Matching with Waiting Times: The German Entry-Level Labor Market for Lawyers

Philipp D. Dimakopoulos (Humboldt University Berlin)
C.-Philipp Heller (Humboldt University Berlin)

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Ludwig-Maximilians-Universität München | Humboldt-Universität zu Berlin
Spokesperson: Prof. Dr. Klaus M. Schmidt, University of Munich, 80539 Munich, Germany
+49 (89) 2180 3405 | info@rationality-and-competition.de

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Philipp D. Dimakopoulos and C.-Philipp Heller *

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Abstract

We study the allocation of German lawyers to regional courts for legal trainee-ships. Because of excess demand in some regions lawyers often have to wait before being allocated. The currently used "Berlin" mechanism is not weakly Pareto efficient, does not eliminate justified envy and does not respect improvements. We introduce a mechanism based on the matching with contracts literature, using waiting time as the contractual term. The resulting mechanism is strategy-proof, weakly Pareto efficient, eliminates justified envy and respects improvements. We extend our proposed mechanism to allow for a more flexible allocation of positions over time.

JEL Classification: D47, D82, C78, H75, I28

Keywords: Many-to-One Matching; Matching with Contracts; Stability, Slot-Specific Choice Functions, Waiting Time, Legal Education

*Address: Humboldt Universität zu Berlin, Wirtschaftswissenschaftliche Fakultät, Spandauer Str. 1, 10178 Berlin. Correspondence: dimakoph@hu-berlin.de, hellerkr@hu-berlin.de. The authors would like to thank Christian Basteck, Helmut Bester, Inacio Bo, Lars Ehlers, Lucien Frys, Rustamdjan Hakimov, Tadashi Hashimoto, Maciej Kotowski, Fuhito Kojima, Morimitsu Kurino, Dorothea Kübler, Roland Strausz and Alex Westkamp as well as participants of the Matching Market Design course at WZB, the 12th Meeting of the Society for Social Choice and Welfare in Boston, the Conference on Economic Design in Istanbul, the World Congress of the Econometric Society in Montreal, the Jahrestagung of the Verein für Socialpolitik in Münster, Seminar participants in Berlin and Stanford, as well as three anonymous referees for helpful comments and suggestions. All remaining errors are of course our own. Financial support from the Deutsche Forschungsgemeinschaft (CRC TRR 190 and RTG1659) and the German Fulbright Commission (Heller) is gratefully acknowledged.

1 Introduction

Many real world matching markets fail to match all participants. Those who are unmatched may either leave the market altogether or wait and participate in a later matching procedure. The example that we study here is the allocation of graduating lawyers to their legal trainee-ship at regional courts in Germany. In this market congestion arises because of excess demand for positions in some parts of the country. This congestion is managed by requiring unmatched applicants to enter a wait list for their trainee-ship. To ensure that lawyers will eventually obtain a position at a court, the priority of a lawyer increases with the acquired waiting time. We assume that lawyers have preferences over where and when they complete their legal trainee-ship. The preferences over time are however ignored by the currently used procedure, which leads to a lack of efficiency, justified envy and a lack of respect of improvements. We propose a new procedure that does not suffer from those shortcomings.

The focus of this work is the trainee-ship allocation problem between graduated lawyers on the one side and courts on the other side. This is an important market as each year there are approximately 8,000 positions for legal trainee-ship in Germany.[1] These numbers are comparable to the (roughly) 20,000 US hospital residency program matches per year studied by Roth and others, e.g. in Roth (1984). In this lawyer market the wage is regulated so it cannot be used to reduce congestion by balancing excess demand.

Unlike in the United States, in Germany lawyers typically begin their legal education as an undergraduate, studying law at a university for around four years. Afterward students take a first state exam, set by the 16 federal states. Following this, students may apply for a legal trainee-ship. Completion of the trainee-ship is necessary to practice law in Germany and a requirement for many jobs in the bureaucracy. It is thus important to ensure access to trainee-ships for all lawyers who wish to complete it.

There is no cooperation across the federal states in terms of having a single national market for positions. This means that each lawyer can in principle ap-

[1] Based on authors' calculations using data from http://www.juristenkoffer.de/rechtsreferendariat/ (Accessed 8. October 2015).

ply for a position in each of the federal states.[2] This leads to sizable congestion, since in the extreme the total number of registered applicants in the system will be several times the number of positions actually demanded by the lawyers. We suspect that multiple applications by some lawyers for positions in several federal states is at least partially responsible for long waiting times as some lawyers apply for positions as a safety option which they are unlikely to take. The authorities seem to be aware of this possibility. Several application forms contain declarations that there are no pending applications to other regional courts of appeals or require applicants to withdraw other applications.[3] If they do not accept a position after they have been offered one, then the application system requires the lawyers to inform the respective authorities in the federal state.[4] The authorities may then decide to allow additional lawyers to begin their trainee-ship at that period. This process of refusing and making new offers takes up time and may leave some positions unused if no other lawyers can be found to take up these positions.[5] Note that this process of formally accepting and rejecting offers could also be addressed by allowing lawyers to formally declare some courts as unacceptable and committing applicants to accepting any position that they were offered. However if the value of the outside option is unknown at the time preferences are submitted, then this will not alleviate the problem of offers being refused.[6]

While the organization of the allocation procedure by federal state, rather than having a national procedure, seems to us to be a major cause of congestion

[2]The allocation of lawyers to courts is organized by regional courts of appeals in each state. Some federal states, notably Bavaria and North Rhine-Westfalia, contain several regional courts of appeals so even within a state there is scope for greater coordination.

[3]See for example the application web page of the regional court of appeals in Munich, https://www.justiz.bayern.de/gericht/olg/m/studiosi/01441/index.php (accessed 8. October 2015).

[4]For example, Art. 4 of the "Verordnung über die Aufnahme in den juristischen Vorbereitungsdienst" of Hamburg states that applicants who have not accepted a position that was offered to them within 10 days, will not be allocated. Furthermore it says that if an applicant does not accept a position twice, the applicant will be excluded from the application procedure and will have to reapply. Last, it says positions which have not been accepted are allocated to applicants next in line.

[5]For example on 6. October 2015 in North Rhine-Westfalia there were 12 positions still to be filled to begin on 1. November 2015. See http://www.olg-duesseldorf.nrw.de/aufgaben/referendarabteilung/09_weiter_info/index.php (accessed 8. October 2015). Congestion problems arising from the need to sequentially inquire about agents acceptance and rejection of offers have been found for example in the market for clinical psychology, Roth and Xing (1997).

[6]This is the case when several federal states run their allocation procedures in parallel.

in the market for lawyers, addressing this problem would require coordinated action of the federal states. Given the difficulty of establishing such cooperation, we here take the approach to consider only an isolated federal state and analyze how the allocation procedure in that federal state should be designed to better handle the congestion resulting from the lack of national coordination. We thus treat the federalized nature of this labor market as an additional constraint to be respected by the market designer, akin to the constraint that monetary flexibility cannot be used to clear some matching markets (e.g. for kidneys or schools[7]).

The number of available positions for the trainee-ship varies by court and usually depends on its size and the budget that has been made available for legal trainees in the budget of the federal state and/or the capacity of the court. This budget is usually set for several starting dates in advance. For example, the relevant administrative order in Berlin states that the capacity needs to be determined for one year in advance while positions may be started in February, May, August and December.[8] In Hamburg, the relevant administrative order states that the number of positions is determined by the number of positions fixed in the budget, which is valid for at least one year. Trainee-ships can start every even-numbered month.[9] In Hessen capacities are set every half a year, while new positions are available in all odd-numbered months.[10]

Due to large numbers of applications in some federal states, not all lawyers applying for a position at a court can be allocated at their desired starting time.[11] The excess demand is managed via a system based on waiting times accumulated by the applicants.[12] Most federal states have a system whereby a lawyer's priority in being allocated a place at a court increases in the number of times that

[7] See Roth, Sönmez and Ünver (2004) and Abdulkadiroğlu and Sönmez (2003)

[8] See Art. 2 (2) in the "Verordnung über die Ausbildungskapazität und das Vergabeverfahren für den juristischen Vorbereitungsdienst" (Stadt Berlin (2004)) for setting capacities. Dates for entry into the trainee-ship are taken from http://www.berlin.de/sen/justiz/gerichte/kg/ausbildung/jur-vorb/bew-verf/ (accessed 6. October 2015).

[9] See Art. 2 and Art. 3 (1) in the "Verordnung über die Aufnahme in den juristischen Vorbereitungsdienst" (Hansestadt Hamburg (2012)) for how capacities are set and the dates when trainee-ships start, respectively.

[10] See the guidelines on the legal trainee-ship for Hesse.

[11] Most application forms ask for the desired entry date of an applicant. Even if applicants could only apply for the next starting date, delaying applications until that date ensures that students can affect the time period for which they are considered.

[12] For example, in Berlin for entry on August 3rd 2015 applicants with a grade of 10 or higher were admitted if they applied 5 months earlier. Those who did their state exam in Berlin were admitted if they applied 10 months earlier, while those who did their state exam elsewhere with a grade below 10 were admitted if they applied 11 months earlier.

lawyer was not matched. For example in Hamburg, grades, waiting time and other concerns are weighted and expressed as a single score for each lawyer.[13] In North Rhine-Westfalia by contrast only the time since the application was received by the regional court of appeals determines the ranking of a candidate.[14] In Hessen 35% of positions are reserved for applicants with the highest waiting time.[15] In Brandenburg 70% of positions are reserved for applicants with the highest waiting time.[16] Thereby it is in principle possible for each lawyer to gain some place in a federal state eventually. Currently, average waiting times can be up to 24 months, depending on the federal state, although it should be noted that in many states waiting time is zero or only a couple of months.[17]

When applying for a position lawyers can typically indicate a preference for a particular regional court.[18] While lawyers can submit rankings over the courts, there is no legal guarantee of being assigned the first choice court.[19] While in general the allocation of lawyers to courts should take into account reported preferences, capacities and priorities, we could not find a clear description of the methods used to allocate lawyers to courts.[20] Some regional courts

[13] Art. 5 of the Aufnahmeverordnung (AVO,Hansestadt Hamburg (2012)) sets rules on how to calculate this score. The base score is the minimum of 6.49 and the grade achieved by the lawyer in the first state exam (Art. 5 (1) AVO). Further points can be added for example for having completed military service, disabilities, having done the state exam in Hamburg and for every 6 months of accumulated waiting time (Art. 5 (2) AVO). In case of ties in the weighted score, Art. 6 (1) AVO instructs to use the grade in the state exam to break ties. Remaining ties are to be broken via lottery according to Art. 6 (2) AVO.

[14] See `https://www.justiz.nrw.de/WebPortal/JM/landesjustizpruefungsamt/juristischer_vorbereitungsdienst/2Einstellung/index.php` (accessed 7. October 2015).

[15] See the Justizprüfungsamt Hessen (2011). Another 50% are reserved for lawyers based on merit and the remaining 15% are reserved for applicants satisfying social criteria.

[16] See Art. 11 (3) of the "Juristenausbildungsgesetz" of Brandenburg (Land Brandenburg (2014)). 20% of positions are given based on waiting time, with the remaining 10% given based on social criteria.

[17] Based on data from `http://www.juristenkoffer.de/rechtsreferendariat/` (Accessed 8. October 2015).

[18] For example lawyers applying to do their trainee-ship in the district of the Dusseldorf (North Rhine-Westfalia) regional court of appeals can apply to the regional courts in Dusseldorf, Duisburg, Kleve, Krefeld, Mönchengladbach or Wuppertal.

[19] For example, Art. 30 (3) of the Lawyer Education Law of North Rhine-Westfalia (Juristenausbildungsgesetz Nordrhein-Westfalen, JAG NRW) states that there is no legal right to a position in a particular district of a regional court of appeals and at a particular time.

[20] For example, in the guidelines on the application in the Dusseldorf district, it simply says that lawyers are allocated to courts following a "comprehensive view" of all applications. This may result in lawyers not getting their first choice so that they are asked to indicate further preferences, (Oberlandesgericht Düsseldorf (2015b)).

of appeal give some additional insights into how lawyers are allocated to particular courts. For example, applicants to Munich are ranked according to a number of criteria.[21] The highest priority is given to applicants having to care for their children, followed by married couples having lived for at least one year in the desired location. Next come those suffering from serious illnesses and then those working as teaching assistants at universities in the desired location. Finally, the length of time that applicants have lived in the desired location is used. There is however no indication in what way those priorities are used.

To analyze the market while accounting for waiting time, we propose a lawyer-court matching problem based on Hatfield and Milgrom (2005).[22] On the one side of the market there are lawyers, who have preferences over assignments to courts over time. Courts on the other side have priorities over lawyers, possibly based on their grade, social criteria and accumulated waiting time, which together with the current time period determines a lawyer's waiting time.[23] A matching mechanism in this context produces an allocation consisting of a subset of contracts, which specify a lawyer, a court and the time period the trainee-ship begins. Capacities of a court in future periods are already known, as we discussed above.

