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ХЕРЦЕГ-НОВИ, 25—31. 8. 1968
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CONTENT — СОДЕРЖАНИЕ

	page стр.
Preface — Предусловие	7—8
Part I часть: General part — Общая часть	9—26
Kurepa Đuro: Opening welcome speech	11
Kurepa Đuro: Реч на открытии Симпозија	12
Kurepa Đuro: Closing speech	15
Kurepa Đuro: Говор на заключительном заседании Симпозија	16
List of members — Список участников	17—21
Program — Программа	23—25
Part II часть: Scientific contributions — Научные статьи	27—339
1. Aarts J. M.: Every Metric Compactification is a Wallman-type Compactification	29—34
2. Аднаджевич Д.: Отношения между некоторыми размерностными функциями топологических пространств	35—37
3. Александров П. С.: О некоторых результатах московских математиков по общей топологии.	38—42
4. Alić Mladen: On Hardy—Luxemburg—Zaanen spaces	43
5. Alo R. A. and Shapiro H. L.: Extensions of Uniformities	44
6. Alo A. Richard: Uniformities and Embeddings	45—59
7. Anderson R. D.: Apparent Boundaries of the Hilbert Cube	60—66
8. Антоновский М.: Структуры, связанные с обобщенными метриками	67—70
9. Bauer F. W.: Cohomology Functors	71—76
10. Behzad M. and Mahmoodian A. A.: On Topological Invariants of the Cartesian Product of Graphs	77
11. Benado S Michael: A propos du problème inverse de la théorie des continus géométriques	78—79
12. Berikachvili N.: Sur les différentielles d'un certain type de suite spectrale	80
13. Bhargava T. N. and Edelman Joel E.: Some Topological Properties of Covered Spaces and Digraphs	81—86
14. Binz E.: Convergence Spaces and Convergence Function Algebras	87—92
15. Boltjanski V. G.: Some Results and Some Problems of the Boolean Algebras Theory	93—97
16. Borsuk Karol: Concerning the Notion of the Shape of Compacts	98—104
17. Bushaw D.: The Scale of a Uniform Space	105—108
18. Chogochvili G.: Sur les groupes d'homologie de l'espace, basés sur une catégorie donnée de complexes	109
19. Ciampa Salvatore: Full Rings of Continuous Real Functions	110—112
20. Császár Akos: Double Compactification and Wallman Compactification	113—117
Császár Klara: New Results on Separation Axioms	118—120
21. Ćirić Ljubomir: A Generalization of Brodski's Theorem	121
22. Dacić Rade: Distributivity and relative complements in the lattice of topologies	122—123
23. Delić Krešimir and Mardesić Sibe: A Necessary and Sufficient Condition for the n -Dimensionality of Inverse Limits	124—129

