Aggregate Information and Organizational Structures

Gorkem Celik (ESSEC Business School)
Dongsoo Shin (Santa Clara University)
Roland Strausz (Humboldt Universität zu Berlin)

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Gorkem Celik\textsuperscript{2}  
Dongsoo Shin\textsuperscript{3}  
Roland Strausz\textsuperscript{4}

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\textsuperscript{2}Economics Department, ESSEC Business School and THEMA Research Center, Cergy Pontoise Cedex, France, Email: celik@essec.fr

\textsuperscript{3}Department of Economics, Leavey School of Business, Santa Clara University, Santa Clara, CA 95053, USA, Email: dshin@scu.edu

\textsuperscript{4}Humboldt-Universität zu Berlin, Institute for Microeconomic Theory, Spandauer Str. 1, D-10178 Berlin (Germany), Email: strauszr@wiwi.hu-berlin.de.
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ABSTRACT

We study an organization with a top management (principal) and multiple subunits (agents) with private information that determine the organization’s aggregate efficiency. Under centralization, eliciting the agents’ private information may induce the principal to manipulate aggregate information, which obstructs an effective use of information for the organization. Under delegation, the principal concedes more information rent, but is able to use the agents’ information more effectively. The trade-off between the organizational structures depends on the likelihood that the agents are efficient. Centralization is optimal when such likelihood is low. Delegation, by contrast, is optimal when it is high.

**JEL Classification:** D82, D86

**Key words:** Agency, Aggregate Information, Organization Design
1 Introduction

Organizational structures, as pointed out by Simon (1973), are “authority mechanisms” that are constructed to process and aggregate organizational information. In some organizations, the communication channels are heavily centralized and top management keeps a strong grip on processing information, while in other organizations, such channels are delegated to subunits and information is aggregated through a chain of hierarchies. Given the importance of utilizing an organization’s information effectively, understanding the pros and cons of different modes of processing information is crucial for the efficiency of an organization.

Using an agency model, this study contrasts the task of aggregating information in a centralized versus a delegated organizational structure. It thereby identifies a new type of incentive problem—manipulation of aggregate information. This incentive problem provides a novel economic rationale for when and why one structure prevails over the other.

The central trade-off in our paper is as follows. When top management centralizes the organization’s communication channels, inducing truthful behavior of the organization’s subunits may lead to the top management’s own misrepresenting behavior—it may have an incentive to manipulate the aggregate information collected from the subunits. We show that this tension between the top management’s and its subunits’ incentives stands in the way of screening, leading to a less effective use of the organization’s information.

When top management delegates aggregation of information to a subunit, the ability to manipulate aggregate information is transferred to that particular subunit. This has two counter-acting effects. On one hand, the top management has to give more information rent to that particular subunit to prevent it from manipulating the aggregate information. On the other hand, the tension between the top management’s and the subunits’ manipulating incentive vanishes, which allows the organization to use information more effectively.

We identify this trade-off by modeling an internal organization with a principal (top management) and two agents (subunits) with private information about their efficiencies (types). The aggregated information of the agents’ types indicates the overall efficiency of the organization. We postulate an organization where the top management’s decisions depend on the organization’s aggregate information, instead of detailed information. In fact, practitioners and organization studies frequently point out top management’s limitations in processing their organization’s entire information in detail.1

1For example, in an interview with Harvard Business Review (Taylor 1991), Percy Barnevik, then CEO of ABB Group, reports that one of the largest obstacles he faces is communication with a large number of the organization’s subunits. See Weick (1995) for an organization study on this issue.
Under centralization, each agent reports his efficiency to the principal directly and no direct communication takes place between the agents. In this organizational structure, each agent can, as in standard screening models, reap information rent by misrepresenting his efficiency and the principal responds by distorting the project size downward in the optimal contract. When these distortions are large, an incentive for the principal arises to manipulate the aggregate information herself. In particular, when both agents report that they are inefficient so that the organization’s aggregate efficiency is low, the principal has an incentive to overstate the aggregate efficiency. In other words, the principal gains ex post by manipulating the aggregate information. We show that reconciling the agents’ and the principal’s incentives hinders an effective use of the agents’ private information and may even prevent its use in the sense that optimal contracts exhibit pooling.

Under delegation, one agent, say agent $\alpha$, becomes the “superior” of the other agent, say agent $\beta$. In this structure, agent $\beta$ first reports his efficiency to agent $\alpha$, who in turn reports the aggregate efficiency to the principal. Since the authority to process the aggregate information is shifted from the principal to agent $\alpha$, the principal faces a loss of control. As a result, the principal must concede larger information rent to this agent. In order to reduce this larger information rent, the principal increases the downward distortions in the optimal project size. There is a gain, however, from the loss of control—a fully separating outcome is restored, implying that the principal can utilize the organization’s information more effectively under delegation than under centralization.

Comparing the two structures, we show that the likelihood of an efficient agent determines the principal’s optimal choice of organizational structure. Our result hinges upon such likelihood because it determines distortions in the project size and thereby the tension between the principal’s and the agents’ manipulating incentives. When the agents are likely to be inefficient, centralization is the optimal organizational structure. By contrast, when the agents are likely to be efficient, the optimal structure is delegation.

The rest of the paper proceeds as follows. In Section 2, we review the related studies. The model is presented in Section 3. In Section 4, we discuss our benchmark to show that, when the principal cannot manipulate the aggregate information, centralization always dominates delegation. In Section 5, we compare centralization and delegation when the principal can manipulate the aggregate information from the agents. In Section 6, we extend our discussion by endogenizing restrictiveness of communication technology in our model. We conclude in Section 7. All proofs are relegated to Appendix.
2 Review of Related Studies

Contrary to the studies that explains organizational structures based on the costs of information processing (e.g. Radner 1992, Bolton and Dewatripont 1994, Qian 1994) or problems of coordination (e.g. Rosen 1982, Harris and Raviv 2002, Hart and Moore, 2005), our paper belongs to the literature that studies organizational structures under ex ante private information. While earlier contributions advocate centralized structures by identifying loss of control under delegation (e.g. Williamson 1967, and McAfee and McMillan 1995), there have been a number of papers identifying situations in which delegation equally matches or even outperforms centralization.

Distinguishing organizational structures on the basis of differences in monitoring rather than information flows, Baron and Besanko (1992) and Melumad et al. (1995) identify necessary conditions under which the vertical hierarchy achieves the same outcome as the horizontal hierarchy. They demonstrate that if top management can monitor transactions between the subunits, then the optimal outcome is independent of the organizational structure. Melumad et al. (1997) show that, when contracts are complex, delegating a contracting authority to an agent brings the organization more flexibility. Severinov (2008) shows that the optimal structure of operation depends on the firm’s production technology.

More closely related to ours are the following studies. Focusing on collusion, Laffont and Martimort (1998) show that contractual delegation enables organizations under limited communication to effectively discriminate transfers among different agents, thus mitigating collusion. Importantly, delegation in their model involves delegation of contracting—under delegation, the principal contracts only with the middle-agent, who in turn, contracts with the bottom-agent. In our model, the principal under delegation still offer contracts to both agents—delegation only affect the information flows within the organization. Our model of delegation is more in line with the “chain of command” model of Friebel and Raith (2004). The authors show that exclusive communication lines to top management induces middle managers to make a sincere effort for recruiting and training subordinates. None of these studies consider the principal’s manipulating incentive under centralization.

The following studies demonstrate the optimality of delegation under some specific form of incomplete contracting. Beaudry and Poitevin (1995) and Olsen (1996) point out that delegation can make it harder to renegotiate. Aghion and Tirole (1997) demonstrate that delegation induces acquisition of useful information for the organization. Olsen and Torsvik (2000) show that a firm’s ability to learn about the difficulty of the tasks workers engage in will induce the firm to give workers more discretion over tasks and weaker incentives. Studies
such as Dessein (2002) and Alonso et al. (2008) show that organizations can benefit from
delegation because it makes better use of private information. Shin and Strausz (2014) show
that delegation mitigates dynamic incentives, when the organization cannot use long term
contracts. Unlike these studies, we focus on organizational structures and the delegation of
information flows rather than decision rights.

Focusing on the manipulation of aggregate information, our paper is related to the
studies on the principal’s manipulating incentives when contracting with multiple agents.
In McAfee and Schwartz (1994), the principal under limited commitment may have an
incentive to renegotiate with an agent at another agent’s expense. Studying agents with
correlated private information, Dequiedt and Martimort (2015) analyze a type of informa-
tion manipulation similar to ours. Akbarpoury and Li (2018) study optimal auctions under
an auctioneer’s manipulating incentives when the bidders cannot observe each other’s bid.
In spirit, this type of manipulations is similar to the manipulation of aggregate informa-
which we study. Celik et al. (2018) show that aggregate information manipulation may lead
to an oversupply of public goods. None of these papers consider the effect of the principal’s
manipulating incentives on organizational structures.