Based on the features of the currently used procedure we introduce the "Berlin" mechanism. This mechanism is not weakly Pareto efficient. We show that this mechanism may lead to allocations where one lawyer justifiably envies another. Furthermore improvements of the ranking achieved by a lawyer may yield an allocation that is worse for that lawyer. However, by construction, the Berlin mechanism achieves an allocation, such that no currently available positions remain unfilled while allocating some lawyers to later positions.

We propose the time-specific choice function, which is a special case of choice functions based on slot-specific priorities of Kominers and Sönmez (2016). Here time-specific means that each court can only accept a fixed number of students to begin their trainee-ship in a given period. Using the time-specific choice functions, the cumulative offer process of Hatfield and Milgrom (2005)

[21] See the criteria for the allocation of trainee-ships, Oberlandesgericht München, (2015).

[22] Other related papers are Hatfield and Kojima (2010), Kominers and Sönmez (2016), Sönmez (2013) and Sönmez and Switzer (2013).

[23] In the district of the regional court of appeals in Dusseldorf, it is explicitly stated that a higher waiting time does not affect the allocation to a desired court (Oberlandesgericht Düsseldorf (2015b)).

is used to find stable allocations. Extending beyond current results, we can show the existence of a lawyer-optimal stable allocation, when lawyers prefer earlier assignments. In cases where lawyers' preferences are unrestricted, no such lawyer-optimal stable allocation need exist.

The time-specific choice function does not satisfy some properties used in the previous literature. Notably it fails to satisfy the unilateral substitutes condition and the law of aggregate demand. Hence we cannot use the results of Hatfield and Kojima (2009) and Hatfield and Kojima (2010). We instead apply the results of Kominers and Sönmez (2016) to show that the time-specific lawyer proposing mechanism is (group) strategy-proof for the lawyers. Moreover, this mechanism is weakly Pareto efficient, eliminates justified envy and respects improvements. Furthermore, our mechanism creates incentives for all lawyers to report verifiable information increasing their priority at a court. However, it may allocate some lawyers to later positions while leaving some currently available positions unfilled. It thus allows current lawyers to obtain better positions at the expense of future lawyers.

We consider another modified version of the matching with contracts model, in which we no longer have time-specific constraints for each court. Instead, courts face only aggregate capacity constraints and are able to shift their positions flexibly over time. This would be applicable if courts had control over their own budgets over a period of some years. We construct the flexible choice function for courts, based on the time-specific choice function. The resulting flexible lawyer-optimal stable mechanism (FLOSM) is (group) strategy-proof, weakly Pareto efficient, while eliminating justified envy and respecting improvements. Furthermore it Pareto dominates the allocation obtained when time-specific capacity constraints need to be respected. It may however violate the time-specific capacity constraints of the courts.

While our model has been developed with the entry-level labor market for lawyers in Germany in mind, there are potentially many more applications of the basic framework. For example, university admissions in Germany for some very competitive courses, such as medicine, often ration places by putting unsuccessful applicants on waiting lists. A certain fraction of all seats is then reserved for those applicants who have waited a sufficient number of periods. Another potential application concerns the allocation of aspiring teachers to teaching traineeship positions at schools, in a system very similar to that of lawyers. The main difference to the market for lawyers is that teachers differ based on their chosen subjects, so that schools' preferences over teachers will be more complex than courts' priorities over lawyers. In addition schools are likely to be strategic players, unlike the courts. A position for math and physics teacher could for example be filled either by one teacher for both subjects or by two teachers each responsible for one of the subjects. Further interesting applications of matching with waiting times are (social or student) house allocation problems. For example, if there are a number of different projects to construct social housing that finished at different, known points in time then our model could be directly applied.

The remainder of this paper is organized as follows. In Section 2 we discuss the relevant literature. The model and some definitions are introduced in Section 3. Using our model, in Section 4 we analyze the currently applied Berlin mechanism and its properties. In Section 5 we propose mechanisms based on matching with contracts. Section 6 concludes. The Appendix contains proofs that are not in the main text.

2 Literature

This paper fits into the research agenda started by Gale and Shapley (1962) on two-sided matching. For a summary of research in this vein until 1990, see Roth and Sotomayor (1990). Two-sided matching has found important applications in the design of labor markets. For examples of the application of two-sided matching to medical entry-level labor markets see Roth (1984), Roth (1991) or Roth and Peranson (1999). More recently a number of papers have applied the original two-sided matching problem to the allocation of seats at universities, for instance Balinski and Sönmez (1999) and, more prominently, to the design of school choice mechanisms (Abdulkadiroğlu and Sönmez, 2003; Abdulkadiroğlu, Pathak, Roth and Sönmez, 2005a; Abdulkadiroğlu, Pathak and Roth, 2005b).

The canonical model of matching with contracts is due to Hatfield and Milgrom (2005),[24] which was later extended by Hatfield and Kojima (2008), Hatfield and Kojima (2009), Hatfield and Kojima (2010) and Hatfield, Kominers,

[24]Fleiner (2003) uses a similar fixed-point approach to find stable matchings.

Nichifor, Ostrovsky and Westkamp (2015a). Some early precursors of this type of model are Crawford and Knoer (1981) and Kelso and Crawford (1982).

In two recent contributions by Sönmez and Switzer (2013) and Sönmez (2013) new applications of the matching with contracts model to the allocation of cadets to branches of the US Army are introduced. Their treatment relies heavily on the recent result of the literature on matching with contracts and shows their practical relevance. In their case the number of years a cadet commits to serve in the army is the contract term. In our model the time at which a lawyer starts her trainee-ship is the contract term. Our work is closely related to and makes use of results in Kominers and Sönmez (2016) who study a more general slot-specific matching with contracts model. We use their results to show (group) strategy-proofness and respect of improvements for our preferred mechanism, the time-specific lawyer offering stable mechanism. We additionally show the existence of a lawyer-optimal stable mechanism by assuming weak impatience for the lawyers.[25] In recent work, Aygun and Turhan (2016) study dynamic reserves in Indian engineering school admission, where some seats might remain unfilled due to affirmative action reserves. Also using the matching with contracts framework and a new choice function for schools, the authors employ privilege types as contractual terms. In our paper we consider the time dimension, giving rise to additional dynamic properties.

There are several papers considering dynamic matching models. Leshno (2015) considers a queuing model in which agents are of two (privately known) types and can be assigned to one of two objects. There is overload in the sense that there are many agents waiting to be assigned an object. This model differs from ours in that the arrival of objects is random, whereas in our model it is known. Furthermore Leshno (2015) assumes that waiting is equally costly for agents, whereas in our model agents differ in time preferences.

Thakral (2015) studies a model similar to Leshno (2015) and ours, where agents are assigned to public housing. In his model houses arrive stochastically over time due to existing tenants moving out of public housing at their discretion. He assumes that agents are weakly impatient in the sense that being assigned public housing earlier is preferred to it being assigned later. Assuming

[25] An alternative route towards our results can be found in Hatfield, Kominers and Westkamp (2015b) who provide conditions for cumulative offer processes to yield stable and strategy-proof mechanisms.

that there is a common ordering of the houses over the agents, he introduces a strategy-proof mechanism that eliminates justified envy and is efficient evaluated at a particular point in time. If the realization of the house arrival process in the model of Thakral (2015) were known, it would correspond to our model in which the available positions in the future are known.

Kadam and Kotowski (2015) consider a two-sided matching model in which agents may have different partners over time. Their model set-up could be formulated in terms of a matching with waiting time model as we propose. The difference to our model is that both sides of the market would be allowed to sign multiple contracts even if all agents can only be matched to one other agent in a given period of time. In addition, they focus on different notions of stability.

Another related literature is the one on dynamic matching markets. Papers in that literature have, to our knowledge, not yet made use of the matching with contracts framework. Damiano and Lam (2005) consider one-to-one matching markets which are repeated over time. Here the outcome is a matching associating one man to a woman for each period. Similarly, Kurino (2009) considers one-to-one repeated matching markets. The focus in the latter paper is on a new notion of credible group-stable dynamic matchings. The paper by Bloch and Houy (2012) considers the allocation of a set of durable objects to agents who successively arrive and live for two periods. Related, Kurino (2014) considers a dynamic house allocation problem in which agents arrive successively and live for two periods. Abdulkadiroğlu and Loertscher (2007) also consider a dynamic house allocation problem. That paper compares static and dynamic mechanisms, finding that the latter can improve welfare upon the former. Another market design application of dynamic matching problems is Kennes, Monte and Tumennasan (2014) who consider the allocation of small children to daycare facilities in Denmark. Our paper differs from these papers insofar as in our paper the outcome is a set of contracts in which each lawyer appears only once, so no lawyer is matched repeatedly. Also, unlike the previous papers we make explicit use of the matching with contracts literature, which might also be fruitfully applied in the papers just mentioned. To apply the matching with contracts framework one would simply need to allow lawyers to hold multiple contracts.

This paper is also related to some papers within the theory of matching which analyze different legal entry-level labor markets. Avery et al. (2001) provide empirical data and discuss possible reconstructions of the market for legal clerkships at US federal courts for graduating law students, primarily addressing the unraveling problem. In Avery et al. (2007), the authors describe the unraveling in this market and relate to the problem of exploding offers. Haruvy et al. (2006) also study dynamics and unraveling inefficiencies of law clerk matching, using experimental and computational investigations to evaluate proposed reforms to the US system. Notably, this market is a decentralized one with no central authority designing an allocation procedure. Additionally, there is some conflict among the judges which prevents an effective coordination to improve the system. In contrast, the market for legal trainee-ships in Germany is centralized within districts of regional courts of appeals. While unraveling does not appear to happen in the allocation of lawyers in Germany, congestion is an important issue. Our paper is thus also related to common themes of the literature on markets suffering various defects (Roth and Xing, 1994; Niederle and Roth, 2003, 2009) and on how to improve the design of markets to overcome these defects (Roth and Peranson, 1999).

Two further related papers are Schummer and Vohra (2013) and Schummer and Abizada (2015). The former paper considers the assignment of landing slots to planes in the event of adverse weather. It shows the lack of incentives to report truthfully the estimated arrival times for flights under the currently used mechanism and proposes a strategy-proof alternative. That paper also highlights the restrictions that notions of incentive compatibility impose on the efficiency of the resulting mechanisms. The landing slot allocation problem as studied in those papers also differs from the lawyer allocation problem studied here. First, the paper assumes that all future arrival times are known by the airlines at the time an allocation is made. Second, the airlines have homogeneous preferences for early arrival at a single airport. So unlike in the present paper, there is only one good to be allocated in any time period.

Last, this paper is also related to other papers analyzing allocation systems in which some participants need to wait before being allocated. Braun et al. (2010) and Westkamp (2013) both study the mechanism used to allocate medical students to universities, where waiting times can be several years. However their models of the allocation procedure are static in the sense that they consider allocations for only one time period.

3 Model

This section introduces the lawyer-court many-to-one matching with waiting time problem. We abstract from complications arising from the fact that lawyers arrive sequentially over time and focus on the case in which a given set of lawyers is to be allocated to courts over several time periods. Each court can only accept a fixed number of lawyers per period.

The lawyer assignment problem consists of the following components:

1. a finite set of periods $T = \{1, ..., t_{max}\}$
2. a finite set of lawyers $I = \{i_1, ..., i_n\}$
3. a finite set of courts $C = \{c_1, ..., c_m\}$
4. a matrix of court capacities $q = (q_{c,t})_{c \in C, t \in T}$
5. lawyers' (strict, rational) preferences $P = (P_i)_{i \in I}$ over $C \times T \cup \{\emptyset\}$, with R_i denoting weak preferences of lawyer i.[26] The domain of preference profiles is denoted $\mathscr{P}$.
6. a list of courts' priority rankings, $\succ = (\succ_c)_{c \in C}$ over I.[27]

We call $(T, I, C, q, P, \succ)$ an instance of a lawyer-court matching with waiting time problem. A **contract** is a triplet $x = (i, c, t) \in I \times C \times T$, specifying a lawyer, a court and the time at which the lawyer begins her trainee-ship at the court. Let $X \subseteq I \times C \times T$ be the set of all feasible contracts. For contract $x = (i, c, t)$ we denote by x_I the lawyer appearing in x, i.e. $x_I = i$. Similarly we denote by x_C and x_T the court and the time period of assignment appearing in contract x, i.e. $x_C = c$ and $x_T = t$. Further, let Y_I be the set of lawyers appearing in some set of contracts $Y \subseteq X$, that is $Y_I = \{i \in I \mid \exists y \in Y s.t. y_I = i\}$.

A subset of contracts $Y \subseteq X$ is an **allocation** if for all $i \in I$, $|\{y \in Y : y_I = i\}| \in \{0, 1\}$ and for all $c \in C$ and $t \in T$, $|\{y \in Y : y_C = c\}| \leq \sum_t q_{c,t}$. In words, an allocation is a set of contracts such that no lawyer appears more than once and there are not more contracts of a court for some period than number of positions

[26] This means that $(c,t) R_i (c',t')$ if and only if either $(c,t) P_i (c',t')$ or $(c,t) = (c',t')$.