	page стр.
24. Dojčinov D.: A Generalization of Tychonoff's Theorem on Compactness	130
25. Dolcher Mario: Relation Between Sequence Convergences and Topologies	131
26. Dold Albrecht: Extension Problems in General Topology and Partitions of Unity	132—136
27. Doyle P. H. and Guinn T.: Some Approximation and Density Theorems	137—139
28. Ďaja Časlav: Einige Eigenschaften der Familie dynamischer Systeme in metri- schen Räumen	140—145
29. van Emde Boas P.: Minimally Generated Topologies	146—152
30. Федорчук В. В.: Бикомпакты с несовпадающими размерностями	153
31. Fleischman W. M.: On Fundamental Open Coverings	154—155
32. Fritsch Rudolf: On Subdivision of Semisimplicial Sets	156—163
33. Frolík Zděnek: Fixed Points of Maps of Extremely Disconnected Spaces	164—167
34. Frolík Zděnek: On the Suslin-graph Theorem	168
35. Gähler Siegfried: Über Verallgemeinerungen von Abstandsräumen	169
36. Gähler Werner: Topologische Untersuchungen in der Flächentheorie	170
37. de Groot J.: Connectedly Generated Spaces	171—175
38. de Groot J., Jensen G. A., Verbeek A.: Superextensions	176—178
39. Hajnal A. and Juhász I.: Discrete Subspaces and de Groot's Conjecture About the Number of Open Subsets	179
40. Hejzman Jan: Partial Products of Uniform Spaces	180—182
41. Henderson W. David: Infinite-dimensional Manifolds	183—185
42. Henderson W. David: Negligible Subsets of Infinite-dimensional Manifolds	186
43. Herrlich Horst: Topological Coreflections	187—188
44. Hofmann Karl Heinrich: The Cohomology Ring of a Compact Abelian Group	189—192
45. Hušek Miroslav: K-complete Proximity Spaces	193—194
46. Inassaridzé H.: Sur l'homologie discrète exacte et la cohomologie compacte exacte	195
47. Isler Romano: On a Problem Concerning Sequential Spaces	196—199
48. Ivanov A. A.: Bitopological Spaces	200—205
49. Javor P.: Continuous Solutions of the Functional Equation $f(x+yf(x)) =$ $=f(x)f(y)$	206—209
50. Juhász I.: On Two Properties Between T_2 and T_3	210
51. Kamps Klaus Heiner: Fibrations and Cofibrations in Categories with Homo- topy System	211—218
52. Katětov M.: On Filters and the Descriptive Theory of Sets and Mappings	219
53. Kečkić Jovan D.: On the Fixed-point Theorem and the Convergence of Cer- tain Sequences	220—221
54. Kodama Yukihiko: On Dimensionally Full-valued Spaces	222—225
55. Koutnik V.: On Convergence in Closure Spaces	226—230
56. Kraus Jürgens F.: Zur Homotopietheorie reduzierter und symmetrischer Pro- dukträume	231—232
57. Krikelis P. B.: Certain relations between generalised Topology and universal Algebra	233—238
58. Kurepa Đuro: Around the General Suslin Problem	239—245
60. Курена Дж.: Некоторые функции на топологических структурах, графах...	246—248
61. Kurke H.: Tensorprodukt von Garben in der kategorietheoretischen Fassung	249
62. Кузминов В.: О произведении паракомпакта на компакт	250—251
63. Mamuzić Z. P.: A Note on the Principle of Prolongation of Identities in Case of Proximity Spaces	252—255

	page стр.
64. Mardešić Sibe: A Locally Connected Continuum which contains no Proper Locally Connected Subcontinuum	256
65. Marjanović Milosav: On topological isometries	257
66. Мишич Миодраг: Две теоремы о многозначных отображениях бикомпактных топологических пространств	258—259
67. Munkholm H. T.: Is there a Borsuk-Ulam theorem for any proper action of finite group on a sphere?	260
68. Nagami Keio: A Note on the Normality of Inverse Limits and Products	261—264
69. Negreponis S.: The Stone Space of the Saturated Boolean Algebras	265—268
70. Novák J.: On Some Topological Spaces Represented by Systems of Sets	269—280
71. Нурекенов Т. К.: Некоторые приложения множества функций типа ограниченных вариаций к решениям интегральных и интегродифференциальных уравнений топологическим методом	271—276
72. Persinger C. A.: Some Results on n -Books in E^3	277—280
73. Произолов В. В.: О сетевидных пространствах	281—284
74. Rehmann Celiar Silva: Sur l'invariance de la connexité	285—290
75. Сарымсаков Т. А.: Применение топологических полуполей к теории вероятностей	291
76. Schaerf Henry M.: Topological Cardinality Theorems	292
77. Schaerf H. M.: Topological relativisation and the continuity of measurable set mappings	293
78. Segal Jack: On Isomorphisms of Complexes	294—295
79. Скляренко Е. Г.: Теория гомологии, удовлетворяющая аксиоме точности.	296
80. Stanković B.: Some Theorems on Fixed Points and Their Applications	297—301
81. Stanojević Momir: A Note on Convergence of Star-shaped Sets	302—304
82. Шведов И.: О зацеплении в топологических пространствах	305—307
83. Тагамлицкий Я.: О топологической индукции	308—309
84. Тихомиров В. М. и Тумаркин Л. А.: О поперечниках компактов	310—316
85. Volčić Alessio: On a Topology Generated by a Subcollection of the Collection of Closed Sets	317—325
86. Vrabec Jože: A Note on Projective Sheaves of Modules	326—330
87. Waldhausen Friedhelm: On the Determination of Some 3-manifolds by their Fundamental Groups Alone	331—332
88. Зайцев В.: Проекционные спектры топологических пространств	333
89. Zarelua A.: On Finite Groups of Transformations	334—339
90. Жижченко А. В.: К теории алгебраических многообразий с произвольными особенностями	340
Парт III часть: Problems — Нерешенные задачи	341—353
Content — Содержание	354—356

