Radiation Research

An International Journal

Editor-in-Chief

R. J. M. Fry

Managing Editor

Martha Edington

Volume 130, 1992

ACADEMIC PRESS, INC.

Harcourt Brace Jovanovich, Publishers

San Diego New York Boston London Sydney Tokyo Toronto

0033-7587/92 $5.00

MADE IN THE UNITED STATES OF AMERICA

This journal is printed on acid-free paper.
∞

RADIATION RESEARCH

OFFICIAL JOURNAL OF THE RADIATION RESEARCH SOCIETY

VOLUME 130, 1992

Councilors, Radiation Research Society 1991–1992

PHYSICS
- W. A. Bernhard, University of Rochester
- A. Chatterjee, Lawrence Berkeley Laboratory

BIOLOGY
- C. R. Geard, Columbia University
- H. D. Thames, University of Texas

MEDICINE
- C. N. Coleman, Harvard Medical School
- M. W. Dewhirst, Duke University

CHEMISTRY
- R. C. Fahey, University of California, San Diego
- M. E. Varnes, Case Western Reserve University

AT-LARGE
- B. F. Kimler, University of Kansas
- J. L. Roti Roti, Washington University

CONTENTS OF VOLUME 130

NUMBER 1, APRIL 1992

NUMBER 3, JUNE 1992

NOTICE

The Subject Index for Volume 130 will appear in the December 1992 issue as part of a cumulative index for the year 1992.

RADIATION RESEARCH 130, 15–25 (1992)

Intermediate Dosimetric Quantities

A. M. KELLERER,*·† K. HAHN,* AND H. H. ROSSI‡

*Radiobiological Institute, University of Munich, Germany; †Institute for Radiation Biology, GSF, Neuherberg, Germany; and ‡105 Larchdale Ave., Upper Nyack, New York 10960

KELLERER, A. M., HAHN, K., AND ROSSI, H. H. Intermediate Dosimetric Quantities. *Radiat. Res.* 130, 15–25 (1992).

The transfer of energy from ionizing radiation to matter involves a series of steps. In wide ranges of their energy spectra photons and neutrons transfer energy to an irradiated medium almost exclusively by the production of charged particles which ionize and thereby produce electrons that can ionize in turn. The examination of these processes leads to a series of intermediate quantities. One of these is kerma, which has long been employed as a measure of the energy imparted in the first of the interactions. It depends only on the fluence of uncharged particles and is therefore—unlike absorbed dose and electron fluence—insensitive to local differences of receptor geometry and composition. An analogous quantity for charged-particle fields, *cema* (converted energy per unit mass), is defined, which quantifies the energy imparted in terms of the interactions of charged particles, disregarding energy dissipation by secondary electrons. Cema can be expressed as an integral over the fluence of ions times their stopping power. However, complications arise when the charged particles are electrons, and when their fluence cannot be separated from that of the secondaries. The resulting difficulty can be circumvented by the definition of *reduced cema*. This quantity corresponds largely to the concept employed in the cavity theory of Spencer and Attix. In reduced cema not all secondary electrons but all electrons below a chosen cutoff energy, Δ, are considered to be absorbed locally. When the cutoff energy is reduced, cema approaches absorbed dose and thereby becomes sensitive to highly local differences in geometry or composition. With larger values of Δ, reduced cema is a useful parameter to specify the dose-generating potential of a charged-particle field 'free in air' or *in vacuo*. It is nearly equal to the mean absorbed dose in a sphere with radius equal to the range of electrons of energy Δ. Reduced cema is a function of the fluence at the specified location at and above the chosen cutoff energy. Its definition requires a modification of restricted linear collision stopping power, L_Δ, and it is recommended that the definition of L_Δ be so changed.

INTRODUCTION

The term "dosimetry" can be taken to refer solely to the determinations of *absorbed dose* (*1*), i.e., the energy absorbed per unit mass in the vicinity of a point in a medium exposed to ionizing radiations. However, in its wider sense dosimetry deals with the processes that link the energy transferred to matter with the radiation *fluence* (*1*), and in this wider sense one can consider certain intermediate quantities that correspond to successive steps of energy transfer. Intermediate quantities account for only part of these successive steps of energy degradation and disregard the remainder. This corresponds to simplifications that are frequently employed in dose calculations, when the resulting inaccuracies lie within the spatial resolution that is required. More importantly, the intermediate quantities are less dependent on receptor geometry than the absorbed dose. They have well-defined values, even if some details of the receptor geometry are left unspecified.

