How (Not) To Argue Against Vague Object

Abstract: In a series of papers, Elizabeth Barnes and Robert Williams have developed a theory of metaphysical indeterminacy for a nonrepresentational understanding of vagueness. According to this theory, metaphysical vagueness is explained as referential indeterminacy with metaphysical source. In particular, an object, $O$, is vague iff “$O$” is referentially indeterminate and the source of indeterminacy is the world itself, not representational or epistemic. An example could be helpful in illustrating the idea.

1 Introduction

In a series of papers, Elizabeth Barnes and Robert Williams have developed a theory of metaphysical indeterminacy for a nonrepresentational understanding of vagueness. According to this theory, metaphysical vagueness is explained as referential indeterminacy with metaphysical source. In particular, an object, $O$, is vague iff “$O$” is referentially indeterminate and the source of indeterminacy is the world itself, not representational or epistemic. An example could be helpful in illustrating the idea.
Commonsensically, Mount Kilimanjaro, $K$, is considered a vague object, in the sense that for some object, $e$, it is indeterminate whether $e$ belongs to $K$. According to BW theory, however, the vagueness of $K$ is explained in terms of referential indeterminacy with metaphysical source; the indeterminacy of part-hood relation between $e$ and $K$ is explained away as “$K$” is referentially indeterminate between $K+$ and $K-$, which are exactly like $K$ except $e$ determinately belongs to $K+$ and is determinately disjointed from $K-$. Furthermore, the source of this indeterminacy is the world itself. Formally, they are committed to the following statements:

1. $D(K = K- \lor K = K+)$
2. $\neg D(K = K-) \land \neg D(K = K+)$
3. $I(K = K-) \land I(K = K+)$

The main attraction of BW characterization of vague object is that it can secure the advantage of supervaluationism in blocking Evans’ *reductio* argument against indeterminate identity. That is, when $b$ is a referentially indeterminate object, the derivation from $I(a = b)$ to $[\lambda x. I(x = a)]b$ is illegitimate. Therefore, Evans’ argument is blocked in the first step.

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4 Here, $e$ could be a simple object (e.g. a partless elementary particles of physics) or an object sufficiently simpler than $K$ (e.g. an electron).

5 An illustration of how the world can be the source of referential indeterminacy can be found in Barnes and Williams (2009)

   “Suppose it were metaphysically unsettled where an object, Table, was located. Suppose that is either located between you and Wardrobe, or is in another room entirely: but there is no fact of the matter which one of these regions it is in. Now introduce the name ‘Front’ to refer to whichever individual is directly in front of you. ‘Front’ will be referentially indeterminate between Table and Wardrobe, and ‘Table = Front’ will be vague. But the source of this referential indeterminacy is not any lack of semantic conventions on your part, but rather the metaphysical indeterminacy in the location of Table” (p. 181).

6 In agreement with BW, determinacy and indeterminacy are understood as modal operators. “$D$” and “$I$” stand for operators “it is determinate that” and “it is indeterminate whether”. Semantical behaviors of $D$ and $I$ are analogues of necessity and contingency operators in alethic modal semantics, respectively. In particular, $IA \iff \neg DA \land \neg D\neg A$. For more details see Parsons and Woodruff (1995), Barnes and Williams (2011).

7 See Evans (1978).

8 Alternatively, if $a$ is referentially indeterminate vague object, the derivation from $\neg I(a = a)$ to $[\lambda x. I(x = a)]a$ is illegitimate. Therefore, as long as indeterminate identity statements involve at least one referentially indeterminate object, Evans’ argument can be blocked. See Lewis (1988) for the details.
Ken Akiba, however, doubts that BW theory is logically coherent. In his recent paper, Akiba provides two rigorous arguments against their characterization of vague object, claiming that it is *ill-conceived and wrong-headed*. In one argument, he objects that BW characterization of metaphysical indeterminacy ultimately generates indeterminate identity between referentially determinate objects, RDI, to which $\lambda$-abstraction is legitimately applicable. Thus, BW theory ultimately fails to block Evans’ argument, or so Akiba argues. *(E-objection)*