3 Model of Internal Organization

We model an organization with a principal who needs two agents, $\alpha$ and $\beta$, to implement a
project. The project of size $q \geq 0$ yields the principal a value $v(q)$, and imposes a cost $\theta^k q$
on agent $k \in \{\alpha, \beta\}$. To ensure interior solutions, we assume that $v(q)$ satisfies the Inada
condition. The project size $q$ is publicly verifiable.

Agent $k$’s cost parameter $\theta^k \in \{\theta_g, \theta_b\}$ is his private information and $\Delta \theta \equiv \theta_b - \theta_g > 0$.
We refer to $\theta^k$ as agent $k$’s “type”. An agent of type $\theta_g$ is “efficient,” and an agent of type $\theta_b$
is “inefficient.” The agents’ types are drawn independently from identical distributions—an
agent is efficient with probability $\varphi \in (0, 1)$, and therefore inefficient with probability $1 - \varphi$.
The probability distribution is public knowledge.

Because the principal needs both agents for the project, the project’s aggregate marginal
cost is $\Theta \equiv \theta^\alpha + \theta^\beta$. Since $\theta^\alpha, \theta^\beta \in \{\theta_g, \theta_b\}$, the project’s “aggregate marginal cost” has the
following three possibilities:

$$\Theta_G \equiv 2\theta_g, \quad \Theta_M \equiv \theta_g + \theta_b, \quad \Theta_B \equiv 2\theta_b.$$ 

Thus, the efficient size of the project, denoted by $q^*$, is characterized by:

$$v'(q^*_\gamma) = \Theta_\gamma, \quad \gamma \in \{G, M, B\}.$$
In order to compensate the agents for their costs, the principal pays each agent a transfer, denoted by \( t^k, k \in \{\alpha, \beta\} \). Given transfers, the principal’s and the agent’s payoff from the project of size \( q \) are respectively:

\[
\pi \equiv v(q) - t^\alpha - t^\beta \quad \text{and} \quad u^k \equiv t^k - \theta^k q.
\]

Reflecting usual employment contracts, we impose a non-slavery condition—each agent can quit and walk away from the organization at any time. An agent will do so if he expects his payoff to be less than his reservation level of zero.

We compare two organizational structures—centralization versus delegation. Under centralization, each agent directly reports his type only to the principal, who subsequently aggregates the information—agents cannot communicate directly with each other.\(^2\) Under delegation, agent \( \beta \) makes a report to agent \( \alpha \), who in turn aggregates the information and reports it to the principal—agent \( \beta \) cannot communicate directly with the principal.

Focusing on the crucial role of aggregating information in organizations, we center our analysis on contracts that are contingent on the aggregate information \( \gamma \in \{G, M, B\} \).\(^3\) Hence, we express the contract as a combination:

\[
\Phi \equiv (q_\gamma, t^\alpha_\gamma, t^\beta_\gamma), \quad \gamma \in \{G, M, B\}.
\]

As mentioned in the introduction, contracting upon aggregate information reflects various reports of practitioners as well as findings in organization studies that top managements tend to work with aggregate, condensed information rather than with the detailed, fine-grained information at the individual level.

Figure 1 illustrates the information flows in the two organizational structures.

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\(^2\)In Section 6, we discuss and motivate these implicit limitations on communication more extensively.

\(^3\)See Laffont and Martimort (1997, 1998) for a similar assumption on contracting.
The timings under centralization and delegation are summarized below.

**Centralization** Under centralization, each agent reports his type directly only to the principal. Once the reports are made, the principal makes an announcement on $\gamma \in \{G, M, B\}$.

1. The principal offers the contract $\Phi \equiv \left(q_\gamma, t^\alpha_\gamma, t^\beta_\gamma\right)$, $\gamma \in \{G, M, B\}$.
2. Each agent makes a report on his type, $\theta^k \in \{\theta_g, \theta_b\}$, to the principal.
3. The principal receives aggregation of the reports and makes a public announcement on $\gamma \in \{G, M, B\}$.
4. The project is implemented and transfers are paid according to $\Phi$.

**Delegation** Under delegation, agent $\beta$ first reports his information to agent $\alpha$, who then sends a report on $\gamma \in \{G, M, B\}$ to the principal.

1. The principal offers the contract $\Phi \equiv \left(q_\gamma, t^\alpha_\gamma, t^\beta_\gamma\right)$, $\gamma \in \{G, M, B\}$.
2. Agent $\beta$ makes a report on his type $\theta^\beta$ to agent $\alpha$, who in turn, makes a report on $\gamma \in \{G, M, B\}$ to the principal.
3. The principal makes a public announcement on $\gamma \in \{G, M, B\}$.
4. The project is implemented and transfers are paid according to $\Phi$.

Again, each agent can quit at any point in the time (the only relevant decision timing in this regard is after stage 3).\footnote{We implicitly assume that if only one agent quits, the project yields no value to the principal but she still has to pay the non-quitting agent according to the contract. Alternatively, we could assume that, as soon as one agent quits, the game ends and all players receive their outside option of zero, but this assumption has the disadvantage that it may yield an additional equilibrium outcome based on the coordination failure that agents reject the contracts, since each thinks the other one will do so.}

In the following two sections, we compare the principal’s maximum payoffs under centralization and delegation. We start analyzing a setup in which the principal cannot manipulate the aggregate information. In this case, the principal always prefers centralization over delegation. However, if efficient agents are relatively likely, the optimal contract under centralization provides the principal with an incentive to manipulate aggregate information. Taking the principal’s incentive to manipulate aggregate information seriously reveals that delegation dominates centralization when it is more likely that the agents are efficient, because in this case the principal’s manipulation incentive is strongest.
4 When the Principal Cannot Manipulate Information

4.1 Centralization

Under centralization, the agents report directly and simultaneously to the principal and are in symmetric positions. As a consequence, an optimal contract exhibits the symmetric structure, $t^a_\gamma = t^b_\gamma = t_\gamma$. Thus, under centralization, we can restrict attention to contracts of the form $(q_\gamma, t_\gamma)$, $\gamma \in \{G, M, B\}$.

In line with the Inada conditions for the value function, the principal wants a strictly positive size of the project regardless of the agents’ types. Since an agent can quit anytime, and in particular after the principal announces the project’s aggregate type $\gamma$, the pair $(q_\gamma, t_\gamma)$ must provide a non-negative rent to each agent for each $\gamma \in \{G, M, B\}$. For an efficient agent, the following participation constraints must be satisfied:

\[
t_G - \theta_g q_G \geq 0 \quad \text{(PC}_G\text{)}
\]

\[
t_M - \theta_g q_M \geq 0, \quad \text{(PC}_M\text{)}
\]

while the constraints below must be satisfied for an inefficient agent’s participation:

\[
t_M - \theta_b q_M \geq 0 \quad \text{(PC}_M\text{)}
\]

\[
t_B - \theta_b q_B \geq 0. \quad \text{(PC}_B\text{)}
\]

The left hand side of the participation constraints above are an agent’s ex post payoffs when he truthfully reports to the principal.

To induce each agent’s truthful report, the following Bayesian incentive compatibility conditions must be satisfied:

\[
\varphi [t_G - \theta_g q_G] + (1 - \varphi) [t_M - \theta_g q_M] \geq \varphi [t_M - \theta_g q_M] + (1 - \varphi) [t_B - \theta_g q_B], \quad \text{(IC}_g\text{)}
\]

\[
\varphi [t_M - \theta_b q_M] + (1 - \varphi) [t_B - \theta_b q_B] \geq \varphi [\max\{t_G - \theta_b q_G, 0\}] + (1 - \varphi) [t_M - \theta_b q_M]. \quad \text{(IC}_b\text{)}
\]

When reporting to the principal, each agent does not know the other agent’s type under centralization. Therefore, an agent’s incentive compatibility constraints are conditional only on his own private information. The left hand side of the constraints express the agent’s expected payoff from reporting truthfully, whereas the right hand side represents an agent’s expected payoff if he decides to misreport his type. Notice that, a misreporting agent may choose to quit. The participation constraints $(\text{PC}_M)$ and $(\text{PC}_B)$, however, imply that an efficient agent will not quit in the case of misrepresenting himself as inefficient, regardless
of the other agent’s type. An inefficient agent, however, may choose to quit after he mis-
represents himself as efficient, depending on the principal’s announcement on $\gamma$. Although
($PC_M$) implies that a misreporting inefficient agent will remain in the organization if the
principal announces that $\gamma = M$, but he may quit if $\gamma = G$ is announced—this is captured
by the first term of RHS in ($IC_b$).

Under centralization, the principal chooses $\Phi = \{q_\gamma, t_\gamma\}$, $\gamma \in \{G, M, B\}$, to solve the following problem:

$$\mathcal{P}^c: \max \limits_\Phi \pi(\Phi) = \varphi^2 [v(q_G) - 2t_G] + 2\varphi (1 - \varphi) [v(q_M) - 2t_M] + (1 - \varphi)^2 [v(q_B) - 2t_B],$$

subject to ($PC_G$) $\sim$ ($IC_b$). The following proposition presents the optimal outcome in $\mathcal{P}^c$.

