Optimal Cost Overruns: Procurement Auctions with Renegotiation

Fabian Herweg (University of Bayreuth)
Marco A. Schwarz (University of Innsbruck)

Discussion Paper No. 56

November 9, 2017
OPTIMAL COST OVERRUNS: 
PROCUREMENT AUCTIONS WITH RENEGOTIATION 

FABIAN HERWEG AND MARCO A. SCHWARZ

Abstract. Cost overrun is ubiquitous in public procurement. We argue that this can be the result of a constrained optimal award procedure: The procurer awards the contract via a price-only auction and cannot commit not to renegotiate. If cost differences are more pronounced for a fancy than a standard design, it is optimal to fix the standard design ex ante. If renegotiation takes place and the fancy design has higher production costs or the contractor’s bargaining position is strong, the final price exceeds the initial price. Moreover, the procurer cannot benefit from using a multi-dimensional auction, i.e., under the optimal scoring auction each supplier proposes the standard design.

Keywords: Auction, Cost Overrun, Procurement, Renegotiation

JEL Codes: D44, D82, H57

1. Introduction

Renegotiation of procurement contracts awarded by public authorities are ubiquitous. The initial contract is awarded via competitive tendering; i.e., via an auction. The terms of the initial contract, however, are often subject to renegotiation with the result that the ultimate price is (by far) higher than the price which the parties initially agreed upon. Prominent recent examples of public procurement projects that are by far more expensive than initially planned are the Elbphilharmonie, a

Date: September 2, 2017.
Acknowledgments: We thank the editor, Masaki Aoyagi, three anonymous referees, Mikhail Anufriev, Alessandro De Chiara, Dominik Fischer, Jacob Goeree, Heiko Karle, Laurent Lamy, Laurent Linnemer, Johannes Maier, Antonio Rosato, Klaus Schmidt, Johannes Schneider, and Christoph Schwaiger for many helpful comments and suggestions. We would also like to thank seminar participants at Carlos III de Madrid, Frankfurt School of Finance & Management, Theoretischer Ausschuss at Basel, X-CREST Paris, University of Bern, University of Munich, and University of Technology Sydney. Part of this research was conducted while the first author visited University of Technology Sydney and Fabian would like to thank the Business School for its great hospitality. Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 and CRC/TRR 190 and from the Austrian Science Fund (SFB F63) are gratefully acknowledged.
concert hall in Hamburg, the Big Dig, a highway artery in Boston, and the North-South metro line in Amsterdam. What is often considered as the most severe case of a cost overrun in modern construction history is the Sydney Opera House. The project was completed 10 years late at a price of 14.6 times the initial price. During the construction of the Sydney Opera House, plenty of design changes had taken place. For instance, the design of the roof has been changed from a relatively flat roof to the fancy ribbed ellipsoidal roof, which increased the cost for the roof by 65%. This and other changes that arguably made the Sydney Opera House more complex are responsible for a part of its cost overrun.

Next to casual observations, empirical studies document that the prices typically increase through renegotiation. According to public opinion, these cost overruns are a sign of inefficient project management by bureaucrats or of strategizing politicians and thus a waste of taxpayers’ money. In contrast to this widespread public opinion, we argue that these seemingly inefficient cost overruns can be the result of a constrained optimal award procedure that minimizes the expected final price for the procurer.

In our model, a procurer needs an indivisible good or service, which can take one of two designs; i.e., a standard (low quality) or a “fancy” (high quality) design (a bridge with two or three traffic lanes). The good can be delivered by several suppliers that may differ in their privately known production costs. Moreover, the ex post efficient design depends on the contractor’s production cost – i.e., on the

---

1Regarding the Elbphilharmonie the accepted offer from the underwriting group in 2006 was 241 million euro. The final price at the hand-over of keys in 2016 was 789 million euro (Fiedler and Schuster 2016). For the Amsterdam metro line the initial budget was set at 1.46 billion euro in 2002 but the costs had risen to 3.1 billion euro in 2009. Recent estimates suggest that it will be completed in 2017 (Chang, Salmon, and Saral 2016). For the Boston highway artery the ultimate price exceeded the initial price by 1.6 billion US dollar (Bajari, Houghton, and Tadelis 2014).

2We define as cost overrun the difference between the final price and the initial price at which the procurement order has been awarded.

3When controlling for inflation, the cost overrun reduces to a factor of 7.5 (Newton, Skitmore, and Love 2014).

4See Newton, Skitmore, and Love (2014) and Drew (1999) for more detailed discussions of the construction and cost increases of the Sydney Opera House.

5Substantial price increases resulting from contract renegotiation are reported by Decarolis (2014) for Italian procurement contracts and by Bajari, Houghton, and Tadelis (2014) for Californian procurement contracts. German procurement contracts and the their cost increases are listed by Fiedler and Schuster (2016). They also report that some projects perform exceptionally well. For instance, the Chemikum, a building of the University of Erlangen-Nuremberg, was completed a year earlier than planned and at a cost of only 80 million euro instead of the planned 140 million euro.
cost type of the supplier who has been awarded with the contract. Initially the procurer runs an auction in order to allocate the contract. Importantly, the contract specified by the auction is a specific performance contract that can be enforced by courts. First, we assume that the procurer can collect bids only on prices and thus has to select one particular design of the good. More precisely, the procurement contract for the given design is awarded to one supplier via a standard auction, e.g., a second-price sealed-bid auction. The design specified in the initial contract may turn out to be inefficient, given the cost type of the contractor. In this case, we assume that the parties engage in Coasian bargaining and implement the efficient design ex post. Renegotiation is expected by the suppliers and thus incorporated in their bidding behavior. The rent the contractor (the supplier who won the auction) receives depends on his cost advantage compared to the second-lowest bidder with regard to the initial design. We assume that cost differences are more pronounced for the fancy than for the standard design. Under this assumption it is optimal for the procurer to fix the standard design ex ante because this enhances competition in the initial auction. In other words, when commitment not to renegotiate is not feasible, it is optimal for the procurer to choose the standard design ex ante and to potentially renegotiate to the fancy design ex post. If the cost for producing the fancy design is higher or if the contractor’s bargaining power is not too weak, the final (renegotiation) price exceeds the initial price; i.e., a cost overrun occurs.

An important feature of our model is that the outcome is always efficient. The supplier who can deliver the ex post efficient design at the lowest cost wins the auction. He benefits most from contract renegotiation and thus bids most aggressively. This implies that the unique goal of the procurer is rent extraction; i.e., choosing the initial design such that the expected final price – for the overall efficient design delivered by the most efficient supplier – is minimized. In other words, the procurer does not face a rent extraction versus efficiency tradeoff.

Due to the assumption of Coasian bargaining ex post, the ex post outcome is always efficient in our baseline model. In two extensions, we augment the baseline model and allow for renegotiation failure. First, we consider the situation where contract renegotiation takes place under asymmetric information. Focusing on a simplified model with only two cost types, we show that the type is always revealed via the bid. The procurer optimally specifies the standard design ex ante and contract renegotiation typically leads to an upward price adjustment. Second, we analyze what happens when there is an exogenous risk that the renegotiation breaks down and the parties are stuck with the initial contract. If this risk is rather low, it is still optimal to choose the standard design ex ante. As the risk becomes
larger, taking into account the situation when renegotiation fails becomes more important, so the optimal initial design is likely to be the fancy one. However, as we demonstrate in an example, upward price adjustments seem to be more likely than downward price adjustments if renegotiation takes place; i.e., if there had been a risk of bargaining breakdown but the parties succeed in finding an agreement.

Finally, we allow for multi-dimensional auctions – i.e., scoring auctions. The procurer now asks for bids containing a price and a design. The supplier who places the bid leading to the highest score – determined by a commonly known scoring rule – wins the auction. The procurer’s initial choice is the scoring function, which we restrict to be linear in price. If the scoring function reflects the procurer’s true preferences, each supplier offers the optimal design given his cost. In this case, contract renegotiation can be avoided. The optimal scoring function, however, does not reflect the procurer’s true preferences. The optimal scoring function is such that any supplier bids the standard design. The most efficient supplier wins the initial auction and the parties are likely to agree to implement the fancy design at a higher price ex post via renegotiation. In other words, a price-only auction for the standard design outperforms scoring auctions, where suppliers place multi-dimensional bids containing a price and a design. The reason is that a multi-dimensional auction allows for differentiation of the suppliers’ bids, which relieves competition between suppliers ex ante and thus leads to higher ultimate prices. This finding is in contrast to the existing literature on scoring auctions that assumes the procurer can commit not to renegotiate the contract (Dasgupta and Spulber, 1989-1990; Che, 1993; Chen-Ritzo, Harrison, Kwasnica, and Thomas, 2005).

The main findings are driven by two behavioral assumptions. First, when submitting bids, suppliers foresee eventual contract renegotiation and bid more aggressively if they can make profits via renegotiation. Second, the procurer is aware of the suppliers’ bidding strategies and designs the initial auction – specifies the initial design – so that the auction is most competitive. Regarding the first assumption, it is widely believed in the construction literature that contractors often bid low on a project and hope to recover the loss through renegotiation (Levin, 1998). This behavior is called opportunistic bidding or bid your claims (Mohamed, Khoury, and Hafez, 2011). That contractors/bidders respond strategically to anticipated ex post changes is also documented in the economics literature, e.g., Bajari, McMillan, and Tadelis (2009); Bajari, Houghton, and Tadelis (2014); Iimi (2013). For instance, Iimi (2013, p. 254) finds that “when submitting bids, firms already foresee certain ex post adjustments [...] with cost overruns expected, firms would likely undercut their bids significantly.” Laboratory evidence that bidders who can gain more from
contract renegotiation bid more aggressively is provided by Chang, Salmon, and Saral (2016). Regarding the second assumption, there is no direct evidence that procurers take advantage of the strategic bidding of suppliers and therefore often initially specify a standard design. There is, however, evidence that procurers often add extra components during construction (Lim 2013).

The paper is structured as follows. After having discussed the related literature, which is done in the following paragraphs, we introduce the model in Section 2. The model is analyzed in Section 3. Asymmetric information and the risk of renegotiation failure are discussed in Section 4 while we extend the baseline model by allowing for multi-dimensional auctions in Section 5. The final Section 6 summarizes our findings and concludes. All proofs are deferred to the Appendix A. Further robustness checks can be found in Appendix B.

Related Literature. Investigation of procurement contracts is an important and classic topic of contract theory. A seminal contribution analyzing procurement and renegotiation is Tirole (1986). He analyzes the contractual relationship between a single procurer and a single supplier with a focus on how initial contracts can enhance non-contractible relationship specific investments.

Dasgupta and Spulber (1989-1990), Che (1993), and Chen-Ritzo, Harrison, Kwasnica, and Thomas (2005) analyze procurement auctions for the case that the procurer can commit not to renegotiate the contract. All three articles show that the optimal scoring auction outperforms price-only auctions. We demonstrate that if this commitment is absent, the optimal price-only auction outperforms scoring auctions.

There is only a small extant literature that analyzes auctions without perfect commitment; i.e., that allows either bidders to renege on their bids or to engage in contract renegotiation. Waehrer (1995), Harstad and Rothkopf (1995), and Roelofs (2002) allow bidders to withdraw the winning bid ex post. In these models suppliers are initially uncertain about their costs and thus may underestimate it. The possibility to default on the initial commitments enhances competition in the auction, which in turn is beneficial to the procurer. Waehrer (1995) also analyzes a scenario

---

Supplementary information:

6 For an excellent discussion of the standard contract theoretical analysis of procurement see Laffont and Tirole (1993).

