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STRUCTURE, APPLICATIONS AND LIMITS OF DYNAMIC PRODUCTION FUNCTIONS OF THE FIRM BASED ON THE INPUT-OUTPUT APPROACH

Hans-Ulrich Küpper

Technische Hochschule Darmstadt, Karolinenplatz 5, D-6100 Darmstadt (F.R.G.)

1. MEANING AND FORMS OF DYNAMIC PRODUCTION MODELS

Central to dynamic models is the concept of variables that refer to different instants and periods of time. Thus, formulating a dynamic model becomes necessary if the production environment changes in the course of time and if its current state affects the production with a delay. Such changes can mainly be traced back to fluctuations in demand as well as in the duration and connection of multi-stage production processes.

This article shows different ways to formulate dynamic production functions. Primarily it is a contribution to production theory. In the second place it is an approach to production planning in order to forecast material requirements for different periods and intervals as well as the need for human and machine labor.

A complete representation of the production process had to be extended into (at least) four dimension: the kinds of goods, their quantities as well as their arrangement in time and in space. Static production models generally represent only the relations between types and quantities of goods. Dynamic production models take the time dimension into account, but the following will neglect the space dimension.

Production theory examines the input–output relations of goods constituting the manufacturing process. The time structure of input–output relations comes to the fore in formulating dynamic production functions, supplementing the type and quantity structure of goods.

To access the time dimension of production, two basic approaches seem feasible. First, the manufacturing process can be represented directly through parameters and variables that are defined as time-dependent – such as production times. Such approaches are labeled as "production time models". Secondly, quantity measurements like input, production and sales volumes may make up the central variables of the model. They all refer to a specific instant or period. While time is treated as a continuum in production time models, it may vary continuously or in discrete steps in production quantity models. Continuous variation leads to difference or differential equations for production quantity variables. This approach is appropriate for very precise modelling of partial processes and long-term global analysis as for growth processes. In contrast, discrete variation divides the planning period into a given number of intervals. No further attention is paid to the processes within an interval. Therefore, this approach seems to be suited for less detailed analysis of interwoven processes which feature discontinuous changes. As this

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article deals with multistage production processes, a model with discrete production quantities will be used.

2. STRUCTURE AND ANALYSIS OF PRODUCTION TIME MODELS

Production and waiting times of jobs, together with idle times of potential factors like machines or persons, are characteristic components of production time models. Equations for the input of potential factors in time are set next to input–output equations for material. Productive and idle times make up the input of the potential factors, machine and human labor. The production time of a potential factor m (e.g., a machine) working at a constant production speed ρ_{pm} per job p can be derived immediately for the period under planning from the production time d_{pm}, which is needed to work the job p on the machine m, and the production quantity r_p of the job p respectively:

$$d_m = \sum_p d_{pm} = \sum_p \rho_{pm} \cdot r_p \qquad \forall m \tag{1}$$

Binary sequence variables y_{qpm} are introduced to determine idle times of a potential factor. They can be defined as:

$$y_{qpm} = \begin{cases} 1, \text{ if the job } q \text{ is worked on the } m\text{th potential factor directly before the job } p \\ 0, \text{ else} \end{cases} \tag{2}$$

Calling the potential factor n that assumes the fth position in the machine sequence of the job q as $n(f_q)$ and the waiting time of the job p ahead of m with w_{pm}, the idle times l_{pm} of the potential factor m before working on job p can be computed using eqn. (3):

$$l_{pm} = \sum_{n=n(f_p=1)}^{n(f_p(m)-1)} (w_{pn} + d_{pn}) + w_{pm} - \sum_{q \neq p} y_{qpm} \sum_{n=n(f_q=1)}^{n(f_q)=m} (w_{qn} + d_{qn}) \qquad \forall p,m \tag{3}$$

This system of equations reflects the relations between the idle, waiting and production times of the potential factors per job as well as the sequencing. Its structure will stand out more clearly if written down in matrix notation. For that end, the production times of the different jobs p on the machine m are consolidated to the vectors $\vec{d}_m = (d_{1m}, \ldots, d_{Qm})$, the waiting times to $\vec{w}_m = (w_{1m}, \ldots, w_{Qm})$, the idle times to $\vec{l}_m = (l_{1m}, \ldots, l_{Qm})$ and the sequence variables to the matrices Y_m:

$$Y_m = \begin{bmatrix} 0 & y_{21m} & \cdots & y_{Q1m} \\ y_{12m} & 0 & \cdots & y_{Q2m} \\ \vdots & \vdots & \ddots & \vdots \\ y_{1Qm} & y_{2Qm} & \cdots & 0 \end{bmatrix} \tag{4}$$

