Reisinger, Markus:

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Markus Reisinger[†]

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[†]Department of Economics, University of Munich, Kaulbachstr. 45, 80539 Munich, e–mail: markus.reisinger@lrz.uni-muenchen.de

Abstract

This paper analyses a two-sided market in which two platforms compete against each other. One side, the advertisers, exerts a negative externality on the other side, the users. It is shown that if platforms can charge advertisers only, a higher degree of competition for users can lead to higher profits because competition on the advertisers' side is reduced. If platforms can charge users as well, profits might increase or decrease, the latter because of increased competition through the additional instrument of the user fee. Nevertheless the equilibrium with user fee is more efficient.

1 Introduction

There are many markets where companies produce services for a group of agents who do not pay for it or pay only a low price. Instead these companies get revenues from advertisers who wish to gain access to potential consumers via the services of these companies. Examples are private radio or television stations[1] which often interrupt their programme to broadcast advertisement.[2] Search engines like Google or Yahoo! or internet portals like GMX often have a multitude of advertisements on their web sites.[3] In the case of radio it is technically impossible to charge listeners for the broadcasting of programmes. In the case of search engines it is not customary to charge users for the services.

This paper studies a model of platform competition in which users dislike advertisement and therefore spend less time to consume services of platforms. On the other hand advertisers wish to gain users' attention to tempt them to buy their products. In equilibrium the level of advertising might be too high or too low compared with the socially optimal one because platform pricing does not internalise the externality which is exerted on users by more advertising. Concerning platform profits a higher degree of competition for users can increase profits because price competition becomes less fierce. Thus a different level of competition on one side influences the level of competition on the other side and may have consequences on profits which are opposite to the one in a one-sided market. If platforms can charge users as well there might be an incentive to subsidise users, i.e. set a negative fee, to attract more users. But since both platforms do so this strategic effect lowers their profits. A prisoner's dilemma situation arises. But if the user fee is positive in equilibrium the additional instrument increases profits.

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[1] To get an idea of the expenditures on advertising, in the US in 2002 approximately $ 50 billion were spent on TV advertising (Kind, Nilssen & Sorgard (2003)) and a thirty second commercial on FOX had an average price of $ 150,000 (Prime Time Pricing Survey, The Advertising Age (2002)).

[2] In the US advertising time ranged from approximately 10 to 15 minutes per hour in 1999 (Television Commercial Monitoring Report (1999)).

[3] For an internet portal advertising is the most important source of revenue since it does not charge users. For example, the internet portal GMX sells a banner on its web site for 20,000 Euros per week (http://www.gmx.de).

The equilibrium with fees for both sides of the market is always efficient. The reason is that with the user fee platforms do now take into account users' utility in their pricing behaviour.

More specifically, we assume that two platforms compete for user time and advertisers. For the advertisers platforms are completely similar while platforms compete for users in a standard Hotelling style. Both sides of the market choose only one platform.[4] Profits of advertisers are increasing in the time users spend on a platform. Users' utility and the time they spend on a platform are decreasing with advertising.[5] Therefore an advertiser causes a negative externality on users of that platform directly and also on all other advertisers on that platform indirectly. If the gains from trading advertisers' goods are high compared to users' utility loss all producers should advertise from a social point of view. If the utility loss is high some of the advertisers should be excluded. The optimal partition of advertisers among platforms is even. The intuition is that if one platform has more than half of the advertisers the externality on all of them is high and can be reduced if some advertisers shift to the other platform.

In a Nash equilibrium the number of advertisers on both platforms is the same but might be too high or too low compared with efficiency. Platforms only internalise the indirect externality that one advertiser exerts on other advertisers but not the direct utility loss of users. This is the case because this externality is incorporated in their pricing behaviour while the externality on users does not influence prices. So if the degree of differentiation between platforms is low competition for users is fierce. But platforms compete for users by reducing their advertisement levels. Thus with low differentiation there is little advertising on platforms. But if the gains from trading advertisers' goods are high this level of advertising is too low compared with the social optimum. If instead platforms are highly differentiated they will lose only few users if they advertise more. In this case the level of

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[4]The assumption that advertisers single-home (use only one platform) is not crucial to the results but simplifies the modeling. See the next section for a longer discussion.

[5]Ferrando, Gabszewicz, Laussel & Sonnac (2003) analyse a model in which some people are advertisement-avoiders while others are advertisement-lovers. But normally commercials are considered a nuisance for users. See Bagwell (2003) or Dukes & Gal-Or (2002).

advertising is too high.

Platforms' profits depend on the level of differentiation as well. If differentiation is relatively high profits fall with an increase in differentiation. The intuition is that platforms have a higher incentive to attract advertisers because users do not switch easily to the other platform. This results in lower prices for advertising. But since both platforms lower their prices the level of advertising stays the same while profits are decreasing. So the strategic effect hurts platforms. This shows that in a two-sided market a lower level of competition on one side can increase the competition on the other side and lead to lower profits. If differentiation is low and competition for users is fierce an increase in the differentiation leads to rising profits. The reason is that advertising levels are low and with a price decrease this level rises, which increases profits.

I also analyse what happens if platforms can charge a user fee. If this user fee is unrestricted, e.g. can either be positive or negative, the efficient outcome is reached. With the possibility of a user fee platforms have two different instruments at hand to make profits. They therefore take users' utility into account as well. Since platforms compete for both sides this leads to the efficient outcome.

In equilibrium it might be the case that this user fee is negative because platforms want to attract users in order to make more profits on advertisers. In this case the additional instrument of a user fee hurts platforms and their profits are lower. If the user fee is positive profits are higher than without a user fee. If the fee is restricted to be positive the efficient outcome cannot be reached in general but only in the case when the user charge would be positive in equilibrium.

Most of the papers in the two-sided markets literature are concerned with participants exerting positive externalities on each other like in the market for credit cards. Examples of these papers are Rochet & Tirole (2003) or Wright (2003). In Section 6 of their paper Rochet & Tirole (2003) briefly analyse a model in which platforms earn revenues from users and advertisers. Platforms are able to use a two-part tariff for both groups of participants. Rochet & Tirole (2003) show that in general both prices depend on the relations between own- and cross-price elastic-

ities.[6]

Recently there has been a growing literature on platform competition for advertisers. A seminal contribution to this literature is the paper of Anderson & Coate (2003). They analyse a model of TV broadcasting and are interested in the question whether two channels will offer the same or different programmes and how much advertisement they will broadcast. They find that dependent on parameter values there can be too little but also too much advertising and also too low or too high a variety of programmes. In their model viewers suffer from advertising with the consequence that they switch to their less preferred programme if this has fewer advertisements. As a result an even distribution of advertisers on platforms is efficient because otherwise some viewers would not watch their preferred programme.