The first major radiological quantity, the *exposure* (*1*), with its (now obsolete) unit, the roentgen, was formulated many decades ago. It served for many years as the only quantification of radiation "dose," although it refers to the amount of ionization which the electrons, generated by X or γ rays in a specified mass of air located at the point of interest, would produce in air.

An analogous quantity that is both more general and more fundamental is the *kerma*, originally formulated by Roesch (*2*). It refers to the first step in the interaction between uncharged particles (e.g. photons or neutrons) and irradiated matter,[1] and it has the same dimension as absorbed dose.

In the subsequent considerations similar quantities that concern further steps in the transfer of radiation energy to matter and that are thus applicable also to charged-particle fields will be defined. The quantities defined below are nonstochastic, i.e., they are the expectation values of quantities that are subject to statistical distributions. The definitions of the stochastic quantities would be largely analogous to those of their expectation values.

THE ENERGY-DEGRADATION PROCESS

Absorbed dose and intermediate quantities, such as kerma, can differ substantially near boundaries of receptors

[1] Roesch proposed the acronym KERM (kinetic energy relased per unit mass); accepting the concept the ICRU added an A to obviate confusion with the German word Kern (nucleus).

0033-7587/92 $3.00

or, generally, when the exposed material or the radiation field is nonuniform. The quantities are, nevertheless, closely related. The spatial dependence of kerma represents the distribution of the absorption of energy of uncharged particles. Absorbed dose represents the spatial distribution that is further degraded by the additional step of energy transport by the released charged particles. The distribution of absorbed dose is a fuzzy image of the distribution of kerma, and, vice versa, the distribution of kerma is a "sharpened" image of absorbed dose. Whenever there is no need to determine the distribution with a spatial resolution better than the charged-particle ranges, the difference between absorbed dose and kerma can be disregarded; kerma is then a suitable approximation to absorbed dose. Analogous considerations apply to other intermediate quantities that will be considered subsequently and that are of interest because they can take the place of kerma as approximations of absorbed dose for charged-particle radiations, or for uncharged radiation, whenever one needs a better approximation than kerma. The interrelationship between the quantities will be referred to as "equality on average," which implies equality in the trivial case of complete equilibrium (*3*, *4*), i.e., of a uniform radiation field in a uniform medium.[2]

The interrelationships between the fluences of various ionizing particles can be expressed by field equations that contain the interaction coefficients (*5*). They can also be represented by diagrams, which facilitate the synopsis of the various channels of energy degradation. Figure 1 is a diagram illustrating major modes of energy degradation when a field of neutrons interacts with matter. The diagram is a simplification that is adequate for intermediate energies of the neutrons. It serves as an example that can be readily translated into analogous diagrams, e.g., for photon fields. At very high energies multifarious interactions involving both nuclear and atomic processes result in more complex modes of energy conversion. At low energies the interactions of slow neutrons, which involve not only conversion of rest mass to kinetic energy but also production of γ radiation, greatly restrict the practical value of intermediate dosimetric quantities, such as kerma. On the other hand, the dissipation of commonly encountered radiations, including photons and charged particles of energies up to a few million electron volts and neutrons of energy between about 10 keV and 10 MeV, occurs predominantly in a relatively simple chain of interactions; in the following the terms neutrons, photons, and charged particles refer to radiation energies within these limits.

