Calculation of the 0^+ and 2^+ Giant Resonances in Ni^{58}

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Abstract. Estimates of the excitation energy, width and sum rule fraction of the collective monopole and quadrupole vibrations indicate that the peak in the $62\,A^{-1/3}$ MeV region is a superposition of the 0^+ and 2^+ modes.

Experiments on the inelastic scattering of electrons and protons on various target nuclei [1] have shown a giant resonance with energy around $62\,A^{-1/3}$ MeV. Angular distributions were consistent with either a 0^+ or a 2^+ assignment. The form factors of the two multipolarities were found to differ from each other when the beam was polarized. Therefore new experiments have been performed on Ni^{58} using a polarized proton beam [2].

In order to identify this giant resonance we present calculations of the collective 0^+ and 2^+ vibrations in Ni^{58} and compare them with experiment.

The excitations of a nucleus can be described within the shell model taking into account one particle – one hole excitations. We restricted our configuration space to the levels shown in Table 2. The single particle energies and corresponding wave functions are taken as eigenvalues and eigenvectors of a Woods Saxon potential diagonalized in the space of the harmonic oscillator functions [3].

The potential parameters are chosen to fit the energies of the shells near the Fermi level to the experimental

excitation energies of the neighbouring nuclei. The parameters defined as in Ref. 3 are given in Table 1. Table 2 shows the single particle energies.

The energies of the collective modes were obtained with the RPA method. To simplify the calculations we reduced the problem to that of a closed shell nucleus by interchanging the neutron $2\,p_{3/2}$ and $2\,p_{1/2}$ level. A zero range residual interaction was used

$$V(1, 2) = V_0\,\delta(\mathbf{r}_1 - \mathbf{r}_2)$$

$$\cdot\,(F_0 + F_0'\,\tau_1\tau_2 + G_0\,\sigma_1\sigma_2 + G_0'\,\sigma_1\sigma_2\,\tau_1\tau_2)$$

Table 1. Parameters of the Woods Saxon potential

	V_0 [MeV]	R_0 [fm]	a_0 [fm]	λ	R_{ls} [fm]	a_{ls} [fm]	R_c [fm]
Protons	−48.56	5.27	0.660	24.5	4.80	0.5	4.9
Neutrons	−49.16	4.94	0.535	28.1	4.84	0.5	0.0

Table 2. Single particle energies for Ni^{58}

Ni^{58}	Proton energy [MeV]	Neutron energy [MeV]
$2\,f_{7/2}$	9.435	5.169
$1\,h_{11/2}$	9.088	4.233
$1\,g_{7/2}$	7.512	2.699
$2\,d_{3/2}$	5.803	1.481
$3\,s_{1/2}$	4.848	−0.080
$2\,d_{5/2}$	4.282	−0.983
$1\,g_{9/2}$	0.301	−5.738
$1\,f_{5/2}$	−2.497	−8.659
$2\,p_{1/2}$	−2.917	−8.189
$2\,p_{3/2}$	−3.417	−10.601
$1\,f_{7/2}$	−8.177	−15.498
$2\,s_{1/2}$	−12.192	−20.486
$1\,d_{3/2}$	−12.553	−21.171
$1\,d_{5/2}$	−16.058	−24.593
$1\,p_{1/2}$	−21.669	−31.586
$1\,p_{3/2}$	−23.261	−33.052

Table 3. Parameters of the residual interaction in Ni⁵⁸

V_0 [MeV fm³]	F_p^{in}	F_p^{ex}	F_n^{in}	F_n^{ex}	G_p	G_n
380	0.65	−1.39	−0.85	−2.29	1.3	−0.3

Table 4. Energy, width and reduced transition probability of the 0⁺ and 2⁺ isoscalar and isovector modes in Ni⁵⁸

L^π	T	$\hbar\omega$ [MeV]	Γ [MeV]	$B(EL)\uparrow$ [e^2 fm⁴]
0⁺	0	17.0	0.72	932
	0	18.0	0.32	49
	1	22.0	1.00	50
	1	28.3	4.70	220
2⁺	0	14.6	3.19	1067
	0	14.9	0.30	81
	1	24.5	4.28	121
	1	26.2	0.87	51

where the parameters V_0, F_0, F_0', G_0 and G_0' were fitted to the experimental energies and transition probabilities of the lowest 2⁺ and 3⁺ modes and the 1⁻ giant resonance. As the Migdal force should be universal we took the parameters which are different inside and outside the nucleus close to the Pb²⁰⁸ values [4]. These are shown in Table 3, defined as in Ref. 3.