[27] These can be thought of as a single score as a function of a lawyer's grade, waiting time and social factors, such as place of birth, current residence or place of study. Since we consider a static setting, we will not consider how these priority rankings might change.

available at that court overall. An allocation $Y \subseteq X$ is **feasible** if for all $c \in C$ and $t \in T$, $|\{y \in Y : y_C = c, y_T = t| \leq q_{c,t}$. Hence an allocation is feasible if each court respects its time-specific capacity constraint for each period.[28] Let $\tilde{X}$ be the set of feasible allocations. For a subset of contracts Y denote by $Y(j)$ the subset of contracts in Y involving agent $j \in I \cup C$ alternatively, if j has no contract in Y then $Y(j)$ is the empty set. Furthermore if Y is an allocation and $j \in I$, let $Y_T(j)$ be the time of start of trainee-ship according to j's contract in Y. We define $Y_C(j)$ accordingly.

A contract x is **acceptable** to lawyer i if $xP_i\emptyset$. We suppose that within the set of courts C there is a court c^G such that $q_{c^G,t} = 0$ for all $t \in T$ and where $\succ_{c^G} = \succ_G$ is the (weak) ranking induced by the lawyers' grades. Similarly we denote by c^W the empty court with a ranking induced by waiting times, $\succ_W$, and by c^S the empty court inducing a ranking by social hardship, $\succ_S$. Note that this modeling choice is not appropriate for all federal states. For example, Hamburg uses a single score to determine which lawyers are allocated.

There are two possible interpretations of our model, the myopic and the fully dynamic interpretation. Under the fully dynamic interpretation, akin to models with overlapping generations of agents, we suppose that in the initial period $t = 1$ it is already determined how many future agents there are, when they "arrive",[29] what their preferences are and how they are ranked. Over short horizons this may be a realistic possibility. However as the horizon that one considers grows, this becomes increasingly unrealistic, especially since lawyers typically only take their state exams in the period before they start applying for positions.

Under the myopic interpretation one only considers the problem of allocating lawyers from a single generation to courts. In that interpretation we abstract away from future generations of lawyers arriving.[30] In the myopic view the capacities of the courts beyond the current period should then not be interpreted as actual physical capacity, but as capacity which has not been reserved for future generations of lawyers. The myopic interpretation ignores the uncertainty involved in deciding how to allocate lawyers when the number and preferences

[28] Note that capacity used in one period does not affect capacity in future periods.

[29] A lawyer arrives in period t if all contracts involving an earlier period of allocation are unacceptable to the lawyer.

[30] We do however incorporate some concern for future generations by considering a basic notion of limiting harm to future generations - early filling, which we define in Subsection 3.1.

of future lawyers are not yet known.

We denote by P_i not only preferences over i's assignment of a court and a time period, but also i's preferences over allocations. These preferences over allocations reflect i's preferences over assignments, so there should be no loss of clarity in this abuse of notation.

A **direct mechanism** ψ is a function $\psi : \mathscr{P} \to \tilde{X}$.[31] Hence ψ associates to each (reported) preference profile an allocation. Note that we treat courts as objects and hence they do not behave strategically, i.e. their priorities over lawyers are assumed to be given. We also take waiting time as given and do not consider changes in priorities arising from accumulated waiting time.

We now describe a few properties that lawyer preferences over the courts and the time of allocation can satisfy.

Definition. *Preferences of lawyer $i \in I$ are* ***weakly impatient*** *if for all $c \in C$, $t, t' \in T$ such that $t < t'$, then $(c,t) R_i (c,t')$.*

A lawyer's preferences are weakly impatient if a lawyer prefers to be allocated an early position at some court to a later position at the same court.

Definition. *Preferences of lawyer $i \in I$ are* ***strictly impatient*** *if for all $c, \tilde{c} \in C$ $t, t' \in T$ such that $t < t'$, then $(c,t) R_i (\tilde{c},t')$.*

Strict impatience is a strengthening of weak impatience. A lawyer having strictly impatient preferences prefers an early position at any court to a later position at *any* court. In practice we do not expect all lawyers' preferences to be strictly impatient. The reason is that there are some regions, e.g. Saxony-Anhalt, in which the average waiting time is zero, while in other regions the average waiting time is strictly positive. This would not be observed if lawyers' preferences were strictly impatient, since in that case those waiting for a position in a desirable region could just switch to a less desirable region without a waiting time and thereby be better off. In addition many of the forms filled in by lawyers when applying for a position allow them to indicate a preferred entry date, which may differ from the next possible starting date.[32] It appears that

[31]In full generality the mechanism should also depend on $(T, I, C, q, \succ)$. We suppress this dependence for simplicity but will highlight whenever it becomes relevant, for example when comparing the outcome of some mechanism when a court's ranking of the lawyers has changed.

[32]See the application form for trainee-ships in the district of the Dusseldorf regional court of appeals, Oberlandesgericht Düsseldorf (2015a)

many lawyers make use of the ability to postpone their starting date.[33] While delay of applicants may be for strategic reasons, delays may be rational if the lawyer plans to obtain an additional qualification, such as a one-year law degree, in the time up to the entry date. Hence it is not clear that lawyers' preferences are either weakly or strictly impatient.

3.1 Properties of Allocations and Mechanisms

To analyze the outcome of different mechanisms it is necessary to be able to talk about properties of allocations. A basic requirement of an allocation is that no lawyer should prefer the outside option to the court that she has been assigned:

Definition. *An allocation $Y \subseteq X$ is **individually rational** if for all $i \in I$, $Y(i) R_i \emptyset$. A mechanism ψ is individually rational if $\psi(P)$ is an individually rational allocation.*

Another basic requirement that any mechanism should satisfy is that it only outputs feasible allocations.

Definition. *A mechanism ψ is **feasible** if $\psi(P)$ is a feasible allocation for all $P \in \mathscr{P}$.*

We next introduce a common notion of fairness:

Definition. *An allocation $Y \subseteq X$ has **no justified envy**, if for any pair of contracts $x, y \in Y$ with $x_I \neq y_I$ and $(x_C, x_T) P_{y_I} (y_C, y_T)$, one of the following conditions holds: $x_I \succ_{x_C} y_I$, $x_I \succ_G y_I$, $x_I \succ_W y_I$ or $x_I \succ_S y_I$. A mechanism ψ **eliminates justified envy** if its outcome $\psi(P)$ has no justified envy for all $P \in \mathscr{P}$.*

An allocation thus is envy-free if, whenever a lawyer prefers some other lawyers' assignment, then that lawyer must have a higher priority at the court she is being assigned to than the former lawyer, a better grade, more waiting time or a higher priority based on social hardship criteria. In standard notions of fairness, usually only the court's priorities are considered. Since the policy maker in our case explicitly uses these other rankings to determine allocations,

[33] See the weighted list of applicants for positions in Hamburg, Hanseatisches Oberlandesgericht (2015). It can be seen that many lawyers have asked their entry date to be postponed for several months.

it appears natural to modify the standard notion of lack of justified envy to incorporate these additional concerns.

The following definition of Pareto dominance is standard.[34]

Definition. *An allocation $Y \subseteq X$ **Pareto dominates** another allocation $\tilde{Y} \subseteq X$ if for all $i \in I$ $Y(i) R_i \tilde{Y}(i)$ and there exists at least one $i \in I$ such that $Y(i) P_i \tilde{Y}(i)$. A mechanism ψ Pareto dominates another mechanism $\tilde{\psi}$ if for all $P \in \mathscr{P}$ $\psi(P)$ Pareto dominates $\tilde{\psi}(P)$.*

It is standard to define Pareto efficiency of an allocation by the absence of another allocation that Pareto dominates it. None of the mechanisms that we study in this paper satisfy this requirement. We thus consider a weaker notion of efficiency:

Definition. *An allocation $Y \subseteq X$ is **weakly Pareto efficient** if there does not exist an individually rational allocation $\tilde{Y} \subseteq X$ such that for all $i \in I$ $\tilde{Y}(i) P_i Y(i)$. A mechanism ψ is weakly Pareto efficient if for all $P \in \mathscr{P}$ $\psi(P)$ is weakly Pareto efficient.*

As usual, a mechanism is strategy-proof if it is a dominant strategy for each agent to truthfully report her preferences to the mechanism:

Definition. *Mechanism ψ is **strategy-proof** if for all $i \in I$, for all $P \in \mathscr{P}$ and for all $\tilde{P}_i \in \mathscr{P}_i$ we have $\psi(P) R_i \psi(\tilde{P}_i, P_{-i})$. Mechanism ψ is **group strategy-proof** if, for any preference profile $P \in \mathscr{P}$, there is no $\tilde{I} \subseteq I$ and $\tilde{P}_{\tilde{I}} = (\tilde{P}_i)_{i \in \tilde{I}}$ such that for all $i \in \tilde{I}$ we have $\psi(\tilde{P}_{\tilde{I}}, P_{-\tilde{I}}) P_i \psi(P)$.*

We next define respect of improvements, first used in the matching literature by Balinski and Sönmez (1999).[35] What that property means is that a lawyer should not receive a worse assignment when her priority has increased at the courts. First we need to define what we mean by an improvement in the priority

[34]Pareto efficiency is only defined with respect to the lawyers' preferences. This is justified by the fact that the courts' priorities are set administratively and therefore do not constitute real preferences. Instead they reflect a desire on by policy-makers to take into consideration grades, waiting time and social criteria. The literature on school choice similarly considers only the preferences of students for Pareto efficiency and treats schools as objects to be allocated (see Abdulkadiroğlu and Sönmez (2003)).

[35]An alternative name for respect of improvements could be priority monotonicity, since it requires that the rank of the outcome achieved by a lawyer is monotone in priority profile improvements.

of a lawyer. In doing so, we will follow closely the presentation in Sönmez (2013).

Definition. *A priority profile $\succ$ is an* ***unambiguous improvement*** *over another priority profile $\succ'$ for lawyer i if:*

- *the ranking of i is at least as good under $\succ$ as under $\succ'$ for any court c,*

- *the ranking of i is strictly better under $\succ$ than under $\succ'$ for some court c,*

- *the relative ranking of other lawyers is the same under $\succ$ and $\succ'$ for any court*

Intuitively, a priority profile improvement of some lawyer means that while all other lawyers' relative rankings among the courts are unchanged, the particular lawyer's ranking is not worse at any court (i.e. there are at most as many lawyers ranked higher than the lawyer as before) and the lawyer's ranking has improved at least at one court. Note that priority profile improvements include improvements in grades, waiting time and social hardship criteria.

Definition. *A mechanism ψ* ***respects improvements*** *if a lawyer never receives a strictly worse assignment as a result of an unambiguous improvement in her court priorities.*

Respect of improvements is a natural property to ask for. Suppose that a better grade for a lawyer leads to an unambiguous improvement in that lawyer's ranking. If respect of improvements did not hold, the lawyer would have received a less preferred position than with the worse grade. This would run counter to the view that law students should be rewarded for good performance in the exams. In addition, some may consider it to be unjust that lawyers obtain a better outcome for themselves despite having a worse grade, compared to another lawyer. Similar arguments can be made for why a mechanism should respect improvements in waiting time and social criteria.

More important, perhaps, is the implicit reliance of existing procedures on waiting time in ranking lawyers. Suppose that under some specified mechanism a lawyer improves her ranking by arriving earlier, then, if the mechanism tries to aid lawyers who arrive early by improving their ranking, this attempt to increase the welfare will hurt those lawyers if the overall mechanism does not respect improvements.

We next formalize the notion that whenever a position is not filled in some period, then no agent who would have been available that period should be assigned later. It seems reasonable to suppose that policy-makers would not be willing to allow some place at a court to go unfilled just to allow a current applicant to obtain a better allocation. This is first because lawyers provide essential work to the court at the time of their trainee-ship and second because in this way more future slots are left open which makes future lawyers (weakly) better off.[36]

Definition. *An allocation $Y \subseteq X$ satisfies* ***early filling*** *if there is no $t \in T$ such that there exists some $c \in C$ such that $|\{y \in Y : y_T = t, y_C = c\}| < q_{c,t}$ and there exists some $i \in I$ such that $Y_T(i) > t$. A mechanism ψ satisfies early filling if for all $P \in \mathscr{P}$, $\psi(P)$ satisfies early filling.*

Early filling appears similar in flavor to the notion of no wastefulness, which is defined as:

Definition. *An allocation $Y \subseteq X$ is* ***wasteful*** *if there exists a time t, a court c and a lawyer i such that $|\{y \in Y : y_T = t, y_C = c\}| < q_{c,t}$, $Y(i) = \emptyset$ and $(c,t) P_i \emptyset$.*

Note that a Pareto efficient allocation is automatically non-wasteful. The following example show that non-wasteful and early filling are logically independent properties:

Example 1. *There are three lawyers i_1, i_2, i_3, two courts c_1, c_2 and two time periods $t = 1, 2$. Each court has a unit of capacity in each period. All contracts are acceptable to all lawyers. The allocation $\{(i_1, c_1, 2), (i_2, c_2, 2), (i_3, c_1, 1)\}$ satisfies non-wastefulness but violates early filling. The allocation $\{(i_1, \emptyset), (i_2, c_1, 1), (i_3, c_2, 1)\}$ satisfies early filling but is wasteful.*

In fact there is a fundamental conflict between early filling and non-wastefulness if it is additionally required that allocations are acceptable to the lawyers.

