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University of Munich
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Vertical Product Differentiation, Market Entry, and Welfare

Markus Reisinger†

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†Department of Economics, University of Munich, Kaulbachstr. 45, 80539 Munich, e-mail: markus.reisinger@lrz.uni-muenchen.de
Abstract

This paper analyses a model of vertical product differentiation with one incumbent and one entrant firm. It is shown that if firms can produce only one quality level welfare in this entry game can be lower than in monopoly. This is the case if qualities are strategic complements because the incumbent may distort its quality downwards. If firms can produce a quality range and practice non-linear pricing welfare in case of entry deterrence is higher than in monopoly because the incumbent enlarges its product line. If entry is accommodated consumer rent increases but the consequences on welfare are ambiguous.
1 Introduction

There are a lot of different ways how an incumbent firm reacts when facing the threat of entry. For example, in the pharmaceutical market after patent expiration some formerly protected monopolists introduced their own generics to keep competitors out of the market\(^1\) while others abstained from such practice and increased its price after entry generic competitors.\(^2\)

Another example is the airline industry. In Canada in fall 2000 the low cost carrier CanJet Airlines entered the Toronto-Halifax market. The reaction of Air Canada, the incumbent, was not to increase its price like in the pharmaceutical industry but to lower its fares.\(^3\) A quite different strategy was pursued by British Airways. Its reaction on the entry of low cost carriers on long haul routes was to reduce economy class capacity and enlarge premium class capacity thereby increasing its average prices.\(^4\) Many flag carriers instead tried to deter entry of low cost airlines by establishing their own ‘no-frills’-airline. This was done by British Airways on short-haul routes with the subsidiary GO. In 2000 the Dutch carrier KLM followed and established Basiq Air and in 2002 the low cost carrier Germanwings was founded.\(^5\) Germanwings is an affiliate company of Eurowings. In turn, Eurowings is controlled by the German flag carrier Lufthansa.

So a couple of questions arise. Why do incumbents pursue so many different strategies to seemingly the same problem, namely threat of entry? Does an incumbent’s strategy differ if it can produce only one quality level or a whole quality range? What are the welfare consequences of this potential competition, i.e. does welfare always increase in such a scenario or is it possible that a protected monopoly is better?

This paper tries to answer these questions in a vertical product differentiation framework. We compare a model where each firm can produce a single quality with one where price discrimination over a quality range is possible. We show

\(^1\)See Hollis (2003).
\(^2\)See Grabowski & Vernon (1992) or Frank & Salkever (1997).
\(^3\)See Gillen (2002).
\(^4\)See Johnson & Myatt (2003).
\(^5\)See Gilroy, Lukas, & Volpert (2003).
that in the single quality case welfare with potential competition can be lower than in monopoly. The intuition is that if qualities are strategic complements the incumbent lowers its quality in comparison to monopoly and produces some middle range quality to deter entry because it is impossible then for an entrant to find a profitable entry segment. Even in case of entry such a quality reduction might be profitable, causing the entrant to produce a low quality and reducing price competition. If qualities are strategic substitutes the incumbent produces higher quality and welfare increases.

If firms can produce a quality range we find that consumer rent with potential competition is higher than under monopoly. The intuition is that in order to deter entry the incumbent enlarges its product line to occupy the lower segment as well. In this case welfare increases as well. If entry cannot be deterred there is a gap between the two firms’ quality ranges which reduces competition. In this case consumer rent always increases because of lower prices while the consequences on welfare are unclear. The reason is that some consumers buy a higher quality but others buy a lower one.

Specifically, we analyse a model of vertical product differentiation with entry. In the first stage the incumbent produces a quality which cannot be changed in the sequel. After observing this quality level the entrant decides if it wants to enter and if so which quality level it wants to produce. In the third stage firms compete in prices dependent on the produced quality levels.\(^6\)

We compare this situation of potential competition with a situation of monopoly. In monopoly the firm produces too low a level of quality. The reason is that the monopolist can only charge one price which is the valuation of the marginal consumer. The valuation of the inframarginal consumer is higher but cannot be represented in the price. In the scenario of potential entry the incumbent can deter entry by varying its quality level. If qualities are strategic complements in the sense of Bulow, Geanakoplos, & Klemperer (1985) a reduction of the incumbent’s quality leads to a reduction of the entrant’s quality which lowers the entrant’s profit.\(^7\)

\(^6\)Throughout the paper we assume that it is more profitable for the incumbent to be the high quality firm than the low quality firm.

\(^7\)In a different terminology which is used by Fudenberg & Tirole (1984) the strategy where the
fixed costs of entry are high enough entry is deterred by a quality reduction and welfare is lower than under monopoly. Even in the case where entry is accommodated it might be profitable for the incumbent to reduce its quality. The entrant lowers its quality as well which results in lessened price competition. So even in case of competition it is possible that welfare is lower than under monopoly. If products are strategic substitutes welfare rises in both cases (entry deterrence and accommodation) because the incumbent increases its quality.\(^8\) We also show that if marginal costs of production are low quality of the incumbent in case of entry is higher than in monopoly. The intuition is that the incumbent wishes to differentiate itself from its competitor by producing a higher quality. If marginal costs are low it is not very costly to do so and quality in case of entry is higher.

We also analyse a model where each firm can produce a whole range of different qualities and engage in second-degree price discrimination. This model is compared with the single quality case and we find that the results differ in some respects. In the model with price discrimination the lowest quality of the incumbent and the highest quality of the entrant are strategic complements. So if the incumbent enlarges its quality range the profit of the entrant decreases. Thus the incumbent’s entry deterrence strategy is to expand its product line which results in a welfare increase because more consumers are served. This is different from the single quality case where welfare in case of entry deterrence can be lower if qualities are strategic complements. If fixed costs of entry are low and the incumbent accommodates entry then we always get a gap between the two product lines of incumbent and entrant in order to reduce price competition. Thus some qualities in the middle range which are produced in monopoly are no longer produced in duopoly. But more qualities in the lower segment are produced in duopoly. The result is that some consumers buy higher quality in duopoly while others buy lower quality. Therefore the consequence on welfare is not clear. By contrast, it can be shown that consumer rent always increases in case of entry due to increased price competition.

\(^8\)In the terminology of Fudenberg & Tirole (1984) this strategy is called ‘Top Dog’.
For both models, single quality case and price discrimination, we provide two empirical examples from different industries where firms’ behaviour is similar to that predicted by our model.

The remainder of the paper is organised as follows. In the next section our model is related to the existing literature. Section 3 presents the model and the equilibrium without price discrimination. Some anecdotal evidence that supports the results is given in Section 4. Section 5 presents the model, the equilibrium, and the welfare consequences if price discrimination is possible. In Section 6 two practical examples for such firm behaviour are given. Section 7 gives a short conclusion and some policy implications. Most proofs of the results are presented in the Appendix.

2 Related Literature

Our model relates to the literatures on vertical product differentiation, second-degree price discrimination, and market entry. We will give the relation to each of the three branches and how our model differs from these literatures in turn.

The literature on quality competition started with the pioneering work of Gabszewicz & Thisse (1979) and Shaked & Sutton (1982). In their models firms are restricted to produce one quality level and compete in prices. In Gabszewicz & Thisse (1979) firms’ qualities are exogenously given while in Shaked & Sutton (1982) firms decide simultaneously about their quality levels in the stage before price competition. Shaked & Sutton (1982) show that firms will produce different quality levels to avoid fierce competition in the last stage of the game. Under some parameter constellations only two firms are active in the market if there exist costs of entry. Shaked & Sutton (1982) were the first to analyse the now common game structure where firms are committed to their quality levels when competing in prices because prices can be changed at will while a quality change involves modifications of the production facilities.

Ronen (1991) analyses a model with a similar framework as Shaked & Sutton (1982) but where a regulation authority can set a minimum quality standard before firms compete in qualities. In his model qualities are strategic complements.
Thus if the minimum quality standard is set (slightly) above the quality which is produced by the low quality firm in a game without restriction, both qualities will rise in equilibrium. Price competition is intensified and all consumers are better off while the high quality firm loses. Ronnen (1991) shows that with an appropriately chosen standard social welfare improves.

Cabrales (2003) looks at the consequence of a price ceiling. He shows that with a lower price ceiling the market share of the high quality variant increases. The reason is that market share depends on the ratio of price to quality. But the quality responds less than proportionally to the price ceiling if the cost function is convex. He applies his model to regulation issues in the pharmaceutical market.

In contrast to these models my paper analyses a sequential move game in the quality decision. It might therefore be possible for the first mover to deter entry by an appropriate quality choice. Also welfare in this sequential structure is compared with a pure monopoly situation.

There are several papers which analyse competition between multiproduct firms. The closest to the model considered here are Champsaur & Rochet (1989) and Johnson & Myatt (2003). Champsaur & Rochet (1989) analyse a duopoly where firms commit in the first stage to a quality range and in the second stage compete in prices for each produced quality. They show that firms produce non overlapping quality ranges (there is always a gap between the two product lines) to reduce price competition. This result appears in my paper as well. The difference is that in my paper quality decisions are taken sequentially and one firm has a first mover advantage. This influences prices and quality ranges and may results in entry deterrence. I also provide a welfare analysis.

Johnson & Myatt (2003) analyse an asymmetric duopoly. One firm (which is called ‘incumbent’ by Johnson & Myatt (2003)) can produce the entire range of qualities while the other (the ‘entrant’) is limited to some range with an upper quality level. So the incumbent can produce upgrade versions. Firms compete simultaneously

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9Spulber (1989) analyses a model where firms are horizontally differentiated on a Hotelling line. He shows that each firm produces the first best quality for the consumer who is located exactly at the firm’s position while qualities for all other consumers are distorted downwards. Stole (1995) in addition to Spulber (1989) considers the case where firms are uncertain about vertical preferences. He finds that a similar result holds in this case.
in quantities for each quality level. As is shown by Johnson & Myatt (2003) the incumbent may produce fewer qualities (‘product line pruning’) or more qualities (‘fighting brands’) in duopoly than in monopoly dependent on the cost function. If marginal revenue is decreasing the quality range is reduced while the quality range might be broader if marginal revenue is increasing in some regions.

A model of market entry in a vertical product differentiation framework is analysed by Donnenfeld & Weber (1995).\textsuperscript{10} In their model there are two incumbents who face the entry threat of a third firm. They show that the equilibrium depends on the level of the fixed costs of entry. If these fixed costs are low entry is accommodated and the incumbents select extreme qualities to reduce price competition. The entrant chooses a quality in the middle.\textsuperscript{11} If fixed costs are in some middle range incumbents deter entry. They do this by producing similar qualities which leads to harsh competition and low profits. If fixed costs are so high that entry is blockaded incumbents choose sharply differentiated products to reduce competition. In contrast to Donnenfeld & Weber (1995), my model analyses the behaviour of only one incumbent but firms can produce quality ranges and engage in second degree price discrimination.

In short, models of vertical product differentiation usually do not consider the possibility of price discrimination if entry is possible. So this paper makes a first attempt to analyse the equilibrium and the welfare consequences of such a strategy.

## 3 The Model without Price Discrimination

This section presents the model where each firm can produce only one quality level.

\textsuperscript{10}For a model of entry deterrence and horizontal preferences see Bonanno (1987).

\textsuperscript{11}A similar result is obtained in Donnenfeld & Weber (1992) in the case without fixed costs. They show that in this case the entrant’s profit is higher than the profit of the incumbent which produces the lower quality.
3.1 Description of the Model

There is a continuum of consumers of mass 1. Each consumer purchases a single unit of a good. If a consumer decides to purchase from firm $i$ she gets a good of quality $q_i$ at price $p_i$. Consumers’ tastes are described by the parameter $\theta$ which is distributed between 0 and 1 with distribution function $F(\theta)$ and density function $f(\theta)$. The utility from purchasing from firm $i$ can therefore be denoted as

$$U(q_i, \theta, p_i) = u(q_i, \theta) - p_i,$$

where $u$ is assumed to be strictly concave in $q$ and in $\theta$ and thrice continuously differentiable. Consumers’ reservation value from not buying is normalised to zero.