My paper revisits their first result in a different type of model. The difference to their paper is that in my model platforms compete directly for advertisers while in their model a change in the commercial price of channel 1 does not influence the commercial price of channel 2. This allows me to analyse the consequences of different degrees of competition on one side for the degree of competition on the other side and on platforms' profits. Anderson & Coate (2003) also analyse the case in which viewers can be charged for watching the programmes. They find that advertising levels are usually lower in this case.

Kind, Nilssen & Sorgard (2003) analyse the broadcasting market as well and are also concerned with the question if competition between channels leads to over- or underprovision of commercials. Like Anderson & Coate (2003) they do not assume direct competition for advertisers. Kind, Nilssen & Sorgard (2003) also find that there can be underprovision of advertising for low degrees of differentiation between platforms. They show as well that a merger between the two channels can improve welfare as it leads to more advertisements.[7]

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[6]For a detailed overview how to model different forms of competition and externalities in two-sided markets see Armstrong (2004). For a model with a monopoly platform see Baye & Morgan (2001).

[7]A paper with a similar basic model is Barros, Kind & Sorgard (2003). They are interested in the consequences of a vertical merger between a platform and a producer. They show that such a merger can be harmful for both firms. This is the case if platforms are close substitutes because the independent platform acts as a free rider on the merger and increases its advertising price.

In a paper of Gal-Or & Dukes (2003) differentiated TV or radio stations also compete for viewers/listeners. They analyse the conditions under which a merger of two stations can be profitable. In their model consumers are averse to advertising but may profit from advertisements by the fact that they are better informed about prices.[8] If two firms merge this results in a higher level of advertising which can drive producers' prices and profits down. Therefore producers can pay less for advertising. This might render a merger unprofitable.

In contrast to the above cited papers my paper analyse a model with competition for both sides, users and advertisers, and not only users. I look at the consequences on pricing behaviour and profits of platforms. As is shown this behaviour can be very different in a two-sided market compared with a one-sided one and also has different effects than competition for only one side.

The remainder of the paper is organised as follows. The next section sets out the basic model. In Section 3 the efficient outcome is presented. Section 4 analyses the equilibrium and compares it with efficiency. Section 5 presents the equilibrium with the possibility of a user charge. In Section 6 an example of pricing behaviour of internet portals is given. A short conclusion is given in Section 7.

2   The Model

The goal is to develop a model in which platforms compete for users (consumers) and advertisers (producers). It is assumed that if platforms are internet portals, radio stations, or television channels consumers have the hardware to get access to these platforms. Advertising causes a negative externality on users but advertisers' profits are increasing in the number of users. In the following the basic model is presented.

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[8]A problem in their model is that this gain for viewers/listeners is not included in the utility function. The reason is that this would complicate the model dramatically and would change some results.

Platforms

There are two platforms $i = 1, 2$. Users cannot be excluded from using the platforms. Therefore platforms cannot make profits from users directly. Instead platforms make profits on advertisers. The profit function of platform i is

$$\Pi_i = p_i n_i.$$

p_i is the price that platform i is demanding from an advertiser for an advert and n_i is the number of advertisers on platform i. Each advertiser can only place one advertisement and has to decide exclusively on which platform she wants to advertise. Thus there is rivalry for advertisers. It is assumed that platform pricing is linear. We also assume that the costs of platforms are zero.[9]

Users

There is a mass of users M. Users are uniformly distributed on a line with length one where platform 1 is located at point 0 and platform 2 located at point 1. Each user decides in favour of only one platform.[10] The utility a user derives from spending time t on platform i is $v(t)$ where $v(t)$ is an increasing and strictly concave function. Users' utility is decreasing in the number of advertisements n_i on platform i. The whole amount of disposable leisure time a user has is $\bar{T}$. So $\bar{T} - t$ is the time a user spends on doing other things during his leisure time. We normalize the utility a user gets from doing this other things to 1 per unit of time.

The maximisation problem of a user who is located at x can be written as[11]

$$\max_{i,t} \quad U_i = \bar{T} - t + v(t) - \gamma t n_i^\lambda - \tau_U |x - x_i| \tag{1}$$

γ is a measure of the nuisance costs of advertising and is the same for all users.

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[9]This assumption is made for simplicity. Relaxing it would change the calculations but not the qualitative results of the model.

[10]This formalisation fits the market for Internet portals or TV broadcasting well. Users or viewers decide in favour of only one portal to do e-mailing or can only watch one programme at the same time.

[11]The advantage of this formulation is that the decision of users how much time to spend on a platform is separated from the decision which platform to use. See Anderson, de Palma & Thisse (1992).

The parameter λ measures the curvature of the utility function in n_i. It is assumed that $\lambda \geq 1$ so utility is weakly concave in n_i. This is a realistic assumption, e.g. one or two commercials on a homepage are not very disturbing but if a web site is full of adverts this does disturb users a lot and the time which is spent on these web site decreases drastically with additional commercials. Lastly, τ_U is the transportation cost parameter and represents the degree of differentiation between both platforms.

If a user has decided in favour of one platform differentiating with respect to t yields

$$v'(t) = 1 + \gamma n_i^{\lambda}. \tag{2}$$

$t^*(n_i)$ is implicitely given by (2) and represents the demand function for time on platform i dependent on n_i. From the Implicit Function Theorem we get the slope of this demand function

$$\frac{\partial t}{\partial n_i} = \frac{\gamma \lambda n_i^{\lambda-1}}{v''(t)} < 0. \tag{3}$$

So the amount of time on platform i is decreasing in n_i.

The indirect utility function of a user x is given by

$$U(x, n_i) = \bar{T} - t(n_i) + v(t_i) - \gamma t(n_i)n_i^{\lambda} - \tau_U |x - x_i|.$$

In the following we set $\bar{T} - t(n_i) + v(t_i) - \gamma t(n_i)n_i^{\lambda} = U_B(n_i)$ so $U(x, n_i) = U_B(n_i) - \tau_U |x - x_i|$. The marginal consumer who is indifferent between both platforms is given by

$$x_m = \frac{1}{2} + \frac{1}{2\tau_U}(U_B(n_1) - U_B(n_2)).$$

We assume that t_U is small enough so that in equilibrium all users use one platform, e.g. $\tau_U \leq 2U_B(N/2)$. Thus a mass of $X_i = Mx_m$ chooses platform 1 and the remaining mass $M(1 - x_m)$ chooses platform 2.

With advertising a producer informs users about the nature and the price of its products. After having seen an advert a consumer knows his willingness to pay for the good. It is assumed that this valuation is the same for all consumers and is K with probability β and 0 with probability $1 - \beta$. For simplicity it is assumed

that it is the same for each advertiser's good.[12] This modeling follows Anderson & Coate (2003). Although this formulation is specific it has the advantage that advertising cannot have a positive value for users because each producer will sell its product at a price K. A lower price does not improve the possibility of a sale. Thus the advertiser's price is equivalent to consumers' valuation and therefore a user's utility of getting aware of a new good is zero. The implication of this formulation is that users do not get informational benefits from using a platform with much advertising.