[36] The notion of early filling requires that if positions are not taken in an early period, then no agent should be allocated in a later period. It thus makes sense to require early filling only if one interprets our model as involving a single cohort of students, rather than overlapping cohorts. In an extended dynamic setting one should amend the definition of early filling to allow positions to be empty even if a lawyer from a later generation takes a position at a later time. Early filling would then only rule out lawyers from the cohort appearing at a time t to take positions after that period if there are empty slots in t.

Lemma 1. *There is no mechanism that is individually rational, non-wasteful and satisfies early filling.*

Proof. Suppose ψ is an individually rational, non-wasteful mechanism. We show that there is an instance of a lawyer-court matching with waiting times problem in which this mechanism necessarily violates early filling. Consider the following example. There are three lawyers i_1, i_2, i_3, two courts c_1, c_2 and two time periods $t = 1, 2$. Each court has a unit of capacity in each period. The only acceptable contracts are: $\{(i_1, c_1, 2), (i_2, c_2, 2), (i_3, c_1, 1)\}$. Then $Y' = \{(i_1, c_1, 2), (i_2, c_2, 2), (i_3, c_1, 1)\}$ is the unique individually rational and non-wasteful allocation, which does not satisfy early-filling. □

In the proof above, both individual rationality and non-wastefulness are required. Without individual rationality, one of the lawyers assigned in period $t = 2$ could have been assigned to an (unacceptable) earlier position. Non-wastefulness is required, since otherwise both lawyers allocated in period $t = 2$ could have been left unassigned.

Usually non-wastefulness is one of the most basic desirable properties that a matching mechanism has to possess. In our application, not being assigned in a particular federal state however likely is the result of having been accepted elsewhere. Therefore, not being assigned appears to us not to harm lawyers to an excessive extent since with a high probability they were accepted elsewhere. Failing to satisfy early-filling can however have a detrimental effect on future generations of lawyers.

4 Berlin Mechanism

We now study the procedure that is currently used in Germany to allocate lawyers to courts, mostly adopting the myopic interpretation of our model. Some aspects of that procedure are reasonably well documented, however the part describing how lawyers are allocated to courts within a period is not. While reported preferences, capacities and priorities are to be taken into account, there is no description of how these are used to find the allocation within a period. Another complication is that lawyers have many strategic options, in addition to reporting preferences over courts. For example they can decide for what entry date they wish to apply. They can refuse to accept an offer that has been made.

They can report verifiable information about social status and other information that affects the priorities they will have. Because of this complexity we decide to model the procedure in a stylized manner that captures the most important features shared by the different allocation procedures as discussed and referenced in the Introduction.

The Berlin mechanism is a **two-stage procedure**. Lawyers are only able to report a ranking over the courts and fix a particular entry date to which we suppose the lawyers have applied.[37] In each given time period, the first stage of the procedure determines the set of lawyers to be considered in this time period, while in the second stage these considered lawyers are matched to open court positions.

The **first-stage lawyer selection procedure** in a given period is often detailed in the relevant regulations, as discussed earlier. This lawyer selection procedure can vary across federal states (see observations in the Introduction), nevertheless in terms of our results, these details do not matter. The important point that the lawyer selection procedure satisfies, is that it selects lawyers based solely on observable characteristics such as grades, waiting time and social criteria while ignoring preferences of the lawyers. We describe here a stylized lawyer selection procedure, which takes as input λ_G, λ_W and λ_S, which are respectively the share of positions to be assigned to lawyers based on grade, waiting time and social hardship criteria. Let $Q^t = \sum_{c=1}^{|C|} q_{c,t}$ be the total capacity of the courts in period t. For period t select the $\lfloor \lambda_S Q^t \rfloor$ lawyers ranked highest according to $\succ_S$.[38] Next, select the $\lfloor \lambda_G Q^t \rfloor$ lawyers ranked highest according to $\succ_G$. Finally, select the $Q^t - \lfloor \lambda_S Q^t \rfloor - \lfloor \lambda_G Q^t \rfloor$ lawyers ranked highest according to $\succ_W$.[39]

For the **second-stage allocation of lawyers to courts** within a given period we make the assumption that the lawyer-proposing deferred-acceptance algo-

[37]In Subsection 4.3 we study the question whether lawyers can strategically delay their application.

[38]We define $\lfloor x \rfloor$ to be the largest integer below x.

[39]Note that $Q^t - \lfloor \lambda_S Q^t \rfloor - \lfloor \lambda_G Q^t \rfloor$ equals $\lfloor \lambda_W Q^t \rfloor$, thereby ensuring that a total of Q^t lawyers gets selected.

rithm of Gale and Shapley (1962) is used.[40] The DA algorithm works as follows, taking as input lawyers' reported preferences and courts' priority rankings over lawyers:

- **Step 1:** Each lawyer applies to her preferred court. Each court considers all applicants and tentatively accepts the ones it ranks highest up to its capacity. All others are rejected.

- **Step k:** Any lawyer who was rejected by a court in the previous step applies to her next most preferred acceptable court or, if all acceptable courts have already rejected her, she is assigned the outside option. Each court considers applicants it tentatively holds from the last step and those who applied in step k and tentatively accepts the ones it ranks highest up to its capacity. All others are rejected.

The DA algorithm eventually stops with all lawyers either assigned to a court or the outside option. We summarize below a number of properties of the DA mechanism that we will use throughout the paper.[41]

Theorem. *(Dubins and Freedman, 1981; Roth, 1982) The Lawyer-Proposing Deferred-Acceptance mechanism is strategy-proof.*

Theorem. *(Balinski and Sönmez, 1999) The Lawyer-Proposing Deferred-Acceptance mechanism lacks justified envy and respects improvements.*

Given the lawyer selection procedure and the DA algorithm, the **Berlin mechanism** proceeds as follows, for ascending integers $t = 1, ..., t_{max}$:[42]

[40] If an unstable mechanism were used instead of the lawyer-proposing DA, then we would immediately have the result that the Berlin mechanism cannot simultaneously satisfy individual rationality, respect of improvements, non-wastefulness and lack of justified envy (see Balinski and Sönmez (1999)). Making this assumption allows us to conclude that any deficiencies we find are likely the result of the way the Berlin mechanism determines the time at which a lawyer is allocated to a court. Consequently, this can be considered as the "most conservative" assumption.

[41] Note that when discussing the elimination of justified envy under the DA mechanism we refer to the standard definition of justified envy, which does depend on the rankings over grade, waiting time and social criteria.

[42] We consider a stylized version of the Berlin mechanism. It does not allow lawyers who could not be assigned at some period to later be allocated. Instead such lawyers are assigned the outside option. We make this assumption here for simplicity. In practice, such lawyers may be considered again by the mechanism in later rounds, with the caveat that they will not gain waiting time following a rejection.

First stage

- **Step t.a:** Select up to Q^t lawyers from the set of lawyers that so far have not been selected, according to the lawyer selection procedure.

Second stage

- **Step t.b:** Selected lawyers submit preferences over courts.[43] Apply the DA algorithm using submitted preferences of the lawyers who have so far been selected in period t and on the courts' priorities. Assign each lawyer to the court assigned under this algorithm. If there are lawyers that were assigned the outside option, go to step $t.c$. If there are no lawyers that were assigned the outside option in this step, go to step $(t+1).a$ or if $t = t_{max}$, end the procedure with all those who were not yet assigned a court being assigned the outside option.

- **Step t.c:** Select as many additional lawyers from those not yet selected as there are unassigned lawyers resulting in Step $t.b$. Repeat Step $t.b$ with those lawyers additionally selected and those that were assigned to a court before.

Intuitively the Berlin mechanism tries to allocate lawyers to the earliest possible period using the DA algorithm. If there are more lawyers than seats at courts for the earliest period there is a first step that determines the set of lawyers to be allocated to courts at the earliest date based on grades, waiting time and social criteria. If at some point a lawyer is allocated to the outside option the mechanism selects an additional lawyer to be allocated in the earliest possible period. Once all positions in the earliest possible period have been filled, the same process is repeated for the subsequent period. From the description of the

[43] In practice lawyers submit their ranking over courts the first time they apply for a position. However it is conceivable that lawyers might contact the *regional court of appeal* to change those submitted preferences. However if such behavior is infrequent it is sensible to assume that preferences over courts are submitted only once by the lawyers. Some federal states explicitly allow lawyers to change their ranking over courts until a position has been offered to them. Allowing lawyers to submit their ranking over courts after the time at which they are allocated has been determined simplifies the strategic analysis of the Berlin mechanism. Given that DA is strategy-proof for lawyers, they will have an incentive to report their ranking over courts in a way that is consistent with their preferences over courts and time, by reporting possibly different rankings over courts for different periods.

Berlin mechanism it follows that the allocation it produces is feasible and that no lawyer receives an allocation that is worse than the outside option.

Lemma 2. *The Berlin mechanism is individually rational and feasible.*

In addition to the previous description of the allocation procedure, there are some peculiarities that may affect its performance, which for now we abstract from in the following theoretical discussion. First, truncated court preferences: In some federal states only two more courts in addition to the most-preferred one can be reported (sometimes with no ordering possible) and if the lawyer is not allocated to any of these three, then her preference list is randomly filled with non-listed courts.[44] Second, endogenous court priorities: lawyers can report a *verifiable* special social connection to some courts, e.g. a spouse or other relatives living in that region etc., leading to higher priority at that court. Third, refusals to accept positions: lawyers are informed of their allocated court, but they can refuse to accept that position. Refusing lawyers are replaced by those still on the waiting list. Usually, refusals lead to non-accrual of waiting time.

4.1 Deficiencies of the Berlin Mechanism

The algorithm as currently used has a number of flaws, mainly associated to the fact that t-preferences are not considered when determining which lawyers are to be allocated in a given time period. While lawyers are able to report different rankings over courts for different periods, any information concerning the trade-off between waiting and obtaining a better court is not used by the mechanism. This has important implications for the efficiency of the mechanism.

Proposition 1. *The Berlin mechanism is not weakly Pareto efficient.*

Proof. Consider the following example.

Example 2. $C = \{c_1\}$, $I = \{i_1, i_2\}$, $T = \{1, 2\}$, $q_{c_1,1} = q_{c_1,2} = 1$ *and* $i_1 \succ_{c_1} i_2$. *But lawyers arrive in the first period and their preferences are* $(c_1, 2) P_{i_1} (c_1, 1)$ *and* $(c_1, 1) P_{i_2} (c_1, 2)$. *Due to the two-stage procedure the higher ranked lawyer* i_1 *is considered for the first period, reports* c_1 *as acceptable and is allocated.*

[44] It is well known that the DA mechanism is more manipulable if lawyers report a ranking over only k courts than if lawyers report a ranking over $k' > k$ courts, see Pathak and Sönmez (2013).

In period two i_2 is selected and allocated. Then, the outcome of the Berlin mechanism is $\{(i_1,c_1,1),(i_2,c_1,2)\}$, which is strictly worse for both lawyers than $\{(i_1,c_1,2),(i_2,c_1,1)\}$.

□

Lack of weak Pareto efficiency means that under the Berlin mechanism there could be situations under which every single lawyer could be made better off. In the above example the inefficiency stems from the fact that the exogenous lawyer selection procedure in effect determines the final allocation of lawyers to courts. Since this allocation does not depend on lawyers' preferences at all, it is not surprising that there are many lawyers that could be made better off. We have the following result.

Proposition 2. *The Berlin mechanism does not eliminate justified envy and does not respect improvements.*

Proof. Consider the following lawyer assignment problem.

Example 3.

[Berlin mechanism does not eliminate justified envy] There are two periods, $t = 1,2$. We have three lawyers $I = \{i_1,i_2,i_3\}$ and two courts $C = \{c_1,c_2\}$. $q_{c_1,1} = q_{c_1,2} = q_{c_2,1} = 1$ and $q_{c_2,2} = 0$. Court priorities are $i_1 \succ_c i_2 \succ_c i_3$ for all $c \in C$. Lawyer preferences are

$$\begin{aligned} i_1 &: (c_1,1)P_{i_1}(c_1,2)P_{i_1}(c_2,1) \\ i_2 &: (c_1,1)P_{i_2}(c_1,2)P_{i_2}(c_2,1) \\ i_3 &: (c_1,1)P_{i_3}(c_2,1)P_{i_3}(c_1,2). \end{aligned}$$

In period 1, in the first stage the two lawyers with highest priority (i_1 and i_2), regardless of their preferences, are selected to be allocated to the two open spots in the first period. Lawyer i_3 is put on hold, increases her waiting time, and will be reconsidered in the next period. In the second stage of period 1 lawyers can report their preferences considering only contracts for this time period $t = 1$. Based on these preferences, i_1 and i_2 are matched to their favorite courts, respecting their priority, and using the deferred acceptance mechanism. In period 2, there is only i_3 who is then allocated.

