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Arms Races and the Onset of War. A Review of Game Theory Models and their Assumptions.

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# 1. Introduction

The study of the causes and consequences of arms races developed during the Cold War, but whether arms races climax in a war or create a stable peace situation has not been answered conclusively so far. Arms races are assumed to have a major impact on states' security but the consent stops here. On the one hand, a build-up by another state and reacting with reinforcing one's own military out of fear can lead to preventive or pre-emptive war. According to this explanation, mutual armaments lead to war. The Anglo-German Naval race before the First World War is often interpreted in this way. On the other hand, massive arms build-ups could also deter another state that is likely to challenge and thus military reinforcement prevents a conflict. Hence, arms races lead to peace. The Cold War is often mentioned by deterrence theorists in this context (Jervis 1976).

One subset in this debate has been the formal mathematical modeling of those opposing arguments. Depending on the assumptions of the models they predict that states either fight or live in peace as a consequence of the arms race. Surprisingly, papers which analyze the question of whether arms races lead to war or to peace in *one formal* model are rare. Louis Richardson (1960) and Brito and Intriligator (1984) used differential equations to analyze arms competition. Brito and Intriligator (1985), Powell (1993), Kydd (1997, 2000), Slantchev (2005), Jackson Morelli (2009) presented further models using game theory.

In contrast the literature of formal models, which deal *separately* with the issue why states race or why states fight, is vast and their assumptions and predictions have also been widely studied. Probably the best developed approach why states race is the repeated Prisoner's Dilemma. But RPD models remain silent on the link between arms races and war. They just analyze whether states race or not (Kydd 2000, 240). James D. Fearon (1995) provided an overview of conditions under which states start a fight and illustrated them with game theoretic explanations. But he does not address whether states start a war as a consequence of an arms race.

Moreover, unlike RPD models and formal models of war, comparisons or outlines of the assumptions in formal models of arms races and war are only superficial and not satisfying. Glaser (2000) provides a general overview on arms races and their causes and consequences which addresses the literature in International Relations, case studies and also, formal models. Since he covers a broad field of research, the part of formal models does not offer a deeper analysis. Dunne and Smith (2007) also outline arms races and the onset of war but their focus lies

on empirical studies. Kydd's textbook (2015) contains a small section on formal arms competition models and their impact on war which is very superficial, though.

Therefore, in my Bachelor Thesis, I will analyze the underlying assumptions of formal models of arms races and war. This should provide a first step to an outline of important modeling conditions. Furthermore shortcomings and potential room for improvements shall be demonstrated. In the first section, a short overview of the non-formal discussion on causes and consequences of arms races in International Relations will be given. Subsequently, I show briefly the common formalization of the question why states race or not and then, a description of the common formal assumptions why states fight or not follows. In the second section, I will provide an overview over the formal models which deal with the relation between arms races and war in *one model*. In the third section, I will reasonably select two of these models in order analyze their underlying formal assumptions und predictions in detail. The focal point will be how arms races affect the likelihood of nations to fight or to come to a peaceful agreement in formal models. Therefore, I will especially focus on the common assumptions under which states fight and whether these criteria also apply with an included arms race. Finally, the achievements and shortcomings of the models shall be compared and discussed.<sup>1</sup>

# 2. Approaching Arms Races and War

I will now describe the two opposing arguments of the causes and consequences of arms races in International Relations. Thereafter, the common formal approaches why states start and continue to race and why states fight or not will be introduced.

#### **2.1.** Arms Races and War in International Relations

Two different views determine the discussion concerning arms races and the onset of war in International Relations: one side states that an increase in build-ups increases the probability of war onset while the opposing view claims that arming is the best option for states to deter potential challengers from attacking and therefore provide peace (Glaser 2000, 251). The first argument is described by the spiral model which is related to realism: states face an anarchic world without a sovereign. In this environment, states are uncertain about the other state's

<sup>&</sup>lt;sup>1</sup> When I talk about arms races I will refer to the definition of Buzan and Herrig:

<sup>&</sup>quot;Arms racing is an abnormally intense condition in relations between states reflecting either or both active political rivalry and mutual fear of the other's military potential in arms dynamic" (Buzan and Herrig 1998, 78)

As war definition I use: "War is organized violence carried on by political units against each other" (Hedley Bull 1977, S. 184).

intentions and can only protect themselves with their own strength if the other state is greedy. Even if it is sure that other states do not have aggressive motives right now, nothing guarantees that they do not have greedy incentives in the future. Therefore, states have to arm in order to defend themselves (Jervis 1976, 62). The problem is that self-protection possibly menaces other states because they cannot be sure whether the weapons procurement is motivated by security seeking or greedy intentions. This is known as the security dilemma where the dominant strategy for both states is to arm. The states' armaments, although intended to achieve security, cause mutual fear and can even lead to war. If there seems to be an advantage to strike first due to technology or strategy, even a security seeking state which is satisfied with the status quo will start a war. The fear of being exploited by the adversary can result in preemptive or preventive attack (ibid., 63-67).

The second approach is based on the deterrence theory: arms races reflect a conflicting bargaining process over the status of an issue at stake. A status quo power faces a challenger who believes that the status quo power is weak in capability or resolve and wants to overturn the status quo. In order to deter the greedy opponent, the content state has to show its ability and willingness to fight (Jervis 1976, 58-61). According to this argument, higher arms levels achieve peace by reducing the opponent's probability to win a war. If the status quo power does not accumulate enough capabilities deterrence fails and war breaks out. This argument is related to power transition theory which assumes, unlike realism, that the world is hierarchically ordered (Organski and Kugler 1980). The more powerful states who establish the fundamental rules and norms in international behavior are more content with the status quo. The weaker and less powerful states represent the potential challengers who want to overturn the status quo and must be deterred from doing so (De Mesquita 2006, 585-587). In contrast to the spiral model, where fear is the reason why states race, in the deterrence model aggression is the main driver of the arms race and the failure of deterrence leads to war.

# 2.2. Formal Explanations of Arms Races

A common game theoretic modeling approach of arms races is the Repeated Prisoners' Dilemma. The RPD model is related to first International Relations argument by Jervis where an anarchic setting of the world leaves defecting as a dominant strategy for both countries. A cooperative equilibrium usually represents arms control agreements whereas defection stands for racing behavior. Although, mutual armament is the common equilibrium, reducing arms can also be sustained by the shadow of the future and punishing strategies such as "Tit-for-Tat" (Axelrod 1984; Downs and Rocke 1990). The main problem with RPD models is that they lack the possibility of an attack. They just provide a formal modeling approach for the question of why states race or not (Kydd 2000, 240). Accordingly, RPD models provide only limited explanations when we analyze how arms races affect the onset of war.

### 2.3. Formal Explanations of War

James D. Fearon (1995) introduced rationalist explanations for war by criticizing that neorealist explanations for war are not completely satisfying since war is costly and risky ex post, even under anarchy. He concedes that anarchy and the security dilemma which are the main drivers of arms races and war in the spiral model might cause arms races and territorial competition but do not explicitly address why rational states engage in war. He claims that positive costs of war always provide a bargaining range and that rational states would be better of avoiding the costs by coming to an agreement ex ante (Fearon 1995, 380). For these considerations to hold three assumptions are needed: the states know that there is some true probability that one state will win the war; States are assumed to be risk-averse or risk-neutral over the issue at stake; The issue is perfectly divisible (ibid., 388). After proving the existence of a bargaining range (with respect to the mentioned assumptions) Fearon addresses three circumstances under which even rational states possibly engage in war, although they would be better off living in peace:

1. Uncertainty about the balance of power, the preferences or the costs of fighting and incentives to misrepresent these criteria the states are not able to choose the right offer because they have wrong assumptions about the bargaining range (ibid., 390-401).

2. Problems in credibly committing to uphold a deal under the lack of a central authority: a firststrike advantage or a large shift of power in the next round of the game reduces the probability of peace (ibid., 401-409).

Jervis also stresses that first-strike advantages cause war in the spiral model (Jervis 1976, 67). In contrast, Fearon claims that first-strike advantages only cause war themselves under extremely odd conditions. It is more likely that the first-strike advantage narrows the bargaining range and in combination with uncertainty or an indivisible issue it is even harder to achieve a peaceful solution (Fearon 1995, 403).

3. Issue Indivisibilities: the states are not able to split the good because it has some strategic value or it is simply not possibly to divide it. Who will sit on the throne can be such a problem (ibid., 382).

I add a fourth condition which Fearon does not mention in his Paper. Nevertheless, I will also consider this modeling assumption because it is important for repeated games.

4. Discounting future payoffs/ shadow of the future: a discount factor refers to how states value future payoffs in repeated games. In RPD models a high discount rate means that the states highly value future payoffs and, therefore, they are more likely to cooperate. In contrast, in bargaining models a higher discount factor can even make war more likely because war is temporary preferred to achieve a secure future (Kydd 2015, 146-147).

Summing up, the two opposing IR arguments provide reasonable rationales why arms races can end up in war or peace but do not include a richer formalization of those explanations. The vast RDP literature provides a formal modeling approach why states start an arms race but does not include a war decision. Fearon's rational explanations for war are a major contribution when it comes to formal explanations of a war but does not contain formal modeling conditions of the causal relationship of arms races and war. I will now summarize the literature which deals with arms races and war in *one formal model* and select two of them in order to exemplarily analyze under which modeling assumptions arms races do lead to war or peace. Therefore, I will especially refer to the formal explanations of war under which states fight, mentioned in point 2.3., and whether these criteria also apply with an included arms race.

# 3. Literature Overview

# **3.1. Differential Equation Models**

# 3.1.1. Richardson 1960: Arms and Insecurity

The "Richardson model" is a common starting for the study of arms races and the onset of war. It consists of two differential equations which describe the rate of change of two countries' missile stocks over time (Richardson 1960, 16). Richardson argued that the likelihood of nations engaging in a war depends on their reaction to each other's weapons build-ups, the economic burden of additional weapons and the state's grievances.

$$\frac{dx}{dt} = ky - ay + g$$

$$\frac{dy}{dt} = kx - \beta y + h$$

 $\frac{dx}{dt}$ ;  $\frac{dy}{dt}$  are the rates of change of the countries missiles over time. The expressions ky and kx stand for each country's additional defense build-ups which are a reaction to the other country's weapons build-ups at time t (Richardson 1960, 16). ay and  $\beta y$  are constants which characterize the fatigue and the economic burden of additional arming. The terms g and h represent the "grievance" of a country. A positive grievance term represents a dissatisfied state and a negative term represents a content state (ibid., 15-16). If the defense coefficients outweigh to economic burden the states end up in a runaway arms race which is identified with war. A stable situation can be reached if the economic burden is higher than the defensive reaction to the other side's build-ups (ibid., 24-28). The implications of the model are similar to spiral model in IR since Richardson states that the main driver of a runaway arms race is the defense armament as a reaction to the other sides build-ups. A central shortcoming of this model is that, although Richardson is concerned about the connection of arms races and war, he does not explicitly model the state's decision to go to war. Hence, he only identifies the unstable runaway arms race situation with war but lackes an explicit description why states decide to fight against each other.