In the other, he provides a formalization of Salmon’s argument, arguing that it is different from Evans’ argument in a crucial way: contrary to Evans’ argument, Salmon’s argument does not use singular terms, but only quantifications and variables. As a result, the ambiguity between the *de re* and *de dicto* readings of indeterminacy, which exists in Evans’ argument, does not occur in Salmon’s argument. In fact, the argument must be understood *de re*. Consequently, BW cannot appeal to referential indeterminacy to block the argument. *(S-objection)*

This paper refutes both arguments. In the next section, I argue that not all cases of vague object characterized in BW theory generate RDI, and hence BW theory is a viable account for vague objects that do not imply RDI. In the last section, I discuss S-objection and argue that contrary to what Akiba claims, validity of his formal proof depends on application of $\lambda$-abstraction. As a result, proponents of BW theory still can block the argument by appealing to referential indeterminacy of vague objects.

## 2 E-objection

To demonstrate any vague object characterized in BW theory ultimately generates RDI, and so is open to Evans’ argument, Akiba chooses an example from Williams (2008a) and shows that it implies RDI. Let us consider Williams’ example.

**Amoebas Example.** Sue, a particular amoeba, splits into two amoebas, Sally and Sandy. One of them is a surviving amoeba and the other is a newly minted amoeba. After the fission, Sally wanders off to the East and Sandy to the West. Williams argues that while Sue and Sandy are vague objects, they do not

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11 See (Williams 2008a, 151–2).
12 Also, note that Sue persists as the surviving amoeba.
generate RDI, so the example is immune from Evans’ argument. Considers the following two scenarios:

1. We can rigidly denote $Sue_1^{13}$ via the definite description “the surviving amoeba”. That is, $Sue_1$ is a vague object which is determinately referred to by “$Sue_1$”. Moreover, $Sandy_1$ in a vague object which is indeterminately referred by “$Sandy_1$”.

2. Alternatively, we can rigidly denote $Sandy_2$ via the definite description “the amoeba that wanders off to the West”. That is, $Sandy_2$ is a vague object which is determinately referred to by “$Sandy_2$”. However, $Sue_2$ in a vague object which is indeterminately referred by “$Sue_2$”.

According to Williams, in both cases, it is indeterminate whether $Sue$ is identical with $Sandy$, but Evans’ argument does not apply. To see this, consider the following true statements:

(1) $I(Sue_1 = Sandy_1)$
(2) $\neg I(Sandy_1 = Sandy_1)$
(3) $I(Sue_2 = Sandy_2)$
(4) $\neg I(Sandy_2 = Sandy_2)$

Since $Sue_1$ and $Sandy_2$ are referentially determinate, $\lambda$-abstraction is legitimately applicable on (1) and (4);

(1*) $[\lambda x.I(x = Sandy_1)] Sue_1$
(4*) $\neg[\lambda x.I(x = Sandy_2)] Sandy_2$

Nonetheless, $Sandy_1$ and $Sue_2$ are referentially indeterminate, i.e. “$Sandy_1$” and “$Sue_2$” do not refer to single objects. Thus, $\lambda$-abstraction is not applicable on (2) or (3), and hence Evans’ argument may not go through.

However, contrary to what Williams claims, Akiba shows that Amoebas example does generate RDI. Consider the following identity statement

(5) $I(Sue_1 = Sandy_2)$

If (1)–(4) are true, (5) is true. Both $Sue_1$ and $Sandy_2$ are referentially determinate, so (5*) is derivable.

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13 Indices represent the numbers of the cases.
14 See Lewis (1988).
\[(5^*) \quad [\lambda x. I (x = \text{Sandy}_2)] \text{Sue}_1\]

Now, we can use Evans' argument to derive $\neg I (\text{Sue}_1 = \text{Sandy}_2)$, contradicting (5).\(^{15}\) Akiba argues that every instance of vague object as characterized in BW theory, like Amoebas example, generates RDI. Consequently, BW theory completely fails to provide a proper characterization of vague object. He writes

"If Williams is to succeed in his defense of vague objects along his original lines, he must come up with another example which has the following feature: it generates true indeterminate identity statements involving referential indeterminacy that results from the existence of vague objects, but it cannot generate true indeterminate identity statements that only involve determinate references to those vague objects. We have no idea how such an example is possible." (2015, 8–9).