Advertisers

There is a mass of advertisers N. Ex ante advertisers are indifferent between both platforms. Advertisers choose only one platform to advertise on. This assumption represents an easy way to model that platforms have to compete for advertisers.[13] If platform i is chosen by an advertiser her profit is

$$P_i = X_i \beta K t(n_i) - p_i.$$

If she decides not to advertise she gets a profit of zero. The value of an advertisement on platform i does positively depend on the time users spend on that platform. The idea is that the more time a user spend on platform i the higher is the possibility that he gets aware of that advertisement and buys the product in the end. The gross value of an advertisement on i is thus $X_i \beta K t(n_i)$. The advertiser has to pay p_i for an advertisement on i. For simplicity it is assumed that production costs for advertisements and products are zero. Again this assumption does not change the qualitative results.

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[12]This stochastic structure is chosen to make the model more realistic and to express that not every user has a positive valuation for each new good he gets aware of through advertising.

[13]The results of the model do not depend on the assumption that advertisers single-home (choose only one platform). What is necessary is that with a price change of platform i the number of advertisers on platform j changes. So if platform i lowers p_i, n_i increases and n_j decreases. One can get the same results with the assumption that advertising firms multi-home (advertise on both platforms) but have only a certain budget for advertising expenditures. So the last unit of this budget can either be spent on one or the other platform. Thus advertisers multi-home but put more commercials on the platform with the lower price.

Game Structure

In the first stage the two platforms decide simultaneously about their prices p_1 and p_2. In the second stage advertisers decide on which platform they want to advertise if on any and users decide how much time they spend on each platform. Then profits and utilities are realised. This completes the description of the model. In the analysis to follow we maintain the following assumption:

$$A1: \quad \beta K > \frac{U_B''(n_i)}{-\left(2\frac{\partial t}{\partial n_i} + \frac{\partial^2 t}{\partial n_i^2} n_i\right)} \quad \forall n_i.$$

The role of this assumption is to guarantee that the gain from trading advertisers' goods is high enough so that $n_i = 0$ is never efficient, e.g. no advertising is never efficient.

3 Efficiency

In this section the optimal number of advertisements on each platform is derived. This result is later compared with the equilibrium outcome of the pricing game. In the analysis of efficiency there are two effects to consider. Firstly, a higher number of advertisements increases the possibility of trade of advertisers' products. Secondly, a higher number of advertisements decreases users' utility and exerts a negative externality on other advertisers. Total welfare is given by

$$
\begin{aligned}
WF = \quad & M\beta K n_1[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_1) - U_B(n_2))]t(n_1) \\
& + M\beta K n_2[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_2) - U_B(n_1))]t(n_2) \\
& + MU_B(n_1)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_1) - U_B(n_2))] \\
& + MU_B(n_2)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_2) - U_B(n_1))] \\
& - \tau_U \int_0^{\frac{1}{2} + \frac{1}{2\tau_U}(U_B(n_1) - U_B(n_2))} x \, dx \\
& - \tau_U \int_{\frac{1}{2} + \frac{1}{2\tau_U}(U_B(n_1) - U_B(n_2))}^{1} (1 - x) \, dx.
\end{aligned}
\tag{4}
$$

The first two terms are the welfare from trading advertisers' products. The third and the fourth term represent the utility of users gross of transportation costs and the fifth and the sixth term are the transportation costs. Differentiating (4) with

respect to n_i, $i = 1, 2$ yields the first order conditions

$$
\begin{aligned}
&\tfrac{1}{2\tau_U} U_B'(n_i) M[\beta K n_i t(n_i) + U_B(n_i)] \\
&+ \tfrac{1}{2} M[\beta K t(n_i) + \beta K n_i t'(n_i) + U_B'(n_i)] \\
&- \tfrac{1}{2\tau_U} U_B'(n_i) M[\beta K n_j t(n_j) + U_B(n_j)] = 0, \\
&\quad i, j = 1, 2.
\end{aligned}
\tag{5}
$$

So the first order condition is the same for both n_1 and n_2. Thus it is efficient if $n_1 = n_2$. The second order condition is globally satisfied because of $A1$. Simplifying (5) yields the following proposition.

Proposition 1

If

$$
\beta K t(N/2) + \beta K(N/2)t'(N/2) + U_B'(N/2) > 0,
\tag{6}
$$

$n_i^{eff} = \frac{N}{2}$ is efficient.

Otherwise, the efficient number of advertisers n_i^{eff}, $i = 1, 2$ is implicitly given by

$$
\beta K t(n_i) + \beta K n_i t'(n_i) + U_B'(n_i) = 0.
\tag{7}
$$

It is therefore efficient if advertisers allocate equally among platforms. The intuition behind this is simple. If we look only at the gains from trade the externality which one advertiser causes on another one is increasing convexly. So if one platform has many advertisers users spend little time on this platform and thus many advertisers gain little attention. To reduce this externality as well as possible it is optimal that each platform has the same number of advertisers. Transportation costs can be reduced with an even partition as well. If βK is high which means that the probability and the welfare gains from trade are high all producers should advertise and $n_i = N/2$. If these gains are lower compared to the utility loss of users, $n_1 + n_2 < N$.

4 Nash Equilibrium

In this section we solve for the Nash equilibrium of the pricing game.

Since platforms can only quote prices to advertisers and are not differentiated from their point of view we are in a standard Bertrand game. The difference is that with a negative externality one platform cannot win all advertisers by undercutting its competitor's price. The platform with the lower price gets more advertisers but this results in a higher externality on all of them and reduces their profits. It is thus optimal for some advertisers to stay on the other platform. Thus platforms earn positive profits in equilibrium.[14]

It turns out that the model is solvable in a similar way as the product differentiation model of Hotelling.

To see this let us assume first that all N producers advertise. Since all advertisers are the same in equilibrium each advertiser must be indifferent between platform 1 and 2. If this would not be the case one platform can increase its price without losing any advertisers which cannot be an equilibrium. Thus we can determine the marginal advertiser who is indifferent between both platforms. She described by

$$
\begin{aligned}
M\beta K t(n_1)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_1) - U_B(N - n_1))] - p_1 \quad &= \\
M\beta K t(N - n_1)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(N - n_1) - U_B(n_1))] - p_2 &
\end{aligned}
\tag{8}
$$

The left hand side is the profit of an advertiser on platform 1 and the right hand side the profit of an advertiser on platform 2 if the number of advertisers are n_1 and $n_2 = N - n_1$.