Proposition 1 Suppose the principal cannot manipulate the aggregate information. Under
centralization, there exists $\varphi^+ \in (0, 1/2)$ and $\varphi^- \in (0, \varphi^+]$ such that the optimal contract,
$\Phi^c$, entails the following:

- For $\varphi > \varphi^+$,

$$v'(q_G^c) = \Theta_G, \quad v'(q_M^c) = \Theta_M + \frac{\varphi}{1 - \varphi} \Delta \theta, \quad v'(q_B^c) = \Theta_B + \frac{2\varphi}{1 - \varphi} \Delta \theta.$$  

An efficient agent gets strictly positive rent, and an inefficient agent gets no rent.

- For $\varphi \leq \varphi^-$,

$$v'(q_G^c) = \Theta_G, \quad 2\varphi v'(q_M^c) + (1 - 2\varphi) v'(q_B^c) = \Theta_B, \quad \text{where} \quad q_B^c = \frac{1 - 2\varphi}{1 - \varphi} q_M^c.$$  

An efficient agent gets strictly positive rent only when $\gamma = M$, and an inefficient agent
gets no rent.

As in the standard screening problems, with an exception of “no distortion at the top,”
the optimal project sizes are distorted downward. As is well-known, an efficient agent has
an incentive to exaggerate his cost of implementation to reap information rent, and in order
to reduce information rent while inducing truthful reports from the agents, the principal
distorts the project sizes in the optimal contract except when both agents are efficient.

When $\varphi$ is large enough ($\varphi > \varphi^+$), an efficient agent receives strictly positive information
rent regardless of the other agent’s type. When $\varphi$ is small ($\varphi \leq \varphi^-$), however, an efficient
agent receives rent only when he is paired with an inefficient agent. Since the agents of
different types receive the same amount of transfer when $\gamma = M$, the efficient agent’s rent
in that case is guaranteed regardless of $\varphi$. Because of this, the principal’s rent provision
when $\gamma = G$ is relatively smaller, and she decreases the amount of this rent as it becomes less likely that an agent is efficient. As a result, for $\varphi$ small enough, although an efficient agent’s expected rent is strictly positive, he gets no rent when the other agent is also efficient.

### 4.2 Delegation

Under delegation, agent $\beta$ reports his type, $\theta^\beta \in \{\theta_g, \theta_b\}$, to agent $\alpha$ who, in turn, reports the aggregate type, $\gamma \in \{G, M, B\}$, to the principal. Each agent’s participation constraints are:

$$t_G^k - \theta_g q_G \geq 0 \quad \text{(PC}_G^k)$$
$$t_M^k - \theta_g q_M \geq 0, \ k \in \{\alpha, \beta\}, \quad \text{(PC}_M^k)$$

for an efficient agent, and

$$t_M^k - \theta_b q_M \geq 0 \quad \text{(PC}_M^k)$$
$$t_B^k - \theta_b q_B \geq 0, \ k \in \{\alpha, \beta\}, \quad \text{(PC}_B^k)$$

for an inefficient agent. Notice that, unlike under centralization, the transfers to the agents cannot be treated symmetrically.

Since agent $\beta$ does not know agent $\alpha$’s type when reporting his own type, his incentive constraints coincide with the incentive constraints under centralization:

$$\varphi \left[ t_G^\beta - \theta_g q_G \right] + (1 - \varphi) \left[ t_M^\beta - \theta_g q_M \right] \geq \varphi \left[ t_M^\beta - \theta_g q_M \right] + (1 - \varphi) \left[ t_B^\beta - \theta_b q_B \right], \quad \text{(IC}_G^\beta)$$
$$\varphi \left[ t_M^\beta - \theta_b q_M \right] + (1 - \varphi) \left[ t_B^\beta - \theta_b q_B \right] \geq \varphi \left[ \max\{t_G^\beta - \theta_b q_G, 0\} \right] + (1 - \varphi) \left[ t_M^\beta - \theta_b q_M \right], \quad \text{(IC}_B^\beta)$$

Different to centralization is that, under delegation, agent $\alpha$ has more information when reporting to the principal, leading to stricter incentive constraints. More specifically, the Bayesian incentive conditions of agent $\beta$ above imply that agent $\alpha$, when he makes a report to the principal, has learned agent $\beta$’s type. Inducing agent $\alpha$’s truthful report, therefore, requires that the following incentive compatibility conditions be satisfied in the optimal contract:

$$t_G^\alpha - \theta_g q_G \geq t_\gamma^\alpha - \theta_g q_\gamma, \ \gamma \in \{M, B\}, \quad \text{(IC}_{G-\gamma}^\alpha)$$
$$t_M^\alpha - \theta_g q_M \geq t_\gamma^\alpha - \theta_g q_\gamma, \ \gamma \in \{G, B\}, \quad \text{(IC}_{M-\gamma}^\alpha)$$
$$t_M^\alpha - \theta_b q_M \geq t_\gamma^\alpha - \theta_b q_\gamma, \ \gamma \in \{G, B\}, \quad \text{(IC}_{M-\gamma}^\alpha)$$
$$t_B^\alpha - \theta_b q_B \geq t_\gamma^\alpha - \theta_b q_\gamma, \ \gamma \in \{G, M\}, \quad \text{(IC}_{B-\gamma}^\alpha)$$
These stricter incentive constraints reflect that, under delegation, agent $\alpha$ has more flexibility to manipulate information. Because agent $\alpha$ knows agent $\beta$’s type when making his report to the principal, the incentive constraints for agent $\alpha$, unlike the constraints for agent $\beta$, have to hold state-by-state rather than only in expected terms.

Under delegation, the principal, chooses $\Phi = \{q_\gamma, t^\alpha_\gamma, t^\beta_\gamma\}$ to solve the following problem:

$$\mathcal{P}^d: \max_{\Phi} \pi(\Phi) = \varphi^2 \left[ v(q_G) - \sum_k t^k_G \right] + 2\varphi(1-\varphi) \left[ v(q_M) - \sum_k t^k_M \right] + (1-\varphi)^2 \left[ v(q_B) - \sum_k t^k_B \right],$$

subject to $(PC^k_\gamma) \sim (IC^\alpha_{B-\gamma})$.

The following proposition presents the optimal outcome in $\mathcal{P}^d$.

**Proposition 2** Suppose the principal cannot manipulate the aggregate information. Under delegation, there exists $\hat{\varphi}^+ \in (0,1/2)$ and $\hat{\varphi}^- \in (0,\hat{\varphi}^+]$ such that the optimal outcome, $\Phi^d$, entails the following:

- For $\varphi > \hat{\varphi}^+$,

  $$v'(q^d_G) = \Theta_G, \quad v'(q^d_M) = \Theta_B + \frac{3\varphi - 1}{1 - \varphi} \Delta \theta, \quad v'(q^d_B) = \Theta_B + \frac{\varphi}{1 - \varphi} \Delta \theta.$$  

  An efficient agent gets strictly positive rent, and an inefficient agent gets no rent.

- For $\varphi \leq \hat{\varphi}^-$, $q^d_B = \frac{1 - 2\varphi}{1 - \varphi} q^d_M$. In addition, there exists $\tilde{\rho} > 1$ such that:

  If $\theta_\beta/\theta_\gamma > \tilde{\rho}$, then $v'(q^d_G) = \Theta_G$,  $2\varphi v'(q^d_M) + (1 - 2\varphi) v'(q^d_B) = \Theta_B + \Delta \theta \varphi^2$, and

  If $\theta_\beta/\theta_\gamma \leq \tilde{\rho}$, then $\varphi(2 - \varphi)v'(q^d_M) + (1 - \varphi)(1 - 2\varphi)v'(q^d_B) = \Theta_B$, where $q^d_M = q^d_G$.

  Agent $\alpha$, when he is efficient, gets strictly positive rent regardless of agent $\beta$’s type.

  Agent $\beta$, when he is efficient, gets strictly positive rent only for $\gamma = M$. An inefficient agent gets no rent.

While the reasoning behind the distorted project sizes is similar to the one under centralization, agent $\alpha$’s information rent is larger under delegation due to his stricter incentive constraints. By delegating the aggregation of information, the principal is relinquishing part of her control to agent $\alpha$. Since agent $\alpha$ ends up possessing more information and makes a report to the principal on behalf of both agents, he has more flexibility to manipulate information, which is the source of larger information rent under delegation. Recall that, for example, when $\varphi$ is small, an efficient agent under centralization receives rent only when
the other agent is inefficient. The same is true for agent $\beta$ under delegation since he does not know agent $\alpha$’s type when making his report. In contrast, the principal, regardless of agent $\beta$’s type, cannot fully extract agent $\alpha$’s information rent, because under delegation agent $\alpha$ knows agent $\beta$’s type when he reports to the principal.

