1,2} \in \mathbb{Z}$, which leads to a D-term potential of the form

$$V_D = \frac{M^4_{\mathrm{Pl}}}{2\,\mathrm{Re}(f)}\xi^2\,. \tag{4.8}$$

Here ξ is the Fayet–Iliopoulos (FI) term of the $U(1) \subset U(N)$ carried by the branes, which is given by

$$\xi = \frac{1}{\mathcal{V}} \int_{\Sigma} J \wedge c_1(L)\,, \tag{4.9}$$

and in (4.8) we have assumed that all charged fields have vanishing vevs. The holomorphic gauge kinetic function for the D7-branes is $f = T + \chi\, S$, where $\chi = \frac{1}{4\pi^2}\int F \wedge F$ denotes the instanton number of the gauge flux on the D7-branes. In the example at hand, the volume is $\mathcal{V} = (t^1)^2 t^2$.

The wrapping numbers (m_1, m_2) and the gauge fluxes (l_1, l_2) are constrained by the generalized Freed–Witten anomaly cancellation conditions (2.50), which in the present case lead to

$$m_1 l_1 q_2 + \big(l_1\, m_2 + l_2\, m_1\big) q_1 = 0\,. \tag{4.10}$$

Using this condition, we find that the FI-parameter can be expressed as

$$\xi = \frac{m_1\, l_1}{q_1 \sqrt{\tau_2}\, \mathcal{V}} \big(q_1\, \tau_1 - 2\, q_2\, \tau_2\big)\,. \tag{4.11}$$

Note that for a supersymmetric minimum, a vanishing F-term implies a vanishing D-term. And indeed, the values (4.3) give a vanishing FI-term. Moreover, ξ also vanishes for the non-supersymmetric minimum in (4.4). Therefore, adding the D-term will not change the position of either extremum, but due to its positive-definiteness it is expected to add positive contributions to the squares of the saxion masses.

We now study in more detail the effect of adding a D-term to the former F-term scalar potential. Concretely, we add

$$V_D = \frac{k}{\tau_1^2 \tau_2} \frac{\big(q_1\, \tau_1 - 2\, q_2\, \tau_2\big)^2}{\big(m_1\, \tau_1 + m_2\, \tau_2\big)\tau_2}\,, \tag{4.12}$$

which is obtained by substituting the various ingredients in (4.8). Here k is a positive numerical prefactor and for the gauge kinetic function we only included the string tree-level part $\mathrm{Re}(f) = m_1\, \tau_1 + m_2\, \tau_2$. As expected, the position of both the supersymmetric (4.3) and the non-supersymmetric (4.4) extrema do not change. Moreover, from the resulting mass matrix it follows that only the mass eigenvalue corresponding to the tachyonic saxion $\tau_{\rm tac}$ receives corrections and can become positive. In the supersymmetric case a tachyonic state will remain, although above the Breitenlohner–Freedman bound. In the non-supersymmetric extremum there is only one negative mass eigenvalue that receives corrections, which is given by (in units of $M_{\rm Pl}^4/(4\pi)$)[5]

$$m^2_{\rm tac} = -\frac{15\, h\, q_1^2\, q_2}{16\, \tilde{\mathfrak{f}}^2} - \frac{375\, q_1^3\, q_2^3\, k}{4\, \tilde{\mathfrak{f}}^3\, (m_1 q_1 + 2 m_2 q_1)}\,. \tag{4.13}$$

We observe that the mass can become positive because $\tilde{\mathfrak{f}} < 0$. For instance, choosing $h = 2$, and $q_1 = q_2 = m_1 = m_2 = 1$, implies that $m^2_{\rm tac}$ will turn positive provided $k > -3\,\tilde{\mathfrak{f}}/50$. We could take for instance $\tilde{\mathfrak{f}} = -10$ and $k = 1$. Thus, the tachyonic mode can be uplifted while the masses of the other moduli do not change. Moreover, as the D-term vanishes in the minimum, the cosmological constant V_0 does not change either.

Swiss cheese Uplifting of tachyons by D-terms also works in the 'swiss-cheese' model of Section 3.30, as we now briefly describe. As in the previous example we introduce N D7-branes

[5] Note that in the following, we have omitted the factor $M_{\rm Pl}^4/(4\pi)$ for ease of notation. It can be re-installed easily by dimensional analysis.

with a $U(1)$ gauge field (4.6) wrapping a four-cycle (4.7). The main difference now is that the non-zero intersection numbers are κ_{111} and κ_{222} which are taken to be equal. Moreover, we expand the Kähler form as $[J] = t^1 D_1 + t^2 D_2$, with $t^2 < 0$. Then, up to normalization, $t^1 = \sqrt{\tau_1}$, $t^2 = -\sqrt{\tau_2}$, and $\mathcal{V} = (\tau_1^{3/2} - \tau_2^{3/2})$. The FI parameter is found to be

$$\xi = \frac{1}{\mathcal{V}}\left(m_1 l_1 \sqrt{\tau_1} - m_2 l_2 \sqrt{\tau_2}\right). \tag{4.14}$$

On the other hand, the Freed–Witten anomaly cancellation condition (2.50) now implies

$$m_1 l_1 q_1 + l_2 m_2 q_2 = 0. \tag{4.15}$$

Substituting in (4.8) then gives

$$V_D = \frac{k\left(q_2\sqrt{\tau_1} + q_1\sqrt{\tau_2}\right)^2}{(m_1\tau_1 + m_2\tau_2)\left(\tau_1^{3/2} - \tau_2^{3/2}\right)^2}, \tag{4.16}$$

where k is again some positive number, and for $\mathrm{Re}\, f$ we took only the tree-level contribution of the gauge kinetic function. The important point is that V_D vanishes not only for the supersymmetric AdS extremum as expected, but also for the non-supersymmetric ones in Table 4, which all happen to have $\sqrt{\tau_1/\tau_2} = |q_1|/|q_2|$. Indeed, cancellation occurs because necessarily $q_1 q_2 < 0$ for the vevs in Table 4 to correspond to true extrema of the F-term potential.

Adding V_D to the F-term potential we find that only the mass eigenvalue corresponding to the tachyonic direction $\tau_{\rm tac} = q_2\tau_1 - q_1\tau_2$ changes. In the non-supersymmetric extremum with only the tachyon $\tau_{\rm tac}$ (third in Table 4) the new mass eigenvalue is given by (in units of $M_{\rm Pl}^4/(4\pi)$)

$$m_{\rm tac}^2 = -\frac{5h(q_1^3 + q_2^3)}{36\tilde{\mathfrak{f}}^2} + \frac{125\,k\,(q_1^3 + q_2^3)^3}{324\tilde{\mathfrak{f}}^3 q_1 q_2 (m_1 q_1^2 + m_2 q_2^2)}. \tag{4.17}$$

Notice that $m_{\rm tac}^2$ can be uplifted precisely because $q_1 q_2 < 0$, while $h > 0$, $\tilde{\mathfrak{f}} < 0$, and $(q_1^3 + q_2^3) > 0$ to keep the saxion vevs positive.

To summarize, we have identified a tachyon uplift mechanism, where a D-term on a D7-brane, the Freed–Witten anomaly conditions, and the nature of the non-supersymmetric minimum nicely conspire to give a positive shift only for the tachyon mass. Let us emphasize that for this uplift mechanism to work, it is essential that ξ vanishes not only for the supersymmetric minimum, but also for the non-supersymmetric one. Note that in concrete string model building, one will also have to take into account tadpole cancellation conditions.

4.2. *Uplift of cosmological constant*

Eventually, also the cosmological constant needs to be uplifted so that the vacuum becomes de Sitter. The common mechanism is to add an extra sector to the theory, which changes the values of the moduli in the minimum in a controlled way, but adding a substantial contribution to the vacuum energy. In the KKLT [7] and the LVS [8] scenario, this can for instance be achieved by adding anti-D3-branes which provide a positive-definite contribution to the potential

$$V_{\rm up} = \frac{\varepsilon}{\mathcal{V}^\alpha}, \tag{4.18}$$

where $\alpha = 2$ for a $\overline{\rm D3}$-brane in the bulk and $\alpha = 4/3$ for a brane located in a warped throat. In the LVS scenario, the F-term contribution to the potential in the AdS minimum scales as

$V_F \sim \mathcal{V}_0^{-3}$ so that for the $\overline{\text{D3}}$-brane to compete one needs $\varepsilon \sim \mathcal{V}_0^{\alpha-3}$ which is small and provides the parameter controlling the slight shift of the minimum after including the uplift potential.

Let us discuss whether such an uplift mechanism also works for the tree-level minima we are working with here. For concreteness, let us first consider this question for Model A that was discussed in Section 3.2.1. To simplify notation, we define $\hat{\mathfrak{f}} := -\tilde{\mathfrak{f}}$ such that all fluxes in this section must be chosen positive to be in the physical regime. Looking at the terms in the scalar potential (3.9), we realize that in the minimum all terms scale as $h\,q^3/\hat{\mathfrak{f}}^2$. Taking into account that for perturbative control we need $\hat{\mathfrak{f}} \gg h, q$, we are led to an uplift term of the form

$$V_{\rm up} = \frac{\varepsilon}{16\,\tau^{\beta}} \qquad \text{with} \quad 0 < \beta < 2\,, \tag{4.19}$$

in order to have $\varepsilon \sim \hat{\mathfrak{f}}^{\beta-2}$ small. Therefore, the two types of $\overline{\text{D3}}$-brane uplifts mentioned above do not work in our case.