Therefore the Berlin mechanism produces the following (unique) outcome $X^{Berlin} = \{(i_1,c_1,1),(i_2,c_2,1),(i_3,c_1,2)\}$ for all $c \in C$. This outcome is not fair

since there exists justified envy of i_2*, i.e.* $(c_1,2)P_{i_2}(c_2,1)$*, although* $i_2 \succ_c i_3$.[45]
[Berlin mechanism does not respect improvements] Consider the previous set-up. If courts' priority orders are changed to $\succ'$*, s.t.* $i_1 \succ'_c i_3 \succ'_c i_2$*, then the resulting allocation under the Berlin mechanism is* $X^* = \{(i_1,c_1,1),(i_2,c_1,2),(i_3,c_2,1)\}$*. If* i_2 *improves, e.g. with a better grade, such that the old priority ranking,* $\succ$*, is recovered, then* X^{Berlin} *would result and* i_2 *would be worse off. Hence the algorithm does not respect improvements.*

□

Finally, we show that the Berlin mechanism may be wasteful.

Proposition 3. *The Berlin mechanism is wasteful.*

Proof. Consider the following lawyer assignment problem.

Example 4. *Suppose there are three lawyers* i_1, i_2 *and* i_3*, two courts* c_1, c_2 *and two time periods* $t = 1,2$*. Each court has one unit of capacity in each time period. All courts rank lawyers the same:* $i_1 \succ_c i_2 \succ_c i_3$*. Lawyers* i_1 *and* i_3 *are strictly impatient and prefer* c_1 *over* c_2 *in both periods. Lawyer* i_2 *is (strictly) impatient and finds only* c_1 *acceptable. Then the outcome of the Berlin mechanism under truth-telling is:* $Y' = \{(i_1,c_1,1),(i_2,\emptyset),(i_3,c_2,1)\}$*. Note that because* i_2 *does not find* $(c_1,1)$ *acceptable, he is assigned the outside option in* $t = 1$*. This is wasteful since court* c_1 *has an empty position in period* $t = 2$ *which lawyer* i_2 *prefers to being unassigned.*

□

Note that Proposition 3 is a corollary of Lemma 1 which shows that no individually rational mechanism is both non-wasteful and fills positions early and Proposition 6, which shows that the Berlin mechanism fills positions early.

4.2 Desirable Properties of the Berlin Mechanism

We have seen that the Berlin mechanism is not weakly Pareto efficient, does not eliminate justified envy, does not respect improvements and further is wasteful for general preferences. One question that could be considered is whether there

[45] Note that the allocation $X^* = \{(i_1,c_1,1),(i_2,c_1,2),(i_3,c_2,1)\}$ is preferred by i_2 and i_3 and weakly preferred by i_1 to X^{Berlin} and hence Pareto dominates it, despite equal courts' rankings of lawyers.

exists a class of preferences for which the Berlin mechanism eliminates justified envy and respects improvements. As it turns out for preferences which are *strictly impatient* the currently used allocation procedure always delivers an allocation without justified envy and respects improvements. This is summarized in the following proposition.

Proposition 4. *Suppose the preferences of each lawyer are strictly impatient and all contracts are acceptable to all lawyers and courts. Then the Berlin mechanism eliminates justified envy.*

Proof. Suppose first lawyers are strictly impatient, all contracts are acceptable to all lawyers and courts and that the Berlin mechanism does not eliminate justified envy. Then there exist lawyers i_1, i_2, contracts x, y resulting in the Berlin mechanism with $x_I = i_1$ and $y_I = i_2$ such that yP_ix and $i_1 \succ_{x_C} i_2$, $i_1 \succ_G i_2$, $i_1 \succ_W i_2$, and $i_1 \succ_S i_2$. Since all contracts are acceptable we have that $x \neq \emptyset$. Since i_1 is higher ranked than i_2 in terms of grade, waiting time and social criteria, the Berlin mechanism must yield contracts such that $y_T \geq x_T$. Strict impatience rules out that $y_T > x_T$, as otherwise i_1 would not prefer y to x. Thus we have $y_T = x_T$. But in period x_T the deferred-acceptance algorithm is used, which is known to eliminate justified envy. This contradicts $i_1 \succ_{x_C} i_2$. $\square$

Note that we need that all lawyer find every court acceptable. If this were not the case, it could happen that lawyer i is selected for a period earlier than lawyer j, but left unassigned. This can happen if there are some courts that lawyer i finds unacceptable and if j later obtains a position at a court that is acceptable. While this result is somewhat encouraging, one should note that in practice it is not obvious that lawyers have preferences that are strictly impatient and find all courts acceptable, which we have already argued does not appear likely.

Unfortunately the above logic cannot be used to show that under strict impatience and acceptability of all contracts, the Berlin mechanism respects improvements. To see this, consider the following example:

Example 5. $I = \{i_1, i_2, i_3, i_4\}$, $C = \{c_1, c_2\}$, $t = 1, 2$ *and* $q_{c_1,1} = q_{c_2,1} = q_{c_1,2} = q_{c_2,2} = 1$. *Preferences of the lawyers are given by* $(c_1, 1)P_i(c_2, 1)P_i(c_1, 2)P_i(c_2, 2)$ *for all* $i \in I$. *There are two priority profiles,* $\succ$

and $\tilde{\succ}$, given by:

$$\begin{array}{lcl} \succ_{c_1}: i_3, i_2, i_1, i_4 & & \tilde{\succ}_{c_1}: i_3, i_2, i_1, i_4 \\ \succ_{c_2}: i_1, i_2, i_3, i_4 & \text{and} & \tilde{\succ}_{c_2}: i_1, i_2, i_3, i_4 \\ \succ_{G}: i_1, i_2, i_3, i_4 & & \tilde{\succ}_{G}: i_2, i_1, i_3, i_4 \\ \succ_{W}: i_2, i_3, i_4, i_1 & & \tilde{\succ}_{W}: i_2, i_3, i_4, i_1 \end{array}$$

Note that the priorities only differ in that i_2 has a better grade under $\tilde{\succ}$. We consider a lawyer selection procedure with $\lambda_G = 0.5$ and $\lambda_W = 0.5$. As $\lambda_S = 0$ by implication, the lawyer selection procedure first selects the lawyer ranked highest according to grade and the highest ranked remaining lawyer according to waiting time. Hence under $\succ$ lawyers i_1 and i_2 are selected and lawyer i_2 gets $(c_1, 1)$. Under $\tilde{\succ}$ lawyers i_2 and i_3 are selected. So i_2 gets $(c_2, 1)$ under $\tilde{\succ}$, meaning that the improvement in the ranking of lawyer i_2 has made her worse off, despite the fact that all agents have strictly impatient preferences.

For the Berlin mechanism to respect improvements we need a further assumption: namely that a single ranking determines which lawyers are allocated for each period.

Proposition 5. *Suppose the preferences of each lawyer are strictly impatient, all contracts are acceptable to all lawyers and courts and $\lambda_G = 1$. Then the Berlin mechanism respects improvements.*

Proof. Let $\tilde{\succ}$ be an unambiguous improvement over $\succ$ for lawyer i and let $x, \tilde{x}$ be the respective assignments obtained under the Berlin mechanism. For a contradiction suppose $xP_i\tilde{x}$. There are three cases. First, suppose $x_T > \tilde{x}_T$. By the Berlin mechanism and all contracts being acceptable there is a constant number $Q_t = \sum_c q_{c,t}$ of agents allocated in period t, which are the $\sum_{s=1}^{t-1} Q_s + 1$ to $\sum_{s=1}^{t} Q_s$ highest ranked agents according to either $\succ_G$ or $\tilde{\succ}_G$. Since $\tilde{\succ}$ is an unambiguous improvement, we must have $\tilde{x}_T \geq x_T$, a contradiction. Second, suppose $\tilde{x} = \emptyset$. From $xP_i\tilde{x}$ it follows that $x \neq \emptyset$. But since all contracts are acceptable, i under $\tilde{\succ}_G$ cannot be ranked higher than $\sum_{s=1}^{t_{max}} Q_s$. But then it must be ranked even lower under $\succ_G$ implying that $x = \emptyset$, a contradiction. Third, suppose $x_T = \tilde{x}_T$. But the deferred-acceptance algorithm satisfies respect of improvements, which contradicts $xP_i\tilde{x}$. □

The Berlin mechanism in Step 1a selects Q_1 lawyers to be allocated via the deferred acceptance algorithm to positions in period $t = 1$. For each unfilled position, another lawyer is selected. Hence either the position will be filled and the algorithm moves to the next period or all remaining lawyers consider the unfilled position to be unacceptable. In the former case the algorithm fills all position for period $t = 1$. In the latter case it does not fill all positions in period $t = 1$ but all lawyers have been assigned to either a position or the outside option. Hence in that case no lawyer will be allocated to a later period. This argument can be extended to any subsequent period, so that either all positions for that period are filled or no positions in subsequent periods are filled. As a result, the final assignment obtained by the Berlin mechanism fills positions early. We summarize this finding in the following proposition.

Proposition 6. *The Berlin mechanism fills positions early.*

One of the deficiencies of the Berlin mechanism was that it sometimes wasted positions. If we require all lawyers to find all positions acceptable, then this is no longer the case.

Proposition 7. *If all agents find all contracts acceptable, then the Berlin mechanisms is non-wasteful.*

Proof. Let Y' be the outcome of the Berlin mechanism for some lawyer-court matching with waiting time problem. Suppose that some lawyer i is not assigned under the Berlin mechanism, but that there exists c,t such that $|\{y \in Y' | y_C = c, y_T = t\}| < q_{c,t}$. By assumption we have that $(c,t) P_i \emptyset$. Note that i cannot have been selected at a step $t' \geq t$, since the fact that there was an empty position at court c for time t implies that more lawyers would have been selected until all positions in period t were filled. In particular, i would have been selected eventually. But then, since i finds (c,t) acceptable, i would have been assigned to it in that step. Hence i must have been selected earlier. Furthermore, i cannot have been selected in step $t' < t$. If i had been selected, i would have been assigned since i finds all courts acceptable. Hence we have a contradiction. $\square$

4.3 Strategic Delay under the Berlin Mechanism

So far we have mainly adopted the myopic interpretation: there is a single cohort of lawyers who simultaneously apply for positions for their legal trainee-ship. This effectively assumes that all lawyers need to apply at the same time. However the Berlin mechanism may lead to incentives for strategically delaying an application. For simplicity, we again abstract away from future generations of lawyers, but allow lawyers to choose the time at which they submit their application. Note that the Berlin mechanism, by its nature, can accommodate agents submitting preferences at various points in time. We adapt the Berlin mechanism by inserting at the very beginning of the Berlin mechanism a step 0, in which each lawyer reports a desired starting time $\tau_i \in T$.

The difference to before is that in each period only those lawyers who wished to be allocated before or in that period are considered in the lawyer selection procedure. Under the Berlin mechanism with reports of starting time, the strategy of each lawyer is now a starting time τ_i as well as her preferences over courts for each period.

The following example shows that agents may have an incentive to delay submitting their preferences:

Proposition 8. *Under the Berlin mechanism, agents have incentives for delaying their application.*

Proof. Consider the following example.

Example 6. *There are two periods, $t = 1,2$. We have lawyers $I = \{i_1, i_2, i_3\}$. There are two courts, i.e. $C = \{c_1, c_2\}$. $q_{c_1,1} = q_{c_1,2} = q_{c_2,1} = 1$ and $q_{c_2,2} = 0$. Court (as well as grade, waiting time and social) priorities are $i_1 \succ_c i_2 \succ_c i_3$ for all $c \in C$. Lawyer preferences are*

$$\begin{aligned} i_1 &: \quad (c_1,1) P_{i_1} (c_1,2) P_{i_1} (c_2,1) \\ i_2 &: \quad (c_1,1) P_{i_2} (c_1,2) P_{i_2} (c_2,1) \\ i_3 &: \quad (c_1,1) P_{i_3} (c_2,1) P_{i_3} (c_1,2) \end{aligned}$$

If all lawyers submit their desired starting time $\tau_i = 1$, the resulting allocation is $\{(i_1,c_1,1), (i_2,c_2,1), (i_3,c_1,2)\}$. However if lawyer i_2 instead reports $\tau_2 = 2$, the outcome of the Berlin mechanism is $\{(i_1,c_1,1), (i_2,c_1,2), (i_3,c_2,1)\}$, which is preferred by lawyer i_2 to the outcome from applying in period $t = 1$. Therefore, lawyer i_2 has an incentive to delay her application.

$\square$

In practice, incentives for strategic delay may be muted by the (uncertain) arrival of future generations of lawyers. If there are sufficiently many highly ranked future generations of lawyers arriving in period $t = 2$, then by delaying her application, agent i_2 might not be assigned at all or later. The motivation of delaying the application in this example is for strategic reasons: it allows lawyer i_2 to obtain a more preferred allocation. In practice students might also wish to delay their entry date for non-strategic reasons. This could happen when they wish to do a PhD or a masters degree before starting their trainee-ship. In such cases the lawyers would have a preference of starting late.