# 3.1.2. Brito and Intriligator 1984: Can Arms Races lead to the Outbreak of War?

Brito and Intriligator 1984 introduce another differential equation model which claims to formalize the connection between an arms race and the onset of war. According to their model, an arms race can lead to war or to peace. Disarmament does not guarantee a peaceful outcome. The authors first implement a model of a missile war which can be seen as a military strategy developed by defense experts (Brito and Intriligator 1984, 63; 66). The hypothetical conflict consists of two rival countries and is modeled by four differential equations which describe the war process.

$$(1) \frac{M_A(t)}{d(t)} = -aM_A - \beta'\beta M_B f_B$$
$$(2) \frac{M_B(t)}{d(t)} = -\beta M_B - a'aM_A f_A$$
$$(3) \frac{C_A(t)}{d(t)} = (1 - \beta')\beta M_B v_B$$

$$(4) \ \frac{C_B(t)}{d(t)} = (1 - a') a M_A v_A$$

 $\frac{M_A(t)}{d(t)}$  and  $\frac{M_B(t)}{d(t)}$  represent the changes in the weapon stocks over time. The expression  $-a'aM_A$  is the number of B's Weapons destroyed by A forces. Similarly,  $-\beta'\beta M_B$  is the number of A's missiles destroyed by B's offensive. The  $\frac{C_A(t)}{d(t)}$  and  $\frac{C_B(t)}{d(t)}$  terms stand for the casualties caused by the opponents' force over time. These casualties consist of  $(1 - a')aM_A$ , the proportion of A's missiles shooted at B cities. Likewise,  $(1 - \beta')\beta M_B$  are the number of B's casualties. The coefficients  $f_B$ ,  $f_A$ ,  $v_B$ ,  $v_A$  describe the effectiveness of the shot missiles. At t = 0, there is no reduction of the countries' weapons stocks and no casualties.

Depending on the least acceptable casualties provoked by the initial number of missiles, states can either follow a deterrence or an attack strategy. Those strategies can be used to derive missile levels where both states deter each other or where both states are able to attack the other state (ibid., 67-69). An arms race causes war whenever the build-up enters an area where one state is able to engage in war (ibid., 73-78).

In contrast to the Richardson model, the Brito and Intriligator model provides a formalization of the war, but it does not explain why states end up in an arms race. The differential equations only show the process of a war and whether this war is acceptable depending on the maximum acceptable rate of casualties. The accumulation of the weapon stocks before t = 0 is not modeled. In addition, the model does not include any payoff reductions caused by the economic burden of the acquisition of weapons. Nevertheless, it is noteworthy that the effectiveness parameters provide a formalization of qualitative and not only quantitative improvements of weapons. According to Brito and Intriligator (2000) technical innovations become more and more important after the Cold War. The model yields the conclusion that a qualitative improvement of the weapons is more dangerous than a quantitative one because more weapons are needed in order to deter a technologically advanced nation (ibid., 78-82). These results are contrary to Huntington (1958) who claimed that quantitative arms races are more harmful.

After reviewing the literature about the differential equation models, I will now turn to the literatue about game theory models. In general, the differential equation models do not include decisions; they just describe a process that happens after entering some initial values. In contrast, the game theoretic approach explicitly models strategic decisions given the choices of another

individual. Therefore, I believe they are more useful when we want to study arms races and their linkage to war. All of the finite horizon models are bargaining models which means that the models include some issue at stake. In contrast, the later explained infinite horizon models do not include bargaining situations. During the review of the game theory models, I will only mention the most important assumptions. The remaining influencing assumptions are summarized in the tables.

#### **3.2.** Game Theory Models

#### **3.2.1.** Finite Horizon Models

Table 1: Modeling Assumptions: Kydd 1997		
Number of Players	$I = \{1,2\};$ Player 1, Player 2	
Number of Stages	3	
Information Structure	Two-sided Incomplete Information: Uncertainty about Preferences	
Offense-Defense-Balance	First-Strike-Advantage	
Discount Factor	No	
Solutions Concept	Perfect Bayesian Equilibrium	
Divisible Good	No	
Risk Attitude	Risk-Neutral	
Status Quo	Yes	
Arms Race	Yes	

#### 3.2.1.1. Kydd 1997: Game Theory and the Spiral Model

Kydd presented a formal modeling approach of Robert Jervis's spiral model in order to analyze whether weapons build-ups serve as a signal that can drive security seeking states more suspicious and even lead to an attack (Kydd 1997, 371-372). The suspicion of the other state's intentions is modeled by two sided incomplete information. The states do not know whether the other state is greedy or security seeking and they do not know what the other states thing about them. A state's belief about how its motives are perceived influences how it interprets the potential build-up of the opponent. This yields four possible types for each player: security seeking and fearful, security seeking and trusting, greedy and trusting and greedy and fearful. Security seekers are content with the status quo and greedy players wish to overturn it. Fearful

players think that the other state is greedy and trusting states believe that the other state is security seeking (ibid., 374). In the first round, the players start with the decision whether to attack or not which is not observable. If neither side attacks, both players can decide to build weapons or to wait. The build-up decision is also not observable to the opponent. The military spending improves the state's probability of winning a war but it is also costly. The game moves to the second round regardless of whether they had built or not and the players choose simultaneously either to attack the other player or to defend (ibid., 379-381). If the first-firststrike advantage is not too high, all players defend in the first round. All greedy types build in the second round in order to improve their military strength. The security seeking and fearful types build as well because they suppose that their opponent is greedy. The only type who is not building is the security seeking and trusting type. All greedy types attack after the build-up round because of their high valuation for war. The security seeking fearful types will only attack if they observe a build-up in the second round. The security seeking and trusting types never build and do not attack. Hence, there is war whenever the two players are greedy and there is no war if both players are security seeking and trusting. Kydd's main point of interest is what happens if the two players are security seeking and fearful types. So, each state is actually satisfied with the status quo but each player believes that the other state is greedy. What happens is that both players build in the first round and the possibility that the other player is really greedy raises. Accordingly, both states attack in the second round because they think the other side is greedy and will attack. Hence, the build-up between two security seeking and fearful types leads to war (ibid., 384-386). Summing up, the main driver of the armament by security seeking fearful types is uncertainty. If the states completely observed each other's decision the states would not arm and fight. The arms race does not reveal the state's true type and uncertainty persist. War happens because the build-ups serve as a signal that enhances the state's belief that the other side is greedy and will attack for sure. The first-strike advantage further enforces the risk of war under uncertainty. In this model, the arms race causes war as the states have to fight if they observe a build-up. Not fighting would make them worse of since they believe to be attacked with a high probability. These results are in accordance with Fearon's modeling assumptions of war: uncertainty, wrong believes and a first-strike advantage are the main drivers of the arms race and the war. Both states would neither arm nor fight under complete information because they could observe that the other side is not greedy. Like in the IR spiral model argument, they would rather live in peace with the status quo.

# 3.2.1.2. Kydd 2000: Arms Races and Arms Control: Modeling the Hawk Perspective

Table 2: Modeling Assumptions: Kydd 2000		
Number of Players	$I = \{1,2\};$ Player 1, Player 2	
Number of Stages	8	
Information Structure	Complete Information Version and Incomplete Information Version: Uncertainty about relative Power	
Offense-Defense-Balance	No	
Discount Factor	No	
Solution Concept	Subgame Perfect Nash Equilibrium; Perfect Bayesian Equilibrium	
Divisible Good	Yes	
Risk Attitude	Risk-Averse	
Status Quo	Yes	
Arms Race	Complete Information Version: No; Incomplete Information Version: Yes	

The model presented by Kydd is one of the first to formalize the relationship between arms races, bargaining and war, in the spirit of deterrence theory (Kydd 2000, 229). The only other model which includes those three components is Brito and Intriligator (1985). However, their model has some drawbacks since its results in the incomplete information version are not subgame perfect (Brito and Intriligator 1985, 953). Kydd focuses on the complete information version, but he also presents an incomplete information version. The model is structured as follows: two states bargain over a perfectly divisible issue space between zero and one and their preferences are strictly opposed. A status quo exists which can be reallocated or maintained depending on the arms levels, the income of the states and the location of the status quo. In the first round, player 1 can make an offer and player 2 can accept or reject it. This is followed by build-up rounds where both players can spend a fraction of their income in military equipment. The military spending not only improves the state's probability of winning a war it also provides the states a better bargaining outcome. Unfortunately, armament causes a loss in consumption and the states cannot spend their whole income on weapons. After both states armed player 1 can make a second offer and player 2 can accept or reject it. At the last stage both players can decide whether to attack or not (Kydd 2000, 233-234). If one state chooses to attack, both sides suffer the costs of a war. In the complete information version, there is always a reachable solution of the issue at stake that both sides prefer over war (ibid., 236). The military spending can be reduced but there is no possibility that it declines to zero. In the incomplete information version, player 2 faces uncertainty about the income and the relative power of player 1. This informational setting can be accompanied by a higher military spending, but the higher armament does not lead to war (ibid., 241). Conclusively, in the complete information version there is neither an arms race nor a war. In the incomplete information version, an arms race can happen in equilibrium, but is never followed by a war. As deterrence theory predicts, arms races do not lead to war in Kydd's model. Moreover, the incomplete information version of the model does not have any war equilibria. This is surprising regarding Fearon's rationalist explanations for war and also contrary to results in Kydd (1997) where uncertainty led to war. In Kydd (2000) uncertainty seems to cause higher armaments but the states are still able to implement a solution which is better than fighting. Eventually, arms races happen but they do not lead to war and deterrence never fails, even under uncertainty.