4.3 Comparison

A direct comparison of the two propositions shows that different contracts are optimal under the different organizational structures. When the principal cannot manipulate the aggregate information, it is relatively straightforward to see that the principal does better under centralization. The intuition is, as mentioned above, that delegation transfers the principal’s control over agent $\beta$ to agent $\alpha$, without bringing her any benefits. A somewhat more technical perspective provides a deeper insight concerning the optimality of centralization, leading to a straightforward formal proof. Under delegation, the incentive constraints for the agent $\alpha$ induces a truthful report regardless of the other agent’s reporting strategy, whereas under centralization the incentive constraint induces a truthful report given the other agent’s reporting strategy. Hence, delegation leads to dominant strategy incentive compatibility constraints for agent $\alpha$’s truthtelling, while under centralization truthtelling leads to a Bayesian incentive compatibility constraint for him. Because Bayesian incentive compatibility constraints are weaker than incentive constraints in dominant strategies, the principal’s problem is less restricted under centralization. As a result, the allocation which the optimal contract under delegation, $\Phi^d$, implements is also feasible under centralization, whereas the allocation which optimal contract under centralization, $\Phi^c$, implements is not feasible under delegation. This observation leads directly to the following corollary.

**Corollary 1** Suppose the principal cannot manipulate the aggregate information. Then, centralization dominates delegation.

5 When the Principal Can Manipulate Information

In the previous section, we derived the optimal contracts under the assumption that, after receiving the agents’ reports, the principal truthfully announces the aggregate information from the agents. As we now argue, this assumption is not innocuous since the optimal contract under centralization, $\Phi^c$, provides the principal with an incentive to manipulate the aggregate information. In particular, the principal, after learning that both agents are inefficient, may benefit from misreporting aggregate costs as $\Theta_M$ rather than $\Theta_B$. Lack
of direct information flows between the agents prevents them from cross-checking their reports, and the principal can achieve such manipulation without being caught out by the agents—making each inefficient agent think that the other agent is efficient. In order to clarify this threat of aggregate information manipulation in centralized organizations, we start this section with revisiting the organization under centralization.

5.1 Centralization

The threat of aggregate information manipulation by the principal can be easily seen in the case where both types are equally likely (\( \varphi = 1/2 \)). In this case, Proposition 1 shows that the optimal contract under centralization, \( \Phi^c \), provides zero rent to an inefficient agent, i.e., \( t^c_B = \theta q^c_B \) and \( t^c_M = \theta q^c_M \). Hence, the principal’s ex post payoffs from a project size \( q_B \) and \( q_M \) are, respectively:

\[
v(q^c_B) - \Theta_B q^c_B \quad \text{and} \quad v(q^c_M) - \Theta_B q^c_M.
\]

Notice that, in Proposition 1, when both types are equally likely (\( \varphi = 1/2 \)), \( q^c_M \) coincides with \( q^*_B \), while \( q^c_B \) is strictly smaller than \( q^*_B \). Since \( q^*_B \) is the unique maximizer of \( v(q) - \Theta_B q \), it is implied that:

\[
v(q^*_B) - \Theta_B q^*_B < v(q^c_B) - \Theta_B q^c_B = v(q^c_M) - \Theta_B q^c_M.
\]

Thus, under the optimal contract \( \Phi^c \), the principal is strictly better off when reported aggregate types are \( \Theta_M \) instead of \( \Theta_B \) for \( \varphi = 1/2 \). We state this insight as the following lemma.

**Lemma 1** Suppose the principal can manipulate the aggregate information and \( \varphi = 1/2 \). Under centralization, the optimal contract, \( \Phi^c \), provides the principal with an incentive to misreport the aggregate type as \( \gamma = M \) when the true type is \( \gamma = B \).

When both agents are inefficient (\( \gamma = B \)), the principal’s announcement of \( \gamma = M \) cannot be detected by the agents as a misrepresentation—since it could very well be that the other agent is efficient, without directly cross checking their reports to the principal, each agent cannot tell whether or not the principal’s announcement is true.\(^5\)

Intuitively, the principal has an incentive to exaggerate the overall efficiency of the organization, because the agents then have to complete the bigger project \( q^*_M \) rather than

\(^5\)In Section 6, we show that if the principal can choose a communication technology before contracting with the agents, she may want to choose a technology that limits communication between the agents.
the smaller project $q_B$. The manipulation as described in the lemma represents a problem in organizations pointed out by management studies—top management’s abuses of its superior information at the expense of lower levels in the hierarchy.\footnote{See Bartolome (1989) for example.} In an organization where information flow is tightly centralized, the top management’s manipulating incentive is an issue since it has superior access to the organization’s bigger picture.

For the principal’s truthful behavior, the following incentive constraint must be satisfied in the optimal contract, in addition to the participation and incentive constraints for the agents:\footnote{Notice that the principal cannot misannounce $\gamma = B$ as $\gamma = G$ since the agents will detect the principal’s misrepresentation in that case. Likewise, when true $\gamma = M$, the principal cannot misrepresent the aggregate type as $\gamma = B$ or $\gamma = G$—if $\gamma = B$ is announced, then the type-$g$ agent will know the principal’s misannouncement, and if $\gamma = G$ is announced, then the type-$b$ agent will know. When $\gamma = G$, the principal can misannounce the aggregate type as $\gamma = M$, but she has no incentive to do so.}

$$v(q_B) - 2t_B \geq v(q_M) - 2t_M. \quad (PIC)$$

When the principal can manipulate the aggregate information, her problem under centralization is:

$$\hat{\mathcal{P}}^c: \max_{\Phi} \pi(\Phi) = \varphi^2 [v(q_G) - 2t_G] + 2\varphi(1 - \varphi) [v(q_M) - 2t_M] + (1 - \varphi)^2 [v(q_B) - 2t_B],$$

subject to $(PIC)$ and the constraints in $\mathcal{P}^c$.

The next lemma makes precise when the principal’s incentive constraint $(PIC)$ matters.

**Lemma 2** Suppose the principal can manipulate the aggregate information. Under centralization, there exist $\varphi^- \in (0, 1/2)$ and $\varphi^+ \in [\varphi^-, 1/2)$ such that:

- For $\varphi < \varphi^-$, the principal’s manipulating incentive is not an issue, i.e., $(PIC)$ is non-binding.
- For $\varphi > \varphi^+$, the principal’s manipulating incentive is an issue, i.e., $(PIC)$ is binding.

The intuition behind Lemma 2 is that the distortion in the project size depends on the likelihood that the agents are efficient. Indeed, when such likelihood is small, the principal expects to provide information rent only with a small probability—for small $\varphi$, distortions in the optimal project sizes are also small. More specifically, for $\varphi$ small enough, $(\varphi < \varphi^-)$, unlike in the case of $\varphi = 1/2$, the project size $q_B^*$ is closer to the first best level $q_B^*$ than $q_M^*$. As a result, the principal has no incentive to manipulate the aggregate information when both agents reports that they are inefficient.
By contrast, when the likelihood that the agents are efficient is high, the probability that the principal has to provide information rent is also high. As a result, distortions in the optimal contract to reduce information rent becomes large. For $\varphi$ large enough ($\varphi > \varphi^+$), as in the case of $\varphi = 1/2$, the project size $q^*_B$ is further away from the first best level $q^*_B$ than $q^*_M$. When both agents report that they are inefficient, the principal prefers to implement project size $q^*_M$ rather than $q^*_B$ in such cases, which leads to her incentive to misrepresent the aggregate information as $\gamma = M$.

An obvious way to dispel the principal’s manipulating incentive is to set the same project sizes for $q_M$ and $q_B$. By doing so, she makes herself indifferent to her announcement on the aggregate information (between $\gamma = B$ and $\gamma = M$ when true $\gamma = B$), and her incentive constraint ($PIC$) is trivially satisfied. The following proposition shows that, when agents are more likely to be efficient, such pooling of project sizes is indeed an optimal to response to the principal’s manipulating incentive.

**Proposition 3** Suppose the principal can manipulate the aggregate information and $\varphi \geq 1/2$. Under centralization, the optimal outcome, $\Phi^*$, entails:

$$v'(\bar{q}_G) = \Theta_G, \quad v'(\bar{q}_M) = v'(\bar{q}_B) = \Theta_B + \frac{2\varphi^2}{1 - \varphi^2} \Delta \theta.$$

An efficient agent gets strictly positive rent, and an inefficient agent gets no rent.

As shown above, under centralization, the principal’s incentive to manipulate the aggregate information arises when it is likely enough that an agent is efficient, and in such a case, the optimal contracts must discourage the principal from misrepresentation. In coping with her own manipulating incentive, the principal may pool the project sizes $q_B$ and $q_M$ in the optimal contract. Proposition 3 shows that such pooling is optimal when it is more likely that the agents are efficient. The optimality of pooling is due to the fact that a separating contract requires the principal to concede larger information rent when both agents are efficient. That is, when the agents are more likely to be efficient, the principal’s own manipulating incentive makes it harder to fine-tune the optimal project sizes according to the available information in the organization.

### 5.2 Delegation and Comparison

Under delegation, the principal receives the aggregate information directly from agent $\alpha$. Any manipulation of the information by the principal is therefore directly detectable by agent $\alpha$, which prevents the principal from misrepresenting the aggregate information. Thus,
the same optimal outcome as in $P^d$ is achieved. Recall from the previous section that, in the absence of the principal’s manipulating incentive, delegation is always dominated by centralization—under delegation, the principal simply needs to provide more information rent to agent $\alpha$, who is granted the authority to aggregate information. In the presence of the principal’s manipulating incentive, however, a trade-off between these structures arises.