Contrary to standard analysis it is not possible to solve (8) for n_1 because users' utility is concave in n_1. To get a solution (8) is solved for p_1 which yields a max-

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[14]It should be mentioned that this result is completely different in a model with positive externalities. If in such models buyers (in our model advertisers) can coordinate on the platform that gives them the highest surplus prices would be driven down to zero because of the standard Bertrand argument. For an overview of this literature see Farrell & Klemperer (2001) or Katz & Shapiro (1994).

imisation problem of platform 1 of

$$\max_{n_1} \quad \Pi_i = \{p_2 + M\beta K t(n_1)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_1) - U_B(N - n_1))] - \\ M\beta K t(N - n_1)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(N - n_1) - U_B(n_1))]\}n_1. \tag{9}$$

Maximising profits for both firms yields two first order conditions. These first order conditions in combination with equation (8) and equation (8) with 1 and 2 reversed yields the equilibrium values of n_i and p_i. After applying the Envelope Theorem, $U'_B(n_i) = -\gamma\lambda n_i^{\lambda-1} t(n_i)$, we get

$$n_i^* = \frac{N}{2}$$

(which is obvious because of symmetry) and

$$p_i^* = \beta K M N \gamma \lambda (N/2)^{\lambda-1}[\frac{t(N/2)^2}{t_U} - \frac{1}{2v''(t(N/2))}].$$

It remains to calculate the equilibrium if $n_1 + n_2 < N$.[15]

The equilibrium of the game is described in the following proposition.

Proposition 2

If $\tau_U \leq \tau_U^1 = \frac{(N/2)^\lambda \gamma \lambda t(N/2)^2}{t(N/2) + \frac{\gamma\lambda(N/2)^\lambda}{v''(t(N/2))}}$ in the unique Nash equilibrium n_i^* is implicitly given by

$$t(n_i) + \frac{\partial t(n_i)}{\partial n_i}n_i - \frac{t(n_i)^2 \gamma \lambda n_i^\lambda}{\tau_U} = 0, \tag{10}$$

where a unique solution $n_i^* \in (0, N/2)$ exists, and

$$p_i^* = \frac{M\beta K t(n_i^*)}{2}. \tag{11}$$

Profits of the platforms are

$$\Pi_i^* = \frac{M\beta K t(n_i^*)}{2}n_i^*. \tag{12}$$

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[15]The method of solution is similar to a standard product differentiation game where consumers' gross surplus from buying is so low that firms are local monopolists. See e.g. Gabszewicz & Thisse (1986).

If $\tau_U^1 < \tau_U \leq \tau_U^2 = \frac{4(N/2)^\lambda \gamma \lambda t(N/2)^2}{t(N/2) + \frac{2\gamma\lambda(N/2)^\lambda}{v''(t(N/2))}}$ in the unique Nash equilibrium $n_i^* = \frac{N}{2}$ and

$$p_i^* = \frac{M\beta K t(N/2)}{2}. \tag{13}$$

Profits of the platforms are

$$\Pi_i^* = \frac{M\beta K t(N/2)}{2} N/2. \tag{14}$$

If $\tau_U > \tau_U^2$ in the unique Nash equilibrium

$$n_i^* = \frac{N}{2} \tag{15}$$

and

$$p_i^* = \beta K M N \gamma \lambda (N/2)^{\lambda-1} \left[\frac{t(N/2)^2}{\tau_U} - \frac{1}{2v''(t(N/2))} \right] \tag{16}$$

Profits of the platforms are

$$\Pi_i^* = \beta K M N \gamma \lambda (N/2)^\lambda \left[\frac{t(N/2)^2}{\tau_U} - \frac{1}{2v''(t(N/2))} \right]. \tag{17}$$

Proof

When calculating the marginal advertiser in equation (8) it was assumed that all producers advertise. But this is only the case if it pays the 'Nth' producer to advertise on a platform instead of not advertising and getting profits of zero.
Thus with a price $p_i^* = \beta K M N \gamma \lambda (N/2)^{\lambda-1} \left[\frac{t(N/2)^2}{\tau_U} - \frac{1}{2v''(t(N/2))} \right]$ this is only the case if

$$\frac{M}{2} \beta K t(N/2) > \beta K M N \gamma \lambda (N/2)^{\lambda-1} \left[\frac{t(N/2)^2}{\tau_U} - \frac{1}{2v''(t(N/2))} \right]$$

or

$$\tau_U > \tau_U^2 = 4 \frac{(N/2)^\lambda \gamma \lambda t(N/2)^2}{t(N/2) + \frac{2\gamma\lambda(N/2)^\lambda}{v''(t(N/2))}}.$$

The next question is what the optimal price of a platform is if it does not have to compete for advertisers because $n_1 + n_2 < N$. In this case the number of advertisers on a platform i depends on p_i and is given by $M\beta K t(n_i) \left[\frac{1}{2} + \frac{1}{2\tau_U} (U_B(n_i) - U_B(n_j)) \right] -$

$p_i = 0$. So the advertiser n_i is the last one whose profit is not negative given a price of p_i. Thus the profit of platform i is

$$\Pi = M\beta K t(n_i) n_i \left[\frac{1}{2} + \frac{1}{2\tau_U}(U_B(n_i) - U_B(n_j))\right]. \qquad (18)$$

Maximising this with respect to n_i for both platforms yields that n_i^* is given by

$$t(n_i) + \frac{\partial t(n_i)}{\partial n_i} n_i - \frac{t(n_i)^2 \gamma \lambda n_i^\lambda}{\tau_U} = 0.$$

which is equation (10).

If $n_i = 0$ the left hand side of (10) is positive because $t(0) > 0$. If $n_i = N/2$ the left hand side is negative because profit function (18) is only relevant if $\tau_U < \tau_U^1$. Thus a solution with $n_i^* \in (0, N/2)$ exists. Since all terms of (10) are continuous functions of n_i this solution is unique.

This n_i^* equals $\frac{N}{2}$ if

$$t(N/2) + \frac{\partial t(N/2)}{\partial n_i} N/2 - \frac{t(N/2)^2 \gamma \lambda (N/2)^\lambda}{\tau_U} = 0$$

or

$$\tau_U = \frac{(N/2)^\lambda \gamma \lambda t(N/2)^2}{t(N/2) + \frac{\gamma \lambda (N/2)^\lambda}{v''(t(N/2))}} = \tau_U^1.$$

So for $\tau_U \leq \tau_U^1 = \frac{(N/2)^\lambda \gamma \lambda t(N/2)^2}{t(N/2) + \frac{\gamma \lambda (N/2)^\lambda}{v''(t(N/2))}}$ n_i^* is given by $t(n_i) + \frac{\partial t(n_i)}{\partial n_i} n_i - \frac{t(n_i)^2 \gamma \lambda n_i^\lambda}{\tau_U} = 0$ and $p_i^* = \frac{M\beta K t(n_i^*)}{2}$.

It remains to calculate what happens if If $\tau_U^1 < \tau_U \leq \tau_U^2 = \frac{4(N/2)^\lambda \gamma \lambda t(N/2)^2}{t(N/2) + \frac{2\gamma \lambda (N/2)^\lambda}{v''(t(N/2))}}$. In this case $n_1^* = n_2^* = N/2$ and both platforms set their prices such that the advertisers have zero utility, e.g.

$$p_i^* = \frac{M\beta K t(N/2)}{2}$$

which is equation (13).