Table 3: Modeling Assumptions: Slantchev 2005		
Number of Players	$I = \{1,2\};$ Player 1, Player 2	
Number of Stages	4	
Information Structure	One-Sided-Incomplete: Uncertainty about Preferences	
Offense-Defense-Balance	Defense-Advantage	
Discount Factor	No	
Divisible Good	No	
Solution Concept	Perfect Bayesian Equilibrium	
Risk Attitude	Risk-Neutral	
Status Quo	Yes	
Arms race	Depends	
Arms race leads to war	Depends	

3.2.1.3. Slantchev 2005: Military Coercion in interstate Crises

Like Kydd 1997, Slantchev only describes an incomplete information model. His focus lies on how states can credibly signal their intentions by the show of force and claims that military movements have tying hands and sunk costs effects (Slantchev 2005, 533). Slantchev analyzes the conditions under which deterrence by arms build-ups succeeds or fails and how armament signals the state's intentions in crises bargaining. He presents a model where two states compete over a territory. The good is not divisible and Player 1 is currently in possession of it. Both states have a valuation for the territory which ranges between zero and one. Player 1 is uncertain about player 2's valuation for the territory. The model consists of four rounds: in the first round, player 1 can decide whether to arm or not. If he does not arm, player 2 gets the territory and the game ends. If he arms, the game continuous and player 2 can also decide whether to arm or not. As in the first round the game ends when player 2 does not build and continues if he builds. During the next stages, both states can decide in sequence whether to attack or not and, in case of an attack, the game ends. The probability of prevailing in war is a function of each side's arms build-ups (ibid., 535-536). In contrast to Kydd 1997 where arms races led to war under uncertainty, and Kydd 2000 where arms races led to peace under complete and incomplete information, Slantchev's model contains multiple equilibria: one involving no arms race and no war, one where mutual build-up takes place but player 1 backs down and one involving war after overserving the build-up (ibid., 537-538). Thus, in this model an arms race can lead to peace or to war. These results are ambiguous from the formal modeling point of view. The solution of the model is complex and it is not so clear how the assumptions influence the arms race-war relation from the first angle. Uncertainty and the presence of an indivisible good normally enable war equilibria but a defense advantage lessens the probability of war. Because Slantchev mainly focuses on the debate over tying hands versus sunk costs in his model, he does not explicitly address the interaction of the arms build-ups and the modeling assumptions. This would require further analyses.

I will now describe infinite horizon models which have the possibility of multiple arming rounds but the states are not bargaining over some good.

Table 4: Modeling Assumptions: Powell 1993		
Number of Players	$I = \{1,2\};$ Player 1, Player 2	
Number of Stages	Infinite	
Information Structure	Complete Information	
Offense-Defense-Balance	Depends on the Parameter	
Discount Factor	Yes	
Solution Concept	Markov Perfect Equilibrium	
Risk Attitude	Depends on the Parameter	
Arms Race	Yes	
Arms Race leads to War	Depends	

#### 3.2.2.1. Powell 1993: Guns, Butter, Anarchy

Powell presents an infinite horizon model in which two states must decide in each period how to allocate their resources and whether to attack or not. The players can either buy weapons to defend themselves or they can consume goods to satisfy domestic needs (guns vs. butter choice). The model contains only a complete information version, so the motivations of the other states are clear. Two main variables in the model are the offense-defense balance and a parameter which describes whether the players are risk-averse, risk-neutral or risk-acceptant. The states alternately decide about the allocation of their military resources and whether to attack or not. It is noteworthy that the military allocations only last two stages and, therefore, they do not accumulate during the game. The game continues as long as neither state attacks. If one of the states decides to attack the probability of prevailing is a relative function of the state's allocations. The loser is defeated as a military power and the victor can save the investment in weapons for all time (Powell 1993, 121-122). Because of the complete information structure and the alternate decision of the states, both can observe the other's allocation perfectly and, therefore, anticipate the decisions in the future periods. Both states arm at their optimal deterrence level and there will be no war at any stage of the game. As the offense-defense balance shifts towards the offense and as the states became more risk-acceptant military spending increases but does not lead to war unless the parameters have extreme values. Thus, higher military spending reduces the status quo payoff because the states have less left to consume and

the probability of war raises but does not lead to war (ibid., 127). A longer shadow of the future leads to higher military allocations which seems peculiar and is contrary to results in RPD models. A deviation by an attack right now means suffering an immediate loss as arming reduces consumption. The incentive to attack is that the states' expected future consumption will exceed the status quo allocation. Thus, the longer the shadow of the future, the higher the expected payoff from deviation right now. The reason for this result is the guns-butter-decision which yields that the states value security or war (higher weapons level) because it promises higher future consumption (ibid., 120). These results confirm that under complete information there will be no war. Even though, the model includes no bargaining, the results are similar to Fearon's rationalist explanations for war: the first-strike advantage and a more risk-averse attitude yield a higher probability for war. The states can anticipate the other states's arming decision and arm for deterrence (Jackson Morelli 2009, 289). Therefore, I would argue that in Powell's model arming leads to peace as the IR deterrence argument predicts. The reason for the arms race is that the states want to secure future consumption. Unlike in RPD models, a greater shadow of the future raises the risk of war and does not lead to cooperation. It is rather the need to satisfy internal claims and the complete information structure with leads to peace. A main shortcoming of this model is the fact that weapons do not accumulate over time. This is very restrictive and distortive assumption since weapons are goods which last long times.

# 3.2.2.2. Jackson Morelli 2009: Strategic Militarization, Deterrence and Wars

Table 5: Modeling Assumptions: Jackson Morelli 2009		
Number of Players	$I = \{1,2\};$ Player 1, Player 2	
Number of Stages	Infinite	
Information Structure	Complete Information with simultaneous decision	
Offense-Defense-Balance	No	
Discount Factor	Yes	
Solution Concept	Markov Perfect Equilibrium	
Risk Attitude	Risk-Neutral	
Arms Race	Yes	
Arms Race leads to War	Depends	

Jackson and Morelli present an infinite horizon model involving two states. The model is similar to Powell's model as both states decide how to allocate their resources and whether to attack or not. But in contrast to Powell's model, the arms decision is made simultaneously before the states decide whether to fight or not. Accordingly, the players cannot observe the other's build-ups and, therefore, not completely anticipate the other's decision. The simultaneous decision creates an uncertain environment about the relative power of the two states. The arming decision is a costly guns-versus-butter choice where build-ups cause a loss in consumption. The build-ups also improve the chance of defeating the opponent forever. A victory means overtaking some recourses of the opponent and never spending own recourses in weapons anymore (Jackson and Morelli 2009, 281). The authors introduce three possible armament levels for the states: low, middle and high. In equilibrium, the states mix over their armament decision because the simultaneous allocation does not allow for pure strategies. This result holds, if the costs of fighting are not overly high and the costs of a build-up are not overly low (ibid., 281-282). A war happens if in some round one state's armament is high and the other state's armament is low. In contrast peace prevails if a middle level meets a high level or the middle level meets a low level. This structure yields mixed strategies: if both states armed at a high level every round, then one could lower its armament to the middle level and save costs without risking an attack. If both armed at the middle level, one could deviate to the low level and again save costs without being attacked. If both armed at the low level, one could choose the high level and defeat the other state with a high probability which guarantees no more arms spending in the future. Hence, both states choose all three levels with some probability in equilibrium in order to keep the other player indifferent over the spending decision. Although, a high level of arming in every period would lead to peace, it reduces overall utility in comparison to the mixed strategy because the military spending reduces consumption. The states rather accept a risk of being attacked in order to satisfy domestic needs than arming to achieve certain deterrence. The arms build-ups only lead to war if a low and a high level build-up meet which happens with some probability. So, both states arm at a certain level in order to deter the other from attacking. The uncertainty caused by the simultaneous decision and the guns vs. butter choice induce the states to accept a risk of war. Therefore, deterrence by building weapons prevails or fails with some probability and, therefore, arming leads to war with some probability (ibid., 282-289). The main problem with this model as with Powell's model is that the weapons do not accumulate over time and are fully depreciated. The build-up decision only influences the likelyhood of winning in the related period which is not suitable for an arms race model. Suppose, player 1 chooses six times a high level while player 2 chooses the middle level. Then player 1 chooses the middle level four times while player 2 chooses the low level. If player 2 would then arm at a high level and player 1 at a low level in the 11<sup>th</sup> round and the players go to war, player 2 would win with a higher probability than player 1. Although, player 1 has always armed at a higher level in the previous rounds, these build-ups do not matter which is not realistic as weapon stocks can last very long times. To be an adequate arms race model, the weapons stocks should accumulate over the periods.

Fearon 2011 included accumulating weapons stocks in an infinite horizon game. In his model, the states either build in every period and deterrence hold or fight at the beginning in order to avoid the costly race (Fearon 2011, 1). But this paper remains unpublished.

I will now summarize the main points, the assumptions and predictions of the models in order explain why I choose Kydd (2000) Slantchev (2005) for a deeper analysis of their assumptions and predictions.

Table 6: Summary of the Predictions			
Arms Race leads to War	Arms Race leads to Peace	Both Outcomes are possible	
Richardson 1960 (Differential Equation Model)	Kydd 2000 (Game Theory Finite Horizon)	Brito & Intriligator 1984 (Differential Equation Model)	
Kydd 1997 (Game Theory Finite Horizon)	Powell 1993 (Game Theory Infinite Horizon)	Slantchev 2005 (Game Theory Finite Horizon)	
		Jackson & Morelli 2009 (Game Theory Infinite Horizon)	

# 4. Summary of the Predictions

The table shows that regardless of whether the models are older or newer, whether they are differential equation models, finite horizon models or infinite horizon models, they all come to different predictions. Most of the models which I introduced above do work with the concept of peace by deterrence. The states armed at a high level to prevent an attack and war is a result of insufficient arming which leads to deterrence failure (Brito and Intriligator (1984), Kydd (2000), Slantchev (2005), Powell (1993), Jackson and Morelli (2009)). The only two models which include a spiral argument are Richardson (1960) and Kydd (1997). In their models, weapons accumulations lead to war.

The models of Richardson (1960) and the Brito & Intriligator (1984), which base on differential equations, do include neither the attack nor the arms race decision. Therefore, they are not suitable for the analysis how arms race affect the state's war decision.

Powell (1993) and Jackson Morelli (2009) provide an infinite horizon game which is useful for arms races since they can last a long time and include many periods of arming. Arms races can lead to war under uncertainty but do not lead to war in the complete information version. A first strike advantage and gambling preferences lead to a higher probability of war. The models lack an important factor: the state's weapons do not accumulate over time. The allocated weapons of the previous periods do not matter for the current situation. Therefore, a state can have a higher probability in prevailing in war although it always armed at a lower rate than its opponent in the previous periods.

