Size Matters - Óverínvestments in a Relational Contracting Setting

Florian Englmaier (LMU Munich)
Matthias Fahn (JKU Linz)

Discussion Paper No. 62

January 8, 2018

Collaborative Research Center Transregio 190 | www.rationality-and-competition.de
Ludwig-Maximilians-Universität München | Humboldt-Universität zu Berlin
Spokesperson: Prof. Dr. Klaus M. Schmidt, University of Munich, 80539 Munich, Germany
+49 (89) 2180 3405 | info@rationality-and-competition.de

Size Matters - "Over"investments in a Relational Contracting Setting*

Florian Englmaier† & Matthias Fahn‡

January 6, 2018

Abstract

The corporate finance literature documents that managers tend to over-invest in their companies. A number of theoretical contributions have aimed at explaining this stylized fact, most of them focusing on a fundamental agency problem between shareholders and managers. The present paper shows that over-investments are not necessarily the (negative) consequence of agency problems between shareholders and managers, but instead might be a second-best optimal response to address problems of limited commitment and limited liquidity. If a firm has to rely on relational contracts to motivate its workforce, and if it faces a volatile environment, investments into general, non-relationship-specific, capital can increase the efficiency of a firm's labor relations.

JEL Codes: C73, D21, D86, G32

Keywords: relational contracts, corporate finance, capital investments

*We thank Dan Barron, Sean Chu, Oscar Contreras, Wouter Dessein, Bob Gibbons, Sebastian Kranz, Jin Li, James Malcomson, Stefanos Mouzas, Mike Powell, Heiner Schumacher, Carmit Segal, Caspar Siegert, Roland Strausz and participants at the 2012 IIOC (Washington), the 2012 Conference of the German Economics Association (Goettingen), the 2013 European Meeting of the Econometric Society (Gothenborg) and the 2016 SIOE meeting (Paris) for helpful comments. Two anonymous referees were very helpful in sharpening the paper's contribution. Financial support by Deutsche Forschungsgemeinschaft through CRC TRR 190 is gratefully acknowledged.

†LMU Munich & CESifo & Organizations Research Group (ORG); florian.englmaier@econ.lmu.de

‡JKU Linz & CESifo; matthias.fahn@jku.at

1 Introduction

A prominent and well established stylized fact in corporate finance is that managers tend to over-invest, or, as Stein (2003) puts it, that they "...have an excessive taste for running large firms, as opposed to simply profitable ones" (p. 119). Numerous theories have been developed to explain this pattern: managers' taste for empire building (see Williamson (1964), Jensen (1986), Jensen (1993)), short-termism of managers who focus on activities the market can easily observe, (see Stein (1989), Bebchuk and Stole (1993)), managerial overconfidence into their own abilities (Roll (1986), Heaton (2002)), or asymmetric information with respect to new investment opportunities (see Inderst and Klein (2007)). All these theories share the perception that over-investments are caused by agency problems between a firm and its management. Hence, mechanisms to reduce free cash-flow – and consequently the management's ability to invest – have been suggested as optimal responses to this perceived fundamental agency problem. However, there is no clear evidence that reducing a firm's free cash-flow and restricting a manager's investment opportunities increases firm value – on the contrary, investors often assess capital investments positively (see McConnell and Muscarella (1985), Myers (2003)).

This paper shows that investments into general, liquidity-generating, capital can have a positive impact on firm value, namely by making it easier to motivate a firm's workforce. If a firm cannot use formal, court-enforceable contracts to provide incentives, and if payments used to compensate its workforce is constrained by the firm's volatile revenues, the scope of incentive provision is limited. Then, the combination of limited commitment and limited liquidity may cause an inefficiently low productivity. In this case, over-investments – investments where marginal costs exceed (direct) marginal benefits – can improve the power of the firm's incentive system because capital investments increase the firm's financial flexibility as well as its commitment by generating additional cash flow. Hence, over-investments partially mitigate contracting frictions, and are not necessarily an inefficient manifestation of intra-firm agency problems.

More precisely, we develop a model where a principal needs physical capital and an agent's effort to produce, and the time horizon is infinite. Effort is potentially exerted in every period and increases output. Capital investments are made at the beginning of the game and increase output as well. The agent has to be motivated to exert effort, however neither his effort nor potential performance signals are verifiable. Therefore, relational contracts where the firm implicitly promises to reward performance have to be employed to incentivize the agent. There, the principal only has an incentive to honor her promises if the net present value of the firm's profits is sufficiently large. This constraint on credible promises limits the level of enforceable effort if the discount factor is small. In addition to this standard credibility problem of relational contracts, we consider the effect of a volatile environment on the principal's ability to make desired payments. It turns out that taking this into account can restrict enforceable effort even for relatively large discount factors. In particular, we assume that the principal is exposed to varying market conditions, i.e., demand may be either high or low. The principal faces a liquidity constraint and can only use generated cash-flow – funds that have been earned by selling the output – to compensate the agent. Under this assumption the agent's compensation is likely to vary with the principal's earnings, and high demand be associated with higher payments to the agent. However, the maximum amount the principal is willing to pay out instead of reneging and shutting down is determined by her expected future profits, independent of the differences in available liquidity between states of the world. A big difference between the revenues in good and bad states, and hence higher payment obligations in the good state, increases the principal's reneging temptation and limits enforceable effort to an inefficiently low level even if the discount factor is close to 1. Therefore, the very combination of a liquidity constraint and the absence of formal enforcement triggers efficiency reductions even for rather large discount factors.

The principal can mitigate this problem by (seemingly excessive) investments into physical capital. We start by assuming that investments are not relationship specific, hence can be re-sold at any time for the initial purchasing price. Higher investment levels raise the output in all states and therefore increase the available cash-flow also in low-demand states, i.e., when the liquidity constraint binds. This direct positive effect of a higher liquidity in bad states is further amplified by an indirect credibility effect that helps in good states: Because a higher effort level can be implemented due to the additional liquidity in low-demand states, expected profits today and in all future periods go up. This allows to credibly promise a higher bonus in high-demand states (recall that effort is restricted by the combination of limited liquidity in bad and limited commitment in good states) and consequently to further increase implemented effort. In this context, over-investments – i.e., capital levels where the marginal investment costs exceed the direct marginal benefits – are potentially optimal.

This result is remarkable in comparison to the "classic" corporate finance literature where over-investments are interpreted as a consequence of agency problems between shareholders and management: Means to *reduce* free cash-flow – like issuing debt – are proposed as (second-best) solutions to the problem of over-investments; see, e.g., Hart and Moore (1995), Zwiebel (1996). We show that the additional cash-flow generated by over-investments can be used to *increase* the firm's financial flexibility and *mitigate* an agency problem between the firm and its workforce.

Because we consider investments into general, non-relationship-specific, capital, over-investments are not optimal for low discount factors, where only limited commitment and not limited liquidity is a problem. This changes in Section 6.1, where we assume that the asset is relationship-specific in a sense that its resale value is smaller than the initially invested amount. Then, investment costs are (partially) sunk and thus not (fully) considered by the principal whenever she faces the decision whether to keep her promises in the relational contract or not. Due to sunk investment costs, and as an increased capital base positively affects future rents, investments into capital improve the enforceability of relational contracts and consequently also attenuate pure credibility problems. Then, over-investments can also be optimal for rather low discount factors. This positive interaction between relationship-specific up-front investments and the enforceability of relational contracts has previously been identified by Halac (2015). She analyzes a setting where a principal and an agent interact repeatedly, and the principal has to make an ex-ante relationship-specific investment. This standard hold-up problem induces the principal to under-invest if she fears that the returns from this investment will ex-post be expropriated by the agent. Then, the inability to use formal contracts may increase the efficiency of the relationship, by a logic similar to ours: A higher relationship-specific investment increases the benefits of keeping the relationship going, reduces the reneging temptation and thereby increases effort in the relational contract. Different from Halac (2015), we show that over-investments not only increase the value of the relationship by creating quasi-rents that are lost after a deviation, but also mitigate problems generated by volatile returns. In addition we show that over-investments can even be optimal with general, non-relationship-specific, capital – a result that is not present in the setup of Halac (2015). In Section 6.1., we further conduct comparative statics with respect to the liquidation value of the asset. If it increases (which implies that the asset becomes *less* relationship specific), the effect on investments is ambiguous, and the scope for over-investments might actually go up: On the one hand, the benefits of over-investments then go down because the associated increase in the firm's outside option is more pronounced for larger investments. On the other hand, the principal's outside option becomes more attractive, which fosters her incentives to deviate and therefore increases the benefits of over-investments.

Our results also appear related to Klein and Leffler (1981). There, up-front relationship specific investments are sunk and so allow future rents to be used as a bond to ensure performance. However, those investments do not affect productivity but instead are necessary to sustain an equilibrium with high performance in a market environment, by dissipating rents and consequently restricting firm entry.

In our benchmark case, we assume that the firm is neither able to enter a credit market, nor to hoard cash in order to deal with liquidity shortages. Especially the latter assumption not only simplifies our analysis, but also seems to describe the reality in a number of instances. There are a variety of reasons for why firms are not able to keep large amounts of cash. For example, short-sighted shareholders may insist on paying out cash or investing it in higher return but less liquid assets. In such cases, this paper argues that a second-best option might be to invest in stable, cash generating business. However, we also show that even if we introduce a credit market (Section 6.2), and even allow for cash holdings (Section 6.3), over-investments continue to be a viable instrument to improve the performance of a firm's incentive system.

Related Literature

The starting point of this paper is the moral hazard principal agent literature which focuses on unobservable effort choice as a determinant of firm profitability (or productivity). An improved solution to the moral hazard problem will, ceteris paribus, increase productivity. While there exists a large literature, building on Holmstrom (1979) and Grossman and Hart (1983), focusing on explicit contracts that reward the agent based on verifiable performance measures, there has been an increased interest in implicit contracts as a way to mitigate the moral hazard problem; see, e.g., Bull (1987), MacLeod and Malcomson (1989), Levin (2003). In a more recent contribution, Gibbons and Henderson (2013) argue that different aspects of relational contracts are responsible for observed persistent performance differences among seemingly similar enterprises that also exist within industrialized countries.

Relational (or implicit) contracts employ repeated-game logic to use observable but unverifiable information and do not rely on explicit, court-enforceable, performance measures to motivate workers. The performance of relational contracts is generally restricted by an insufficient discounted future value of the relationship. This credibility problem hence ceteris paribus is more severe if players have rather low discount factors. The present paper introduces a liquidity problem and shows that the efficiency of relational contracts can also be restricted by volatile returns, even if discount factors are rather large. In the recent past there have been a number of papers investigating richer dynamics and the effect of stochastic shocks on the efficiency and stability of implicit contracts – see, e.g., Li and Matouschek (2013), Englmaier and Segal (2011) – to which the present paper relates. In addition, we relate to some recent papers linking a firm's financing conditions and decisions to the enforceability of relational contracts. Contreras (2013) analyzes how relational contracts formed between a firm and its supplier interact with the quality of financial markets. Fahn, Merlo, and Wamser (2017) show how equity financing helps to enforce relational contracts. Debt increases a firm's reneging temptation because some of the negative consequences of breaking implicit promises can be shifted to creditors. In a related vein, Barron and Li (2015) explore how the negative effect of debt on the enforceability of relational contracts affects firm dynamics. They show that it is optimal for firms to first meet its financial obligations at the expense of having low compensation and effort levels in early periods.

2 Model Setup

We first characterize the basic model. In the next subsection we will describe the informational structure of the game. There is one principal ("she") and one agent ("he"). The principal needs two inputs for production, capital and the agent's effort. Time is discrete, the time horizon infinite; principal and agent are risk-neutral and share a common discount factor $\delta \in (0, 1)$. In the first period of the game, $t = 0$, the principal makes capital investments $k \in [0, \overline{k}]$, where $\overline{k}$ is assumed to be large enough. Capital investments are associated with marginal investment costs of 1. They can either be funded by the principal herself or raised from (passive) outside investors who provide equity. To sharpen our arguments, we abstract from any agency conflicts between the principal and outside shareholders, and assume that the latter automatically receive their fair share of residual profits. Furthermore, we assume that the asset is not relationship-specific and can be resold at the end of every period, for a value k. Below, in Section 6.1, we allow for asset specificity in the sense that the resale value is below k. We also assume that the asset can only be liquidated as a whole and not parts of it, and that the game is over once the asset has been sold.

In every period $t = 1, 2, ...$, the firm makes a short-term employment offer to the agent; this offer consists of a fixed wage $w_t \geq 0$ and the promise to make a contingent bonus payment $b_t \geq 0$. This bonus promise provides the agent with incentives and is paid at the principal's discretion.

The agent's decision whether to accept an offer or not is captured by $d_t \in \{0, 1\}$, where $d_t = 1$ describes an acceptance and $d_t = 0$ a rejection. After accepting an offer, the agent makes his effort choice e_t. Effort is continuous, $e_t \in [0, \overline{e}]$, where $\overline{e}$ is assumed to be large enough, and associated with linear effort costs $c \cdot e_t$, where $c > 0$.

If the agent rejects the principal's offer, the principal consumes her outside option $\overline{\pi} \geq 0$, and the agent consumes his outside option $\overline{u} \geq 0$ in the respective period.

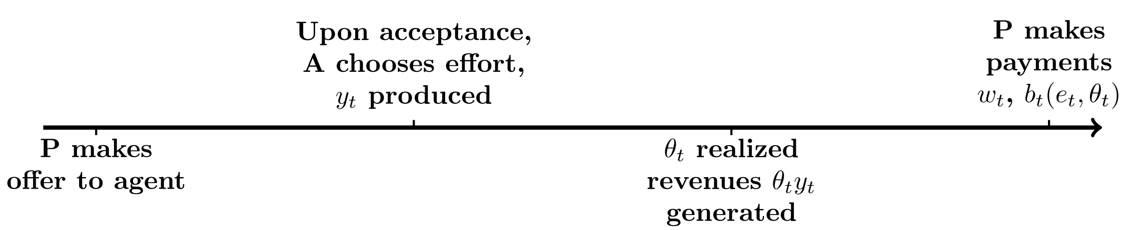
The output generated in period t is $y_t = f(e_t, k)$. $f(e_t, k)$ is a continuous function in both arguments, with $f_e, f_k > 0$ and $f_{ee}, f_{kk} < 0$. For simplicity, and without affecting any of our qualitative results, we further assume $f_{ek} = 0$, with one exception: $f(0, k_t) = 0$ for all $k_t \geq 0$. Therefore, at least some effort by the agent is needed for a productive use of the asset.