7 There is also a small literature that analyzes screening mechanisms if the principal (the procurer) cannot commit not to renegotiate; e.g., Beaudry and Poitevin (1995). This literature typically assumes that there is only one buyer and one seller and focuses on the constraints limited commitment power imposes on the implementable allocations.

8 The effects of limited liability on more general mechanisms than auctions are investigated by Burguet, Gauza, and Hauk (2012).
where the procurer and the winner renegotiate a new contract. Here, however, renegotiation takes place after the default of the winner and thus the initial contract has no impact on the outcome of renegotiation.

A similar form of renegotiation is analyzed by Wang (2000) and Shachat and Tan (2015). In these models the procurer either accepts the lowest bid or rejects all bids. In case of rejection, the procurer negotiates with the supplier who placed the lowest bid; i.e., if renegotiation takes place the initial contract concluded by the auction is not binding. In such a setup renegotiation always leads to lower prices, which is exactly the opposite from what we study.

The initial contract has an impact on the outcome of renegotiation in Chang, Salmon, and Saral (2016). Here, suppliers’ production costs have an ex-ante unknown common component. Some of the suppliers are wealth constrained, while other have deep pockets and this is private information of each supplier. Allowing for contract renegotiation is advantageous to wealth-constrained suppliers who can credibly threaten to default. The prices increase with renegotiation in order to avoid bankruptcy of the contractor who is faced by unexpectedly high costs. In our model, the parties agree to a different design of the project ex post, which is often more costly to produce and thus the final price exceeds the initial price.

A few papers directly deal with the issue of cost overruns. Birulin and Izmalkov (2013) analyze what shares of a price are optimally paid before and after a potential extra cost to the supplier realizes when suppliers are protected by limited liability. This paper is orthogonal to ours, because it does not provide an explanation for cost overruns but rather assumes its existence. Closer to our work is Ganuza (2007). Here, suppliers are differentiated à la Salop (1979). The buyer does not know her preferences – her location – but can invest in obtaining a noisy signal. A procurement order for the expected optimal design is awarded via a price-only auction. Ex post, the buyer’s preferences are common knowledge and the winner of the auction can make a take-it-or-leave-it renegotiation offer. The main result is that the buyer under-invests in learning her preferences because this enhances competition in the initial auction. This is related to our result that the initial design is chosen to enhance competition in the initial auction. There are, however, crucial differences. For instance, in Ganuza (2007) the parties renegotiate the contract because of new incoming information about the buyer’s preferences. Cost overruns are almost automatic because the ex post optimal design will with probability one be different from the ex ante design and the contractor has the lowest production cost for the ex ante

\footnote{A similar model is analyzed by Birulin (2014).}
design. By contrast, in our model there is no uncertainty about the buyer’s preferences and depending on the ex ante design, the price will be renegotiated downward or upward. In other words, cost overruns are the outcome of a strategic decision in our model. Moreover, Gamuzal (2007) analyzes solely price-only auctions, while we investigate also multi-dimensional auctions. A similar model where the procurer’s preferences are initially unknown and suppliers are horizontally differentiated is analyzed by De Chiara (2015). In his model, the procurer is indifferent between all initial designs if contract renegotiation is efficient.

Finally, costly renegotiation of incomplete procurement contracts is analyzed by Bajari and Tadelis (2001) and Herweg and Schmidt (2017). The former paper analyzes when fixed-price contracts outperform cost-plus contracts, while the latter one derives conditions so that bilateral negotiations outperform procurement auctions.

2. The Model

2.1. Players and Payoffs. A procurer $P$ (she), say a government agency, wants to buy one unit of an indivisible good, e.g., a bridge. The good can be produced and delivered in one of two designs $x \in \{x_L, x_H\}$. Design $x_L$ is a standard design of rather low quality, while $x_H$ is a “fancy” design of high quality. The procurer’s gross valuation of the good is denoted by $v(x)$, with $v(x_H) > v(x_L)$. Thus, if the procurer obtains design $x$ at price $p$, her ex post utility is

\begin{equation}
    u = v(x) - p.
\end{equation}

There are $n \geq 2$ suppliers that can produce the good required by the procurer. A supplier’s production cost depends on the design $x$ and his cost type $\theta \in [\bar{\theta}, \bar{\theta}] \equiv \Theta$, and is denoted by $c(x, \theta)$. Ex ante, the cost type $\theta$ is private information of each supplier. The $n$ cost types are drawn independently according to an identical cumulative distribution function $F(\theta)$. Let the corresponding probability density be $f(\theta)$, with $f(\theta) > 0$ for all $\theta \in \Theta$. When design $x$ is delivered at price $p$, the

\footnote{We conjecture that scoring auctions are optimal in the model of horizontally differentiated suppliers analyzed by Gamuzal (2007). A scoring auction allows the procurer to select the ideal supplier with a high probability but nevertheless achieves strong competition by handicapping suppliers that can propose designs closer to the procurer’s ideal one. As we demonstrate, in our model with vertically differentiated suppliers, the procurer cannot benefit from using a multi-dimensional auction.}

\footnote{An empirical analysis of incomplete procurement contracts is provided by Crocker and Reynolds (1993). They argue that contracts are left incomplete intentionally to economize on the cost of the ex ante design.}
COST OVERRUNS IN PROCUREMENT

contractor’s, i.e., the selected supplier’s, ex post payoff is

\[ \pi = p - c(x, \theta). \]

All parties are assumed to be risk neutral and the outside option utilities are all set equal to zero.

We assume that a supplier’s cost function is differentiable with respect to the supplier’s type and satisfies the following assumption.

**Assumption 1.** For all \( \theta_i, \theta_j \in \Theta \) with \( \theta_i < \theta_j \):

- (i) \( c(x_L, \theta_i) < c(x_L, \theta_j) \);
- (ii) \( c(x_H, \theta_i) - c(x_L, \theta_i) < c(x_H, \theta_j) - c(x_L, \theta_j) \).

Part (i) of Assumption [1] is purely labeling; types are ordered according to how efficient they can produce the standard (low-quality) design \( x_L \). According to part (ii), the cost advantage is increasing in the complexity of the design; i.e., the difference in production costs between two types is larger for the fancy design \( x_H \) than for the standard design \( x_L \). This implies that for the implementation of the fancy design, selecting an efficient supplier is more important. Assumption [1] implies that the cost for producing design \( x_H \) is also increasing in the type \( \theta \). In order to see this, note that \( c(x_H, \theta) \) can be written as

\[ c(x_L, \theta) + [c(x_H, \theta) - c(x_L, \theta)]. \]

This does not imply, however, that for any type, production of the fancy design \( x_H \) is more expensive than production of the standard design \( x_L \).

Finally, we assume that the ex post efficient design, \( x^*(\theta) \in \arg \max_{x \in \{x_L, x_H\}} \{v(x) - c(x, \theta)\} \), for the lowest and the highest type differs. This implies that the ex post efficient design is

\[ x^*(\theta) = \begin{cases} x_H & \text{for } \theta < \tilde{\theta}, \\ \{x_L, x_H\} & \text{for } \theta = \tilde{\theta}, \\ x_L & \text{for } \theta > \tilde{\theta}, \end{cases} \]

where \( \tilde{\theta} \) is implicitly defined by \( v(x_H) - c(x_H, \tilde{\theta}) = v(x_L) - c(x_L, \tilde{\theta}) \). Let the social surplus generated by the ex post efficient design be

\[ S(\theta) = \max_{x \in \{x_L, x_H\}} \{v(x) - c(x, \theta)\}, \]

which is decreasing in \( \theta \).
2.2. Award Procedure and Renegotiation. The procurement contract is awarded to one supplier, called the contractor, via an auction. For now, we focus on simple price-only auctions. In other words, the procurer can collect only price bids for a given design. For the ease of exposition, we assume that the procurer uses a second-price sealed-bid auction to award the contract for a pre-specified design $x \in \{x_L, x_H\}$. As we will explain below, the (reduced form) auction game that we analyze satisfies the assumptions of the revenue equivalence principle and thus the restriction to second-price auctions is without loss of generality.

When specifying the initial auction, the procurer has only one choice variable, the initial design $\bar{x} \in X$. The procurement order for the good with design $\bar{x}$ is auctioned off between the $n$ suppliers. Each supplier $i$ places a secret price bid $p_i$. The supplier with the lowest bid is selected as the contractor and the specified price equals the second lowest bid. If the lowest bid is made by several suppliers, one of these suppliers is selected at random as the contractor.

With the procurer being restricted to simple auctions and suppliers’ types being stochastic, the initial design $\bar{x}$ may not be optimal given the contractor’s type ex post. In this case there is scope for renegotiation. We posit that the contractor’s type is observed by the procurer after the award of the contract and thus the parties engage in (efficient) Coasian bargaining ex post.

**Assumption 2.** After the award of the initial contract but before contract renegotiation takes place, the contractor’s type $\theta$ is observed by the procurer.

In practice, the contractor starts working on the project before the parties agree to renegotiate. The procurer monitors the contractor and thus obtains an informative signal – next to the contractor’s bid – about his efficiency. For simplicity we focus on the extreme case where the contractor’s type is perfectly observed. Doubtlessly, this is a strong assumption but it allows us to simplify the exposition significantly. The surplus from renegotiation is split between the procurer and the contractor.

12The restriction to price-only auctions is relaxed in Section 5.
13Alternatively the procurer might receive a signal about the contractor’s type due to information acquisition which would have been too costly before the selection of a certain supplier.
14If the true type of the contractor is observed ex post, the procurer could use a mechanism that makes payments contingent on the true type; c.f. Skrzypacz (2013). As one anonymous referee correctly pointed out, the procurer can implement the first-best allocation without leaving a rent to suppliers by using a “shoot the liar”-mechanism: The contractor is punished severely for not having reported the true type ex ante. Such mechanisms are hardly observed in practice. Our paper is more in the spirit of the literature on incomplete contracts and therefore does not allow for such mechanisms. We restrict our attention to auctions because those are commonly observed in practice.
according to the generalized Nash bargaining solution (GNBS), i.e., the renegotiation contract is
\[
(\hat{x}, \hat{p}) \in \arg \max_{x \in \{x_L, x_H\}, p \in \mathbb{R}} [p - c(x, \theta) - d_C]^\alpha \times [v(x) - p - d_P]^{1-\alpha},
\]
where \(\alpha \in (0, 1)\) denotes the contractor’s relative bargaining power ex post \(^{15}\) The disagreement payoffs of the two parties are determined by the initial contract \((x, p)\) \(^{16}\)
\[
\begin{align*}
d_C & = p - c(x, \theta) \\
d_P & = v(x) - p.
\end{align*}
\]

2.3. Timing of the Game. At stage 1, nature draws each supplier’s cost type \(\theta\). The procurer selects the initial design \(\bar{x} \in \{x_L, x_H\}\) that she seeks to purchase via a price-only auction. At stage 2, the auction is executed; i.e., each supplier places a price bid and the supplier who demanded the lowest price is awarded with the contract \((x, p)\), where \(p\) is the second lowest price bid. At the beginning of stage 3, the procurer observes the contractor’s cost type and the parties engage in Coasian bargaining and agree to implement a renegotiation contract \((\hat{x}, \hat{p})\). The sequence of events is depicted in Figure 1.

As equilibrium concept, we employ perfect Bayesian equilibrium in symmetric strategies.