In contrast to Powell (1993) and Jackson and Morelli (2009), the models provided by Kydd (1997, 2000) and Slantchev (2005) only include one round of mutual arming. All three models allow for bargaining over an issue at stake and show how arming works as a signal. In Kydd (1997), under uncertainty and a first-strike advantage an arms race leads to war as the IR spiral model predicts. However, in Kydd (2000) arms races do not lead to war, even under uncertainty. The good is perfectly divisible and deterrence never fails. Slantchev (2005) contains multiple possible equilibria in an uncertain environment where states bargain over an indivisible good and face a defense advantage. Mutual armament can be avoided; arms races can either lead to war or to peace.

I decided to analyze Kydd (2000) and Slantchev (2005), although they both include only one round of arming. Nevertheless, I think it is quite interesting how those models yield such different outcomes: Kydd (2000) has no equilibria where an arms race is followed by war, even under uncertainty. This result is opposed to the common findings in game theory models of war

initiation where uncertainty provides war equilibria. Therefore, it would be very interesting to find out whether the arms race is the reason why peace prevails even under uncertainty. Kydd (2000) is further the only model with a complete and an incomplete information version. Hence, it should be possible to see how the information structure affects the arms race – war decision in the same model. Moreover, it is the only model which contains a divisible good. In contrast, in Slantchev's model, states bargain over an indivisible good. By comparing Kydd (2000) and Slantchev (2005), it might be possible to deduce the influence of the structure of the good on the arms race-war decision. As Slantchev (2005) has many possible equilibria, it is very interesting how the arms race affects or whether armament causes those different equilibria. Therefore, I will exemplarily analyze and compare Kydd (2000) and Slantchev (2005). The theoretical and mathematical assumptions and their predictions about how arms races affect whether states fight or not will be studied in the following section.

# 5. Modeling

# 5.1. Kydd 2000: Arms Races and Arms Control: Modeling the Hawk Perspective

I will first analyze the complete information version and second the incomplete information version. Player 1 will be called "he" and player 2 "she". In the complete information version, I will exemplarily describe the mathematical solution of the attack decision, the second bargaining round and the military spending decision of player 2. Since, those choices suffice to give an impression how the game is solved. For the military spending decision of player 1, the first bargaining round and the incomplete information version, I will restrict myself to the description of the assumptions and their impact on the arms-war decision.

#### 5.1.1. The Setting

The model describes an interstate dispute over the good x  $\epsilon$  [0,1] among two states, S<sub>1</sub>, S<sub>2</sub>. The states' interests are strictly opposed: S<sub>1</sub>'s utility is increasing in x and S<sub>2</sub>'s utility is decreasing in x. Thus, S<sub>1</sub> receives the payoff (x) and S<sub>2</sub> the payoff (1 - x). The status quo distribution is denoted x<sup>0</sup>. The game starts with a bargaining round where S<sub>1</sub> makes an offer (x') and S<sub>2</sub> can accept or reject it. The game then moves to the arms building decision regardless of the outcome in the bargaining round. Player S<sub>1</sub> starts the build-up and decides to allocate military equipment m<sub>1</sub>  $\epsilon$  [0,1]. In the next stage, player S<sub>2</sub> builds m<sub>2</sub>  $\epsilon$  [0,1]. The model includes a "guns-butter"

decision since the military equipment m<sub>i</sub> is a fraction of the states' national income y<sub>i</sub> and causes a loss in consumption. The states' absolute level of spending is denoted  $M_i = m_i * y_i$  and their relative power is described by  $p(M_1, M_2)$ . The relative power increases with  $M_1$  and decreases with  $M_2$ . Player S<sub>1</sub>'s chance of winning is denoted p and player S<sub>2</sub>'s chance of winning is (1-p). If  $M_2$  increases, the relative power decreases for  $S_2$  but since its chance of winning (1-p) is strictly increasing with a smaller p, higher military allocation for S<sub>2</sub> still increases its chance of winning. The build-up rounds are followed by a second bargaining round where player  $S_1$  offers  $(x'') \in [0,1]$  and  $S_2$  accepts or rejects it. Afterwards, the game moves to the attack decision and player S1 can decide whether to attack or not. If he attacks the game ends, if not, player S2 can choose between attacking and not attacking. If neither player attacks, the agreements of the second bargaining round remains in place and if there was no agreement, the agreement of the first bargaining round holds. If no agreement was reached in the first round, the status quo remains in place. In case of a war, the winner gets the entire good and the loser is left with nothing. The costs of war for both sides are  $w_i$  and these costs are low enough so if the players were certain to win, they would attack. The payoffs for war are  $u_i(p, m_i) - w_i$  and the peace payoffs are  $u_i(x, m_i)$  (Kydd 2000, 234)."The utility functions are continuous, twice differential, and increase with one's own consumption and chance of winning with diminishing marginal returns" (Kydd 2000, 234).

Three important assumptions are already given here: First, the good over which the states bargain is divisible since all intermediate outcomes (x) between zero and one are available. Second, the states are fully informed over the situation in the complete information version of the game. Third, the players are risk-averse. These three assumptions normally facilitate the achievement of a peaceful bargain in game theory models of war (Fearon 1995).

# 5.1.2. The Structure of the Game

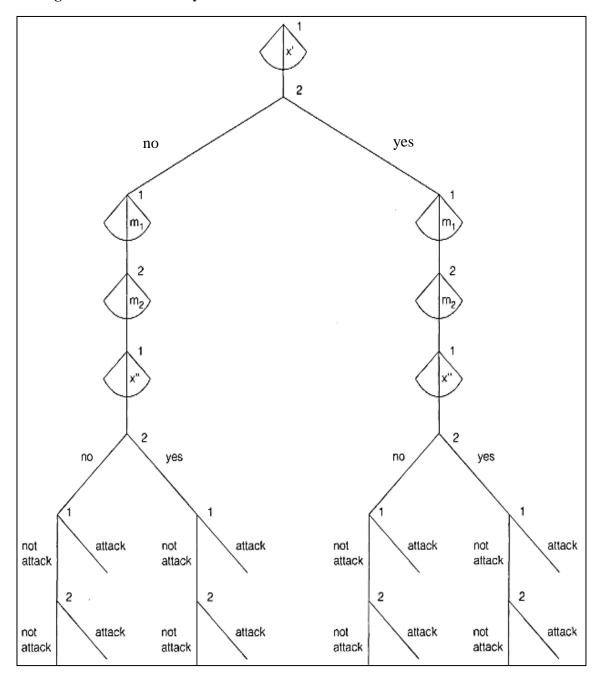


Figure 1: Game Tree: Kydd 2000

Source: Kydd, Andrew (2000): Arms Races and Arms Control. Modeling the Hawk Perspective. In: *American Journal of Political Science* 44 (2), S. 233.

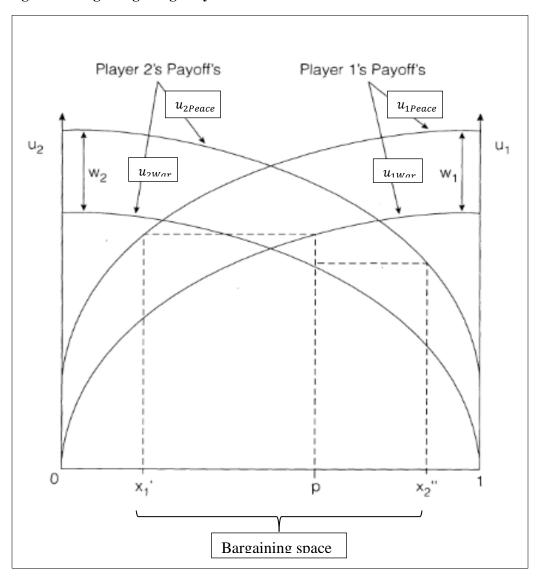
#### 5.1.3. Analyzing the Game

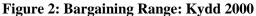
#### 5.1.3.1. Complete Information Version

#### **The War Decision**

For player 2 not to attack it must be the case that  $u_{2peace}(x, m_2) \ge u_{2war}(p, m_2) - w_2$  and for player 1  $u_{1peace}(x, m_1) \ge u_{1war}(p, m_1) - w_1$ . The payoff functions for war and for peace differ in p and x, but they still have the same slope. The y-intercept is lower for the war payoff than for the peace payoff because of the positive costs for war  $w_i$ . Hence, if x = p, the peace utility is always higher than the war utility. In order to achieve equal war and peace utilities, p, the utility of winning the war, must be greater than x for  $S_1$  and smaller than x for  $S_2$ . These conditions define the reservation values of x for the second bargaining round where the states are indifferent between fighting and accepting the peaceful solution. The bottom line, the smallest portion of the good that states are willing to accept instead of fighting, are denoted  $x_1''$  for S<sub>1</sub> and  $x_2''$  for S<sub>2</sub>. S<sub>1</sub> does not accept any solution smaller than  $x_1''$  and  $S_2$  does not accept any solution greater than  $x_2''$ (since  $S_2$ 's preferences, 1 - x, are strictly opposed to  $S_1$ 's preferences). Accordingly,  $S_1$  would attack if  $x < x_1''$  and S<sub>2</sub> would attack if  $x > x_2''$ . Since,  $x_1'' < p$ , and  $x_2'' > p$  both states are better or at least not worse of accepting a result that lies between  $x_1''$  and  $x_2''$  to war. This solution is possible because all bargains within this range are attainable. The allocation of x lies between this bargaining range, regardless of whether x describes the first or the second bargaining offer or the status quo allocation that remains in place. Figure 2 shows the bargaining range for player  $S_1$ and  $S_2$ . The horizontal axis is describing the whole issue space which ranges between zero and one. The partition of x and probability of winning p range also between zero and one and both are shown on the horizontal axis as well. The utility for player 2 is noted on the left hand side and the utility for player 1 on the right hand side. The utility for player 1 increases as x and p increase while the utility for player 2 decreases as x and p increase because the preferences are strictly opposed. The slopes for the war and the peace payoff functions are the same (the slope for player 2's functions is negative since the preference are opposed) and the positive costs of war ensure that the peace payoffs lie always above the war payoffs for any given point on the horizontal axis. Hence, the maximum utility for player 1 is reached as x and p equal one and the maximum utility for player 2 is reached as x and p equal zero. The dotted lines show the levels of indifference

between war and peace for both players for a given p. Since  $S_1$ 's and  $S_2$ 's utility for peace are better than for war at any point on the horizontal axis, less of x than p is needed for  $S_1$  to reach the same utility and more of x than p for  $S_2$  is necessary to gain the same utility. As already stated, both prefer any distribution of the good between  $x_1''$  and  $x_2''$  to war (Kydd 2000, 234-235).