After producing, the principal sells the output and generates revenues $\theta_t y_t$. $\theta_t \in \{\theta^l, \theta^h\}$ is a parameter specifying the demand conditions for the principal's output, with $0 < \theta^l < \theta^h$, and is realized after the output has been produced. High demand is realized with probability p, low demand with $1 - p$. These probabilities are independent over time, i.e., there is no persistency in demand conditions. After the sale of output, payments w_t and b_t are made. Hence, the bonus can be contingent on the realization of θ (in addition to chosen effort), i.e., $b_t(e_t, \theta_t)$, whereas w_t is fixed by assumption.

We assume that the principal is liquidity constrained: all funds used to compensate the agent must be earned via the sale of its products. This implicitly assumes that the principal does not retain profits earned in earlier periods and has no access to credit markets. We relax the first assumption in Section 6.3 and show that it does not drive our results. The second assumption is relaxed in Section 6.2. Furthermore, potential outside shareholders are not able to inject additional funds into the firm in later periods. In this case, note that if they were able at the beginning of the game to not only provide funds for physical investments but to leave cash reserves in the firm to cover later shortages, our results would not be qualitatively affected (see Section 6.3.).

Finally, note that although we do not further analyze potential interactions with outside shareholders, their presence would not affect the principal's incentives in her relationship with the agent. Resulting dividend payments would just scale down all components of the constraints proportionally and hence cancel out (see Fahn, Merlo, and Wamser (2017)).

The timing within every period t is summarized in the following graph:



Information, Payoffs, Strategies and Equilibrium

Generally, we assume that no contingent formal contracts are feasible. However, there are no informational asymmetries between players, hence agency problems only arise because formal contracts cannot be used to motivate the agent.[1] More precisely, a formal, court-enforceable contract can neither be based on the agent's effort e_t, on output y_t, revenues $\theta_t y_t$, nor on the realization of the demand parameter θ. Still, all these aspects, as well as acceptance decisions d_t, and wage and bonus payments can be observed by the principal and by the agent.

Presuming that the asset is never liquidated, the principal's payoff in any period $t \geq 1$ is

$$\Pi_t = \mathrm{E}\left\{ \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[\overline{\pi} + d_\tau \left(\theta_\tau f\left(e_\tau,\, k\right) - \left(w_\tau + b_\tau(e_t, \theta_\tau)\right) - \overline{\pi} \right) \right] \right\},$$

where expectation is over the realizations of θ. Furthermore,

$$\Pi_0 = -k + \delta \Pi_1.$$

The agent's expected discounted payoff stream equals

$$U_t = \mathrm{E}\left\{ \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[\overline{u} + d_\tau \left(w_\tau + b_\tau(e_t, \theta_\tau) - c \cdot e_t - \overline{u} \right) \right] \right\}.$$

We only consider pure strategies, and define a relational contract as a *Subgame Perfect Equilibrium (SPE)* where after every history strategies determine a Nash Equilibrium. More precisely, we are interested in a SPE that maximizes the principal's expected profits at the beginning of the game, i.e., Π_0.

In the following, we focus on equilibria where the employment offer is accepted by the agent ($d_t = 1$).

In Appendix A, in Lemma 1, we show that, under the assumption that no formal contracts based on θ_t can be written, we can without loss of generality focus on stationary contracts that are independent of calendar time, as well as past realizations of demand shocks. Hence, effort, w_t, and b_t are constant over time, allowing us to omit time subscripts t. This is driven by shocks being distributed i.i.d., by effort being chosen *before* the state of the world is realized, and by the principal – the party facing the liquidity constraint – being able to reap the whole surplus. Hence, only equilibrium bonus payments might vary over time, depending on the respective realization of θ. There, b^h is the equilibrium bonus given θ^h is observed,

[1] See Englmaier and Segal (2011) or Li and Matouschek (2013) for an analysis of situations where shocks to the firm are not observable to the workforce.

and b^l the bonus for θ^l. Finally, we set the outside options $\overline{\pi} = \overline{u} = 0$. This assumption does not affect our results qualitatively, but simplifies our analysis given the firm's liquidity constraints.

3 Maximization Problem and Constraints

Our objective is to find levels of capital k, (stationary) effort e, as well as a compensation package (w, b^l, b^h) to maximize

$$\Pi_0 = -k + \delta\Pi,$$

where

$$\Pi = \frac{p(\theta^h f(e,k) - b^h) + (1-p)(\theta^l f(e,k) - b^l) - w}{1-\delta}$$

is the principal's expected discounted equilibrium payoff stream in any period $t \geq 1$. Note that an equilibrium that maximizes the principal's profits involves no liquidation on the equilibrium path.

The following constraints have to be satisfied to enforce a stationary SPE. First, it must be optimal for the agent to accept an employment offer. This is captured by an individual rationality (IR) constraint,

$$U \geq 0, \tag{IR}$$

where $U = w + pb^h + (1-p)b^l - c \cdot e + \delta U$ is the agent's expected discounted equilibrium payoff stream. An incentive compatibility (IC) constraint must hold for equilibrium effort e^*. For given bonus payments b^l (after $\theta = \theta^l$) and b^h (after $\theta = \theta^h$), this constraint equals

$$pb^h + (1-p)b^l - c \cdot e^* + \delta U \geq 0. \tag{IC}$$

There, we assume that the agent receives no further future offer after selecting $e = 0$.[2] Because $w \geq 0$, (IR) is automatically implied by the (IC) constraint.

Furthermore, because of the non-verifiability of effort and output, it must be in the interest of the principal to actually pay out b^l and b^h to the agent, which is characterized by dynamic enforcement (DE) constraints. There, if she fails to make a promised payment, we assume a reversion to the static Nash equilibrium.[3] This is characterized by no payments being made

[2] This is based on the assumption that once an agent deviated, the principal assumes he will not exert effort in the future as well. The analysis would be identical, though, if the principal believed that an agent's deviation was a singular event, which is driven by the agent not receiving a rent in any profit-maximizing equilibrium (derived below).

[3] Following Abreu (1988): The static Nash Equilibrium determines the lower bound on the principal's profits and should hence constitute her punishment following observable deviations.

to the agent, who in return chooses $e = 0$. Therefore, if the principal reneges on a bonus payment, she further will immediately shut down and consume the liquidation value of the asset, k.

The two dynamic enforcement (DE) constraints, one for b^l and one for b^h, are

$$-b^l + \delta\Pi \geq k \tag{DEl}$$

and

$$-b^h + \delta\Pi \geq k. \tag{DEh}$$

In addition, a liquidation must never be optimal for the principal, i.e., $\Pi \geq k$. Given that bonus payments are non-negative, though, this condition is automatically implied by the firm's (DE) constraints.

Since the right hand sides of (DEl) and (DEh) are identical, only one of them has to be considered, depending on whether b^l or b^h is larger.

Furthermore, payments must not violate the principal's liquidity constraints, which state that payments in any period cannot exceed respective revenues:

$$w + b^l \leq \theta^l f(e, k) \tag{Ll}$$

and

$$w + b^h \leq \theta^h f(e, k). \tag{Lh}$$

Finally, note that it must also not be optimal for the principal to liquidate the firm and compensate the agent with the resulting funds. This however, is implied by the stated constraints because a liquidation of the firm implies that no production takes place in any future period.

Before characterizing equilibrium effort, we first derive the value of e that maximizes the (unconstrained) total surplus. In the following, we denote this efficient – or first-best – effort level e^{FB}. For a given capacity level k, e^{FB} is characterized by

$$\left(p\theta^h + (1-p)\theta^l\right) f_e(\cdot) - c = 0. \tag{FB}$$

To keep the analysis interesting, we impose Assumption 1, implying that operating is strictly optimal for the principal.

Assumption 1: There exists a $\tilde{k} > 0$ such that $\delta\frac{f(e^{FB},\tilde{k})\left(p\theta^h+(1-p)\theta^l\right)-e^{FB}c}{1-\delta} - \tilde{k} > 0$

Given the concavity of the production function, Assumption 1 implies that investment levels that satisfy such a condition also exist if first-best effort cannot be enforced.

Furthermore, we want the firm's liquidity constraints to potentially have bite. Hence, we impose Assumption 2:

Assumption 2: $ce^{FB} > \theta^l f(e^{FB}, k)$

If Assumption 2 was violated, the firm would never be constrained by a lack of liquidity and could always set $b^h = b^l = ce$.

4 Equilibrium Effort e^*

In this section, we derive the (profit-maximizing) equilibrium effort level – denoted e^* – and in particular explore how it is affected by the principal's liquidity constraints. As a first step, we can show that the (IC) constraint binds in any profit-maximizing equilibrium, and that it is further optimal to set $w = 0$ (see Lemma 1 in Appendix A). These results follow from the observability of effort and our focus on a profit-maximizing equilibrium. Hence, the principal will aim at maximizing the total surplus subject to the constraints derived above.

Equilibrium effort is characterized in Proposition 1.

Proposition 1 *For a given capital level k, the firm chooses equilibrium effort* e^* *to maximize* $\Pi = \frac{\left(p\theta^h+(1-p)\theta^l\right)f(e,k)-ec}{(1-\delta)}$*, subject to*

$$ce^* \leq \frac{\delta p^2}{1-\delta+\delta p}\theta^h f(e^*, k) + (1-p)\theta^l f(e^*, k) - \frac{(1-\delta)p}{(1-\delta+\delta p)}k, \tag{DE-L}$$

and

$$ce^* \leq \delta f(e^*, k)\left(p\theta^h + (1-p)\theta^l\right) - (1-\delta)k. \tag{DE}$$

There exist values $\underline{\delta}$ *and* $\overline{\delta}$*, with* $\underline{\delta} < \overline{\delta} < 1$*, such that*

- $e^* = e^{FB}$ *for* $\delta \geq \overline{\delta}$
- $e^* < e^{FB}$ *for* $\underline{\delta} \leq \delta < \overline{\delta}$*, and* e^* *is determined by the binding (DE-L) constraint*
- $e^* < e^{FB}$ *for* $\delta < \underline{\delta}$*, and* e^* *is determined by the binding (DE) constraint.*

Proof: See Appendix B.

The (DE-L) constraint is obtained by adding (Ll) (multiplied with $1-p$) and (DEh) (multiplied with p), whereas the (DE) constraint follows from (DEh) and (DEl) provided $b^h = b^l = ce^*$. Concerning the intuition behind Proposition 1, note that the principal can only implement e^{FB} with $b^h > b^l$ – because having $b^h = b^l = ce^{FB}$ would require $ce^{FB} \leq \theta^l f(e^{FB}, k)$ which is ruled out by Assumption 2. Put differently, the binding liquidity constraint in the low-demand state forces the principal to promise the agent a larger bonus in the high-demand state. The maximum size of b^h, however, is determined by the (DEh) constraint. Because the principal's willingness to reward the agent is determined by *expected* discounted future rents and not by current profits, her reneging temptation is higher in the high than in the low state. If δ is sufficiently large, though, (DEh) has no bite, and e^{FB} can be enforced. For $\delta \in [\underline{\delta}, \overline{\delta})$, (DEh) binds because the principal cannot credibly promise to pay a sufficiently high bonus. Therefore, the interaction of constrained credibility and constrained liquidity restricts enforceable effort. Note that $\overline{\delta}$ not only depends on the surplus of the relationship, but also on the volatility of earnings. This aspect will be made more precise in the next section.

For $\delta < \underline{\delta}$, cash shortage is no longer a problem but instead the principal has a credibility problem also when demand is low. Then, (DEl) bites as well, which makes it optimal to set $b^h = b^l = e^*c$, yielding identical (DEh) and (DEl) constraints.

These results can be matched to empirically documented regularities. Proposition 1 also implies that on average, firms with a higher discount factor have a larger variation in *residual* cash flows because a binding liquidity constraint forces many of them to pass all their revenues on to the agent if demand is low. On the other hand, if $\delta < \underline{\delta}$, the liquidity constraint does not bind, and free cash flow is also available if demand is low. Given we expect firms to pay higher dividends to their shareholders in case they have more free cash flow, we predict dividend payments of firms with a higher discount factor to vary more than of firms with a lower discount factor. There, Michaely and Roberts (2012) show that privately held firms in the UK smooth dividends significantly less than their publicly listed counterparts, and respond more to transitory earnings shocks. Michaely and Roberts (2012) conjecture that this might be due to agency problems, which are more prominent in publicly held firms. We offer an alternative but related explanation, because of a notion that privately-held firms are supposed to have larger discount factors: Privately held firms are often assumed to focus more on long-term goals – in particular with respect to their employment relationships – compared to publicly listed firms. Take family firms, where for example a study by Price Waterhouse Cooper (2012) identifies a larger commitment to jobs, which leads "family-run businesses ... to have more loyalty toward their staff – people are not just a number" (PriceWaterhouseCooper

(2012), p. 6). This makes it more likely that (DE-L) is the relevant constraint, implying more variation in dividend payments.

In a next step, we flesh out that it is not only players' impatience (i.e., a low δ) that limits the power of incentives if the firm faces a liquidity constraint. This is different from "standard" contributions like MacLeod and Malcomson (1989) or Levin (2003), and also Halac (2015), where the enforceability of relational contracts is solely determined by players' credibility and in particular their discount factors. We can show that the liquidity constraint might bind – and hence effort be restricted to inefficiently low levels – even if the principal is arbitrarily patient. This is the case if the principal's cash-flow is very uncertain in the sense that the firm makes high profits with a small probability.