<div align="right">q.e.d.</div>

The profit function is continuous but has two kinks. In the following we provide some comparative static analyses. First let us look at a change in the transportation cost parameter τ_U.

Proposition 3

Platform profits are increasing in τ_U as long as $\tau_U \leq \tau_U^1$.

Profits are independent of τ_U if $\tau_U^1 < \tau_U \leq \tau_U^2$ and profits are decreasing in τ_U if $\tau_U > \tau_U^2$.

Proof

If $\tau_U \leq \tau_U^1$ profit is given by (12) and the optimal number of advertisers is given by (10). As was shown in the proof of Proposition 2 (10) is the first order condition for the maximisation problem of platform i with respect to n_i. Applying the Implicit Function Theorem to (10) yields that $\text{sign}(\frac{\partial n_i}{\partial \tau_U}) = \text{sign}(\frac{\partial (10)}{\partial \tau_U}) = -\frac{1}{2(\tau_U)^2}\beta K M n_i t(ni) U'_B(n_i)$, which greater than zero. Differentiating equation (12) with respect to τ_U gives $\frac{\partial \Pi_i}{\partial \tau_U} = \frac{M\beta K}{2}(t(n_i) + \frac{\partial t(n_i)}{\partial n_i}n_i)\frac{\partial n_i}{\partial \tau_U}$. So $\text{sign}(\frac{\partial \Pi_i}{\partial \tau_U}) = \text{sign}(t(n_i) + \frac{\partial t(n_i)}{\partial n_i}n_i)$. By equation (10), $t(n_i) + \frac{\partial t(n_i)}{\partial n_i}n_i - \frac{t(n_i)^2 \gamma \lambda n_i^\lambda}{\tau_U} = 0$. Since the last term of the left hand side is negative $t(n_i) + \frac{\partial t(n_i)}{\partial n_i}n_i > 0$ which yields $\frac{\partial \Pi_i}{\partial \tau_U} > 0$.

If $\tau_U^1 < \tau_U \leq \tau_U^2$ profit is given by (14). Here $n_i^* = \frac{N}{2}$ and therefore (14) is independent of τ_U.

If $\tau_U > \tau_U^2$ profit is given by (17). In this case $\frac{\partial \Pi_i}{\partial t_U} = -\beta K M N \gamma \lambda (N/2)^\lambda [\frac{t(N/2)^2}{(\tau_U)^2}] < 0$.

<div align="right">q.e.d.</div>

The intuition behind this result is the following. τ_U represents the level of differentiation between the two platforms from the perspective of the users. If τ_U is small platforms have to compete fiercely for users. They do this by reducing their

amount of advertising. Thus prices are high and only few producers advertise. If τ_U increases prices decrease. But profit is rising because more advertiser choose to advertise on the platforms.[16] In this region platforms do not compete for advertisers since $n_1 + n_2 < N$. But when τ_U reaches τ_U^1 all producers are advertising and competition for advertisers starts. In the region between τ_U^1 and τ_U^2 profits stay the same since it does not pay for one platform to lower prices. But if τ_U rises further competition for advertisers lowers prices. The reason is that it pays platforms to attract more advertisers because fewer consumers will switch to the other platform. This strategic effect drives prices down. But also profits are lower because both firms lower their prices and n_i^* stays the same.

This shows that in a two-sided market with negative externalities a lower degree of competition on one side can increase the competition on the other side and lead to lower profits. This is never possible in a standard market with only one side. It is also possible to derive a comparative static result with respect to γ, the nuisance cost of advertising.

Proposition 4

If $\tau_U \leq \tau_U^2$ platform profits are decreasing in γ but if $\tau_U > \tau_U^2$ the effect of a change in γ on profits is ambiguous.

Proof

First look at the case $\tau_U \leq \tau_U^1$. Equation (10) is the first order condition of the maximisation problem of platform i. By applying the Implicit Function Theorem we have

$$\text{sign}(\tfrac{\partial n_i}{\partial \gamma})$$
$$= \text{sign}(\tfrac{\partial t(n_i)}{\partial \gamma} + n_i \tfrac{\partial^2 t(n_i)}{\partial n_i \partial \gamma} + n_i U_B'(n_i) \tfrac{1}{\tau_U} \tfrac{\partial t(n_i)}{\partial \gamma})$$
$$= \text{sign}(\tfrac{n_i^\lambda (1+\lambda)\tau_U - n_i^{2\lambda} t(n_i)\gamma\lambda}{v''(t(n_i))})$$
$$= \text{sign}(-(1+\lambda)\tau_U + n_i^\lambda \gamma\lambda t(n_i)).$$

Multiplying (10) with $\frac{\tau_U}{t(n_i)}$ yields

$$\tau_U + \frac{\partial t(n_i)}{\partial n_i} n_i \frac{\tau_U}{t(n_i)} - t(n_i)\gamma\lambda n_i^\lambda = 0.$$

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[16] A similar result is obtained by Barros, Kind & Sorgard (2003) in a different model.

Thus $\tau_U > t(n_i)\gamma\lambda n_i^\lambda$ and therefore $\tau_U(1 + \lambda) > t(n_i)\gamma\lambda n_i^\lambda$. This shows that $\frac{\partial n_i}{\partial\gamma} < 0$.

If $\tau_U \leq \tau_U^1$ profit is given by (12). Differentiating (12) with respect to γ yields

$\frac{\partial\Pi_i}{\partial\gamma} = \frac{M\beta K}{2}[t(n_i)\frac{\partial n_i}{\partial\gamma} + \frac{\partial t(n_i)}{\partial\gamma}n_i]$.

Differentiating $\frac{\partial t(n_i)}{\partial\gamma}$ yields $\frac{n_i^\lambda}{v''(t(n_i))} < 0$ and thus $\frac{\partial\Pi_i}{\partial\gamma} < 0$.

If $\tau_U^1 < \tau_U \leq \tau_U^2$ profit is given by (14). Differentiating yields $\frac{\partial\Pi_i}{\partial\gamma} = \frac{M\beta KN}{4}\frac{\partial t(n_i)}{\partial\gamma} < 0$.

If $\tau_U > \tau_U^2$ profit is given by (17). Differentiating profit with respect to γ yields

$\text{sign}(\frac{\partial\Pi_i}{\partial\gamma}) = \text{sign}(2t(n_i)^2(v''(t(n_i)))^2 - t(n_i)v''(t(n_i)) + 4t(n_i)\gamma v''(t(n_i))^2\frac{\partial t(n_i)}{\partial\gamma} + \gamma\tau_U\frac{\partial t(n_i)}{\partial\gamma}v'''(t(n_i)))$.