In sum, Akiba’s failure to see how an example of a vague object without generating RDI in BW theory is possible leads him to conclude that his objection destroys Williams’ defense of vague objects. I call it “immodest conclusion”.\(^{16}\)

In the rest of this section, I argue that while Akiba is right that Amoebas example does generate RDI and hence is open to Evans’ argument, his immodest conclusion is untenable. In particular, I present two examples of vague objects characterized in BW theory without generating RDI, which refute his conclusion.

### 2.1 Against E-objection

Providing an example of a vague object without facing E-objection requires to understand which aspect of Amoebas example entails RDI. That is to say, we need to understand which aspect of the example allows to fix reference relations rigidly. It is not hard to see that for both Sue and Sandy there are definite descriptions that allow to designate them rigidly. Hence, an example of vague object that does not generate RDI should not be committed to definite descriptions that allow to rigidly denote indeterminately identical objects.

Recall Kilimanjaro example. The following statements are true:

1. $I(K = K^+)$
2. $I(K = K^-)$

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\(^{15}\) Note that Williams is committed to (4*) and the modal framework S5 as the logic of in/determinacy operators.

\(^{16}\) Later in this section, I argue for a modest conclusion of E-objection.
However, while there are definite descriptions that rigidly designate \( K^+ \) and \( K^- \), there is no definite description that rigidly denotes \( K \). As a result, \( \lambda \)-abstraction is not applicable to the above statements. So Evens’ argument cannot be applied to this example.

For the second example, I show that even Amoebas example can be modified in order to not generate RDI. The modification only requires replacing “Sally wanders off to the East and Sandy to the West” with the following:

After the fission, Sally and Sandy wander off to different directions, i.e. one goes to East and the other goes to West.

In the modified version of the example still the following statements are held

a) \( I (\text{Sandy} = \text{Sue}) \)

b) \( \sim I (\text{Sandy} = \text{Sandy}) \)

Nonetheless, the definite description “the amoeba that wanders off to the West” does not rigidly refer to \( \text{Sandy} \), since \( \text{Sandy} \) might be the amoeba that wanders off to the West as well. Consequently, \( \lambda \)-abstraction cannot be legitimately applied to (a) or (b). Hence the modified version of Amoebas example is immune from Evans’ argument.

\[ I(K = K)^{17} \]

\[ \sim I(K = K) \]

\[ I(K* = K) \]

\[ I(K* = K^+) \]

\[ I(K* = K^-) \]

\[ \sim I(K* = K*) \]

Similar to \( K \), there is no definite description which could rigidly designate \( K^* \). As a result, \( \lambda \)-abstraction is not applicable on above statements either.

\[ I(K* = K) \]

\[ I(K* = K+) \]

\[ I(K* = K^-) \]

\[ \sim I(K* = K*) \]

\[ \sim I(K* = K) \]

\[ I(K* = K^+) \]

\[ I(K* = K^-) \]

\[ \sim I(K* = K*) \]

\[ I(K* = K) \]

\[ I(K* = K^+) \]

\[ I(K* = K^-) \]

\[ \sim I(K* = K*) \]
The two examples show that it is perfectly plausible to provide examples of vague objects characterized in BW theory without generating RDI. Therefore, Akiba’s immodest conclusion is untenable: E-objection neither destroys BW theory nor does it show that their project is ill-conceived and wrong-headed.

Yet, a more modest conclusion can be drawn from E-objection. Indeed the objection leads us to a better understanding of limitations of BW theory. Namely, BW theory is unable to characterize all types of vague objects. Cases of vague objects that generate RDI are not coherently representable in BW theory. This is a disadvantage of this theory, particularly, when it is compared to other theories of ontic vagueness. Parsons (2000), Wilson (2013) and Abasnezhad and Hosseini (2014) provide theories of ontic vagueness and they block Evans’ argument without appealing to referential indeterminacy.20 This means, they can characterize cases of vague objects that generates RDI without facing Evans’ argument. Therefore, in this respect, they are preferable over BW theory.

3 S-objection

In his second objection, Akiba appeals to Salmon’s argument, claiming that it is different from Evans’ argument in a crucial way:21

“Salmon uses quantification and variables where Evans uses singular terms. Thus, the ambiguity between the de re and de dicto readings of ‘it is indeterminate whether a is identical with b,’ which exists in the original Evans’ argument, does not exist in the Salmon argument; the argument must be understood de re.” (2015, 9).