Proposition 2 *Fix an arbitrary effort level $\hat{e} > \frac{\theta^l f(\hat{e},k)}{c}$, and assume a discount factor $\delta \geq \underline{\delta}$, i.e., the (DE) constraint can be omitted. Then, for fixed values θ^l and c, as well as for a fixed per-period surplus $f(\hat{e})\left(p\theta^h + (1-p)\theta^l\right) - c\hat{e}$, there exists a $\underline{p}$ such that for $p < \underline{p}$, constraint* (DE-L) *does not hold for $\hat{e}$.*

Proof: See Appendix B.

A reduction of p, accompanied by an increase of θ^h in order to keep the surplus for a given effort level fixed will eventually lead to a violation of (DE-L).[4] This result is driven by the combination of liquidity and dynamic enforcement constraints. When constrained liquidity has bite, larger shares of the compensation package must be shifted to high-demand states, ceteris paribus increasing the temptation to renege. Hence, the (DE-L) constraint is more likely to bind if the expected surplus generated in the relationship is high, and if the firm operates in a high-risk environment. Absent (Ll), the enforceability of an effort level e would – for a given discount factor δ – only depend on the future surplus, independent of the exact specification of p and θ.

To sum up, this section establishes a new potential enforcement problem induced by a combination of the standard credibility problem in relational contracts with liquidity constraints. Even if the principal is very patient, her commitment in the relational contract is limited if earnings are volatile.

5 Optimal Capital Choice and the Scope for Over-Investments

We now derive the principal's optimal capital choice. At the beginning of the game, the principal sets the optimal investment level k^* to maximize $\Pi_0 = -k + \delta\Pi$, taking into account

[4]Note that given effort, θ^l, c, and surplus stay constant and also $\underline{\delta}$ is unaffected.

the direct effect of k on output, but also potential indirect effects of k on equilibrium effort in later periods. If equilibrium effort is at e^{FB}, i.e., if neither (DE) nor (DE-L) bind, the latter aspect does not affect equilibrium capital k^* due to the envelope theorem. In this case, the optimal capital level is defined by

$$\frac{\partial \Pi_0}{\partial k} = -1 + \frac{\delta}{1-\delta}\left(p\theta^h + (1-p)\theta^l\right) f_k = 0. \tag{1}$$

In the following, let $\tilde{k}$ denote the capital level characterized by (1).[5] We can show that capital is above $\tilde{k}$ if (DE-L) binds. There, we make use of the critical discount factors $\overline{\delta}$ and $\underline{\delta}$ (as defined in Proposition 1), which determine if e^{FB} can be implemented, or if equilibrium effort is restricted by a binding (DE) or (DE-L) constraint.

Proposition 3 *The optimal capital level k^* is given by one the following cases:*

- *$k^* > \tilde{k}$ if (DE-L) binds at $\tilde{k}$, i.e., if δ is such $\underline{\delta} \leq \delta < \overline{\delta}$*
- *$k^* = \tilde{k}$ if either e^{FB} can be implemented ($\delta \geq \overline{\delta}$), or if (DE) binds ($\delta < \underline{\delta}$) at $\tilde{k}$.*

Proof: See Appendix B.

If the (DE-L) constraint binds, i.e., if $\underline{\delta} \leq \delta < \overline{\delta}$, over-investments are optimal because a higher value of k increases output and thereby the available cash-flow in each state of the world. Then a larger share of the agent's compensation can be shifted to the low-demand state, thereby also relaxing the principal's (DEh) constraint. More precisely, the benefits of having a capital level above $\tilde{k}$ consist of a direct liquidity effect and an indirect credibility effect. The direct liquidity effect is caused by more available cash in periods where the firm faces a negative demand shock and where the liquidity constraint binds. There, starting from $\tilde{k}$ (which is associated with an effort level $\tilde{e} < e^{FB}$) and increasing capital by a small dk increases the feasible bonus payment by $db^l = \theta^l \left(f(\tilde{e}, \tilde{k} + dk) - f(\tilde{e}, \tilde{k})\right)$. Consequently, higher effort can be implemented, with $de = (1-p)\frac{\theta^l\left(f(\tilde{e},\tilde{k}+dk)-f(\tilde{e},\tilde{k})\right)}{c}$, increasing per-period profits by $d\pi = p\theta^h f(\tilde{e}+de, \tilde{k}+dk) + (1-p)\theta^l f(\tilde{e}+de, \tilde{k}+dk) - c\,(\tilde{e}+de)$. This raises total profits Π_0, because the costs of having a capacity above $\tilde{k}$ are of second order at the margin, whereas the benefits of having larger effort are of first order due to $\tilde{e} < e^{FB}$.

Moreover, the direct liquidity effect also gives rise to an indirect credibility effect which helps the firm in high-demand periods where it is *not* restricted by a lack of liquidity. In these periods, the principal wants to pay a higher bonus b^h, but cannot credibly promise to

[5] Because $f_{ek} = 0$, $\tilde{k}$ is independent of effort.

do so because her discounted future profits, $\delta\Pi$, are not sufficiently high compared to her outside option, k. The relaxed liquidity constraint and associated increase of expected *future* effort induced by a higher capacity increase $\delta\Pi$ by more than it increases k (which, again, is because the benefits of higher effort are of first order, whereas the costs of a higher capacity are of second order at the margin). Consequently, the principal can credibly promise a higher b^h, thereby further increasing implemented effort and equilibrium profits. Concluding, the combination of a direct liquidity and an indirect credibility effect renders over-investments optimal for $\delta \geq \underline{\delta}$.

If (DE) binds, over-investments are not optimal. Higher levels of k increase on-path profits, but also the liquidiation value. The first aspect relaxes, the second tightens the (DE) constraint. For $k^* = \tilde{k}$, both effects just offset each other. However, note that if the liquidation value of the asset is lower than k, over-investments are also optimal if the (DE) constraint binds. This aspect is further explored below, in Section (6.1).

Also note that, because $f_{ek} = 0$, $\tilde{k}$ is independent of effort, and hence the optimal capital the same when e^{FB} is implemented and when (DE) binds. For $f_{ek} \neq 0$, the capital level in both cases would not be identical, however in each case still characterized by condition (1). Only with a binding (DE-L) constraint, k^* is above the level specified by condition (1), irrespective of the sign of f_{ek}.

Concluding, we show that the very lack of free cash-flow in some states renders over-investments ex-ante optimal. This result stands in contrast to the classic corporate finance literature (see, e.g., Hart and Moore (1995) or Zwiebel (1996)), where a *reduction* of free cash-flow is regarded as a potential remedy to overcome over-investment problems.

6 Extensions and Robustness

6.1 Asset Specificity

To focus on the effect of capital investments on a relational contract with liquidity constraints – an interaction that has to our best knowledge not been identified before – we have assumed that the asset's outside value corresponds to the invested amount. This assumption serves to isolate our main contribution that investments in *general* capital can improve the performance of relational contracts. In this section, we analyze the effects of the investment being (partially) relationship specific. Then, over-investments are optimal even for a binding (DE) constraint. The reason is that investments increase revenues in all future periods. Since investment costs are (partially) sunk, the difference between profits in and out-of equilibrium increases, and promises made to the agent become more credible. Such a result has been identified before, for example by Halac (2015). She shows that the hold-up problem (generating under-investments into relationship-specific assets) can be less severe in a relationship where relational contracts have to be used ex-post.

Here, we assume that the resale value of the asset is γk, with $\gamma \in [0,1]$. Everything we derived so far remains unaffected, only that k is replaced by γk. Hence, (DE) and (DE-L) constraints, which determine enforceable effort, become

$$e^* c \leq \delta \left(p\theta^h + (1-p)\theta^l\right) f(w^*, k) - (1-\delta)\gamma k \tag{DE}$$

and

$$e^* c \leq (1-p)\theta^l f(e^*, k) + p\left(\delta\Pi - \gamma k\right). \tag{DE-L}$$

A larger relationship specificity of the asset, i.e., a smaller γ, reduces the principal's reneging temptation because the liquidation value of the asset (which the principal consumes after a deviation) is lower. Therefore, both constraints are relaxed, and a higher effort level can be implemented.

Optimal effort and investment levels, as well as the effect of the degree of asset specificity γ on (over-)investments are given in Proposition 4:

Proposition 4 *There exist values $\underline{\delta}(\gamma)$ and $\overline{\delta}(\gamma)$, with $\underline{\delta}(\gamma) < \overline{\delta}(\gamma) < 1$, such that*

- $e^* = e^{FB}$ *for* $\delta \geq \overline{\delta}(\gamma)$; *in this case,* $k^* = \tilde{k}$, *where* $\tilde{k}$ *is characterized by* (1) .
- $e^* < e^{FB}$ *for* $\underline{\delta}(\gamma) \leq \delta < \overline{\delta}(\gamma)$, *and* e^* *is determined by the binding (DE-L) constraint; in this case,* $k^* > \tilde{k}$ *for all* $\gamma \leq 1$.
- $e^* < e^{FB}$ *for* $\delta < \underline{\delta}(\gamma)$, *and* e^* *is determined by the binding (DE) constraint; in this case,* $k^* > \tilde{k}$ *for all* $\gamma < 1$.

Furthermore, $d\underline{\delta}(\gamma)/d\gamma > 0$ *and* $d\overline{\delta}(\gamma)/d\gamma > 0$; *if either (DE) or (DE-L) binds,* k^* *might increase or decrease in* γ.

Proof: See Appendix B.

Concerning the marginal effect of the asset's relationship specificity on optimal investments, note the following: First, critical discount factors are increasing in γ, hence a larger γ increases the parameter range where constraints bind and over-investments are generally optimal. This is because a larger γ increases the principal's outside option and hence tightens the relevant constraints. If $\delta < \overline{\delta}(\gamma)$, a higher γ has two opposing effects. On the one hand, it tightens the principal's constraints, which amplifies the benefits of over-investments. On the other hand, the increase in the principal's outside option is more pronounced for a larger k^*, hence a larger γ reduces the benefits of over-investments. The latter effect is relatively stronger in case the (DE) constraint binds, i.e., for rather low discount factors. For intermediate discount factors such that (DE-L) is the relevant constraint, the marginal effect of γ on k^* is more likely to be positive. In this case, over-investments also provide additional cash in low-demand states, whereas the increase of the principal's outside option only matters for high-demand states.

6.2 Principal has Access to a Credit Market

In this section, we explore whether the availability of a credit market can solve the problem of constrained liquidity, sticking to the assumption that the asset's outside value is γk. We argue that even if a competitive credit market where repayment can be contingent on the state of the world exists, over-investments remain optimal for many firms. Potentially, the principal can benefit from a credit market if her (DE-L) constraint binds (if the DE constraint binds, i.e., if effort is solely restricted by the principal's lack of credibility, borrowing obviously does not help). Then, the liquidity constraint could be relaxed by borrowing in low-demand and repaying loans in high-demand states. In the following, we will therefore assume an intermediate discount factor where the (DE-L) constraint binds (given no credit is taken by the firm), whereas (DE) is slack.

Generally, the principal needs an incentive to repay the firm's debt – supposing that a default leads to a termination of the firm or that at least the principal has no access to any future profits generated by it. Here we assume that the asset can be used as collateral for a credit (otherwise, a credit market could not increase implementable effort). Then, the credit market can effectively be used to smooth payments to the agent and thereby relax the principal's limited liability constraint. Consequently, a credit market can actually help to enforce higher effort levels, however over-investments will still be optimal in many instances.

Assume there is a competitive credit market for short-term credit where creditors also have a discount factor δ, and that repayment can be made contingent on the state of the world. Furthermore, repayment is required in every high-demand period (even if the principal has not borrowed in the previous period; this "smoothing" of repayments is optimal because of the principal's DE constraint in high-demand states). A credit market with an interest larger than the rate reflecting time preferences, or with repayment not being fully contingent on the state of the world would reduce its benefits and get us closer to our baseline situation.

We denote the amount borrowed by the principal in a low-demand state by D, and the amount repaid in a high-demand state by R. Because the credit market is competitive, $pR-(1-p)D=0$, hence $R=(1-p)D/p$.

First, note that for a given effort level, the principal's profits are naturally unaffected by the existence of the credit market, $\Pi=\frac{p\left(\theta^h f(e,k)-b^h-R\right)+(1-p)\left(\theta^l f(e,k)-b^l+D\right)}{1-\delta}$, where by construction $pR=(1-p)D$.

Furthermore, recall that the constraints that determine (DE-L) are (Ll) and (DEh), which – taking the consequences of a credit market into account – now amount to

$$b^l \leq \theta^l f(e^*,k)+D \tag{Ll}$$

and

$$-b^h-R+\delta\Pi \geq \max\left\{\gamma k-R,\, 0\right\}. \tag{DEh}$$

The firm also needs an incentive to make the payment R (i.e., $-R+\delta\Pi \geq \max\left\{\gamma k-R,\, 0\right\}$ must hold), which however is implied by (DEh). Enforceable effort again is determined by the (DE-L) constraint, which is obtained by adding (Ll) (multiplied with $1-p$) and (DEh) (multiplied with p), taking into account that an agent's (IC) constraint will bind. Hence,

$$-e^*c+\delta p\Pi+(1-p)\theta^l f(e^*,k)+(1-p)D-pR-\max\left\{p\gamma k-pR,\, 0\right\} \geq 0. \tag{DE-L}$$

The left hand side of the (DE-L) constraint is maximized for chosing $D=\frac{p}{(1-p)}\gamma k$, i.e., such that $R=\gamma k$. Larger values cannot be collateralized and hence do not further relax the constraint.

This implies that the principal uses the credit market if the (DE-L) constraint binds in order to increase implemented effort. She will either borrow an amount smaller than $\frac{p}{(1-p)}\gamma k$, such that the (DE-L) constraint just binds (and either e^{FB} can be implemented or (DE) has become the relevant constraint), or she will borrow an amount $D=\frac{p}{(1-p)}\gamma k$ (if (DE-L) still binds at this debt level).

Over-investments remain optimal unless e^{FB} can be implemented. Moreover, note that there is an additional benefit of over-investments in case the (DE-L) constraint binds at $D=\frac{p}{(1-p)}\gamma k$. Over-investments relax the "constraint" $D \leq \frac{p}{(1-p)}\gamma k$. Hence, having a larger value of k creates more collateral and allows to take more debt in order to smooth payments.