The resulting excitation energies of the isoscalar 0⁺ and 2⁺ modes can be approximated by $65\,A^{-1/3}$ MeV and $56\,A^{-1/3}$ MeV respectively, which is consistent with calculations on other nuclei [4].

The width of these collective modes results mainly from their coupling to single particle continuum states. It can be expressed by [5]

$$\Gamma = \frac{2\pi}{\Delta E}\sum_i |(\hbar\omega - \varepsilon_i)X_i|^2$$

where the sum runs over all continuum states of the chosen configuration space; ε_i is the single particle energy and ΔE the energy range of continuum states around the energy $\hbar\omega$ of the collective mode. X_i is the RPA amplitude.

Table 4 shows energy, width and reduced transition probability of the collective vibrations.

The collectivity of the excitations is expressed by the fraction of the corresponding sum rules which these modes exhaust. The isoscalar monopole and quadrupole sum rules are given by [6]

$$\sum_f \hbar\omega_f\, \bar{B}(EL)_f\uparrow = \frac{(2L+1)^2\,\hbar^2\,AR^2}{(L+2)\,\pi\,m}; \qquad L = 0, 2.$$

The sum runs over all final states that can be excited by the multipole operator; A is the mass number, R the rms radius of the charge distribution of the nucleus and

$$\bar{B}(EL)_f\uparrow = \left| \left\langle L_f^+ \left\| \sum_{i=1}^A r_i^2 Y_L^{(i)} \right\| 0^+ \right\rangle \right|^2$$

is a modified reduced transition probability, the sum including neutron states.

Summing over all isoscalar excitations with energy up to 24 MeV, one finds that most of the sum rule is exhausted: 75.0 % for $L=0$ and 90.5 % for $L=2$.

The scattering experiments [2] show a peak which extends from 13 to 18 MeV. The calculated energies and widths of both multipole resonances are thus compatible with the experimental data.

Satchler [2] carried out a DWBA calculation of the differential cross section for the excitation of the monopole as well as the quadrupole mode. For the normalization of the curves he assumed the maximum transition strength allowed by the sum rules.

The differential cross section was measured in the energy region 14.6 − 16.7 MeV. Including the 0⁺ mode at 17.0 MeV we find in this region only 49.6 and 36.8 % of the 0⁺ and 2⁺ sum rules respectively. As the 2⁺ mode at 14.6 MeV is lying at the edge of the considered region and has a large width we took only
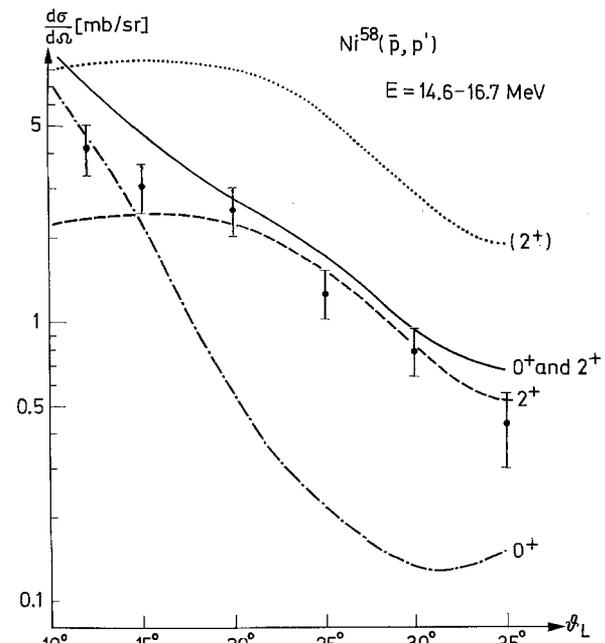
Fig. 1. Cross section for the giant 0⁺ and 2⁺ resonances; the data as well as the relative values of the theoretical predictions are taken from Ref. 2. The dotted DWBA curve labelled with (2⁺) is normalized as done in Ref. 2 and is shown for comparison. The full line is a superposition of the 0⁺ and 2⁺ curves with the proper normalization

half of its strength when computing the 2^+ sum rule fraction.

Fig. 1 shows the renormalized DWBA curves compared with the experimental cross section. The 2^+ curve is in good agreement with the experiment for larger angles while the behavior at small angles is better described by the 0^+ mode. The full line denotes the superposition of the 0^+ and 2^+ cross sections and is in fair agreement with the experimental data in the total angular range considered. If we take the energy region from 12.7–23.7 MeV we find a similar result.

Thus is strong evidence that the isoscalar giant resonance excited in proton scattering experiments is a superposition of a monopole and a quadrupole mode.

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