With identical machine sequences (flow shop production), the following system of equations evolves:

$$\begin{bmatrix} \vec{l}_1 \\ \vec{l}_2 \\ \vdots \\ \vec{l}_M \end{bmatrix} = \begin{bmatrix} E-Y_1 & 0 & \cdots & 0 \\ E-Y_2 & E-Y_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ E-Y_M & E-Y_M & \cdots & E-Y_M \end{bmatrix} \cdot \begin{bmatrix} \vec{w}_1 \\ \vec{w}_2 \\ \vdots \\ \vec{w}_M \end{bmatrix} + \begin{bmatrix} -Y_1 & 0 & \cdots & 0 \\ E-Y_2 & -Y_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ E-Y_M & E-Y_M & \cdots & -Y_M \end{bmatrix} \cdot \begin{bmatrix} \vec{d}_1 \\ \vec{d}_2 \\ \vdots \\ \vec{d}_M \end{bmatrix} \tag{5}$$

Job shop production complicates this system of equations. Nevertheless, Seelbach [1] has shown how it can be modified to compute the idle and waiting times for each sequencing alternative (i.e., for each given combination of sequence variables) with relative ease.

Using the production times according to eqn. (1) and the idle times from eqn. (3), the operation times r_m of the potential factors m in the planning period can be computed:

$$r_m = \sum_{p=1}^{Q} (d_{pm} + l_{pm}) \qquad \forall\, m \tag{6}$$

In production models of this kind the variables refer to jobs. The lot sizes can be treated – observing eqn. (1) – as variables. The number of jobs, though, has to be given. Allowing a variable number of jobs implies setting an upper limit to the number of jobs for each planning period. This extension complicates the model substantially. The serial production structure imposes another limitation.

3. STRUCTURE OF DYNAMIC PRODUCTION QUANTITY MODELS

3.1 Basic properties

Characteristic variables of the production quantity models to be formulated are

- the production quantity r_i^t of product type i in interval t
- the sales volume x_i^t of product type i in interval t
- the input quantity r_{ij}^t of the ith good that is used in interval t to manufacture good j.

The planning period is divided into $t = 1, \ldots, T$ intervals of equal length. To simplify discussion, we assume that no stocking takes place. The following equation holds for each good i:

$$r_i^t = \sum_j r_{ij}^t + x_i^t \qquad \forall\, i, t \tag{7}$$

As in the input–output approach, the product quantity of good i manufactured in interval t is composed of the quantities needed for domestic production and those for sale. Defining quantities of original factors of production like materials, human or machine labor as output of purchasing processes, eqn. (8) compresses the whole input–output volume of a production process. The input–output relations of consecutive partial processes ij play a dominant role in deriving the production function. They can be represented using eqn. (8):

$$r_{ij}^t = f_{ij}^\theta(\ldots) \cdot r_j^{t+\theta} \qquad \forall\, i, j, t, \theta \tag{8}$$

The partial dynamic production function f_{ij}^θ indicates how much of good i has to be used in interval t to produce one unit of product j in the interval $t+\theta$. The term "process duration" denotes the time between input and output of the process and is signified by θ. Assuming the most straightforward case, f_{ij}^θ takes the form of a constant coefficient a_{ij}^θ.

An important premise of the approach is based on the assumption that process time can be fixed as integer multiples of the basic interval unit, independent of the quantity produced. The interval division chosen in advance entails that the production period can only be approximated. In addition, the influence of production volume on process time cannot be (at least not immediately) accounted for. Different or similar goods i can be employed in different earlier intervals to produce a good j in t. In contrast to production time models, complex production structures can be represented without difficulties.

By inserting eqn. (8) into eqn. (7) for all relevant process times we obtain the basic equations of the production quantity model:

$$r_i^t = \sum_j \sum_{\theta=0}^{\Omega} a_{ij}^\theta \cdot r_j^{t+\theta} + x_i^t \qquad \forall\, i, t \tag{9}$$

Solving the system of eqns. (9) for $i=1, \ldots, J$ production factors, production and sales volumes and for all intervals $t=1, \ldots, T$ leads to the dynamic production function of a firm.