We can nonetheless study the above uplift for general β. Working at linear order in the small parameter ε, one can show that the values in the stable, non-supersymmetric minimum are shifted as

$$\begin{aligned} \tau_0 &= \frac{6\,\hat{\mathfrak{f}}}{5\,q} + \varepsilon\, \frac{3^{2-\beta}\beta}{5^{2-\beta}\,2^{\beta+1}}\, \frac{\hat{\mathfrak{f}}^3}{h\,q^4} \left(\frac{q}{\hat{\mathfrak{f}}}\right)^{\beta} + O(\varepsilon^2)\,, \\ s_0 &= \frac{\hat{\mathfrak{f}}}{h} - \varepsilon\, \frac{3^{2-\beta}\beta}{5^{2-\beta}\,2^{\beta+1}}\, \frac{\hat{\mathfrak{f}}^3}{h^2\,q^3} \left(\frac{q}{\hat{\mathfrak{f}}}\right)^{\beta} + O(\varepsilon^2)\,. \end{aligned} \tag{4.20}$$

The value of the scalar potential at the minimum gets shifted as

$$V_0 = -\frac{25\,h\,q^3}{216\,\hat{\mathfrak{f}}^2} + \frac{\varepsilon}{16}\left(\frac{5q}{6\hat{\mathfrak{f}}}\right)^{\beta} + O(\varepsilon^2)\,. \tag{4.21}$$

Therefore, we could uplift to $V_0 = 0$ for

$$\varepsilon \simeq \frac{2^{\beta+1}5^{2-\beta}}{3^{3-\beta}}\,\frac{hq^3}{\hat{\mathfrak{f}}^2}\left(\frac{\hat{\mathfrak{f}}}{q}\right)^{\beta}, \tag{4.22}$$

which is small in the perturbative regime $\hat{\mathfrak{f}} \gg h, q$. Inserting this value back into (4.20), we find

$$\tau_0 = \frac{\hat{\mathfrak{f}}}{q}\left(\frac{6}{5} + \frac{\beta}{3}\right), \qquad s_0 \simeq \frac{\hat{\mathfrak{f}}}{h}\left(1 - \frac{\beta}{3}\right), \tag{4.23}$$

so that the correction term is of the same order as the initial value. Therefore, it is not clear whether the $O(\varepsilon^2)$ corrections are actually subleading. Performing a numerical analysis we find that, indeed, choosing ε sufficiently large to uplift to a de-Sitter vacuum (4.22), the minimum gets destabilized for $\beta \gtrsim 1/4$. The same numerical behavior is found for Model B. For $\beta = 1/4$ and a specific choice of fluxes, Fig. 2 shows a plot of the potential around the uplifted de Sitter minimum. We note that no linear approximation in ε was done here.

Let us also check how the mass eigenvalues of Model A change due to the uplift. Recall from (3.17) that the masses of the moduli in the stable AdS vacuum scale as $M^2_{{\rm mod},i} = \mu_i\, \frac{hq^3}{\hat{\mathfrak{f}}^2}\, \frac{M^2_{\rm Pl}}{4\pi 2^4}$ with the numerical factors $\mu_i = (6.2,\ 1.7\,;\ 3.4,\ 0)$. In the uplifted Minkowski vacuum the scaling remains the same and the numerical factors decrease slightly to $\mu_i^{\rm up} = (5.3,\ 0.8\,;\ 2.9,\ 0)$.

Of course, the question now is which string theoretical effect can generate an effective uplift potential of the required type. As we have discussed, the introduction of just $\overline{\text{D3}}$-branes does not

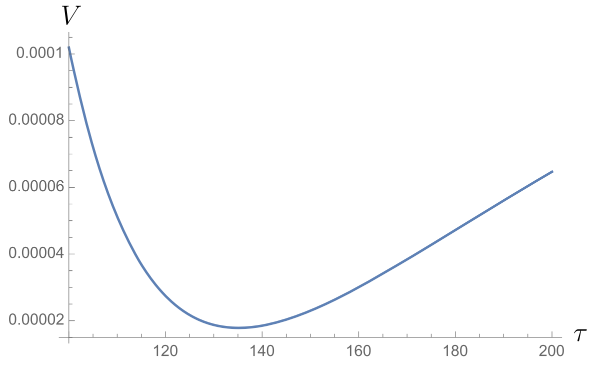

Fig. 2. The exact potential $V(\tau, s_0, c_0, \rho_0)$ for $h = 2$, $q = 1$ and $\hat{\mathfrak{f}} = 100$ and the uplift term $V_{\rm up} = 0.0013/(16\,\tau^{\frac{1}{4}})$. Here the numerical minimum lies at $\tau_0 = 135.13$, $s_0 = 40.60$ and $2c_0 + \rho_0 = 0$. The uplifted minimum is de Sitter.

give the appropriate power β. One could envision more complicated uplift sectors, where also matter-field contributions to D-terms and F-terms play a role.[6] However, for our purposes here, we just state that the uplift procedure is in principle possible, but needs more care than in the KKLT and LVS scenarios.

5. Physical aspects of the scaling vacua

In this section, we study several phenomenological aspects and problems of the flux vacua constructed above. After discussing issues concerning the dilute-flux limit, we investigate the mass hierarchies of our models in several examples. We then consider particle-physics questions, in particular, we compute soft terms for a MSSM-like D-brane setup.

5.1. A note on the dilute flux limit

Describing the string flux compactifications investigated above in an orientifolded $\mathcal{N} = 2$ gauged-supergravity framework can only be an approximation, where fluxes are considered as (small) perturbations around the flux-less Calabi–Yau geometry. The superpotential and the induced scalar potential describe in an effective four-dimensional framework, how the system reacts upon turning on fluxes that give extra contributions to the ten-dimensional equations of motion. Usually, one hopes that the appearance of minima of $V_{\rm eff}$ signals new solutions of the full ten-dimensional string equations of motion.

In the case that only NS–NS and R–R three-form fluxes are turned on, it has been shown that the backreaction gives a warped Calabi–Yau geometry [112]. Since, due to the no-scale structure, the Kähler moduli are unstabilized, one can take the large-volume limit, in which the fluxes become diluted and the effective gauged supergravity description becomes a controlled approximation.

For the models discussed here, also the Kähler moduli are stabilized by turning on non-geometric fluxes, so that the backreacted geometry is not explicitly known. Even though the Kähler moduli are stabilized in terms of fluxes in the perturbative regime, a priori it is not clear

[6] For a D-term with such low β the modular weight of the matter field Kähler metric needs to be positive. For abelian heterotic orbifolds this can occur for the twisted sector fields [111].

whether a dilute-flux limit really exists. In fact, the expectation from generalized geometry and double field theory is that non-geometric flux changes the space from a smooth manifold to a T-fold where the transition functions between two charts are given by a T-duality transformation (for reviews see [113–115]). Since the latter identifies small and big radii, it would be surprising if a dilute-flux limit did exist because the geometry could then be better and better approximated by a flat torus or Ricci-flat Calabi–Yau space, respectively.

Let us investigate this point for the class of models presented in this paper. For that purpose, we focus on Model A as being realized on the isotropic six-torus with fixed complex structure modulus $U=1$. We then consider the flux kinetic terms in the ten-dimensional Einstein-frame action, including also the non-geometric Q-flux [116–118]

$$S=\frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-g}\Big(\mathcal{L}^{HH}+\mathcal{L}_1^{QQ}+\mathcal{L}_2^{QQ}+\mathcal{L}^{HQ}+\mathcal{L}^{\mathrm{RR}}\Big)\,, \tag{5.1}$$

with the various contributions given by

$$\begin{aligned}
\mathcal{L}^{HH}&=-\frac{e^{-\phi}}{12}H_{ijk}\,H_{i'j'k'}\,g^{ii'}g^{jj'}g^{kk'}\,, &\qquad \mathcal{L}^{HQ}&=\frac{1}{2}H_{mni}\,Q_{i'}{}^{mn}\,g^{ii'}\,,\\
\mathcal{L}_1^{QQ}&=-\frac{e^{\phi}}{4}Q_k{}^{ij}\,Q_{k'}{}^{i'j'}\,g_{ii'}g_{jj'}g^{kk'}\,, &\qquad \mathcal{L}_2^{QQ}&=-\frac{e^{\phi}}{2}Q_m{}^{ni}\,Q_n{}^{mi'}\,g_{ii'}\,,\\
\mathcal{L}^{\mathrm{RR}}&=-\frac{e^{\phi}}{12}\mathfrak{F}_{ijk}\,\mathfrak{F}_{i'j'k'}\,g^{ii'}g^{jj'}g^{kk'}\,.
\end{aligned} \tag{5.2}$$

In [118] it was shown explicitly that this action coincides with the one derived in double field theory. Moreover, upon dimensional reduction it gives the scalar potential generated by the superpotential and tree-level Kähler potential reviewed in Section 2. With the fluxes being integers, in Model A the dilaton and metric behave as

$$e^{-\phi}\sim s\sim\frac{\hat{\mathfrak{f}}}{h}\,,\qquad g\sim\sqrt{\tau}\sim\frac{\hat{\mathfrak{f}}^{\frac{1}{2}}}{q^{\frac{1}{2}}}\,,\qquad g^{-1}\sim\frac{q^{\frac{1}{2}}}{\hat{\mathfrak{f}}^{\frac{1}{2}}}\,, \tag{5.3}$$

where $\hat{\mathfrak{f}}=-\tilde{\mathfrak{f}}$. Hence, all the kinetic terms in (5.2) scale in the same way as

$$\mathcal{L}^{HH}\sim\mathcal{L}_1^{QQ}\sim\mathcal{L}_2^{QQ}\sim\mathcal{L}^{HQ}\sim\mathcal{L}^{\mathrm{RR}}\sim\frac{hq^{\frac{3}{2}}}{\hat{\mathfrak{f}}^{\frac{1}{2}}}\,. \tag{5.4}$$

Therefore, in the large radius limit, $\hat{\mathfrak{f}}\gg 1$, all terms are suppressed and one could think that there exists a dilute flux limit. However, in order to control the backreaction of the fluxes on the geometry, the essential quantity is not the action but the energy–momentum tensor $T_{ij}=\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta g^{ij}}$, appearing on the right-hand side of the Einstein equation. Now, it turns out that all contributions to T_{ij} scale in the same way, namely

$$T_{ij}^{HH}\sim T_{1\,ij}^{QQ}\sim T_{2\,ij}^{QQ}\sim T_{ij}^{HQ}\sim T_{ij}^{\mathrm{RR}}\sim h\,q\,. \tag{5.5}$$

Therefore, the backreaction of the fluxes on the metric is of order one and is not diluted in the limit $\hat{\mathfrak{f}}\gg 1$. On the other hand, the backreaction is also not substantially large, i.e. we do not have $T_{ij}\sim\hat{\mathfrak{f}}^p$ for some positive power p. Therefore, it can be claimed that we are on the boundary of controlling/non-controlling the backreaction.