Each arrow in the diagram in Fig. 1 represents energy conversion between the different forms of energy; the term

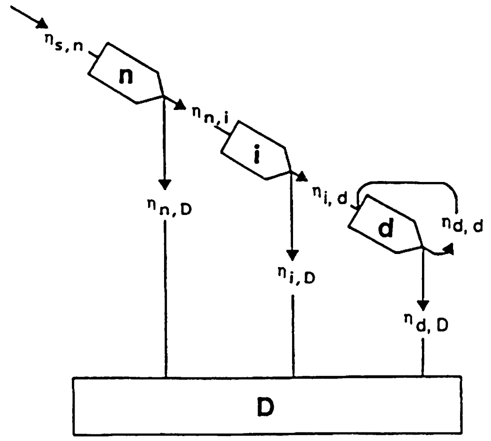

FIG. 1. Energy degradation diagram for neutron radiation. The pointed enclosures represent kinetic energy of neutrons (n), ions (i), and secondary electrons (d). The rectangle represents absorbed dose. The arrows symbolize energy conversion, i.e. energy converted per unit mass during the time of interest: $\eta_{s,n}$, neutron energy released from unspecific source; $\eta_{n,D}$, energy expended by neutrons against binding energy; $\eta_{n,i}$, energy transferred from neutrons to kinetic energy of ions; $\eta_{i,D}$, energy expended by ions against binding energy; $\eta_{i,d}$, energy transferred from ions to kinetic energy of secondary electrons; $\eta_{d,D}$, energy expended by secondary electrons against binding energy; $\eta_{d,d}$, energy transferred from secondary electrons to kinetic energy of secondary electrons.

energy conversion denotes here the energy transformed per unit mass during a specified time interval. It must be noted that the connecting lines do not refer to spatial transport of energy, but to energy conversions taking place in interactions at a point. The pointed enclosures symbolize kinetic energy of neutrons (n), charged recoils—in this example ions (i)—and secondary electrons (d)[3] which mediate energy transport in the medium. The rectangle (D) represents energy removed from the field of ionizing radiation; this is energy expended against binding energy, but it includes also energy of particles or photons that is considered as energy absorbed, because the energy transport is no longer by ionizing radiation. The term "ionizing radiation" is used here, although the definition of ionization in a condensed material is vague, and the flow terms into the rectangle in the diagram are correspondingly uncertain. The subsequent considerations will lead to more specific definitions.

Each symbol for energy conversion is given two indices that identify the forms of energy between which the transition occurs. For example, $\eta_{s,n}$ stands for the conversion of energy (per unit mass) from an unspecified source to kinetic energy of neutrons.

[2] There is no exact equality on average between absorbed dose and kerma, but this is merely a technicality in the definition of kerma that will be considered below under *Kerma*.

[3] The adjective "primary" applies in the subsequent discussions to all charged particles except those that are liberated by charged particles. Electrons liberated by charged particles are here called secondary electrons; since s is used for "source," the symbol d (δ rays) is used for "secondary" electrons.

Thus the diagram might refer to a solution containing a neutron-emitting radionuclide in which the kinetic energy $\eta_{s,n}$ of neutrons has been generated per unit mass. This is predominantly transformed into kinetic energy of ions in the conversion $\eta_{n,i}$ and to a small part expended against binding energy in $\eta_{n,D}$. In a further degradation step kinetic energy of charged primaries is partly transformed into kinetic energy of secondary electrons in $\eta_{i,d}$ and partly expended against binding energy in $\eta_{i,D}$.

As stated, the rectangle represents energy transferred from ionizing radiation to the exposed material, and hence the absorbed dose, D, is equal to the sum of the conversions terminating at the rectangle.

Energy conservation requires that under complete equilibrium the source terms, i.e. $\eta_{s,n}$ plus any other source terms, equal the absorbed dose. Furthermore, the influx equals the efflux for each of the kinetic energy compartments. The absorbed dose, therefore, also equals

$$D = \eta_{s,n} = \eta_{n,D} + \eta_{n,i}. \quad (1)$$

These relationships apply, as stated, only under complete equilibrium or as spatial averages over a sufficiently large region of the exposed material. They indicate the interconnection between various intermediate dosimetric quantities. Kerma will be considered as the first example.