6.3 Firms can Accumulate Retained Earnings

So far, we have assumed that the firm cannot retain its high-demand earnings in order to relax the liquidity constraint in low-demand periods. In this section, we show that over-investments can still be optimal if the principal is able to retain earnings. Doing so potentially allows the principal to temporarily increase the implemented effort level, and might or might not be optimal if the principal's liquidity constraint binds. However, over-investments continue to be part of the principal's optimal investment decisions whenever effort is restricted. Further note that the following can also be applied to analyze the possibility of keeping part of the initial investment as a cash reserve to cover later shortages.

To simplify matters, we first assume that accumulated cash is kept by the principal and does not generate interest payments. Later, we show that over-investments may also be optimal if the principal can keep the retained cash in an interest-bearing current account. Now, retained earnings can be used in two ways: On the one hand, effort can be increased until the next realization of θ^l. On the other hand, a given effort level can be sustained for more than just one subsequent realization of θ^l. We relegate a general characterization of the principal's optimal behavior to Appendix C (where we stick to the assumption of Section 6.1 and assume that the asset's outside value is γk) and only present the main result in this section.

Proposition 5 *Retaining earnings is optimal if and only if in the situation without this possibility, the (DE-L) constraint binds (i.e., if δ is between $\underline{\delta}$ and $\overline{\delta}$, as defined above in Proposition 1)* ***and*** *the following condition is satisfied:*

$$\left(\frac{\delta\,(1-p)}{[1-\delta p]}p\theta^h + (1-p)\theta^l\right) f_e - c > 0. \tag{2}$$

In this case, maximum effort is characterized by

$$\left(\frac{\delta\,(1-p)}{[1-\delta p]}p\theta^h + (1-p)\theta^l\right) f_e - c = 0. \tag{3}$$

Proof: See Appendix C.

Holding cash reserves is never optimal if only (DE) constraints bind. However, a binding liquidity constraint alone is only a necessary and not a sufficient condition for making it optimal to retain earnings. Instead, the costs of holding cash reserves – delayed consumption – might still be too high compared to the benefits of having higher effort in the future, which is the case if condition (2) does not hold.

In Appendix C, we further show that after a cash stock has been built up and the principal is hit by a number of negative shocks, effort is decreased gradually. Then, each low-demand period triggers an effort reduction until all retained earnings have been used up. This parallels results in Li and Matouschek (2013) where implemented effort levels gradually decrease with every adverse shock hitting a firm.

Note that the exact properties of these results rely on a number of simplifying assumptions. First, we assume that the liquidity constraints in high-demand states never bind. This implies that one high-demand period is sufficient to replenish cash reserves to the principal's preferred (maximum) level, which gives rise to a "quasi-stationary" equilibrium. Without this assumption, the level of retained earnings after a high-demand period might also be a function of the stock of cash reserves preceding this period. Furthermore, we assume that the principal can consume her retained earnings following a deviation. Therefore, retained earnings not only relax the firm's liquidity constraints, but also tighten its dynamic enforcement constraints. This delivers an additional dimension of potential benefits of over-investments: For $\gamma < 1$, those are relationship-specific in a way that retained earnings are not because higher investments tighten dynamic enforcement constraints to a lesser degree than retained earnings do.[6] In any case, we can show that the possibility to accumulate cash reserves does not eliminate over-investments.

Proposition 6 *Assume retaining earnings is possible. Then over-investments are optimal if in the situation without this possibility, either (DE) or (DE-L) constraints bind.*

Proof of Proposition 6: If (DE) constraints bind, holding cash is not optimal and the situation is equivalent to above – over-investments are an optimal response to relax the constraint if $\gamma < 1$. If holding cash is optimal due to a binding (DE-L) constraint and because condition (2) holds, effort never exceeds the level characterized by condition (3). Hence, it is below the first best. The rest directly follows from the proof to Proposition 3.■

All of these arguments can also be applied to a setting where we allow the principal to keep cash reserves at the beginning of the game: Assume that the principal or outside investors not only have the possibility to invest into the physical asset, but are able to leave cash in the firm to make up for later shortages. Then, the tradeoff still amounts to keeping cash reserves in order to increase future effort versus instantaneous consumption. Therefore, over-investments would remain optimal along the lines of Proposition (6).

Finally, note that retained earnings are also costly for the firm because they do not generate any interest income and hence cause a first-order loss in profits, whereas over-investments at the margin only entail a second-order loss. This changes once retained earnings can be kept in a current account and generate interest. In the following, we argue that as long as the discount rate does not fully make up for discounting, over-investments remain

[6] This is apparent in the respective formulations that can be found in Appendix C.

optimal. Again, we relegate an in-depths analysis of this case to Appendix C and just present the main results: If the interest rate is denoted by r, maximum effort is characterized by

$$\frac{\delta p\,(1-p)}{(1-\delta p(1+r))}\theta^h f_e + \frac{(1-p)\theta^l f_e - c}{(1+r)} = 0,$$

with maximum effort being increasing in r. Indeed, for $r = \frac{1-\delta}{\delta}$, i.e., the interest rate completely makes up for discounting, the condition becomes $p\theta^h f_e + (1-p)\theta^l f_e - c = 0$, and maximum effort is at the first-best. But even then, over-investments will generally remain optimal – because for effort always being at the first-best with probability 1, the firm would need an infinite amount of cash reserves from the beginning of the game. In all other cases, there would be a positive probability that the firm eventually runs out of cash and has to reduce effort below the first-best level. But then, over-investments are optimal because those only entail a second-order profit loss at the margin. Furthermore, an interest rate $r = (1-\delta)/\delta$ would mostly be associated with fully efficient capital markets – whereas in reality capital markets are regarded to entail at least some inefficiencies.

Internal capital markets have been identified as an instrument to mitigate a firm's exposure to inefficient external capital markets. If firms (or divisions within firms) pool their resources, they might use their funds more efficiently and thereby improve capital allocation. But internal capital markets are also associated with inefficiencies; see Stein (1997), or Inderst and Laux (2005). Therefore, this paper can be used to argue that over-investments might be an appropriate "internal" alternative to address inefficient external capital markets.

7 Discussion and Conclusion

The present paper has shown that observed over-investments are not necessarily the (negative) consequence of agency problems between shareholders and managers. Instead, they might actually be a second-best optimal response to contracting frictions: In situations where firms face volatile market conditions and hence varying cash-flow streams, and where they cannot rely on court-enforceable contracts to motivate their workforce but have to use relational contracts instead, "excessive" capital investments relax liquidity constraints by increasing the firm's cash-flow base.

We did not allow for different kinds of investments. In our setting, firms facing binding (DE-L) constraints prefer investment opportunities with less volatile cash flows, even at the cost of lower expected returns. I.e., these firms would abstain from making R&D-type (high-risk/high return) investments and rather grow their business in a conservative way.

Hence, there is potentially an additional indirect cost of low contract enforcement quality in a country: reduced R&D activity and on the macro level reduced growth.[7]

[7] We are grateful to Bob Gibbons for pointing out this implication.

A Maximization Problem, Constraints, and Proof of Stationarity

Note that it is sufficient to regard equilibrium effort as well as compensation as a function of the history of past shocks. The reason is our focus on pure strategies. Denote the history at the beginning of period t as $\theta^{t-1} = \{\theta_1, \theta_2, ..., \theta_{t-1}\}$, with $\theta_t \in \{\theta^l, \theta^h\}$, and $\theta^0 = \emptyset$. Then, expected payoff streams can be written as

$$\begin{aligned}\Pi(\theta^{t-1}) =& p\left[\theta^h f\left(e(\theta^{t-1}), k\right) - e(\theta^{t-1}) b^h(\theta^{t-1})\right] \\ &+ (1-p)\left[\theta^l f\left(e(\theta^{t-1}), k\right) - e(\theta^{t-1}) b^l(\theta^{t-1})\right] \\ &- e(\theta^{t-1}) w(\theta^{t-1}) + \delta\left[p\Pi\left(\theta^{t-1}, \theta^h\right) + (1-p)\Pi\left(\theta^{t-1}, \theta^l\right)\right]\end{aligned}$$

and

$$\begin{aligned}U(\theta^{t-1}) =& w(\theta^{t-1}) + pb^h(\theta^{t-1}) + (1-p)b^l(\theta^{t-1}) - e(\theta^{t-1})c \\ &+ \delta\left[pU\left(\theta^{t-1}, \theta^h\right) + (1-p)U\left(\theta^{t-1}, \theta^l\right)\right],\end{aligned}$$

where $b^h(\theta^{t-1})$ is the bonus paid for history θ^{t-1} given a high shock is realized in period t. Equivalent definitions hold for $b^l(\theta^{t-1})$, $\Pi\left(\theta^{t-1}, \theta_t\right)$ and $U\left(\theta^{t-1}, \theta_t\right)$.

Then, the firm's objective function is to choose k as well as $e(\theta^{t-1})$, $w(\theta^{t-1})$, $b^h(\theta^{t-1})$ and $b^l(\theta^{t-1})$ to maximize

$$\Pi_0 = -k + \delta\Pi(\theta^0),$$

subject to the following constraints, which must be satisfied for every history θ^{t-1}:

$$U(\theta^{t-1}) \geq 0 \tag{IRA}$$

$$pb^h(\theta^{t-1}) + (1-p)b^l(\theta^{t-1}) - e(\theta^{t-1})c + \delta\left[pU\left(\theta^{t-1}, \theta^h\right) + (1-p)U\left(\theta^{t-1}, \theta^l\right)\right] \geq 0 \tag{IC}$$

$$b^l(\theta^{t-1}) \leq \delta\Pi\left(\theta^{t-1}, \theta^l\right) \tag{DEl}$$

$$b^h(\theta^{t-1}) \leq \delta\Pi\left(\theta^{t-1}, \theta^h\right) \tag{DEh}$$

Given both (DE) constraints, the firm's individual rationality constraint, $\Pi(\theta^{t-1}) \geq 0$, is automatically satisfied.

$$w(\theta^{t-1}) + b^l(\theta^{t-1}) \leq \theta^l f(e(\theta^{t-1})) \tag{Ll}$$

$$w(\theta^{t-1}) + b^h(\theta^{t-1}) \leq \theta^h f(e(\theta^{t-1})) \tag{Lh}$$

Lemma 1 *The (IC) constraint binds for every history* θ^{t-1}.

Proof of Lemma 1: To the contrary, assume there is a history $\tilde{\theta}^{t-1}$ where (IC) does not bind. At this point, reduce $b^h(\tilde{\theta}^{t-1})$ as well as $b^l(\tilde{\theta}^{t-1})$ by a small $\varepsilon > 0$ such that (IC) for history $\tilde{\theta}^{t-1}$ is still satisfied. Furthermore, increase $w(\tilde{\theta}^{t-1})$ by ε and leave everything else unchanged. This has no impact on Π_0, as well as $\Pi(\theta^{t-1})$ and $U(\theta^{t-1})$ for *any* history θ^{t-1}, hence does not affect any (IRA) constraint. Furthermore, all (Ll) and (Lh) constraints remain unchanged. Finally (DEh) and (DEl) for history $\tilde{\theta}^{t-1}$ are relaxed and unaffected for any other history.

■

Using the results of Lemma A1 gives $U(\theta^{t-1}) = w(\theta^{t-1})$,

$$\begin{aligned}\Pi(\theta^{t-1}) =& p\theta^h f\left(e(\theta^{t-1}), k\right) + (1-p)\theta^l f\left(e(\theta^{t-1}), k\right) - e(\theta^{t-1})c \\ &+ \delta\left[p\Pi\left(\theta^{t-1}, \theta^h\right) + (1-p)\Pi\left(\theta^{t-1}, \theta^l\right)\right]\end{aligned}$$

and $b^h(\theta^{t-1}) = \frac{e(\theta^{t-1})c-(1-p)b^l(\theta^{t-1})}{p}$.

Furthermore, the remaining constraints are

$$w(\theta^{t-1}) \geq 0 \tag{IRA}$$

$$b^l(\theta^{t-1}) \leq \delta\Pi\left(\theta^{t-1}, \theta^l\right) \tag{DEl}$$

$$\frac{e(\theta^{t-1})c - (1-p)b^l(\theta^{t-1})}{p} \leq \delta\Pi\left(\theta^{t-1}, \theta^h\right) \tag{DEh}$$

$$b^l(\theta^{t-1}) \leq \theta^l f\left(e(\theta^{t-1}), k\right) \tag{Ll}$$

$$\frac{e(\theta^{t-1})c - (1-p)b^l(\theta^{t-1})}{p} \leq \theta^h f\left(e(\theta^{t-1}), k\right) \tag{Lh}$$

This allows us to prove Lemma 2:

Lemma 2 *$w(\theta^{t-1})(= U(\theta^{t-1})) = 0$ for every history θ^{t-1}. Furthermore, contracts are stationary in a sense that effort as well as bonus and wage payments in equilibrium are independent of the history of shocks θ^{t-1}.*

Proof of Lemma 2: We first show that a (constrained) surplus-maximizing equilibrium is stationary and subsequently that the principal can extract the full rent.

In a surplus-maximizing equilibrium, none of the effort levels can optimally be above e^{FB}. Furthermore, the surplus is increasing in $e(\theta^{t-1})$ for any possible history as long as effort there is inefficiently low. Now, take any equilibrium effort level $e(\theta^{t-2})$ and assume that $e(\theta^{t-2}, \theta^h) \neq e(\theta^{t-2}, \theta^l)$. If $e(\theta^{t-2}, \theta^h) > e(\theta^{t-2}, \theta^l)$, replacing $e(\theta^{t-2}, \theta^l)$ with $e(\theta^{t-2}, \theta^h)$ would violate no constraint and increase the surplus. If $e(\theta^{t-2}, \theta^l) > e(\theta^{t-2}, \theta^h)$, on the other hand, replacing $e(\theta^{t-2}, \theta^h)$ with $e(\theta^{t-2}, \theta^l)$ would violate no constraint and increase the surplus. Hence effort in any period is independent of previous shock realizations and might only be history-dependent based on the number of observed shocks, i.e., on timing. There, however, note that the structure of the game is stationary. This implies that the highest effort level that is enforceable in any period can be implemented in all other periods as well, and it is surplus-maximizing to choose the maximum feasible effort (subject to $e \leq e^{FB}$) in every period. Therefore, it is without loss of generality to also have payments b^h, b^l and w history-independent.