3.2 Interval-oriented production function

The matrix form points at possibilities to determine the production function and shows its structure. In this notation, equations can be arranged according to intervals or products. Using interval order, the production quantities of one interval t are aggregated to a vector $\vec{r}^t = (r_1, \ldots, r_J)$, the sales quantities to $\vec{x}^t = (x_1, \ldots, x_J)$ respectively. The production coefficients of the same process duration θ are packed into the direct demand matrix $A^\theta = \|a_{ij}^\theta\|$. With T intervals, the longest process duration to be taken into consideration can reach $\theta = T-1$. Now we get the systems of eqns. (10):

$$\begin{bmatrix} \vec{r}^1 \\ \vec{r}^2 \\ \cdot \\ \cdot \\ \vec{r}^T \end{bmatrix} = \begin{bmatrix} E-A^0 & -A^1 & -A^2 & \ldots & -A^{T-1} \\ 0 & E-A^0 & -A^1 & \ldots & -A^{T-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \ldots & E-A^0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \vec{x}^1 \\ \vec{x}^2 \\ \cdot \\ \cdot \\ \vec{x}^T \end{bmatrix} \qquad (10)$$

Starting from the equations of the last interval T, it can be solved recursively to give the production quantity vector $\vec{r}^t$. For any given interval t, it follows:

$$\vec{r}^t = \sum_{\kappa=0}^{T-t} A_\kappa^* \, \vec{x}^{t+\kappa} \qquad \forall t \qquad (11)$$

where
$$A_0^* := (E-A^0)^{-1}$$
$$A_\kappa^* := \sum_{v=1}^{\kappa} (E-A^0)^{-1} \cdot A^v \cdot A_{\kappa-v}^*$$

The law of development for the A^*-matrices becomes clearer by writing them down up to, say, A_3^*:

$$\begin{aligned} A_0^* &= (E-A^0)^{-1} \\ A_1^* &= (E-A^0)^{-1} \cdot A^1 \cdot A_0^* \\ A_2^* &= (E-A^0)^{-1} \cdot A^1 \cdot A_1^* + (E-A^0)^{-1} \cdot A^2 \cdot A_0^* \\ A_3^* &= (E-A^0)^{-1} \cdot A^1 \cdot A_2^* + (E-A^0)^{-1} \cdot A^2 \cdot A_1^* + (E-A^0)^{-1} \cdot A^3 \cdot A_0^* \end{aligned} \qquad (12)$$

The system of eqns. (11) represents the dynamic input–output relations both for original and derivative production factors. Hence it comprises the dynamic production function of a firm. It may be simplified if we can rule out process durations of zero or if they are all equal to one.

The system ordered according to intervals can always be solved recursively because reflux to an earlier time is impossible. At the worst, $(E-A_0)$ has to be inverted. Partial processes with zero duration constitute a marginal case of reality. On the other hand, a recursive solution of a system arranged by products is possible only with an acyclic production structure. Therefore, the interval order seems to be preferable to determine production quantities.

4. APPLICATIONS AND LIMITS OF DYNAMIC PRODUCTION QUANTITY MODELS

The dynamic production function 11 can be used both for analysis in production theory and for production planning. A simple example is shown in Fig. 1. In it, each circle of the gozintograph represents a (partial) production process, the lines show the direct input–output relations with constant production coefficient a_{ij}^θ. The figure represents a multistage manufacturing process. In the (partial) processes 1 up to 3, three original production factors are purchased, while in the processes 9 through 11 three sales products x_9, x_{10} and x_{11} are produced. The

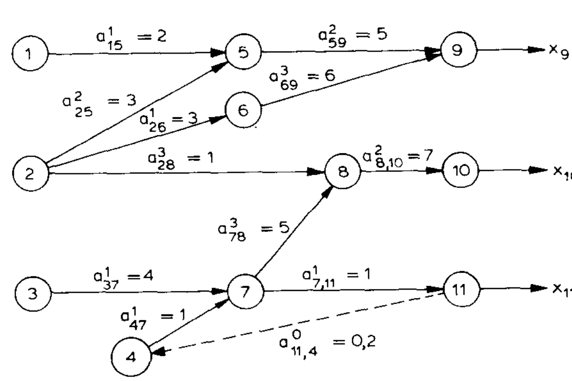

Fig. 1. Example of a multistage production structure.

analysis assumes that good 4 is purchased from the outside or (in the case of a cyclical structure) can be supplied by re-use of final product 11.