INTERMEDIATE QUANTITITES

Kerma

The kerma, K, is the sum of the initial kinetic energies of charged particles liberated by uncharged particles per unit mass of irradiated material (1). Hence neutron kerma is equal to the term $\eta_{n,i}$ in Fig. 1 and, as each of the flow terms, it can be expressed as an integral in kinetic energy, T, over fluence and an interaction coefficient:

$$K = \eta_{n,i} = \int_{T_{min}}^{T_{max}} T\varphi_n(T)\mu_{tr}(T)dT \quad (2)$$

$\varphi_n(T)dT$ is the fluence due to neutrons of energy between T and $T + dT$; we will subsequently use the term fluence spectrum (in energy). The term $\mu_{tr}(T)$ is the mass–energy transfer coefficient (1) in the specified material. One concludes that the kerma is, under the condition of complete equilibrium, slightly less than the absorbed dose, $D = \eta_{n,D} + \eta_{n,i}$. The missing term $\eta_{n,D}$ is, however, insignificant. For photons there are added complexities. Bremsstrahlung and pair production can, at higher photon energies, make kerma larger on average than absorbed dose.

Unlike charged particles, uncharged ionizing particles have substantial mean free paths between collisions, and this implies that the fluence of uncharged particles is only gradually changed—due to absorption and scattering—when small receptors are introduced into a radiation field. A dosimetric quantity, such as kerma, that is defined purely in terms of the fluence of uncharged particles and their interaction coefficients therefore has values that pertain to small exposed objects without critical dependence on their size or shape. Kerma thus can also be specified for a material other than that at the point of interest (e.g., tissue kerma in free air) and it is defined even in the absence of material (e.g., kerma for any material in outer space).

Absorbed dose has no similar properties. Whenever its values are quoted as in vacuo, "free in air," or "in small receptors," intermediate quantities are in fact meant, and this imprecision leads frequently to confusion. The use of kerma avoids this problem for photon and neutron fields. For charged-particle fields intermediate quantities are often used by implication, but there are no formal definitions, and such definitions will therefore be introduced.

Cema

The diagram in Fig. 1 suggests that, in analogy to kerma which relates to the energy expended by uncharged particles in the liberation of charged particles, one can also define a quantity, C, relating to the energy expended by these charged particles in turn. Thus the absorbed dose caused by the primaries is equal on average to $\eta_{i,D} + \eta_{i,d}$ which can therefore be used as the intermediate dosimetric quantity. One obtains the equation

$$C = \eta_{i,D} + \eta_{i,d} = \frac{1}{\rho}\int_{T_{min}}^{T_{max}} \varphi_i(T)L(T)dT, \quad (3)$$

where $\varphi_i(T)$ is the fluence spectrum in energy of the charged primaries, and $L(T)$ is the unrestricted LET, i.e., their *linear collision stopping power*.[4] In the example of neutrons the charged primaries are the recoil ions. In the general case they are all charged particles except the secondary electrons.

The definition in Eq. (3) differs from that of kerma in a major aspect: the kinetic energy released in the liberation of secondary electrons is not the sole, dominant component; the energy expended against the binding energy of electrons is of comparable importance, and the inclusion of the term $\eta_{i,D}$ is therefore essential.

C equals absorbed dose on average but shows somewhat different spatial variations. The kerma, K, disregards the energy transport by the comparatively long-ranged charged particles immediately produced by uncharged particles. C disregards merely—as is common in the continuous slowing-down approximation (CSDA)—the energy dissipation by the secondaries, which have short ranges. The differ-

[4] To simplify notation the more explicit symbol L_{∞} is replaced by L.

ences between D and C therefore are substantially smaller and more local than those beween D and K.

The inclusion of the important term $\eta_{i,D}$ obviates the use of the name charged-particle kerma for C. One can instead speak of converted energy per unit *mass* and accordingly use the term *cema* for C.

Applicability of Cema

The general acceptance of kerma as a quantity in its own right is comparatively recent. But the quantity has, of course, been used widely as a substitute or approximation for absorbed dose, even by those who did not see it as a separate quantity. Before it had become common to refer to kerma or "shielded" kerma in the atomic bomb dosimetry, one encountered designations such as "tissue dose free in air." Dose calculations for X-ray, γ-ray, or neutron therapy beams are other examples. In such calculations one frequently disregards the energy transport by charged particles, i.e., one computes the distributions of kerma in the organs of interest. But one usually calls the result an absorbed-dose distribution.