Finally, assume that $w > 0$. Now a reduction of w by ε and an increase of $pb^h + (1-p)b^l$ by $\delta\varepsilon$ is feasible, does not violate any constraint and increases the firm's profits. ■

Note that Lemmas 1 and 2 establish that it is not optimal to promise the agent a higher continuation payoff in a low-demand state, i.e., paying him a rent in a future period when demand is high. The reason is that this tightens the high-state (DE) constraints equivalently and hence does not allow for higher effort levels. This would change if formal contracts based on the demand state could be written. Then, it might be optimal to let a low-demand realization be followed by a promise to offer a high fixed wage in the next period conditional on the demand then being high.

B Omitted Proofs

Proof of Proposition 1. Recall that the firm's objective is to maximize $\Pi_0 = -k + \delta\Pi$, subject to the relevant constraints. For a given value of k, however, e is chosen to maximize Π. Furthermore, we show in Appendix A that (IC) and (IRA) constraints bind. Hence, we can use $w = 0$ and $pb^h + (1-p)b^l - ec = 0$, giving the problem

$$\max_e \Pi = \frac{p\theta^h f(e,k) + (1-p)\theta^l f(e,k) - ec}{1-\delta},$$

subject to

$$b^l \leq \delta\Pi - \gamma k \tag{DEl}$$

$$\frac{ec - (1-p)b^l}{p} \leq \delta\Pi - \gamma k \tag{DEh}$$

$$b^l \leq \theta^l f(e,k) \tag{Ll}$$

$$\frac{ec - (1-p)b^l}{p} \leq \theta^h f(e,k). \tag{Lh}$$

It follows that one of (DEl) and (Ll), as well as one of (DEh) and (Lh) generally can be omitted. In a next step, we show that (Lh) cannot bind in equilibrium. To the contrary, assume it binds. Then, either (Ll) or (DEl) must bind as well because otherwise, b^l could be increased and (Lh) relaxed without violating any constraint. First, assume that (Ll) binds together with (Lh). This, however, would imply that $\Pi = 0$, which is not possible in a profit-maximizing equilibrium with positive effort. Now, assume that (DEl) binds together with (Lh). From section 3, we know that setting $b^h \geq b^l$ is optimal. Hence, (DEh) has to bind as well, implying $b^h = b^l$. Then, $\theta^h f(e,k) = b^h = b^l = \theta^l f(e,k)$, which - due to $\theta^h > \theta^l$ - is not possible for $e > 0$.

Now, consider all effort levels with $\theta^l f(e,k) \geq \delta\Pi - k$. In this case, (Ll) is automatically satisfied given (DEl). Adding (DEh) (multiplied with p) and (DEl) (multiplied with $(1-p)$) proves the necessity of (DE-L). Sufficiency immediately follows: Assuming (DE-L) holds, there always exists a $b^l \geq 0$ such that (DEh) and (DEl) are satisfied.

For effort levels $\theta^l f(e,k) < \delta\Pi - k$ (DEl) is automatically satisfied given (Ll). In this case, necessity and sufficiency of (DE) are obtained equivalently as for (DE-L).

To prove that $e^* \leq e^{FB}$, we set up the Lagrange function,

$$\begin{aligned} L =& \frac{p\theta^h f(e,k) + (1-p)\theta^l f(e,k) - ec}{1-\delta} + \lambda_{DE}\left[\delta f(e,k)\left(p\theta^h + (1-p)\theta^l\right) - (1-\delta)k - ec\right] \\ &+ \lambda_{DEL}\left[\frac{\delta p^2}{1-\delta+\delta p}\theta^h f(e,k) + (1-p)\theta^l f(e,k) - \frac{(1-\delta)p}{(1-\delta+\delta p)}k - ec\right], \end{aligned}$$

giving the first order condition

$$\frac{\partial L}{\partial e} = \left(p\theta^h f_e + (1-p)\theta^l f_e - c\right)\left(\tfrac{1}{1-\delta} + \delta\lambda_{DE} + \lambda_{DEL}\right) - \lambda_{DE}c(1-\delta) - \lambda_{DEL}p\theta^h f_e \tfrac{1-\delta}{1-\delta+\delta p} = 0.$$

Hence, if either (DE-L) or (DE) binds, $\left(p\theta^h f_e + (1-p)\theta^l f_e - c\right) > 0$, and e^* is inefficiently small.

Concerning values $\underline{\delta}$ and $\overline{\delta}$, we first establish the existence of $\overline{\delta}$. To do so, we show that both constraints are relaxed for larger values of δ, that e^{FB} can be enforced if δ is sufficiently large, and that (DE-L) is the relevant constraint to enforce e^{FB}. To prove the first aspect, we obtain the first partial derivatives of the right hand sides of the (DE) and (DE-L) constraints with respect to δ, $f(e,k)\left(p\theta^h + (1-p)\theta^l\right) + k$ and $\frac{p^2}{(1-\delta+\delta p)^2}\left(\theta^h f(e,k) + k\right)$. Both expressions are positive, hence (DE) and (DE-L) are relaxed by larger values of δ. Furthermore, e^{FB} can be enforced for δ sufficiently large because (DE) and (DE-L) converge to $e^*c \leq p\theta^h f(e^*,k) + (1-p)\theta^l f(e^*,k)$ for $\delta \to 1$, which holds for e^{FB} because of Assumption 1. To show that (DE-L) is the relevant constraint to enforce e^{FB}, we set up (DE-L) for e^{FB} and let it hold as an equality:

$$e^{FB}c = \frac{\delta p^2}{1-\delta+\overline{\delta} p}\theta^h f(e^{FB},k) + (1-p)\theta^l f(e^{FB},k) - \frac{(1-\delta)p}{(1-\delta+\delta p)}k.$$

Solving this expression for $(1-\delta)k$, and substituting it into the (DE) constraint for e^{FB}, yields

$$e^{FB}c - f(e^{FB},k)\theta^l \geq 0,$$

which holds due to Assumption 2.

This, together with previous results, establishes the existence of $\overline{\delta}$ above which e^{FB} can be implemented, and that for discount factors slightly below $\overline{\delta}$, (DE-L) binds and (DE) is slack.

To establish the existence of $\underline{\delta}$, take an arbitrary effort level $\tilde{e}$ that is supposed to be enforced. There, (DE-L) is the relevant constraint if the right hand side of (DE) is smaller than the right hand side of (DE-L), or if

$$\delta \leq \frac{\theta^l + \frac{\gamma k}{f(\tilde{e},k)}}{(p\theta^h + (1-p)\theta^l) + \frac{\gamma k}{f(\tilde{e},k)}}. \tag{4}$$

.

Condition (4) provides a treshold $\underline{\delta}$ for a given effort level $\tilde{e}$. This is not sufficient to complete the proof, though, because $\underline{\delta}$ is a function of enforceable effort, which itself is a function of δ. Therefore, we show that, for a given effort level, a lower δ tightens (DE) by more than it tightens (DE-L). This implies that for discount factors below $\underline{\delta}$ (which is characterized by (4) holding as an equality), (DE) is the relevant constraint. We rewrite both constraints as

$$0 \leq \delta p^2\theta^h f(e^*,k) + (1-\delta+\delta p)(1-p)\theta^l f(e^*,k) - (1-\delta+\delta p)\, e^*c - (1-\delta)pk \qquad \text{(DE-L)}$$

and

$$0 \leq \delta f(e^*,k)\left(p\theta^h + (1-p)\theta^l\right) - e^*c - (1-\delta)k. \qquad \text{(DE)}$$

The partial derivative of the right hand side of (DE-L) with respect to δ equals

$$p^2\theta^h f(e^*,k) - (1-p)^2\theta^l f(e^*,k) + e^*c\,(1-p) + pk \qquad (5)$$

and the partial derivative of the right hand side of (DE) with respect to δ equals

$$f(e^*,k)\left(p\theta^h + (1-p)\theta^l\right) + k. \qquad (6)$$

(5) is smaller than (6) if

$$e^*c < f(e^*,k)\left\{p\theta^h + \theta^l + (1-p)\theta^l\right\} + k.$$

This condition holds provided (DE), hence for all potential equilibrium levels of effort. ■

Proof of Proposition 2. As we fix the surplus, as well as θ^l and $\hat{e}$, a decrease in p has to be compensated by an appropriate increase in θ^h. More precisely, taking the total derivative of the per-period surplus, $f(\hat{e},k)\left(dp\theta^h + pd\theta^h - dp\theta^l\right) = 0$, implies $\frac{d\theta^h}{dp} = -\frac{(\theta^h-\theta^l)}{p}$.

Take an arbitrary high-state probability $\overline{p}$ where constraint (DE-L) is satisfied for effort $\hat{e}$ (if such a $\overline{p} < 1$ does not exist for $\hat{e}$, we are done). For any probability $p^* < \overline{p}$, always counterbalanced by an increase of θ^h that keeps the surplus constant, the right hand side of (DE-L) equals

$$\frac{\delta}{1-\delta+p^*\delta}(p^*)^2\left(\theta^h + d\theta^h\right) f(\hat{e},k) + (1-p^*)\theta^l f(\hat{e},k) - \frac{(1-\delta)p^*}{(1-\delta+\delta p^*)}k$$

$$= \frac{\delta}{1-\delta+p^*\delta}(p^*)^2\left(\theta^h - (\theta^h-\theta^l)\int_{p^*}^{\overline{p}}\frac{1}{p}dp\right) f(\hat{e},k) + (1-p^*)\theta^l f(\hat{e},k) - \frac{(1-\delta)p^*}{(1-\delta+\delta p^*)}k$$

$$= \frac{\delta}{1-\delta+p^*\delta}(p^*)^2\left(\theta^h - (\theta^h-\theta^l)\ln\overline{p} + (\theta^h-\theta^l)\ln p^*\right) f(\hat{e},k) + (1-p^*)\theta^l f(\hat{e},k) - \frac{(1-\delta)p^*}{(1-\delta+\delta p^*)}k$$

For $p^* \to 0$, the last expression becomes

$\frac{\delta}{1-\delta(1-p^*)}(\theta^h-\theta^l)\frac{\ln p^*}{\frac{1}{(p^*)^2}}f(\hat{e},k)+\theta^l f(\hat{e},k)=\frac{\delta}{1-\delta}(\theta^h-\theta^l)\frac{(p^*)^2}{-2}f(\hat{e},k)+\theta^l f(\hat{e},k)=\theta^l f(\hat{e},k).$

Since $\theta^l f(\hat{e},k)<\hat{e}c$ by assumption, effort $\hat{e}$ will eventually not be enforceable anymore.

Finally, we have to show that the right hand side of (DE-L) is increasing in p.

There,

$\frac{d}{dp}=\delta\frac{2p(1-\delta+p\delta)-p^2\delta}{(1-\delta+p\delta)^2}\theta^h f(e,k)+\frac{p^2\delta}{1-\delta+p\delta}\frac{d\theta^h}{dp}f(e,k)-\theta^l f(e,k)-\frac{(1-\delta)^2}{(1-\delta+\delta p)^2}k$

$=\frac{(1-\delta)}{(1-\delta+p\delta)^2}\left[\delta\left(\theta^h f(e,k)p+(1-p)\theta^l f(e,k)\right)-\theta^l f(e,k)-(1-\delta)k\right]$. Since $\theta^l f(e,k)<ce$, this expression is positive provided the (DE) constraint holds. ■

Proof of Proposition 3. In order to maxime $\Pi_0=-k+\frac{\delta}{1-\delta}\left[\left(p\theta^h+(1-p)\theta^l\right)f(e,k)-ec\right]$, we have to take into account the effect of k on equilibrium effort in later periods.

Therefore, the first-order condition becomes

$$\frac{d\Pi_0}{dk}=-1+\frac{\delta}{1-\delta}\left[\left(p\theta^h+(1-p)\theta^l\right)f_k+\left(\left(p\theta^h+(1-p)\theta^l\right)f_e-c\right)\frac{de^*}{dk}\right]=0.$$

If $e^*=e^{FB}$ at $k=\tilde{k}$, where $\tilde{k}$ is the level characterized by $-1+\frac{\delta}{1-\delta}\left(p\theta^h+(1-p)\theta^l\right)f_k=0$, the second term in squared brackets equals zero, and $k^*=\tilde{k}$.

Now, assume that $e^*<e^{FB}$ at $k=\tilde{k}$, which implies that either (DE) or (DE-L) binds.

In the first case, e^* is characterized by $e^*c=\delta f(e^*,\tilde{k})\left(p\theta^h+(1-p)\theta^l\right)-(1-\delta)\tilde{k}$ and

$$\frac{de^*}{dk}\mid_{\tilde{k}}=-(1-\delta)\frac{\frac{\delta}{(1-\delta)}f_k\left(p\theta^h+(1-p)\theta^l\right)-1}{\delta f_e\left(p\theta^h+(1-p)\theta^l\right)-c}. \tag{7}$$

The denominator of (7) must be negative because otherwise, a larger effort level would relax (DE), contradicting that it binds. The numerator of (7), as well as de^*/dk, are positive for $k<\tilde{k}$, negative for $k>\tilde{k}$, and equal to zero for $k=\tilde{k}$. Therefore, $\frac{d\Pi_0}{dk}=0$ for $\tilde{k}$, and $k^*=\tilde{k}$.