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ON SUBDIVISION OF SEMISIMPLICIAL SETS

§ 1 Introduction

The regular subdivision $\Delta' X$ of a semisimplicial set X can be easily defined in a purely combinatorial way; this has been done by Barratt [1] and Kan [7]. To investigate the geometrical meaning of this let us first consider especially the two degeneracy maps from a 2-simplex to an 1-simplex and their subdivisions. We find out that there can't be a natural homeomorphism between the geometric realizations of a semisimplicial p -simplex and its regular subdivision, although the underlying spaces are nothing but geometric p -simplices. As the category \underline{S} of semisimplicial sets and semisimplicial maps is the completion of the category of semisimplicial simplices and semisimplicial maps with respect to colimits, such a natural homeomorphism would be necessary and sufficient for the existence of a natural homeomorphism between $|X|$ and $|\Delta' X|$, the geometric realizations of a semisimplicial set X and its regular subdivision $\Delta' X$. So we have

Theorem 1. *There exists no natural equivalence between the functors $|?|: \underline{S} \rightarrow \underline{CW}$ and $|\Delta' ?|: \underline{S} \rightarrow \underline{CW}$ ("CW" denotes the category of CW-complexes and continuous maps).*

Now the question arises if there is any homeomorphism between $|X|$ and $|\Delta' X|$. The answer to this question is in my mind far away from being trivial—as many people believed a long time—and is first given in my paper [3] as a special case of a general result on a class of various subdivisions. Subsequent to this Puppe has found an explicit formula, which—as we proved in [4]—gives a homeomorphism in the regular case.

The question mentioned above has also suggested my paper [2] and my aim here is to outline the content of [2] and [3].

2 Standard division functors

Standard division functors are introduced in [2].

Definition 1. *A "standard division functor" is a pair (U, u) consisting of a functor*

$$U: \underline{\Delta} \rightarrow \underline{S}$$

and a family

$$u = (u_p / p \text{ non-negative integer})$$

such that the following conditions are satisfied:

(i) u_p is a homeomorphism $|U[p]| \rightarrow \Delta_p$ for each non-negative integer p

(ii) $|\Delta\beta| \circ u_p = u_q \circ |U\beta|$

for each injective map $\beta: [p] \rightarrow [q]$ of $\underline{\Delta}$. (Here the notation must be explained: $\underline{\Delta}$ denotes the category of non-empty finite ordered sets and weak order preserving maps and $[p]$ the set of the numbers $0, 1, 2, \dots, p$, that means the ordered set of $p+1$ elements, for each non-negative integer p ; the maps of $\underline{\Delta}$ are symbolized by small greek letters; we shall briefly write " $\beta \in \underline{\Delta}$ " to indicate that β is a map of $\underline{\Delta}$. $\Delta: \underline{\Delta} \rightarrow \underline{S}$ means the functor which assigns to each object $[p]$ of $\underline{\Delta}$ the semisimplicial p -simplex $\Delta[p]$ and to each $\beta \in \underline{\Delta}$ the semisimplicial map $\Delta\beta$; it is the simplest example for a standard division functor. Finally Δ_p denotes the geometric p -simplex.)

Given a standard division functor (U, u) one can identify each $|U[p]|$ with Δ_p by means of (i). Then we have two CW-structures on Δ_p , the one is induced by the simplices of $\Delta[p]$, the other by the simplices of $|U[p]|$; the same is to say that the underlying spaces of the CW-complexes $|\Delta[p]|$ and $|U[p]|$ coincide. From this point of view (ii) assures that $|U[p]|$ is a CW-subdivision of $|\Delta[p]|$, that means that each cell of $|U[p]|$ lies in a cell of $|\Delta[p]|$.

Each standard division functor (U, u) can be extended uniquely to a continuous functor from \underline{S} to itself, which we denote—by abuse of notation—also by " U ". Such a so-called "*division functor*" has the following properties:

Proposition 1. U preserves the fundamental group and the homology groups up to natural equivalence.

Proposition 2. U preserves coverings.

A semisimplicial map f is said to be a "*weak homotopy equivalence*" if f induces an isomorphism of the fundamental groups and \tilde{f} , the universal covering of f , induces isomorphisms of the homology groups. This definition is justified by the fact that the geometric realization of a semisimplicial weak homotopy equivalence is indeed a homotopy equivalence. From the propositions 1 and 2 now it follows at once

Proposition 3. U preserves weak homotopy equivalences.