The first two terms are positive the third term is negative and the fourth term is unclear. So profit may increase or decrease in γ.

$$\text{q.e.d.}$$

γ represents the nuisance costs of advertising. So one would guess that profit should decrease in γ because consumers spend less time on the platforms. Proposition 4 states that this is only true if platforms do not compete for advertisers, i.e. if τ_U is low. In this case each user spends less time on platforms which results in a lower possibility of trade of advertisers' goods and thus in lower prices. But if τ_U is high and platforms compete for advertisers, profit might increase in γ. The intuition is that with a high τ_U platforms have an incentive to lower their prices to attract new advertisers. This reduces profits. With a higher nuisance cost this effect is dampened because each platform makes lower profits on new advertisers and thus prices might be higher compared with a lower γ. Thus a higher γ causes two effects on prices. The first is that users are more disturbed by commercials which reduces prices. The second is that competition is reduced which increases prices. The consequences on profits are therefore not clear.

Now let us turn to the comparison of the Nash equilibrium with the efficient outcome.

Proposition 5

If $\beta Kt(N/2) + \beta KN/2t'(N/2) + U_B'(N/2) > 0$ advertising is efficient if $\tau_U \geq \tau_U^1$ and there is too little advertising if $\tau_U < \tau_U^1$.

If there exists n_i s.t. $\beta Kt(n_i) + \beta Kn_it'(n_i) + U_B'(n_i) = 0$, there can be too much or too little advertising in equilibrium.

There is too little advertising if $\tau_U < \min[\tau_U^1, \beta K n_i^{eff} t(n_i^{eff})]$ and too much if $\tau_U > \min[\tau_U^1, \beta K n_i^{eff} t(n_i^{eff})]$.

Only if $\tau_U = \beta K n_i^{eff} t(n_i^{eff}) \leq \tau_U^1$ the equilibrium is efficient.

Proof

From Proposition 1 the optimal number of advertisers on each platform is given by (6) or (7). First look at the case where there exists an $n_i < N/2$ such that (7) holds.

In the Nash equilibrium of the game $n_i^{eq} = N/2$ if $\tau_U > \tau_U^1$. Thus it follows that $n_i^{eq} > n_i^{eff}$ if $\tau_U > \tau_U^1$.

If $\tau_U < \tau_U^1$, n_i^{eq} is given by the first order condition (10). If we insert n_i^{eff} in this first order condition we get from (7)

$$\frac{\gamma \lambda t(n_i^{eff})(n_i^{eff})^{\lambda-1}}{\beta K} - \frac{\gamma \lambda t(n_i^{eff})^2 n_i^{eff}}{\tau_U} \quad \begin{pmatrix} > \\ = \\ < \end{pmatrix} \quad 0$$

or

$$\tau_U \quad \begin{pmatrix} > \\ = \\ < \end{pmatrix} \quad \beta K n_i^{eff} t(n_i^{eff}).$$

So if $\tau_U > \beta K n_i^{eff} t(n_i^{eff})$ the left hand side of equation (10) is than zero at n_i^{eq} but it is greater zero at n_i^{eff}. Thus $n_i^{eq} > n_i^{eff}$.

If $\tau_U < \beta K n_i^{eff} t(n_i^{eff})$ equation (10) is zero at n_i^{eq} but it is lower than zero at n_i^{eff}. Thus $n_i^{eq} < n_i^{eff}$.

It therefore follows that $n_i^{eq} > n_i^{eff}$ if $\tau_U > \min[\tau_U^1, \beta K n_i^{eff} t(n_i^{eff})]$ and $n_i^{eq} < n_i^{eff}$ if $\tau_U < \min[\tau_U^1, \beta K n_i^{eff} t(n_i^{eff})]$. Only in the case when $\tau_U = \beta K n_i^{eff} t(n_i^{eff}) \leq \tau_U^1$ the equilibrium is efficient.

Now look at the case where $n_i^{eff} = N/2$. We know that in equilibrium $n_i^{eq} = N/2$ if $\tau_U \geq \tau_U^1$ and $n_i^{eq} < N/2$ if $\tau_U < \tau_U^1$.

The proposition follows.

q.e.d.

This shows that it depends on the level of τ_U whether the equilibrium is efficient or not.[17] If τ_U is low competition for users is fierce and therefore advertising levels are low. Platforms do not take into account the users' utility loss from an additional commercial but only the indirect externality on all advertisers. The reason is that only this indirect externality can be reflected in their pricing behaviour. If competition for users is harsh many users switch to the competitor if one platform has an additional advertiser. Thus the advertising level is lower than the efficient level. From (6) and (7) we know that τ_U does not play a role in determining the efficient advertising level. But it is the important variable for platform competition. In the case when not all producers should advertise there can be too much advertising if τ_U is high because competition for users is low.

If all producers should advertise there can only be too little advertising. This is the case if competition is fierce with the same line of reasoning as before.

5 Pricing Behaviour of Internet Portals

In this section we discuss the pricing behaviour of two internet portals, namely AOL and GMX. We argue that the structure of their commercial prices fits the results of the preceding section quite well.

Both AOL and GMX are portals where members have access to free e-mail, get informed about cheap offers of products and can inform themselves about specific topics in so called affinity groups. It is costless to become a member of theses portals. The portals get revenues from members only if these buy some services from the portals, like sending SMS or printing pictures. Usually these services are sold at low prices.

The most important source for profits of the portals is advertising. There are different forms of advertising on both portals but the most common ones are banners on their web site. AOL sells a full-size banner on its homepage for 15 Euros per thousand eye-balls, a half-size banner is sold for 10 Euros. A full-size banner on

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[17]In a model of TV-advertising a similar result is obtained by Kind, Nilssen & Sorgard (2003).

the logout-page of AOL costs only 7 Euros, a half-size banner 5 Euros.[18] A similar pricing structure can be observed at GMX.[19] At GMX a logout banner costs 15 Euros while a comparable banner on the homepage costs 24 Euros.[20]

Where does this difference come from? Since these prices are per thousand eyeballs one cannot argue that homepage prices are higher because more people are watching the homepage. Instead a reason can be found from the arguments of the preceding section. To attract advertisers portals have to attract users at first. But before a user decides which portal to use he will compare the homepages of the portals. If one site is plain while the other one is full of commercials while both portals can be used for free he will most likely decide in favour of the plain one. Thus competition for users takes mainly place on the homepages. This can explain the high prices for the homepage banners. Thus homepages of portals do usually have few advertisements on it.

By contrast, only if a user has already decided to use a portal he will see the logout page. So there is no more competition for users and prices for logout-banners are cheap. For example, on the portal GMX usually four advert banners are on the logout page but at most one the start page.

This provides some evidence that the degree of competition for users has a high influence on commercial prices.

6 User Charge

In some markets it is not only possible for platforms to make money on advertisers but to charge users for the consumption of platforms' services as well. Examples are pay-TV channels and newspapers. For example in Europe direct broadcast satellite channels like Canal Plus or Premiere are partially financed by user charges. This is also of policy interest since in the TV case it is becoming technically easier to exclude viewers.

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[18]See http://www.aol.de/mediaspace/preise/preistabelle/contentview.jsp

[19]See http://media.gmx.net/de/cgi/preise?LANG=de&AREA=homepage.