Similar considerations apply to cema. It, too, is an obvious concept in computations. It is used as a better approximation of absorbed dose than kerma whenever one requires higher spatial resolution in computations for X-ray, γ-ray, or neutron beams. It is employed in most computations for charged-particle beams, and it is, of course, part of the CSDA, which is a computation of cema that disregards energy-loss straggling. Another, less obvious example for the implicit use of cema is the concept of the distribution, D_L, of dose in linear collision stopping power which is employed in the definition of the quality factor (*6, 7*). The absorbed dose is produced by the fluence of all charged particles, but D_L refers—even if this is usually not stated—only to the primary charged particles, excluding secondary electrons even of high energies. D_L is in reality the distribution, C_L, of cema in L.

Cema is thus a rigorously defined quantity to replace the somewhat ambiguous but frequently invoked concept of absorbed dose "under electron equilibrium." Its applicability includes the cases in which one needs to state the dose-generating potential *in vacuo* or free in air of charged particles from radioactive sources, accelerators, or cosmic radiation. As with kerma, its value can be stated in air or *in vacuo* for any specified material. A "dose-rate constant" for a radionuclide is, in fact, either a kerma rate constant or a cema rate constant.

Kerma and cema need to be recognized as quantities that differ from absorbed dose, but they are applicable because of their approximate equality to the (mean) absorbed dose in receptors, or in receptor subregions of "intermediate" size. The term intermediate refers to characteristic distances that need to be qualified. For kerma the distances lie between the ranges of the charged recoils and the mean free paths of the uncharged particles. For cema they are considerably smaller and lie between the ranges of the secondary electrons and the ranges of the charged primaries.

Cema is readily applicable to ions or other charged particles except electrons. For electrons, complications arise because the ranges of the secondary electrons can be comparable to those of the primaries, and also because it may be difficult in certain cases to distinguish between the fluence of primary and secondary electrons. A modified concept, reduced cema, is therefore required, and it will be considered in the subsequent section.

The exclusion of the secondary electron fluence is essential in the definition of cema, and it is instructive to quantify this condition. To obtain cema, one needs to integrate the linear energy transfer over the spectral distribution of fluence, $\varphi_e(T)$, in the kinetic energy of primary electrons

$$C = \frac{1}{\rho}\int_{T_{\min}}^{T_{\max}} \varphi_e(T)L(T)dT. \qquad (4)$$

If one were to use instead the spectral distribution of the total electron fluence, $\varphi(T)$, one would obtain a different quantity,

$$C' = \frac{1}{\rho}\int_{T_{\min}}^{T_{\max}} \varphi(T)L(T)dT. \qquad (5)$$

The quantities in Eqs. (4) and (5) are equal for an electron beam *in vacuo* that is not accompanied by secondary electrons. However, in matter the expression in Eq. (5) is substantially larger on average than the absorbed dose, because some of the energy transmitted by the incident fluence is added repeatedly, as it is dissipated by successive generations of electron radiation. C' is therefore not generally a meaningful quantity. Figure 2 illustrates, for the electrons released by monoenergetic photons, the substantial difference between the integrals in Eqs. (4) and (5); it also illustrates the broad overlap of the secondary and the primary electron fluence spectrum. In calculations the primary fluence can generally be separated from the fluence of secondary electrons which extends up to one-half of the maximum electron energy.[5] Examples are dosimetric calculations such as the extension of the Bragg–Gray principle by Laurence (*8*) or Spencer and Attix (*9*) in terms of the CSDA, or modified CSDA computations that account for the production of secondary electrons in terms of averages. In measurements, however, it may often be impossible to

[5] The convention that a secondary electron cannot have more energy than the parent electron means that the maximum energy of a secondary electron is $(T - b_{\min})/2$, where $b_{\min}$ is the minimum binding energy. However, $b_{\min}$ can be neglected in this context.

separate the primary electron fluence from the fluence of secondaries. C is therefore not a very suitable quantity for photon or electron radiations; it cannot be evaluated on the basis of the electron–fluence spectrum at a given point and it can, in fact, have different values for the same fluence and the same energy distribution of fluence, depending on the fraction of fluence that is due to secondary electrons. The distinction becomes unnecessary in a further dosimetric quantity that involves the last phase of energy dissipation, and that is meaningful for all ionizing radiations.