4.1 Analysis of the distribution of production on the time scale

Applying the production function developed to the example shows how the distribution of sales on the time axis influences the manufacturing process. The effects will be illustrated in the examples of Fig. 2 for continuous, but constant sales quantities (examples no. 1 and 3) and for changing sales quantities (examples no. 2 and 4). Furthermore, it is analyzed how a cyclical structure affects the production and the purchasing quantities.

Using the production function 11 we can analyze how the time distribution of production settles after a changed level of (relatively) constant or stationary changing sales volumes. The volume of calculation required is determined – according to eqn. (11) – which A^*-matrices have to be computed. This depends on the longest lead time τ needed to complete a final product. In an acyclic structure all matrices A^*_κ turn to zero matrices for $\kappa > \tau$. Looking at the gozintograph of Fig. 1 we find that maximum lead time adds up to $\tau = 6$ intervals (following processes 3-7-8-10). Therefore A^*_7 becomes zero. On the other hand, production cycles are continuously repeated. If the cycle allows complete domestic production, the re-use quantities needed for a final product get smaller and smaller the further away they are in time from the final product.

The first example shows the constant purchasing quantities of the goods 1 through 4 which are needed to produce the given sales quantities of the final products 9* through 11*. Because of the maximum lead time of 6 intervals, purchase of goods 3 and 4 must be started in the first interval in order to get the final product 10 finished in the 7th interval. In the second example the changing sales quantities cause corresponding earlier changes in purchasing quantities of the original factors 1 through 4.

This simple approach determines how much has to be produced at the latest to realize a certain production program. If original production factors and intermediate products are storable, it becomes possible to move up their acquisition and production to an earlier interval. Then the production function has to be extended by stock variables – which is accomplished easily. They project the added discretion in the distribution of production in time on a given marketing program.

Advancing this approach, Trossmann [2] has shown how the entirety of production programs to generate a given marketing program can be formulated as dynamic input correspondences. This provides a foundation for a complete analysis of production alternatives, a way to look for satisfactory or optimal distributions in time.

4.2 Determination of scheduled requirements

In direct materials budgeting we know methods to determine the material requirements of the following different time intervals (periods). Those methods in many cases assume equal duration for each partial process. The dynamic production function 11 includes that case and extends direct materials budgeting to varying process durations.

Forecasting requirements seems to make sense beyond raw materials and supplies. It can be applied to predict the need for human and machine labor. Thus, this production function can be used as a comprehensive approach to forecast distributions of production factors with given marketing programs in quantity and time.

Example No. 1, acyclic; 9*, 10*, 11* = product quantities for sale

i \ t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1				100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0			
2		140.0	470.0	470.0	470.0	470.0	470.0	470.0	470.0	470.0	470.0	330.0				
3	2920.0	2920.0	2920.0	2920.0	2920.0	2920.0	2920.0	2920.0	2920.0	2920.0	120.0	120.0	120.0	120.0		
4	730.0	730.0	730.0	730.0	730.0	730.0	730.0	730.0	730.0	730.0	30.0	30.0	30.0	30.0		
5					50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0		
6				60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0			
7		730.0	730.0	730.0	730.0	730.0	730.0	730.0	730.0	730.0	730.0	30.0	30.0	30.0	30.0	
8					140.0	140.0	140.0	140.0	140.0	140.0	140.0	140.0	140.0	140.0		
9*							10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10*							20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
11*			30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0

Example No. 2, acyclic

i \ t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1				100.0	150.0	200.0	250.0	300.0	300.0	250.0	200.0	150.0	100.0			
2		70.0	435.0	635.0	835.0	1035.0	1200.0	1165.0	965.0	765.0	565.0	330.0				
3	1440.0	2160.0	2880.0	3600.0	4320.0	4340.0	3660.0	2960.0	2240.0	1520.0	100.0	80.0	60.0	40.0		
4	360.0	540.0	720.0	900.0	1080.0	1085.0	915.0	740.0	560.0	380.0	25.0	20.0	15.0	10.0		
5					50.0	75.0	100.0	125.0	150.0	150.0	125.0	100.0	75.0	50.0		
6				60.0	90.0	120.0	150.0	180.0	180.0	150.0	120.0	90.0	60.0			
7		360.0	540.0	720.0	900.0	1080.0	1085.0	915.0	740.0	560.0	380.0	25.0	20.0	15.0	10.0	
8					70.0	105.0	140.0	175.0	210.0	210.0	175.0	140.0	105.0	70.0		
9*							10.0	15.0	20.0	25.0	30.0	30.0	25.0	20.0	15.0	10.0
10*							10.0	15.0	20.0	25.0	30.0	30.0	25.0	20.0	15.0	10.0
11*			10.0	15.0	20.0	25.0	30.0	35.0	40.0	40.0	35.0	30.0	25.0	20.0	15.0	10.0