Allowing lawyers to choose the time period in which they apply may alleviate some concerns regarding the negative properties of the Berlin mechanism. However the following example shows that there are equilibria under the Berlin mechanism that are not weakly Pareto efficient.

Proposition 9. *There are Nash equilibrium outcomes under the Berlin mechanism with strategic delay that are weakly Pareto inefficient.*

Proof. Consider the following example.

Example 7. $C = \{c_1, c_2\}$, $I = \{i_1, i_2\}$, $q_{c_1,1} = q_{c_2,2} = 1$, $q_{c_1,2} = q_{c_2,1} = 0$ *and* $i_1 \succ_{c_1} i_2$, $i_2 \succ_{c_2} i_1$ *and* $i_1 \succ_G i_2$ *and* $\lambda_G = 1$. *Preferences are:* $(c_2, 2)P_{i_1}(c_1, 1)$ *and* $(c_1, 1)P_{i_2}(c_2, 2)$. *Let* $\tau_1, \tau_2 \in \{1, 2\}$ *be the desired starting dates of the two lawyers, respectively. Note that reported preferences over courts are not relevant in this example. Then* $\{(\tau_1 = 1), (\tau_2 = 2)\}$ *is a Nash equilibrium strategy profile. The outcome associated with this strategy profile is* $\{(i_1, c_1, 1), (i_2, c_2, 2)\}$. *To see that this strategy profile is indeed a Nash equilibrium, suppose* i_1 *deviated to report* $\tau_1 = 2$. *Then no lawyer would be allocated in the first period. In the second period, lawyer* i_2 *would still be allocated to* c_2 *due to her higher priority at the court. Lawyer* i_1 *would be left unallocated. Hence* i_1 *does not gain from this deviation. Next suppose* i_2 *deviates to report* $\tau_2 = 1$. *Then only* i_1 *is selected to be allocated in the first period, while* i_2 *is still allocated in the second period. Hence* i_2 *is indifferent. Thus* $\{(t_1 = 1), (t_2 = 2)\}$ *constitutes a Nash equilibrium. To see that this is not Pareto efficient, note that if* i_1 *and* i_2 *switched allocations such that* $\{(i_1, c_2, 2), (i_2, c_1, 1)\}$, *both would be better off.*

□

Note however that there are multiple equilibria in the example we considered. For example the profile $\{(\tau_1 = 2), (\tau_2 = 1)\}$ would result in a Pareto efficient Nash equilibrium outcome in the example used above.

5 Stable Mechanisms

5.1 Choice Functions and their Properties

In the previous section we have seen that the currently employed procedure of allocating lawyers to their trainee-ships has some serious deficiencies. In this section we propose a procedure which overcomes these problems. Our approach is to first take the court (or grade, waiting time and social) priorities as used in the current procedure and then to construct choice functions, as in the matching with contracts literature. Having constructed the choice functions we can then use the cumulative offer process of Hatfield and Milgrom (2005) to find a stable allocation. Specifying appropriate choice functions for the lawyers does not present a difficulty since a lawyer will simply choose her most preferred contract from the set of available contracts. The choice functions for the courts are somewhat harder to define.

We will denote general choice functions of some agent $j \in I \cup C$ as Ch_j which associates for each offer set $Y \subseteq X$ some contracts involving j. When we write $\mathrm{Ch}_i(Y)$ then the choice function of an agent $i \in I$ from the offer set Y is meant, whereas $\mathrm{Ch}_c(Y)$ denotes the choice function of a court $c \in C$ from the offer set. A lawyer i's choice function $\mathrm{Ch}_i(Y)$ specifies for each set of contracts $Y \subseteq X$ which contract the lawyer chooses and is given by

$$\mathrm{Ch}_i(Y) \equiv \max_{P_i} Y.$$

The above formulation says that lawyer i will choose from set Y the contract naming lawyer i that is maximal according to the lawyer's preferences P_i. If Y does not contain a contract with i then $\mathrm{Ch}_i(Y) = \emptyset$.

While there are many possible choice functions that are conceivable for the courts, we restrict attention to slot-specific choice functions as in Kominers and Sönmez (2016). Each court c has a set S_c of slots where $|S_c| = \sum_{t \in T} q_{c,t}$. Each slot $s \in S_c$ has an associated priority ordering Π_c^s over the set of contracts involving court c, where we denote the profile of slot-specific priority orderings of court c by $\Pi_c = \cup_{s \in S_c} \Pi_c^s$. In our setting it is natural to suppose that each court has $q_{c,t}$ slots of type t. We let S_c^t be the set of slots of type t and thus we have $S_c = \cup_{t \in T} S_c^t$. Furthermore for each court c there is a precedence order $\rhd_c$ over slots in S_c. The interpretation of $\rhd_c$ is that for slots $s, s' \in S_c$ if $s \rhd_c s'$ then slot s

is filled before slot s', where we make precise what filling a slot before another one means below. Given the slot-specific priorities and the precedence order over slots, a court's slot-specific choice function $\mathrm{Ch}_c(Y;\rhd_c,\Pi_c)$ is constructed as follows. Consider slots in order of their precedence $\rhd_c$. Each slot s chooses its most preferred contract according to Π_c^s from those contracts that have been offered and are not yet associated to any lawyer chosen by any slot with higher precedence.

For court c the model set-up does not prescribe a unique slot-specific choice function that is consistent with the priority $\succ_c$ and the time-specific capacity constraints. While we know a court's priority ordering over lawyers and its capacity constraints $q_{c,t}$, this does not imply a single slot-specific choice function. There are potentially many different slot-specific choice functions, differing both in the precedence order $\rhd_c$ as well as in the slot priority orders Π_s. We introduce below the **time-specific choice function** $\mathrm{Ch}_c^{ts}(\cdot)=\mathrm{Ch}_c(\cdot;\rhd^{ts},\Pi(\succ_c))$, for which each slot of type t finds only contracts involving period t acceptable and ranks acceptable contracts according to the court's priority ordering $\succ_c$.[46] The precedence order $\rhd^{ts}$ is such that any slot of type t has precedence over any slot of type t' if $t<t'$, i.e. for all $s\in S_c^t$ and $s'\in S_c^{t'}$ such that $t<t'$ we have $s\rhd^{ts}s'$. Slots of the same type can be ordered arbitrarily without loss of generality since their priority orderings are identical. The reason for referring to this as the time-specific choice function is that it makes choices of contracts based on constraints, which specify for each time period the number of contracts that can be held. For any set of available contracts Y the choice of court c from Y, $\mathrm{Ch}_c^{ts}(Y)$, is thus given by the following procedure:

- **Step 0:** Reject all contracts $y\in Y$ with $y_C\neq c$.

- **Step t** $\in\{\mathbf{1},...,\mathbf{t_{max}}\}$: Consider contracts $y\in Y$ with $y_T=t$. Accept one by one contracts of the highest priority lawyers according to $\succ_c$ until $q_{c,t}$ contracts have been accepted. If a contract of lawyer y_I has been accepted, reject all other contracts y' with $y'_I=y_I$. Once $q_{c,t}$ contracts have been accepted, reject all other contracts y with $y_T=t$. If there are no contracts which have not yet been considered, end the algorithm. Unless $t=t_{max}$ move to the next step $t+1$. If $t=t_{max}$ end the algorithm.

[46] Since each lawyer has only one contract available for each period, this completely determines the slot's priority ordering.

We will make use of the following definitions of unilateral and bilateral substitutes from Hatfield and Kojima (2010):

Definition. *Contracts are **unilateral substitutes** for court c if there do not exist contracts $x, z \in X$ and a set of contracts $Y \subseteq X$ such that $z_I \notin Y_I$, $z \notin \mathrm{Ch}_c(Y \cup \{z\})$ and $z \in \mathrm{Ch}_c(Y \cup \{x, z\})$.*

Consider a situation in which for some lawyer i there is only one contract, say z, in the available set of a court that is not chosen by the court. Then the choice function of the court satisfies unilateral substitutes if and only if that contract is also not chosen when some other contract, say x, is added to the available set.

Definition. *Contracts are **bilateral substitutes** for court c if there do not exist contracts $x, z \in X$ and a set of contracts $Y \subseteq X$ such that $z_I, x_I \notin Y_I$, $z \notin \mathrm{Ch}_c(Y \cup \{z\})$ and $z \in \mathrm{Ch}_c(Y \cup \{x, z\})$.*

Bilateral substitutes is a less strict requirement on choice functions. Consider a situation in which for some lawyer i there is only one contract, z, in the available set, that is not chosen by the court. Then consider adding another contract, x, to the available set, such that the lawyer of that new contract did not previously have a contract in the available set. The court's choice function satisfies bilateral substitutes if and only if the contract z of lawyer i is still rejected out of the larger set of available contracts.

The following irrelevance of rejected contracts property as defined by Aygün and Sönmez (2012) will be needed:

Definition. *Choice functions satisfy **irrelevance of rejected contracts** (IRC) for court c if for all $Y \subset X$ and for all $z \in X \setminus Y$, we have $z \notin \mathrm{Ch}_c(Y \cup \{z\})$ implies $\mathrm{Ch}_c(Y) = \mathrm{Ch}_c(Y \cup \{z\})$.*

Irrelevance of rejected contracts simply means that the availability of contracts which are not chosen does not matter for choices.

Although we will rely on the results of Kominers and Sönmez (2016) to establish strategy-proofness of the cumulative offer process for a particular choice function, other choice functions that we introduce in this paper satisfy the law of aggregate demand, first introduced by Hatfield and Milgrom (2005):

Definition. *The choice function of court $c \in C$ satisfies the* ***law of aggregate demand*** *if for all $X' \subseteq X'' \subseteq X$, $|\mathrm{Ch}_c(X')| \leq |\mathrm{Ch}_c(X'')|$.*

The law of aggregate demand intuitively says that when more contracts are available to a court, then the court does not choose to accept fewer contracts. We can now state Lemma 3:

Lemma 3. *(Kominers and Sönmez, 2016) The time-specific choice functions satisfy bilateral substitutes and IRC.*

In general, Kominers and Sönmez (2016) have shown that slot-specific choice functions satisfy neither unilateral substitutes nor the law of aggregate demand. However since we consider a particular slot-specific choice function it could potentially satisfy these conditions. However the next two examples show that this is not the case.

Example 8. *Let $T = \{1,2\}$, $Y = \{(i_2,c,2)\}$ and $x = (i_2,c,1)$, $z = (i_1,c,2)$. Furthermore let $i_2 \succ_c i_1$ and $q_{c,1} = q_{c,2} = 1$. Then we have under a time-specific choice function $z \notin \mathrm{Ch}_c^{ts}(Y \cup \{z\}) = \{(i_2,c,2)\}$. However we have $z \in \mathrm{Ch}_c^{ts}(Y \cup \{x,z\}) = \{(i_2,c,1),(i_1,c,2)\}$, which contradicts unilateral substitutes.*

Example 9. *Let $Y = \{(i_1,c,1),(i_2,c,2)\}$, $i_2 \succ_c i_1$ and $q_{c,1} = q_{c,2} = 1$. Then we have $\mathrm{Ch}_c^{ts}(Y) = \{(i_1,c,1),(i_2,c,2)\})$ but we also have $\mathrm{Ch}_c^{ts}(Y \cup \{(i_2,c,1)\}) = \{(i_2,c,1)\}$. Hence adding the contract $(i_2,c,1)$ to the set of contracts Y reduces the total number of contracts chosen.*[47]

The unilateral substitutes as well as the law of aggregate demand condition is used by Hatfield and Kojima (2010) and Aygün and Sönmez (2012) to prove (group) strategy-proofness and the rural hospitals theorem for the cumulative offer process. The unilateral substitutes condition is also used to show the existence of a doctor-optimal stable matching. Nevertheless we are able to show that despite of the failure of the unilateral substitutes condition, this result continues to hold in our model. The key to this result is to assume that the preferences of lawyers satisfy the weak impatience property. With that property a situation such as the one in the example above cannot arise. There we had that a contract

[47]We thank Christian Basteck for this example and for correcting a previously incorrect lemma.

of lawyer i_2 for a late period was available without contracts of the same lawyer for all earlier time periods being available. Adding one of these earlier time periods then caused lawyer i_1 to be accepted when i_1 was previously rejected. If lawyers however propose early contracts before later ones, such a situation cannot arise in the cumulative offer process.