3. The Analysis

3.1. Contract Renegotiation and Bidding Behavior. We start the analysis with the renegotiation game. Suppose the procurer awarded a supplier with cost type \(\theta\) with the contract \((x, p)\). If the design \(x\) is not the efficient design given the contractor’s cost type, \(x \neq x^*(\theta)\), then there is scope for renegotiation. For instance, if \(x = x_L\) and the contractor’s type \(\theta < \tilde{\theta}\), the social surplus can be increased by

\(^{15}\)For a detailed description of the Nash bargaining solution see Muthoo (1999). A non-cooperative foundation for the Nash bargaining solution is provided by Binmore, Rubinstein, and Wolinsky (1986).

\(^{16}\)Exactly the same findings are obtained with an alternative bargaining game, where the GNBS is replaced by a take-it-or-leave-it offer game. With probability \(\alpha\) the contractor can make a take-it-or-leave-it offer in the renegotiation game, while with probability \((1 - \alpha)\) the procurer can.
adjusting the design from $x = x_L$ to $x^*(\theta) = x_H$. The additional surplus is split between the two parties according to their relative bargaining power. The outcome of renegotiation is characterized in the following lemma.

**Lemma 1** (Renegotiation outcome). Suppose Assumption 2 holds and let the contractor’s type be $\theta$. Ex post, for any initial contract $(\bar{x}, \bar{p})$, the procurer and the contractor agree to trade design $\hat{x} = x^*(\theta)$ at price

$$\hat{p}(x, p, \theta) = p + \alpha[v(x^*(\theta)) - v(x)] + (1 - \alpha)[c(x^*(\theta), \theta) - c(x, \theta)].$$

A supplier taking part in the auction is aware that the contract may be renegotiated ex post. In particular, he knows that if he wins the auction, he may obtain additional profits generated by contract renegotiation. These additional profits from renegotiation are incorporated in a supplier’s bidding behavior. Supplier $\theta$’s ex post payoff from being awarded with the procurement contract $(\bar{x}, \bar{p})$ is

$$\pi(\bar{x}, \bar{p}, \theta) = \hat{p}(\bar{p}, \theta) - c(x^*(\theta), \theta)$$

$$= p + \alpha[v(x^*(\theta)) - c(x^*(\theta), \theta)] - \alpha v(x) - (1 - \alpha) c(x, \theta).$$

(9)

It is important to note that the outcome of the final stage of the game, the renegotiation game, is deterministic – it is fully determined by the GNBS. For a given initial design $\bar{x}$, the reduced form auction game simplifies to a standard auction. Defining an adjusted type

$$\psi(\theta | x) = c(x, \theta) - \alpha \{ [v(x^*(\theta)) - c(x^*(\theta), \theta)] - [v(x) - c(x, \theta)] \},$$

(10)

allows us to write a supplier’s profit from winning as $\pi = p - \psi$. Importantly, there is a one-to-one mapping from the type $\theta$ into an adjusted type $\psi$. Under any standard auction format, the lowest adjusted type, which is also the lowest true type, will win the auction. For a given initial design $x$, the revenue equivalence principle holds; i.e., the expected initial price is identical for all standard auction formats. The final allocation is also independent of the used auction format because Lemma 1 holds irrespective of how the initial contract was determined.17

As explained above, we illustrate our findings for the case of a second-price auction. Here, the price bid affects directly the probability of winning the auction but only indirectly the price the supplier receives when being awarded with the contract. Thus, placing the lowest feasible bid that allows the supplier to break-even, even when he is awarded with the contract at a price equal to his bid, is optimal. The equilibrium bidding behavior is formally described in the next lemma.

17In Appendix B we formally establish that the revenue equivalence principle holds in our model.
Lemma 2 (Outcome of the auction). Suppose that Assumptions 1 and 2 hold and that the procurement order for design $x \in \{x_L, x_H\}$ is awarded via a second-price sealed-bid auction. In the unique equilibrium in undominated strategies, each supplier uses the bidding function

$$p(x, \theta) = c(x, \theta) - \alpha \{v(x^*(\theta)) - c(x^*(\theta), \theta)\} - [v(x) - c(x, \theta)],$$

which is continuous and strictly increasing in $\theta$. The supplier with the lowest type wins the auction.

It is important to note that – according to Lemma 2 – the auction selects the most efficient supplier. In other words, productive efficiency is always guaranteed by a second-price auction even if contract renegotiation is feasible. This relies on the assumption that a more efficient type has not only lower production costs for producing design $x$ but also generates a higher surplus by adjusting the design via contract renegotiation. Moreover, by Lemma 1, the ex post design is always efficient. Hence, overall efficiency is always achieved; i.e., the most efficient type delivers the efficient design. This implies that the procurer’s problem is solely a problem of rent extraction. She wants to procure the efficient design from the efficient supplier at the lowest feasible price. There is no tradeoff between rent extraction and efficiency.

3.2. Constrained Optimal Auction. The procurer solely cares about the ultimate price she has to pay for the good. The initial price – i.e., the price specified in the procurement contract, is determined by the auction and depends on the cost of the second lowest type, type $\theta^2$. It is given by

$$p(x, \theta^2) = \alpha v(x) + (1 - \alpha)c(x, \theta^2) - \alpha S(\theta^2).$$

The procurer’s ex post utility, for given realizations of $\theta^1$ and $\theta^2$, is

$$u(x, \theta^1, \theta^2) = v(x^*(\theta^1)) - p(\theta^2) - \alpha[v(x^*(\theta^1)) - v(x)]$$

$$- (1 - \alpha)[c(x^*(\theta^1), \theta^1) - c(x, \theta^1)]$$

$$= (1 - \alpha)S(\theta^1) + \alpha S(\theta^2) - (1 - \alpha)[c(x, \theta^2) - c(x, \theta^1)].$$

The first part of the procurer’s ex post utility can be written as $S(\theta^2) + (1 - \alpha)[S(\theta^1) - S(\theta^2)]$; i.e., the procurer obtains the whole surplus generated by the second most efficient type due to the competitive award procedure. On top of that, the procurer obtains the share $1 - \alpha$ of the rents that are generated by the excess efficiency of type $\theta^1$ compared to type $\theta^2$. This, however, is only half the story. Different supplier types benefit differently from contract renegotiation ex post. The most efficient
type benefits more from contract renegotiation than the second most efficient type because he can produce design $\bar{x}$ at lower cost. Therefore, the contractor obtains a rent which equals his advantage from contract renegotiation as compared to type $\theta^2$, plus the share $\alpha$ of the additional surplus that he generates, $\alpha[S(\theta^1) - S(\theta^2)]$.

Now, we can state the first main finding of the paper.\footnote{\textsuperscript{18}If the procurer can specify a maximum bid $R$ – i.e., only price bids $p \leq R$ are allowed in the second-price auction – and this maximum bid is publicly announced before suppliers place their bids, it is still optimal for the procurer to specify $x = x_L$ initially \cite{HerwegSchwarz2016}.}

**Proposition 1.** Suppose that Assumptions 1 and 2 hold. Then, the procurer optimally chooses design $\bar{x} = x_L$. Renegotiation takes place if and only if $\theta^1 < \tilde{\theta}$. If this is the case, the ultimate price exceeds the initial price – i.e., $\hat{p} > \bar{p}$, if and only if either (i) $c(x_H, \theta^1) \geq c(x_L, \theta^1)$, or (ii) $c(x_H, \theta^1) < c(x_L, \theta^1)$ and $\alpha > \hat{\alpha}(\theta^1)$, where

$$\hat{\alpha}(\theta^1) \equiv \frac{c(x_L, \theta^1) - c(x_H, \theta^1)}{v(x_H) - v(x_L) + c(x_L, \theta^1) - c(x_H, \theta^1)} \in (0, 1).$$

The contractor’s bid already reflects that renegotiation may take place. In other words, part of the contractor’s profits made by contract renegotiation are competed away in the initial auction. The profits from renegotiation that are not competed away can be decomposed into two parts. The first part is the additional surplus the contractor generates compared to the second most efficient supplier, $S(\theta^1) - S(\theta^2)$. The second part, $c(\bar{x}, \theta^2) - c(\bar{x}, \theta^1)$, is due to the fact that the contractor’s disagreement payoff is higher than the one of the second most efficient supplier; i.e., the contractor can produce $\bar{x}$ at lower costs than all other suppliers. From the procurer’s perspective, the first part is a random variable, which does not depend on her choice variable, the initial project design $\bar{x}$. The second part, on the other hand, depends on the initial design. The more complex the initial design is, the larger is the difference in disagreement payoffs between suppliers of different types. Hence, in order to minimize this difference, the procurer optimally specifies the standard (low-quality) design ex ante.\footnote{\textsuperscript{19}If $\alpha = 1$, i.e., the contractor has all the bargaining power, the procurer is indifferent between $x = x_L$ and $x = x_H$. In this case the revelation of the contractor’s type is irrelevant and we are in the standard framework: The payoffs are pinned down by the utility of the least efficient type in combination with the incentive constraints; i.e., all types are revealed truthfully \cite{Myerson1981}.} If the type of the winner of the auction is sufficiently efficient (low), then contract renegotiation takes place and the parties agree to trade the fancy design $x_H$ ex post. If the fancy design is more costly to produce for any type, renegotiation always leads to a price increase. If the winner, type $\theta^1$, can produce the fancy design at lower cost than the standard design, then contract renegotiation leads to a price increase only if the contractor’s bargaining
power at the renegotiation stage is not too weak. If the procurer’s bargaining power is strong, then a cost underrun occurs.

The next result is readily obtained by noting that the probability of contract renegotiation is given by \( \text{prob}(\text{reneg}) = 1 - [1 - F(\bar{\theta})]^n \).

**Corollary 1.** Contract renegotiation is more likely (in the sense of set inclusion), the more suppliers participate in the auction.

When renegotiation is inefficient or there is a risk it might break down, however, there is a tradeoff and the optimal ex ante design is not necessarily the standard one any more, as we demonstrate in Subsection 4.2.

4. **Asymmetric Information and Breakdown of Renegotiation**

We derived our findings under a couple of strong assumptions. In particular, we assumed that renegotiation takes place under symmetric information, which implies that there is no risk that renegotiation may fail. In this section we address this issue by first relaxing Assumption 2 and second by introducing an exogenous risk of renegotiation failure.

4.1. **Renegotiation with Asymmetric Information.** In this subsection we assume that contract renegotiation may take place under asymmetric information. More precisely, the procurer does not observe the contractor’s cost type before the two parties engage in contract renegotiation; i.e., we relax Assumption 2. The procurer may be able to deduce the contractor’s type from the bids, which she observes. In order to keep this signaling model as simple as possible, we restrict attention to the case of binary types \( \theta \in \{\theta_1, \theta_2\} \equiv \Theta \). As before the types are independently drawn from the same distribution. Let \( \text{prob}(\theta = \theta_1) = q \in (0, 1) \).

The good can be delivered in one of two designs, \( x \in \{x_L, x_H\} \), with \( v(x_H) > v(x_L) \). The type \( \theta \) orders suppliers according to their efficiency in the production of the standard design \( x_L \):

\[
(13) \quad c(x_L, \theta_1) < c(x_L, \theta_2).
\]

Moreover, we assume that the efficient design is different across the two types

\[
(14) \quad x^*(\theta) = \begin{cases} x_H & \text{for } \theta = \theta_1; \\ x_L & \text{for } \theta = \theta_2. \end{cases}
\]

First, this assumption implies that \( c(x_H, \theta_2) > c(x_L, \theta_2) \). Second, it implies that

\[
(15) \quad c(x_H, \theta_2) - c(x_L, \theta_2) > v(x_H) - v(x_L) > c(x_H, \theta_1) - c(x_L, \theta_1).
\]
Finally, (13) and (14) imply that \( S(\theta_1) > S(\theta_2) \). The above assumptions correspond to Assumption 1 from the general model with a continuum of types.