We proceed by making a few assumptions on the utility and the distribution function.

1. **Single Crossing Property**
   - $u_{q\theta}(q, \theta) > 0$
2. **Monotone Hazard Rate Condition**
   - $\partial F(\theta) / \partial \theta \leq 0$.

A1 is the single crossing property. It states that utility and marginal utility go in the same direction if $\theta$ increases. It implies that indifference curves cross only once. This assumption is standard in the literature. A2 imposes two technical assumptions that guarantee that the second order conditions are satisfied. A3 is a standard assumption in the adverse selection literature and is called monotone hazard rate condition. It is satisfied by many distribution functions like the uniform distribution, the normal distribution etc.

There are two firms $i = 1, 2$. Firm 1 is the incumbent and firm 2 the potential entrant. If a firm decides to produce quality $q$ it has to incur development costs $c(q)$ with $c'(q) > 0$ and $c''(q) > 0$.\footnote{$c(q)$ satisfies the standard Inada-conditions $\lim_{q \to 0} c'(q) = 0$ and $\lim_{q \to \infty} c'(q) = \infty$.} $c(q)$ is the same for both firms. Marginal costs are denoted $v$ and are the same for both firms as well.

The game structure is as follows. The game has three stages. In stage 1 firm 1 chooses $q_1$. Firm 2 decides about market entry in stage 2 after observing the choice...
of firm 1. If firm 2 decides not to enter firm 1 is a monopolist in stage three and
decides about $p_1$. If firm 2 enters it has to incur fixed costs of market entry of $F_{13}$
and chooses $q_2$ in stage 2. Firm 1 observes $q_2$ and in stage 3 both firms set their
prices $p_1$ and $p_2$ conditional on $q_1$ and $q_2$.

The important feature of the model is that both firms are committed to the quality
they produce. In particular it is not possible for firm 1 to make a later change in
the quality to which it has committed in stage 1.\textsuperscript{14} This time structure represents
the idea that it is easy and almost costless possible to change prices but it takes a
considerable amount of time and costs to change the quality of a good.\textsuperscript{15}

\section*{3.2 Monopoly Situation}

First let us look at the monopoly case as a benchmark which is later compared
with the results of the entry game. So suppose firm 1 is a monopolist and there
is no potential entrant. In other words stage 2 of the game does not exist and
firm 1 chooses first $q_1$ and then $p_1$. Let the marginal consumer who is served by
the monopolist be called $\theta_{mon}^m$. If quality is $q_1$ this marginal consumer is given by
$u(q_1, \theta_{mon}^m) - p_1 = 0$. So all types $\theta_{mon}^m \leq \theta \leq 1$ are buying from the monopolist
while all types $\theta < \theta_{mon}^m$ are not buying. In the last stage the monopolist chooses
its price given quality $q_1$. The maximisation problem is thus

$$\max_{p_1} \Pi_1 = \int_{\theta_{mon}^m}^1 [p_1 - vq_1] f(\theta) d\theta - c(q_1).$$

Since $\theta_{mon}^m$ is determined by $p_1$ it is convenient to make a change in the decision
variables and let $\theta_{mon}^m$ be the decision variable. Thus we have

$$\max_{\theta_{mon}^m} \Pi_1 = \int_{\theta_{mon}^m}^1 [u(q_1, \theta_{mon}^m) - vq_1] f(\theta) d\theta - c(q_1).$$

\textsuperscript{13}These entry costs might contain advertising expenditures to inform consumers about the en-
trant’s product, investment in transportation channels and so on.

\textsuperscript{14}For a model where such commitment is only partially possible see Henkel (2003).

\textsuperscript{15}This line of reasoning is followed in most models of vertical product differentiation, see for
This results in a first order condition of

$$\frac{\partial \Pi_1}{\partial \theta_{mon}} = -f(\ell_{mon}^m)[u(q_1, \ell_{mon}^m) - vq_1] + (1 - F(\ell_{mon}^m))u_\theta(q_1, \theta_{mon}^m) = 0. \quad (1)$$

Because of Assumption A3 the second order condition is globally satisfied.

The first order condition as usual states that the marginal gain from serving an additional consumer type (first term) is equal to the loss on all other consumers because of the price reduction (second term).

Turning to the first stage where the firm decides about quality $q_1$ we get a first order condition of

$$\frac{\partial \Pi_1}{\partial q_1} = (1 - F(\theta_{mon}^m))[u_{q_1}(q_1, \theta_{mon}^m) - v] - c'(q_1) = 0. \quad (2)$$

The second order condition is globally satisfied because of $u_{q_1q_1}(q_1, \theta_{mon}^m) < 0$ and $c''(q_1) > 0$. Thus we get that $\theta_{mon}^{m*}$ is given by (1), $p_{mon}^{m*} = u(q_{mon}^{m*}, \theta_{mon}^{m*})$ and $q_{mon}^{m*}$ is given by (2).

A comparison of the monopolistic outcome with the welfare maximising outcome yields

**Proposition 1**

Compared with the welfare-maximizing $\theta_{m}^{WF}$ and $q_{1}^{WF}$ a monopolist serves too few consumers, $\theta_{m}^{mon*} > \theta_{m}^{WF}$, and provides too low a quality $q_{1}^{WF} > q_{1}^{mon*}$.

**Proof**

See the Appendix.

The result that too few consumers are served by a monopolist is standard. The intuition for the quality distortion is that the monopolist can charge only one price namely $p_{1}^{mon*} = u(q_{1}^{mon*}, \theta_{m}^{mon*})$ for its produced quality. So by increasing quality it can only increase its price by the amount that the utility of the marginal consumer

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16Because of the Envelope Theorem terms with $\frac{\partial \Pi_1}{\partial \theta_{mon}} \frac{\partial \theta_{mon}^m}{\partial q_1} = 0$ and can therefore be ignored in the first order condition.
rises. But the utility of all types $\theta > \theta_{\text{mon}}^*$ rises more from a quality increase than the utility of the marginal consumer because of the single crossing property. Thus from a welfare point of view quality in monopoly is too low. Since the monopolist also serves too few consumers the downward distortion of quality is intensified.

### 3.3 Potential Competition

Now let us turn to the three stage game in which firm 2 can enter the market in stage 2. In the following let us define $q_2(q_1)$ as the best answer of firm 2 if it enters in response to firm 1 producing $q_1$. Before starting with the analysis we need two additional assumptions:

\[ A4 : \Pi_2(q_{\text{mon}}^*, q_2(q_{\text{mon}}^*)) > 0 \]
\[ A5 : \Pi_1(q_H^1, q_2(q_H^1)) > \Pi_1(q_L^1, q_2(q_L^1)) \]
whenever $q_H^1 > q_2(q_H^1)$ and $q_L^1 < q_2(q_L^1)$.

The first assumption states that the profit of firm 2 is positive if firm 1 produces its optimal monopoly quality. The assumption is made to avoid the uninteresting case that it is an equilibrium if firm 1 produces its monopoly quality and firm 2 stays out of the market. In the terminology of Bain (1956) this would mean that entry is blockaded. Assumption $A5$ states that firm 1’s profit is higher if it is the high quality firm, i.e. produces such a quality in stage 1 that the optimal response of firm 2 is to produce a lower quality in stage 2.

As usual the game is solved by backwards induction.

In the third stage there are two possibilities. Either firm 2 has entered in stage 2 and there is competition or firm 2 stayed out of the market and firm 1 is a monopolist. If firm 1 is a monopolist the marginal consumer is determined in the same way as in the last subsection and $\theta_{\text{mon}}^*$ is given by (1) given the quality $q_1$ firm 1 has produced in stage 1 (which is different from $q_{\text{mon}}^*$ because of Assumption $A4$.)

If firm 2 has entered the market in stage 2 firms compete for consumers in stage 3. Because of Assumption $A5$ firm 1 will always produce a quality $q_1$ such that it is optimal for firm 2 to produce $q_2 < q_1$. It is therefore apparent that firm 1
will serve higher consumer types. The marginal consumer $\theta^{d_{m1}}$ who is indifferent between buying from firm 1 and buying from firm 2 is given by $u(q_1, \theta^{d_{m1}}) - p_1 = u(q_2, \theta^{d_{m1}}) - p_2$ or $p_1 = p_2 + u(q_1, \theta^{d_{m1}}) - u(q_2, \theta^{d_{m1}})$. Thus firm 1’s profit function is given by

$$
\Pi_1 = \int_{\theta^{d_{m1}}}^{1} [p_2 + u(q_1, \theta^{d_{m1}}) - u(q_2, \theta^{d_{m1}}) - vq_1] f(\theta) d\theta - c(q_1).
$$

Maximising this with respect to $\theta^{d_{m1}}$ yields

$$
\frac{\partial \Pi_1}{\partial \theta^{d_{m1}}} = -f(\theta^{d_{m1}})[p_2 + u(q_1, \theta^{d_{m1}}) - u(q_2, \theta^{d_{m1}}) - vq_1] + (1 - F(\theta^{d_{m1}}))(u_\theta(q_1, \theta^{d_{m1}}) - u_\theta(q_2, \theta^{d_{m1}})) = 0. \quad (3)
$$

The second order condition is globally satisfied because of Assumptions A2 and A3.

Concerning firm 2 the marginal consumer $\theta^{d_{m2}}$ who is indifferent between buying at firm 2 and buying nothing is given by $u(q_2, \theta^{d_{m2}}) - p_2 = 0$ or $p_2 = u(q_2, \theta^{d_{m2}})$. Thus the profit function of firm 2 is

$$
\Pi_2 = \int_{\theta^{d_{m2}}}^{1} [u(q_2, \theta^{d_{m2}}) - vq_2] f(\theta) d\theta - c(q_2) - F.
$$

The first order condition is

$$
\frac{\partial \Pi_2}{\partial \theta^{d_{m2}}} = -f(\theta^{d_{m2}})[u(q_2, \theta^{d_{m2}}) - vq_2] + (1 - F(\theta^{d_{m2}}))u_\theta(q_2, \theta^{d_{m2}}) = 0. \quad (4)
$$

Again because of Assumption A3 the second order condition is satisfied.

In equilibrium marginal consumers $\theta^{*}_{m1}$ and $\theta^{*}_{m2}$ are given by (3) and (4) and equilibrium prices are given by $p^*_1 = u(q_2, \theta^{*}_{m2}) + u(q_1, \theta^{*}_{m1}) - u(q_2, \theta^{*}_{m1})$ and $p^*_2 = u(q_2, \theta^{*}_{m2})$.\footnote{Variables marked with a star indicate equilibrium values of the game after firm 2 has entered.}

Now let us look at stage 2 and suppose for the moment that firm 2 has entered. In this case firm 2 maximises its profit with respect to $q_2$ which yields

$$
\frac{\partial \Pi_2}{\partial q_2} = (F(\theta^{*}_{m1}) - F(\theta^{*}_{m2}))(u_q(q_2, \theta^{*}_{m2}) - v) + [u(q_2, \theta^{*}_{m2}) - vq_2] f(\theta) \frac{\partial \theta^{*}_{m1}}{\partial q_2} - c'(q_2) = 0. \quad (5)
$$
The second order condition is satisfied because of $u_{qq}(q, \theta) < 0$ and $c''(q) > 0$. $q_2^*$ is given by (5) and since $\theta_{m_1}^*$ is dependent on $q_1$, $q_2^*$ is dependent on $q_1$ as well.