[20]Prices are higher at GMX than at AOL because banners are bigger and the form of advertising is fancier.

In our model the possibility of a user charge can be incorporated in an easy way. In the following we assume that each platform i can charge users a fee c_i for its services. Then platform profit is given by

$$\Pi_i = p_i n_i + X_i c_i.$$

The indirect utility of a user who is located at x and uses platform i is given by

$$U(x, n_i) = T - t(n_i) + v(t_i) - \gamma t(n_i) n_i^\lambda - c_i - \tau_U |x - x_i|.$$

Again, as in Section 2 we set $T - t(n_i) + v(t_i) - \gamma t(n_i) n_i^\lambda = U_B(n_i)$ so $U(x, n_i) = U_B(n_i) - c_i - \tau_U |x - x_i|$. The assumption that all users choose one platform is maintained so $\tau_U \le 2(U_B(N/2) - c_i^*)$.

The marginal user is then given by

$$x_m = \frac{1}{2} + \frac{1}{2\tau_U}(U_B(n_1) - U_B(n_2) + c_2 - c_1).$$

Conducting the same analysis as before gives a maximisation problem of platform 1 of

$$\max\nolimits_{n_1, c_1} \quad \Pi_1 = \{p_2 + \beta K M t(n_1)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_1) - U_B(N - n_1)) + c_2 - c_1] - $$
$$M\beta K t(N - n_1)[\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(N - n_1) - U_B(n_1))]\} n_1$$
$$+ c_1 M(\tfrac{1}{2} + \tfrac{1}{2\tau_U}(U_B(n_1) - U_B(n_2) + c_2 - c_1))$$

if all producers advertise. Formulating the first order condition and solving for p_i and c_i yields

$$p_i^* = M\gamma\lambda(N/2)^{\lambda-1}[t(N/2) - \frac{N\beta K}{2v''(t(N/2))}]$$

and

$$c_i^* = \tau_U - \beta K N t(N/2).[21]$$

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[21]For the moment we assume that c_i can be positive or negative.

The profit of the platform is given by

$$\Pi_i^* = M\gamma\lambda(N/2)^\lambda[t(N/2) - \frac{N\beta K}{2v''(t(N/2))}] + \frac{M}{2}[\tau_U - t(N/2)N\beta K].$$

Comparing this profit with the profit without a user charge we get

$$\Pi_{\text{with charge}} = \Pi_{\text{without charge}} + [1 - t(N/2)\frac{N\beta K}{\tau_U}][M\lambda\gamma t(N/2)(N/2)^\lambda + \tau_U\frac{M}{2}].$$

Thus the profit with user charge is higher if $1 - t(N/2)\frac{N\beta K}{\tau_U} > 0$. But this is exactly the formula for the user charge to be positive.

The profits in the case that not all producers advertise are computed in the same way as in Section 3. This leads to the following equilibrium.

Proposition 6

If $\beta K \leq \frac{(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^\lambda}{v''(t(N/2))}}$ then n_i^* is implicitly given by

$$t(n_i) + ni\frac{\partial t}{\partial n_i} - t(n_i)\frac{n_i^{\lambda-1}\gamma\lambda}{\beta K} = 0, \tag{19}$$

where a unique solution $n_i^* \in (0, N/2)$ exists, and

$$p_i^* = \frac{M}{2}\beta Kt(n_i^*) \tag{20}$$

and

$$c_i^* = \tau_U - \beta Kn_i^*t(n_i^*). \tag{21}$$

Profits of the platforms are

$$\Pi_i^* = \frac{1}{2}M\tau_U \tag{22}$$

If $\frac{(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^\lambda}{v''(t(N/2))}} < \beta K \leq \frac{2(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^\lambda}{2v''(t(N/2))}}$

then $n_i^* = \frac{N}{2}$,

$$p_i^* = \frac{M}{2}\beta Kt(N/2) \tag{23}$$

and

$$c_i^* = \tau_U - \beta K N t(N/2). \qquad (24)$$

Profits of the platforms are

$$\Pi_i^* = \frac{1}{2} M \tau_U. \qquad (25)$$

If $\beta K > \frac{2(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^\lambda}{2v''(t(N/2))}}$
then $n_i^* = \frac{N}{2}$,

$$p_i^* = M\gamma\lambda(N/2)^{\lambda-1}[t(N/2) - \frac{N\beta K}{2v''(t(N/2))}] \qquad (26)$$

and

$$c_i^* = \tau_U - \beta K N t(N/2). \qquad (27)$$

Profits of the platforms are

$$\Pi_i^* = M\gamma\lambda(N/2)^\lambda[t(N/2) - \frac{N\beta K}{2v''(t(N/2))}] + \frac{M}{2}[\tau_U - t(N/2)N\beta K]. \qquad (28)$$

Proof

If platforms set prices $p_i^* = M\gamma\lambda(N/2)^{\lambda-1}[t(N/2) - t(N/2)\frac{N\beta K}{2v''(t(N/2))}]$ the condition under which N producers advertise is given by

$$\frac{M}{2}\beta K t(N/2) - p_i^* = M\gamma\lambda(N/2)^{\lambda-1}[t(N/2) - \frac{N\beta K}{2v''(t(N/2))}] > 0$$

or

$$\beta K > \frac{2(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2) + \frac{\gamma\lambda(N/2)^\lambda}{2v''(t(N/2))}}.$$

In this case $p_i^* =$ and c_i^* are given by (26) and (27).

If not all N producers advertise there is no competition for advertisers. Thus each platform set the price $p_i = \frac{M}{2}\beta K t(n_i)$. The maximisation problem of platform i is

thus

$$\max_{n_i, c_i} \qquad \Pi_i = n_i \frac{M}{2} \beta K t(n_i)$$

which yields that n_i^* is implicitly given by (19) and $c_i^* = \tau_U - \beta K N t(n_i^*)$.

For the same reason is in the proof of Proposition 2 a unique solution $n_i^* \in (0, N/2)$ exists.

Inserting $n_i = N/2$ in (19) gives $t(N/2) + N/2 \frac{\partial t}{\partial n_i} - t(N/2) \frac{(N/2)^{\lambda-1} \gamma \lambda}{\beta K} = 0$ or $\beta K = \frac{(N/2)^{\lambda-1} \gamma \lambda t(N/2)}{t(N/2) + \frac{\gamma \lambda (N/2)^\lambda}{v''(t(N/2))}}$.

Thus if $\beta K \leq \frac{(N/2)^{\lambda-1} \gamma \lambda t(N/2)}{t(N/2) + \frac{\gamma \lambda (N/2)^\lambda}{v''(t(N/2))}}$ then n_i^* is given by (19). If $\beta K > \frac{(N/2)^{\lambda-1} \gamma \lambda t(N/2)}{t(N/2) + \frac{\gamma \lambda (N/2)^\lambda}{v''(t(N/2))}}$ then $n_i^* = N/2$ and $p_i^* =$ and c_i^* are given by (23) and (24).