Now, contract renegotiation takes place under asymmetric information and thus we cannot apply the GNBS in order to determine the outcome of renegotiation. Instead, we posit that the contractor can make a take-it-or-leave-it (TIOLI) offer with probability \( \alpha \in (0, 1) \). With the converse probability, \( 1 - \alpha \), the procurer can make a TIOLI offer at the renegotiation stage.

The equilibrium concept we employ is perfect Bayesian equilibrium in pure strategies. The analysis will focus mainly on separating equilibria. In a separating equilibrium the two types place different bids in the auction and thus the type is revealed to the procurer before renegotiation takes place.

For the sake of the argument suppose the procurer sets \( \bar{x} = x_L \). In a separating equilibrium, the procurer can perfectly deduce the contractor’s type from his bid. If the contractor’s type is \( \theta_2 \), there is no scope for renegotiation. Suppose the contractor’s type is \( \theta_1 \). If he can make the offer at the renegotiation stage, he offers \( \hat{x} = x_H \) at

\[
\hat{p}^C = p + [v(x_H) - v(x_L)],
\]

which is accepted by the procurer. If, on the other hand, the procurer can make the offer, she offers \( \hat{x} = x_H \) at

\[
\hat{p}^P = p + [c(x_H, \theta_1) - c(x_L, \theta_1)].
\]

The offer is accepted by type \( \theta_1 \). Note that this offer would be rejected by type \( \theta_2 \). Thus, the final expected price of a supplier of type \( \theta_1 \) is

\[
\hat{p} = p + \alpha[v(x_H) - v(x_L)] + (1 - \alpha)[c(x_H, \theta_1) - c(x_L, \theta_1)].
\]

At the auction stage, each supplier of type \( \theta_1 \) takes into account that contract renegotiation will occur. A candidate for equilibrium bidding strategies are the bids that allow a supplier just to break-even if the bid determines the price at which the supplier has to deliver the good:

\[
p(\theta|x_L) = \begin{cases} 
    c(x_H, \theta_1) - \alpha[v(x_H) - v(x_L)] - (1 - \alpha)[c(x_H, \theta_1) - c(x_L, \theta_1)] & \text{if } \theta = \theta_1; \\
    c(x_L, \theta_2) & \text{if } \theta = \theta_2.
\end{cases}
\]

Notice that \( p(\theta_1, x_L) < p(\theta_2, x_L) \); i.e., if there is a supplier of type \( \theta_1 \), then a type \( \theta_1 \) wins the auction. As we show in the appendix, none of the supplier types has an incentive to deviate from the above bidding strategy.
If, on the other hand, the procurer specifies $x = x_H$ initially, then the candidate bidding function for a separating equilibrium is:

$$\bar{p}(\theta|x_H) = \begin{cases} c(x_H, \theta_1) & \text{if } \theta = \theta_1; \\ \alpha c(x_L, \theta_2) + (1 - \alpha)c(x_H, \theta_2) + \alpha[v(x_H) - v(x_L)] & \text{if } \theta = \theta_2. \end{cases} \quad (20)$$

Again, it can be shown that $\bar{p}(\theta_1|x_H) < \bar{p}(\theta_2|x_H)$. Hence, irrespective of the initial design, if there is a supplier of type $\theta_1$, then a supplier of this type wins the auction.

The procurer optimally specifies $\bar{x} = x_L$ if and only if $E[u(x_L, \theta_1, \theta_2)] > E[u(x_H, \theta_1, \theta_2)]$, which is equivalent to

$$c(x_H, \theta_2) - c(x_L, \theta_2) > c(x_H, \theta_1) - c(x_L, \theta_1). \quad (21)$$

The above inequality is always satisfied under the imposed assumptions.

**Proposition 2** (Asymmetric Information). Suppose the contractor’s type is not observed ex post and that $\theta \in \{\theta_1, \theta_2\}$. Then, there are separating equilibria with bidding functions (19) and (20). For these bidding strategies, the procurer optimally specifies the standard design $x = x_L$ initially. Contract renegotiation to the fancy design $x_H$ takes place with probability $1 - (1 - q)^n$. In case of contract renegotiation, there is a cost overrun if and only if either (i) the contractor can make the TIOLI offer, or (ii) the procurer can make the TIOLI offer and $c(x_H, \theta_1) > c(x_L, \theta_1)$. Moreover, a pooling equilibrium does not exist.

Proposition 2 shows that the finding of Proposition 1 is not an artifact due to Coasian bargaining ex post. With more than two types, however, we cannot expect to obtain a fully separating equilibrium. In particular, more efficient types will have an incentive to mimic less efficient types that do not differ with respect to the efficient design. In such a case the procurer may make a renegotiation offer which is accepted only by sufficiently efficient types. Now, a tradeoff between rent extraction and efficiency may arise. This tradeoff is absent in our baseline model with renegotiation under complete information but also in the model with asymmetric information and only two types.

4.2. **Risk of Breakdown of Renegotiation.** In the baseline model and in the simple model with binary types and asymmetric information, the efficient design is always implemented ex post. In the following, we will show that our main findings are robust toward introducing frictions of contract renegotiation. In order to do so, we augment the baseline model with a continuum of types.

We model the imperfection of contract renegotiation in the following simple way. With fixed exogenous probability $b \in [0, 1)$ the parties cannot reach an agreement
ex post – i.e., renegotiation fails. In this case the initial contract \((x, p)\) is executed.

With the converse probability \(1 - b\) the parties reach an agreement and the outcome is determined by the GNBS. The parameter \(b\) measures how intricate or how costly contract renegotiation is. For \(b = 0\) the model collapses to the one previously analyzed\(^{20}\).

The analysis of the model with a risk of renegotiation breakdown proceeds by the same steps as the analysis of Section \(3\).

**Proposition 3.** Suppose that Assumptions \(4\) and \(5\) hold and that renegotiation fails with probability \(b \in [0, 1)\). The procurer optimally chooses the design \(\bar{x}\) that solves

\[
\max_{\bar{x}} \mathbb{E}_{\theta_1, \theta_2}\left[ b[v(\bar{x})] - c(\bar{x}, \theta_2) \right] - (1 - \alpha)(1 - b)[c(\bar{x}, \theta_2) - c(\bar{x}, \theta_1)]
\]

The procurer now faces a tradeoff. On the one hand, as before, she wants to minimize the cost advantage that the most efficient supplier has in comparison to the second most efficient supplier in the production of design \(x\). This is achieved by setting \(\bar{x} = x_L\). On the other hand, the procurer has an incentive to choose as initial design the design that is optimal when the second most efficient supplier obtains the contract. This is intuitive because if renegotiation fails the procurer obtains the surplus that is generated by the second most efficient type. This is likely to be achieved by the fancy design \(x_H\). If \(b\) is sufficiently low, the former concern dominates the latter and \(x = x_L\) is optimal. As the risk of renegotiation failure increases, the optimal ex ante design becomes (weakly) more complex\(^{21}\).

If \(x = x_H\) is optimal, then ex post renegotiation may lead to a downward price adjustment even if the conditions for upward price adjustments from Proposition \(4\) are met. How likely upward and downward price adjustments are, is intricate to characterize without further assumptions on the type distribution, the feasible designs, and the cost functions. Therefore, we will present the results of a simple numerical example in the following.

**Example 1.** Let the procurer’s gross benefit be \(v(x_H) = 2\) and \(v(x_L) = 1\). The cost functions are \(c(x_H, \theta) = 3\theta\) and \(c(x_L, \theta) = \theta\). The types of the \(n \geq 3\) suppliers are drawn independently from the uniform distribution with support \([0, 1]\). The ex post

\(^{20}\)Ganuza (2007) uses the same approach to model transaction costs of contract renegotiation. In his interpretation there are transaction costs associated with renegotiation and these costs are stochastic. With probability \(1 - b\) the transaction costs are zero, while with probability \(b\) the transaction costs are prohibitively high so that renegotiation does not take place.

\(^{21}\)We can interpret \(b\) as a bargaining inefficiency multiplier. As renegotiation becomes more costly, the optimal ex ante design becomes (weakly) more complex.
efficient design is $x^*(\theta) = x_H$ for $\theta \leq 1/2$ and $x^*(\theta) = x_L$ for $\theta \geq 1/2$; i.e., the marginal type is $\hat{\theta} = 1/2$.

The procurer optimally specifies design $x = x_L$ initially if and only if

\begin{equation}
\hat{b} \leq \frac{2 - 2\alpha}{n - 1 - 2\alpha} \in (0, 1].
\end{equation}

As long as the risk of renegotiation failure is not too high, the procurer prefers to specify the standard design ex ante. The stronger the contractor’s bargaining power $\alpha$, the lower is the critical threshold $\hat{b}$; i.e., a strong bargaining position of the contractor makes it less likely that $x = x_L$ is optimal.

For $b > \hat{b}$ the procurer sets $x = x_H$. In this case, if renegotiation takes place, it leads to a downward adjustment of the price; i.e., a cost underrun. This happens if $\theta^1 > \hat{\theta}$. Thus, the conditional probability (conditional on renegotiation taking place) for a cost underrun is $\rho^D = \text{prob}(\theta^1 > \hat{\theta}) = \frac{1}{2^n}$. This conditional probability is rather low already for a moderate number of competitors; e.g., for $n = 6$ we have $\rho^D \approx 1.6\%$.

For $b \leq \hat{b}$ the procurer specifies $x = x_L$. Now, if renegotiation takes place, the price is adjusted upwards and thus we observe a cost overrun. The conditional probability for a cost overrun is $\rho^U = \text{prob}(\theta^1 < \hat{\theta}) = 1 - \frac{1}{2^n}$. This conditional probability is high already for a moderate number of competitors; e.g., for $n = 6$ we have $\rho^U \approx 98.4\%$.

The example illustrates that even when there is a risk of renegotiation failure – and thus a rational for the procurer to choose a more complex design than $x_L$ – ex post adjustments leading to a cost underrun are unlikely to occur. The procurer selects $x = x_H$ only if $b$ is rather high, which implies that it is unlikely that the parties are able to renegotiate the contract. Moreover, even when the parties are able to renegotiate, contract renegotiation takes place only if the most efficient type is rather inefficient, i.e., only if $\theta^1 > \hat{\theta}$, which is a rather unlikely event.

5. Scoring Auctions

So far we assumed that the procurer has to specify the good she wants to procure completely ex ante, i.e., before the auction takes place. In the auction, the procurer collected bids only on prices and the supplier who offered the lowest price has been awarded with the contract. Different types of suppliers do not only have different production costs but also differ in the optimal design – i.e., the design that maximizes
the joint surplus. Therefore, it may be profitable for the procurer to ask suppliers for bids on price and design.\footnote{Scoring auctions where bids are multi-dimensional (e.g., price and quality) are analyzed by Che (1993) and Asker and Cantillon (2008). An excellent short review of this literature is provided by Asker and Cantillon (2010).}

5.1. The Model with Multi-Dimensional Auctions. In the following we consider a second-score auction.\footnote{We show that our results hold for first-score auctions in Appendix B.} Each supplier places a bid containing a price $p \in \mathbb{R}$ and a design $x \in \{x_L, x_H\}$. Each bid $(x, p)$ is mapped into a single score. The supplier who placed the bid giving rise to the highest score wins the auction and is required to match the highest rejected score – i.e., the second highest score. The outcome $(x, p)$ determines a binding specific-performance contract between the procurer and the winner (the contractor). Nevertheless, this contract can be renegotiated after the auction as before.