The upshot is that the existence of a full string theory uplift of the solutions found using the effective supergravity action, is on a less firm ground than for no-scale models with only NS–NS and R–R three-form fluxes H and $\mathfrak{F}$. Nevertheless, absence of an argument for dilute fluxes logically does not rule out that the effective string (double field) theory still provides a sort of consistent truncation of the full dynamics of the theory. This is still an unsettled open question in double field theory.

5.2. Moduli spectroscopy

We now look at the moduli spectroscopy of the flux-scaling vacua, i.e. we investigate whether one has control over the desired hierarchy of mass scales

$$M_{\rm Pl} > M_{\rm s} > M_{\rm KK} > M_{\rm mod}\,. \tag{5.6}$$

The first two hierarchies are evident. In order to trust the four-dimensional supergravity approximation we are using, the masses of the moduli should also be smaller than the Kaluza–Klein scale. One can define an additional mass-scale which is related to the energy density in the uplift potential

$$M_{\rm up} = (V_{\rm up})^{\frac{1}{4}} = |V_0|^{\frac{1}{4}}\,. \tag{5.7}$$

In the flux-scaling models we had large fluxes f_L guaranteeing that the moduli are in their perturbative regime and other fluxes f_S that we usually choose to be of order one. Moreover, there are further order one coefficients entering the Kähler potential, once we specify a concrete Calabi–Yau manifold.

Let us now formalize what we mean by parametrical control: A scale M_1 is called *parametrically larger* than a scale M_2, denoted as $M_1 \underset{p}{\gtrsim} M_2$, if it occurs that $M_2/M_1 \to 0$ for $f_L \to \infty$. The two scales are called *parametrically equal*, $M_1 \underset{p}{\simeq} M_2$, if $M_2/M_1 \to O(1)$ for $f_L \to \infty$. This distinguishes the case where one has parametric control over the relative size of two mass scales from the case when their relative size is just a numerical coincidence. It can happen that even though $M_1 \underset{p}{\simeq} M_2$ one of the order one fluxes f_S can guarantee parametric control. If that is the case we mention it explicitly. It is also possible that in our examples it just happens that the numerical prefactors are such that $M_1 > M_2$. In this case, we say that M_1 is numerically larger than M_2 and denote it as $M_1 \underset{n}{\gtrsim} M_2$.

We observe that in all the models we have studied, we have demanded that moduli are stabilized in the perturbative regimes as $\tau, s, v \underset{p}{\gtrsim} 1$, which lead us to the following relations for the mass scales

$$M_{\rm up}^2 \underset{p}{\simeq} M_{\rm mod}\, M_{\rm Pl}\,, \qquad M_{\rm up} \underset{p}{\gtrsim} M_{\rm s}\,. \tag{5.8}$$

The first relation can be viewed as a generic prediction for this class of models, where the second relation rather indicates that the energy density in the uplift potentially exceeds the scale where we can confidently use the effective supergravity description. In Appendix A we provide a model, showing that one can have $M_{\rm up} \underset{p}{\lesssim} M_{\rm s}$, once one gives up the requirement that all $\tau, s, v \underset{p}{\gtrsim} 1$. We will come back to this point in Section 6, when we discuss inflation. In that context the two relations receive a different interpretation.

The relative sizes of $M_{\rm s}$, $M_{\rm KK}$ and $M_{\rm mod}$ turn out to be model dependent. Let us discuss three representative examples.

Model A We first discuss Model A of Section 3.2.1. Using (3.3) we can calculate the Kaluza–Klein and the string scale. For the tachyon-free vacuum 3 of Model A we obtain

$$\frac{M_{\rm s}}{M_{\rm KK}} = 2\pi \left(\frac{2\tau}{s}\right)^{\frac{1}{4}} = 2\pi \left(\frac{12}{5}\right)^{\frac{1}{4}} \left(\frac{h}{q}\right)^{\frac{1}{4}} . \tag{5.9}$$

Therefore, to have the string scale parametrically higher than the KK-scale, we need to require $h > q$. This means $\tau > s$ so that α'-corrections to the tree-level Kähler potential are indeed subleading. The ratio of the KK-scale to the moduli mass scale comes out as

$$\frac{M_{\rm KK}}{M_{\rm mod}} = \frac{10}{6\sqrt{\mu_i\, hq}} . \tag{5.10}$$

Since both ratios do not depend on the very large flux $\hat{\mathfrak{f}}$, we would write that $M_{\rm s} \underset{p}{\simeq} M_{\rm KK} \underset{p}{\simeq} M_{\rm mod}$. However, by choosing for the order one fluxes $h > q$ we can at least guarantee $M_{\rm s} \underset{p}{\gtrsim} M_{\rm KK}$. However, the KK-scale is not separated from the flux induced moduli masses.

Model D The same problem appears for model D of Section 3.3.2. The scales for the tachyon-free vacuum are

$$M_{\rm s}^2 = \mu_{\rm s}\, \frac{(\tilde{h}^1)^{\frac{1}{2}} (\tilde{q}^1)^{\frac{3}{2}}}{\hat{\mathfrak{f}}_1 \tilde{\mathfrak{f}}^0} \, \frac{M_{\rm Pl}^2}{4\pi \cdot 2^7} , \qquad M_{\rm KK}^2 = \mu_{\rm KK}\, \frac{(\tilde{q}^1)^2}{\hat{\mathfrak{f}}_1\, \tilde{\mathfrak{f}}^0} \, \frac{M_{\rm Pl}^2}{4\pi \cdot 2^7} , \tag{5.11}$$

with $\mu_{\rm s} = 2274$ and $\mu_{\rm KK} = 36$. For the ratio of the Kaluza–Klein scale and the moduli masses we find

$$\frac{M_{\rm mod}^2}{M_{\rm KK}^2} = \mu\, \frac{(\tilde{h}^1)(\tilde{q}^1)(\hat{\mathfrak{f}}_1)^{\frac{1}{2}}}{(\tilde{\mathfrak{f}}^0)^{\frac{1}{2}}} , \tag{5.12}$$

with the prefactor μ of order one. Recall that we had to scale $\hat{\mathfrak{f}}_1 \gg \tilde{h}^1, \tilde{q}^1, \tilde{\mathfrak{f}}^0 \approx O(1)$ in order to be in the weak-coupling and large-radius regime. Then the KK-scale becomes parametrically lighter than the heavy moduli, $M_{\rm mod} \underset{p}{\gtrsim} M_{\rm KK}$.

Model C Finally, let us present a model where parametric control over the ratios in principle is not in conflict with the perturbative regime. Using (3.3) in Model C, the string and Kaluza–Klein scale are computed as

$$M_{\rm s}^2 = \mu_{\rm s}\, \frac{h^{\frac{1}{2}} q^{\frac{3}{2}}}{\mathfrak{f}_0\, \tilde{\mathfrak{f}}^1} \, \frac{M_{\rm Pl}^2}{4\pi \cdot 2^7} , \qquad M_{\rm KK}^2 = \mu_{\rm KK}\, \frac{q^2}{\mathfrak{f}_0\, \tilde{\mathfrak{f}}^1} \, \frac{M_{\rm Pl}^2}{4\pi \cdot 2^7} , \tag{5.13}$$

with $\mu_s = 84$ and $\mu_{\rm KK} = 1.4$. For the ratio of the Kaluza–Klein and the string scale we obtain

$$\frac{M_{\rm KK}^2}{M_{\rm s}^2} = 0.016 \left(\frac{q}{h}\right)^{\frac{1}{2}} , \tag{5.14}$$

whereas the ratio of the moduli masses and the Kaluza–Klein scale is

$$\frac{M_{\rm mod}^2}{M_{\rm KK}^2} \sim \frac{h\, q\, (\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}{\mathfrak{f}_0^{\frac{1}{2}}} , \tag{5.15}$$

so that for large enough $\mathfrak{f}_0$ one can ensure that the moduli are lighter than the Kaluza–Klein scale, i.e. $M_{\rm KK} \underset{p}{\gtrsim} M_{\rm mod}$. To summarize, for this model we find the following controlled hierarchy of mass scales

$$M_{\rm Pl} \underset{p}{\gtrsim} M_{\rm up} \underset{p}{\gtrsim} M_{\rm s} \underset{p}{\gtrsim} M_{\rm KK} \underset{p}{\gtrsim} M_{\rm mod} . \tag{5.16}$$

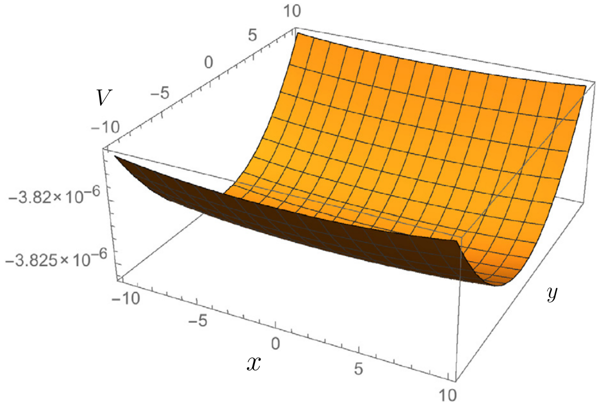

Fig. 3. The potential $V(x, y)$ around the minimum, where x is pointing in the direction of the lightest axionic modulus and y in the direction of the lightest saxionic modulus.