Firm 2 only enters if

$$\int_{\theta_{m_1}^*}^{\theta_{m_2}^*} [u(q_2^*, \theta_{m_2}) - vq_2^*]f(\theta)d\theta - c(q_2^*) > F.$$ 

Firm 1 in stage 1 does not enter if

$$\int_{\theta_{m_1}^*}^{\theta_{m_2}^*} [u(q_2^*, \theta_{m_2}) - vq_2^*]f(\theta)d\theta - c(q_2^*) > F.$$ 

$F$ is high enough then firm 1 also has the possibility to choose $q_1$ in such a way that firm 2 does not enter. Let us denote the quality that deters entry of firm 2 by $q_1^{ED}$. It is given by

$$\int_{\theta_{m_1}^*}^{\theta_{m_2}^*} [u(q_2^*(q_1^{ED}), \theta_{m_2}) - vq_2^*(q_1^{ED})]f(\theta)d\theta - c(q_2^*(q_1^{ED})) = F.$$ 

If firm 1 produces this $q_1^{ED}$ it is a monopolist in stage 3 and earns profits of

$$\Pi_1^{ED} = \int_{\theta_{m_1}^{(q_1^{ED})}}^{\theta_{m_2}^{(q_1^{ED})}} [u(q_1^{ED}, \theta_{m}^{(q_1^{ED}))}) - vq_1^{ED}]f(\theta)d\theta - c(q_1^{ED}).$$ 

Thus firm 1 engages in entry deterrence if and only if

$$\Pi_1^{ED} = \int_{\theta_{m_1}^{(q_1^{ED})}}^{\theta_{m_2}^{(q_1^{ED})}} [u(q_1^{ED}, \theta_{m}^{(q_1^{ED}))}) - vq_1^{ED}]f(\theta)d\theta - c(q_1^{ED}) = \Pi_1^{duo}.$$ 

We are now in a position to state the equilibrium of the game:

- If $\Pi_1^{ED} > \Pi_1^{duo}$ then firm 1 chooses $q_1^{ED}$, firm 2 does not enter in stage 2 and
  $$p_1^* = u(q_1^{ED}, \theta_{m}^{*})$$ where $\theta_{m}^{*}$ is given by (1) with $q_1 = q_1^{ED}$.
- If $\Pi_1^{ED} \leq \Pi_1^{duo}$ then $q_1^{*}$ is given by (6), firm 2 enters in stage 2 and $q_2^{*}$ is given by (5). $\theta_{m_1}^{*}$ and $\theta_{m_2}^{*}$ are given by (3) and (4) and
  $$p_1^* = u(q_2^{*}, \theta_{m_2}^{*}) + u(q_1^{*}, \theta_{m_1}^{*}) - u(q_2^{*}, \theta_{m_1}^{*})$$ and $p_2^* = u(q_2^{*}, \theta_{m_2}^{*})$. 


Now this equilibrium with potential competition can be compared with the monopoly equilibrium. First look at the case where firm 2 enters. In this case fixed costs of market entry are so low that it does not pay for firm 1 to choose \( q_{1}^{*} \) such that firm 2 does not enter. Instead firm 1 sets \( q_{1}^{*} \) according to (6).

**Proposition 2**

\( q_{1}^{*} > q_{1}^{\text{mon}} \) if and only if

\[
v < u_{q_{1}}(q_{1}^{\text{mon}}, \theta_{m_{1}}) - \frac{\partial u_{q_{1}}}{\partial q_{1}}[u_{q_{2}}(q_{2}^{*}, \theta_{m_{1}}) + u_{q_{2}}(q_{2}^{*}, \theta_{m_{2}}) - u_{\theta}(q_{2}^{*}, \theta_{m_{2}})]\left(\frac{1}{F(\theta_{m_{1}}^{\text{mon}})} - \frac{1}{F(\theta_{m_{1}})}\right) - c'(q_{1}) = 0,
\]

(7)

**Proof**

\( q_{1}^{\text{mon}} \) is given by (2),

\[
\frac{\partial \Pi_{1}}{\partial q_{1}} = (1 - F(\theta_{m_{1}}^{\text{mon}}))(u_{q_{1}}(q_{1}, \theta_{m_{1}}^{\text{mon}}) - v) - c'(q_{1}) = 0,
\]

while \( q_{1}^{*} \) is given by (6),

\[
\frac{\partial \Pi_{1}}{\partial q_{1}} = (1 - F(\theta_{m_{1}}^{*}))(u_{q_{1}}(q_{1}, \theta_{m_{1}}^{*}) - v - (u_{q_{2}}(q_{2}^{*}, \theta_{m_{1}}^{*}) + u_{q_{2}}(q_{2}^{*}, \theta_{m_{2}}^{*}) - u_{\theta}(q_{2}^{*}, \theta_{m_{2}}^{*})\frac{\partial u_{q_{2}}}{\partial q_{2}}\frac{\partial q_{2}^{*}}{\partial q_{1}} - c'(q_{1}) = 0.
\]

Evaluated at \( q_{1}^{\text{mon}} \), (6) becomes

\[
[F(\theta_{m_{1}}^{\text{mon}}) - F(\theta_{m_{1}}^{*})](u_{q_{1}}(q_{1}^{\text{mon}}, \theta_{m_{1}}^{*}) - v) - (u_{q_{2}}(q_{2}^{*}, \theta_{m_{1}}^{*}) + u_{q_{2}}(q_{2}^{*}, \theta_{m_{2}}^{*}) - u_{\theta}(q_{2}^{*}, \theta_{m_{2}}^{*})\frac{\partial u_{q_{2}}}{\partial q_{2}}\frac{\partial q_{2}^{*}}{\partial q_{1}}),
\]

which can be greater or smaller than zero. Solving for \( v \) yields

\[
u_{q_{1}}(q_{1}^{\text{mon}}, \theta_{m_{1}}^{*}) - [(u_{q}(q_{2}^{*}, \theta_{m_{1}}^{*}) + u_{q_{2}}(q_{2}^{*}, \theta_{m_{2}}^{*}) - u_{\theta}(q_{2}^{*}, \theta_{m_{2}}^{*})\frac{\partial u_{q_{2}}}{\partial q_{2}}\frac{\partial q_{2}^{*}}{\partial q_{1}})\left(\frac{1}{F(\theta_{m_{1}}^{\text{mon}})} - \frac{1}{F(\theta_{m_{1}}^{*})}\right)] = v.
\]

If \( ' > ' \) is true the first derivative of the profit function of firm 1 after
entry is increasing at $q_1^{\text{mon}}$ while it is zero at $q_1^*$. Since the function is globally concave $q_1^{\text{mon}} < q_1^{\text{duo}}$.

q.e.d.

This shows that the quality level of the incumbent increases after entry if and only if marginal costs are lower than a given threshold. At first glance one may have guessed that the quality level of firm 1 in duopoly is always higher achieving a higher degree of differentiation from the entrant’s quality. But with high marginal costs this is not true. The reason is that in case of competition it is harder for the incumbent to extract consumer rent. Thus it does not pay to produce high quality if this comes at high costs.

More specifically, let us have a closer look at inequality (7). It is obvious from equation (2) that $u(q_1^{\text{mon}}, \theta_m^*) > v$. Thus if the term $[-\frac{\partial q_2}{\partial q_1} \left( (u_{q_2}(q_2^*, \theta_m^1) + u_{q_2}(q_2^*, \theta_m^2) - u_\theta(q_2^*, \theta_m^2) \right) \frac{\partial q_2}{\partial q_1}] \left( \frac{\frac{1}{F(\theta_m^{\text{mon}})} - F(\theta_m^*)}{\frac{1}{F(\theta_m^{\text{mon}})} - F(\theta_m^*)} \right)$ is greater than zero the right hand side of (7) is higher than the left hand side and we have $q_1^* > q_1^{\text{mon}}$. To get an intuition for the result suppose that $\theta_m^1 < \theta_m^{\text{mon}}$ (and thus $F(\theta_m^1) < F(\theta_m^{\text{mon}})$). Then this term is positive if $\frac{\partial q_2}{\partial q_1} < 0$, i.e. qualities are strategic substitutes. In this case an increase in $q_1^*$ has a favourable impact for firm 1 on $q_2^*$, namely a reduction of $q_2^*$. Thus $q_1^*$ unambiguously increases with competition. If instead $\frac{\partial q_2}{\partial q_1} > 0$ the qualities are strategic complements. In this case it might be optimal for firm 1 to set $q_1^* < q_1^{\text{mon}}$ to induce firm 2 to lower its quality as well. Firm 1 will do so if variable costs are high because then costs can be reduced and competition is lowered by the reaction of firm 2.

To gain some insights into welfare comparisons between monopoly and potential competition we have to give a bit more structure to the model.

**Proposition 3**

Let $u(q, \theta) = \theta q$. If qualities are strategic substitutes welfare unambiguously rises with entry.

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18In the next proposition it is shown that this is always the case if $u(q, \theta) = \theta q$. 


Proof

See the Appendix.

If \( u(q, \theta) = \theta q \) firm 1 serves more consumers in duopoly than in monopoly. The reason is that the quality deflated price \( \frac{p^*_1}{q^*_1} \) is lower. This follows from the fact that \( \theta^*_m < \theta^*_{m1} \) which for the specific utility function means that \( \frac{p^*_1 - p^*_2}{q^*_1} < \frac{p^*_{1m}}{q^*_{1m}} \), and the fact that \( \frac{p^*_2}{q^*_2} < \frac{p^*_{1m}}{q^*_{1m}} \). Taken together this implies that \( \frac{p^*_1}{q^*_1} < \frac{p^*_{1m}}{q^*_{1m}} \). Thus more consumers are buying from the incumbent. If its quality in duopoly is higher as well then welfare in duopoly is for sure higher. This is the case if qualities are strategic substitutes because then firm 2 reduces its quality as reaction to a quality increase of firm 1, which is profitable for firm 1. It should be mentioned that if qualities are strategic substitutes welfare necessarily increases. But the "only if" statement is not true. Even in case if qualities are strategic complements welfare can rise because more consumers are buying in duopoly. But it is also possible that welfare decreases because the incumbent reduces its quality and this quality reduction effect dominates the effect that more consumers are served.

Now let us turn to the case where firm 1 deters entry of firm 2. In this case firm 1 produces \( q^*_{1E} \) and is a monopolist thereafter. From Proposition 1 we know that a monopolist distorts quality downwards. So whether welfare in case of entry deterrence is higher than welfare in a pure monopoly situation depends on \( q^*_{1E} \) in comparison with \( q^*_{1m} \). If \( q^*_{1E} > (\leq) q^*_{1m} \) welfare in case of entry deterrence is higher (lower). But this depends on the reaction of \( q^*_2 \) on \( q^*_1 \). If e.g. \( \frac{\partial q^*_2}{\partial q^*_1} < 0 \) the incumbent has to increase its quality to keep the entrant out of the market. \( p^*_{1m} \) is always given by \( p^*_{1m} = u(q^*_{1m}, \theta^*_{m1}) \). Thus a change in \( q^*_{1m} \) leads to a change in \( p^*_{1m} \) of \( u_{q^*_1}(q^*_{1m}, \theta^*_{m1}) \) but \( \theta^*_{m1} \) stays unchanged and we get the following proposition.

Proposition 4

If qualities are strategic substitutes welfare in case of entry deterrence

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19 This result is obtained in many models of quality competition, see e.g. Bae & Choi (2003) or Banerjee (2003). In these models quality is exogenous. In the paper here it is shown that this result holds for endogenous quality choice as well.
is higher than in a protected monopoly. If qualities are strategic complements the reverse is true.

This shows that the threat of entry can either increase or decrease welfare depending on the strategic reaction of firm 2 to the quality of firm 1. In the most general model it is impossible to assess whether qualities are strategic substitutes or complements. But we can make a general conclusion in the specific framework of Mussa & Rosen (1978). In their model \( \theta \) is uniformly distributed, \( u(q, \theta) = \theta q \), \( v = 0 \), and \( c(q) = \frac{1}{2}q^2 \).

**Proposition 5**

In the linear-uniform-quadratic case of Mussa & Rosen (1978) qualities are strategic complements.

**Proof**

See the Appendix.