Proposition 4 Suppose the principal cannot manipulate the aggregate information. Then, there exists $\varphi^c > \varphi^-$ and $\varphi^d > \varphi^+$ such that:

- For $\varphi < \varphi^c$, centralization dominates delegation.
- For $\varphi \geq \varphi^d$, delegation dominates centralization.

As shown in Lemma 2, the principal’s manipulating incentive arises only when the likelihood that the agents are efficient, $\varphi$, is large enough. Therefore, for $\varphi$ small, centralization remains the prevailing structure. As $\varphi$ becomes larger, the principal’s manipulation incentive arises, and a trade-off between the two structures starts to emerge. Under delegation, although the principal must provide more information rent due to a loss of control, the optimality of separating types demonstrates that delegation allows the principal to use the available information within the organization more effectively than centralization.

6 Unlimited Communication and Collusion

In modeling centralization, we postulated that the agents cannot directly communicate with each other. This limitation on direct communication between agents is crucial for our result, because the type of information manipulation that we consider is avoidable when the agents can directly communicate with each other—the agents could then, by simply cross-checking their reports, detect the principal’s manipulation of the aggregate information.

Even though these limits on communication seem natural in large organizations, where it is infeasible for an agent to cross-check the reports of all other agents, we provide in this section an endogenous argument for organizations to restrict such unlimited communication. The gist of this argument is that allowing direct communication between agents may invite collusion, and dealing with such collusion is more costly to the principal than dealing with her own manipulating incentive. Indeed, organization studies point out that communication facilitates collusion, stressing that group behaviors are frequently observed in organizations.
where communication among their members are less restricted.\footnote{See Mintzberg (1979) for example.} Organization theory also points out the connection of unwanted communication and collusion among agents.\footnote{See Laffont and Rochet (1997) among others.}

To see the potential of collusion under centralization, recall from Proposition 1 that, without information manipulation, the optimal contract under centralization, $\Phi^c$, yields an efficient agent a strictly larger payoff when the other agent is inefficient than when the other agent is efficient:

$$2t_M^c - \Theta_Gq_G^c > 2t_G^c - \Theta_Gq_G^c. \quad \text{(CIC)}$$

This inequality implies that, when both are efficient, the agents can increase their payoff if they coordinate their reports such that one of them reports to be efficient, while the other misreports his type as inefficient. An implementation of this collusive agreement requires however communication between agents for some coordination to learn each other’s types—given $\Phi^c$, an efficient agent has no incentive to misreport his type as inefficient to the principal unless he knows that the other agent will report his type as efficient.

To analyze collusion under asymmetric information, we follow Laffont and Martimort (1997) and introduce a third party side-contractor who, given the principal’s “grand contract”, coordinates collusion between asymmetrically informed agents. The side-contractor’s objective is to maximize the expected joint payoff of the agents. Given the principal’s contract under centralization, $\Phi = \{q, t, \gamma\}$, $\gamma \in \{G, M, B\}$, the side-contractor’s offer to the agents specifies a collusive reporting function to the principal,

$$\hat{\gamma}: \{g, b\} \times \{g, b\} \rightarrow \{G, M, B\},$$

with the interpretation that if agent $\alpha$ reports type $\theta^\alpha \in \{g, b\}$ to the side-contractor and agent $\beta$ reports type $\theta^\beta \in \{g, b\}$, then the side-contractor reports $\hat{\gamma}(\theta^\alpha, \theta^\beta)$ to the principal. Laffont and Martimort (1997) allows the side-contract to specify side-transfers, but, in our framework, the threat of collusion has bite without side-transfers. Hence, our concept of collusion is weaker than the concept in Laffont and Martimort (1997).\footnote{The weaker the concept of collusion, the easier the principal can prevent it. Hence, showing that collusion is already problematic in this weaker form emphasizes the problem of collusion. Indeed, in our proof we consider an even weaker form of collusion because we impose the additional restriction that the side-contractor treats the agents equally.}

Indeed, as formally shown in the next proposition, a necessary condition for the principal’s contract to be collusion-proof is:

$$t_G - \theta_gq_G \geq t_M - \theta_gq_M. \quad \text{(CIC)}$$
As a result, an upperbound on the principal’s expected payoff is the solution of the following problem:

$$\mathcal{P}^u: \max_{\Phi} \pi(\Phi) = \varphi^2 [v(q_G) - 2t_G] + 2\varphi(1 - \varphi) [v(q_M) - 2t_M] + (1 - \varphi)^2 [v(q_B) - 2t_B],$$

subject to (CIC) and the constraints in $\mathcal{P}^c$. Comparing the optimal outcome in $\mathcal{P}^u$ to those in the previous sections leads to the following result.

**Proposition 5** *Suppose the organization’s communication technology is the principal’s choice and unlimited communication between the agents enables the agents to collude. Then it is suboptimal to allow unlimited communication between them.*

As mentioned above, although unlimited communication between the agents removes the principal’s manipulating incentive under centralization, it provides the agents with more flexibility to manipulate their private information through collusion. Our result here shows that although limiting communication among subunits in an organization causes top management’s manipulating incentive under centralization, it is less costly to the organization since unlimited communication among subunits opens the door to collusive behavior that lowers the organization’s optimal outcome.

7 Conclusion

In this paper, we have analyzed the optimal structure of an organization when information can be manipulated, not only by the agents who possess private information, but also by the principal who aggregates the information. Under centralization, a tension between the principal’s and the agent’s incentives arises, which may lead to pooling in the optimal contract—under centralization, an organization prone to aggregate information manipulation cannot use all the available information of its subunits effectively. Under delegation, although the principal must provide more rent to the agent to whom is delegated the information aggregation, the optimal contract is separating—under delegation, an organization can use the information of its subunits more effectively. The trade-off between information rent and the effective use of information determines the optimal structure of the organization. Its outcome depends on the extent to which the agents’ private information leads to distortions, and therefore the likelihood that agents are efficient—centralization is optimal when such likelihood is low, whereas delegation is optimal when it is high.
Appendix

Proof of Proposition 1.

Instead of solving $\mathcal{P}^c$, we first solve the relaxed problem

$$\max_{\Phi} \pi(\Phi) \text{ s.t. } (IC_g), (PC_M), (PC_B).$$

First note that since $\pi(\Phi)$ is strictly decreasing in $t_G$, the constraint $(IC_g)$ binds for any solution of this relaxed problem—since otherwise one could raise the objective by lowering $t_G$ without affecting $(IC_g)$ and $(PC_M)$. Second, note that since $\pi(\Phi)$ is strictly decreasing in $t_B$, also $(PC_B)$ binds for any solution—since otherwise one could raise the objective by lowering $t_B$, as this change relaxes $(IC_g)$ and does not affect $(PC_M)$. Finally, also $(PC_M)$ binds for any solution, since otherwise one could lower $t_M$ by $\delta > 0$ and raise $t_G$ by $(1 - 2\varphi)/\varphi\delta$. This change does not affect $(IC_g)$ and $(PC_B)$, but raises the objective by $2\varphi\delta$.

A binding $(PC_M), (PC_B)$ and $(IC_g)$ give the following expressions for the transfers:

$$t_G = \theta_g q_G + \frac{2\varphi - 1}{\varphi} \Delta \theta q_M + \frac{1 - \varphi}{\varphi} \Delta \theta q_B, \quad t_M = \theta_b q_M, \quad t_B = \theta_b q_B. \quad (A1)$$

Substituting these transfers in the objective $\pi(\Phi)$ and optimizing with respect to the project sizes gives:

$$v'(q_G^c) = \Theta_G, \quad v'(q_M^c) = \Theta_M + \frac{\varphi}{1 - \varphi} \Delta \theta, \quad v'(q_B^c) = \Theta_B + \frac{2\varphi}{1 - \varphi} \Delta \theta, \quad (A2)$$

implying that $q_G^c > q_M^c > q_B^c$.

We next check whether this solution to the relaxed problem also satisfies the neglected constraints, $(PC_M), (IC_b)$, and $(PC_G)$. Notice first that $(PC_M)$ implies $(PC_M)$. Also, by $(A1)$ the constraint $(IC_b)$ simplifies to:

$$0 \geq \varphi \max\{0, t_G - \theta_b q_G^c\},$$

which holds because, by $(A1)$ and $q_G^c > q_M^c > q_B^c$, it follows that:

$$t_G - \theta_b q_G^c = [(2\varphi - 1)(q_M^c - q_G^c) + (1 - \varphi)(q_B^c - q_G^c)] \Delta \theta/\varphi < 0.$$

Finally, to check $(PC_G)$, let $f^c(\varphi) \equiv (2\varphi - 1)q_M^c + (1 - \varphi)q_B^c$, so that the relaxed solution satisfies $(PC_G)$ if and only if $f^c(\varphi) \geq 0$. Because for any $\varphi \in [1/2, 1)$, it holds $f^c(\varphi) > 0$ and since $f^c(0) = q_B^c - q_M^c < 0$, continuity implies that there exists at least one $\tilde{\varphi} \in (0, 1/2)$
such that $f^c(\tilde{\varphi}) = 0$. Let $\tilde{\varphi}^+ \in (0, 1/2)$ be the largest (supremum) $\tilde{\varphi}$ such that $f^c(\tilde{\varphi}) = 0$, and let $\tilde{\varphi}^- \in (0, 1/2)$ be the smallest (infimum) $\tilde{\varphi}$ such that $f^c(\tilde{\varphi}) = 0$.