Reduced Cema

The final step in the interaction of ionizing radiations and matter consists in the transfer of energy to electrons. When this is merely excitation or results in liberated electrons of insufficient kinetic energy to cause ionization, the energy has been said to be imparted to (or absorbed by) the medium.

Kerma disregards energy transport by charged particles; one can say that charged particles are treated as if they dissipated their energy on the spot (*10*). Cema disregards merely the energy transport by secondary electrons. As stated, it is a useful quantity in calculations, but it employs the distinction between primary electrons and secondary electrons, which is artificial because a secondary electron resembles in all its properties—except its origin—a primary electron of the same energy. A more tangible criterion is therefore desirable for photon or electron radiations. Such a criterion is a suitably chosen cutoff energy of the electrons, and the adoption of such a cutoff corresponds to the convention adopted in the cavity theory of Spencer and Attix (*9*).

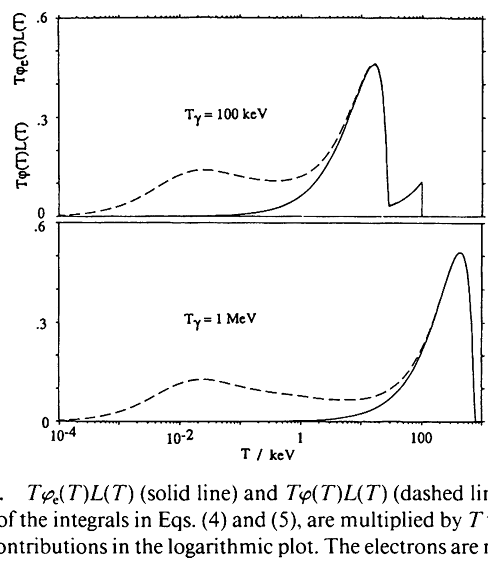

FIG. 2. $T\varphi_e(T)L(T)$ (solid line) and $T\varphi(T)L(T)$ (dashed line), the arguments of the integrals in Eqs. (4) and (5), are multiplied by T to indicate relative contributions in the logarithmic plot. The electrons are released by unscattered photons in water. The photon energies are 0.1 and 1 MeV. The area under the solid curve is normalized to unity. For details of the computations see Appendices.

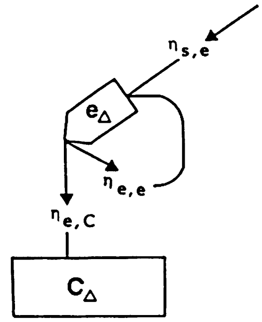

FIG. 3. Modified energy degradation diagram for electrons. The symbol e_Δ represents kinetic energy of "fast" electrons, i.e., of electrons with energy larger than Δ. The rectangle stands for reduced cema. The arrows symbolize energy converted per unit mass during the time of interest: $\eta_{s,e}$, energy of fast electrons released from unspecified source; $\eta_{e,C}$, energy expended by fast electrons against binding energy and kinetic energy of "slow" electrons emerging from interactions; $\eta_{e,e}$, energy transferred from fast electrons to kinetic energy of their fast secondary electrons.

In the quantity *reduced cema*, C_Δ, Δ represents a cutoff for the energy of electrons. One excludes electrons below this cutoff from the radiation field, as if they dissipated their energy on the spot. Their energy is thus counted with the energy imparted to matter. Rather than disregarding energy transport by secondary electrons, one disregards transport by all electrons below the chosen energy Δ. In particular, all secondary electrons below the cutoff energy are considered to be absorbed locally, and all secondary electrons above it, not absorbed locally. To indicate the modified convention, the symbol C_Δ is used instead of D in the diagram of Fig. 3, which relates to a radiation where electrons are the only charged particles. A cutoff is, in fact, also implied in the definition of absorbed dose which invokes the notion of ionizing particles, even though there has been no numerical specification of the value of this cutoff, i.e., of the minimum kinetic energy for the different types of particles.