As a result, BW solution for Evans’ argument does not work for Salmon’s argument, or so Akiba argues. Let us start with a brief description of Salmon’s argument.

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20 Parsons (2000) and Abasnezhad and Hosseini (2014) both reject legitimacy of the application of λ-abstraction, without appealing to referential indeterminacy. Wilson (2013), on the other hand, argues that committing to vague objects does not imply a commitment to indeterminate identity, so Evans’ argument is not an issue at all.

21 As it is argued in the last section, BW theory can explain cases of vague objects that do not generate RDI, without facing evens’ argument. Accordingly, in this section, I restrict my discussion to these cases only and show that BW characterization of them is immune from Akiba’s second objection as well.
“Suppose that there is a pair of entities x and y...such that it is vague...whether they are one and the very same thing. Then this pair (x, y) is quite definitely not the same pair as (x, x), since it is determinately true that x is one and the very same thing as itself. It follows that x and y must be distinct.” (1981, 243)

Akiba provides the following formalization of the argument

1. \( \forall x (x = x) \)  \hspace{1cm} \text{Law of identity}
2. \( \forall x \forall y [(x = y) \rightarrow \forall P (Px \rightarrow Py)] \)  \hspace{1cm} \text{Leibnitz’s law}
3. \( \neg D(x = y) \land \neg D(s = y) \)  \hspace{1cm} \text{Reductio premise}^{22}
4. \( \neg D(x = y) \)  \hspace{1cm} \text{From (3)}
5. \( (x = x) \)  \hspace{1cm} \text{Instantiation of (1)}
6. \( D(x = x) \)  \hspace{1cm} \text{From (5) by rule of Determination}^{23}
7. \( (x = y) \rightarrow [D(x = x) \rightarrow D(x = y)] \)  \hspace{1cm} \text{Instantiation of (1)}
8. \( \neg (x = y) \)  \hspace{1cm} \text{From (4), (6), and (7)}^{24}

He argues that the argument will remain valid when variables are replaced with singular terms. That is to say, for every instance of a vague object characterized in BW, the argument proves that it generates contradiction. Moreover, Akiba thinks that since there is no application of \( \lambda \)-abstraction in the argument, appealing to referential indeterminacy does not help in blocking the argument. Therefore, referential indeterminacy does not play any significant role in a proper defense of vague objects.

In the rest of this paper, I show that, contrary to Akiba’s claim, validity of the argument actually depends on application of \( \lambda \)-abstraction. In particular, the correct instantiation of (2) requires application of \( \lambda \)-abstraction. Then, I consider two possible ways of completing the argument with \( \lambda \)-abstraction and argue that proponent of BW theory can reject both.

### 3.1 Against S-objection

Consider the derivation from (2) to (7). Note that \( P \) in the second premise is a monadic predicate, however, it is instantiated with a relation (i.e. identity) within the scope of a sentential operator (i.e. \( D \)), in (7). Obviously a direct

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22 Recall that IA iff \( \neg D \neg A \land \neg D \neg A \).
23 It is analogue of Necessitation in alethic modal logic.
24 Again, it is presupposed that within S5 from (8) we can derive \( D \neg (x = y) \) contradicting (3).
instantiation from (2) to (7) is not permitted. That is, the argument requires more details to become valid. In fact, the correct instantiation of \( P \), which is relevant here, is the monadic predicate \( \lambda z. D(z = x) \). This is, a direct consequence (2) is not (7) but

\[ (x = y) \to (\lambda z. D(z = x))(x \to \lambda z. D(z = x))(y) \]

So contrary to Akiba’s claim, \( \lambda \)-abstraction is required for the argument to be valid. Nevertheless, contrary to (4), (6) and (7) in the original proof, (4), (6) and (7’) do not directly entail \( \neg(x = y) \). Thus, still further modifications are needed to complete the argument. Using (7’), the proof can be completed in two general ways.