Since all scales differ only by a relative factor of $O(10)$, they are very sensitive to numerical prefactors. For concreteness let us make the choice

$$\mathfrak{f}_0 = 3200\,, \qquad \tilde{\mathfrak{f}}^1 = 1\,, \qquad h = -2\,, \qquad q = -1\,, \tag{5.17}$$

and analyze the moduli around the minimum with values

$$\tau = 275\,, \qquad s = 110\,, \qquad v = 18\,, \qquad u = c = \rho = 0\,. \tag{5.18}$$

Using $M_{\rm Pl} = 2.44 \cdot 10^{18}$ GeV, the string and KK-scale come out as

$$M_{\rm s} \sim 1.17 \cdot 10^{16}\,{\rm GeV}\,, \qquad M_{\rm KK} \sim 1.25 \cdot 10^{15}\,{\rm GeV}\,. \tag{5.19}$$

The masses of the saxion moduli are

$$M_i^{\rm sax} \sim \big(2.9,\ 1.2,\ 1.0\big) \cdot 10^{14}\,{\rm GeV}\,, \tag{5.20}$$

and the masses of the two heavy axions are

$$M_i^{\rm ax} \sim \big(2.5,\ 0.23\big) \cdot 10^{14}\,{\rm GeV}\,. \tag{5.21}$$

Note that the second axion is the lightest (massive) axion and therefore could be a candidate for an inflaton. In Fig. 3 we show the potential around the minimum, in the directions of the lightest and the second-lightest modulus.

5.3. Tunneling between flux branches

Another potential problem could be the occurrence of substantial tunneling between the various branches in the flux landscape. Such tunnelings are induced by domain walls changing the value of the fluxes and were discussed to be a potential problem also to maintain axion-monodromy inflation for a long enough period [32].

Let us estimate this tunneling rate for Model A using the formulas from Coleman–De Luccia for the thin-wall approximation [119,120]. The tunneling amplitude can be computed from

$$P \sim e^{-S_{\rm CD}}\,, \qquad S_{\rm CD} \sim \frac{B}{(1+A^2)^2}\,, \tag{5.22}$$

with coefficients given by

$$B = \frac{27\pi^2\sigma^4}{2(\Delta V)^3}\,, \qquad A = \frac{\sqrt{3}\sigma}{2\sqrt{\Delta V}\,M_{\rm Pl}}\,. \tag{5.23}$$

Here σ denotes the tension of the domain wall and ΔV the potential difference on the left and the right-hand side of the domain wall.

For Model A the fluxes we have turned on are the NS–NS and R–R three-form fluxes and the non-geometric Q-flux. Thus, following [38], the corresponding domain walls are the NS5-brane, the D5-brane and the non-geometric 5^2_2-brane [121,122] wrapped on three-cycles of the Calabi–Yau manifold. Due to the tadpole cancellation conditions, we cannot change a single flux but only all three fluxes at the same time. Therefore, we expect the corresponding domain wall to be a bound state of those three kinds of branes. Let us compute the action for each individual brane, where as we will see it is sufficient to consider the NS5-brane and the D5-brane.

Via dimensional reduction of the corresponding world-volume actions, we find

$$\sigma_{\rm D5} \sim M_s^3\, s^{\frac{1}{4}}\, \tau^{\frac{3}{4}}\,, \qquad \sigma_{\rm NS5} \sim M_s^3\, s^{\frac{5}{4}}\, \tau^{\frac{3}{4}}\,. \tag{5.24}$$

Using $\Delta V \sim V \sim M^2_{\rm mod}\, M^2_{\rm pl}$ and the expressions for $M_{\rm s}$ and $M_{\rm KK}$ for Model A, we obtain for a D5-domain wall

$$B_{\rm D5} \sim \frac{M_{\rm s}^8}{M^2_{\rm Pl}\, M^6_{\rm mod}} \sim \frac{1}{\hat{\mathfrak{f}}^2 h q^3}\,, \qquad A^2_{\rm D5} \sim \frac{M_{\rm s}^4}{M^2_{\rm Pl}\, M^2_{\rm mod}} \sim \frac{1}{\hat{\mathfrak{f}}^2}\,, \tag{5.25}$$

and for a NS5 domain wall

$$B_{\rm NS5} \sim \frac{M^{12}_{\rm KK}\, M^2_{\rm Pl}}{M^8_{\rm s}\, M^6_{\rm mod}} \sim \frac{\hat{\mathfrak{f}}^2}{h^5 q^3}\,, \qquad A^2_{\rm NS5} \sim \frac{M^6_{\rm KK}}{M^4_{\rm s}\, M^2_{\rm mod}} \sim \frac{1}{h^2}\,, \tag{5.26}$$

where $\hat{\mathfrak{f}} = -\tilde{\mathfrak{f}}$. This is the result just for unit one domain walls, i.e. those changing $\Delta\hat{\mathfrak{f}} = \Delta h = 1$. As mentioned we need to satisfy the tadpole cancellation conditions $h\,\hat{\mathfrak{f}} = $ const. and $q\,\hat{\mathfrak{f}} = $ const. The solution to these constraints is $\hat{\mathfrak{f}} \to \kappa\hat{\mathfrak{f}}$ while $h \to \kappa^{-1}h$ and $q \to \kappa^{-1}q$. For the vacuum energy to decrease, we need $\kappa > 1$. As a consequence, the relevant domain wall must change the three fluxes as $\Delta\hat{\mathfrak{f}} \sim \hat{\mathfrak{f}}$, $\Delta h \sim h$ and $\Delta q \sim q$ so that also the individual tensions scale as $\sigma^{(\kappa)}_{\rm D5} \sim \hat{\mathfrak{f}}\sigma_{\rm D5}$ and $\sigma^{(\kappa)}_{\rm NS5} \sim h\sigma_{\rm NS5}$. Including these factors, one finds

$$B^{(\kappa)}_{\rm D5} \sim B^{(\kappa)}_{\rm NS5} \sim \frac{\hat{\mathfrak{f}}^2}{hq^3}\,, \qquad (A^{(\kappa)})^2_{\rm D5} \sim (A^{(\kappa)})^2_{\rm NS5} \sim 1\,, \tag{5.27}$$

i.e. both branes have parametrically the same action. Therefore, assuming that the bound state is essentially at threshold, the tunneling amplitude scales as

$$P \sim \exp\left(-\hat{\mathfrak{f}}^2\right), \tag{5.28}$$

so that one has parametric control to suppress such tunneling transitions.

5.4. Soft masses of MSSM on D7*-branes*

In this section, we study some of the particle-physics aspects of the scaling-type minima presented in the previous sections. Since the vacua generically break supersymmetry, we are in particular interested in the induced supersymmetry breaking scale in some D7-brane sector supporting the Standard Model. We have already seen that the gravitino mass scale is of the same order as the moduli masses. For models of large-field inflation, this scale is of the order 10^{14}–10^{15} GeV, thus leading to a very high supersymmetry breaking scale. Motivated by the sequestered scenario in the LVS framework [92], we can therefore ask whether also here it is possible to obtain soft-masses that are smaller than the gravitino mass.

5.4.1. Bulk scenario

We now add an extra four-cycle to the geometry and compute the soft gaugino and sfermion masses for magnetized D7-branes wrapping that cycle and supporting the MSSM. For concreteness we consider the Model C described in Section 3.3.1. Due to the FW anomalies, we cannot place a magnetized D7-brane on the single four-cycle. Thus, in order to also allow a sector where the Standard Model can be supported, we need to introduce additional four-cycles into the geometry. In order to avoid brane deformation moduli, we deform the geometry such that we introduce a further del Pezzo surface so that the volume form is given by the familiar swiss-cheese type

$$\mathcal{V} = \tau^{\frac{3}{2}} - \tau_s^{\frac{3}{2}} . \tag{5.29}$$

Moreover, to avoid Freed–Witten anomalies, the superpotential should not depend on the Kähler modulus T_s.

We now assume that the MSSM is realized on stacks of magnetized D7-branes wrapping the added four-cycle and proceed as in the analysis of the sequestered LVS scenario [92]. The axion ρ_s in T_s becomes massive via the Stückelberg mechanism and the Kähler modulus τ_s appears in an induced Fayet–Iliopoulos term that shrinks the del Pezzo surface to zero size $\tau_s \sim 0$. Since W does not depend on T_s and $K^{\overline{T}_s, i}\partial_i K = -2\tau_s \sim 0$, we find $F^{T_s} = 0$.