Example No. 3, cyclic (' = *quantity of good no. 4 which has to be purchased*)

i \ t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1				100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0			
2		140.0	470.0	470.0	470.0	470.0	470.0	470.0	470.0	470.0	470.0	330.0				
3	3648.9	3648.9	3644.4	3644.4	3622.0	3622.0	3509.8	3509.8	2948.8	2948.8	144.0	144.0	120.0	120.0		
4	912.2'	912.2'	911.1	911.1	905.5	905.5	877.4	877.4	737.2	737.2	36.0	36.0	30.0	30.0		
5					50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0		
6				60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0			
7	---	912.2	912.2	911.1	911.1	905.5	905.5	877.4	877.4	737.2	737.2	36.0	36.0	30.0	30.0	
8					140.0	140.0	140.0	140.0	140.0	140.0	140.0	140.0	140.0	140.0		
9*							10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10*							20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
11	---	---	212.2	212.2	211.1	211.1	205.5	205.5	177.4	177.4	37.2	37.2	36.0	36.0	30.0	30.0
11*			30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0

Example No. 4, cyclic (' = *quantity of good no. 4 which has to be purchased*)

i \ t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1				100.0	150.0	200.0	250.0	300.0	300.0	250.0	200.0	150.0	100.0			
2		70.0	435.0	635.0	835.0	1035.0	1200.0	1165.0	965.0	765.0	565.0	330.0				
3	2221.7	3079.7	3908.5	4598.7	5142.5	4993.5	4112.5	3267.5	2262.4	1537.6	112.0	88.0	60.0	40.0		
4	555.4'	769.9'	977.1	1149.7	1285.6	1248.4	1028.1	816.9	565.6	384.4	28.0	22.0	15.0	10.0		
5					50.0	75.0	100.0	125.0	150.0	150.0	125.0	100.0	75.0	50.0		
6				60.0	90.0	120.0	150.0	180.0	180.0	150.0	120.0	90.0	60.0			
7	---	555.4	769.9	977.1	1149.7	1285.6	1248.4	1028.1	816.9	565.6	384.4	28.0	22.0	15.0	10.0	
8					70.0	105.0	140.0	175.0	210.0	210.0	175.0	140.0	105.0	70.0		
9*							10.0	15.0	20.0	25.0	30.0	30.0	25.0	20.0	15.0	10.0
10*							10.0	15.0	20.0	25.0	30.0	30.0	25.0	20.0	15.0	10.0
11	---	---	205.4	244.9	277.1	274.7	235.6	198.4	153.1	116.9	40.6	34.4	28.0	22.0	15.0	10.0
11*			10.0	15.0	20.0	25.0	30.0	35.0	40.0	40.0	35.0	30.0	25.0	20.0	15.0	10.0

Fig. 2. Examples of production quantities for different distributions of sales.

4.3 Analysis of cyclical production processes

Partial processes – like 4-7-11-4 (shown as dotted lines in Fig. 1) – that are part of one production cycle can be carried out only one after the other. Therefore, to start such a process, good no. 4 or good no. 11 (in Fig. 1) has to be acquired from the outside. As the example shows, domestic production is reached only after a settling process. In examples 3 and 4 good no. 4 must be purchased in the first two intervals. After this time, the cycle bears fully and the quantity of the re-used good is completely self-produced. One can also see that most of no. 11 is needed as an input in the 4th process. In example 3 the quantities of the goods 3, 4, 7 and 11 in the cycle diminish during the intervals 5 through 14 as production quantities are only computed up to the 16th interval. This shows that the effect of the cycle reaches back very far. In comparing example 3 with 1 and 4 with 2, it is significant that domestic production via cycles pushes up the need for original production factors. For example, the quantity needed of good 3 is about 25% higher in the cyclic example (no. 3) than in the acyclic (no. 1). In this way the examples demonstrate the facilities to analyze dynamic effects on purchasing and production processes.