The procurer does not choose a design when particularizing the auction. She specifies a scoring function, $G : \{x_L, x_H\} \times \mathbb{R} \rightarrow \mathbb{R}$, that maps bids into a single score. We focus on quasi-linear scoring functions of the form

$$G(x, p) = g(x) - p,$$

which implies that the procurer effectively chooses $\Delta_g \equiv g(x_H) - g(x_L)$.

If the procurer can commit not to renegotiate the contract, the optimal quasi-linear scoring function implements the second-best allocation (Che, 1993). Here, the procurer cannot commit not to engage in contract renegotiation. However, as we will show below, if the scoring function represents the procurer’s true preferences, i.e., $g(x) \equiv v(x)$, contract renegotiation can be avoided.

5.2. The Analysis of Multi-Dimensional Auctions. As before, we solve the game by backward induction. The outcome of the renegotiation game is independent of the award procedure. In other words, Lemma 1 still holds and the implemented design will always be ex post efficient. Thus, the ex post utility of a supplier of type $\theta$ who has been awarded procurement contract $(\bar{x}, \bar{p})$ is

$$\pi(\bar{x}, \bar{p}, \theta) = \hat{p}(\bar{x}, \bar{p}, \theta) - c(x^*(\theta), \theta)$$

$$= \bar{p} + \alpha S(\theta) - \alpha v(\bar{x}) - (1 - \alpha) c(\bar{x}, \theta).$$

Optimal bidding behavior in the second-score auction is described by the following result.
Lemma 3. Suppose that Assumptions 1 and 2 hold. The (reduced) second-score auction game has a dominant strategy equilibrium. The equilibrium bid of each supplier of type $θ$ is

$$x^b(θ) ∈ \arg\max_{x ∈ X} \{g(x) − αv(x) − (1 − α)c(x, θ)\},$$

$$p^b(θ) = αv(x^b(θ)) + (1 − α)c(x^b(θ), θ) − αS(θ).$$

By Lemma 3, each supplier bids the optimal design, $x^b(θ) = x^∗(θ)$, if the scoring function represents the procurer’s true preferences – i.e., if $v(x) ≡ g(x)$. If, on the other hand, the scoring function does not reflect the true preferences of the procurer, then it is likely that suppliers propose designs that are not efficient.

According to Lemma 3, the score offered by a supplier of type $θ$ amounts to

$$G(θ) ≡ g(x^b(θ)) − p^b(θ)$$

$$= g(x^b(θ)) − αv(x^b(θ)) − (1 − α)c(x^b(θ), θ) + αS(θ).$$

As before, the most efficient type places the bid that leads to the highest score and thus wins the auction. Thus, there is no tradeoff between rent extraction and efficiency as there is for scoring auctions with commitment (Che, 1993); the procurer’s problem is solely a problem of rent extraction.

Lemma 4. Suppose that Assumption 1 holds. Then for all $θ_1 < θ_2$ it holds that:

$$G(θ_1) > G(θ_2).$$

The winner of the auction, type $θ^1$, has to match the second highest score but is otherwise free in its choice of $(x, p)$. Thus, the winner of the auction chooses the initial contract $(x, p)$ in order to maximize

$$p + αS(θ_1) − αv(x) − (1 − α)c(x, θ)$$

subject to $g(x) − p = G(θ_2)$. Hence, the initial contract specifies $x(θ_1, θ_2) = x^b(θ_1)$ and $p(θ_1, θ_2) = g(x^b(θ_1)) − G(θ_2)$. The ultimate price paid by the procurer is

$$h(θ_1, θ_2) = g(x^b(θ_1)) − g(x^b(θ_2)) + αv(x^b(θ_2)) + (1 − α)c(x^b(θ_2), θ_2)$$

$$− αS(θ_2) + α[v(x^∗(θ_1)) − v(x^b(θ_1))]$$

$$+ (1 − α)[c(x^∗(θ_1), θ_1) − c(x^b(θ_1), θ_1)].$$

5.3. The Optimality of Price-Only Auctions. We use equation (25) to derive the procurer’s ex post utility – for given realizations of $θ^1$ and $θ^2$ –, which is given
by
\[ u(\theta^1, \theta^2) = v(x^*(\theta^1)) - \hat{p}(\theta^1, \theta^2) \\
= (1 - \alpha)S(\theta^1) + \alpha S(\theta^2) \\
+ \{ g(x_b(\theta^2)) - \alpha v(x_b(\theta^2)) \} \\
- \{ g(x_b(\theta^1)) - \alpha v(x_b(\theta^1)) \}. \] (26)

If the designs chosen by the most efficient and the second most efficient type are the same, i.e., \( x_b(\theta^1) = x_b(\theta^2) \), equation (26) is highly reminiscent to equation (12) from the case of price-only auctions. In general, the first part of the procurer’s ex post utility reflects that the procurer obtains the whole surplus generated by the second most efficient type due to the competitive award procedure. In addition she obtains the share \( 1 - \alpha \) of the rents that are generated by the excess efficiency of type \( \theta^1 \) compared to type \( \theta^2 \). As before, the procurer has to leave a rent to the most efficient type which reflects type \( \theta^1 \)’s advantage in the initial auction compared to type \( \theta^2 \). Notice that the sum of the two terms in curly brackets is always negative. This advantage is now more complex than simply the difference in production costs for design \( x \) due to the applied scoring auction.

The procurer chooses the scoring function \( g(\cdot) \) – or more precisely the difference \( \Delta_g = g(x_H) - g(x_L) \) – that maximizes her expected payoff \( E[u(\theta^1, \theta^2)] \). As it turns out, the optimal scoring auction coincides with the optimal price-only auction.

**Proposition 4.** Suppose that Assumptions 1 and 2 hold. Then, any optimal quasi-linear scoring rule specifies
\[ \Delta_g \leq \alpha[v(x_H) - v(x_L)] + (1 - \alpha)[c(x_H, \theta) - c(x_L, \theta)]. \]
Each supplier type \( \theta \) bids \( x_b(\theta) = x_L \). Renegotiation takes place if and only if \( \theta^1 < \hat{\theta} \). If this is the case, the ultimate price exceeds the initial price – i.e., \( \hat{p} - \hat{p} > 0 \), if and only if either (i) \( c(x_H, \theta^1) \geq c(x_L, \theta^1) \), or (ii) \( c(x_H, \theta^1) < c(x_L, \theta^1) \) and \( \alpha > \hat{\alpha}(\theta^1) \).

According to Proposition 4, if the buyer is unable to commit not to renegotiate, she cannot benefit from using a scoring auction. A scoring auction by its multi-dimensionality allows suppliers to differentiate their bids, which reduces price competition. In other words, a more efficient supplier can offer a design that leads to a higher score than a less efficient supplier. By doing so the more efficient supplier may be able to win the auction even if his price bid is relatively high. This makes the usage of a scoring auction expensive and thus less attractive to the procurer. Hence, a price-only auction where the procurer collects price bids for a given design
is optimal. The given design is rather simple, so that the differences between suppliers regarding their costs for delivering this design are relatively low. This enhances the competition at the auction stage and leads to a very low initial price. Even though the ex post price can be significantly higher than the initial price, the effect on the initial price dominates.

The following result follows immediately from Proposition 4.

**Corollary 2.** Suppose that Assumptions 1 and 2 hold and that \( c(x_L, \theta) \leq c(x_H, \theta) \). Then, a scoring function that is independent of the proposed design, i.e., \( \Delta g = 0 \), is optimal.

Instead of fixing an initial design, the procurer can also use a scoring auction where suppliers are free to propose any \( x \in \{x_L, x_H\} \). The award of the contract, however, is solely based on the price bid – i.e., the supplier who placed the lowest price bid is awarded with the contract. According to Corollary 2 such a scoring auction is optimal if the fancy design has higher production costs than the standard design for all supplier types (this is a sufficient but not a necessary condition).

Finally, note that if the scoring function represents the procurer’s true preferences, each supplier \( \theta \) bids the efficient design \( x^*(\theta) \). In this case, there is no scope for renegotiation. Avoiding renegotiation, however, is not in the procurer’s interest. This is due to the fact that we assume efficient – Coasian – bargaining ex post and that the gains from renegotiation are incorporated in the initial price bids.

### 5.4. Multi-Dimensional Auctions and Asymmetric Information

Consider the binary types model of Subsection 4.1; i.e., the procurer cannot directly observe the contractor’s cost type before renegotiation takes place. The difference to Subsection 4.1 is that we now allow the procurer to run a multi-dimensional auction. Again, we restrict attention to quasi-linear scoring rules. The next result establishes that the bidding functions from the case with Coasian bargaining ex post are also part of an equilibrium when renegotiation potentially takes place under asymmetric information (type maybe revealed via the bid).

**Proposition 5.** The suppliers’ equilibrium bids \((x^b(\theta), p^b(\theta))\) specified in Lemma 3 remain equilibrium bids under renegotiation with asymmetric information. Moreover, a pooling equilibrium does not exist.

This implies that the optimal scoring function is not affected by relaxing the assumption of Coasian bargaining ex post; i.e., relaxing Assumption 2. In other
words, the scoring function derived in Proposition 4 is also optimal in the case where the procurer has to deduce the contractor’s type from the bid.

6. Conclusion

We analyzed competitive procurement mechanisms in an environment where the procurer is unable to commit not to renegotiate the contract ex post. Moreover, the cost function of the supplier who has been awarded with the initial contract is publicly observed ex post. Hence, if the initial design turns out to be ex post inefficient the parties adjust the initial design to the ex post efficient one; i.e., the parties engage in Coasian bargaining. We showed that the constrained optimal award procedure is a price-only auction. The procurer awards the contract for the standard design via a price-only auction. Ex post, the fancy design may be implemented via contract renegotiation. If this is the case, the ultimate price typically is higher than the initial price determined by the auction.

The findings of the paper rely on a couple of assumptions that often will not all be satisfied in practice. Hence, we do not argue based on these results that most of the projects with severe cost overruns that we observe in practice are always the result of efficient award procedures. However, our main assumption that commitment not to renegotiate is not feasible seems to be realistic. For instance, complex construction projects often cannot be executed exactly the way as initially specified, so contract renegotiation has to take place. This paper shows that severe cost overruns are not necessarily a sign of inefficient award procedures or project completion.