Focusing on the supersymmetry breaking strictly stable minimum (3.37), using the formalism from [123], let us compute the soft supersymmetry breaking masses on the magnetized D7-branes wrapping the small cycle. The gaugino masses are then given by

$$M_a = \frac{1}{2}\,(\mathrm{Re}\, f_a)^{-1} F^i\, \partial_i f_a\, , \tag{5.30}$$

with

$$F^i = e^{\frac{K}{2}} K^{i\bar{j}} D_{\bar{j}}\overline{W}\, , \tag{5.31}$$

and where $f_a = T_s + \chi_a S$ is the holomorphic gauge kinetic function for the D7-brane. Here $\chi_a = \frac{1}{4\pi^2}\int F_a \wedge F_a$ is again the instanton number of the gauge flux on the D7-branes. Using $F^{T_s} = 0$ and evaluating F^S, we find for the gaugino masses

$$M_a^2 = \mu_a\, \frac{h_1(q_1)^3}{(\mathfrak{f}_0)^{\frac{3}{2}}(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}\, \frac{M_{\mathrm{Pl}}^2}{4\pi\cdot 2^4} \sim M_{\frac{3}{2}}^2\, , \tag{5.32}$$

with $\mu_a = 12$. The sfermion masses are given by

$$M_\alpha^2 = M_{\frac{3}{2}}^2 + V_0 - F^{\bar{i}} F^j \partial_{\bar{i}} \partial_j \log Z_\alpha\, , \tag{5.33}$$

where Z_α is the Kähler metric for the matter-field. It was argued in [124] that for magnetized branes on a small shrinkable cycle, at tree-level one has $Z_\alpha = k_\alpha/\tau$. Assuming also an uplift mechanism to $V_0 = 0$, the sfermion masses become

$$M_\alpha^2 = M_{\frac{3}{2}}^2 - \frac{(F^T)^2}{4\,\tau^2} = \mu_\alpha \,\frac{h_1(q_1)^3}{(\mathfrak{f}_0)^{\frac{3}{2}}(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}\,\frac{M_{\rm Pl}^2}{4\pi\cdot 2^4} \sim M_{\frac{3}{2}}^2\,, \tag{5.34}$$

with $\mu_\alpha = 28$. Thus, we realize that all soft masses are of the same order as the gravitino mass. The shrinkable cycle itself therefore does not lead to sequestering. Opposed to the sequestered LVS scenario [92], the main difference is that here $F^S \neq 0$ already at tree-level.

5.4.2. *Sequestered scenario*

As we have seen, in order to achieve a suppression for the soft-masses, we need $F^S = 0$ at tree-level and we need to turn on a further small correction that can induce $F^S \neq 0$. In our model search so far we have not[7] obtained a non-supersymmetric model with $F^S = 0$. Therefore, we now consider the non-supersymmetric minimum of Model B discussed in Section 3.2.2, where we will be able to enforce $F^T = 0$ for a subset of fluxes. If we now place an unmagnetized D7-brane on the four-cycle, we have a toy model for the situation we are interested in.

Gravity-mediated gaugino masses Let us estimate the size of the gaugino masses, once we take into account the $(\alpha')^3$-correction to the Kähler potential as

$$K = -2\log\left[\left((T+\overline{T}) + \frac{\kappa}{4\,(S+\overline{S})}(G+\overline{G})^2\right)^{\frac{3}{2}} + \frac{\xi_p}{2}\,s^{\frac{3}{2}}\right] - \log(S+\overline{S})\,, \tag{5.35}$$

where $\xi_p = -\frac{\chi(\mathcal{M})\zeta(3)}{2\,(2\pi)^3}$. The superpotential (3.19) for this model is not changed. The values of the auxiliary fields $F^i = e^{\frac{K}{2}} K^{i\bar{j}} F_{\bar{j}}$ at the minimum are

$$\begin{aligned} F^T &= e^{\frac{K}{2}}\,\frac{8\,i}{25}\,\frac{\tilde{\mathfrak{f}}^2}{q}\,\frac{8x+3}{1+x}\,, \qquad F^S = -e^{\frac{K}{2}}\,\frac{8\,i}{5}\,\frac{\mathfrak{f}^2}{h}\,\frac{1}{(1+x)}\,,\\ F^G &= e^{\frac{K}{2}}\,\frac{16\,i}{5}\,\frac{\mathfrak{f}^2}{f}\,\frac{x}{x+1}\,, \end{aligned} \tag{5.36}$$

with $x = \frac{f^2}{\kappa h q}$. Thus, we see that we can force $F^T = 0$ by choosing $8x + 3 = 0$. This is only possible for negative κ since otherwise we would leave the physical regime and $s_0, \tau_0 < 0$.

To assess the order of magnitude of the α'-correction first notice that (3.3) implies $M_{\rm KK}/M_{\rm s} \sim s^{\frac{1}{4}}/\mathcal{V}^{\frac{1}{6}}$ for the ratio of the KK and the string scale. Using that the volume is $\mathcal{V} \propto \left(2\tau + \frac{\kappa}{2s}\psi^2\right)^{\frac{3}{2}}$, and substituting the values of the moduli in the minimum (3.21), we then find

$$\frac{M_{\rm KK}}{M_{\rm s}} \sim \left(\frac{q}{h}\right)^{\frac{1}{4}}\,, \tag{5.37}$$

where $x = -\frac{3}{8}$ has been used. Inserting in (5.35) we then conclude that the α'-correction is small compared to the tree-level term provided $\xi_p (q/h)^{3/2} \ll 1$, which can be achieved taking $h \gg q$.

A numerical analysis shows that in this regime the former minimum gets slightly shifted and that the main contribution to the new value of F^T, denoted F^T_ξ, comes from plugging in the values of the old minimum in the corrected expression for F^T. Thus at linear order in ξ_p, the induced vacuum expectation value of the auxiliary field F^T is parametrically given by

[7] In Section 3.3.5 with $h^{1,2}_- = 2$, we found a model with $F^{U_1} = 0$ (see (3.70)), which does not help here as only F^T and F^S appear in the gaugino or sfermion masses. Nevertheless, it shows that accidentally a zero auxiliary field is in principle possible.

$$\frac{F^T_\xi}{F^T_0} \sim \xi_p \left(\frac{q}{h}\right)^{\frac{3}{2}} \sim \xi_p \left(\frac{M_{\rm KK}}{M_{\rm s}}\right)^6 , \tag{5.38}$$

where F^T_0 is the size of the tree-level F-term (5.36) for $x \neq -\frac{3}{8}$ and where we used (5.37). The gravity-mediated gaugino masses can now be expressed as

$$M_a \sim \left(\frac{M_{\rm KK}}{M_{\rm s}}\right)^6 M_{\frac{3}{2}} , \tag{5.39}$$

which is suppressed relative to the gravitino mass scale by a high power of the ratio of the KK-scale to the string scale.

Anomaly-mediated gaugino masses With the tree-level gravity-mediated gaugino masses vanishing at leading order, the one-loop generated anomaly-mediated gaugino masses are expected to be generically larger than the next-to-leading order tree-level masses. In the sequestered LVS scenario, it turned out that even the leading-order anomaly-mediated contribution vanishes due to an extended no-scale structure. Let us estimate this contribution in our model. The anomaly-mediated gaugino masses are given by [125]

$$\begin{aligned} M_a^{\rm anom} = -\frac{g^2}{16\pi^2}\Big(&(3\,T_G - T_R) M_{\frac{3}{2}} - (T_G - T_R)(\partial_i K) F^i \\ &- \frac{2\,T_R}{d_R} F^i\, \partial_i \log\det Z_{\alpha\beta}\Big), \end{aligned} \tag{5.40}$$

where $T_G = N$ is the Dynkin index of the adjoint representation of $U(N)$ and T_R is the Dynkin index of some matter representation R of dimension d_R. In our simple case of unmagnetized D7-branes, there is no charged matter so that the above formula simplifies. Indeed, there is no cancellation between the first and second term and we obtain

$$M_a^{\rm anom} = \frac{1}{16\pi^2\, {\rm Re}(f_a)}\,\frac{8}{3}\, N M_{\frac{3}{2}} = \frac{1}{(4\pi)^{\frac{3}{2}}}\,\frac{16\,N}{9}\,\frac{M_{\rm KK}\, M_{\frac{3}{2}}}{M_{\rm Pl}} . \tag{5.41}$$

Therefore, we still get a suppression, which generically will be weaker than the next-to-leading order gravity-mediated one (5.39). For instance, for $M_{\rm s} \sim 10^{16}$ GeV, $M_{\rm KK} \sim 10^{15}$ GeV and $M_{\frac{3}{2}} \sim 10^{14}$ GeV, we find $M_a \sim 10^8$ GeV and $M_a^{\rm anom} \sim 10^{11}$ GeV. Therefore, one can get gaugino masses in the intermediate regime.

As argued in [92], the computation of other soft terms is sensitive to higher-order corrections to the matter-field metric and to the uplift, so that we are not pursuing this question here further. Of course, what we have presented is just a toy model, as the brane wrapping the four-cycle is non-chiral and presumably will carry extra massless deformation modes (that also have to be stabilized). The purpose of our analysis was to show how one can arrange for a situation where the gaugino masses are induced by higher-order corrections, and can therefore be parametrically smaller than the gravitino mass scale. This is important for string model building, if one wants to have the supersymmetry breaking scale for the MSSM smaller than the GUT or inflation scale. With the supersymmetry breaking scale in the intermediate regime, one can realize the scenario of [126], where gauge coupling unification is obtained by the F-theory motivated scenario proposed in [127].