So in the uniform-linear-quadratic case welfare decreases with potential entry if fixed costs from entry are high enough such that entry is deterred. The reason is that the incumbent distorts its quality further downwards so that it is not profitable for the entrant to occupy the low quality segment and therefore the entrant stays out of the market. But this downward distortion of quality lowers welfare. In Section 5 this result will be contrasted with a model where both firms can produce many different quality levels.

### 4 Discussion

The preceding analysis points to cost-and demand-function-based reasons for an incumbent to increase or decrease its quality and price after entry. In this section we turn to a discussion of some empirical examples from different markets that give anecdotal evidence for our results.
4.1 Pricing of Pharmaceuticals after Generic Entry

In the market for pharmaceuticals, patents protect drug developers after the development of a new pharmaceutical. The aim of these patents is to give developing firms an incentive to develop new pharmaceuticals because they can earn monopoly rents during the patent period. After the expiry of the patent, entry of generic drugs is possible. In the US the Watchman-Hax Act in 1984 makes it easier for generic firms to enter the market. This makes the pharmaceutical market a suitable example for applying the results of the previous section.

By Proposition 2 our theory predicts that if variable marginal costs of production are low, quality and price of the brand-name drug should increase after entry. In the production of pharmaceuticals marginal costs are very low compared with research and development costs. For example, in the US the pharmaceutical industry has spent the largest fraction of its sales receipts to research and development among all US industries with comparable data (US Federal Trade Commission (1985)). So one would predict that prices increase after generic entry. This is confirmed by empirical studies. Scherer (2000) gives an example of the expiry of the product patent covering the cephalosporin antibiotic cephalexin in April 1987. This was sold under the brand name Keflex. After entry the price of Keflex rose from around $60 (per 100 capsules) to $85 in 1990. During this time the prices of generics went down from $30 to $15. Frank & Salkever (1997) looked at 45 drugs which faced generic competition for the first time after the Watchman-Hax Act. They found that brand-name prices increased by 50% five years after generic entry. Similar pricing patterns were obtained in the studies by Grabowski & Vernon (1992) and Scherer (1996). This supports the prediction that if variable marginal costs are low prices will rise after entry.

Rising quality is a bit harder to explain because normally quality of drugs stayed unchanged. But the brand-name producers tried to increase consumers’ perceived quality via advertising during the period of patent protection. Scherer (2000) states

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20The reason is that testing requirements for generics have been relaxed. It is only necessary to demonstrate that the drug has the same ingredients as the original, that the formulation was absorbed in the blood stream at more or less the same time, and to document good manufacturing practices of the generic firm. See Scherer (2000), p. 1321.
that in 1998 in the USA producers spend about $1 billion on direct-to-consumer advertising.\textsuperscript{21} This amount can not only be seen as informative advertising but is also done to convince consumers of the product’s quality and to separate from generics. After entry, advertisement was reduced because of the fear that this would also spur the sales of the new competitors. Thus in the market for pharmaceuticals brand-name producers did not increase the real quality of the drugs. Instead they increased perceived quality when faced with the threat of entry.

4.2 The Market for Fragrance and Cosmetics

In Singapore for a long time cosmetics were sold exclusively by authorised distributors and listed retailers. These firms demand high prices and had high price-cost margins. For example, consumers had to pay $35 to $38 for a lipstick at cosmetic counters of department stores but it costs only US $0.50 to manufacture a lipstick.\textsuperscript{22} These lipsticks are imported from the US or Europe so one had to add transportation costs. Still price cost margins were high.

In the late 1980s the parallel importer B&N entered the market. B&N imported the same products as the authorised distributors but had a simple business strategy, namely price cuts. It sold a Christian Dior lipstick at $19 or $20\textsuperscript{23} and in general offered the cosmetics up to 50% below the prices of listed retailers. The products are qualitatively similar but disadvantages for B&N were that the company was unknown at the beginning of their business and that authorised distributors placed their products on premium space and had set up cosmetic counters at department stores. What was the reaction of distributors to the entry of B&N? Beside negative advertising about parallel imports and lawsuits their main response consisted in price cuts. For example they lowered the lipstick price from $34 to $28.\textsuperscript{24}

In contrast to the pharmaceutical market in the market for fragrance and cosmetics marginal costs play the important role compared to development costs. The only source of development costs is the building up of connections to importers. But

\textsuperscript{21}See also Caves, Whinston & Hurwitz (1991).
\textsuperscript{22}See Lee, Lim & Tan (2001).
\textsuperscript{23}"Parallel Importers Make Cosmetic Firms See Red", The Straits Time, October 7, 1994, p.44.
\textsuperscript{24}"Parallel Imports: Copyright Owners Fight Back", The Straits Time, August 12, 1996, p.31.
the main bulk of costs a retailer has to bear are the delivering costs of lipsticks, the advertising costs, and the rents to be paid to department stores for display on premium space. In this respect the retailers also reduced the quality of their offers. They set up fewer cosmetic counters in stores and spend less money on costly advertising. But especially with cosmetics and fragrance the conveyed life-style of the products is very important and this can be mainly given by advertising. Since the retailers do not manufacture the cosmetics themselves the physical quality if the products stays the same. But the quality was reduced from the perspective of the consumers since the products are no longer displayed on premium space and are less advertised. Thus the observations in this market go in line with the predictions of our theory that an incumbent’s price and quality decrease if marginal variable costs are high.

5 The Model with Price Discrimination

This section analyses a model where firms can produce many different qualities which can be sold at different prices. The results of this model are later compared with the results of Section 3.

5.1 Model Framework

Consumers’ utility functions, the distribution of preferences, firms’ cost functions, and the game structure is the same as in Section 3. The only difference is that each firm can now produce not only one quality but many different qualities which are sold at different prices. We are therefore in a problem of adverse selection. We assume that for each quality a firm produces it has to bear development costs \( c(q) \) and variable costs \( v \). Assumptions A1, A2, and A3 are kept as well.

\(^{25}\)See Lee, Lim & Tan (2001).

\(^{26}\)Theoretically the assumption of development costs for each quality is necessary to avoid that firm 1 can commit costless to the whole range of qualities. If this is possible we get trivial equilibria in which firm 2 is always kept out of the market.
5.2 Monopoly Situation

As in Section 3 before solving the game consider the benchmark case where firm 1 is a monopolist. In this case we are in a standard mechanism design problem of second-degree price discrimination. The firm’s problem is to choose the optimal quality-payment schedule and the marginal consumer $\theta_{m}^{mon}$ subject to the standard participation and incentive compatibility constraints,

$$\max_{q(\theta), p(\theta), \theta_{m}^{mon}} \Pi_1 = \int_{\theta_{m}^{mon}}^{1} [p(\theta) - vq(\theta)] f(\theta) d\theta - \int_{\theta_{m}^{mon}}^{1} c(q) d\theta$$

s.t.

$$u(q(\theta), \theta) - p(\theta) \geq 0 \quad \forall \theta \geq \theta_{m}^{mon}$$

$$u(q(\theta), \theta) - p(\theta) \geq u(q(\hat{\theta}), \theta) - p(\hat{\theta}) \quad \forall \theta, \hat{\theta} \geq \theta_{m}^{mon}.$$

The equilibrium is characterised in the following lemma:

**Lemma 1**

The optimal $q(\theta)^{mon*}, p(\theta)^{mon*}, \theta_{m}^{mon*}$ are given by the following equations:

$$p^{mon*}(\theta) = u(q^{mon*}(\theta), \theta) - \int_{\theta_{m}^{mon*}}^{\theta} \frac{\partial u(q^{mon*}(\tau), \tau)}{\partial \theta} d\tau, \quad (8)$$

$$\frac{\partial u(q^{mon*}(\theta), \theta)}{\partial q} - \left(1 - F(\theta)\right) \frac{\partial^2 u(q^{mon*}(\theta), \theta)}{\partial q \partial \theta} - \nu - \frac{\nu c'(q^{mon*}(\theta))}{f(\theta)} = 0, \quad (9)$$

$$[u(q^{mon*}(\theta_{m}^{mon*}), \theta_{m}^{mon*}) - vq^{mon*}(\theta_{m}^{mon*})] f(\theta_{m}^{mon*}) - c(q^{mon*}(\theta)) = (1 - F(\theta_{m}^{mon*})) \frac{\partial u(q^{mon*}(\theta_{m}^{mon*}), \theta_{m}^{mon*})}{\partial \theta_{m}^{mon*}}, \quad (10)$$

**Proof**

See the Appendix.

The first two equations are standard in second degree price discrimination. The first states that the price for each type $\theta$ is the utility type $\theta$ gets from buying a
good of quality $q^{\text{mon}}(\theta)$ minus a term which is increasing in $\theta$. So higher types get a higher utility to prevent them from choosing the contract designed for the lower types. The second equation states that the quality a type $\theta$ gets is increasing in $\theta$ but is always lower than the optimal quality except for $\theta = 1$. This is the famous ‘no-distortion-at-the-top-result’. The third equation states that the marginal consumer is characterised in such a way that the net gain of serving him (the left hand side of (10)) is exactly equal to the loss that occurs to the firm because it has to give a higher rent to the inframarginal consumers (the right hand side of (10)).

Concerning welfare the firm offers a whole range of qualities where higher types get higher quality. But except for the highest type quality is distorted downwards.

### 5.3 Analysis of the Duopoly Situation

In the following we denote the quality range of firm 1 $Q_1 = [q_1^-, q_1^+]$ and the quality range of firm 2 $Q_2 = [q_2^-, q_2^+]$. $Q_2(Q_1)$, as in Section 3, is the best response of firm 2 after entry if firm 1 produces a quality range $Q_1$. If the quality ranges do not overlap, i.e. the lowest quality of firm $i$, $q_i^-$, is higher than the highest quality of firm $j$, $q_j^+$, we say that $Q_i > Q_j$.

Again before starting with the analysis of the entry game we make two assumptions which are modifications of assumptions A4 and A5 of Section 3.

\[ A4' : \quad \Pi_2(Q_1^{\text{mon}}, Q_2(Q_1^{\text{mon}})) > 0. \]

$Q_1^{\text{mon}}$ is the quality range firm 1 produces in the monopoly case and $A4'$ states that firm 2 enters if firm 1 produces $Q_1^{\text{mon}}$.

\[ A5' : \quad \Pi_1(Q_1^H, Q_2(Q_1^H)) > \Pi_1(Q_1^L, Q_2(Q_1^L)) \]

whenever $Q_1^H > Q_2(Q_1^H)$ and $Q_1^L < Q_2(Q_1^L)$.

Assumption $A5'$ states that it is profitable for firm 1 to be the high quality firm, i.e. producing a quality range which is above the one of firm 2.

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\[ ^{27} \text{In Lemma 2 we show that in equilibrium this is always the case.} \]
Again the game is solved by backwards induction. First look at the case where firm 2 did not enter in stage 2. By the same calculations as in the monopoly case we get that prices are $p^*(\theta) = u(q(\theta), \theta) - \int_{\theta_m}^{\theta} \frac{\partial u(q(\tau), \tau)}{\partial \theta} d\tau$. This prices are independent of the ends of the quality range firm 1 has produced in stage 1.