**First version**

1. \( (x = y) \to (\lambda z. D(z = x))(x \to \lambda z. D(z = x))(y) \)  
   **Instantiation of (2); \([P/\lambda z. \Delta(z = x)]\)**
2. \( (x = y) \to [D(x = x) \to D(x = y)] \)  
   **From (7’)**
3. \( \neg(x = y) \)  
   **From (4), (6), and (7)**

**Second version**

4. \( \neg D(x = y) \)  
   **From (3)**
5. \( \neg [\lambda z. D(z = x)]y \)  
   **(4) \( \lambda \)-abstraction**
6. \( (x = x) \)  
   **Instantiation of (1)**
7. \( D(x = x) \)  
   **From 5 By rule of Necessitation**
8. \( \lambda z. D(z = x) x \)  
   **(6) \( \lambda \)-abstraction**
9. \( (x = y) \to (\lambda z. D(z = x))(x \to \lambda z. D(z = x))(y) \)  
   **Instantiation of (2); \([P/\lambda z. \Delta(z = x)]\)**
10. \( \neg(x = y) \)  
    **From (4’), (6’), and (7’)**

I argue that there are good reasons for proponents of BW theory to reject both arguments. This is because the move from (2) to (7’) presupposes that every object has the property of being determinately self-identical. However, this presupposition is unacceptable in BW theory when variables are substituted with proper names. Recall that vague objects, as characterized in BW theory, are

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25 It is worth noting that Evans was careful not to make Akiba’s mistake. In fact, if the direct instantiation from (2) to (7) was legitimate, Evans also would derive \( (a = b) \to [I(a = b) \to I(a = a)] \) from Leibnitz’s law, which was all he needed to finish his argument. That is, his discussion of property abstraction would be completely redundant.
referentially indeterminate due to their ontological status. As a result, for a vague object \(a\), it is determinate that \((a = a)\), but it is not the case that \(a\) has the property of “being determinately identical with \(a\),” \([\lambda x . D(x = a)]\). In fact, this is uncontroversial among BW and Akiba. For instance, in Amoebas example, Akiba agrees that \(Sandy_2\) does not have the property of “being determinately identical with \(Sandy_2\).”\(^{26}\) Now, let me explain how this point can be used to block both versions of the argument.

**Against the first version**

Instantiate \(x\) and \(y\) in the first argument with \(a\) and \(b\), respectively, and let \(a\) be a vague object. As explained above, in BW theory \(D(a = a)\) is true and \([\lambda z . D(z = a)]a\) is false. As a result, \([D(a = a) \rightarrow D(a = b)]\) does not follow from \((\lambda z . D(z = a)]a \rightarrow [\lambda x . D(x = a)]b\). That is, the derivation from (7) to (7) is illegitimate in BW theory. Therefore, proponents of BW can reject the validity of the first argument.\(^{27}\)

**Against the second version**

Consider the second argument in which \(x\) and \(y\) are instantiated by \(a\) and \(b\), respectively, and let \(a\) be a vague object. Proponents of BW theory can reject the derivation from (6) to (6'). As explained, \(D(a = a)\) is true and \([\lambda a . D(a = z)]a\) is false, which implies the derivation from \(D(a = a)\) to \([\lambda a . D(a = z)]a\) is illegitimate.\(^{28}\) Therefore, the second argument can be blocked as well.

To conclude: I concur with Akiba that Amoeba example generates RDI to which makes it open to Evans’ argument. However, he is wrong in claiming that ‘this destroys Williams’s defense of vague objects’; since there are cases of vague objects characterised in BW theory which do not imply RDI. Furthermore, BW theory can also block Akiba’s formalization of Salmon’s argument by appealing to referential indeterminacy, since validity of the argument depends on application of \(\lambda\)-abstraction. Therefore, BW theory is not a complete failure, rather its explanatory power is restricted to the cases of vague objects that do not generate RDI.

\(^{26}\) Similar to the property of “being indeterminately identical to \(Sandy_2\).”

\(^{27}\) Note that according to BW, possible worlds, which are called Precisification, are determinate. However, even if \([\lambda z . D(a = z)]a\) would be considered neither true nor false, still the derivation from (7) to (7) could be rejected in an proper three-valued semantics, e.g. Strong Kleene semantics.

\(^{28}\) Alternatively, when \(b\) is vague, the derivation from (4) to (4') is illegitimate.
References