Hence, (A1) and (A2) characterize the principal’s optimal contract for any $\varphi \geq \tilde{\varphi}^+$.

Since, for the case $\varphi < \tilde{\varphi}^-$, the above characterization violates $(PC_G)$, we next consider the (less) relaxed problem

$$\max_{\Phi} \pi(\Phi) \text{ s.t. } (IC_g), (PC_M), (PC_B), (PC_G),$$

where we know that, given $\varphi < \tilde{\varphi}^-$, the constraint $(PC_G)$ binds for any solution. Repeating the arguments of the beginning of this proof shows that, again, $(PC_B)$ and $(PC_M)$ bind at any solution of this (less) relaxed problem. Given that $(PC_M), (PC_B)$, and $(PC_G)$ bind, also $(IC_g)$ binds, since maximizing the relaxed problem when disregarding $(IC_g)$ yields the candidate solution $q_G = q_G^*$; $q_M = q_B^*, q_B = q_B^*$, which violates $(IC_g)$. Hence, for any solution (A1) holds. Together with $(PC_G)$ binding, this implies that $(1 - \varphi)q_B = (1 - 2\varphi)q_M$.

It follows that, with constraints $(IC_g), (PC_M), (PC_B)$, and $(PC_G)$ all binding, we can rewrite the principal’s problem as:

$$\max_{q, \varphi^2} [v(q_G) - \Theta_G q_G] + 2\varphi(1 - \varphi) [v(q_M) - \Theta_B q_M] + (1 - \varphi)^2 [v(q_B) - \Theta_B q_B q_M], (A3)$$

where

$$q_B(q_M) = \frac{1 - 2\varphi}{1 - \varphi} q_M.$$

Substituting out $q_B(q_M)$ in (A3) and optimizing with respect to the project sizes yields:

$$v'(q_G^*) = \Theta_G, \quad 2\varphi v'(q_M^*) + (1 - 2\varphi)v'(q_B^*) = \Theta_B, \text{ where } q_B^* = \frac{1 - 2\varphi}{1 - \varphi} q_M^*.$$ 

To check $(IC_b)$, note again that it is satisfied if $t_G - \theta bq_B^* \leq 0$. Using (A1) and the relationship $(1 - \varphi)q_B^* = (1 - 2\varphi)q_M^*$, we have:

$$t_G - \theta bq_B^* = -\Delta \theta q_G < 0.$$ 

Thus, as specified in the proposition, for both $\varphi < \tilde{\varphi}^-$ and $\varphi \geq \tilde{\varphi}^+$ we have characterized the optimal contract. The agents’ rents follow from the binding constraints.

**Proof of Proposition 2.**

Similar to the proof of Proposition 1, we make a conjecture about the relevant constraints and optimize the objective function under this subset of constraints. We then verify whether the solution satisfies the other constraints. In particular, we conjecture that incentive
constraints, \((IC^\alpha_g)\) and \((IC^\alpha_{B-M})\), and the participation constraints, \((PC^\alpha_M)\), \((PC^\alpha_B)\), \((PC^\beta_M)\) and \((PC^\beta_B)\) are binding. This yields the following expressions for transfers:

\[
\begin{align*}
    t^G_G &= \theta_g q_G + \Delta_\theta q_M, \\
    t^G_M &= \theta_g q_M, \\
    t^B_B &= \theta_b q_B.
\end{align*}
\]  

(A4)

After substituting these transfers in the objective function, an unconstrained optimization over the remaining variables yields:

\[
v'(q^d_G) = \Theta_G, \quad v'(q^d_M) = \Theta_B + \frac{3\varphi - 1}{2(1 - \varphi)} \Delta_\theta, \quad v'(q^d_B) = \Theta_B + \frac{\varphi}{1 - \varphi} \Delta_\theta,
\]  

(A5)

implying that \(q^d_G > q^d_M > q^d_B\). Since \(\theta_g < \theta_b\), (A4) implies that \((PC^\alpha_G)\), \((PC^\alpha_M)\) and \((PC^\beta_M)\) are satisfied. Also, (A4) together with \(q^d_G > q^d_M > q^d_B\) implies that \((IC^\alpha_{G-B})\), \((IC^\alpha_{M-\gamma})\), \((IC^\alpha_{B-\gamma})\) and \((IC^\beta_B)\) are satisfied. Hence, it remains to check whether the solution also satisfies \((PC^\beta_G)\). Using (A4), it holds \(t^\beta_M - \theta_g q_G \geq 0\) if and only if \((2\varphi - 1)q^d_M + (1 - \varphi)q^d_B \geq 0\). Hence, let \(f^d(\varphi) \equiv (2\varphi - 1)q^d_M + (1 - \varphi)q^d_B\), so that this solution satisfies \((PC^\beta_G)\) only if \(f^d(\varphi) \geq 0\). Because for any \(\varphi \in [1/2, 1]\), it holds \(f^d(\varphi) > 0\) and since \(f^d(0) = q^*_B - q^*_M < 0\), continuity implies that there exists at least one \(\hat{\varphi} \in (0, 1/2)\) such that \(f^c(\hat{\varphi}) = 0\). Let \(\hat{\varphi}^+ \in (0, 1/2)\) be the largest (supremum) \(\hat{\varphi}\) such that \(f^c(\hat{\varphi}) = 0\), and let \(\hat{\varphi}^- \in (0, 1/2)\) be the smallest (infimum) \(\hat{\varphi}\) such that \(f^c(\hat{\varphi}) = 0\).

Then, it follows that, for \(\varphi > \hat{\varphi}^+\), (A4) together with (A5) fully characterizes the optimal contract as presented in Proposition 2. For \(\varphi < \hat{\varphi}^-\), the solution characterized above violates \((PC^\beta_G)\), implying that this participation constraint also binds at the optimum. Under (A4) the constraint \((PC^\beta_G)\) simplifies to:

\[
(1 - \varphi)q_B = (1 - 2\varphi)q_M.
\]  

(A6)

With \((IC^\beta_g)\), \((IC^\beta_{G-M})\), \((PC^\alpha_M)\), \((PC^\alpha_B)\), \((PC^\beta_M)\), \((PC^\beta_B)\) and \((PC^\beta_G)\) binding, the principal's problem rewrites as:

\[
\max_{q_G, q_M, q_B} \varphi^2 [v(q_G) - \Theta_G q_G - \Delta_\theta q_M] + 2\varphi(1-\varphi) [v(q_M) - \Theta_B q_M] + (1-\varphi)^2 [v(q_B(q_M)) - \Theta_B q_B(q_M)],
\]

where \(q_B(q_M) = (1 - 2\varphi)q_M/(1 - \varphi)\) from (A6). The first order conditions with respect to \(q_G\) and \(q_M\) imply that the optimal project sizes are characterized by:

\[
v'(q^d_G) = \Theta_G, \quad 2\varphi v'(q^d_M) + (1 - 2\varphi)v'(q^d_B) = \Theta_B + \varphi^2 \Delta_\theta \text{ and } (1 - \varphi)q^d_B = (1 - 2\varphi)q^d_M, \quad (A7)
\]

implying that \(q^d_M > q^d_B\). Unlike centralization where the optimal contract only needs to be Bayesian incentive compatible, delegation requires that the optimal contract be incentive
compatible in dominant strategy for agent $\alpha$ since he learns agent $\beta$'s type when he makes a report to the principal. Thus, it is needed that $q'^d_{G} \geq q'^d_{M} \geq q'^d_{B}$ to satisfy all of the ignored constraints. In fact, the solution characterized in (A7) may not satisfy $(IC_{M-G}^{\alpha})$ and $(IC_{B-G}^{\alpha})$ if $q'^d_{G} \geq q'^d_{M}$ does not hold, since $(IC_{M-G}^{\alpha})$ and $(IC_{B-G}^{\alpha})$ with the transfers for agent $\alpha$ in (A4) require that:

$$0 \geq \Delta \theta(q_{M} - q_{G}).$$

We next show that for any $\varphi \leq \hat{\varphi}^-$, there exists $\hat{\rho} > 1$ such that $q'^d_{G} \geq q'^d_{M}$ if and only if $\theta_{b}/\theta_{g} \geq \hat{\rho}$. From (A7), $q'^d_{G}$ is defined by $v'(q'^d_{G}) = \Theta_{G}$, while $q'^d_{M}$ is implicitly defined by the following equation:

$$2\varphi v'(q'^d_{M}) + (1 - 2\varphi)v'((1 - 2\varphi)q'^d_{M}/(1 - \varphi)) = \rho \Theta_{G} + \varphi^2(\rho - 1)\theta_{g},$$

(A8)

where $\rho \equiv \theta_{b}/\theta_{g} > 1$. Note that for $\rho = 1$ we have $q'^d_{M} < q'^d_{G}$, while for $\rho$ large enough we have $q'^d_{M} < q'^d_{G}$. The result then follows from noting that $\partial q'^d_{M}/\partial \rho < 0$, so there exists a unique $\hat{\rho} > 1$ such that $q'^d_{G} > q'^d_{M}$ if and only if $\rho > \hat{\rho}$. To see $\partial q'^d_{M}/\partial \rho < 0$, note that by the implicit function theorem, it follows from (A8) that:

$$\frac{\partial q'^d_{M}}{\partial \rho} \left[ 2\varphi v''(q'^d_{M}) + \frac{(1 - 2\varphi)^2}{1 - \varphi}v''(q'^d_{B}) \right] = \Theta_{G} + \varphi^2\theta_{g},$$

and since $v''(\cdot) < 0$, the term within the bracket in the LHS of the equation is negative. Since the RHS of the equation is positive, it follows that $\partial q'^d_{M}/\partial \rho < 0$. Hence, for $\varphi < \hat{\varphi}^-$ and $\theta_{g}/\theta_{b} > \hat{\rho}$, the solution in (A7) characterizes the optimal project sizes.