Source: Kydd, Andrew (2000): Arms Races and Arms Control. Modeling the Hawk Perspective. In: *American Journal of Political Science* 44 (2), S. 235.

#### **The Second Bargaining Round**

The relative power p is determined by the absolute levels of military spending  $M_1, M_2$  which can be written as  $M_i = m_i * y_i$  – the income  $y_i$  multiplied by the share of income  $m_i$  which is spend in military capabilities. The bottom lines,  $x_1''$  and  $x_2''$ , of the players are also an implicit function of state's military spending. Thus, as  $m_i$  increases the bargaining range shifts either to the right as  $m_1$  increases or to the left as  $m_2$  increases. I will now describe under what conditions a revision of x takes place or the status quo or the former agreement remains in place.

*Revision in favor of*  $S_1$ : If the former share x for player 1 is smaller than  $x''_1$ ,  $x < x''_1$ ,  $S_1$  would attack when the game moves to the attack decision. But in the second bargaining round, player 1 can make an offer which lies in the bargaining range and makes both players better off than fighting. This offer will be  $x''_2$  because if player 2 would reject the offer end leave the former distribution, player 1 would attack, which makes player 2 worse off. Although the revision share is worse for player 2 than the former share, the new distribution is still better for her than fighting.

## No revision in the second bargaining round:

If the former distribution lies within the bargaining range, neither side has an incentive to attack in the next round and the already implemented distribution will remain. Player 2 will reject any offer worse than the status quo and player 1 has no incentive to attack in the next round even if player 2 rejects his offer. Thus, neither side has a credible threat and there is no revision in the second bargaining round.

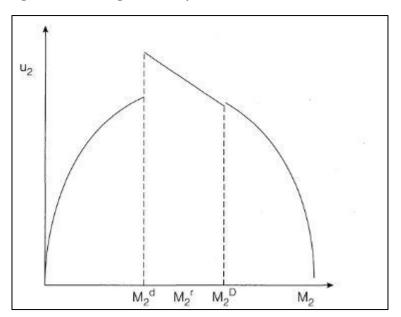
#### *Revision in favor of player* S<sub>2</sub>:

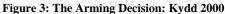
If the status quo is located to the right of  $x_2''$ ,  $x > x_2''$ , player 2 would be better off fighting in the next round than living with the status quo. Since a redistribution of the good that shifts x to the left is preferred by both players to war, S<sub>1</sub> will anticipate that and offer  $x_2''$ . S<sub>2</sub> will accept because  $x_2''$  will make her better off than fighting. S<sub>1</sub> could also offer  $x < x_2''$  and player 2 would accept it. Although this offer is still better for S<sub>1</sub> than war,  $x = x_2''$  makes him better off than offering  $x < x_2''$  (ibid., 235).

#### **The Arming Decision**

#### Player 2's choice, $m_2$ .

In general, there are three possible levels of spending for player 2 which are shown in figure 3:





Source: Kydd, Andrew (2000): Arms Races and Arms Control. Modeling the Hawk Perspective. In: *American Journal of Political Science* 44 (2), S. 236.

First, player 2 can spend so little that player 1 will make a demand in the bargaining round. That means player 2 spends between 0 and a critical value denoted  $M_2^d$ . The  $M_2^d$  stands for the deterrence level spending of player 2. In this case, the bargaining range is located at the right of the status quo,  $x < x_1''$ , and player 1 will make a demand in the next round while player two makes a concession.

Second, player 2 spends  $M_2^d$  which is the level of spending by  $S_2$  that makes  $S_1$  indifferent between making a demand and obtaining the status quo. That means, player 2 spends enough to deter player 1. The equation that derives this level is:  $u_1(x, m_1) \ge u_1(p(M_1, M_2^d)m_1) - w_1$ . Player 2 spends enough, so that the war payoff to the right side of the equation yields the same utility than the status quo or the first bargaining round allocation. Hence, when  $S_2$  spends  $M_2^d$  the status quo x lies within the bargaining range where no one has an incentive to fight after the bargaining round. No credible threat exists. If player 2 spends above  $M_2^d$ , it does not increase but rather decrease her utility as further military spending is expensive and x is still located in the bargaining range. The bargaining range is shifting to the left though, but the status quo will still be accepted by both players in the following round since neither has a credible threat to fight if the offer were rejected.

Third, player 2 can spend so much,  $M_2^D$ , so she can make a demand on her own. Accordingly the bargaining range shifts so massively to the left that  $x > x_2''$  and player 2 has a credible threat to fight. Player 2 spends enough to solve the following condition:  $u_2(x, m_2) \le u_2(p(M_1, M_2^d)m_2) - w_2$ . The utility of the implemented allocation is smaller than the utility of fighting. Player 1 would then offer  $x = x_2''$  in the second bargaining round which makes both better off than fighting.

#### The optimal allocation:

To find the optimal spending decision for player 2 we have to look for her best spending decision in case of war given her income and the spending level of S<sub>1</sub>. This level defines where p is located and as a consequence where  $x_1''$  and  $x_2''$  are located. If the absolute optimal spending level  $M_2^r$  for war is located within the deterrence zone, player 2 always prefers to allocate the deterrence level. If  $M_2^r$  is below or above the deterrence level, we have to compare the utilities for making a concession or making a demand with the deterrence spending utility. In case of  $u_2(x, m_2^d) > u_2(p(M_1, m_2^r y_2)m_2^r) - w_2$  player 2 will spend the deterrence level spending. The richer player 2 is, the more likely it is that she will make a demand and vice versa. Note, that the model has no genuine challenger. Both states can make a demand if they are not satisfied with the status quo and if they are rich enough (ibid., 236).

#### Player 1's Choice and the first Bargaining Round

Player 1's arming choice is very similar to player 2's arming choice. There are also three ranges of spending: If he spends a low amount player 2 will make a demand and he has to make a concession. If he spends at the deterrence level the status quo will remain in place and if he invests enough the make a demand player 2 will make a concession. It depends on his income how much he will spend in weapons. Military allocations do not only increase the players' payoffs in case of a war and their bargaining result of x but also decrease their utility as arming is costly. A richer player 1 can afford to arm at a high level and achieve a better bargain while a poor player rather cedes some portion of the good in order to save military spending.

Player 1's offer in the first bargaining round depends on whether his utility for a revision arms race is higher or lower than his utility to arm for deterrence for a given status quo. If the status quo  $x^0$  lies within a certain range where player 1 is satisfied with the status quo and the necessary deterrence spending to maintain it,  $x^0$  remains in place. If the status quo  $x^0$  is low, player 1 does not have to spend much to achieve deterrence but he is also doing bad on x. Hence, he would like to spend the revision amount in the arming decision in order to achieve a better bargain. If  $x^0$  has nearly reached a value of one, he is very happy with the issue but has to spend too much for deterrence in the next round. Therefore, player 1 is better off releasing some x to player 2 and spending less on military equipment. In the last two cases where player 1 is not satisfied and revision takes place, player 1 can perfectly anticipate the following decisions. Therefore, he will choose the optimal allocation of x and both players can arm for deterrence and save resources because they do not have to spend the revision amount in the arming decision amount in the arming decision (ibid., 236-238).

Conclusively, in the complete information version, there will be no war. Given that the players do are risk-averse and the good is divisible, the peace solution is always better than the war payoff. Hence, there is a bargaining range and a peaceful distribution is obtainable that makes both better off if one player is dissatisfied. The players can anticipate the decisions and wisely avoid the war and the arms race. According to Kydd, this outcome shows that arms races are just as inefficient as war and that the states have an incentive to avoid them. The players can anticipate the outcome under complete information and come to an agreement as Fearon (1995) in "rationalist explanations for war" predicts. Therefore, neither an arms race nor a war takes place.

I think this a quite reasonable conclusion but according to Fearon, states can completely avoid war under complete information and save the costs. However, armament is reduced in Kydd's model but not completely avoided. Whether reduced mutual build-ups to keep the status quo are not an arms race depends on the definition of arms races. As Buzan and Herring (1978) claim, arms races reflect underlying conflicts and/or mutual fear between states. If picking a new status quo in the first round is equal to a resolved conflict, the deterrence armament cannot be called an arms race because it does not reflect an underlying dispute anymore. Therefore, I agree with Kydd that the deterrence armament does not equal to an arms race.

#### 5.1.3.2. Incomplete Information

Uncertainty is modeled over player 1's income which can be one of two types: high income  $y_1^h$  and low income  $y_1^l$ . Since the income of the players determines who much they spend in military equipment, player 2 faces also uncertainty about the relative power of the states. In contrast, Player 2's income is common knowledge. According to Kydd uncertainty over the income refers to "the overall utility of a country to bear the costs of arms racing, now and into the future" (Kydd 2000, 238). The costs of war and the preferences over the issue at stake are clear. It is assumed that in the complete information version the income levels are chosen such that the status quo lies within the bargaining range of the first round if player 1 is poor. Therefore, a deterrence equilibrium will take place and the status quo will be maintained. If player 1 were rich, he would make a demand and player 2 would accept. In the next round, both players arm for deterrence to keep the new allocation.

The solution of the incomplete version is very similar to complete information version:

The attack decisions of the players and the second bargaining round are not affected by the uncertainty because the information about the income of player 1 is already revealed by the arms race. The absolute military spending is a function of the states' income and the share of the states invest in armament. Hence, the military spending of the states reveals how rich player 1 is and, as the game proceeds to the second bargaining round both players are able to reach a bargain which makes both better off than fighting. Since the good is divisible, all intermediate outcomes are obtainable and war is avoided. In contrast to the complete information game, arms races can happen in the incomplete information version. An arms race equilibrium is only possible if player 2 is skeptical that player 1 is rich. Both types of player 1 make a demand in the first round and player 2 will reject it. If player 1 is really rich, he arms for revision of the status quo in the second bargaining round. If player 1 is poor, he arms for deterrence and the status quo will remain in place. A bluff is not possible in this equilibrium since player two forces player one to reveal his type.

If player 2 is trusting, she will accept player one's demand. Both players arm at the deterrence level to maintain the recently reallocated x. The arms race is avoided although there is still a chance that player 1 is not really rich. Hence, a bluff is possible in this equilibrium.

Summing up, even though there is incomplete information the states do not engage in war. Uncertainty over the state's income/relative power can lead to an arms race but does not result in war. The arms race reveals the strength of the informed player and both players are able to find a solution under complete information. A bluff is possible but will not be called if player 2 is not skeptical whether player 1 is really rich (ibid., 238-239).