6. Axion monodromy inflation

We now turn to string-cosmological properties of our models. Recall that a large tensor-to-scalar ratio points towards an inflationary scenario with the slow rolling occurring for large field values $\Theta/M_{\rm Pl} \sim 1$–10. Therefore, in any UV complete theory of gravity one has to control higher order corrections. Axions with their perturbative shift-symmetries are good candidates and various scenarios have been proposed, ranging from natural inflation, over N-flation to axion monodromy inflation. The latter can be naturally realized in string theory, where the very same scalar potential that stabilizes the moduli can also give rise to the axion potential. Here the shift symmetry of the axion is spontaneously broken by the choice of fluxes in the background, thus giving rise to an effective potential and an unwrapping of the compact field range for the axion.

In this section we investigate whether the flux scaling models we have discussed can provide working examples to realize F-term axion monodromy inflation in set-ups with consistent moduli stabilization. As a matter of fact, this was our initial motivation to look into this part of the string/gauged supergravity landscape in more detail. Please recall the challenges for such a construction that have been listed in the introduction.

By including all closed string moduli in the tree-level flux superpotential, we have available many of the axions that have been put forward as inflaton candidates in the literature. In general, the eventual inflaton Θ will be a linear combination of some of the following axions:

- There is the universal axion c from the axio-dilaton superfield. It was proposed [39] that if this axion is part of the inflaton, it can provide an appealing reheating mechanism.
- The Kähler moduli T_α contain the R–R four-form axions ρ_α. As opposed to KKLT and the LVS scenario these moduli are also stabilized at tree-level by turning on non-geometric Q-flux [45,54].
- In [38,44] the proposal was to consider the two-forms B_2 or C_2 as the inflaton. This can be realized by generating an F-term potential for odd Kähler moduli G^a by turning on geometric flux.
- In the large complex structure limit, extra geometric shift symmetries arise so that the quasi-axions $\mathrm{Re}(U) = u$ can also be considered as inflaton candidates [40].

The purpose of this section is not to construct a fully-fledged cosmological model, but, continuing the analysis of [49,50], to study the question whether an axion can realize large field inflation.

In the following the inflaton is considered to be an initially massless axion, which receives a parametrically smaller mass by turning on additional fluxes. Since fluxes are of order one, the hierarchy occurs by turning on large fluxes λ for the heavy moduli. Thus, we have a (flux) parameter available by which we can control both the mass hierarchy of the inflaton and the heavy moduli as well as its backreaction on the other moduli. In fact it has been shown in [50] that for $\lambda \gg 1$ the backreaction is under control and that one obtains the naive polynomial scalar potential. We find that this is in principle possible but that, in all examples we have looked at, the KK-scale becomes lighter than the moduli masses.

In [73] we analyze a toy model for this scenario, where the backreaction can be taken into account analytically. There, changing a parameter analogous to λ interpolates between chaotic, linear and Starobinsky-like inflation.

6.1. Brief review on large field inflation

In this section we review the basics of large field inflationary models. (For more details see for instance [128].) In general, one can distinguish convex and concave scenarios. The prototype examples of the first type are models with polynomial scalar potentials, like for instance chaotic inflation governed by a quadratic potential. Such models would have been the best candidates to explain the BICEP2 [10] result $r = 0.2$. However, due to the PLANCK 2015 data [12,13] this is explained by the foreground dust contamination of the signal and substituted by the upper bound $r < 0.113$. Moreover, the reported values for the spectral index and its running are $n_s = 0.9667 \pm 0.0040$ and $\alpha_s = -0.002 \pm 0.013$, respectively. As a consequence, potentials $V \sim \Theta^p$ with $p \geq 2$ are disfavored. Instead, the recent results point towards concave models. The Bayesian analysis reviewed in [129] also indicates that plateau-like potentials are the best class of models fitting the current data. Nonetheless, in the following we will investigate how polynomial inflation is realized in our fluxed vacua.

Let us recall the cosmological data needed for our discussion. For a polynomial potential appearing in the single field Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Theta\, \partial^\mu \Theta + \mu^{4-p}\, \Theta^p\,, \tag{6.1}$$

the slow-roll parameters

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta = \left(\frac{V''}{V}\right), \tag{6.2}$$

can be computed as

$$\epsilon = \frac{1}{2}\,\frac{p^2}{\Theta^2}\,, \qquad \eta = \frac{p\,(p-1)}{\Theta^2}\,. \tag{6.3}$$

The number of e-foldings is expressed as

$$N_e = \int_{\Theta_{\rm end}}^{\Theta_*} \frac{V}{V'}\, d\Theta = \frac{1}{p}\int_{\Theta_{\rm end}}^{\Theta_*} \Theta\, d\Theta \simeq \frac{\Theta_*^2}{2\,p}\,. \tag{6.4}$$

Thus, we can write $N_e \sim \frac{p}{4\epsilon}$. Therefore, for the spectral indices and the tensor-to-scalar ratio one obtains

$$\begin{aligned} n_s &= 1 + 2\eta - 6\epsilon \sim 1 - \frac{(p+2)}{2N_e}\,, \\ n_t &= -2\epsilon \sim -\frac{p}{2N_e}\,, \\ r &= 16\epsilon \sim \frac{4p}{N_e}\,. \end{aligned} \tag{6.5}$$

For a quadratic potential, that is $p = 2$, and 60 e-foldings, this leads to

$$n_s \sim 0.967\,, \qquad n_t \sim -0.017\,, \qquad r = 0.133\,, \tag{6.6}$$

which is excluded by the recent measurements from Planck and from BICEP2 at 95% confidence level [12,13]. The amplitude of the scalar power spectrum $\mathcal{P} = 2.142 \cdot 10^{-9}$ can be written as follows

$$\mathcal{P} \sim \frac{H_{\rm inf}^2}{8\pi^2 \epsilon M_{\rm Pl}^2}\,, \tag{6.7}$$

which leads to a Hubble constant during inflation of $H_{\rm inf} \sim 9.14 \cdot 10^{13}$ GeV. Using $V_{\rm inf} = 3M_{\rm Pl}^2 H_{\rm inf}^2$, we can extract the mass scale of inflation as $M_{\rm inf} = V_{\rm inf}^{1/4} \sim 1.96 \cdot 10^{16}$ GeV, which is of the order of the GUT scale. Finally, the mass of the axion $M_\Theta^2 = 3\eta H_{\rm inf}^2$ comes out as $M_\Theta \sim 1.45 \cdot 10^{13}$ GeV.

From a stringy point of view, for realizing single field inflation in a controlled way, one needs the hierarchy of string theoretic and inflationary scales[8]

$$M_{\rm Pl} > M_{\rm s} > M_{\rm KK} > M_{\rm inf} \sim M_{\rm mod} > H_{\rm inf} > |M_\Theta|\,. \tag{6.8}$$

As we have seen, for large field inflation we have $H \sim 10^{14}$ GeV so that between the Hubble-scale and the Planck-scale there are only four orders of magnitude for all the other scales. Clearly, to achieve and control such a sensitive hierarchy is a major challenge for string theory.

6.2. *Realization of polynomial inflation*

Following the procedure suggested in [49], the idea is to generate a non-trivial scalar potential for the axion Θ (in the following called inflaton) by turning on the additional fluxes $f_{\rm ax}$, while the former fluxes are scaled by a large number λ. Thus the total superpotential reads

$$W_{\rm inf} = \lambda\, W + f_{\rm ax}\, \Delta W\,. \tag{6.9}$$

In [49] this procedure led to a parametrically lighter axion mass.[9] We will see that for our case, where now all moduli appear in W, the situation is different and more subtle.

6.2.1. *Parametrically light axion for Model C*

As a first attempt we consider the superpotential

$$W_{\rm inf} = \lambda\Big(- \mathfrak{f}_0 - 3\tilde{\mathfrak{f}}^1 U^2 - h_1 U\, S - q_1 U\, T\Big) + i\,(h_0 S + q_0 T)\,. \tag{6.10}$$

Let us first proceed under the assumption that the mass for Θ can be parametrically smaller than the masses of all the other moduli and that the backreaction of ΔW on the values of the moduli in the old minimum (3.37) is negligible. Then we can analyze the problem by first integrating out all heavy moduli and computing an effective potential for $\Theta \sim c$. In practice this means determining the scalar potential induced by $W_{\rm inf}$ and then inserting the values (3.37) for the moduli. In this way we obtain

$$V_{\rm eff}(c) = \frac{1}{2^7}\Big(A\,(c - B)^2 + V_0\Big)\,, \tag{6.11}$$

with

$$A = \frac{2^{\frac{15}{4}}}{5^{\frac{5}{4}} \cdot 3^{\frac{1}{2}}}\, \frac{q_1 h_1 (h_1 q_0 - q_1 h_0)^2}{\mathfrak{f}_0^{\frac{7}{2}}\,(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}\,, \qquad B = \frac{q_1 \mathfrak{f}_0 \lambda}{2(q_1 h_0 - h_1 q_0)}\,, \tag{6.12}$$

[8] For a recent discussion of these hierarchies see [130].