For $\varphi < \hat{\varphi}^-$ and $\theta_{g}/\theta_{b} \leq \hat{\rho}$ the solution in (A7) violates $(IC_{M-G}^{\alpha})$ and $(IC_{B-G}^{\alpha})$ implying that $q_{G} = q_{M}$ in the optimal contract, and from all binding constraints, we have:

$$t'^\alpha_{G} = \theta_{b}q_{M} = \theta_{g}q_{M} + \Delta \theta q_{M}, \quad t'^\beta_{G} = \theta_{g}q_{M},$$

$$t'^\alpha_{M} = \theta_{b}q_{M}, \quad t'^\beta_{M} = \theta_{b}q_{M},$$

$$t'^\beta_{B} = \theta_{b}q_{B}.$$

After substituting for the transfers in the objective function, the principal’s problem is to maximize:

$$[1 - (1 - \varphi)^2] \left[ v(q_{M}) - \Theta_{B}q_{M} \right] + (1 - \varphi)^2 \left[ v(q_{B}) - \Theta_{B}q_{B} \right],$$

subject to (A6). It follows that the optimal project sizes are characterized by:

$$\varphi(2 - \varphi)v'(q'^d_{M}) + (1 - \varphi)(1 - 2\varphi)v'(q'^d_{B}) = \Theta_{B}, \quad \text{where } q'^d_{B} = \frac{1 - 2\varphi}{1 - \varphi}q'^d_{M}. \blacksquare$$
Proof of Corollary 1.

The proof directly follows from comparing $\mathcal{P}^c$ and $\mathcal{P}^d$. The incentive compatibility constraints in $\mathcal{P}^d$ are stronger and therefore the principal’s choices are more restricted in $\mathcal{P}^d$ compared to $\mathcal{P}^c$. ■

Proof of Lemma 1.

The proof directly follows from the discussion. ■

Proof of Lemma 2.

In order to show that there exists $\varphi^- \in (0,1/2)$ such that the constraint $(PIC)$ does not bind, we verify that the optimal contract as identified in Proposition 1 satisfies $(PIC)$ for all $\varphi$ smaller than some $\varphi^- > 0$. To see this, first recall from Proposition 1 that, for $\varphi \in (0,\bar{\varphi})$, the solution is characterized by:

$$2t_B^c = \Theta_B q_B^c; \quad 2t_M^c = \Theta_B q_M^c; \quad 2\varphi v'(q_M^c) + (1 - 2\varphi)v'(q_B^c) = \Theta_B; \quad \text{and} \quad q_B^c = \frac{1 - 2\varphi}{1 - \varphi} q_M^c. \quad (A9)$$

Hence, for $\varphi \to 0$ we have $q_B^c = q_M^c = q_B^*$, and with these values, $(PIC)$ is satisfied in equality. Using this, we show that $(PIC)$ is non-binding for $\varphi$ small enough. Defining the function $q_M(x) = (1 - \varphi)x/(1 - 2\varphi)$, (A9) implies that $q_B^*$ is implicitly defined by:

$$2\varphi v'(q_M(q_B^*)) + (1 - 2\varphi)v'(q_B^*) = \Theta_B.$$

Differentiating the expression with respect to $\varphi$ yields:

$$2v'(q_M) + 2\varphi v''(q_M) \left[ \frac{1}{(1 - 2\varphi)^2 q_B^*} + \frac{1 - \varphi}{1 - 2\varphi} \frac{\partial q_B^*}{\partial \varphi} \right] - 2v'(q_B^*) + (1 - 2\varphi)v''(q_B^*) \frac{\partial q_B^*}{\partial \varphi} = 0.$$

Thus, we have:

$$\frac{\partial q_B^*}{\partial \varphi} \bigg|_{\varphi=0} = \frac{2[v'(q_B^*) - v'(q_M^*)]}{v''(q_B^*)} \bigg|_{\varphi=0} = \frac{2[v'(q_B^*) - v'(q_M^*)]}{v''(q_B^*)} = 0,$$

where the second equality follows from $q_B^* = q_M^* = q_B^*$ for $\varphi = 0$. Now, differentiating the last equation in (A9), we have:

$$\frac{\partial q_M^c}{\partial \varphi} = \frac{1 - 2\varphi}{1 - \varphi} \frac{\partial q_M^c}{\partial \varphi} - \frac{1}{1 - \varphi} q_M^c,$$

and therefore:

$$\frac{\partial q_M^c}{\partial \varphi} \bigg|_{\varphi=0} = \frac{1}{1 - \varphi} q_B^* > 0,$$
since $\partial q_B^c / \partial \varphi = 0$ and $q_M^c = q_B^c$ at $\varphi = 0$. That is, at $\varphi = 0$, (PIC) is satisfied with $q_B^c = q_M^c = q_B^*$ and $\partial q_M^c / \partial \varphi > 0 = \partial q_B^c / \partial \varphi$, which implies that (PIC) is strictly satisfied for $\varphi > 0$ close to zero. Since $\Phi^c$ violates (PIC) at $\varphi = 1/2$ from Lemma 1, there exists $\varphi^- \in (0, 1/2)$ such that (PIC) is satisfied for $\varphi < \varphi^-$. 

To see that $\Phi^c$ violates the constraint for $\varphi \geq 1/2$, consider $q_M^c$ characterized in Proposition 1. Again, at $\varphi = 1/2$, we have $q_M^c = q_B^c$ and by Lemma 1, (PIC) is violated. By the implicit function theorem, it follows for $\varphi > 1/2$ that:

$$\frac{\partial q_M^c}{\partial \varphi} = \frac{\Delta \theta}{\nu'(q_M^c)(1-\varphi)^2} < 0,$$

where the inequality follows from $\nu''(\cdot) < 0$. As a result, we have for $\varphi > 1/2$ that $q_B^* > q_M^c$. Also, Proposition 1 implies $q_M^c > q_B^c$, and thus it follows from the concavity of $v(q) - \Theta bq$ that the ranking $q_B^* > q_M^c > q_B^c$ implies:

$$\max_q v(q) - \Theta Bq = v(q_B^c) - \Theta Bq_B^c > v(q_M^c) - \Theta Bq_M^c > v(q_B^c) - \Theta Bq_B^c.$$

This establishes that (PIC) is violated for all $\varphi \geq 1/2$. By continuity, there exists some $\varphi^+ \in [\varphi^-, 1/2)$ such that (PIC) is violated for all $\varphi > \varphi^+$. ■

**Proof of Proposition 3.**

For $\varphi \geq 1/2$, Lemma 2 shows that (PIC) is a binding constraint at the optimum. Since $(IC_2)$, $(PIC)$, and $(PC_B)$ are also binding, binding (PIC) can be rewritten as:

$$v(q_B) - \Theta Bq_B = v(q_M) - \Theta Bq_M,$$

and hence the principal’s payoff $\pi(\Phi)$ can be rewritten as:

$$\varphi^2 \left[ v(q_G) - \Theta Gq_G - 2 \left( \frac{2\varphi - 1}{\varphi} \Delta \theta q_M + \frac{1 - \varphi}{\varphi} \Delta \theta q_B \right) \right] + (1 - \varphi)^2 \left[ v(q_B) - \Theta Bq_B \right],$$

which is to be maximized subject to (A10). Note that for $\varphi = 1/2$ the objective function simplifies to:

$$[v(q_G) - \Theta Gq_G - \Delta \theta q_B] / 4 + 3 [v(q_B) - \Theta Bq_B] / 4,$$

which is independent of $q_M$. Maximizing this expression with respect to $q_G$ and $q_B$, and setting $q_M = q_B$ satisfies (A10) and yields a maximizer that coincides with the expression in the proposition.