Now let us turn to the case where firm 2 entered in stage 2.\(^{28}\) We have to determine the prices given that firm 1 produces quality range $Q_1$ and firm 2 produces quality range $Q_2$.\(^{29}\)

For simplicity let us assume first that $q_1^- > q_2^+$. We will later show that this is always the case. The marginal consumer $\theta_m$ who is indifferent between buying $q_1^-$ and $q_2^+$ is given by $u(q_1^-, \theta_m) - p_1(\theta(q_1^-)) = u(q_2^+, \theta_m) - p_2(\theta(q_2^+))$ or $p_1(\theta(q_1^-)) = p_2(\theta(q_2^+)) + u(q_1^-, \theta_m) - u(q_2^+, \theta_m)$. Firm 1’s maximisation problem in stage 3 can be written as

$$\max_{p_1(\theta), p_1(q_1^-)} \Pi_1 = \int_{\theta(q_1^-)}^{\theta(q_1^-)} \left[ p_1(\theta) - u(q(\theta)) \right] f(\theta) d\theta + \int_{\theta_m}^{\theta(q_1^-)} \left[ p_1(\theta_m) - u(q(\theta)) \right] f(\theta) d\theta - \int_{\theta(q_1^-)}^{\theta(q_1^-)} c(q(\theta)) d\theta$$

subject to

$$u(q(\theta), \theta) - p_1(\theta) \geq u(q_2^+, \theta) - p_2(q_2^+) \quad \forall \theta \geq \theta_m$$

$$u(q(\theta), \theta) - p_1(\theta) \geq u(q(\hat{\theta}), \theta) - p_1(\hat{\theta}) \quad \forall \theta, \hat{\theta} \geq \theta_m,$$

where $\theta(q_1^-)$ is the highest type who buys quality $q_1^-$.\(^{30}\)

Firm 2’s maximisation problem in stage 3 is

$$\max_{p_2(\theta), p_2(q_2^+)} \Pi_2 = \int_{\theta_m}^{\theta(q_2^+)} \left[ p_2(\theta) - u(q(\theta)) \right] f(\theta) d\theta + \int_{\theta_m}^{\theta(q_2^+)} \left[ p_2(\theta) - u(q(\theta)) \right] f(\theta) d\theta - \int_{\theta(q_2^+)}^{\theta(q_2^+)} c(q(\theta)) d\theta$$

subject to

$$u(q(\theta), \theta) - p_2(\theta) \geq u(q_1^-, \theta) - p_1(q_1^-) \quad \forall \theta < \theta_m$$

$$u(q(\theta), \theta) - p_2(\theta) \geq u(q(\hat{\theta}), \theta) - p_2(\hat{\theta}) \quad \forall \theta, \hat{\theta} < \theta_m,$$

where $\theta(q_2^+)$ is the lowest type who buys quality $q_2^+$.\(^{31}\)

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\(^{28}\)The analysis in this section draws heavily on Champsaur & Rochet (1989). The difference is that firms choose qualities simultaneously in Champsaur & Rochet (1989) while in my model qualities are chosen sequentially. But the analysis of the second and the third stage is quite similar.

\(^{29}\)In principle we should analyse the third stage for arbitrary $(Q_1, Q_2)$. However, this is clearly impossible to do. But one can put the restriction on $(Q_1, Q_2)$ that there is never a whole in one of two quality ranges for the same reason as for the monopolist. For a discussion on that issue and why it is reasonable to conduct the analysis in the way it is done in this chapter see Champsaur & Rochet (1989).
Solving for $p_1(\theta)$ and $p_2(\theta)$ yields for the same reasons as in Lemma 4.1

$$p_1(\theta) = u(q(\theta), \theta) - \int_{\theta(q_1^-)}^{\theta} \frac{\partial u(q(\tau), \tau)}{\partial \theta} d\tau + p_1(q_1^-) - u(q_1^-, \theta(q_1^-)), \quad (11)$$

and

$$p_2(\theta) = u(q(\theta), \theta) - \int_{\theta(m_2)}^{\theta} \frac{\partial u(q(\tau), \tau)}{\partial \theta} d\tau + p_2(q_2^+) - u(q_2^+, \theta(q_2^+)). \quad (12)$$

Plugging this back in the profit function and solving for $p_1(q_1^-)$ and $p_2(q_2^+)$ gives

$$p_1^*(q_1^-) = vq_1^- + \frac{1 - F(\theta_m)}{f(\theta_m)} [u_\theta(q_1^-, \theta_m) - u_\theta(q_2^+, \theta_m)], \quad (13)$$

$$p_2^*(q_2^+) = vq_2^+ + \frac{F(\theta_m) - F(\theta_{m2})}{f(\theta_m)} [u_\theta(q_1^-, \theta_m) - u_\theta(q_2^+, \theta_m)]. \quad (14)$$

Having solved stage 3 of the game we can go back one stage to stage 2 where firm 2 chooses its optimal quality range. The problem of firm 2 is thus

$$\max_{q(\theta), q_2^+, \theta_{m2}} \Pi_2 = \int_{\theta_{m2}}^{\theta(q_2^+)} [u(q(\theta), \theta) - \int_{\theta_{m2}}^{\theta} \frac{\partial u(q(\tau), \tau)}{\partial \theta} d\tau + p_2^*(q_2^+) - u(q_2^+, \theta(q_2^+)) - vq(\theta) - \frac{c(q(\theta))}{f(\theta)}] f(\theta) d\theta +$$

$$\int_{\theta_{m2}}^{\theta(q_2^+)} [p_2^*(q_2^+) - vq_2^+] f(\theta) d\theta.$$

Differentiating with respect to $\theta_{m2}$ and $q(\theta)$ yields

$$(F(\theta_m) - F(\theta_{m2})) (\frac{\partial u(q^*(\theta), \theta_{m2})}{\partial \theta_{m2}}) - \frac{f(\theta_{m2})}{f(\theta_m)} [u_\theta(q_1^-, \theta_m) - u_\theta(q_2^+, \theta_m)] = \frac{c(q(\theta_{m2}))}{f(\theta_{m2})} + p_2^*(q_2^+) - u(q_2^+, \theta(q_2^+)) - vq(\theta_{m2})] \quad (15)$$

and

$$\frac{\partial u(q^*(\theta), \theta)}{\partial \theta} - \left( \frac{1 - F(\theta)}{f(\theta)} \right) \frac{\partial^2 u(q^*(\theta), \theta)}{\partial \theta \partial \theta} - u - \frac{c'(q^*(\theta))}{f(\theta)} = 0,$$

$\forall \theta$ with $\theta(q_2^+) > \theta \geq \theta_{m2}$. \quad (16)

Before differentiating with respect to $q_2^+$ it is helpful to decompose the profit func-
tion as Champsaur & Rochet (1989) do. Inserting $p^*_2(q^+_2)$ in $\Pi_2$ yields

$$\Pi_2 = \frac{(F(\theta_m) - F(\theta_{m2}))^2}{f(\theta_m)} \left[ u_\theta(q^+_2, \theta_m) - u_\theta(q^+_2, \theta_m) \right] + \int_{\theta_{m2}}^\theta [u(q(\theta), \theta) - \int_{\theta_{m2}}^\theta \frac{\partial u(q(\tau), \tau)}{\partial \tau} d\tau - u(q^+_2, \theta(q^+_2))] - vq(\theta) + \alpha q^+_2 - \frac{c(q(\theta))}{f(\theta)} \right] f(\theta) d(\theta).$$

(17)

The first term is dependent on $q^-_1$ and $q^+_2$, while the second term (the integral term) is independent of $q^-_1$. In the following we denote the integral term by $I(q^+_2)$. This decomposition also shows that $q^+_2$ is only dependent on $q^-_1$ but not on the other qualities firm 1 produces. The first order condition for $q^+_2$ is thus given by

$$-\frac{(F(\theta_m) - F(\theta_{m2}))^2}{f(\theta_m)} u_{\theta q}(q^+_2, \theta_m) + \frac{\partial I(q^+_2)}{\partial q} = 0$$

(18)

It is now possible to show that $q^+_2 < q^-_1$.

**Lemma 2**

There is always a gap between the quality ranges of firm 1 and firm 2.

**Proof**

See the Appendix.

This result is different to Champsaur & Rochet (1989). If firms decide simultaneously about their qualities there can be equilibria where the quality ranges overlap and firms make zero profits with these overlapping qualities. If development costs exists firms would make losses with overlapping qualities and they may decide not to produce them even in the simultaneous move game. Despite this, in the sequential move game even if $c(q) = 0$ product ranges would never overlap.

30Champsaur & Rochet (1989) call the first term pure differentiation profit and the second term pure segmentation profit.

31Champsaur & Rochet (1989) assume that there are no development costs, i.e. $c(q) = 0$. If such development costs exists firms would make losses with overlapping qualities and they may decide not to produce them even in the simultaneous move game. Despite this, in the sequential move game even if $c(q) = 0$ product ranges would never overlap.
So firm 2’s profit is increasing if firm 2 produces a smaller quality range. But this also implies that \( \frac{\partial q_2^+}{\partial q_1^-} > 0 \) so the lowest quality of firm 1 and the highest quality of firm 2 are strategic complements. Firm 1 will take this into account in its decision of \( Q_1 \) in stage 1.

Let us now turn to stage 1. As in Section 3 firm 1 has two possibilities either to accommodate entry or to deter entry. Let us look at each case in turn. If fixed costs are low firm 1 finds it optimal to accommodate entry. Decomposing firm 1’s profit function in the same way as firm 2’s profit function before we get a maximisation problem of

\[
\max_{q_1^-} \Pi_1 = \frac{(1-F(\theta_m))^2}{f(\theta_m)} [u_\theta(q_1^-, \theta_m) - u_\theta(q_2^+, \theta_m)] + \\
\int_{\theta(q_1^-)}^{\theta(q_1^+)} [u(q(\theta), \theta)] - \int_{\theta(q_1^-)}^{\theta(q_1^+)} \left[ \frac{\partial u(q(\theta), \theta)}{\partial q} + \frac{\partial u(q(\theta), \theta)}{\partial \theta} \right] d\theta - u(q_1^-, \theta(q_1^-)) \\
- v q(\theta) + v q_1^- - c'(q_1^-) f(\theta) d(\theta).
\]

In the following we call the integral term \( I(q_1^-) \).

We get two first order conditions

\[
\frac{\partial u(q^*(\theta), \theta)}{\partial q} - \left( \frac{1-F(\theta)}{f(\theta)} \right) \frac{\partial^2 u(q^*(\theta), \theta)}{\partial q \partial \theta} - v - \frac{c'(q^*(\theta))}{f(\theta)} = 0, \quad \forall \theta \quad \text{with} \quad \theta(q_1^-) \leq \theta \leq 1. \tag{20}
\]

and

\[
- \frac{1-F(\theta_m)^2}{f(\theta_m)} (u_\theta(q_1^-, \theta_m) - u_\theta(q_2^+, \theta_m) \frac{\partial q_2^+}{\partial q_1^-}) + \frac{\partial I(q_1^-)}{\partial q} = 0. \tag{21}
\]

From the first of these two equations it is apparent that all types \( \theta(q_1^-) \leq \theta \leq 1 \) get the same quality as in monopoly because this equation coincides with equation (9). All types \( \theta_m \leq \theta < \theta(q_1^-) \) get a higher quality because they buy \( q_1^- \) which is above \( q(\theta) \) in the monopoly case given by equation (9).

The term \( u_\theta(q_2^+, \theta_m) \frac{\partial q_2^+}{\partial q_1^-} \) in equation (21) is greater than zero because we know that \( \frac{\partial q_2^+}{\partial q_1^-} > 0 \). This expresses that with a change in \( q_1^- \) firm 1 can change firm 2’s reaction in stage 2. In the model of Champsaur & Rochet (1989) this term does not exist because qualities are chosen simultaneously. Thus the incumbent produces a larger quality range than with a simultaneous quality choice to shift firm 2’s upper quality downwards.
Let us now look at the case where firm 1 deters entry of firm 2. From equation (17) we know that \( q_2^+ \) does only depend on \( q_1^- \) and from equation (19) \( \frac{\partial \Pi_2}{\partial q_1} > 0 \). So if firm 1 wants to deter entry it has to enlarge its quality range compared with the monopoly situation. The intuition is straightforward. There is less space in the product range left for firm 2 because firm 1 has occupied more quality levels and if fixed entry costs \( F \) are high enough firm 2 founds it not profitable to enter. Let us denote the quality range \( Q_1 \) which deters entry by \( Q_{ED}^1 = [q_{ED1}, q_1^+] \) where \( Q_{ED}^1 \) is given by \( \Pi_2(Q_{ED}^1, Q_2(Q_{ED}^1)) = 0 \).