Finally, in this model an arms race never leads to war. In the complete information version of the game, the positive costs of war and non-gambling preferences provide a bargaining range which is similar to Fearon's findings. The states arm for deterrence and a solution can be implemented according to the states strength which is shown by their armament. The good is divisible which makes it easier to find a solution because some partition is acceptable for both states. In the incomplete information version, the arms race reveals the states strength and the war decision is not influenced by uncertainty. Thus, Kydd stresses that the model shows no causal link between arms races and war. The states arm primarily to achieve their international goals if the states cannot completely observe their strength and to obtain a better bargain. This result is similar to Gray (1974) who claims that arms races are often means to substantiate diplomatic weight. They are not intended to guarantee a victor in the first place (Glaser 2000, 252; Gray 1974, 214-215). Gray further claims that arms races might be a substitute for war (Gray 1974, 216) to show power and therefore war itself is not necessary to reveal the state's strength. Although, Kydd sees no causal link between arms races and war, I would say that the arms race leads to peace since it transmits information which enables the states to come to an agreement short of war.

I will now turn to the analysis of Slantchev's model which includes much more possible equilibria than the Kydd model. Arms races can here either lead to war or to peace or arms races and war can be completely avoided.

# 5.2. Slantchev 2005: Military Coercion in Interstate Crises

Slantchev's focus lies on how states can credibly signal their intentions by the show of force. In a world where states have private information about their valuations for the good, costs or capabilities, arms build-ups can alter incentives and change the expected payoffs of the states. Therefore, arms build-ups are not completely sunk but also have tying hands effects because they increase the probability of winning and the state's war payoff (Slantchev 2005, 533-534).<sup>2</sup> For the following analysis, the tying hands or sunk discussion of armament is secondary because I will focus on the impact of mutual build-ups on war. Therefore, the signaling effect of the arms

<sup>&</sup>lt;sup>2</sup> For an explanation of tying hands and sunk costs see Fearon 1997

race and its effects on war in the model will be my focus.<sup>3</sup> Player 1 will be called "he" and player "she" in the following description.

# 5.2.1. The Setting

The Model characterizes an interstate dispute over a territory between two players (states),  $S_1$  and  $S_2$ . The status quo power  $S_1$  is currently in possession of the territory and his valuation for the area  $v_1$  lies between zero and one:  $v_1 \in (0,1)$ . The valuation of the revisionist state  $v_2$  is private information and  $S_1$  believes that  $v_2$  is distributed on the interval [0,1]. Therefore,  $S_1$ 's belief about  $S_2$ 's valuation and S1's own valuation are common knowledge whereas the exact valuation of player 2 is his private information. Three important assumptions are already given here: the states bargain over an indivisible good which is valued between zero and one. The model has one-sided incomplete information and the states are risk-neutral (ibid., 535-536). Uncertainty and indivisible goods make peaceful solutions more difficult in bargaining models (Fearon 1995)

<sup>&</sup>lt;sup>3</sup> It is further important to mention that in all of the previously introduced models the probability of prevailing in a war is a function of the armament.

## 5.2.2. The Structure of the Game

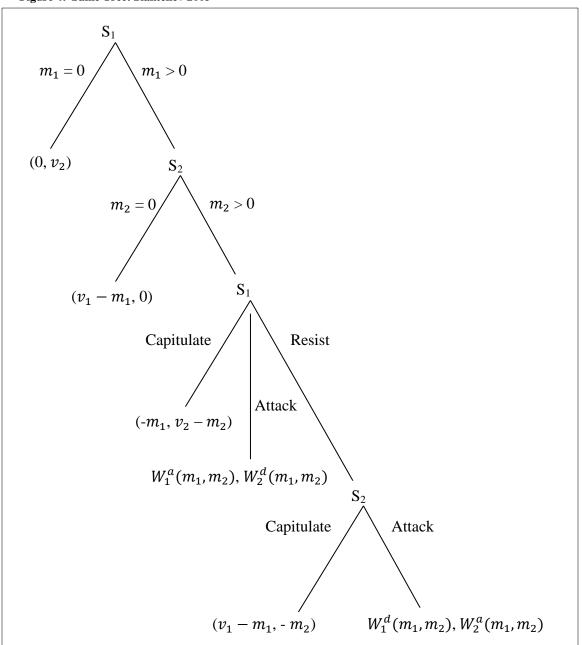


Figure 4: Game Tree: Slantchev 2005

At the first stage  $S_1$  chooses his military allocation level  $m_1$ . If  $m_1 = 0$ ,  $S_1$  gives up any claim for the territory and gets nothing. Hence,  $S_2$  is now in possession of the good which gives him the payoff  $v_2$  and the game ends. In case of  $m_1 > 0$ , which means that  $S_1$  invests in defense,  $S_2$  can either back down  $m_2 = 0$  or arm  $m_2 > 0$  and start a crises. If  $m_2 = 0$ ,  $S_2$  capitulates, the game ends with payoffs  $v_1 - m_1$  for  $S_1$ , his valuation for the territory minus the build-up costs, and 0 for S<sub>2</sub>. In this case, S<sub>1</sub> is still the owner of the territory. For all  $m_2 > 0$  the game moves to the next stage, where S<sub>1</sub> can decide whether to capitulate, attack preemptively or resist. Capitulation means that S<sub>1</sub> relinquishes the territory and the payoffs are  $-m_1$  and  $v_2 - m_2$ . If S<sub>1</sub> attacks preemptively, the war payoffs for the countries are  $W_1^a(m_1, m_2)$  for S<sub>1</sub> and  $W_2^d(m_1, m_2)$  for S<sub>2</sub> and the game ends. If S<sub>1</sub> resists, the game proceeds to the last stage and S<sub>2</sub> can decide whether to capitulate or attack. The capitulation decision leaves the payoffs  $v_1 - m_1$  and  $-m_2$  and S<sub>1</sub> keeps the territory. After an attack decision the game ends with the war payoffs  $W_1^d(m_1, m_2)$  and  $W_2^a(m_1, m_2)$ .

The war payoffs of the third and the fourth stage depend on the level of mobilization  $m_i$ ,  $m_{-i}$ , the costs of fighting  $c_i \in (0,1)$ , the players valuations for the territory and the offense-defense-balance  $\lambda$ . The probability that player i prevails  $\frac{\lambda m_i v_i}{\lambda m_i + m_{-i}}$  is a function one's own armament in relation to the overall military allocations weighed with  $\lambda$ , the offense-defense-balance. If  $\lambda = 1$ , there is no first-strike advantage, if  $\lambda > 1$ , offense dominates  $W_i^a > W_i^d$  and if  $\lambda < 1$ , defense dominates  $W_i^d > W_i^a$  (ibid., 536).

$$W_i^a(m_1, m_2) = \frac{\lambda m_i v_i}{\lambda m_i + m_{-i}} - c_i - m_i$$
$$W_i^d(m_1, m_2) = \frac{m_i v_i}{m_i + \lambda m_{-i}} - c_i - m_i$$

The model contains a defense advantage which depends on the military technology (ibid., 536 fn.9). According to the construction of the offense-defense balance in the model, the parameters not only favor a defense advantage but the attacking state also faces an offense disadvantage at the same time. Slantchev does not give any further theoretical explanation why he introduces a defense-advantage. He only claims that the results would be much more involved but do not change if one introduces an offense advantage (ibid., 536). According to Jervis theory of the offense-defense balance it is easier for the status quo power to defend (Jervis 1978, 194). In the model, only S<sub>1</sub>, the status quo power enjoys the defense advantage. Although, S<sub>2</sub> would theoretically also benefit in case of being attacked, the construction of the model prevents this scenario. As the analysis will show, S<sub>1</sub> never attacks because of the defense-advantage. Therefore, S<sub>2</sub> can never defend and only S<sub>1</sub>, the status quo power, can profit from the defense-advantage as Jervis states.

#### 5.2.3. Analyzing the Game:

The solution concept is perfect Bayesian equilibrium, which requires that strategies are sequentially rational given the beliefs, and that beliefs are consistent with the strategies, and derived from Bayes rule whenever possible (Kydd 2015, 93). I will not describe the solution of the whole game but rather the most important steps to understand realization of the different equilibria. I will mainly focus on the arms race leads to peace and the arms race leads to war equilibria.

 $S_2$  would attack  $S_1$  at the fourth stage, only if her expected payoff is as good as capitulating. That means  $W_2^a(m_1, m_2) \ge -m_2$ . Solving the equation for  $v_2$  leaves the following result:

$$W_2^a(m_1, m_2) \ge \frac{\lambda m_2 v_2}{\lambda m_2 + m_1} - c_2 - m_2 = -m_2$$
$$v_2 \ge c_2 + \frac{c_2 m_1}{\lambda m_2} \equiv \gamma(m_1, m_2) > 0$$

Accordingly, all types  $v_2 \ge \gamma(m_1, m_2)$  attack, if S<sub>1</sub> resists and all  $v_2 < \gamma(m_1, m_2)$  capitulate.

Turning now to  $S_1$ 's decision: If  $S_1$  resists, the game proceeds to the fourth stage and his payoff would be:

$$R_1(m_1, m_2) = G(\gamma)(v_1 - m_1) + (1 - G(\gamma)) W_1^d(m_1, m_2)$$

 $R_1(m_1, m_2)$  stands for S<sub>1</sub>'s resisting payoff and G( $\gamma$ ) characterizes S<sub>1</sub>'s updated belief.

If S<sub>1</sub> resists and S<sub>2</sub> capitulates at the fourth stage, S<sub>1</sub> would receive  $v_1 - m_1$  which is larger than S<sub>1</sub>'s payoff if S<sub>2</sub> would attack at the fourth stage  $W_1^d(m_1, m_2)$ . Because of the defense advantage the payoff  $W_1^d(m_1, m_2)$  (S<sub>1</sub> resists at the third stage and S<sub>2</sub> attacks at the fourth stage) is larger than  $W_1^a(m_1, m_2)$  (S<sub>1</sub> preempts at third stage and the game ends). Leaving the following payoff order for S<sub>1</sub>:

$$v_1 - m_1 > W_1^d > W_1^a$$

The resisting payoffs  $v_1 - m_1$  and  $W_1^d$  are both better than the preemption payoff  $W_1^a$ , so  $S_1$  always favors resisting over preemption regardless of his posterior belief.

$$R_1(m_1, m_2) > W_1^a(m_1, m_2)$$
 for any  $\lambda < 1$ 

Hence,  $S_1$  either resists or capitulates at the third stage (ibid., 536). The defense advantage achieves that  $S_1$  has no incentive to preempt. According to Fearon a defense advantage widens the bargaining range and it would be easier for both states to come to an agreement without fighting (Fearon 1995, 403).