We next show that, for $\varphi > 1/2$, a solution satisfies $q_M = q_B$. To see this, note first that, for $\varphi > 1/2$, expression (A11) is strictly decreasing in $q_M$. Moreover note that (A10) is
satisfied whenever $q_M = q_B$. These two observations imply that project sizes with $q_M > q_B$ are not optimizing (A11), since it yields less payoff than project sizes with $q_M = q_B$. Likewise, $q_B > q_M$ is not optimal for the following reason. Using (A10), we can express (A11) as:

$$\varphi^2 \left[ v(q_G) - \Theta_G q_G - 2 \left( \frac{2\varphi - 1}{\varphi} \Delta \theta q_M + \frac{1 - \varphi}{\varphi} \Delta \theta q_B \right) \right] + (1 - \varphi^2) [v(q_M) - \Theta_B q_M]. \tag{A12}$$

Thus, the solution maximizes (A12) subject to (A10). Note however that (A12) is decreasing in $q_B$. Project sizes with $q_B > q_M$ does not maximize (A12) subject to (A10), since it yields less than project sizes with $q_B = q_M$ which satisfies (A10).

For an optimal solution, we therefore have $q_B = q_M$ so that (A10) is satisfied and (A11) simplifies to:

$$\varphi^2 \left[ v(q_G) - \Theta_G q_G + 2 \Delta \theta q_M \right] + (1 - \varphi^2) [v(q_G) - \Theta_B q_M].$$

Again, optimizing with respect to $q_G$ and $q_M$ and setting $q_B = q_M$ yields the expression in the proposition. 

**Proof of Proposition 4.**

From Lemma 2, $\pi(\Phi^c) = \pi(\Phi^d)$ for $\varphi \leq \varphi^-$, and hence by Corollary 1, $\pi(\Phi^c) > \pi(\Phi^d)$ at $\varphi = \varphi^-$. Continuity then implies the existence of $\varphi^c > \varphi^-$, such that for $\varphi \leq \varphi^c$, $\pi(\Phi^c) \geq \pi(\Phi^d)$. To see the existence of $\varphi^d$, recall first from Proposition 3 that, for $\varphi \geq 1/2$, the optimal $q_M$ and $q_B$ are bunched in $\Phi^c$. For $\varphi \geq 1/2$, it can be easily verified that $\Phi^c$ satisfies all constraints in $P^d$, and hence can be implemented in $P^d$. Since $\Phi^c \neq \Phi^d$ and $\Phi^c$ is not a solution to $P^d$, it follows, for $\varphi \geq 1/2$, that $\pi(\Phi^d) > \pi(\Phi^c)$. By continuity there exists a $\varphi^d > \varphi^+$ such that for all $\varphi > \varphi^d$, $\pi(\Phi^d) \geq \pi(\Phi^c)$. 

**Proof of Proposition 5.**

As noted in footnote 10, imposing more constraints on the side contractor relaxes the collusion proofness constraints on the principal. Since our objective is to obtain an upperbound of the principal’s expected payoff under collusion (to compare that expected payoff with the principal’s expected payoff in $\tilde{P}^c$ and $P^d$), we impose the additional constraint that the side-contractor treats the agents equally—in particular, $\tilde{\gamma}(g,b) = \tilde{\gamma}(b,g) = \tilde{\gamma}(M)$, as well as $\tilde{\gamma}(g,g) = \tilde{\gamma}(G)$ and $\tilde{\gamma}(b,b) = \tilde{\gamma}(B)$. Thus, given the principal’s contract $\Phi$, the restricted side-contract is $\phi = \tilde{\gamma}(\gamma)$, $\gamma, \gamma \in \{G, M, B\}$. The side-contract is Bayesian incentive
compatible if the following conditions hold:

\[
\varphi[t\bar{\gamma}(G) - \theta_g q\bar{\gamma}(G)] + (1 - \varphi)[t\bar{\gamma}(M) - \theta_g q\bar{\gamma}(M)] \\
\geq \varphi[t\bar{\gamma}(M) - \theta_g q\bar{\gamma}(M)] + (1 - \varphi)[t\bar{\gamma}(B) - \theta_g q\bar{\gamma}(B)]
\]  
(A13)

\[
\varphi[t\bar{\gamma}(M) - \theta_b q\bar{\gamma}(M)] + (1 - \varphi)[t\bar{\gamma}(B) - \theta_b q\bar{\gamma}(B)] \\
\geq \varphi[t\bar{\gamma}(G) - \theta_b q\bar{\gamma}(G)] + (1 - \varphi)[t\bar{\gamma}(M) - \theta_b q\bar{\gamma}(M)]
\]  
(A14)

The participation in the side-contracting requires that:

\[
\varphi \left[ t\bar{\gamma}(G) - \theta_g q\bar{\gamma}(G) \right] + (1 - \varphi) \left[ t\bar{\gamma}(M) - \theta_g q\bar{\gamma}(M) \right] \\
\geq \varphi \left[ t_G - \theta_g qG \right] + (1 - \varphi) \left[ t_M - \theta_g qM \right]
\]  
(A15)

\[
\varphi \left[ t\bar{\gamma}(M) - \theta_b q\bar{\gamma}(M) \right] + (1 - \varphi) \left[ t\bar{\gamma}(B) - \theta_b q\bar{\gamma}(B) \right] \\
\geq \varphi \left[ t_M - \theta_b qM \right] + (1 - \varphi) \left[ t_B - \theta_b qB \right]
\]  
(A16)

The RHSs of the participation constraints, (A15) and (A16), are an agent payoffs if he rejects the side-contract. If any agent rejects the side-contract, both agents make reports according to their best interest according to the principal’s contract. A reporting function \(\bar{\gamma}(\gamma)\) is feasible if it Bayesian incentive compatible and individually rational for participation for each agent of any type. We define the principal’s contract collusion-proof if there does not exist a feasible reporting function \(\bar{\gamma}(\gamma)\) for which at least one individual rationality constraint is strictly satisfied. We restrict our attention to the principal’s collusion-proof contracts, which by the collusion-proofness principle of Laffont and Martimort (1997) is without loss.

Next, we show that the principal’s contract that exhibits \(t_G - \theta_g qG < t_M - \theta_g qM\) is not collusion-proof. In doing so, we consider the two collectively exhaustive cases: (i) where \(t_B - \theta_b qB \geq t_M - \theta_b qM\) and (ii) \(t_B - \theta_b qB < t_M - \theta_b qM\). For case (i), consider the side-contract \(\bar{\gamma}(G) = \bar{\gamma}(M) = M\) and \(\bar{\gamma}(B, b) = B\). With \(t_G - \theta_g qG < t_M - \theta_g qM\), this side-contract strictly satisfies the participation constraint (A15). It also satisfies (A16) with equality. In addition, since the principal’s contract \(\Phi\) is Bayesian incentive compatible, the side-contract also satisfies (A13). Finally, note that since \(t_B - \theta_b qB \geq t_M - \theta_b qM\) for case (i), the side contract also satisfies (A14). This establishes that the principal’s contract is not collusion-proof for case (i) if \(t_G - \theta_g qG < t_M - \theta_g qM\). For case (ii), consider the side-contract \(\bar{\gamma}(G) = \bar{\gamma}(M) = \bar{\gamma}(B) = M\). With \(t_G - \theta_g qG < t_M - \theta_g qM\), this side-contract also strictly
satisfies the participation constraint (A15). Since \( t_B - \theta g q_B < t_M - \theta q_M \) for case (ii), it also strictly satisfies (A16). Moreover, Bayesian incentive compatibility of the principal’s contract \( \Phi \) implies that the side-contract also satisfies (A13) and (A14). This establishes that the principal’s contract is not collusion-proof for case (ii) if \( t_G - \theta g q_G < t_M - \theta q_M \).

Thus, a direct corollary is that \( (CIC) \) is a necessary condition for the principal’s contract \( \Phi \) to be collusion-proof.

Since \( (CIC) \) is a necessary condition for collusion-proofness, we can use it to obtain an upper bound on the principal’s payoff from the optimal contract in \( P^u \). The binding constraints in \( P^u \) are \( (CIC), (PC_M), (PC_B) \) and \( (IC_g) \). It is straightforward to verify that other constraints are satisfied by the solution without them. From the binding constraints, the transfers are:

\[
t_G = \theta g q_G + \Delta \theta q_M, \quad t_M = \theta q_M, \quad t_B = \theta q_B,
\]

and the binding \( (IC_g) \) reduces to \( q_M = q_B \). After substituting for the transfers with \( q_M = q_B \) in the objective function, optimization gives the project sizes in \( P^u \), characterized by:

\[
v'(q_G^u) = \Theta_G, \quad v'(q_M^u) = v'(q_B^u) = \Theta_B + \frac{2 \varphi^2}{1 - \varphi^2} \Delta \theta.
\]

The optimal outcome in \( P^u \) satisfies all the constraints in \( P^c \) and \( P^d \). Thus, the expected payoff from \( \Phi^u \) can be implemented in \( P^c \) and \( P^d \). Since \( \Phi^u \neq \Phi^c \) (\( \Phi^u = \Phi^c \) for \( \varphi \geq 1/2 \) and \( \Phi^u \neq \Phi^d \), it follows that \( \Phi^u \) is dominated by \( \Phi^c \) and \( \Phi^d \).
References


