

The inadequacy of nontransitive solutions to paradox

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(Ripley, 2012) has argued that a nontransitive substructural logic (NT) provides both a solution to semantic paradoxes and preserves full classical logic. Here we argue that NT fulfils these goals only inadequately.

We distinguish between weak and strong inconsistency: A language is weakly inconsistent for some formula ϕ iff it proves both $\vdash \phi$ and $\phi \vdash$. It is strongly inconsistent iff it proves the empty sequent. Although NT is strongly consistent, it remains weakly inconsistent. For even without Cut, NT derives both $\vdash T^\top \lambda^\top$ and $T^\top \lambda^\top \vdash$ for the Liar sentence λ .

According to Ripley, the characteristic of any paradoxical sentence ψ is that both $\vdash \psi$ and $\psi \vdash$ are provable. However, one can show that NT is weakly inconsistent for many central theorems of classical logic as well; including the law of noncontradiction, excluded middle and identity. This is straightforward in first-order logic when these principles are understood in terms of the Truth-predicate. With second-order logic, this can be extended to hold of these theorems understood with arbitrary predicates.

There are two problems. First, NT fails to provide an adequate distinction between paradoxical and nonparadoxical sentences. For classical theorems bear the characteristic of paradoxes. Second, it casts doubt on the classicality of NT. Intuitively, it is not sufficient to merely prove all theorems of classical logic. For even an inconsistent system with explosion will do this. One would further have to ensure that the system does not prove anything weakly inconsistent

with these theorems—again, NT fails to do so.

References

Ripley, D. (2012) Conservatively extending classical logic with transparent truth. *Review of Symbolic Logic*. 5 (2):354-378.