[9] An alternative idea with $\lambda = 1$ was promoted in [50], where certain assumptions about tunings in the string landscape had to be made.

and

$$V_0 = -\frac{7 \cdot 2^{\frac{7}{4}}}{5^{\frac{5}{4}} \cdot 3^{\frac{3}{2}}} \frac{\lambda^2 h_1 q_1^3}{\mathfrak{f}_0^{\frac{3}{2}} (\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}} + O(\mathfrak{f}_0^{-\frac{5}{2}})\,. \tag{6.13}$$

Therefore, the inflaton receives a large vacuum expectation value. In particular, inserting its value back into the superpotential (6.10) one realizes that the two terms λW and $f_{\rm ax} \Delta W$ scale in the same way with the fluxes. Therefore, one expects that the backreaction on the old minimum is substantial. This is confirmed by a numerical analysis of the scalar potential. As a consequence, the effect of this form of ΔW is not under parametric control and therefore it is not a good candidate for a deformation. The problem is the resulting linear term in Θ in the effective scalar potential, whose prefactor relative to the quadratic term is generically of the order λW_0. To control the backreaction on the former minimum, one needs an effective potential, where the prefactor becomes zero, as was generically the case in [49].

Let us consider a different deformation of the superpotential, now generated by turning on non-geometric P flux

$$W_{\rm inf} = W + \Delta W = \lambda\Big(- \mathfrak{f}_0 - 3\tilde{\mathfrak{f}}^1 U^2 - h_1 U\, S - q_1 U\, T \Big) - p_0\, S\, T\,. \tag{6.14}$$

In this case the effective scalar potential becomes

$$V_{\rm eff} = \frac{1}{2^7}\Big(A\, c^4 + B\, c^2 + C \Big)\,, \tag{6.15}$$

with

$$A = \frac{2^{\frac{7}{4}}}{5^{\frac{5}{4}} \cdot 3^{\frac{1}{2}}} \frac{p_0^2 h_1^3 q_1}{\mathfrak{f}_0^{\frac{7}{2}} (\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}\,, \qquad B = \frac{2^{\frac{3}{4}}}{5^{\frac{9}{4}} \cdot 3^{\frac{1}{2}}} \frac{p_0 h_1 q_1 (20\lambda h_1 q_1 + 73\sqrt{10} p_0 \tilde{\mathfrak{f}}^1)}{\mathfrak{f}_0^{\frac{5}{2}} (\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}\,, \tag{6.16}$$

and

$$C = \frac{2^{\frac{15}{4}}}{5^{\frac{5}{4}} \cdot 3^{\frac{3}{2}}} \frac{q_1 (-h_1^2 q_1^2 \lambda^2 + 21 p_0 h_1 q_1 \tilde{\mathfrak{f}}^1 \lambda + 90 p_0^2 (\tilde{\mathfrak{f}}^1)^2)}{h_1 \mathfrak{f}_0^{\frac{3}{2}} (\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}\,. \tag{6.17}$$

For all fluxes and λ being positive, the effective potential has a global minimum at $c = 0$. Therefore, in this case we expect that the backreaction of the P-flux term on the other moduli can be made small. In this minimum the mass of the canonically normalized inflaton is computed as

$$M_\Theta^2 = \mu_\Theta \frac{p_0 q_1 (\tilde{\mathfrak{f}}^1)^{\frac{1}{2}} (20 h_1 q_1 \lambda + 73\sqrt{10} p_0 \tilde{\mathfrak{f}}^1)}{h_1 \mathfrak{f}_0^{\frac{3}{2}}} \frac{M_{\rm Pl}^2}{4\pi \cdot 2^7}\,, \tag{6.18}$$

with $\mu_\Theta = 1.6$. Therefore, in the regime $20 h_1 q_1 \lambda \gg 73\sqrt{10} p_0 \tilde{\mathfrak{f}}^1$ the mass of the inflaton can be made parametrically smaller than the mass of the heavy moduli

$$\frac{M_\Theta^2}{M_{\rm mod}^2} \sim \frac{p_0 \tilde{\mathfrak{f}}^1}{h_1 q_1 \lambda}\,. \tag{6.19}$$

In order to realize chaotic inflation with a quadratic potential, we have to ensure that for $c \sim 10\, M_{\rm Pl}$ the quartic term in the canonically normalized $V_{\rm eff}$ can be neglected. Indeed, as long as

$$c^2 \ll \frac{q_1 h_1 \lambda}{10\, p_0 \tilde{\mathfrak{f}}^1}\,, \tag{6.20}$$

the effective potential is dominated by the quadratic term, where the same combination of fluxes as in (6.19) appears. Therefore, we conclude that for sufficiently large λ we can gain parametric control over the scales in the inflaton sector while also guaranteeing a quadratic potential. Note that the ratio of the KK-scale and the moduli masses behaves as

$$\frac{M^2_{\rm mod}}{M^2_{\rm KK}} \sim \frac{\lambda^2 h_1 q_1 (\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}{\mathfrak{f}_0^{\frac{1}{2}}}\,. \tag{6.21}$$

For large λ, it becomes impossible to keep the KK-scale larger than the heavy moduli mass, while still having a string scale of the order of the GUT scale. We can summarize these findings for realizing quadratic inflation by

$$M_{\rm mod} \underset{p}{\gtrsim} M_\Theta \quad \Longrightarrow \quad M_{\rm mod} \underset{p}{\gtrsim} M_{\rm KK}\,. \tag{6.22}$$

6.2.2. Axion potential for Model D

As a second example let us also consider Model D, which was designed such that a deformation of the superpotential of the type already mentioned in (6.10) can generate a parametrically small mass for the so far massless axion. We then choose the deformation

$$W_{\rm inf} = \lambda\Big(\hat{\mathfrak{f}}_1 U + i\,\tilde{\mathfrak{f}}^0 U^3 + 3i\,\tilde{h}^1 U^2 S + 3i\,\tilde{q}^1 U^2 T\Big) + i\,(h_0 S + q_0 T)\,. \tag{6.23}$$

After integrating out the massive moduli we obtain an effective potential

$$V_{\rm eff}(c) = \frac{1}{2^7}\Big(Ac^2 + V_0 + O(\mathfrak{f}_1^{-\frac{3}{2}})\Big) \tag{6.24}$$

with

$$A = \frac{324\cdot 2^{\frac{1}{4}}}{5^{11/4}}\,\frac{\tilde{q}^1\tilde{h}^1(\tilde{h}^1 q_0 - \tilde{q}^1 h_0)^2}{\hat{\mathfrak{f}}_1^{\frac{7}{2}}(\tilde{\mathfrak{f}}^0)^{\frac{1}{2}}}\,, \tag{6.25}$$

where $V_0 \sim \hat{\mathfrak{f}}_1^{-\frac{1}{2}}$ is the value given in (3.45) multiplied by λ^2. Since this effective potential is minimized at $c = 0$ and the mass of the inflaton is parametrically smaller than the mass of the heavy moduli, we expect that one can trust this approximation. Taking into account the kinetic term for c the mass for the canonically normalized inflaton becomes

$$M^2_\Theta = \mu_\Theta\,\frac{\tilde{q}^1(\tilde{h}^1 q_0 - \tilde{q}^1 h_0)^2\,(\tilde{\mathfrak{f}}^0)^{\frac{1}{2}}}{\tilde{h}^1\,(\hat{\mathfrak{f}}_1)^{\frac{5}{2}}}\,\frac{M^2_{\rm Pl}}{4\pi\cdot 2^7}\,, \tag{6.26}$$

with $\mu_\Theta = 10$. Therefore, due to

$$\frac{M^2_\Theta}{M^2_{\rm mod}} \sim \frac{(\tilde{h}^1 q_0 - \tilde{q}^1 h_0)^2\,(\tilde{\mathfrak{f}}^0)^2}{\lambda^2(\tilde{h}^1\,\tilde{q}^1)^2\,\hat{\mathfrak{f}}_1^2}\,, \tag{6.27}$$

the mass of the inflaton can be made parametrically smaller than the mass of the heavy moduli by choosing the flux $\lambda\mathfrak{f}_1$ large enough. However, we come into conflict with the separation of the KK and moduli scales. Recalling the ratio of the Kaluza–Klein scale and the moduli masses (5.12),

$$\frac{M^2_{\rm mod}}{M^2_{\rm KK}} \sim \frac{\lambda^2\,(\tilde{h}^1)(\tilde{q}^1)(\hat{\mathfrak{f}}_1)^{\frac{1}{2}}}{(\tilde{\mathfrak{f}}^0)^{\frac{1}{2}}}\,, \tag{6.28}$$

we can derive the relation

$$\frac{M^2_{\Theta}}{M^2_{\rm mod}} \cdot \frac{M^8_{\rm mod}}{M^8_{\rm KK}} \sim \lambda^6 (\tilde{h}^1)^2 (\tilde{q}^1)^2 (\tilde{h}^1 q_0 - \tilde{q}^1 h_0)^2 \geq 1\,. \tag{6.29}$$

We want both quantities on the right to be smaller than one, but this is not compatible with the mass scales we derived for this model.