In the second case where (DE-L) binds, e^* is characterized by

$e^*c=\frac{\delta p^2}{1-\delta+\delta p}\theta^h f(e^*,\tilde{k})+(1-p)\theta^l f(e^*,\tilde{k})-\frac{(1-\delta)p}{(1-\delta+\delta p)}\tilde{k}$ and

$$\frac{de^*}{dk}\mid_{\tilde{k}}=-\frac{\frac{p}{1-\delta+\delta p}(1-\delta)\left[\frac{\delta}{(1-\delta)}f_k\left(p\theta^h+(1-p)\theta^l\right)-1+\frac{1-p}{p}\theta^l f_k\right]}{\frac{\delta p^2}{1-\delta+\delta p}\theta^h f_e+(1-p)\theta^l f_e-c}. \tag{8}$$

The denominator of (8) must be negative because otherwise, a larger effort level would relax (DE-L), contradicting that it binds. The numerator of (8), as well as de^*/dk, are positive for any $k\leq\tilde{k}$. Therefore, $\frac{d\Pi_0}{dk}>0$ for any $k\leq\tilde{k}$, and $k^*>\tilde{k}$. ■

Proof of Proposition 4. Now, the relevant constraints are

$$ec \leq \delta \left(p\theta^h + (1-p)\theta^l\right) f(e,k) - (1-\delta)\gamma k \tag{DE}$$

and

$$ec \leq \left[(1-p)\theta^l + \frac{\delta p^2}{(1-\delta+p\delta)}\theta^h\right] f(e,k) - \frac{(1-\delta)p}{(1-\delta+p\delta)}\gamma k. \tag{DE-L}$$

The thresholds $\overline{\delta}(\gamma)$ and $\underline{\delta}(\gamma)$, as well as according properties, are obtained as in the proof to Proposition (1), only that k is replaced with γk.

Furthermore, $\frac{\partial \underline{\delta}}{\partial \gamma} = \frac{k}{f(e,k)} \frac{p\left(\theta^h - \theta^l\right)}{\left[\left(p\theta^h + (1-p)\theta^l\right) + \frac{\gamma k}{f(e,k)}\right]^2}$ and $\frac{\partial \overline{\delta}}{\partial \gamma} = \frac{(1-\overline{\delta})\left(1-\overline{\delta}+p\overline{\delta}\right)k}{p\theta^h f(e^{FB},k) + p\gamma k} > 0$, where the last step uses the binding (DE-L) constraint.

In a next step, we solve for the optimal investment level. The first-order condition of the principal's maximization problem equals

$$\frac{d\Pi_0}{dk} = -1 + \frac{\delta}{1-\delta}\left[\left(p\theta^h + (1-p)\theta^l\right) f_k + \left(\left(p\theta^h + (1-p)\theta^l\right) f_e - c\right)\frac{de^*}{dk}\right] = 0.$$

For $\delta \geq \overline{\delta}(\gamma)$, effort is at its first best and the investment level is characterized by

$$-1 + \frac{\delta}{1-\delta}\left[\left(p\theta^h + (1-p)\theta^l\right) f_k\right] = 0.$$

Hence, there are no over-investments.

For $\delta < \underline{\delta}(\gamma)$, (DE) binds and (DE-L) is slack. Therefore, $e^* < e^{FB}$, and $\frac{de^*}{dk} = -(1-\delta)\frac{\frac{\delta}{(1-\delta)} f_k\left(p\theta^h + (1-p)\theta^l\right) - \gamma}{\delta f_e\left(p\theta^h + (1-p)\theta^l\right) - c}$. This implies that also for a binding (DE), $k^* > \tilde{k}$, and over-investments are optimal. Furthermore, the (FOC) becomes

$$-\delta f_e \left(p\theta^h + (1-p)\theta^l\right)(1-\gamma) - \delta\left(p\theta^h + (1-p)\theta^l\right) f_k c + c\,(1-\delta\gamma) = 0.$$

This condition, together with the binding (DE) constraint,

$$\delta f(e^*,k^*)\left(p\theta^h + (1-p)\theta^l\right) - e^* c - (1-\delta)\gamma k^* = 0,$$

determines effort e^* and investment k^*.

Both conditions allow to compute $dk^*/d\gamma$, with

$$\frac{dk^*}{d\gamma} = \frac{\begin{vmatrix} -\delta\left(p\theta^h + (1-p)\theta^l\right) f_{ee}\,(1-\gamma) & -\delta\left(f_e\left(p\theta^h + (1-p)\theta^l\right) - c\right) \\ \delta f_e\left(p\theta^h + (1-p)\theta^l\right) - c & (1-\delta)k \end{vmatrix}}{\begin{vmatrix} -\delta\left(p\theta^h + (1-p)\theta^l\right) f_{ee}\,(1-\gamma) & -\delta\left(p\theta^h + (1-p)\theta^l\right) f_{kk} c \\ \delta f_e\left(p\theta^h + (1-p)\theta^l\right) - c & \delta f_k\left(p\theta^h + (1-p)\theta^l\right) - (1-\delta)\gamma \end{vmatrix}},$$

where the denominator must be positive in order to satisfy the second order condition for a maximum. Hence, the sign of $\frac{dk^*}{d\gamma}$ is determined by the sign of the numerator, which equals

$-\delta \left(p\theta^h + (1-p)\theta^l\right) f_{ee} \left(1-\gamma\right) \left(1-\delta\right)k$

$+\delta \left(f_e \left(p\theta^h + (1-p)\theta^l\right) - c\right) \left(\delta f_e \left(p\theta^h + (1-p)\theta^l\right) - c\right).$

The first line is positive because $f_{ee} < 0$. The second line must be negative, for the following reason: $f_e \left(p\theta^h + (1-p)\theta^l\right) - c > 0$ because $e^* < e^{FB}$. $\delta f \left(p\theta^h + (1-p)\theta^l\right) - c < 0$ because otherwise, increasing e^* would relax the (DE) constraint, contradicting that it binds. Generally, the impact of the positive effect is larger if γ is rather small, whereas the impact of the negative second term is larger if effort is rather small.

Now, assume that δ is such that $\underline{\delta}(\gamma) \leq \delta < \overline{\delta}(\gamma)$, hence (DE-L) binds and (DE) is slack. Therefore $e^* < e^{FB}$, and $\frac{de^*}{dk} = -\frac{\frac{p}{1-\delta+\delta p}(1-\delta)\left[\frac{\delta}{(1-\delta)} f_k \left(p\theta^h+(1-p)\theta^l\right)-\gamma+\frac{1-p}{p}\theta^l f_k\right]}{\frac{\delta p^2}{1-\delta+\delta p}\theta^h f_e+(1-p)\theta^l f_e-c}$. This implies that $k^* > \tilde{k}$, and over-investments are optimal. Furthermore, the (FOC) becomes

$$-\delta f_k p\theta^h c - \left(1-\delta\right) f_e(1-p)\theta^l + \left(1-\delta\right) c - \delta p \left(1-\gamma\right) \left[f_e \left(p\theta^h + (1-p)\theta^l\right) - c\right] = 0.$$

This condition, together with the binding (DE-L) constraint,

$$\left[(1-p)\theta^l + \frac{\delta p^2}{(1-\delta+p\delta)}\theta^h\right] f(e^*, k^*) - e^*c - \frac{(1-\delta)p}{(1-\delta+p\delta)}\gamma k^* = 0,$$

determines effort e^* and investment k^*.

Both conditions allow to compute $dk^*/d\gamma$, with

$$\frac{dk^*}{d\gamma} = \frac{\begin{vmatrix} -\left(1-\delta\right) f_{ee}(1-p)\theta^l - \delta p \left(1-\gamma\right) \left[f_{ee} \left(p\theta^h + (1-p)\theta^l\right)\right] & -\delta p \left[f_e \left(p\theta^h + (1-p)\theta^l\right) - c\right] \\ \left[(1-p)\theta^l + \frac{\delta p^2}{(1-\delta+p\delta)}\theta^h\right] f_e - c & \frac{(1-\delta)p}{(1-\delta+p\delta)} k^* \end{vmatrix}}{\begin{vmatrix} -\left(1-\delta\right) f_{ee}(1-p)\theta^l - \delta p \left(1-\gamma\right) f_{ee} \left(p\theta^h + (1-p)\theta^l\right) & -\delta f_{kk} p\theta^h c \\ \left[(1-p)\theta^l + \frac{\delta p^2}{(1-\delta+p\delta)}\theta^h\right] f_e - c & \left[(1-p)\theta^l + \frac{\delta p^2}{(1-\delta+p\delta)}\theta^h\right] f_k - \frac{(1-\delta)p}{(1-\delta+p\delta)}\gamma \end{vmatrix}},$$

where the denominator must be positive in order to satisfy the second order condition for a maximum. Hence, the sign of $\frac{dk^*}{d\gamma}$ is determined by the sign of the numerator, which equals

$-\left(1-\delta\right) f_{ee}(1-p)\theta^l - \delta p \left(1-\gamma\right) f_{ee} \left(p\theta^h + (1-p)\theta^l\right) \frac{(1-\delta)p}{(1-\delta+p\delta)} k^*$

$+\left[\left((1-p)\theta^l + \frac{\delta p^2}{(1-\delta+p\delta)}\theta^h\right) f_e - c\right] \delta p \left[f_e \left(p\theta^h + (1-p)\theta^l\right) - c\right].$

The first line is positive because $f_{ee} < 0$. The second line must be negative, for the following reason:

$f_e \left(p\theta^h + (1-p)\theta^l\right) - c > 0$ because $e^* < e^{FB}$.

$\left((1-p)\theta^l + \frac{\delta p^2}{(1-\delta+p\delta)}\theta^h\right) f_e - c < 0$ because otherwise, increasing e^* would relax the (DE-L) constraint, contradicting that it binds. ■

C Optimal Firm Behavior Given Cash Holdings are Possible

Starting from the equilibrium we derived in Section 4, assume that whenever demand conditions are high, the principal can retain some of her earnings. These cash reserves are used to increase effort from the next period on. After a number of subsequent low-demand periods, the cash reserves are used up and effort is down at its original level e^* - until the next high-demand period. In the following, we analyze to what extent retaining earnings is optimal for the principal (for the general case where the resale value of the asset is γk, with $\gamma \leq 1$).

Define $m \geq 1$ as the number of periods a higher effort level can *at least* be enforced, i.e., m is the subsequent number of low-demand periods after which all cash reserves are used up. Furthermore, define the total retained amount as s_m, and the effort level in the first period after s_m has been accumulated as e_m.

Now, assume that s_m has been retained and effort raised to e_m in the following period. If the firm faces a low-demand shock in this period, the agent is compensated accordingly. However, some of the retained cash is needed, and available funds go down to s_{m-1}. Furthermore, effort in the next period will be e_{m-1}. This process is continued until either all cash reserves are used (and effort is at $e_0 = e^*$) or a high-demand shock allows to fill up cash reserves and increase effort to e_m again. To keep the problem tractable, we assume that income in a high-demand state is sufficiently large such that the optimal amount s_m can be retained in one high-demand state, i.e., we impose the following assumption:

Assumption A1: Assume the firm can retain earnings. Then, (Lh) does not bind in a profit-maximizing equilibrium.

Assumption A1 implies that only one high-demand state is needed in order to replenish the firm's desired cash stock.

Profits

We write payoffs as functions of the remaining subsequent low-demand shocks before all cash reserves are used up. Profits given retained earnings are at its maximum level are denoted $\Pi(m)$, since m has been defined as the number of periods an effort level above e^* can at least be enforced. Hence,

$$\Pi(m) = p\left[\theta^h f(e_m, k) - b_m^h + \delta\Pi(m)\right] + (1-p)\left[\theta^l f(e_m, k) - b_m^l + (s_m - s_{m-1}) + \delta\Pi(m-1)\right].$$

Note that bonus payments are the amounts actually paid out to the agent. Therefore, the reduction of cash holdings in a low-demand state enters the principal's profits positively.

Furthermore, profits after $j < m$ subsequent low-demand periods are

$$\begin{aligned}\Pi(m-j) =& p\left[\theta^h f(e_{m-j},k) - b^h_{m-j} - (s_m - s_{m-j}) + \delta\Pi(m)\right] \\ &+ (1-p)\left[\theta^l f(e_{m-j},k) - b^l_{m-j} + (s_{m-j} - s_{m-j-1}) + \delta\Pi(m-j-1)\right].\end{aligned}$$

After $m-1$ subsequent low-demand period, higher effort can only be enforced for a maximum of one more low-demand period. Then,

$$\Pi(1) = p\left[\theta^h f(e_1,k) - b^h_1 - (s_m - s_1) + \delta\Pi(m)\right] + (1-p)\left[\theta^l f(e_1,k) - b^l_1 + s_1 + \delta\Pi(0)\right].$$

Finally,

$$\Pi(0) = p\left(f(e_0,k)\theta^h - b^h - s_m + \delta\Pi(m)\right) + (1-p)\left(f(e_0,k)\theta^l - b^l + \delta\Pi(0)\right)$$

are profits given the firm has used all its cash.

Objective

The objective is to find levels of $m \geq 0$, e_{m-j} ($j \leq m$) and the respective amounts of cash that maximize $\Pi(0)$, given the constraints derived below. Hence, we maximize profits given no cash is initially available. It will turn out, though, that the respective strategy also maximizes $-s_m + \delta\Pi(m)$, the principal's objective given cash could also be raised at the beginning of the game (when capital k is invested).

Constraints

The following constraints have to be satisfied. For all $j \in \{0, ..., m\}$, dynamic enforcement constraints for low- and high-demand states must hold:

$$b^l_{m-j} \leq \delta\Pi(m-j-1) - s_{m-j-1} - \gamma k \tag{DEl(j)}$$

and

$$b^h_{m-j} \leq \delta\Pi(m) - s_m - \gamma k. \tag{DEh(j)}$$

Cash holdings enter the above constraints since if the principal reneges on payments promised to the agent, it will also be optimal to consume retained earnings. Furthermore, note that $s_{-1} = s_0 = 0$ and $\Pi(-1) \equiv \Pi(0)$.

Since (Lh) is satisfied by assumption, liquidity constraints must only hold for low-demand states.

For all $j \in \{0, ..., m\}$, we have

$$b^l_{m-j} \leq \theta^l f(e_{m-j}, k) + (s_{m-j} - s_{m-j-1}), \tag{Ll(j)}$$

where $s_{-1} = 0$ in LLl(m).