<div align="right">q.e.d.</div>

The profit can now be compared with the profit if a user charge is not possible.

Proposition 7

Suppose that platforms can set an unrestricted user charge. If this user charge is positive in equilibrium profits are higher than without the user charge.

Proof

To prove the proposition we have to compare the highest profit without a user charge with the lowest profit with user charge.

Because of Proposition 2 the highest profit without a user charge is given by $\Pi_{\text{without charge}} = \frac{M \beta K t(N/2)}{2} N/2$.

The lowest profit with user charge is $\Pi_{\text{with charge}} = \frac{1}{2} M \tau_U$. This is the case because $M \gamma \lambda (N/2)^\lambda [t(N/2) - \frac{N \beta K}{2 v''(t(N/2))}] + \frac{M}{2}[\tau_U - t(N/2) N \beta K]$, which is the profit with user charge if $\beta K > \frac{2(N/2)^{\lambda-1} \gamma \lambda t(N/2)}{t(N/2) + \frac{\gamma \lambda (N/2)^\lambda}{2 v''(t(N/2))}}$, is higher than $\frac{1}{2} M \tau_U$ if $\beta K > \frac{2(N/2)^{\lambda-1} \gamma \lambda t(N/2)}{t(N/2) + \frac{\gamma \lambda (N/2)^\lambda}{2 v''(t(N/2))}}$.

Now comparing $\Pi_{\text{with charge}} = \frac{1}{2} M \tau_U$ with $\Pi_{\text{without charge}} = \frac{M \beta K t(N/2)}{2} N/2$ yields that $\Pi_{\text{with charge}} > \Pi_{\text{without charge}}$ if and only if $\tau_U > N/2 \beta K t(N/2)$. But this exactly the condition for the user fee to be positive.

<div align="right">q.e.d.</div>

We have shown that profits always increase if the user charge is positive. But if it is negative profits might be lower than without this possibility. The intuition behind this result is the following. If platforms have the possibility to set a user charge there are two different ways to do that. The first is to set a higher commercial price to get rid of some advertisers and then make profits on users with a positive user charge. This is the case if $\tau_U > \beta K N t(N/2)$. Both platforms set a higher p_i^* so none of them loses many advertisers. But they set $c_i^* > 0$ as well which results in higher profits. The second possibility is to subsidise users with a negative fee in order to attract more advertisers.[22] But since both platforms do so in equilibrium they reduce their advertiser price as well and profits are lower than without a user charge. Thus a prisoner's dilemma situation arises.[23] Profits would be higher if the additional instrument of the user charge were not available.

Differentiating with respect to τ_U yields that $\frac{\partial \Pi_i^*}{\partial \tau_U} = \frac{M}{2} > 0$. So in contrast to the case without user charge profits are always increasing in τ_U. The reason is that p_i^* is independent of τ_U while c_i^* is increasing in τ_U. Thus if platforms can charge both sides of the market the degree of competition on one side is only reflected in the price of that side.

Let us turn now to the welfare analysis.

Proposition 8

If platforms can set an unrestricted user charge the equilibrium is efficient.

Proof

If $\beta K \leq \frac{(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^\lambda}{v''(t(N/2))}}$ then in equilibrium n_i^* is given by (19). But since $U_B'(n_i) = -\gamma\lambda n_i^{\lambda-1}t(n_i)$ equation (19) is the same as $\beta K t(n_i)+\beta K n_i t'(n_i)+U_B'(n_i) = 0$ which is equation (7), the condition for efficiency if $n_i^{eff} < N/2$.

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[22]A similar way of reasoning is given by Rochet & Tirole (2003). In two-sided markets the platforms charge prices such that the side with the higher demand elasticity is subsidised by the side with the lower demand elasticity.

[23]Similar effects are at work in Anderson & Leruth (1993) and Thisse & Vives (1988) where an additional pricing instrument hurt firms.

If $\beta K > \frac{(N/2)^{\lambda-1}\gamma\lambda t(N/2)}{t(N/2)+\frac{\gamma\lambda(N/2)^{\lambda}}{v''(t(N/2))}}$ then $n_i^* = N/2$. Routine manipulations of the inequality yields that this inequality is the same as $\beta K t(N/2)+\beta K N/2 t'(N/2)+U_B'(N/2) > 0$. But this is inequality (6), the condition for efficiency if $n_i^{eff} = N/2$.

q.e.d.

Why does the additional instrument of a user charge lead to the efficient outcome? The intuition is that platforms now take users' utility directly into account and not only indirectly in the commercial prices. Since there is competition for users both platforms set the fees in such a way that users allocate efficiently. On the side of the advertisers there is Bertrand competition (although profits are positive). Advertisers are allocated efficiently as well. The reason is that the efficient allocation helps both firms to get higher revenue. Thus with a second instrument at hands competition for users and advertisers leads to the efficient outcome.

Up to now we assumed that the user charge is unrestricted so it can be negative. But in many situations this is not practicable. TV watchers are not paid by stations or internet users are not subsidised by portals. If the user fee is restricted to be positive this means in our analysis that $c_i^* = \max\{0, \tau_U - \beta K N t(n_i^*)\}$. If $\tau_U < \beta K N t(n_i^*)$ this constraint is binding. In this case it would be optimal for platforms in a symmetric equilibrium to set $c_i^* = 0$. But this exactly what we observe in many markets. Take again the case of an internet portal. For them it would be technically no problem to charge users if they want to get access to their site. Instead they do not require users to pay a fee in order to attract many users and make profits on advertisers.

In terms of welfare if a user charge has to be positive the equilibrium is no longer generally efficient but only if $\tau_U > \beta K N t(n_i^*)$. So if the constraint on c_i^* is binding the result is the same as in the case without a user charge. To reach efficiency both pricing instruments have to be unrestricted.[24]

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[24]For a discussion of policy implications for two-sided markets see Evans (2004).

7   Conclusion

This paper analysed a model of platform competition in which each advertiser exerts a direct negative externality on users and indirect one on all other advertisers on the same platform. It was shown that the number of advertisements in equilibrium can be too high or too low compared with the efficient one. Profits of platforms can increase if they become less differentiated because this leads to lower competition on the advertisers' side. If platforms can set a user charge as well profits increase only if this charge is positive in equilibrium. A prisoner's dilemma result is possible. But welfare is always higher with a user charge. We have also given anecdotal evidence that supports our results in an example of pricing behaviour of internet portals .

An interesting suggestion for further research might be to analyse the dynamics of such a two-sided market. Usually if people are used to one internet portal or read a newspaper for several years they would not switch easily if another one has fewer advertisements. People form habits. It would be interesting to analyse how such habit formation might change the results. A new platform which enters the market after the others (such as Google in the search engine case) needs a very low level of advertising to induce consumers to switch. This is what was actually observed for Google. So the question arises if this low level of advertising will persist or vanish over time.

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