Appendix A. Proofs and Calculations

Proof of Lemma 7. First, we show that the parties agree to trade \( x^*(\theta) \). In contradiction, let \((\hat{x}, \hat{p})\) with \( \hat{x} \neq x^*(\theta) \) be the outcome of renegotiation. The resulting generalized Nash product is

\[
GNP(\hat{x}, \hat{p}) = [\hat{p} - c(\hat{x}, \theta) - d_C]^\alpha \times [v(\hat{x}) - \hat{p} - d_P]^{1-\alpha}
\]

Consider the alternative contract with design \( x^*(\theta) \) and price \( p^* = \hat{p} + v(x^*(\theta)) - v(\hat{x}) \). By construction, the procurer is indifferent between the two contracts. The contractor’s net payoff under the alternative contract is

\[
p^* - c(x^*(\theta), \theta) - d_C.
\]

This is less clear if there is a risk of breakdown of renegotiation: When the risk of such a breakdown is high, scoring auctions where different types propose different designs are better at implementing the ex post efficient design, which is likely to be in the procurer’s interest.
Hence, the contractor prefers the alternative contract if and only if
\[
\hat{p} + v(x^*(\theta)) - c(x^*(\theta), \theta) - v(\hat{x}) - d_C \geq \hat{p} - c(\hat{x}, \theta) - d_C
\]
(A.3)
\[\iff v(x^*(\theta)) - c(x^*(\theta), \theta) \geq v(\hat{x}) - c(\hat{x}, \theta),\]
which holds by the definition of \(x^*(\theta)\) and the fact that \(\bar{x}\) does not maximize the social surplus. Thus, \(GNP(x^*(\theta), p^*) > GNP(\hat{x}, \hat{p})\) a contradiction to the assumption that \((\hat{x}, \hat{p})\) is the outcome of renegotiation.

Taking the partial derivative of the generalized Nash product with respect to \(p\) yields
\[
\frac{\partial GNP}{\partial p} = \alpha \left[ \frac{v(x^*(\theta)) - p - v(x) + p}{p - c(x^*(\theta), \theta) - p + c(x, \theta)} \right]^\alpha
\]
\[\quad - (1 - \alpha) \left[ \frac{p - c(x^*(\theta), \theta) - p + c(x, \theta)}{v(x^*(\theta)) - p - v(x) + p} \right]^{1-\alpha}.
\]
We set the partial derivative equal to zero and solve for the renegotiation price
\[
\hat{p} = p + \alpha[v(x^*(\theta)) - v(x)] + (1 - \alpha)[c(x^*(\theta), \theta) - c(x, \theta)].
\]
(A.4)

**Proof of Lemma 2**

It is a well-known result that in a second-price auction it is a (weakly) dominant strategy for each bidder to bid his type. Placing a bid equal to the type, corresponds to placing a price bid so that the profit equals zero in our setup. Placing a higher bid reduces the probability of winning the auction without affecting the price \(\bar{p}\). A lower bid is not optimal because in the additional cases where the supplier now wins the auction, he makes losses.

It remains to be shown that \(\theta_1 < \theta_2\) implies \(p(\bar{x}, \theta_1) < p(\bar{x}, \theta_2)\). This property of the bidding function follows immediately from Assumption [1]. Note that \(S(\theta) \equiv \max_x \{v(x) - c(x, \theta)\}\) and thus \(S(\theta_1) > S(\theta_2)\) by Assumption [1].

**Proof of Proposition 1**

The procurer’s expected utility ex ante is
\[
\mathbb{E}[u(x, \theta^1, \theta^2)] = \mathbb{E} \left[(1 - \alpha)S(\theta^1) + \alpha S(\theta^2) - (1 - \alpha)[c(\bar{x}, \theta^1) - c(\bar{x}, \theta^1)] \right].
\]
(A.6)

The expected utility is maximized by the design \(\bar{x} \in \{x_L, x_H\}\) that minimizes
\[
\mathbb{E}[c(\bar{x}, \theta^2) - c(\bar{x}, \theta^1)].
\]
(A.7)

By Assumption [1] the above expression is minimized for \(x = x_L\).

Renegotiation takes place if and only if \(\theta^1 < \tilde{\theta}\). From Lemma [1] it is readily obtained that \(\hat{p} - p > 0\) if and only if
\[
\alpha[v(x_H) - v(x_L)] + (1 - \alpha)[c(x_H, \theta^1) - c(x_L, \theta^1)] > 0,
\]
(A.8)
which completes the proof. □

**Proof of Corollary 1.** The result is shown in the main text. □

**Proof of Proposition 2.** The proof proceeds in three steps. First, we posit that the procurer specifies \( \bar{x} = x_L \) initially. Thereafter, we posit that \( \bar{x} = x_H \). Finally, we show that there does not exist a pooling equilibrium. For the analysis we distinguish two cases, \( \bar{x} = x_L \) and \( \bar{x} = x_H \).

**Case (I): Procurer sets \( \bar{x} = x_L \)**

The offers made at the renegotiation stage are derived in the main text, which allow us to derive the expected profits. At the auction stage, the expected profit of a supplier of type \( \theta \) is

\[
\pi(p, x_L, \theta) = \begin{cases} 
  p + \alpha[v(x_H) - v(x_L)] + (1 - \alpha)[c(x_H, \theta) - c(x_L, \theta)] - c(x_H, \theta_1) & \text{if } \theta = \theta_1; \\
  p - c(x_L, \theta_2) & \text{if } \theta = \theta_2.
\end{cases}
\]

A candidate for equilibrium bidding strategies are the bids that allow a supplier just to break-even if the bid determines the price at which the supplier has to deliver the good.

\[
p(\theta | x_L) = \begin{cases} 
  c(x_H, \theta_1) - \alpha[v(x_H) - v(x_L)] - (1 - \alpha)[c(x_H, \theta_1) - c(x_L, \theta_1)] & \text{if } \theta = \theta_1; \\
  c(x_L, \theta_2) & \text{if } \theta = \theta_2.
\end{cases}
\]

The bid of type \( \theta_1 \) can be written as

\[
p(\theta_1 | x_L) = c(x_L, \theta_1) - \alpha \left\{v(x_H) - c(x_H, \theta_1) - v(x_L) + c(x_L, \theta_1)\right\}. \tag{A.10}
\]

This implies \( p(\theta_1, x_L) < p(\theta_2, x_L) \); i.e., if there is a supplier of type \( \theta_1 \), then a type \( \theta_1 \) wins the auction.

**Incentives to deviate:** Does the bidding strategy \( \{19\} \) constitute an equilibrium?

First, consider a supplier of type \( \theta_2 \). When bidding \( p(\theta_2 | x_L) \) the supplier can win only if all competitors are also of type \( \theta_2 \). In this case he wins with probability \( 1/n \).

The expected profit is \( \pi = 0 \).

(i) Bidding \( p > p(\theta_2 | x_L) \): The supplier never wins and thus makes a zero profit with certainty.

(ii) Bidding \( p \in (p(\theta_1, x_L), p(\theta_2 | x_L)) \): The supplier wins the auction if all competitors are of type \( \theta_2 \). The price is \( p = c(x_L, \theta_2) \). Behavior at the renegotiation stage depends on the out-of-equilibrium belief of the procurer. If the procurer believes that a supplier who bids \( p \in (p(\theta_1, x_L), p(\theta_2 | x_L)) \) is of type \( \theta_2 \), she does not make an offer at the renegotiation stage. If she believes that...
the supplier is with positive probability of type \( \theta_1 \), then she offers \( \bar{x} = x_H \) at price \( \hat{p}^P \). This offer is accepted by a type \( \theta_1 \) contractor but rejected by our type \( \theta_2 \) contractor. If the contractor can make an offer, he does not propose to change the design (changing the design decreases the generated surplus). Hence, irrespective of who can make the offer and the procurer’s belief the initial contract is executed. The expected utility of the considered supplier is \( \pi = 0 \).

(iii) Bidding \( p \leq p(\theta_1|x_L) \): Due to similar arguments as made in case (ii) the initial contract is always executed (either renegotiation fails or is not proposed in the first place). Now, however, the supplier wins for sure. If there is a competitor of type \( \theta_1 \), then the initial and final price is \( p = p(\theta_1|x_L) \). At this price the supplier of type \( \theta_2 \) makes a loss and thus his expected profit from this bid is negative as well.

To sum up, a supplier of type \( \theta_2 \) has no incentives to deviate.

Now, we consider a supplier of type \( \theta_1 \). When bidding \( p(\theta_1|x_L) \) the supplier makes a strictly positive profit if all competitors are of type \( \theta_2 \). If at least one rival is of type \( \theta_1 \) all the expected rents from contract renegotiation are competed away.

(i) Bidding \( p > p(\theta_2|x_L) \): The supplier never wins and thus makes a zero profit.

(ii) Bidding \( p \in (p(\theta_1|x_L), p(\theta_2|x_L)) \): The supplier wins if all rivals are of type \( \theta_2 \) the price is \( p = c(x_L, \theta_2) \). If the supplier won the auction and can make the TIOLI offer, he offers \( \hat{x} = x_H \) at \( \hat{p} = \hat{p}^C \). This offer is accepted by the procurer. If, on the other hand, the procurer can make the TIOLI offer, she either offers \( \hat{x} = x_H \) at \( \hat{p} = \hat{p}^P \) or does not propose a renegotiation contract. Depending on her beliefs about the supplier’s type, the former or the latter strategy is optimal. In both cases, the contractor does not benefit from contract renegotiation. He benefits only if he can make the offer. Thus, bidding \( p \in (p(\theta_1|x_L), p(\theta_2|x_L)) \) is not strictly preferred to bidding \( p(\theta_1|x_L) \).

(iii) Bidding \( p < p(\theta_1|x_L) \): Now, the supplier wins for sure. In the additional cases where the supplier now wins, he makes an expected profit of at most zero. If contract renegotiation takes place, the offered contracts are the same as in case (ii).

A type \( \theta_1 \) supplier has no incentive to deviate.

The proposed equilibrium is not the unique separating equilibrium. As it is well-known, the second-price auction has many equilibria and this holds true also in our model with asymmetric information at the renegotiation stage.
Procurer’s expected profit: The procurer’s realized profit depends on the most efficient type $\theta^1$ and the second most efficient type $\theta^2$. We denote the expected profit by $\mathbb{E}[u(x, \theta^1, \theta^2)]$. We distinguish three cases.

(i) $\theta^1 = \theta^2 = \theta_2$: In this case, renegotiation does not take place and the procurer’s utility amounts to

$$u(x_L, \theta_2, \theta_2) = v(x_L) - c(x_L, \theta_2)$$

(A.11)

(ii) $\theta^1 = \theta^2 = \theta_1$: In this case, renegotiation takes place. The procurer obtains $x_H$ at price $\hat{p}$ (in expectations) and the initial price is $p = p(\theta_1, x_L)$. Thus, the procurer’s utility is

$$u(x_L, \theta_1, \theta_1) = v(x_H) - c(x_L, \theta_2) - \alpha[v(x_H) - v(x_L)] - (1 - \alpha)[c(x_H, \theta_1) - c(x_L, \theta_1)],$$

which can be simplified to

$$u(x_L, \theta_1, \theta_1) = v(x_H) - c(x_H, \theta_1)$$

(A.12)

(A.13)

(iii) $\theta^1 = \theta_1$ and $\theta^2 = \theta_2$: In this case, renegotiation takes place as well but the initial price is $p = p(\theta_2, x_L)$. Thus, the procurer’s utility is

$$u(x_L, \theta_1, \theta_2) = v(x_H) - c(x_L, \theta_2) - \alpha[v(x_H) - v(x_L)] - (1 - \alpha)[c(x_H, \theta_1) - c(x_L, \theta_1)],$$

which can be simplified to

$$u(x_L, \theta_1, \theta_2) = v(x_H) - c(x_H, \theta_1) + \alpha S(\theta_2) - (1 - \alpha)[c(x_L, \theta_2) - c(x_L, \theta_1)].$$

(A.14)

Case (II): Procurer sets $x = x_H$

Now, there is no scope for renegotiation for type $\theta_1$. If the contractor is of type $\theta_2$ and can make the TIOLI renegotiation offer, he proposes $\hat{x} = x_L$ at

$$\hat{p}^C = p - [v(x_H) - v(x_L)].$$

(A.15)