Much deeper is the following result:

Theorem 2. If X is a semisimplicial set, then the CW-complexes $|X|$ and $|UX|$ have the same homotopy type; more precisely: if $[?]: CW \rightarrow CWh$ denotes the projection onto the homotopy category of CW-complexes, then the functors $[U?]$ and $[?]$ are naturally equivalent.

To obtain this result we need an interesting device, which is explained in the following section.

§ 3 Non—degenerate semisimplicial sets

Definition 2. A semisimplicial set X is "non—degenerate" if no non—degenerate simplex of X has a degenerate face. If X and Y are non—degenerate semisimplicial sets, a semisimplicial map $f: X \rightarrow Y$ is "non—degenerate" if f maps non—degenerate simplices of X on non—degenerate simplices of Y .

Non—degenerate semisimplicial sets and non—degenerate semisimplicial maps from a subcategory \underline{P} of \underline{S} and one can prove:

Proposition 4. P is a reflective subcategory of S .

That means that the embedding functor $E: \underline{P} \rightarrow \underline{S}$ is left adjoint to a functor $\underline{S} \rightarrow \underline{P}$, the reflector R . To prove this one has to define to each semisimplicial set \underline{X} a non—degenerate semisimplicial set $R\underline{X}$ and a semisimplicial map $X: R\underline{X} \rightarrow \underline{X}$ such that for each semisimplicial map $f: Y \rightarrow \underline{X}$ with Y non—degenerate there exists a unique non—degenerate semisimplicial map $f': Y \rightarrow R\underline{X}$ with $f = r\underline{X} \circ f'$ ¹⁾.

Having done this, a simple straightforward computation yields

Proposition 5. $r\underline{X}$ induces in a natural way isomorphisms of the fundamental groups and all homology groups, and

Proposition 6. R preserves coverings.

From this two propositions it follows at once

Theorem 3. $r\underline{X}$ is a weak homotopy equivalence ²⁾.

In our context the meaning of the category \underline{P} is due to

Proposition 7. If \underline{X} is a non—degenerate semisimplicial set, then the spaces $|X|$ and $|UX|$ are homeomorphic; more precisely: the functors $|E?|: \underline{P} \rightarrow \underline{CW}$ and $|UE?|: \underline{P} \rightarrow \underline{CW}$ are naturally equivalent.

We omit the proof of this proposition; now theorem 2 is an easy consequence of theorem 3, proposition 3 and proposition 7.

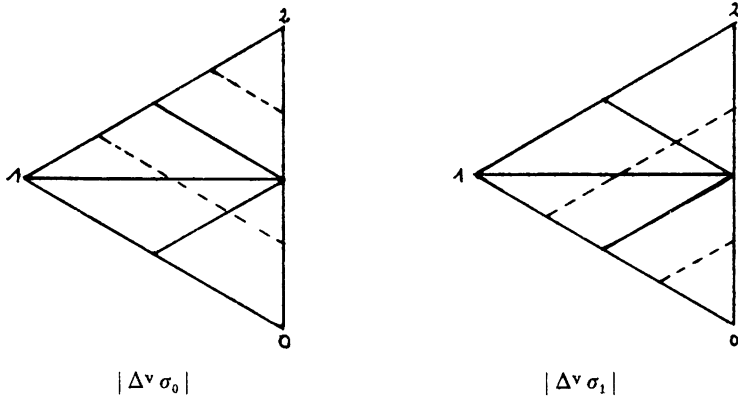
§ 4 Examples

1) The "regular" or "barycentric" subdivision of the semisimplicial simplices induces a standard division functor. It was already described by Kan [7]; he uses the symbol " Δ' " for the functor $\Delta \rightarrow S$, but the symbol " Sd " for the extended division functor, which we according to our conventions denote also by " Δ' "; we mentioned it in § 1.

¹⁾ For this situation the terminology "coreflective" seems to become standard; but obeying the demand for logical consistency we use "reflective" in accordance with the book of Mitchell [9].

²⁾ In some papers Giever [5] and Hu [6] have studied the space $|RX|$; they denoted it by " PX " and called it "geometric realization of X ", but they had not defined the semisimplicial set $R\underline{X}$ explicitly. Then Kodama [8] has constructed the map $|r\underline{X}|$ —he denoted it by " pX "—and a homotopy inverse to it, but by using the fact that $|\Delta'X|$ can be interpreted as CW —subdivision of $|X|$, which was not proved at that time.