In addition the (IC) constraints must hold (where we already take into account that agents receive no rent), namely

$$pb^h_{m-j} + (1-p)b^l_{m-j} - e_{m-j}c \geq 0$$

for all $j \in \{0, ..., m\}$.

For the same reasons as before, the (IC) constraint will bind on the equilibrium path, giving $b^l_{m-j} = \frac{e_{m-j}c - pb^h_{m-j}}{1-p}$.

Results

In this section, we first assume that it is optimal to accumulate strictly positive cash reserves (which implies $m \geq 1$) and derive properties of a profit-maximizing equilibrium. Then, we work out conditions under which it is actually optimal to retain earnings.

First, we show that given retaining earnings is optimal, (DEl) constraints can be omitted.

Lemma 3 *Assume $m \geq 1$. Then, all (DEl) are automatically implied by the respective (Ll) constraints.*

Proof of Lemma 3: First, we plug $b^l_{m-j} = \frac{e_{m-j}c - pb^h_{m-j}}{1-p}$ into profits, which yields the set of constraints

$$\frac{e_{m-j}c - pb^h_{m-j}}{1-p} \leq \delta\Pi(m-j-1) - s_{m-j-1} - \gamma k \tag{DEl(j)}$$

$$b^h_{m-j} \leq \delta\Pi(m) - s_m - \gamma k \tag{DEh(j)}$$

$$\frac{e_{m-j}c - pb^h_{m-j}}{1-p} \leq \theta^l f(e_{m-j}, k) + (s_{m-j} - s_{m-j-1}). \tag{Ll(j)}$$

The left hand sides of DEl(j) and Ll(j) constraints are the same. Therefore, if

$$\delta\Pi(m-j-1) - \gamma k \geq \theta^l f(e_{m-j}, k) + s_{m-j},$$

DEL(j) constraints are implied by Ll(j) constraints.

To the contrary, assume there is a $j^* \geq 0$ where $\delta\Pi(m-j^*-1)-\gamma k < \theta^l f(e_{m-j^*}, k)+s_{m-j^*}$.

Reduce all s_{m-j} with $j \leq j^*$ by a small $\varepsilon > 0$. This tightens Ll(j^*) which however will still hold for ε sufficiently small. Ll(j) constraints for all j besides j^* remain unaffected. Furthermore, profits change by $\Delta\Pi(0) = p\varepsilon\frac{1-(\delta(1-p))^{j^*+1}}{1-\delta(1-p)}$, hence go up.

It remains to show that no other constraint is violated by this operation. Note that

$$\Delta\Pi(m) = -\varepsilon\left(\delta(1-p)\right)^{j^*}(1-p),$$

$$\Delta\Pi(m-j^*) = -\frac{\varepsilon(1-p)}{1-\delta(1-p)}\left(1-\delta+\delta p\left(\delta(1-p)\right)^{j^*}\right)$$

and, for any $1 < k < m$,

$$\Delta\Pi(m-(j^*-k)) = \frac{\delta p\Delta\Pi(m) - \varepsilon\left(\delta(1-p)\right)^{k}[1-p](1-\delta)}{1-\delta(1-p)}$$

and

$$\Delta\Pi(m-(j^*+k)) = \frac{p\varepsilon\left(1-\left(\delta(1-p)\right)^{j^*+1}\right)}{1-\delta(1-p)}.$$

Now we can show that no constraint is violated if all s_{m-j} with $j \leq j^*$ are reduced by $\varepsilon > 0$:

- DEh(j) are relaxed as each right hand side changes by $\varepsilon\left(1-\left(\delta(1-p)\right)^{j^*+1}\right) > 0$
- DEl(0) is relaxed as its right hand side changes by $\varepsilon\left(1-\left(\delta(1-p)\right)^{j^*+1}\right) > 0$
- DEl(j^*) is relaxed, as its right hand side changes by $\varepsilon\left(\frac{(1-\delta)(1-\delta(1-p))+\delta p\left(1-(\delta(1-p))^{j^*+1}\right)}{1-\delta(1-p)}\right) > 0$
- DE(j^*+k) are relaxed as each right hand side changes by $\frac{p\varepsilon\delta\left(1-(\delta(1-p))^{j^*+1}\right)}{1-\delta(1-p)} > 0$
- DE(j^*-k) are relaxed as each right hand side changes by $\varepsilon\frac{(1-\delta)\left(1-(\delta(1-p))^{k+1}\right)+\delta p\left(1-(\delta(1-p))^{j^*+1}\right)}{1-\delta(1-p)} > 0$.

Hence, reducing all s_{m-j} with $j \leq j^*$ by a small $\varepsilon > 0$ does not violate any constraint but increases profits. ■

Lemma 3 implies that all DEl constraints can be omitted *provided cash holdings are optimal*. This allows us to simplify the problem by adding DEh(j) (multiplied with p) and LLl(j) (multiplied with $(1-p)$) constraints for each j, which gives a set of necessary and sufficient constraints (sufficiency follows from the same reasoning as in the situation without retained earnings):

$$p\left(\delta\Pi(m)-s_m-\gamma k\right)+(1-p)\theta^l f(e_{m-j},k)-e_{m-j}c+(1-p)\left(s_{m-j}-s_{m-j-1}\right)\geq 0, \quad \text{(DE-L(j))}$$

with $s_0=s_{-1}=0$.

In a next step, we show that given holding cash is optimal, all DE-L constraints must bind:

Lemma 4 *If $s_m>0$, DE-L(j) constraints bind for each $j\in\{0,...,m\}$.*

Proof of Lemma 4: Assume there is a j^* where DE-L(j^*) does not bind. Then, the respective effort level can be increased without violating any constraint, thereby also increasing profits. ■

Lemma 4 allows us to plug the binding DE-L constraints

$$\delta p\Pi(m)=e_0c-(1-p)\theta^l f(e_0,k)+p\left(s_m+\gamma k\right)$$

and

$$\left(s_{m-j}-s_{m-j-1}\right)=\frac{e_{m-j}c-\delta p\Pi(m)-(1-p)\theta^l f(e_{m-j},k)+p\left(s_m+\gamma k\right)}{(1-p)}$$

into profits, giving

$$\Pi(0)=\frac{p}{1-\delta(1-p)}\left(f(e_0,k)\theta^h+\gamma k\right),$$

$$\Pi(m-j)=p\theta^h f(e_{m-j},k)+p\left(s_{m-j}+\gamma k\right)+\delta(1-p)\Pi(m-j-1),$$

and

$$\Pi(m)=\sum_{i=0}^{m-1}\left(\delta(1-p)\right)^i\left(p\theta^h f(e_{m-i},k)+p\left(s_{m-i}+\gamma k\right)\right)+\left(\delta(1-p)\right)^m\frac{p}{1-\delta(1-p)}f(e_0,k)\theta^h.$$

In a next step, we can show that effort is gradually going down once cash reserves are used in low-demand periods:

Lemma 5 *$e_{m-j+1} > e_{m-j}$ for all $j \in \{1, ..., m\}$.*

Proof of Lemma 5: First, we show that $e_{m-j+1} \geq e_{m-j}$. To the contrary, assume there is a $\tilde{j}$ with $e_{m-\tilde{j}+1} < e_{m-\tilde{j}}$. Replace both effort levels and reduce $s_{m-\tilde{j}}$ to keep DE-L($\tilde{j}+1$) unaffected, i.e.,

$$\Delta s_{m-\tilde{j}} = \theta^l f(e_{m-\tilde{j}}, k) - \frac{e_{m-\tilde{j}}c}{(1-p)} - \left(\theta^l f(e_{m-\tilde{j}+1}, k) - \frac{e_{m-\tilde{j}+1}c}{(1-p)}\right).$$

Note that this leaves DE-L($\tilde{j}$) as well as other constraints unaffected. However, this change increases $\Pi(m)$ and thereby $\Pi(0)$:

$$\begin{aligned}
\frac{\Delta\Pi(m)}{(\delta(1-p))^{\tilde{j}-1}} &= p\theta^h \left(f(e_{m-j}, k) - f(e_{m-j+1}, k)\right) + \delta(1-p)p\theta^h \left(f(e_{m-j+1}, k) - f(e_{m-j}, k)\right) \\
&+ p\left[\theta^l f(e_{m-j}, k) - \frac{e_{m-j}c}{(1-p)} - \left(\theta^l f(e_{m-j+1}, k) - \frac{e_{m-j+1}c}{(1-p)}\right)\right] \\
&\geq p\theta^h \left(f(e_{m-j}, k) - f(e_{m-j+1}, k)\right) + (1-p)p\theta^h \left(f(e_{m-j+1}, k) - f(e_{m-j}, k)\right) \\
&+ p\left[\theta^l f(e_{m-j}, k) - \frac{e_{m-j}c}{(1-p)} - \left(\theta^l f(e_{m-j+1}, k) - \frac{e_{m-j+1}c}{(1-p)}\right)\right] \\
&= p\left(p\theta^h f(e_{m-j}, k) + (1-p)\theta^l f(e_{m-j}, k) - e_{m-j}c\right) \\
&- p\left(p\theta^h f(e_{m-j+1}, k) + (1-p)\theta^l f(e_{m-j+1}, k) - e_{m-j+1}c\right) > 0,
\end{aligned}$$

where the last inequality follows from $e_{m-j} > e_{m-j+1}$, and from both effort levels being inefficiently low.

To complete the proof, it remains to show that $e_{m-j+1} = e_{m-j}$ is not possible. To the contrary, assume there is a $\hat{j}$ with $e_{m-\hat{j}+1} = e_{m-\hat{j}}$. Now, marginally increase $e_{m-\hat{j}+1}$ and marginally decrease $e_{m-\hat{j}}$. Furthermore, $ds_{m-\hat{j}} = \theta^l \frac{\partial f(e_{m-\hat{j}+1},k)}{\partial e} - \frac{c}{1-p}$ is set to keep DE-L($\hat{j}+1$) and DE-L($\hat{j}$) unaffected. Then,

$$\begin{aligned}
\frac{d\Pi(m)}{(\delta(1-p))^{\hat{j}-1}} &= p\theta^h \frac{\partial f(e_{m-\hat{j}+1}, k)}{\partial e} + \delta(1-p)\left(-p\theta^h \frac{\partial f(e_{m-\hat{j}+1}, k)}{\partial e} + p\left(\theta^l \frac{\partial f(e_{m-\hat{j}+1}, k)}{\partial e} - \frac{c}{1-p}\right)\right) \\
&\geq p\left(p\theta^h \frac{\partial f(e_{m-\hat{j}+1}, k)}{\partial e} + (1-p)\theta^l \frac{\partial f(e_{m-\hat{j}+1}, k)}{\partial e} - c\right) > 0.
\end{aligned}$$

■

Finally, we can show that maximum effort e_m is independent of m.

Lemma 6 *Maximum effort e_m is characterized by*

$$\frac{\delta(1-p)}{(1-\delta p)} p\theta^h \frac{\partial f(e_m, k)}{\partial e_m} + (1-p)\theta^l \frac{\partial f(e_m, k)}{\partial e_m} - c = 0.$$

Proof of Lemma 6: Note that if holding cash is optimal, all DE-L(j) constraints bind for a given m, which implies that

$p\left(\delta\Pi(m)-s_m-\gamma k\right)+(1-p)\theta^l f(e_{m-j},k)-e_{m-j}c+(1-p)\left(s_{m-j}-s_{m-j-1}\right)=0$

can be used to obtain the necessary cash for all levels of e_{m-j}. Then, the objective is to maximize $\Pi(0)=\frac{p}{1-\delta(1-p)}\left(f(e_0,k)\theta^h+\gamma k\right)$, which is obtained by using binding (DE-L) constraints. Put differently, effort levels e_{m-j} are chosen to maximize e_0. But holding cash can only be optimal if DE-L(0) binds, in which case e_0 is determined by $p\left(\delta\Pi(m)-s_m-\gamma k\right)+(1-p)\theta^l f(e_0,k)-e_0c=0$. Hence, the objective can be reformulated in a way to choose effort levels e_{m-j} in order to maximize the left hand side of this condition, therefore $\delta\Pi(m)-s_m$, and determined by setting $\frac{\partial(\delta\Pi(m)-s_m)}{\partial e_{m-j}}=0$.

Furthermore, solving DE-L(m-j) for cash levels s_{m-j} yields

$s_{m-j}=-\sum_{i=j}^{m-1}\frac{p(\delta\Pi(m)-s_m-\gamma k)+(1-p)\theta^l f(e_{m-i},k)-e_{m-i}c}{(1-p)}$.

This is done iteratively, starting with

$s_1=-\frac{p(\delta\Pi(m)-s_m-\gamma k)+(1-p)\theta^l f(e_1,k)-e_1c}{(1-p)}$,

$s_2=-\frac{p(\delta\Pi(m)-s_m-\gamma k)+(1-p)\theta^l f(e_2,k)-e_2c}{(1-p)}-\frac{p(\delta\Pi(m)-s_m-\gamma k)+(1-p)\theta^l f(e_1,k)-e_1c}{(1-p)}$, and so on.

Therefore,

$s_m=-\sum_{i=0}^{m-1}\frac{p(\delta\Pi(m)-s_m-\gamma k)+(1-p)\theta^l f(e_{m-i},k)-e_{m-i}c}{1-p}$.

Furthermore, note that $\frac{\partial s_{m-j}}{\partial e_m}=0$ for $j>0$ – because $\frac{\partial(\delta\Pi(m)-s_m)}{\partial e_{m-j}}=0$ must be satisfied provided $s_m>0$.

This implies $\frac{\partial s_m}{\partial e_m}=-\frac{(1-p)\theta^l\frac{\partial f(e_m,k)}{\partial e_m}-c}{1-p}$.