We conclude from this analysis that our attempts to gain parametric control over axion masses were only half-successful. By turning on additional fluxes it was possible to find slightly shifted minima, where the lightest axion was parametrically smaller than all the other moduli. However, it was not possible at the same time to keep the heavy moduli lighter than the KK-scale. All this is reflected in the simple formula

$$M_{\rm mod} \underset{p}{\gtrsim} M_{\Theta} \quad \Longrightarrow \quad M_{\rm mod} \underset{p}{\gtrsim} M_{\rm KK}\,. \tag{6.30}$$

We close this section with two remarks:

- These tree-level flux induced potentials clearly provide a generic framework for realizing F-term axion monodromy inflation in type IIB string theory, while also controlling the masses of other relevant moduli. They contain all the closed string axions that have been proposed in the literature as inflaton candidates. The models discussed in this section involved the three axions $\{\rho, c, u\}$. Note in particular, that in contrast to the no-go results from [49], for the generic superpotential involving also the Kähler moduli, the universal axion c could also be present in the linear combination for the inflaton. Moreover, by turning on also the geometric fluxes, generically the orientifold odd axions $\mathrm{Im}(G) = C_2$ would appear in the inflaton. We did not explicitly discuss an example of this class, as the two models presented in Sections 3.2.2 and 3.3.3 were plagued by massless saxions.
- In [73] we analyze backreaction issues. While the models discussed in the present section are already quite involved, in [73] a simple toy model based on Model A is defined, for which the backreaction can be solved analytically. For very large values of a parameter similar to λ, indeed the potential becomes effectively quadratic while for decreasing values the effect of the flattening becomes more and more visible. First, one gets an effective linear potential while for values of $O(1)$ the potential becomes Starobinsky-like. However, in this regime also the hierarchy between the inflaton and the heavy moduli mass diminishes so that one is actually dealing with a model of multi-field inflation.[10]

7. Conclusions and outlook

In this paper we have proposed a certain large scale scenario of tree-level moduli stabilization. We considered the class of non-supersymmetric, strictly stable minima of the scalar potential generated by type IIB orientifolds on CY three-folds with non-trivial geometric and non-geometric fluxes turned on. This gives the scalar potential of orientifolded $\mathcal{N} = 2$ gauged supergravity and also involves the orientifold odd moduli made up by the B_2 and C_2 two-forms.

[10] We thank Francisco Pedro for pointing this out to us.

We have presented an algorithm to construct such minima. Their characteristic feature is a certain scaling with the fluxes that allows to parametrically control many properties of the vacuum. For instance, it is easy to guarantee that all moduli are stabilized in the perturbative regime where higher order corrections are suppressed. We have started our investigation with simple models with a few moduli. By going to more involved models, we encountered the appearance of tachyonic states. For multiple Kähler moduli we identified a general mechanism, involving the addition of certain D7-branes, which allows to uplift a class of tachyonic modes. For models with multiple complex structure moduli such a mechanism is still an open question. We also mention that our model search is not exhaustive.

All the vacua considered are of AdS type. Since all moduli are stabilized at string tree-level, identifying a proper uplift mechanism for the cosmological constant is a more involved task. We provided a possible uplifting term but could not justify how it could arise. It would be interesting to really find a stringy realization of this type of uplift. In the literature it has been asserted that there exist de Sitter vacua for non-geometric flux models. It would be interesting to find out whether extending the set of fluxes in our models can lead to dS minima, while maintaining other desirable properties.

We also addressed some phenomenological issues. Since all moduli are stabilized at tree-level, the whole physics is expected to happen at the high scale. For ultra-large fluxes one could in principle lower the moduli masses and the gravitino mass scale. We have computed soft supersymmetry breaking masses on MSSM-like D7-brane set-ups. Generically, the supersymmetry breaking scale is given by the gravitino mass which is of the same scale as the moduli masses. One can arrange for sequestering of the gravity mediated terms, but then anomaly mediation happens to be the dominant contribution. This allows for a further suppression of the soft masses down to e.g. the intermediate regime.

A technical problem is that models with non-geometric fluxes do not admit a proper dilute flux limit. Thus, one cannot argue that the minima found in the effective four-dimensional theory can be uplifted to true solutions of the ten-dimensional string equations of motion or double field theory. This is an open issue whose eventual clarification relies on further progress in the understanding of non-geometric backgrounds in e.g. generalized geometry and double field theory. Another generic feature is that to achieve parametric control over the perturbative regime, the uplift mass-scale is larger than the string scale.

In the final section, we applied our results to the study of inflation, more concretely to F-term monodromy inflation with the inflaton given by an axion. In particular we asked the question whether by an appropriate scaling of the fluxes one can engineer viable models with polynomial potentials. We find that this is in principle possible via turning on additional fluxes, though at the expense of introducing large flux quanta and of making the moduli masses larger than the KK-scale. One way of interpreting these difficulties we realized in controlling all mass scales at the same time, is that maybe string theory, as a UV complete theory, wants to tell us that one has to give up at least some of the usual order of scales. A proper description of large field inflation in string theory might require to take some of the Kaluza–Klein and string states into account from the very beginning.

Acknowledgements

We thank D. Baumann, G. Dall'Agata, K. Choi, X. Gao, A. Hebecker, O. Loaiza-Brito, A. Lukas, D. Lüst, L. Martucci, A. Mazumdar, F. Pedro, F. Quevedo, P. Shukla, S. Theisen, P. Vaudrevange and T. Weigand for helpful discussions. A.F. is supported by the Alexander von

Humboldt Foundation grant VEN/1067599 STP. E.P. is supported by the MIUR grant FIRB RBFR10QS5J and the Padua University Project CPDA144437.

Appendix A. A model with parametric control $M_s > M_{up}$

We again consider a model with $h^{2,1}_- = 1$ and $h^{1,1}_+ = 1$, that can also be related to the isotropic six-torus with Käher potential

$$K = -3\log(T+\overline{T}) - \log(S+\overline{S}) - 3\log\big(U+\overline{U}\big)\,. \tag{A.1}$$

For the flux superpotential we now choose

$$W = \hat{\mathfrak{f}}_0 - i\,\mathfrak{f}_1 U + h\,U^3\,S + q\,U^3\,T\,, \tag{A.2}$$

where $h = \tilde{h}^0$, $q = \tilde{q}^0$ and $\hat{\mathfrak{f}}_0 = -\mathfrak{f}_0$. Analyzing the scalar potential, we find a non-supersymmetric, non-tachyonic minimum with stabilized moduli

$$\begin{aligned}
\tau &= \frac{5^{\frac{1}{2}}}{2^{\frac{3}{4}}\cdot 3^{\frac{11}{4}}}\frac{\mathfrak{f}_1^3}{q\,\hat{\mathfrak{f}}_0^2}\,, \qquad s = \frac{2^{\frac{5}{4}}\cdot 5^{\frac{1}{2}}}{3^{\frac{15}{4}}}\frac{\mathfrak{f}_1^3}{h\,\hat{\mathfrak{f}}_0^2}\,, \qquad hc + q\rho = -\frac{2}{27}\frac{\mathfrak{f}_1^3}{\hat{\mathfrak{f}}_0^2}\,,\\
v &= \frac{3\cdot 6^{\frac{1}{4}}\cdot 5^{\frac{1}{2}}}{(5+\sqrt{6})}\frac{\hat{\mathfrak{f}}_0}{\mathfrak{f}_1}\,, \qquad u = -\frac{3\cdot\sqrt{6}}{(5+\sqrt{6})}\frac{\hat{\mathfrak{f}}_0}{\mathfrak{f}_1}\,.
\end{aligned} \tag{A.3}$$

To stay in the physical region we take all fluxes positive. It is interesting to notice that in this example the flux tadpoles are positive.

The value of the cosmological constant in the AdS minimum is

$$V_0 = -\frac{6561\cdot 3^{\frac{1}{4}}}{50\cdot 5^{\frac{1}{2}}\cdot 2^{\frac{3}{4}}}\frac{h\,q^3\,\hat{\mathfrak{f}}_0^7}{\mathfrak{f}_1^9}\frac{M_{\rm Pl}^4}{4\pi}\,. \tag{A.4}$$

For the ratio of the string scale and the uplift scale it then follows

$$\frac{M_{\rm s}^4}{M_{\rm up}^4} = 54\cdot 6^{\frac{3}{4}}\cdot 5^{\frac{1}{2}}\cdot\pi^3\,\frac{\hat{\mathfrak{f}}_0}{\mathfrak{f}_1^3}\,, \qquad \frac{M_{\rm KK}^4}{M_{\rm up}^4} = \frac{9\cdot 3^{\frac{3}{4}}\cdot 5^{\frac{1}{2}}}{2^{\frac{5}{4}}\cdot\pi}\frac{q\,\hat{\mathfrak{f}}_0}{h\,\mathfrak{f}_1^3}\,, \tag{A.5}$$

so that for $\hat{\mathfrak{f}}_0/\mathfrak{f}_1^3 > 1$ we have gained parametric control. However, in this regime parametrically we get $\tau, s \underset{p}{\lesssim} 1$.

To give a concrete example, let us choose the numerical values $\hat{\mathfrak{f}}_0 = 1$ and $\mathfrak{f}_1 = 3$, leaving h and q free. For the moduli we readily obtain

$$\tau = \frac{1.75}{q}\,, \quad s = \frac{2.33}{h}\,, \quad v = 0.47\,, \quad u = -0.33\,, \quad c+\rho = -2\,, \tag{A.6}$$

and for the mass scales

$$\frac{M_{\rm Pl}}{M_{\rm s}} = \frac{1.78}{h^{\frac{1}{4}}q^{\frac{3}{4}}}\,, \qquad \frac{M_{\rm s}}{M_{\rm up}} = 4.8\,, \qquad \frac{M_{\rm KK}}{M_{\rm up}} = \frac{0.69\,q^{\frac{1}{4}}}{h^{\frac{1}{4}}}\,. \tag{A.7}$$

This shows that it is not possible to have all moduli in the perturbative regime while also attaining $M_{\rm up} < M_{\rm KK}$. For the purpose of realizing models of large field inflation, this means that we get parametric control of $M_{\rm inf} \underset{p}{\lesssim} M_{\rm KK}$ only for $\tau \underset{p}{\lesssim} 0.3$.

After all, independent of parametrically controlling certain ratios of scales or not, in models of large field inflation the mass scales are pushed to the threshold of having control. Moreover, with all scales being close together, extra numerical factors matter.

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