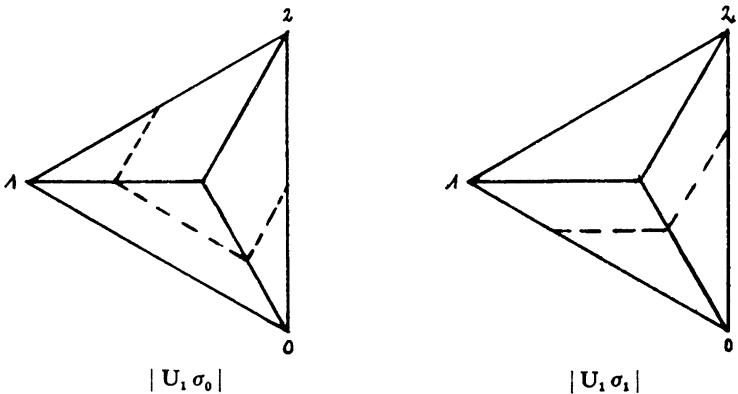
2) The "natural" subdivision. We denote the corresponding functor $\underline{\Delta}'_1 \rightarrow \underline{S}$ by " Δ^\vee ". Its effect on $[2]$ and the two degeneracy maps $\sigma_e: [2] \rightarrow [1]$ ($e = 0, 1$) can be illustrated by the following pictures:



(The fully traced line segments indicate the cell structure of $\underline{\Delta}_2 = |\Delta^\vee[2]|$; $|\Delta^\vee \sigma_0$ resp. $|\Delta^\vee \sigma_1|$ identifies the dotted line segments and just so their parallels to a point.)

In this case the condition (ii) of definition 1 is satisfied for all $\beta \in \underline{\Delta}$, not only for the injective ones. The name "natural" is justified by the fact that for all semisimplicial sets X the CW-complexes $|\Delta^\vee X|$ and $|X|$ are naturally homeomorphic. The disadvantage of this functor is that we know no method to approximate continuous maps by semisimplicial maps by means of it. The approximation constructed by Kan [7] can't be transferred.

3) The "r-skeleton-preserving" subdivision (r non-negative integer) has got its name from the fact that for all semisimplicial sets X there is a natural semisimplicial isomorphism between $U_r(X^r)$ and X^r , where U_r denotes the corresponding division functor and X^r the r -skeleton of X . The effect of U_1 on $[2]$ and the two degeneracy maps $\sigma_e: [2] \rightarrow [1]$ ($e = 0, 1$) can be illustrated by the following pictures:



(Again the fully traced line segments indicate the cell structure, here that of $\Delta_2 = |U_1[2]|$; $|U_1\sigma_0|$ resp. $|U_1\sigma_1|$ identifies the dotted line segments and just so their parallels to a point; moreover $|U_1\sigma_1|$ identifies the whole left upper triangle to one point.)

§ 5 Natural transformations

One can ask now if the natural equivalence in the category \underline{CWh} of theorem 2 is induced by a natural transformation in \underline{CW} . We are not able to give a general answer to this question. Here we list the partial results we have obtained.

Proposition 8. *Each natural transformation between $|U\Delta?|$ and $|\Delta?|$ induces natural equivalence in the homotopy category \underline{CWh} .*

Proposition 9. *The natural transformations between $|U\Delta?|$ and $|\Delta?|$ and between $|U\Delta?|$ and $|\Delta?|$ correspond in an one-to-one fashion.*

So it suffices to consider natural transformations between the functors $|U\Delta?|$ and $|\Delta?|$. We know almost nothing about natural transformations $|U\Delta?| \rightarrow |\Delta?|$, therefore let us deal with natural transformations $|U\Delta?| \rightarrow |\Delta?|$. Such a natural transformation can be given by a sequence t_0, t_1, t_2, \dots of maps $t_i: \Delta_i \rightarrow \Delta_i$ such that certain commutativities hold. Then one can prove:

Proposition 10. *Each natural transformation t_0, t_1, t_2, \dots is uniquely determined by t_1 .*

The essential device for proving this is the following almost trivial

Lemma. *Let V be a topological space, $f, g: V \rightarrow \Delta_n$ continuous maps, i_0, i_1 distinct elements of $[n-1]$ and*

$$|\Delta\sigma_{i_e}| \circ f = |\Delta\sigma_{i_e}| \circ g, \text{ for } e = 0, 1$$

($\sigma_{i_e}: [n] \rightarrow [n-1]$ denotes the i_e -th degeneracy map). *Then holds: $f = g$.*

This lemma also yields

Proposition 11. *Each triple t_0, t_1, t_2 with*

$$|\Delta\beta| \circ t_m = t_n \circ |U\beta|$$

for $0 \leq m, n \leq 2$ and $\beta \in \underline{\Delta}$ such that $|\Delta\beta| \circ t_m$ and $t_n \circ |U\beta|$ are defined can be extended to a natural transformation $|U\Delta?| \rightarrow |\Delta?|$.