Important for the analysis of cycles, the static solution of the production quantity model comes out not only if the time dimension is neglected by setting all process durations to zero. It is also the limit as the number of intervals approaches infinity (with all process durations the same and constant sales volume). If we only have, say, process durations of one and if $(A^1)^T$ disappears for $T \to \infty$, eqn. (11) becomes:

$$\vec{r}^t = \lim_{T\to\infty} \sum_{\kappa=0}^{T-t} A^*_\kappa \cdot \vec{x} = \lim_{T\to\infty} (E + A^1 + (A^1)^2 + (A^1)^3 + \ldots) \cdot \vec{x} = (E - A^1)^{-1} \cdot \vec{x} \qquad \forall t \qquad (13)$$

4.4 Limits of dynamic production quantity models

Because process durations have to be given as multiples of one interval, the bearing of production quantity on the distribution of production in time can only be represented approximately. In reality, especially with job shop production, the process duration of a job depends on the production volume. Therefore the given process durations can represent the average values for the sum of waiting and production times of the jobs. The production volume resulting from production function 11 may exceed capacities available in that interval. Furthermore, the model does not preclude that a new process is started for one type of good in intervals immediately adjacent regardless if the duration of a single process is larger than one interval.

The influence of production quantities on process durations can be implicitly introduced with capacity limits for each good and interval. They force earlier production of goods needed for a certain marketing product if interval capacity has already been used up by demand scheduled earlier. Moreover, heuristic procedures aiming at an optimal shifting of production volume to intervals with free capacities – similar to critical path methods – could be developed.

The limited representation of time structure seems to rule out applications to lot size and sequencing problems. Effects of lot size-dependent preparation times and sequence-dependent waiting times on process duration cannot be inserted into the production function because process durations have to be given.

5. FINAL COMPARISON AND EVALUATION OF THE MODELS

The production time and production

quantity models differ in applications and limits. Production time models allow a precise representation of the production process in time if a serial production structure and the number of jobs are given. Compared to that, production quantity models are best suited to model branched processes. While production time models can express the time structure of the production process as a function of a quantity structure (that must be given to a large extent), the production process in time and its dependence on lot sizes can only be approximated in dynamic production quantity models.

Production time models immediately represent the dependence of the production process in time on production quantity and scheduling. But it incurs the drawback that the number of jobs and a serial production structure have to be given – important elements of the quantitative structure. Production time models hence primarily cover the effect of quantity on time structures. They cannot represent two-way relationships between quantity and time structure in a manner suited to practical application.

Instead, production quantity models allow relatively easy analysis of the effects that alternate sales volume and production structure have on the distribution of production in time and quantity. They represent the dependence of the purchasing and production quantities during several intervals and periods on given sales volumes and process durations.

To a certain degree both approaches complement each other because one is more oriented towards time structure, the other more towards quantity structure. Production quantity models are better suited for mid-term and rough planning of scheduled production programs while production time models are preferable for short-term, detailed planning of the actual production process.

Extending each of the two kinds of models to incorporate the parameters treated as production variables given leads – like the combination of the two types of models Osswald [3] suggests – to extremely complex models. They contain huge numbers of binary variables and an equal amount of restrictions. Because of that they cannot be applied to practical problems – at least to date. The importance of the models sketched rests with the introduction of time as a dimension into the system of production theory. For practical applications, the dynamic production quantity model seems to be suited in particular to approximate scheduled input and production quantities. It could provide an instrument – easily implemented as an interactive computer program – to analyze distribution of production on the time axis, aiding in finding good production schedules.

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3 Osswald, J., 1979. Produktionsplanung bei losweiser Fertigung. Gabler, Wiesbaden.

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LIST OF SYMBOLS

Variables

- d = production time
- l = idle time (of a machine)
- r = production quantity or operation time of a potential factor
- w = waiting time (of a job)
- x = sales volume in an interval
- y = binary sequence variable

Subscripts

- m = potential factor (e.g., machine)
- i,j = product type
- p,q = job
- t = time interval

Coefficients

- a = production coefficient (input quantity for one unit)
- ρ = production speed
- θ = process duration