The offer is accepted by the procurer. If the procurer can make the offer, she offers $\hat{x} = x_L$ at

$$\hat{p}^P = p - [c(x_H, \theta_2) - c(x_L, \theta_2)].$$

(A.16)

This offer is accepted by type $\theta_2$. The expected final price of a supplier of type $\theta_2$ is

$$\hat{p} = p - \alpha[v(x_H) - v(x_L)] - (1 - \alpha)[c(x_H, \theta_2) - c(x_L, \theta_2)].$$

(A.17)
At the auction stage, the expected profit of a supplier of type $\theta$ is

$$\pi(p, x_H, \theta) = \begin{cases} p - c(x_H, \theta_1) & \text{if } \theta = \theta_1; \\ p - \alpha[v(x_H) - v(x_L)] - (1 - \alpha)[c(x_H, \theta_2) - c(x_L, \theta_2)] - c(x_L, \theta_2) & \text{if } \theta = \theta_2. \end{cases}$$

(A.18)

The candidate for an equilibrium bidding function is:

$$p(\theta|x_H) = \begin{cases} c(x_H, \theta_1) & \text{if } \theta = \theta_1; \\ \alpha c(x_L, \theta_2) + (1 - \alpha)c(x_H, \theta_2) + \alpha[v(x_H) - v(x_L)] & \text{if } \theta = \theta_2. \end{cases}$$

(A.19)

The bid of type $\theta_2$ can be written as

$$p(\theta|x_H) = c(x_H, \theta_2) + \alpha[v(x_H) - c(x_H, \theta_2)] - \alpha[v(x_L) - c(x_L, \theta_2)].$$

(A.19)

Note that $p(\theta_1|x_H) < p(\theta_2|x_H)$ is equivalent to

$$\alpha \{ [v(x_L) - c(x_L, \theta_2)] - [v(x_H) - c(x_H, \theta_2)] \} < c(x_H, \theta_2) - c(x_H, \theta_1).$$

(A.20)

The left-hand side of (A.20) is positive because $x^*(\theta_2) = x_L$. Thus, if condition (A.20) is satisfied for $\alpha = 1$, then it is satisfied for all $\alpha \in (0,1)$. For $\alpha = 1$ inequality (A.20) is equivalent to

$$v(x_L) - c(x_L, \theta_2) < v(x_H) - c(x_H, \theta_1),$$

(A.21)

which is always satisfied ($S(\theta_2) < S(\theta_1)$).

This result implies that if there is a supplier of type $\theta_1$, then a supplier of type $\theta_1$ wins the auction.

**Incentives to deviate:** Does the bidding strategy (20) constitute an equilibrium? First, we consider a supplier of type $\theta_2$. When bidding $p(\theta|x_H)$ the supplier wins only if all competitors are also of type $\theta_2$ and he is the one who is selected randomly. In this case, renegotiation takes always place but the expected profit equals zero.

(i) Bidding $p > p(\theta_2|x_H)$: The supplier never wins and thus makes a zero profit.

(ii) Bidding $p \in (p(\theta_1|x_H), p(\theta_2|x_H))$: Now the supplier wins with certainty if all competitors are of type $\theta_2$. The initial price is $p = p(\theta_2|x_H)$. If the contractor can make the renegotiation offer, he offers $\hat{x} = x_L$ at $\hat{p} = \hat{p}^C$. In this case he makes a positive profit. If the procurer can make the TIOL offer, she either offers $\hat{x} = x_L$ at $\hat{p} = \hat{p}^P$ or no renegotiation contract (depending on her out-off-equilibrium beliefs). In either case, the contractor does not make any profit at the renegotiation stage and thus makes a loss. Overall, the price $p = p(\theta_2|x_H)$ is such that the positive profits and the negative profits just cancel out in expectation. The supplier’s expected profit from this bid is zero.
(iii) Bidding $p \leq p(\theta_1|x_H)$: Now, the supplier may win also in cases where some competitors are of type $\theta_1$. In these cases, however, the supplier makes a loss in expectations. The renegotiation offers are the same as in case (ii) but the initial price can be $p = p(\theta_1|x_H)$.

A supplier of type $\theta_2$ has no incentives to deviate.

Now, we consider a supplier of type $\theta_1$. When bidding $p(\theta_1|x_L)$ the supplier makes a strictly positive profit if all competitors are of type $\theta_2$. If at least one rival is of type $\theta_1$ all the expected rents from contract renegotiation are competed away.

(i) Bidding $p > p(\theta_2|x_H)$: The supplier never wins and thus makes a zero profit.

(ii) Bidding $p \in (p(\theta_1|x_H), p(\theta_2|x_H))$: The supplier wins only if all competitors are of type $\theta_2$. If the supplier wins and the procurer can make a TIOLI offer, the procurer’s offer depends on her out-of-equilibrium beliefs. If she believes the contractor is of type $\theta_1$, she does not make an offer. If she believes he is of type $\theta_2$ (with positive probability), she offers $\hat{x} = x_L$ at $\hat{p} = \hat{p}^P$. This offer, however, is rejected by the supplier of type $\theta_1$ because

\begin{equation}
(A.22) \quad p - c(x_H, \theta_2) + c(x_L, \theta_2) - c(x_L, \theta_1) < p - c(x_H, \theta_1).
\end{equation}

(iii) Bidding $p \leq p(\theta_1|x_H)$: For such a bid the supplier wins the auction for sure. In the additional cases where he wins, the initial (and final) price is $\bar{p} = c(x_H, \theta_1)$. Renegotiation does not take place or is unsuccessful by similar reasoning as in case (ii). Hence, in the additional cases where the supplier wins, he makes a zero profit.

A supplier of type $\theta_1$ has no strict incentive to deviate.

**Procurer’s expected profit:** The procurer’s realized profit depends on the most efficient type $\theta_1$ and the second most efficient type $\theta_2$. We distinguish three cases.

(i) $\theta_1 = \theta_2 = \theta_2$: In this case, renegotiation takes place and the initial price is $p = p(\theta_2|x_H)$. The procurer’s (expected) utility is

\begin{equation}
(A.23) \quad u(x_H, \theta_2, \theta_2) = v(x_L) - c(x_L, \theta_2) = S(\theta_2).
\end{equation}

(ii) $\theta_1 = \theta_2 = \theta_1$: In this case, renegotiation does not take place. The initial and final price is $p = c(x_H, \theta_1)$. Thus, the procurer’s utility is

\begin{equation}
(A.24) \quad u(x_H, \theta_1, \theta_1) = v(x_H) - c(x_H, \theta_1) = S(\theta_1).
\end{equation}
(iii) \( \theta^1 = \theta_1 \) and \( \theta^2 = \theta_2 \): In this case, renegotiation does not take place and the initial and final price is \( p = p(\theta_2|x_H) \). Thus, the procurer’s utility is

\[
\begin{align*}
  u(x_H, \theta_1, \theta_2) &= v(x_H) - \alpha c(x_L, \theta_2) - (1 - \alpha) c(x_H, \theta_2) - \alpha [v(x_H) - v(x_L)] \\
  &= (1 - \alpha) S(\theta_1) + \alpha S(\theta_2) - (1 - \alpha) [c(x_H, \theta_2) - c(x_H, \theta_1)].
\end{align*}
\]

(A.25)

Comparison of profits: The procurer optimally specifies \( \bar{x} = x_L \) if and only if

\[
\mathbb{E}[u(x_L, \theta^1, \theta^2)] > \mathbb{E}[u(x_H, \theta^1, \theta^2)],
\]

which is equivalent to

\[
(A.26) \quad c(x_H, \theta_2) - c(x_L, \theta_2) > c(x_H, \theta_1) - c(x_L, \theta_1).
\]

The above inequality is always satisfied under the imposed assumptions, which proves the first part of the proposition.

Pooling equilibria: It remains to be shown that there does not exist a pooling equilibrium. To see this consider the case \( \bar{x} = x_L \). The only candidate for a pooling equilibrium is the bid \( \bar{p} = c(x_L, \theta_2) \). There is no scope for renegotiation for type \( \theta_2 \) and thus this type never makes a renegotiation offer. If a type \( \theta_1 \) wins and can make the offer, he proposes \( \hat{x} = x_H \) at price \( \hat{p} = p + v(x_H) - v(x_L) \). This is accepted by the procurer. If the procurer can make the offer, the optimal offer – independent of her belief – is \( \hat{x} = x_H \) at price \( \hat{p} = p + c(x_H, \theta_1) - c(x_L, \theta_1) \). This offer is accepted by a contractor of type \( \theta_1 \) but rejected by a contractor of type \( \theta_2 \). In expectations a type \( \theta_1 \) contractor makes a strictly positive profit at the renegotiation stage. Hence, he has an incentive to place a bid \( p < c(x_L, \theta_2) \). This increases his probability of winning – from \( 1/n \) to \( 1 \) – without affecting his (expected) profit in case he wins the auction.

A similar argument can be made also for the case \( \bar{x} = x_H \).

\[ \square \]

Proof of Proposition \[3\]. If renegotiation takes place, then the outcome is characterized by Lemma \[1\]. The expected ex post utility of the contractor from contract \((x, p)\) is

\[
\pi(x, p, \theta) = (1 - b)[\hat{p}(p, \theta) - c(x^*(\theta), \theta)] + b[p - c(x, \theta)]
\]

(A.27)

\[
= p + (1 - b)\alpha [S(\theta) - v(x)] - [1 - \alpha(1 - b)] c(x, \theta).
\]

From the above expression the next result is readily obtained.

Lemma 5. The symmetric equilibrium bidding strategy is

\[
p(\theta) = (1 - b)\alpha [v(x) - S(\theta)] + [1 - \alpha(1 - b)] c(x, \theta).
\]
The above lemma can be proven by the usual steps (as in the proof of Lemma 2).