We can also give necessary and sufficient conditions that a continuous map t_1 induces a natural transformation. But we do not know if for each standard division functor (U, u) there exists a map t_1 , which satisfies these conditions. This may at most depend on certain hypotheses about $U\sigma_e$ in case $\dim \sigma_e = 2$ ($e = 0, 1$). In our example the existence of such natural transformations can be easily established; there are even natural transformations $U \rightarrow \Delta$. To end this section we mention that there is an infinite number of natural transformations $|U\Delta?| \rightarrow |\Delta?|$, if there is one.

§ 6 Standard homotopies

Now we turn to the main problem: We want to show that under certain further assumptions on a given division functor U the CW -complexes $|UX|$ and $|X|$ are homeomorphic X being any semisimplicial set. Such a homeomorphism will be constructed inductively, so we arrive at the problem to continue a given map of the boundary of Δ_n onto itself over the whole geometric simplex Δ_n such that the interior of Δ_n is mapped homeomorphically onto itself: We have to stuff holes. To this end we need

Definition 3. Let V be a topological space. A homotopy $h_t: V \rightarrow V$ is "stuffing" if $h_0 = \text{id} V$ and h_t is a homeomorphism for all $t < 1$

By means of a stuffing homotopy one can stuff holes:

Proposition 12. Given a stuffing homotopy $h_t: S^n \rightarrow S^n$ there exists an extension $h: B^{n+1} \rightarrow B^{n+1}$ of h_1 such that the interior of B^{n+1} is mapped homeomorphically onto itself. Moreover there is a stuffing homotopy $H_t: B^{n+1} \rightarrow B^{n+1}$ such that $H_1 = h$ and $H_t v = h_t v$ for all $v \in S^n$ and $t \in [0, 1]$. ("Sⁿ" denotes the n-sphere and "Bⁿ⁺¹" the (n + 1)-ball.)

Now let be given a fixed division functor U ; we describe the additional condition:

Definition 4. A "standard homotopy (for U)" is a family $(l_t \beta | \beta \in \underline{\Delta})$ of stuffing homotopies $l_t \beta: \Delta_{\dim \beta} \rightarrow \Delta_{\dim \beta}$ such that the following conditions are satisfied:

$$(10) \quad l_t(i d) = i d,$$

$$(11) \quad |\Delta \beta| \circ l_t(\alpha \beta) = l_t \alpha \circ |\Delta \beta| \text{ for injective } \beta \in \underline{\Delta},$$

$$(12) \quad l_t(\alpha \beta) \circ (l_t \beta)^{-1} \text{ single-valued (and therefore a continuous map)?}$$

$$(13) \quad |\Delta \beta| \circ l_1(\alpha \beta) = l_1 \alpha \circ |U \beta|;$$

from (13) it follows that (for all suitable degeneracy maps $\sigma_i \in \underline{\Delta}$) $l_1(\beta \sigma_i) \circ l_1(\sigma_i)^{-1}$ maps each line segment parallel to the line segment between the i -th and $(i + 1)$ -st vertex of $\Delta_n = |\Delta[n]|$ on such a line segment. We demand further:

$$(14) \quad \text{for each such line segment this map is weakly monotone.}$$

We do not know if there exists a standard homotopy for each division functor U . If there is a standard homotopy for a given U , then it follows at once that the sequence

$$l_1([0] \rightarrow [0]), l_1([1] \rightarrow [0]), l_1([2] \rightarrow [0]), \dots$$

represents a natural transformation $|U ?| \rightarrow |?|$, which we call the "corresponding natural transformation".