We are now in a position to describe the equilibrium of the game:

- If \( \Pi_{ED}^1 > \Pi_1^{duo} \) then \( Q_1^* = [q_{1}^{ED}, q_1^+] \) where \( q^*(\theta) \) is given by (9), firm 2 does not enter in stage 2 and prices are given by \( p^*(\theta) = u(q^*(\theta), \theta) - \int_{\theta}^{q^*(\tau)} \frac{\partial u(q^*(\tau), \tau)}{\partial \theta} d\tau \).

- If \( \Pi_{ED}^1 \leq \Pi_1^{duo} \) then \( Q_1^* = [q_1^-, q_1^+] \) where \( q^*(\theta) \) is given by (20), \( q_1^- \) is given by (21). Firm 2 enters in stage 2 and produces a quality range of \( Q_2^* = [q_2^-, q_2^+] \) where \( q^*(\theta) \) is given by (16), \( q_2^+ \) is given by (18) and \( \theta_{m2}^* \) is given by (15). Prices of the firm are given by (11), (12), (13), and (14).

This equilibrium can be compared with the monopoly outcome with regard to consumer rent and welfare. First we analyse the case where firm 2 enters. Comparing welfare of market entry with welfare under pure monopoly we get the following proposition.

**Proposition 6**

Welfare in case of market entry is higher than under monopoly if and only if

\[
\int_{\theta(q_1^-)}^{\theta(q_1^+)} [u(q_1^-, \theta(q_1^-)) - u(q(\theta), \theta) - vq_1^- + vq(\theta) + \frac{c(q(\theta))}{f(\theta)}] f(\theta) d\theta - c(q_1^-) - F \\
+ \int_{\theta(q_2^-)}^{\theta(q_2^+)} [u(q(\theta), \theta) - vq(\theta) - \frac{c(q(\theta))}{f(\theta)}] f(\theta) d\theta \\
> \int_{\theta(q_2^-)}^{\theta(q_2^+)} [u(q(\theta), \theta) - u(q_2^-, \theta(q_2^-)) + vq_1^- - vq(\theta) - \frac{c(q(\theta))}{f(\theta)}] f(\theta) d\theta + c(q_2^+) .
\]  

(22)

**Proof**

See the Appendix.
If firm 2 enters some consumers stay with firm 1, others switch to firm 2, while a third group which has not bought in monopoly does now buy from firm 2. Types \( \theta \) with \( \theta(q^-_1) \leq \theta < 1 \) stay at firm 1 and get the same quality as in monopoly. This can be seen from the first order conditions of the quality maximisation, (9) and (20). Consumers between \( \theta_{duo_m}^* \) and \( \theta(q^-_1) \) are consuming higher quality in duopoly, namely \( q^-_1 \), than in monopoly. This leads to a rise in welfare. But consumers between \( \theta_{duo_m}^* \) and \( \theta(q^+_1) \) are now getting a lower quality, \( q^+_2 \), than in monopoly because they buy from firm 2. Consumers with a \( \theta \) below \( \theta(q^+_2) \) but above \( \theta_{mon^*}^m \) buy the same quality as before since equations (9) and (16) coincide. Customer types \( \theta_{mon^*}^m > \theta \geq \theta_{m2}^* \) have not bought in monopoly but are buying now from firm 2.

Thus we have two sources for a welfare increase, namely that more consumers are served and that types between \( \theta_{duo_m}^* \) and \( \theta(q^-_1) \) buy higher quality. But there are two sources for a welfare loss as well, namely that types between \( \theta(q^+_2) \) and \( \theta_{duo_m}^* \) buy lower quality and the fixed costs of entry \( F \). The overall effect on welfare is therefore ambiguous.

But we can say more about consumer rent.

**Proposition 7**

Consumer rent in case of market entry is always higher than in monopoly.

**Proof**

See the Appendix.

The intuition behind this result is simple. In monopoly the marginal consumer \( \theta_{mon^*}^m \) gets zero rent. But in duopoly there is competition for this consumer. Thus he gets a positive utility. But because the incentive compatibility constraints have to be satisfied this leads to an increase of the rents for all types above. Since more consumers are served in duopoly utility for the types below \( \theta_{mon^*}^m \) weakly increases as well.

Now let us turn to the case where firm 1 deters entry. As was already mentioned firm 1 deters entry by enlarging its product line and producing more qualities than in the monopoly case. So more people are served. But since the incentive compat-
ility constraints must be satisfied this results in lower prices for all consumers who bought already in the monopoly case. Thus we get the following proposition.

**Proposition 8**

If the incumbent can produce a range of qualities welfare and consumer rent in case of entry deterrence are higher than in case of pure monopoly.

The proof is omitted.

This result can be contrasted with the result of Section 3 where firms can produce only one quality level. If in that case qualities are strategic complements welfare in case of entry deterrence is lower because the incumbent distorts its quality downwards. In case of price discrimination the lowest quality of the incumbent and the highest one of the entrant are strategic complements. This results in an enlargement of the quality range in the segment of low qualities and increases welfare. The rent for every consumer who buys is higher than in monopoly as well because only the marginal one gets zero utility and prices for the ’old’ consumers are lower to prevent them from buying lower qualities.

It is also interesting to investigate under which conditions it is more profitable for an incumbent to deter entry than to accommodate entry.

**Proposition 9**

There exists a threshold value $v'$. If $v < v'$ the incumbent deters entry, if $v \geq v'$ entry is accommodated.

**Proof**

The incumbent’s profit if entry is deterred is given by

$$\Pi_{1}^{ED} = \int_{\theta_{m}(q^{ED})}^{1} [u(q(\theta), \theta) - vq(\theta) - \int_{q^{ED}}^{\theta} \frac{\partial u(q^{*}(\tau), \tau)}{\partial \theta} d\tau - \frac{c(q(\theta))}{f(\theta)}] f(\theta) d\theta.$$
If entry is accommodated profit is given by

\[
\Pi_{1}^{duo} = \int_{\theta_{m}}^{\theta_{m}^{+}} \left[ 1 - F(\theta_{m}) \right] \left[ u(\theta_{m}, q_{m}^{+}) - u(\theta_{m}, q_{m}^{-}) \right] f(\theta) d\theta + \int_{\theta_{m}^{-}}^{\theta_{m}^{-}} \left[ u(q_{m}^{-}, \theta_{m}) - u(q_{m}^{-}, \theta_{m}) \right] f(\theta) d\theta.
\]

Thus entry is deterred if \( \Pi_{1}^{ED} > \Pi_{1}^{duo} \). Rearranging terms yields

\[
\int_{\theta_{m}^{-}}^{\theta_{m}^{-}} \left[ u(q(\theta), \theta) \right] - \int_{\theta_{m}^{-}}^{\theta_{m}^{-}} \frac{\partial u(q(\theta), \theta)}{\partial \theta} d\theta - vq(\theta) + vq_{m}^{-} + \frac{1}{f(\theta)} \left[ u(\theta_{m}, q_{m}^{-}) - u(\theta_{m}, q_{m}^{-}) \right] f(\theta) d\theta.
\]

The right hand side is independent of \( v \) while the left hand side is strictly decreasing in \( v \). Thus there exists a value \( v' \) below which the left hand side is higher and above which the right hand side is higher.

q.e.d.

Thus if \( v \) is small the incumbent deters entry. The intuition is that in order to deter entry the incumbent has to enlarge its product line. This is costly for him. But if costs are small it pays the incumbent to bear these costs to enjoy monopoly power afterwards. If instead costs are high this enlargement is not profitable. The incumbent reduces its product line to save on costs but faces competition from the entrant. In the next section we provide two examples that seem to fit very well with the results of our theory.

6 Empirical Examples

As in Section 4 in this section we present two empirical examples from different industries that seem to resonate well with our theory.
6.1 Airline Industry

In Europe deregulation of the air transportation market started in the late 1980’s and lasted till 1993. The European Council of Ministers decided to launch three ‘liberalisation packages’ but only the last one which was launched in 1993 really caused market liberalisation. After this package each airline was allowed to offer services with no restrictions either on prices or on routes.32

One of the most striking developments of this deregulation was the entry of the so called ‘no-frills’-airlines or low cost carriers starting in summer 1995 with Ryanair.33 These low cost carriers offer little or no services but demand prices which are very cheap.34 Also the low cost carriers mainly fly to secondary airports like Stansted instead of Heathrow in London or Frankfurt-Hahn instead of Frankfurt. So the quality of these low cost carriers is obviously below that of the established airlines.

Usually all established airlines engage in second degree price discrimination. So there can be two possible reactions of the established airlines to this entry threat. They can either expand their quality range to deter entry in the low quality segment or accommodate entry and reduce their quality range to lessen competition. From Proposition 4.9 we would predict that if variable costs are high the reaction would be a contraction of the quality range while if costs are low entry would be deterred by introducing an own low cost carrier. In the airline industry there are examples of both practices.

On long-haul routes the U.K. carrier British Airways focused on the business traveller segment and reduced its quality range.35 The aim of British Airways was to offer premium services and facilities to charge higher prices and attract a higher number of business travellers. The segment of the leisure travellers was given away to the low cost carriers.

On short-haul routes costs are to some extent cheaper than on long-haul routes.

33 For an extensive overview of low cost carriers in Europe see Gilroy, Lukas, & Volpart (2003).
34 Recently there was an offer of Ryanair to fly from Salzburg (Austria) to London with return flight for 1 Cent. Although the time of the flight was not attractive it is hard to imagine such an offer five years ago.
For example, on intercontinental flights by regulation three or four pilots are needed instead of only two as on continental flights and also more board personnel. This results in higher personnel costs. After a long-haul flight an airline is obliged to maintain the aircraft because the engine has worked for a long time and the risk of a crash is increased.\footnote{36}{See Doganis (2001).} This causes fewer capacity utilization of a long-haul plane and therefore higher costs. As predicted by our theory the strategy of many established airlines on short-haul routes was very different than the one on long-haul routes. As an example we take the case of Lufthansa in Germany. In October 2002 the low cost carrier Germanwings was founded which is an affiliate company of Eurowings. In turn, Lufthansa holds 24.9\% of Eurowings and has the option to enlarge its share up to 49\%.\footnote{37}{See Gilroy, Lukas, & Volpert (2003).} Germanwings operates mainly on routes in Germany which are offered by Lufthansa as well. So the foundation of Germanwings can be seen as an entry deterrence strategy of Lufthansa to occupy the lower market segment and to deter entry of competing low cost carriers.\footnote{38}{As mentioned in the introduction a similar strategy was pursued by British Airways and KLM.}

A different interpretation for the introduction of a low cost carrier by an established airline is given by Johnson & Myatt (2003). They argue that these low cost carriers are introduced as fighting brands to other competitive low cost airlines. Without entry of these competitors the subsidiary would not have been founded because of negative effects on core operations but after entry the low quality segment is opened and the established airline finds it profitable to enter. This might be true in case of GO which was purchased by Easyjet in 2002. But in case of Lufthansa, Germanwings was clearly introduced to deter entry of other low cost airlines and up to now no independent low cost airline has entered the German market.

### 6.2 Brand-Controlled Generics in the Pharmaceutical Market

In Subsection 4.1 we gave some evidence that prices of brand-name drugs increased after the entry of generics. However, some patent-holding firms pursued
a different strategy namely to introduce a 'branded generic', i.e. the same drug under a different label. These branded generics were introduced shortly before patent expiration and were priced below the prices of the branded drugs. Hollis (2003) reports that the success of these branded generics in Canada was very impressive. While in the 1980’s they had only a tiny share of total generic sales this share has grown to 34.6% in 1999 which is an amount in money terms of approximately $500 million. The reason was obviously to deter entry of generic competitors as Scherer (2000) states:

In this way they (brand-name firms) gained a "first mover advantage" in the generic market, secured the leading share of generic sales, and perhaps thereby discouraged some would-be generic suppliers from entering and driving prices even lower.