To discuss elimination of justified envy, we follow Sönmez (2013) in defining fairness of a choice function.[48]

Definition. *For any court c, choice function* Ch_c *is **fair** if for any set of contracts* $Y \subseteq X$*, and any pair of contracts* $x, y \in Y$ *with* $x_C = y_C = c$*,* $y_I \succ_c x_I$*,* $y_T = x_T$ *and* $x \in \mathrm{Ch}_c(Y)$*, then there exists* $z \in \mathrm{Ch}_c(Y)$ *such that* $z_I = y_I$*.*

In words, a choice functions of a court is fair if it chooses one lawyer's contract but not another lawyer's contract, although the latter enjoys a higher priority at that court, this can only be if the latter lawyer has another contract which is chosen by that court. We then have the following Lemma 4:

Lemma 4. *The time-specific choice function* Ch_c^{ts} *is fair.*

We now define stability, the central concept of the two-sided matching literature since Gale and Shapley (1962).

Definition. *An allocation* $Y \subseteq \tilde{X}$ *is **stable** with respect to choice functions* $(\mathrm{Ch}_c)_{c=1}^{|C|}$ *if we have:*

1. individual rationality: $\mathrm{Ch}_i(Y) = Y(i)$ for all $i \in I$ and $\mathrm{Ch}_c = Y(c)$ for all $c \in C$; and

2. there is no court $c \in C$ and a blocking set $Y' \neq \mathrm{Ch}_c(Y)$ such that $Y' = \mathrm{Ch}_c(Y \cup Y')$ and $Y' R_i Y$ for all $i \in Y'_I$.

Hence an allocation is stable if each lawyer prefers the assignment to being allocated no contract, each court chooses its assignment over some subset of that assignment and there is no set of contracts such that a court would rather choose that set of contract, the blocking set, when this and the allocation are available, such that the lawyers having contracts in the blocking set weakly prefer those contracts over their assignment. Under the assumption that courts use the time-specific choice function $\mathrm{Ch}_c^{ts}(\cdot)$ stable allocations are feasible. Stability is not

[48]Note that this is a different concept from fairness of an allocation.

a desiderata per se in our model. In the original literature on two-sided matchings stability was seen as important in explaining whether matching procedures would systematically lead to unraveling (Roth, 1984, 1991). In our case the regional courts are not strategic players and the priorities according to which they evaluate lawyers are determined by the mechanism designer. This precludes the possibility of courts contracting with lawyers around the centralized mechanism. However, stability matters in our context as stability implies other desirable properties of mechanisms. An allocation $Y \subseteq \tilde{X}$ is the **lawyer-optimal stable allocation** if every lawyer weakly prefers it to any other stable allocation.

5.2 Cumulative Offer Process

We now introduce the **cumulative offer process** (COP) as defined in Hatfield and Kojima (2010), which is a generalization of the deferred-acceptance algorithm of Gale and Shapley (1962).

The cumulative offer process takes as input the (reported) preferences of the lawyers as well as the choice function of each court.

- **Step 1:** One (arbitrarily chosen) lawyer offers her first choice contract x_1. The court that is offered the contract, $c_1 = (x_1)_C$, holds the contract if it is acceptable and rejects it otherwise. Let $A_{c_1}(1) = \{x_1\}$, and $A_c(1) = \emptyset$ for all $c \neq c_1$.

In general,

- **Step k $\geq$2:** One of the lawyers for whom no contract is currently held by any court offers her most preferred contract, say x_k, that has not been rejected in previous steps. Let $c_k = (x_k)_C$, hold $\mathrm{Ch}_c\left(A_{c_k}(k-1) \cup \{x_k\}\right)$ and reject all other contracts. Let $A_{c_k}(k) = A_{c_k}(k-1) \cup \{x_k\}$ and $A_c(k) = A_c(k-1)$ for all $c \neq c_k$.

Now we apply Theorem 1 of Hatfield and Kojima (2010) to show that the cumulative offer process, as just described, in conjunction with the time-specific choice function produces a stable allocation.

Theorem. *[Hatfield and Kojima (2010)] Suppose the choice functions of the court used in the cumulative offer process satisfy bilateral substitutes. Then the cumulative offer process produces a stable allocation.*

The existence of a stable matching is the minimum requirement that we ask of an algorithm. By the above result and the fact that the time-specific choice functions satisfy bilateral substitutes, using the time-specific choice functions when running the COP yields a stable allocation. Hatfield and Kojima (2010) further show that if one strengthens the assumptions to unilateral substitutes for the choice functions used, then one can show that the cumulative offer process produces the lawyer-optimal stable allocation. In our case however the time-specific choice functions do not satisfy unilateral substitutes.

Nevertheless one can adapt Theorem 4 of Hatfield and Kojima (2010), as modified by Aygün and Sönmez (2012), which is used in Theorem 5 of Hatfield and Kojima (2010) to show the existence of a lawyer-optimal stable allocation (doctor-optimal in their terminology). To do so, it is sufficient to make an assumption on the preferences of the lawyers, rather than on the choice functions used by the courts. Namely we will assume that lawyers are weakly impatient. Previous results in the matching with contracts literature usually proceeded by restricting the choice functions used by the side of the market which could accept multiple contracts to obtain results, while placing essentially no restrictions on the other side of the market. Here we depart from this approach and relax the restrictions placed on the choice functions used by the side of the market which can accept several contracts (the courts) and instead put some restrictions on the single-contract side (lawyers) of the market. Both approaches, as we will see, lead to similar results.

Lemma 5. *A contract z that is rejected by a court c at any step of the cumulative offer process using the time-specific choice function Ch_c^{ts}, cannot be held by court c in any subsequent step.*

The key to our proof of this result lies in the specific choice function that we use. This causes lawyers, when a contract of theirs is rejected, to either propose to a new court or to propose to some court at which the lawyer was previously rejected. So if some court c has multiple offers, say z and z' of some lawyer i and holds z, then it will, when receiving a new contract offer from some other lawyer j, never reject z while simultaneously accepting z'. In the proof we heavily rely on Aygün and Sönmez (2012).

With this result in hand, we can now state the following lemma:

Lemma 6. *Suppose lawyer preferences are weakly impatient. The outcome of the cumulative offer process using the time-specific choice function* Ch_c^{ts} *produces the lawyer-optimal stable allocation.*

The proof is essentially the same proof as the one of the corresponding Theorem 5 in Hatfield and Kojima (2010) and Aygün and Sönmez (2012). Assuming weak impatience again allows us to relax the unilateral substitutes assumption and instead use the time-specific choice functions which only satisfy the bilateral substitutes assumption. The reason that this works is that because of weak impatience, any sets of available contracts that the courts will have to make choices from are sets such that if a contract x of some lawyer i is available for period t, then contracts for any earlier and feasible period for that lawyer i will also be available. On this restricted domain of sets of available contracts unilateral substitutes essentially holds for the time-specific choice function, allowing the proofs by Hatfield and Kojima (2010) to go through, with some modifications. Note that we only needed to make use of the assumption of weak impatience for proving Lemma 6.

The result in Lemma 6 is a new result, which is not implied by any of the results in Kominers and Sönmez (2016), since they consider more general slot-specific choice functions than we do here. For general slot-specific choice functions a lawyer-optimal stable allocation is not guaranteed to exist and even when such an allocation exists, the COP is not guaranteed to find it. Lemma 6 above shows that under weak impatience, a lawyer-optimal stable allocation is guaranteed to exist and that it is found by the COP. The following example shows that without weak impatience, the existence of a lawyer-optimal stable allocation is no longer guaranteed.

Example 10. *Let* i_1 *and* i_2 *prefer* $(c,2)$ *to* $(c,1)$ *and assume* $i_1 \succ i_2$ *with* $q_{c,1} = q_{c,2} = 1$. *Then the allocation* $Y = \{(i_1,c,1),(i_2,c,2)\}$ *is stable, while the COP produced the also stable allocation* $Y' = \{(i_1,c,2),(i_2,c,1)\}$. *Notice that* i_2 *prefers* Y, *while* i_1 *prefers* Y', *i.e. neither allocation is weakly Pareto efficient.*

5.3 Properties of the Time-specific Lawyer Offering Stable Mechanism

The time-specific lawyer offering stable mechanism (TSLOSM), ψ^{ts}, is defined to be that mechanism which associates with each preference profile the outcome of the COP using the time-specific choice functions. We will refer to this mechanism as the time-specific stable mechanism. We have the following result:

Proposition 10. *The time-specific lawyer offering stable mechanism is stable and (group) strategy-proof. If in addition lawyers' preferences are weakly impatient, then the time-specific stable mechanism is lawyer-optimal stable.*

Stability and (group) strategy-proofness follow directly from Theorem 3 in Kominers and Sönmez (2016). The second part follows from applying our Lemma 6. Instead of applying Theorem 3 in Kominers and Sönmez (2016), an alternative way of obtaining the first part of the above results when lawyers' preferences are weakly impatient is to adapt results in Hatfield and Kojima (2009) making use of the fact that under weak impatience, a lawyer-optimal stable allocation is guaranteed to exist. An important corollary of the time-specific stable mechanism being group strategy-proof is that it leads to weak Pareto efficiency as in Hatfield and Kojima (2009).

Corollary 1. *The time-specific lawyer offering stable mechanism is weakly Pareto efficient.*

Proof. Suppose otherwise. Hence there exists an allocation $Y \subseteq X$ such that for all $i \in I$ we have $Y(i) P_I \psi^{ts}(P)$. Let $\tilde{P}_i$ be the preference profile for each lawyer $i \in I$ that lists $Y(i)$ as the only acceptable contract. Then we have that $\psi^{ts}(\tilde{P}) = Y$. This implies that we have found a coalition of lawyers, namely all lawyers, that can jointly deviate to make all its member strictly better off. This contradicts ψ^{ts} being group strategy-proof. $\square$

One of the problems in the current procedure, the Berlin mechanism, is that lawyers may be worse off by improving their ranking, for example by obtaining a better grade or having waited longer. The next proposition shows that this is not the case for the cumulative offer process using the time-specific choice function.

Proposition 11. *The time-specific lawyer offering stable mechanism respects improvements.*

The intuition behind the proof of this result, which is simply an application of Theorem 4 in Kominers and Sönmez (2016), is as follows. Let $\succ_1$ be an unambiguous improvement over $\succ_2$ for lawyer i and let ψ_1 be the associated mechanism. Similarly for $\succ_2$. Suppose the COP were run initially excluding lawyer i under $\succ_1$, which will lead to some allocation X^1. After this, lawyer i proposes contracts in order of preference. This process will terminate for some contract offer x^k, which is i's assignment under the mechanism ψ_1. Running the algorithm under $\succ_2$ without lawyer i will lead to the same initial allocation X^1 since only the ranking of lawyer i has changed. Letting i propose contracts however will lead to the same rejections occurring since $\succ_1$ is an unambiguous improvement over $\succ_2$ until x^k is offered by i, which by assumption is the final allocation under $\succ_1$ but which may nevertheless be rejected under $\succ_2$. From this it follows that i cannot do worse under $\succ_1$ than under $\succ_2$. Note that the priorities based on grades, accumulated waiting time and social criteria do not directly enter the time-specific lawyer offering stable mechanism. Hence changes in those rankings will leave the outcome of the mechanism unchanged, which is consistent with respect of improvements.

This is an important result since it implies that targeted efforts to improve the allocation obtained by specific lawyers through an improvement of their ranking can never hurt these lawyers who those efforts are intended to help. One implication is that when the ranking depends positively on grades, then lawyers are rewarded for better grades by an improvement in their assignment.

The fact that the time-specific stable mechanism respects improvements has a further implication in our application. Lawyers, in the current system, may report to have a special social relationship to a court. For example, having children grants higher priority for regional courts in Bavaria, as discussed above. Consider now a game which first asks lawyers to report any such information. In a second stage, the priorities of each court would be adjusted to reflect those reports, in case the information lawyers have reported has been verified. In case lawyers do have special social relationship to a court, but do not report it, the choice function remains unaffected. Then we have the following result, which follows by noting that reporting this information leads to an unambiguous improvement in the priority of a lawyer at a court. Since the time-specific stable mechanism respects improvements, reporting this information, holding the strategies of everyone else fixed, cannot make a lawyer worse off, but may lead to an improvement. Hence the following corollary is obtained:

Corollary 2. *Each lawyer has an incentive to report verifiable information increasing her priority at a court under the time-specific stable mechanism.*

The above corollary shows that respect of improvements is closely linked to the incentive compatibility of revealing hard information.[49] Another desirable property that the time-specific stable mechanism satisfies is elimination of justified envy.

Proposition 12. *The time-specific stable mechanism eliminates justified envy.*

Proof. To see that the time-specific stable mechanism eliminates justified envy, let $x, y \in Y \subseteq \tilde{X}$ be two contracts obtained by the time-specific stable mechanism such that $x_I \neq y_I$ and $(x_C, x_T) P_{y_I} (y_C, y_T)$. Then since by the cumulative offer process y_I must have offered (y_I, x_C, x_T) at some step during the process, it must have been rejected. But the only way that (y_I, x_C, x_T) had been rejected while x was accepted is when $x_I \succ_{x_C} y_I$, which implies that the time-specific stable mechanism eliminates justified envy. □

The time-specific mechanism proposed in this section however does not fill positions early. The reason is an inherent conflict between stability and early filling. To see this consider the following example.