Finally, using

$\Pi(m)=\sum_{i=0}^{m-1}\left(\delta(1-p)\right)^i\left(p\theta^h f(e_{m-i},k)+p\left(s_{m-i}+\gamma k\right)\right)+\left(\delta(1-p)\right)^m\frac{p}{1-\delta(1-p)}f(e_0,k)\theta^h$

and computing $\frac{\partial(\delta\Pi(m)-s_m)}{\partial e_m}=0$, allows to characterize e_m:

$$\frac{\delta(1-p)}{(1-\delta p)}p\theta^h\frac{\partial f(e_m,k)}{\partial e_m}+(1-p)\theta^l\frac{\partial f(e_m,k)}{\partial e_m}-c=0.$$

■

This result shows that a larger value of m does not increase maximum effort, but rather "smoothes" the process of effort reductions after negative demand shocks.

Finally, we can prove Proposition 5.

Proof of Proposition 5: Lemma 6 gives condition (3) for maximum effort. Furthermore, note that if retaining earnings is optimal, then $e_m>e_{m-1}>...>e^*$. Hence, if maximum enforceable effort without cash reserves satisfies $\left(\frac{\delta(1-p)}{[1-\delta p]}p\theta^h + (1-p)\theta^l\right) f_e - c \leq 0$, holding cash cannot be optimal. ■

Here, we do not aim for solving for the optimal m, i.e. the maximum number of subsequent negative shocks until cash reserves are used up. The optimal level of m would again be determined by maximizing $\delta\Pi(m) - s_m$. To get around integer problems, we would first treat m as a continuous variable, set $\frac{\partial(\delta\Pi(m)-s_m)}{\partial m} = 0$ and get the optimal m as the largest integer for which $\frac{\partial(\delta\Pi(m)-s_m)}{\partial m} \geq 0$.

Cash Holdings Generate Interest

Now, we derive the properties of an equilibrium if cash holdings generation interest $r \geq 0$. We assume that $r \leq \frac{1-\delta}{\delta}$ and that if indifferent between holding cash and consuming today, the principal consumes today. Without the first part of the assumption, it would be strictly optimal to never consume and instead save all profits. The second part simplifies the analysis in case $r = \frac{1-\delta}{\delta}$, i.e., when the interest rate just makes up for discounting.

We stick to the assumption that the principal is able to retain her desired amount s_m in one high-demand period, i.e., liquidity constraints never bind in high-demand states. If this were not the case, the benefits of holding cash would be muted.

As before, define the retained amount as s_m, and the effort level in the first period after s_m has been accumulated as e_m.

Then,

$$\Pi(m) = p\left[\theta^h f(e_m, k) - b_m^h + rs_m + \delta\Pi(m)\right] + (1-p)\left[\theta^l f(e_m, k) - b_m^l + ((1+r)s_m - s_{m-1}) + \delta\Pi(m-1)\right].$$

There, s_{m-1} is the amount of cash kept for the next period (in which the principal can spend $(1+r)s_{m-1}$). Note that interest income is consumed by the principal if high-demand states are followed by high-demand states. However, interest income is never consumed in a low-demand state. If this were the case, a reduction of savings in previous periods would be optimal.

To further simplify our analysis, we also assume that $(1+r)s_{m-1} \leq s_m$. This implies that the principal only consumes interest income in high-demand states which follow high-demand states.

Therefore,

$$\Pi(m-j) = p\left[\theta^h f(e_{m-j}, k) - b_{m-j}^h - (s_m - (1+r)s_{m-j}) + \delta\Pi(m)\right] + (1-p)\left[\theta^l f(e_{m-j}, k) - b_{m-j}^l + ((1+r)s_{m-j} - s_{m-j-1}) + \delta\Pi(m-j-1)\right].$$

After $m-1$ subsequent low-demand period, higher effort can only be enforced for a maximum of one more low-demand period. Then,

$$\Pi(1) = p\left[\theta^h f(e_1,k) - b_1^h - (s_m - (1+r)s_1) + \delta\Pi(m)\right] + (1-p)\left[\theta^l f(e_1,k) - b_1^l + (1+r)s_1 + \delta\Pi(0)\right].$$

Finally,

$$\Pi(0) = p\left(f(e_0,k)\theta^h - b^h - s_m + \delta\Pi(m)\right) + (1-p)\left(f(e_0,k)\theta^l - b^l + \delta\Pi(0)\right)$$

are profits given the firm has used all its cash.

Constraints

For all $j \in [0, m]$, dynamic enforcement constraints equal

$$b_{m-j}^l \leq \delta\Pi(m-j-1) - s_{m-j-1} - \gamma k \tag{DEl(j)}$$

and

$$b_{m-j}^h \leq \delta\Pi(m) - s_m - \gamma k. \tag{DEh(j)}$$

They are unaffected by the interest rate. This is because, for example, the (DE) constraint in a high-demand state equals

$$-b_{m-j}^h - (s_m - (1+r)s_{m-j}) + \delta\Pi(m) \geq \gamma k + (1+r)s_{m-j},$$

where the term that includes interest payments cancels out.

As before, $s_{-1} = s_0 = 0$ and $\Pi(-1) \equiv \Pi(0)$.

Since (Lh) is satisfied by assumption, liquidity constraints for all $j \in \{0, ..., m\}$, which are only needed in low-demand states, equal

$$b_{m-j}^l \leq \theta^l f(e_{m-j},k) + ((1+r)s_{m-j} - s_{m-j-1}). \tag{Ll(j)}$$

Finally, binding (IC) constraints deliver $b_{m-j}^l = \frac{e_{m-j}c - pb_{m-j}^h}{1-p}$.

Results

As before, all (DEl) are automatically implied by the respective (Ll) constraints if cash holdings are optimal, and the following (DE-L) constraints are necessary and sufficient for implementing equilibrium effort levels:

$$p\left(\delta\Pi(m)-s_m-\gamma k\right)+(1-p)\theta^l f(e_{m-j},k)-e_{m-j}c+(1-p)\left((1+r)s_{m-j}-s_{m-j-1}\right)\geq 0, \tag{DE-L(j)}$$

with $s_0=s_{-1}=0$.

These DE-L constraints must bind, which delivers profit levels

$$\Pi(0)=\frac{p}{1-\delta(1-p)}\left(f(e_0,k)\theta^h+\gamma k\right),$$

$$\Pi(m-j)=p\theta^h f(e_{m-j},k)+p\left((1+r)s_{m-j}+\gamma k\right)+\delta(1-p)\Pi(m-j-1),$$

and

$$\Pi(m)=\sum_{i=0}^{m-1}\left(\delta(1-p)\right)^i\left[p\theta^h f(e_{m-i},k)+p\left((1+r)s_{m-i}+\gamma k\right)\right]+\left(\delta(1-p)\right)^m\frac{p}{1-\delta(1-p)}f(e_0,k)\theta^h.$$

Still, effort is gradually going down once cash reserves are used in low-demand periods, hence $e_{m-j+1}>e_{m-j}$ for all $j\in\{1,...,m\}$.

Finally, we characterize maximum effort e_m in case retained earnings generate interest payments.

Lemma 7 *Maximum effort e_m is characterized by*

$$\frac{\delta p\,(1-p)}{(1-\delta p(1+r))}\theta^h\frac{\partial f(e_m,k)}{\partial e_m}+\frac{(1-p)\theta^l\frac{\partial f(e_m,k)}{\partial e_m}-c}{(1+r)}=0. \tag{9}$$

Proof of Lemma 7: Binding DE-L(j) constraints yield

$p\left(\delta\Pi(m)-s_m-\gamma k\right)+(1-p)\theta^l f(e_{m-j},k)-e_{m-j}c+(1-p)\left((1+r)s_{m-j}-s_{m-j-1}\right)=0$,

which can be used to obtain the necessary cash for all levels of e_{m-j}. Then, the objective is to maximize $\Pi(0)=\frac{p}{1-\delta(1-p)}\left(f(e_0,k)\theta^h+\gamma k\right)$. Thus, effort levels e_{m-j} are chosen in order to maximize e_0. Holding cash can only be optimal if DE-L(0) binds, in which case e_0 is determined by $p\left(\delta\Pi(m)-s_m-\gamma k\right)+(1-p)\theta^l f(e_0,k)-e_0c=0$. Hence, effort levels e_{m-j} are optimally chosen to maximize $\delta\Pi(m)-s_m$ and determined by setting $\frac{\partial(\delta\Pi(m)-s_m)}{\partial e_{m-j}}=0$.

Solving DE-L(m-j) for cash levels s_{m-j} yields

$s_{m-j}=-\sum_{i=j}^{m-1}\frac{p(\delta\Pi(m)-s_m-\gamma k)+(1-p)\theta^l f(e_{m-i},k)-e_{m-i}c}{(1+r)^{i+1-j}(1-p)}$.

Therefore,

$s_m = -\sum_{i=0}^{m-1} \frac{p(\delta\Pi(m)-s_m-\gamma k)+(1-p)\theta^l f(e_{m-i},k)-e_{m-i}c}{(1+r)^{i+1}(1-p)}$.

Furthermore, note that $\frac{\partial s_{m-j}}{\partial e_m} = 0$ for $j > 0$ because $\frac{\partial(\delta\Pi(m)-s_m)}{\partial e_{m-j}} = 0$ must be satisfied provided $s_m > 0$.

This implies $\frac{\partial s_m}{\partial e_m} = -\frac{(1-p)\theta^l \frac{\partial f(e_m,k)}{\partial e_m} - c}{(1+r)(1-p)}$.

Finally, using

$$\Pi(m) = \sum_{i=0}^{m-1} (\delta(1-p))^i \left[p\theta^h f(e_{m-i},k) + p\left((1+r)s_{m-i} + \gamma k\right)\right] + (\delta(1-p))^m \frac{p}{1-\delta(1-p)} f(e_0,k)\theta^h,$$

and computing $\frac{\partial(\delta\Pi(m)-s_m)}{\partial e_m} = 0$, allows to characterize e_m:

$$\frac{\delta p\,(1-p)}{(1-\delta p(1+r))}\theta^h \frac{\partial f(e_m,k)}{\partial e_m} + \frac{(1-p)\theta^l \frac{\partial f(e_m,k)}{\partial e_m} - c}{(1+r)} = 0. \tag{10}$$

■

References

Abreu, D. (1988): "On the Theory of Infinitely Repeated Games with Discounting," *Econometrica*, 56(2), 383–396.

Barron, D., and J. Li (2015): "Financing Constraints and Relational Contracts," *mimeo.*

Bebchuk, L., and L. Stole (1993): "Do short-term objectives lead to under- or over-investment in long-term projects," *Journal of Finance*, 48, 719–729.

Bull, C. (1987): "The Existence of Self-Enforcing Implicit Contracts," *The Quarterly Journal of Economics*, 102(1), 147–159.

Contreras, O. (2013): "Relational Contracts, Financing Constraints, and Social Welfare," *mimeo.*

Englmaier, F., and C. Segal (2011): "Cooperation and Conflict in Organizations," *mimeo.*

Fahn, M., V. Merlo, and G. Wamser (2017): "The Commitment Role of Equity Financing," *mimeo.*

Gibbons, R., and R. Henderson (2013): "What do managers do? Exploring persistent performance differences among seemingly similar enterprises," *Handbook of Organizational Economics*, pp. 680–731.

Grossman, S., and O. Hart (1983): "An Analysis of the Principal-Agent Problem," *Econometrica*, 51(1), 7–45.

Halac, M. (2015): "Investing in a Relationship," *RAND Journal of Economics*, 46(1), 165–185.

Hart, O., and J. Moore (1995): "Debt and Seniority: An Analysis of the Role of Hard Claims in Constraining Management," *American Economic Review*, 113, 1–41.

Heaton, J. (2002): "Managerial Optimism and Corporate Finance," *Financial Management*, 31, 33–45.

Holmstrom, B. (1979): "Moral Hazard and Observability," *The Bell Journal of Economics*, 10(1), 74–91.

Inderst, R., and M. Klein (2007): "Innovation, endogenous overinvestment, and incentive pay," *RAND Journal of Economics*, 38(4), 881–904.

Inderst, R., and C. Laux (2005): "Incentives in Internal Capital Markets," *RAND Journal of Economics*, 36(1), 215–228.

Jensen, M. (1986): "Agency Costs of Free Cash Flows, Corporate Finance, and Takeovers," *American Economic Review*, 76, 323–329.

——— (1993): "The Modern Industrial Revolution, Exit, and the Failure of Internal Control Systems," *Journal of Finance*, 48, 831–880.

Klein, B., and K. B. Leffler (1981): "The Role of Market Forces in Assuring Contractual Performance," *Journal of Political Economy*, 89(4), 615–641.

Levin, J. (2003): "Relational Incentive Contract," *The American Economic Review*, 93(3), 835–857.

Li, J., and N. Matouschek (2013): "Managing Conflicts in Relational Contracts," *The American Economic Review*, 103(6), 2328–2351.

MacLeod, W., and J. Malcomson (1989): "Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment," *Econometrica*, 57(2), 447–480.

McConnell, J., and C. Muscarella (1985): "Corporate capital expenditure decisions and the market value of the firm," *Journal of Financial Economics*, 14, 399–422.

Michaely, R., and M. Roberts (2012): "Corporate Dividend Policies: Lessons from Private Firms," *Review of Financial Studies*, 25(3), 711–746.

Myers, S. (2003): "Financing of Corporations," *Handbook of the Economics of Finance*, pp. 215–253.

PriceWaterhouseCooper (2012): "Family Firm: A Resilient Model for the 21st Century," *PwC Family Business Survey*.

Roll, R. (1986): "The Hubris Hypothesis of Corporate Takeovers," *Journal of Business*, 59, 97–216.

Stein, J. (1989): "Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior," *Quarterly Journal of Economics*, 104, 655–669.

——— (1997): "Internal Capital Markets and the Competition for Corporate Resources," *Journal of Finance*, 52(1), 111–133.

——— (2003): "Agency, Information and Corporate Investment," *Handbook of the Economics of Finance*, pp. 111–165.

Williamson, O. (1964): *The Economics of Discretionary Behavior: Managerial Objectives in a Theory of the Firm*. Prentice Hall, Englewood Cliffs/NJ.

Zwiebel, J. (1996): "Dynamic capital structure under management entrenchment," *American Economic Review*, 86, 1197–1215.