The most efficient type \( \theta^1 \) wins the auction and the initial price is determined by the second most efficient type \( \theta^2 \), which is given by

\[(A.28) \quad p(\theta^2) = (1-b)\alpha[v(x) - S(\theta^2)] + [1 - \alpha(1-b)]c(x, \theta^2).\]

The final price is given by

\[(A.29) \quad \hat{p}(\theta^1, \theta^2) = \alpha(1-b)v(x) - \alpha(1-b)S(\theta^2) + [1 - \alpha(1-b)]c(x, \theta^2)\]
\[\quad + \alpha[v(x^*(\theta^1)) - v(x)] + (1 - \alpha)[c(x^*(\theta^1), \theta^1) - c(x, \theta^1)].\]

The procurer’s ex post utility for given realizations of \( \theta^1 \) and \( \theta^2 \) is

\[(A.30) \quad u(\bar{x}, \theta^1, \theta^2) = b[\bar{v}(\bar{x}) - p(\theta^2)] + (1-b)[v(x^*(\theta^1)) - \hat{p}(\theta^1, \theta^2)].\]

Inserting the expressions for \( \hat{p} \) and \( p \) in the procurer’s utility and rearranging yields

\[(A.31) \quad u(\bar{x}, \theta^1, \theta^2) = (1-b)[(1-b)\alpha S(\theta^1) + \alpha S(\theta^2)] \]
\[\quad - (1-b)(1-\alpha)[c(x, \theta^2) - c(x, \theta^1)] + b[v(x) - c(x, \theta^2)].\]

Noting that the procurer maximizes \( \mathbb{E}[u(x, \theta^1, \theta^2)] \) by choosing \( \bar{x} \) completes the proof.

\[\Box\]

**Supplementary Calculations to Example 1.** When choosing \( \bar{x} = x_L \), the procurer’s expected payoff is

\[(A.32) \quad \mathbb{E}_{\theta^1, \theta^2} \left[ b(1 - \theta^2) - (1-\alpha)(1-b)(\theta^2 - \theta^1) \right].\]

On the other hand, when choosing \( \bar{x} = x_H \), her expected payoff amounts to

\[(A.33) \quad \mathbb{E}_{\theta^1, \theta^2} \left[ b(2 - 3\theta^2) - (1-\alpha)(1-b)3(\theta^2 - \theta^1) \right].\]

Thus, the procurer optimally chooses \( \bar{x} = x_L \) iff

\[(A.34) \quad \mathbb{E}_{\theta^1, \theta^2} \left[ b(2\theta^2 - 1) + (1-\alpha)(1-b)2\theta^2 - (1-\alpha)(1-b)2\theta^1 \right] \geq 0,\]

which is equivalent to

\[(A.35) \quad 2(1-\alpha + \alpha b)\mathbb{E}_{\theta^2}[\theta^2] \geq 2(1 - \alpha)(1-b)\mathbb{E}_{\theta^1}[\theta^1] + b.\]

By using the distribution of the lowest and the second lowest type realization (order statistics), the expected values are

\[(A.36) \quad \mathbb{E}_{\theta^1}[\theta^1] = \frac{1}{1+n} \quad \mathbb{E}_{\theta^2}[\theta^2] = \frac{2}{1+n}.\]

Hence, \( \bar{x} = x_L \) is optimal iff

\[(A.37) \quad b \leq \frac{2 - 2\alpha}{n - 1 - 2\alpha} =: \hat{b}.\]
Proof of Lemma 3. Each supplier has an incentive to place the bid $(x^b, p^b)$ that maximizes the score $G(x, p)$ subject to the supplier’s break-even constraint.

Bidding a lower score reduces the probability of winning without affecting the concluded contract in case the supplier wins the auction. As in a second-price auction, the concluded contract is independent of the bid placed by the winner.

Bidding a higher score increases the probability of winning. In the additional cases where the supplier now wins, he has to match a score at which he makes losses.

Hence, the optimal bid solves:

$$\max_{x, p} g(x) - p$$

s.t.  

$$p + \alpha S(\theta) - \alpha v(x) - (1 - \alpha) c(x, \theta) \geq 0.$$ 

The solution is $x^B(\theta)$ and $p^b(\theta)$, which concludes the proof; see also Che (1993).

Proof of Lemma 4.

$$G(\theta_2) = g(x^b(\theta_2)) - \alpha v(x^b(\theta_2)) - (1 - \alpha) c(x^b(\theta_2), \theta_2) + \alpha S(\theta_2)$$

$$< g(x^b(\theta_2)) - \alpha v(x^b(\theta_2)) - (1 - \alpha) c(x^b(\theta_2), \theta_1) + \alpha S(\theta_1)$$

$$\leq g(x^b(\theta_1)) - \alpha v(x^b(\theta_1)) - (1 - \alpha) c(x^b(\theta_1), \theta_1) + \alpha S(\theta_1)$$

$$= G(\theta_1).$$

The first inequality follows from Assumption 1 and the second inequality holds by the definition of $x^b(\cdot)$.

Proof of Proposition 4. The procurer’s payoff maximization problem can be restated as a minimization problem. The optimal scoring function minimizes

$$(A.38) \quad \mathbb{E}_{\theta_1, \theta_2} \left\{ g(x^b(\theta_1)) - \alpha v(x^b(\theta_1)) - (1 - \alpha) c(x^b(\theta_1), \theta_1) \right\}$$

$$- \left\{ g(x^b(\theta_2)) - \alpha v(x^b(\theta_2)) - (1 - \alpha) c(x^b(\theta_2), \theta_2) \right\}.$$ 

By the definition of $x^B(\cdot)$, the term (A.38) is (weakly) larger than

$$(A.39) \quad \mathbb{E}_{\theta_1, \theta_2} \left\{ (1 - \alpha) [c(x^b(\theta_2), \theta_2) - c(x^b(\theta_1), \theta_1)] \right\}.$$ 

Hence, it is optimal that $x^b(\theta_2) = x^b(\theta_1) = x_L$. In this case, (A.38) coincides with its lower bound and the lower bound is minimized. That the lower bound is minimized for $x^b(\theta_2) = x^b(\theta_1) = x_L$ follows from Assumption 1(ii).

A supplier of type $\theta$ places a design bid $x^b(\theta) = x_L$ iff

$$(A.40) \quad \alpha [v(x_H) - v(x_L)] + (1 - \alpha) [c(x_H, \theta) - c(x_L, \theta)] \geq g(x_H) - g(x_L).$$
By Assumption [1(ii)], if the above inequality holds for \( \theta = \theta \), then it holds for all \( \theta \in \Theta \). This establishes the first part of the proposition. The statement regarding the price adjustment follows directly from the proof of Proposition [1].

\[ \square \]

**Proof of Corollary [2]** The result follows directly from Proposition [4].

\[ \square \]

**Proof of Proposition [5]** As in Subsection 4.1, we assume that there are two different types, \( \theta \in \{ \theta_1, \theta_2 \} \), and rather than applying the GNBS, we assume that the contractor can make a TIOLI offer with probability \( \alpha \in (0, 1) \) and the procurer can do so with a probability \( 1 - \alpha \). Again, we assume that \( c(x_L, \theta_1) < c(x_L, \theta_2) \) and that the efficient design is different across the two types.

\[ x^*(\theta) = \begin{cases} x_H & \text{for } \theta = \theta_1; \\ x_L & \text{for } \theta = \theta_2. \end{cases} \]

Recall that these assumptions imply

\[ c(x_H, \theta_2) > c(x_L, \theta_2), \]

\[ c(x_H, \theta_2) - c(x_L, \theta_2) > v(x_H) - v(x_L) > c(x_H, \theta_1) - c(x_L, \theta_1), \]

and \( S(\theta_1) > S(\theta_2). \)

As equilibrium concept, we employ perfect Bayesian equilibrium in pure strategies.

Now, we proceed with the actual proof. Suppose each supplier with type \( \theta \) bids

\[ x^b(\theta) \in \arg \max_{x \in X} \{ g(x) - \alpha v(x) - (1 - \alpha)c(x, \theta) \}, \]

\[ p^b(\theta) = \alpha v(x^b(\theta)) + (1 - \alpha)c(x^b(\theta), \theta) - \alpha S(\theta). \]

Then, the procurer can perfectly deduce the contractor’s type from his bid and/or the initial contract. There are up to four different cases: (i) \( x^b(\theta_1) = x^b(\theta_2) = x_H \), (ii) \( x^b(\theta_1) = x^b(\theta_2) = x_L \), (iii) \( x^b(\theta_1) = x_H, x^b(\theta_2) = x_L \), and (iv) \( x^b(\theta_1) = x_L, x^b(\theta_2) = x_H \). For cases (i) and (ii), the analysis is analogous to the analysis of price-only auctions in Subsection 4.1, where \((x^b(\theta), p^b(\theta))\) as specified in Lemma [3] are optimal.

Case (iii): Suppose that \( x^b(\theta_2) = x_L \) and \( x^b(\theta_1) = x_H \). In equilibrium, there will be no renegotiation. Type \( \theta_2 \) does not want to deviate, because then he would make losses. If a supplier with type \( \theta_1 \) imitates type \( \theta_2 \) and all other suppliers are type \( \theta_2 \) (otherwise he could not win the auction if all others play their equilibrium strategy), he would win the auction with a probability \( \frac{1}{n} \), and could make a TIOLI offer with probability \( \alpha \). Hence, he would make an expected profit of \( \frac{2}{n}[v(x_H) - v(x_L) - c(x_H, \theta_1) + c(x_L, \theta_2)] \). If he played his equilibrium strategy, his profits in that case would be \( g(x_H) - p^b(\theta_1) - g(x_L) + p^b(\theta_2) = g(x_H) - g(x_L) - [c(x_H, \theta_1) - c(x_L, \theta_2)] \).
Because $x^b(\theta_1) = x_H$, we have $g(x_H) - g(x_L) - [c(x_H, \theta_1) - c(x_L, \theta_2)] \geq \alpha [v(x_H) - v(x_L)] - \alpha [c(x_H, \theta_1) - c(x_L, \theta_2)] > \frac{\alpha}{n} [v(x_H) - v(x_L) - c(x_H, \theta_1) + c(x_L, \theta_2)]$. Thus, deviation is not profitable. In fact, because the inequality is strict, there does not exist a pooling equilibrium.

Now we show that case (iv) does not exist. Suppose that $x^b(\theta_2) = x_H$. Thus, $g(x_H) - \alpha v(x_H) - (1 - \alpha) c(x_H, \theta_2) \geq g(x_L) - \alpha v(x_L) - (1 - \alpha) c(x_L, \theta_2) \Leftrightarrow g(x_H) - g(x_L) \geq \alpha [v(x_H) - v(x_L)] + (1 - \alpha) [c(x_H, \theta_2) - c(x_L, \theta_2)]$. Because $c(x_H, \theta_2) - c(x_L, \theta_2) > c(x_H, \theta_1) - c(x_L, \theta_1)$, it follows that $x^b(\theta_1) = x_H$.

\[\square\]

**Appendix B. Revenue equivalence**

Here we show that the procurer’s expected utility is the same for all standard auctions and their corresponding scoring auctions. In particular, this implies that all of our findings hold for first-price auctions and first-score auctions. We rely on the findings of Asker and Cantillon (2008).

First, let us define a supplier’s pseudotype,

\[ (B.1) \quad k(\theta) = \max_x \{ g(x) - c(x, \theta) + \alpha \{ [v(x^*(\theta)) - c(x^*(\theta), \theta)] - [v(x) - c(x, \theta)] \} \}, \]

and let $z(k, (k_-)$ be the allocation rule, i.e., the probability that a supplier with pseudotype $k$ is awarded the contract, given the other players have the pseudotypes $k$ (and play their equilibrium strategies). Let $z(k)$ denote the expected probability that a supplier with pseudotype $k$ is awarded the contract.

**Proposition 6** (Asker and Cantillon (2008), Theorem 2). Any two scoring auctions with a quasi-linear scoring rule $G(x, p)$ that use the same allocation rule $z(k, (k_-)$, and yield the same expected payoff for the lowest pseudotype $\min_{\theta} \{ k \}$ generate the same expected utility for the buyer.

**Proof.** With our definition of pseudotypes, our setting is included in Asker and Cantillon (2008), except for the assumption in Asker and Cantillon (2008) that the scoring rule be strictly increasing in the quality (i.e., the design). However, all arguments leading up to their Theorem 2 do not rely on that assumption. \[\square\]

It follows immediately:

**Corollary 3.** First- and second-price auctions with the same ex-ante design generate the same expected utility for the buyer in our setting.

**Corollary 4.** First- and second-score auctions with the same quasi-linear scoring rule $G(x, p)$ generate the same expected utility for the buyer in our setting.

Thus, our results carry over to first-price and first-score auctions.
References


University of Bayreuth, CESifo, and CEPR
E-mail address: fabian.herweg@uni-bayreuth.de

University of Munich, University of Innsbruck
E-mail address: Marco.Schwarz@uibk.ac.at