Example 11. *There are two periods $t = 1,2$ and two courts $c = c_1, c_2$, each with one position in each period. In the first period, there are two lawyers $I = \{i_1, i_2\}$, with common preferences for each $i \in I$: $(c_1,1)P_i(c_1,2)P_i(c_2,1)P_i(c_2,2)$. Both courts have priorities such that lawyer $i_1 \succ_c i_2$. The outcome of the cumulative offer process using the time-specific choice function results in the allocation $\{(i_1, c_1, 1), (i_2, c_1, 2)\}$, which leaves the position in period 1 at court c_2 unoccupied even though lawyer i_2 is given a position in period 2, thereby violating early filling.*

[49] Aygün and Bo (2013) in their analysis of college admissions with affirmative action in Brazil study the incentives of different disadvantaged groups to disclose their status to the mechanism. For example, it is desired by the policy-makers in Brazil that students from ethnic minorities who went to a public high school are given higher priority. However such students may decide not to reveal their status as an ethnic minority. In the currently used procedure in Brazil they sometimes do not have an incentive to do so. The mechanism proposed by Aygün and Bo (2013) makes it optimal for students to reveal this information truthfully.

The example shows that (lawyer-optimal) stable outcomes might not satisfy the early filling properties. There is a trade-off between preferences of lawyers from different time periods: On the one hand, a mechanism finding a stable outcome over lawyers from all periods might not fill positions early and by this make future lawyers worse off. On the other hand, guaranteeing early filling could benefit future lawyers at the costs of earlier lawyers, but would not be stable and might violate other desirable properties, such as strategy-proofness.

5.4 Flexible Choice Functions

The discussion in the previous sections assumed that each court c could only accept $q_{c,t}$ lawyers in time period t. This assumption was made because the number of positions at each court is determined by the budget of the federal state several periods into the future so that the courts cannot flexibly set their own capacity for each period. In this subsection we consider the possibility of allowing each court to flexibly determine how to allocate total capacity, which is assumed to be fixed over several periods. Hence, we no longer have a time-specific capacity constraint but instead have for each court a global constraint on the total number of lawyers that can be accepted. In other words, we relax the requirement that a mechanism produces a feasible allocation.

We continue to consider slot-specific choice functions and build on the time-specific choice function Ch_c^{ts} to develop the flexible choice function Ch_c^{flex}. As before there are $q_{c,t}$ copies of a slot of type t and the slots with a lower t have higher precedence. Under the time-specific choice function Ch_c^{ts} each slot's priority ordering was such that only periods of the associated time period were deemed acceptable and acceptable contracts were ranked according to $\succ_c$. Under Ch_c^{flex} each slot's priority ordering Π_s ranks highest the contracts of the associated time period. Next highest are ranked the contracts of period 1 (or period 2 in case we are considering slots of the period 1 type), followed by those contracts of the next highest period and so on.[50] Contracts of the same period are ranked according to $\succ_c$. As a consequence each slot, irrespective of its types, considers all contracts acceptable. Note that this allows for choices that violate the time-specific capacity constraints of the courts.

[50]The ranking of contracts not involving period t by slots of type t does not matter for our results, so long as all contracts involving period t are ranked highest.

Since the flexible choice function is also a slot-specific choice function and since contracts, given a fixed period, are ranked according to $\succ_c$ by slots, the cumulative offer process using these choice functions is a strategy-proof, weakly Pareto efficient mechanism that eliminates justified envy and respects improvements. The proofs are similar to those for the TSLOSM and are omitted. We call this new mechanism the flexible lawyer offering stable mechanism (FLOSM) and denote it by ψ^{flex}.

FLOSM does not necessarily satisfy the time-specific capacity constraints of the courts, while the TSLOSM satisfies these constraints. Hence there may be allocations that can be reached by FLOSM that violate feasibility under TSLOSM but are preferred over allocations feasible under TSLOSM.

Proposition 13. *Fix a lawyer matching problem* $(T,I,C,q,P,\succ)$*. Then we have for all i that* $\psi^{flex}(P)(i)R_i\psi^{ts}(P)(i)$*.*

Proposition 13 says that all lawyers weakly prefer the outcome of the flexible lawyer offering stable mechanism over the outcome of the time-specific lawyer offering stable mechanism. The intuition behind this result is that the time-specific choice function can be obtained from the flexible choice function via truncation strategies. Truncation strategies by one side of the market make the other side of the market weakly worse off. Hence we conclude that relaxing time-specific capacity constraints and suitably adapting the choice functions used by the courts has the potential of making lawyers better off.[51]

We interpret the flexible choice function as corresponding to cases in which a court is given greater budgetary freedom with respect to when to open trainee-ship positions. In practice there may be other reasons for having time-specific constraints that make adjustments to capacities over time difficult. For example, class room sizes could constraint how many lawyers may begin their trainee-ship in any given period of time.

6 Conclusion

While the above description of the Time-Specific Lawyer Offering Mechanism is rather theoretical, from a practical point of view it could be interpreted in two

[51] This could happen for example by allowing courts to transfer funding for trainee-ship positions over time in response to demand, rather than sticking to an exogenously given budget for each time period.

ways.

First, as a mechanism of perfect foresight, the matching for all lawyers at all courts in all time periods is finalized already in period 0. This requires grades, preferences etc. of arriving lawyers to be known already at time 0. If courts before time t are unacceptable to some lawyer i, then we can think of i arriving at time t. This interpretation could be realistic for short time horizons, however less so for longer horizons.

The second interpretation is that the mechanism is run every single period, whereby lawyers are allocated to positions now and in the future, ignoring future arrivals. Then, once a lawyer is allocated to a future position, this seat remains "reserved" for the current lawyer. In that case one needs to additionally analyze how many future positions one allows to be assigned today.

An interesting extension of our model would be to consider how our proposed mechanism behaves when it needs to be applied for each period over a number of periods. Dur and Kesten (2014) consider a problem in which a set of students is to be matched to colleges, but in which the set of colleges is partitioned. They show that when the assignment happens sequentially, it is inherently difficult to have a mechanism be non-wasteful, and strategy-proof while eliminating justified envy and respecting improvements. Such results would also apply in a dynamic version of our model in which the time-specific lawyer-optimal mechanism were applied repeatedly. A related problem, that we have ignored so far, is how to manage capacity. While we assumed that capacities were given exogenously, in a dynamic procedure with excess demand one may want to reserve some capacity at some courts to ensure that future agents are not unduly disadvantaged by earlier agents taking these positions. Future research should address this question.

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Appendix

A Omitted Proofs

Proof. **[Lemma 4]** Suppose to the contrary that for court c and a set of contracts Y with elements $y,x \in Y$ such that $y_C = x_C = c$, $y_I \succ_c x_I$, $y_T = x_T$ and $x \in \text{Ch}_c^{tisp}(Y)$ but that there does not exist $z \in \text{Ch}_c^{tisp}(Y)$ with $z_I = y_I$. Note that in particular this implies that $y \notin \text{Ch}_c^{tisp}(Y)$. Then since such a z does not exist, it must be that in step $y_T = t$ of the procedure to construct Ch_c^{tisp}, y has not yet been rejected. So in step t both x and y are still available. Now x is accepted in step t since $x \in \text{Ch}_c^{tisp}(Y)$ while y is rejected, since $y \notin \text{Ch}_c^{tisp}(Y)$. This contradicts $y_I \succ_c x_I$, since the procedure to construct the time-specific choice function would have selected the contract of the agent with the better ranking. $\square$

Proof. **[Lemma 5]** Towards a contradiction let k' be the first step a court c holds a contract z that was previously rejected at step $k < k'$. As z is rejected at step k, it was on hold by court c at step $(k-1)$ or it was offered to court c at step k. In either case no other contract of lawyer z_I could be on hold by court c at step $(k-1)$. But then, since z is the first contract to be held after an earlier rejection, court c cannot have held another contract by lawyer z_I at step k. That is $z_I \notin \left[\mathrm{Ch}_c^{tisp}(A_c(k))\right]_I$. Since z is rejected at step k, this means that for all $x \in \mathrm{Ch}_c^{tisp}(A_c(k))$ with $x_T = z_T$, we must have $x_I \succ_c z_I$. Let $\left[\mathrm{Ch}_c^{tisp}(A_c(k))\right](z_T)$ denote the set of such contracts in time z_T. Given the definition of Ch_c^{tisp}, $z \in \mathrm{Ch}_c^{tisp}(A_c(k'))$ implies that some contract $x \in \left[\mathrm{Ch}_c^{tisp}(A_c(k))\right](z_T)$ can no longer have been under consideration in step t of the procedure to find the court's choice. But for that to have happened, it must be that some contract y with $y_I = x_I$ and $y_T < x_T$ has been accepted in step k'. But this cannot be since by assumption z is the first contract that was rejected and subsequently accepted and because x_I cannot have offered a contract in step k' since a contract of x_I was held by the court in period $k' - 1$. Hence a contradiction. □

Proof. **[Lemma 6]** To prove the lemma, it is sufficient to show that for any stable allocation $X' \subseteq \tilde{X}$ and any contract $z \in X'$, contract z is not rejected by the cumulative offer algorithm when the time-specific choice function is used. To obtain a contradiction, suppose not. Let k be the first step where court $c = z_C$ rejects contract z, and let $Y = \mathrm{Ch}_c^{tisp}(A_c(k))$. Then by IRC, $z \notin \mathrm{Ch}_c^{tisp}(Y \cup \{z\})$. Then by lemma 5, $z_I \notin Y_I$. As k is the first step a contract in any stable allocation is rejected, every lawyer in Y_I weakly prefers their contract in Y to their contract in X' which is stable by assumption. We then consider two cases:
Case 1: $z \notin \mathrm{Ch}_c^{tisp}(Y \cup X')$. In this case, court c blocks allocation X' together with lawyers in Y_I, contradicting stability of X'.
Case 2: $z \in \mathrm{Ch}_c^{tisp}(Y \cup X')$. But this cannot be, since for any $x \in \mathrm{Ch}_c^{tisp}(Y)$ with $x_T = z_T$, we have for all $s < t$, $(x_I, c, s) \in Y$ by weak impatience.
Therefore the addition of contracts cannot result in z being chosen when both Y and X' are available due to the way the time-specific choice function is constructed. A contradiction. □

For the proof of Proposition 13 we make use of an associated lawyer-slot matching market as in Kominers and Sönmez (2016). A lawyer-slot matching market is constructed from a lawyer-court allocation problem in which the courts have slot-specific choice functions $\mathrm{Ch}_c(\cdot; \rhd_c, \Pi_c)$ as follows. The contract set X is extended to the set Z defined by $Z \equiv \{(x,s) : x \in X \textit{ and } s \in S_{x_C}\}$. Slot priorities $\tilde{\Pi}^s$over contracts in Z are derived from priorities Π_c^s over contracts in X. This means that $(x,s)\tilde{\Pi}^s(x',s)$ if and only if $x\Pi_c^s x'$. A lawyer's preferences $\tilde{P}_i$ over contracts in Z remain the same as preferences P_i over contracts in X

with ties between the same contract at different slots broken according to the precedence order. This means that $(x,s)\tilde{P}_i(x',s')$ if and only if either xP_ix' or $[x = x' \textit{ and } s \rhd_{x_C} s']$.

Proof. **[Proposition 13]** For any instance of a lawyer-court allocation problem we construct the lawyer-slot matching problem as follows. By Theorem A.1 of Kominers and Sönmez (2016) the outcome of the lawyer offering stable mechanism in a lawyer-court matching with waiting time problem in which courts use slot-specific choice functions, corresponds to the outcome of the lawyer offering stable mechanism in the associated lawyer-slot matching market.

Suppose now that one slot $s \in S_c$ in the lawyer-slot market truncates from its priority ordering its lowest ranked contract, say x. If that contract was not part of the allocation under the lawyer offering stable mechanism without the truncation, then this truncation has no effect on the final allocation. If that contract was part of the allocation under the lawyer offering stable mechanism without the truncation, then the lawyer x_I applies to her next highest ranked slot, s'. The slot s' will either accept lawyer x_I's contract (in the process possibly rejecting another contract of another lawyer) or reject it. In either case there will be a finite chain of rejections of contracts. All lawyers involved in this rejection chain will receive a worse allocation than without the truncation of slot s according to their preferences over slots. There are now two possibilities. Either all lawyers find the new allocation worse only because of the tie-breaking induced by the precedence order $\rhd_c$. In that case the lawyers in the original lawyer-court matching with waiting time problem are unaffected by the truncation. If the new allocation is worse because of a change in the court and time period allocated to a lawyer, then lawyers in the original lawyer-court matching will be worse off.

A similar logic applies to any further truncation by any slot. Each truncation makes the lawyers weakly worse off. Consider now the case in which each slot's priorities have been truncated to only find contracts involving a time period corresponding to the slot's type acceptable. In that case the outcome of the lawyer-optimal stable mechanism in the lawyer-slot matching problem corresponds to the outcome of the lawyer-proposing stable mechanism under the time-specific choice functions. By the previous arguments, all lawyers weakly prefer the allocation without any truncations. □