§ 7 The main theorem

Main theorem. *Let U be a division functor with standard homotopy. Then X being any semisimplicial set there is a homeomorphism $|UX| \rightarrow |X|$ which is homotopic to the map $|UX| \rightarrow |X|$ deduced from the corresponding natural transformation.*

Here we can only give the idea of the proof. To do this we need a more explicit description of the spaces $|UX|$ and $|X|$ for a given semisimplicial set X . They are quotient spaces of $FX = \Sigma_p X_p \times \Delta_p$, where X_p denotes the set of p -simplices of X provided with the discrete topology and Σ the topological sum. We obtain $|UX|$ by taking the equivalence relation which is generated by

$$(x\beta, v) \sim (x, |U\beta|v)$$

and $|X|$ by taking that which is generated by

$$(x\beta, v) \sim (x, |\Delta\beta|v)$$

for $x \in X_q$, $\beta: [p] \rightarrow [q]$ in $\underline{\Delta}$ (as X is a semisimplicial set, β induces a map from X_q to X_p and $x\beta$ denotes the image of x under this map).

Therefore a continuous map $|UX| \rightarrow |X|$ can be constructed if there is given a family $(h_x/x \in X)$ of continuous maps $h_x: \Delta_{\dim x} \rightarrow \Delta_{\dim x}$ with $|\Delta\beta| \circ h_{x\beta} = h_x \circ |U\beta|$ for all $x \in X$ and all $\beta \in \underline{\Delta}$ such that $x\beta$ is defined. It is easy to show that a map $|UX| \rightarrow |X|$ constructed in such a way is a homeomorphism iff h_x maps the interior of Δ_p homeomorphically onto itself for each non-degenerate $x \in X_p$. In order to establish the first part of the main theorem one has to construct such a family $(h_x/x \in X)$ and it is obvious that the given standard homotopy plays an essential part in this construction.

§ 8 Examples of standard homotopies

We indicate how a standard homotopy for the regular subdivision can be given. Let be $\beta \in \underline{\Delta}$ and $p = \dim \beta$. $l_1\beta$ maps the cells of $|\Delta'[p]| = \Delta_p$ —they are simplices—linearly and $l_1\beta$ is the linear connection between the identity and $l_1\beta$. So it suffices to show as $l_1\beta$ maps the vertices of $|\Delta'[p]|$. Any vertex b of $|\Delta'[p]|$ corresponds to a O -simplex of $\Delta'[p]$ that is an injective map $\mu: [m] \rightarrow [p]$ of $\underline{\Delta}$. As $p = \dim \beta$ the composition $\beta\mu$ is defined; $\beta\mu$ can be uniquely decomposed in an injective and a surjective part; let us denote the latter by ϱ . We define a right inverse $\hat{\rho}$ to ϱ by setting

$$\hat{\rho}(i) = \max \varrho^{-1}(i).$$

Then we take

$$l_1\beta(b) = |\mu\hat{\rho}|$$

(we interpret $\mu\hat{\rho}$ to be a O -simplex of $\Delta'[p]$ and denote by $|\mu\hat{\rho}|$ the corresponding vertex of $|\Delta'[p]|$, which is obviously a point of Δ_p),

For the other examples of standard division functors we have given there exist standard homotopies, too; in the case of natural subdivision it is easy to see that one can take the identity.

References

- [1] M. G. Barratt: *Simplicial and semisimplicial complexes*, Princeton seminar notes 1956.
- [2] R. Fritsch: *Zur Unterteilung semisimplizialer Mengen*, I, Math. Zeit. 108 (1968/69), 329-367.
- [3] R. Fritsch: *Zur Unterteilung semisimplizialer Mengen*, II, Math. Zeit. 109 (1969), 131-152.
- [4] R. Fritsch—D. Puppe: *Die Homöomorphie der geometrischen Realisierungen einer semisimplizialen Menge und ihrer Normalunterteilung*, Arch. Math. (Basel) 18 (1967), 508-512.
- [5] J. B. Giever: *On the Equivalence of Two Singular Homology Theories*, Ann. of Math. 51 (1950), 178-190.
- [6] S. Hu: *On the Realizability of Homotopy Groups and their Operations*, Pacific J. Math. 1 (1951), 583-602.
- [7] D. M. Kan: *On c. s. s. complexes*, Amer. J. Math. 79 (1957), 449-476.
- [8] Y. Kodama: *A Relation between Two Realizations of Complete Semi-simplicial Complexes*, Proc. Japan Acad. 33 (1957), 536-540.
- [9] B. Mitchell: *Theory of Categories*, Academic Press, New York and London, 1965.