S<sub>2</sub>'s equilibrium behavior depends on the payoffs that she can get from attacking at her final decision node, from capitulating at the second stage or from compelling S<sub>1</sub>. Compelling describes certain capitulation by S<sub>1</sub> at the third stage after he has observed some allocation level  $\overline{m_2}$  by S<sub>2</sub>. S<sub>2</sub> would never mobilize at a higher level than her own valuation. That means  $\overline{m_2} \ge 1$  levels will always be suboptimal for S<sub>2</sub> because  $v_2 - m_2$  is the best outcome S<sub>2</sub> could reach after S<sub>1</sub> has mobilized. Thus, Slantchev does not implement a budget constraint but  $v_2$  serves as an upper bound for  $m_2$  (ibid., fn. 10). S<sub>2</sub>'s strategy can be described by three cutpoints which divide her valuation types into ranges where S<sub>2</sub> behaves the same way. Type  $\beta(m_1)$  stands for indifference between war and assured compellence, type  $\alpha(m_1)$  describes whether she is indifferent between war and capitulation. Depending on the location of the cutpoints and the valuation of S<sub>2</sub> the states capitulate, fight, deter the other state or compel a concession. I will now describe how Slantchev deduces those cutpoints (ibid., 536-537).

## $\beta(m_1) \rightarrow$ indifference between war and assured compellence:

$$W_2^a(m_1, m_2^*(m_1, \beta(m_1))) = \beta(m_1) - \overline{m}_2(m_1)$$

The left side of the equation describes the war payoff which depends on the optimal allocation of  $m_2$  for the type  $\beta(m_1)$  given that  $S_1$  has armed at some  $m_1$  and given that player  $S_1$  has resisted. The optimal level  $m_2^*(m_1, \beta(m_1))$  can be calculated by the derivation with respect to  $v_2$  and solving for  $m_2$ :

$$\frac{d W_2^a(m_1, m_2(v_2))}{dv_2} = \frac{d \frac{\lambda m_2 v_2}{\lambda m_2 + m_1} - c_2 - m_2}{dv_2} = \sqrt{\frac{m_1 v_2}{\lambda}} - \frac{m_1}{\lambda} > 0$$
$$\leftrightarrow m_2^*(m_1, v_2) = \sqrt{\frac{m_1 v_2}{\lambda}} - \frac{m_1}{\lambda} > 0$$

 $m_2^*$  is the optimal allocation level for S<sub>2</sub> if she wants to fight for sure some  $m_1$ . We can now insert  $m_2^*(m_1, v_2) = \sqrt{\frac{m_1 v_2}{\lambda} - \frac{m_1}{\lambda}}$  into the war payoff and substitute  $v_2$  by  $\beta(m_1)$ . To solve for

the cutpoint  $\beta(m_1)$ , where S<sub>2</sub> is indifferent between fighting with her optimal military allocation and compelling S<sub>1</sub>, the war payoff must equal the assuered compellence payoff :

$$W_2^a(m_1, m_2^*(m_1, \beta(m_1))) = \beta(m_1) - \overline{m}_2(m_1)$$

Solving for  $\beta(m_1)$ :

$$\leftrightarrow \beta(m_1) = \frac{(m_1 + \lambda [\overline{m_2}(m_1) - c_2])^2}{4\lambda m_1}$$

# All types v<sub>2</sub> > β(m<sub>1</sub>) prefer assured compellence to war and all v<sub>2</sub> ≤ β(m<sub>1</sub>) prefer war to assured compellence.

This results from the first derivations of the optimal war payoff and the assured compellence payoff with respect to  $v_2$ .

$$\frac{d W_2^a(m_1, m_2^*(m_1, v_2))}{d v_2} = 1 - \sqrt{\frac{m_1}{\lambda v_2}} < 1$$
$$\frac{d (v_2) - \bar{m}_2(m_1)}{d v_2} = 1$$

The derivation of the war payoff is smaller than 1 while the derivation of the compellence payoff equals 1. It follows that the payoffs for any  $v_2 > \beta(m_1)$  must be larger for compellence because the payoff derived by the compellence function is increasing faster than the payoff from war. At  $v_2 = \beta(m_1)$  S<sub>2</sub> is indifferent between war and compellence but for any  $v_2 > \beta(m_1)$  the compellence term yields a higher utility because it is increasing faster. Thus all  $v_2 > \beta(m_1)$  are better off choosing compellence. The following cutpoints are solved the same way.

#### $\alpha(m_1) \rightarrow$ indifference between capitulation and assured compellence:

The type  $\alpha(m_1)$  is indifferent between assured compellence (payoff  $v_2 - m_2$ ) and capitulation, (payoff 0) if  $\overline{m_2}(m_1)$  was observed. Substituting  $v_2$  by  $\alpha(m_1)$  and  $m_2$  by  $\overline{m_2}(m_1)$  that yields:

$$\alpha(m_1) - \overline{m}_2(m_1) = 0$$

The equation is simply solved:

$$\leftrightarrow \ \alpha(m_1) = \overline{m}_2(m_1)$$

Because the derivative of the compellence payoff  $\frac{d v_2 - \overline{m}_2(m_1)}{d v_2} = 1$  is larger than the first order condition of the capitulation payoff  $\frac{d 0}{dv_2} = 0$ ,

# → all types $v_2 \ge \alpha(m_1)$ prefer assured compellence to capitulation and all types $v_2 < \alpha(m_1)$ prefer capitulation to assured compellence.

#### $\delta(m_1) \rightarrow$ indifference between war and capitulation

The optimal war payoff  $W_2^a(m_1, m_2)$  has to be equated with the capitulation payoff, 0:

$$W_2^a\left(m_1,m_2^*\left(m_1,\delta(m_1)\right)\right)=0$$

Substituting  $v_2$  by  $\delta(m_1)$  and inserting  $m_2^*$  leaves:

$$\leftrightarrow \ \delta(m_1) = c_2 + \frac{2\sqrt{c_2m_1}}{\lambda} + \frac{m_1}{\lambda}$$

According to Slantchev, the war payoff strictly increases in type<sup>4</sup> which yields that

→ all 
$$v_2 < \delta(m_1)$$
 prefer capitulation to optimal war and all  $v_2 \ge \delta(m_1)$  prefer war to capitulation (ibid., 537).

The equilibrium behavior depends on the relation between the types  $\beta(m_1), \alpha(m_1)$  and  $\delta(m_1)$ , and the location of S<sub>2</sub>'s valuation. I will not mathematically show how Slantchev deduces the two possible locations of the cutpoints. Instead, I will focus on the interpretation of the two following cases and their corresponding equilibria:

1.  $\alpha(m_1) \leq \delta(m_1)$ ; indifference between assured compellence and capitulation  $\leq$  indifference between war and capitulation

<sup>&</sup>lt;sup>4</sup> Im am not sure whether the claim that the war payoff strictly increases in type is clear: Inserting different values into the derivation of the war payoff with respect to  $v_2$ , provides also some negativ values. I think that the war payoff only strictly increases in type if

if  $m_2^*(m_1, v_2) > 0$ , but this would require some further proof.

2.  $\delta(m_1) < \alpha(m_1)$  and  $\alpha(m_1) < \beta(m_1)$ ; indifference between war and capitulation < indifference between assured compellence and capitulation < indifference between war and assured compellence

In the first case  $\alpha(m_1) \leq \delta(m_1)$  the type who is indifferent between assured compellence and capitulation has a lower valuation than the type who is indifferent between war and assured compellence. The states do not fight, S<sub>2</sub> either capitulates at the second stage, if she is a low valuation type, or she coerces a concession from S<sub>1</sub> at the third stage if she is a higher valuation type. In the former case, S<sub>1</sub> keeps the territory and in the second case S<sub>2</sub> coerces S<sub>1</sub> to relinquish the territory. All types of S<sub>2</sub> whose valuation is smaller than  $\alpha(m_1)$  capitulate and all types whose valuation is larger than  $\alpha(m_1)$  mobilize at the compellence level  $\overline{m}_2(m_1)$  in equilibrium<sup>5</sup>. S<sub>1</sub> capitulates for sure if he observes an allocation level  $\overline{m}_2(m_1)$  (ibid., 537).

A bluff by  $S_2$ , which means that she arms at  $\overline{m}_2(m_1)$  although she would not fight at the last stage, is possible in this equilibrium but will not be called. This equilibrium is only obtainable if  $S_1$  is expected to capitulate which depends on  $S_2$ 's willingness to fight.  $S_2$ 's willingness to fight again depends on her costs of fighting and  $S_1$ 's armament. Counterintuitively, bluffing by is  $S_2$  is only possible if her costs of fighting are high (this parameter is observable), she is a "weak" type. The reason is the impact that a "weak"  $S_2$  has on  $S_1$ 's arming decision. If  $S_1$  thinks  $S_2$  is "weak" he concludes that she will not mobilize at a high level. Therefore, he does not spend much on armament to reduce the costly allocation. These high costs only enable the bluff because the probability of being forced to make a concession is much lower if  $S_2$  is weak. The probability that  $S_2$  has armed at the compellence level  $\overline{m}_2(m_1)$  while she would not fight if  $S_1$  resists, is very low. Accordingly,  $S_1$  does not resist if he observes  $\overline{m}_2(m_1)$  even though it is still possible that  $S_2$ is bluffing. The bluff works because only a small group of low valuation types of  $S_2$  is willing to pay the costs for the expensive signal (ibid., 537-538, 541).

Eventually, there are two possibilities: first, deterrence by  $S_1$  works and  $S_2$  capitulates. Therefore, no arms race and no war takes place because  $S_2$  does not respond to  $S_1$ 's armament. She gives up at the second stage. Although, I interpret the build-up of only one side not as an arms race, higher military allocations by  $S_1$  never increase the risk of war in this equilibrium. Second,  $S_2$  arms and coerces a redistribution of the territory because  $S_1$  capitulates. In this case, arms build-ups lead to peace. If  $\alpha(m_1) \leq \delta(m_1)$  higher armament of the states always provide a peaceful solution

<sup>&</sup>lt;sup>5</sup> provided that  $\overline{m}_2(m_1)$  is feasible

because one of them capitulates. Uncertainty and indivisibility of the good do not lead to war because build-ups serve as a signal of resolve and achieve deterrence.