However, not all brand name producers introduced these pseudo-generics. In the US a study of the U.S. Congressional Budget Office (1998) reports that among 112 drugs with generic competition only 13 sold its own generic products. But this is in line with the predictions of our theory that not all firms expand their product line to deter entry but only those with low costs. In Canada in the 1990’s, Altimed, a joint venture of three brand-name firms, was created. The purpose of this joint venture was to sell branded generics. For this three firms after the joint venture it was easily and cheaply possible to sell generics. In contrast, in the US such a joint venture was not created so brand name pharmaceutical firms have to bear higher costs of introducing their own generics. This might be a reason why many of them found it profitable to accommodate entry of generic competitors.

7 Conclusion

The reactions of incumbents on entry threats are very different. Some firms accommodate entry and prune their product line while others deter entry and ex-

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40 An important source for these costs is the fear of destroying the brand name. This fear was not by present in case of Altimed because it emerged as an own brand rapidly.
pand their product line. In the single quality case post-entry prices of incumbents in some markets are higher than pre-entry prices while in other markets they are lower.

This paper analysed a model of vertical product differentiation where an incumbent and an entrant can either produce a single quality or a quality range. We show that in the single quality case the behaviour of the incumbent depends on the cost function and on the nature of strategic competition (whether qualities are strategic complements or strategic substitutes). We have shown that if qualities are strategic complements the incumbent deters entry by reducing its quality which leads to a welfare loss compared with monopoly. In case of entry accommodation quality might be lowered as well to cause a quality reduction of the entrant and reduce price competition. With low marginal costs quality of the incumbent increases after entry which results in a welfare gain. Also if qualities are strategic substitutes the incumbent increases its quality to differentiate itself from the entrant. If firms can produce a quality range the results are different. To deter entry the incumbent has to enlarge its quality range and this leads to a welfare increase. If entry is accommodated the consequences on welfare are not clear because some consumers buy a higher quality while others buy a lower one.

We have not provided a substantial empirical analysis but have given examples from different industries that seem to fit well with the predictions of our theory. Since we relate the results to firm’s cost functions which are observable in many industries we give predictions which are potentially testable.

To conclude the paper we want to discuss some policy implications resulting from our theory. Let us first look at the case where production of a quality range is possible. In this case we find that the effects on welfare are positive in case of entry deterrence and unclear in case of entry accommodation but consumer rent increases in both cases.41 This leads to the conclusion that deregulation and potential entry have positive consequences in industries in which it is possible to produce a qual-

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41We have not done a welfare comparison between the case of entry deterrence and entry accommodation. This is an interesting topic for further research because it can provide some policy implications, e.g. if it should be allowed for incumbents to establish a subsidiary brand which produces a downgrade version of the product.
ity range. Thus governments should pursue the policy of free market entry and reduce legal barriers like it was done in the deregulation of the airline industry in the US and Europe.

The effects in the single quality case are not so clear. Whether welfare increases with potential entry depends heavily on the nature of competition. But normally it is hard to assess if products are strategic complements or substitutes. Thus governments should be careful in deregulating such markets because potential competition does not necessarily lead to a welfare gain.
8 Appendix

8.1 Proof of Proposition 1

We first show that the monopolist provides too low quality. Welfare is given by

\[ WF = \int_{\theta_{WF}}^{1} [u(q, \theta) - vq] f(\theta) d\theta - c(q). \]

For a given \( q \) welfare is maximised if

\[ \frac{\partial W F}{\partial \theta_{WF}} = u(q, \theta_{WF}) - vq = 0. \]

(23)

In monopoly \( \theta_{m}^{\text{mon}} \) is given by

\[ u(q, \theta_{m}^{\text{mon}}) - vq = 1 - F(\theta_{m}^{\text{mon}}) f(\theta_{m}^{\text{mon}}) u_{\theta}(q, \theta_{m}^{\text{mon}}). \]

(24)

The left hand side of equation (24) is greater 0 while it is 0 in equation (23). Since \( u_{\theta}(q, \theta_{m}^{\text{mon}}) > 0 \) it follows that \( \theta_{m}^{WF} < \theta_{m}^{\text{mon}} \). Thus for a given \( q \) the monopolist serves too few consumers.

Maximising welfare with respect to quality yields

\[ \frac{\partial}{\partial q} \int_{\theta_{WF}}^{1} [u(q, \theta) f(\theta) d\theta] = (1 - F(\theta_{m}^{WF})) v + c'(q). \]

(25)

The equivalent formula for the monopolist is

\[ (1 - F(\theta_{m}^{\text{mon}}))(u_{q}(q, \theta_{m}^{\text{mon}}) - v) = c'(q). \]

(26)

If both qualities were the same we can solve both equations (25) and (26) for \( c(q) \) and get

\[ \frac{\partial}{\partial q} \int_{\theta_{WF}}^{1} [u(q, \theta) f(\theta) d\theta] - (1 - F(\theta_{m}^{WF})) v = (1 - F(\theta_{m}^{\text{mon}}))(u_{q}(q, \theta_{m}^{\text{mon}}) - v). \]
This can be written as

\[
\frac{\partial}{\partial q} \int_{\theta_{\text{mon}}^m}^{1} [u(q, \theta) - u(q, \theta_{\text{mon}}^m)] f(\theta) d\theta + \frac{\partial}{\partial q} \int_{\theta_{\text{WF}}^m}^{\theta_{\text{mon}}^m} [u(q, \theta) f(\theta) d\theta] = (F(\theta_{\text{mon}}^m) - F(\theta_{\text{WF}}^m)) v.
\]

The second term on the left hand side is the increase in utility for all consumers between \(\theta_{\text{mon}}^m\) and \(\theta_{\text{WF}}^m\) from a marginal increase in \(q\). The term on the right hand side is the increase in variable costs if consumers between \(\theta_{\text{mon}}^m\) and \(\theta_{\text{WF}}^m\) are served. Thus the second term on the left hand side must be higher than the right hand side because otherwise it would not have been welfare maximising to serve consumers between \(\theta_{\text{mon}}^m\) and \(\theta_{\text{WF}}^m\). Since the first term on the left hand side is positive as well we get that the first order condition for \(q_{\text{WF}}\) is positive at \(q_{\text{mon}}\). Thus \(q_{\text{WF}} \geq q_{\text{mon}}\).

Turning back to the comparison of marginal consumers we have shown in equations (23) and (24) that if \(q_{\text{WF}} = q_{\text{mon}}\) then \(\theta_{\text{WF}}^m = \theta_{\text{mon}}^m\). But now we know that \(q_{\text{WF}} > q_{\text{mon}}\). A comparison of the left hand sides of (23) and (24) shows that for \(\theta_{\text{WF}}^m = \theta_{\text{mon}}^m\) the left hand side of (23) is higher. But since the right hand side of (24) is higher it follows that \(\theta_{\text{WF}}^m < \theta_{\text{mon}}^m\).

q.e.d.

8.2 Proof of Proposition 3

If \(u(q, \theta) = \theta q\) the marginal consumer in the monopoly case is given by \(\theta_{m} q_{1}^{mon} - p_{1}^{mon} = 0\) or \(\theta_{m} = p_{1}^{mon} / q_{1}^{mon}\). This yields a first order condition for \(\theta_{m}\) of

\[
1 - F(\theta_{m}^{mon}) - f(\theta_{m}^{mon})(\theta_{m}^{mon} - v) = 0.
\]

In duopoly the marginal consumer \(\theta_{\text{duo}}^{m_1}\) who is indifferent between buying from firm 1 and buying from firm 2 is given by \(\theta_{\text{duo}}^{m_1} q_1 - p_1 = \theta_{\text{duo}}^{m_1} q_2 - p_2\) or \(\theta_{\text{duo}}^{m_1} = \frac{p_1 - p_2}{q_1 - q_2}\). The first order condition for the incumbent is then

\[
F \left( \frac{p_1 - p_2}{q_1 - q_2} \right) - f \left( \frac{p_1 - p_2}{q_1 - q_2} \right) \frac{p_2 - v q_2}{q_1 - q_2} - F \left( \frac{p_2}{q_2} \right) - f \left( \frac{p_2}{q_2} \right) \left( \frac{p_2}{q_2} - v \right) = 0
\]
or

\[ 1 - F(\theta^*_m) - f(\theta^*_m)(\theta^*_m - \frac{p_2 - v}{q_1 - q_2}) = 0. \]  

(27)

Evaluating (27) at \(\theta^*_{mon}\) yields

\[ vq_2 - p_2 < 0. \]

Since the profit function is globally concave in \(\theta\) this shows that \(\theta^*_{mon} > \theta^*_m\) so more consumers are buying from firm 1 in duopoly.

Now we know that \(F(\theta^*_{mon}) > F(\theta^*_m)\). Thus the term \(\frac{1}{F(\theta^*_m) - F(\theta^*_{mon})}\) on the right hand side in inequality (7) is positive. If qualities are strategic substitutes, \(\frac{dq_2}{dq_1} < 0\), the right hand side of inequality (7) is always higher than the left hand side since \(v < u_{q_1}(q^*_1, \theta^*_m)\). It follows that \(q^*_1 > q^*_{1\	ext{mons}}\).

Up to now we have shown that in duopoly quality of the incumbent is higher than in monopoly and that in duopoly more consumers are served by the incumbent. Because firm 2 is present as well there are some people who are not consuming in monopoly but consume in duopoly from firm 2. So the only source for a welfare loss can be the fixed costs \(F\). But firm 2 only enters if \(\Pi_2 > 0\). Since \(p^*_2 = u(q^*_2, \theta^*_m)\), \(\Pi_2\) must be lower than the welfare gain because consumers between \(\theta^*_m\) and \(\theta^*_1\) still get a rent. Thus the welfare gain which is induced by firm 2 is higher than \(F\). Altogether welfare must have been increased.

q.e.d.

### 8.3 Proof of Proposition 5

Let us look at the case \(\theta\) uniformly distributed, \(u(q, \theta) = \theta q, v = 0\), and \(c(q) = \frac{1}{2}q^2\).

Solving the first order conditions in the third stage of the game, equations (3) and (4), we get

\[ p_1 = \frac{2q_1(q_1 - q_2)}{4q_1 - q_2} \quad p_2 = \frac{q_2(q_1 - q_2)}{4q_1 - q_2}. \]

Inserting these values in the first order condition of firm 2 in stage 2, we get from equation (5)

\[ \frac{2q_1 - q_2}{4q_1 - q_2} - \frac{q_1 - q_2}{4q_1 - q_2} \left( \frac{2q_1 - q_2}{4q_1 - q_2} - \frac{6q_1^2}{(4q_1 - q_2)^2} \right) + \frac{q_1 - q_2}{4q_1 - q_2} \left( -\frac{q_1}{4q_1 - q_2} + \frac{6q_1^2}{(4q_1 - q_2)^2} \right) - q_2 = 0. \]
Simplifying and totally differentiating yields

\[ dq_1[64q_1^2(1 - q_2) + 2q_2^3(2 - q_2) + q_1q_2(64q_2 - 50)] = dq_2[64q_2^2(q_1 - q_2) + 25q_1^2 + 36q_1q_2^2 - 4q_2(q_1 + q_2)]. \]
Both terms in brackets are always positive since \( q_1 > q_2 \). Thus we get \( dq_2 \frac{dq_1}{dq_1} > 0 \). q.e.d.

### 8.4 Proof of Lemma 1

The first step in this proof is to replace the incentive compatibility constraint

\[
 u(q(\theta), \theta) - p(\theta) \geq u(q(\hat{\theta}), \theta) - p(\hat{\theta}) \quad \forall \theta, \hat{\theta} \geq \theta_{mon}^m
\]

by

\[
 \frac{dq(\theta)}{d\theta} \geq 0 \quad \forall \theta \in [\theta_{mon}^m, 1]
\]  

(28)

and

\[
 \frac{\partial u(q(\theta), \theta)}{\partial q} \frac{dq(\theta)}{d\theta} + \frac{dp(\theta)}{d\theta} = 0 \quad \forall \theta \in [\theta_{mon}^m, 1].
\]  

(29)

This step is a standard one in the theory of adverse selection and the proof of it can be found in many textbooks. See e.g. Fudenberg & Tirole (1991, chapter 7) or Schmidt (1995, chapter 4).

We know that

\[
 U(\theta) = u(q(\theta), \theta) - p(\theta).
\]

Using (29) we get

\[
 \frac{dU(\theta)}{d\theta} = \frac{\partial u(q(\theta), \theta)}{\partial q} \frac{dq(\theta)}{d\theta} + \frac{dp(\theta)}{d\theta} = \frac{\partial u(q(\theta), \theta)}{\partial \theta}.
\]

Integrating both sides of this equation yields

\[
 U(\theta) = U(\theta_{mon}^m) + \int_{\theta_{mon}^m}^{\theta} \frac{\partial u(q(\tau), \tau)}{\partial \theta} d\tau.
\]

Because firm 1 wants to maximise the payoff from consumers, the participation constraint must bind for \( \theta = \theta_{mon}^m \), which implies \( U(\theta_{mon}^m) = 0 \) and therefore

\[
 U(\theta) = \int_{\theta_{mon}^m}^{\theta} \frac{\partial u(q(\tau), \tau)}{\partial \theta} d\tau.
\]

Equation (8) follows.

Now we have determined the prices for a given quality range. In the first stage
the marginal consumer $\theta_m$ and the quality $q(\theta)$ assigned to each type has to be determined.

The maximisation problem of firm 1 can be written as

$$\max_{q(\theta), \theta_m} \int_{\theta_m}^{1} [u(q(\theta), \theta) - vq(\theta) - \int_{\theta_m}^{\theta} \frac{\partial u(q(r), r)}{\partial r} dr] f(\theta) d\theta - \int_{\theta_m}^{1} c(q(\theta)) d\theta$$

s.t. $\frac{dq(\theta)}{d\theta} \geq 0$.

After integration by parts we get

$$\max_{q(\theta), \theta_m} \int_{\theta_m}^{1} \left[ u(q(\theta), \theta) - vq(\theta) - \frac{1 - F(\theta)}{f(\theta)} \frac{\partial u(q(\theta), \theta)}{\partial \theta} - \frac{c(q(\theta))}{f(\theta)} \right] f(\theta) d\theta. \quad (30)$$

Pointwise differentiation with respect to $q(\theta)$ yields (9).

Differentiation with respect to $\theta_m$ yields (10).

Because of Assumptions in $A_1, A_2,$ and $A_3$ all second order conditions and condition (28) are satisfied.

q.e.d.

8.5 Proof of Lemma 2

From equation (18) we know that the first order condition for $q_2^+$ is given by

$$\frac{\partial I(q_2^+)}{\partial q_2^+} = [u_q(q_2^+, \theta_2^+)) - v]f(\theta_2^+)) - u_q(q_2^+, \theta_2(\theta_2^+))\left[F(\theta(q_2^+)) - F(\theta_2^+)- \frac{c'(q_2^+)}{f(\theta)}\right] = \frac{\partial I(q_2^+)}{\partial q_2^+}.$$

We have to show that the derivative of the profit function with respect to $q_2^+$ is negative at $q_2^+ = q_1^-$. Integrating by parts and differentiating the term in the integral, $I(q_2^+)$, with respect to $q_2^+$ we get

$$\frac{\partial I(q_2^+)}{\partial q_2^+} = [u_q(q_2^+, \theta_2^+)) - v]f(\theta_2^+)) - u_q(q_2^+, \theta_2(\theta_2^+))\left[F(\theta(q_2^+)) - F(\theta_2^+)- \frac{c'(q_2^+)}{f(\theta)}\right]$$

$$= \frac{\partial}{\partial \theta} \left[ -u_q(q_2^+, \theta_2(q_2^+)) + v]f(\theta(q_2^+)) - F(\theta_2^+))\right] - \frac{c'(q_2^+)}{f(\theta)}.$$
Thus

\[
\frac{\partial \Pi_2(q^*_m, q^*_m = q^*_m)}{\partial q^*_2} = -[u_q(q^*_2, \theta(q^*_2)) - v]\int_{\theta_m}^{\theta(q^*_2)} \left[ F(\theta(q^*_2)) - F(\theta_m) \right] d\theta = \frac{(F(\theta(q^*_2)) - F(\theta_m))^2}{f(\theta)} u_{\theta q}(q^*_2, \theta_m) - \frac{c(q^*_2)}{f(\theta)} u_{\theta q}(q^*_2, \theta_m),
\]

where the first equality follows from the fact that

\[
\int_{\theta_m}^{\theta(q^*_2)} \frac{\partial}{\partial \theta} \left( [-u_q(q^*_2, \theta(q^*_2)) + v][F(\theta(q^*_2)) - F(\theta_m)] \right) d\theta = [-u_q(q^*_2, \theta(q^*_2)) + v][F(\theta(q^*_2)) - F(\theta_m)]
\]

and the second equality follows from equation (16).

We know that \(\theta(q^*_2) < \theta_m\) so it remains to check that \(u_{\theta q}(\frac{F(\theta) - F(\theta_m)^2}{f(\theta)})\) is increasing in \(\theta\).

We have

\[
u_{\theta q}[2(F(\theta) - F(\theta_m)) - \frac{(F(\theta) - F(\theta_m))^2}{f(\theta)} f'(\theta)] + u_{\theta q} \left( \frac{(F(\theta) - F(\theta_m))^2}{f(\theta)} \right) > 0
\]

because of Assumptions A2 and A3.

q.e.d.

8.6 Proof of Proposition 6

We first show that \(\theta_{m2} < \theta_{m}^{\text{mon}}\).

\(\theta_{m2}\) is given by the first order condition

\[
(F(\theta_{m}^*) - F(\theta_{m2}))(\frac{\partial u(q^*(\theta_{m}^*), \theta_{m}^*)}{\partial \theta_{m}^*}) - \frac{f(\theta_{m2})}{f(\theta_{m})} [u_{\theta q}(q^*_1, \theta_m^*) - u_{\theta q}(q^*_2, \theta_m)] = f(\theta_{m2})[u(q(\theta_{m2}), \theta_{m2}) - \frac{c(q(\theta_{m2}))}{f(\theta_{m2})}] + p_2(\theta_{m}^*) - u(q^*_2, \theta(q^*_2)) - vq(\theta_{m2}).
\]

\(\theta_{m}^{\text{mon}}\) is given by the first order condition

\[
[u(q_{m}^{\text{mon}^*}(\theta_{m}^{\text{mon}^*}), \theta_{m}^{\text{mon}^*}) - vq_{m}^{\text{mon}^*}(\theta_{m}^{\text{mon}^*})]f(\theta_{m}^{\text{mon}^*}) - c(q_{m}^{\text{mon}^*}(\theta^*)) = (1 - F(\theta_{m}^{\text{mon}^*})) \frac{\partial u(q_{m}^{\text{mon}^*}(\theta_{m}^{\text{mon}^*}), \theta_{m}^{\text{mon}^*})}{\partial \theta_{m}^{\text{mon}^*}}.
\]
Inserting $\theta^\text{mons}$ in the first order condition for $\theta_{m2}$ yields

$$-\frac{f(\theta^\text{mons})}{f(\theta_m)} [u_\theta(q^-_1, \theta_m) - u_\theta(q^+_2, \theta_m)]$$

$$< f(\theta^\text{mons}) p_2(\theta_m) - u(q^+_2, \theta(q^+_2)) + (1 - F(\theta_m)) \frac{\partial u(q^\text{mons}(\theta^\text{mons}), \theta^\text{mons})}{\partial \theta_{m2}}.$$ 

Thus $\theta_{m2} < \theta^\text{mons}$, more consumers are served after entry than in pure monopoly.

Now let us turn to the welfare comparison. Consumers with $\theta(q^-_1) \leq \theta \leq 1$ and with $\theta^\text{mons} \leq \theta < \theta(q^+_2)$ get the same quality under monopoly and under duopoly. This is obvious because equations (9) and (16) and also equations (9) and (20) coincide. Consumers between $\theta_m$ and $\theta(q^-_1)$ consume a higher quality in duopoly, namely $q^-_1$, than in monopoly, while consumers between $\theta(q^+_2)$ and $\theta^\text{mons}$ consume a lower one, namely $(q^+_2)$. Therefore we have that welfare under market entry is only higher if

$$\int_{\theta(q^-_1)}^{\theta^\text{mons}} [u(q^-_1, \theta(q^-_1)) - q^{-1}_1] f(\theta)d\theta - c(q^-_1) - F$$

$$+ \int_{\theta(q^+_2)}^{\theta^\text{mons}} [u(q^+_2, \theta(q^+_2)) - q^+_2] f(\theta)d\theta - c(q^+_2) + \int_{\theta_{m2}}^{\theta^\text{mons}} [u(q(\theta), \theta) - q(\theta) - \frac{c(q(\theta))}{f(\theta)}] f(\theta)d\theta$$

$$> \int_{\theta(q^-_1)}^{\theta(q^-_1)} [u(q(\theta), \theta) - q(\theta) - \frac{c(q(\theta))}{f(\theta)}] f(\theta)d\theta.$$

Rearranging terms yields equation (22).

q.e.d.

### 8.7 Proof of Proposition 7

All types $\theta(q^-_1) < \theta \leq 1$ get the same quality in duopoly than monopoly but have to pay a price of

$$p^\text{dmo}_1(\theta) = u(q(\theta), \theta) - \int_{\theta(q^-_1)}^{\theta} \frac{\partial u(q(\tau), \tau)}{\partial \theta} d\tau + p_1(\theta_m) - u(q^-_1, \theta(q^-_1))$$

This can also be written as $p^\text{dmo}_1(\theta) = p^\text{mons}_1(\theta) + p_1(\theta^\text{dmo}) - u(q^-_1, \theta(q^-_1)) < p^\text{mons}_1(\theta)$. Thus the price in duopoly is lower than in monopoly.

Types $\theta \leq \theta(q^+_2)$ get the same quality in duopoly as in monopoly if they are served in both cases. The price under duopoly is $p^\text{dmo}_2(\theta) = p^\text{mons}_1(\theta) + p_2(\theta^\text{dmo}) - u(q^+_2, \theta(q^+_2)) < p^\text{mons}_1(\theta)$ and thus below the price in monopoly. Since in duopoly
more consumer types are served as well, the consumer rent for types $\theta \leq \theta(q^+_2)$ is weakly higher in duopoly than in monopoly.

The utility for types $\theta(q^-_1) \geq \theta > \theta(q^+_2)$ in monopoly is given by $\int_{\theta_{\text{mon}}}^{\theta} \frac{\partial u(q(\tau),\tau)}{\partial \theta} d\tau$. with $\theta$ increasing utility is increasing by $\frac{\partial u(q(\theta),\theta)}{\partial \theta}$. In duopoly for types $\theta > \theta(q^+_2)$ utility is $u(q^+_2, \theta(q^+_2)) - p_2(q^+_2)$, and for types $\theta(q^-_1) \geq \theta$ utility is given by $u(q^-_1, \theta(q^-_1)) - p_1(q^-_1)$. Starting at type $\theta(q^+_2)$, if $\theta$ increases utility increases by $u_\theta(q^+_2, \theta)$ up to $\theta_{\text{duo}}$ and by $u_\theta(q^-_1, \theta)$ from $\theta_{\text{duo}}$ up to $\theta(q^-_1)$. But since we know that $U(\theta(q^+_2))_{\text{duo}} > U(\theta(q^+_2))_{\text{mon}}$ and $U(\theta(q^-_1))_{\text{duo}} > U(\theta(q^-_1))_{\text{mon}}$ for all types in between $\theta(q^+_2)$ and $\theta(q^-_1)$ utility in duopoly must be higher than in monopoly as well. Thus consumer rent increases.

q.e.d.
References