In the second case,  $\delta(m_1) < \alpha(m_1)$  and  $\alpha(m_1) < \beta(m_1)$  the type who is indifferent between war and assured compellence has a lower valuation than the type who is indifferent between assured compellence and capitulation. War, capitulation and compellence are possible and all challenges of S<sub>2</sub> are genuine. All  $v_2 \le \delta < a < \beta$  prefer capitulation to assured compellence and war. All  $v_2 \in (\delta, \beta]$  prefer war to capitulation and to assured compellence and all  $v_2 > \beta$  prefer assuered compellence to war and capitulation. All types  $v_2 \in (\delta, \beta)$  arm at the optimal war level  $m_2^*(m_1, v_2)$  and all types  $v_2 > \beta$  arm at the assured compellence level  $\overline{m}_2(m_1)$ . The mid valuation types  $v_2 \in (\delta, \beta)$  are the most dangerous ones because their valuation is not low enough to be deterred by S<sub>1</sub>'s allocation and they are also not willing to compel S<sub>1</sub> because it is too costly compared to their valuation. If S<sub>1</sub> observes an armament  $m_2^*(m_1, v_2)$  he separates fully and infers S<sub>2</sub>'s type with certainty and both players fight with complete information. In this case the revelation of the type by the armament leads to war because both players do not come to a solution which is better than fighting. One could argue that after the armament decision it is not the incomplete information which leads to war but rather the complete information that is revealed by the armament. This result is contrary to the incomplete information version of Kydd 2000 where the revelation of the type always leads to peace because the players are able to find a solution with the additional information (ibid., 538). Bluffing is not possible in this case. If  $S_2$ had low costs, she is "strong", a low mobilization level of S<sub>1</sub> would ensure his capitulation and therefore, he eliminates the risk of a bluff ex ante. He arms at a higher level and bluffing becomes too expensive for  $S_2$ , even if it is certain succeed (ibid., 541). War happens in this equilibrium because S1 faces a strong S2 and he arms at a higher level to prevent a concession. S2 also arms at a higher level but this does not suffice to achieve assured compellence. As  $m_1$  raises, the costs to coerce  $S_1$  to give up the territory become very high and makes assured compellence inefficient for S<sub>2</sub>. Since the state's higher levels of armament increased their war payoff compared to the capitulation payoff, they prefer to fight after the information is revealed. Therefore, mutual armament leads to war in this equilibrium.

Summing up, in the first case where  $\alpha(m_1) \leq \delta(m_1)$  (the type who is indifferent between assured compellence and capitulation has a lower valuation than the type who is indifferent

between war and assured compellence) deterrence by  $S_1$  works or the arms race leads to peace because  $S_1$  capitulates. Although, we have incomplete information and an indivisible good the states never fight because the armament serves a signal of strength. In the second case where  $\delta(m_1) < \alpha(m_1)$  and  $\alpha(m_1) < \beta(m_1)$  (the type who is indifferent between war and assured compellence has a lower valuation than the type who is indifferent between assured compellence and capitulation) war can happen after the build-up. The armament of  $S_2$  perfectly reveals her type and if she is a mid-valuation type, the states go to war with complete information.

#### 6. Discussion

In the Kydd model, complete information, a perfectly divisible good and no gambling preferences provide peace without an arms race. The deterrence arming perfectly operates as a measure of strength to come to an optimal agreement for both states. Hence, the states find a solution without any arms race or war. Therefore, in the complete information version arms build-ups cannot have any impact on war. But it is remarkable though that the states cannot completely avoid the deterrence arming since it is their main instrument to achieve an acceptable bargain. In the incomplete information version, an arms race is possible, but it does not lead to war. Therefore, Kydd denies any causal link between arms races and war in his model. I think the arms race leads to peace since the unknown component, the income and the state's relative power, is revealed. The arms race is the reason why the states are able to achieve a solution short of war.

In my opinion, the no war equilibria result is massively promoted by the assumptions of the model especially under uncertainty. The divisible good facilitates a peaceful agreement most notably under incomplete information. Although an arms race increases the war payoff for both states in this information setting, a bargaining range exists because war is still costly and the states have an incentive to avoid it. If the good were not divisible or only some intermediate outcomes were reachable, war might happen. I think this is the scenario in Slantchev model, where the states fight after the revelation of the information. In his model, there is no possibility to split the good according to the state's strength. Furthermore, an initial attack decision before the states arm is not modeled. But this decision gave both players the option to avoid a costly arms race. Kydd himself stressed in his formalization of the spiral model that states can attack at any time and therefore they should also have the option to preventively attack (Kydd 1997, 380).

Under uncertainty, the information whether a state is strong or not would not be revealed at this point and uncertainty would influence this decision. A war but not an arms race equilibrium would be imaginable in this case. Moreover, a majority of game theoretic war models include uncertainty over the costs of war which is interpreted as a state's resolve (Kydd 2015, 93). If Kydd included uncertainty over the costs of war, this information would not be revealed by the arms race. The states' believes would influence the attack, the bargaining and the build-up decision and a war equilibrium could be possible. Summing up, other assumptions might lead to an arms race which causes war. Nevertheless, such claims remain speculative and difficult to prove without actually modeling it. Whether the arms race would be the main driver under such different assumptions requires an explicit formulation of a model.

Slantchev on the other hand analyzes a one-sided incomplete information model. The states do not have gambling preferences but the good, a territory, is indivisible which makes a bargain difficult. Like in the Kydd model, the arming serves as a signal of strength in bargaining. But in Slantchev's model the uncertainty affects the attack decision and war equilibria are possible. Depending on the costs of war, the states' valuations and the other state's arming there are many possible outcomes: the status quo state either deters the challenger and no arms race and no war takes place; the status quo state arms, the challenger arms but the status quo state decides not to attack and the arms race leads to peace; both states arm and decide to fight.

Since Slantchev does not provide an analysis of a complete information version, it is difficult to estimate whether ex ante uncertainty is the main driver of an arms race war causality. It is very interesting though, that in an incomplete information model the revelation of information through arming does not always lead to peace. In the assured compellence equilibrium where both players arm and player 1 capitulates mutual armament leads to peace because player 2 armed at a high enough level to coerce player 1. The reallocation of the territory happens peacefully and uncertainty lasts during the game but does not lead to war. In the war equilibrium on the other hand, the armament of player 2 reveals her type and shows that her valuation is not high enough to arm at a compellence level. The players go to war with complete information because it yields to highest payoff for both. If the good were divisible I think it would be possible to split the territory after the armament revealed the type of player 2 like in Kydd's model. Since all information were perfectly known, the states do not have gambling preferences and war is costly a bargaining range would exist. But only one state can possess the territory in Slantchev's model

and both players are not able to come to an agreement although all information is known. Hence, in my opinion the indivisibility of the good is a major reason why the states fight after they armed. In addition, like in Kydd's model, an initial attack choice is not modeled and there is no possibility of a preventive war. Another similarity is that it is not possible not to arm and to maintain the status quo. Player 1 in Slantchev model has to arm in order to keep the territory he possess at the beginning of the game. In Kydd's model the players also arm at the deterrence level to keep the status quo.

Conclusively, in both models an arms race reveals information about the players' strength and has great influence on the information structure especially under incomplete information. It is remarkable that uncovered information within the model yields different results depending on the nature of the good and the revealed parameters. Although ex ante incomplete information is one of the reasons for war in game theory models, complete information which is revealed in the model does not necessarily prevent war. Even though, war is still costly and risky.

### 7. Conclusion

The literature showed that there are only a few game theory models which deal with the question whether arms races cause war or peace. I reviewed them by shortly noting their underlying formal assumptions which cause the arms race. I paid special attention under which assumptions arms races cause war. Whether these results are distinct to the predictions of formal models, which only deal with war onset without arms races, was also mentioned. As a result, the models in the overview contained very different modeling conditions and came to varying predictions regardless of their structure, solution concept or publication year. I decided to analyze Kydd (2000) and Slantchev (2005) because they came to opposing predictions: Kydd (2000), on the one hand, does not have any war equilibria even under uncertainty over relative power. Therefore, an arms race does not lead to war in his model. Slantchev (2005), on the other hand, provides multiple possible equilibria in one model with incomplete information: arms races can either lead to war or to peace; an arms race and war can be avoided. The analysis of both models showed that arming is one means which the states use to show force and to achieve a better bargain. In the complete information version of Kydd's model, the states neither engaged in an arms race nor fought. Under uncertainty, an arms race reveals the relative power of both states and they can achieve a bargain since all intermediate outcomes of the issue at stake are reachable. However, in Slantchev's model it depends on the players' valuations and the costs of fighting whether the type of the informed player will be revealed by the arms race. This revelation does not necessarily lead to peace because the states are not able to split the good. Eventually, an arms race can uncover information but it depends on the parameter which is revealed and the structure of the good whether this information provides peace. An indivisible good seems to make it much more difficult to achieve a solution without fighting, even if the arms race ensures complete information within the model. In general, it is not finally possible to clearly state which assumptions are the reason for an arms race-war equilibrium. In Slantchev's model, it seems that uncertainty over the states valuation and an indivisible good constitute the main drivers but, this conclusion can be obsolete in a model with uncertainty over the costs of war.

For generalizing statements, much more different models with different modeling conditions have to be implemented. Not only varying uncertainty parameters would be valuable, but also a study whether more than one round of arming in bargaining models provide different result. In addition, the weapons should accumulate over time, like in Fearon's unpublished paper (Fearon 2011). Moreover, none of the game theory models described in the literature overview includes an explicit treatment of qualitative improvements. Since technical innovations become more important (Brito and Intriligator 2000, 51) it would be suitable to include such improvements in a model. An offense-defense-balance addresses differences in military equipment by modeling whether states arm to fight (offense advantage) or to deter (defense advantage). But the offense-defense-balance does not address whether qualitative improvements or quantitative improvements are more dangerous. The Brito and Intriligator (1984) model which I introduced concludes that a qualitative arms race is more dangerous but it is based on differential equations and does not include decisions. In contrast, Huntington (1958) suggests quantitative improvements are more dangerous than qualitative progresses. It would be fruitful to include this discussion in game theory models.

On a final note, it is not conclusively resolved under which assumptions arms races lead to war or to peace and there is room for much more game theory models on arms races and war. Bargaining models with more rounds of arming, accumulating weapons stocks and a measurement of technological improvements could be the first step.

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## **Declaration of Authorship**

I hereby declare that the thesis submitted is my own unaided work. All direct or indirect sources used are acknowledged as references. This paper was not previously presented to another examination board and has not been published.

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