It is instructive to consider the limit $\Delta = 0$ which has been invoked by Alm Carlsson[6] (*3*, *4*) and earlier by Spencer (*11*). In this case there is no further transport of energy, and C_0 corresponds closely to the absorbed dose

$$C = \frac{1}{\rho} \sum \int_{T_{\min}}^{T_{\max}} \varphi(T)\Lambda_0(T)\mathrm{d}T \approx D. \tag{6}$$

Here the summation extends over all kinds of particles in the radiation field, although the summation indices are omitted. The fluences and energies are φ and T, respectively, and $\Lambda_0(T)$ is the energy expended per unit distance by a particle (of specified type and of energy T) against

[6] Alm Carlsson (*4*) derives equations for absorbed dose under special conditions of equilibrium and arrives at essentially the same intermediate quantities, which are formally defined in the present article.

binding energy. Since the contribution of uncharged particles can usually be neglected, Λ_0 can be taken to be the linear collision stopping power of charged particles minus the kinetic energy of electrons released per unit distance.

In an irradiated medium the bulk of the absorbed dose is contributed by low-energy electrons (Fig. 4) In the example of monoenergetic photons, the electrons are the only type of charged particles. However, the fluence of very low-energy electrons in general cannot be evaluated with sufficient precision. A suitable intermediate quantity should therefore be independent of the fluence of very low-energy electrons, and this suggests the use of the quantity reduced cema, C_Δ, with a finite cutoff, Δ, where Δ is appreciably larger than the ionization threshold. The intermediate quantity C_Δ then equals the absorbed dose on average, but can deviate from it locally over spatial distances up to the range of electrons with energy Δ.

For fast neutrons or ions, $C_\Delta = C$ if Δ exceeds the maximum energy of secondary electrons. Usually this will be an adequate condition, and the subsequent consideration of C_Δ can therefore be restricted to the case where electrons are the only charged particles. This simplifies the discussion, but the extension to mixed fields of charged particles that include ions will be straightforward.

The explicit definition of C_Δ in terms of the collision cross sections is given in the Appendixes. A simplified formulation in terms of the continuous slowing-down approximation will be used here.

An electron can contribute to C_Δ in two ways. It can expend energy against binding forces and in creating secondary electrons of kinetic energy less than Δ; this contribution will be called the LET term. It can also in the course of the degradation process arrive at a kinetic energy below Δ, and will then be discounted from the radiation field. This latter contribution to C_Δ will be called the track-end term.

The LET term can be related to the interaction coefficient, Λ_Δ, which will be called the *reduced stopping power*. In analogy to Λ_0 it is defined as the difference between the linear collision stopping power and the sum of all kinetic energies larger than Δ of secondary electrons created per unit distance. The binding energy of these secondaries constitutes a difference between Λ_Δ and the restricted stopping power, L_Δ.[7] The restricted stopping power excludes all energy losses in excess of Δ. The reduced stopping power excludes merely the kinetic energies of secondary electrons in excess of Δ. The difference is small for large values of Δ, but for small values of Δ, comparable to the binding energies, it is considerable and Λ_Δ is then indeed more meaningful than Λ_Δ (*12*). The ratios Λ_Δ/L and L_Δ/L are shown in Fig. 5 for electrons as a function of their energy.

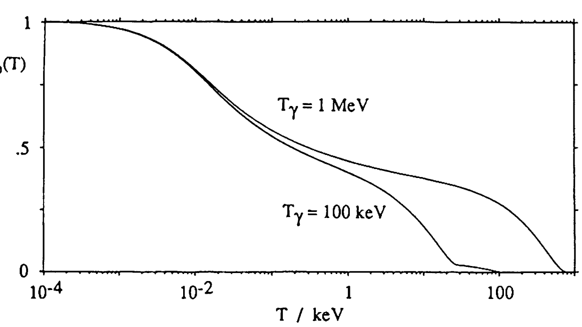

FIG. 4. The relative contribution, $C_0(T)$, to C_0 by electrons with energy above T; it equals the fractional part of the integral in Eq. (6) from T to infinity. The electrons are released by unscattered photons in water. The photon energies are 0.1 and 1 MeV.

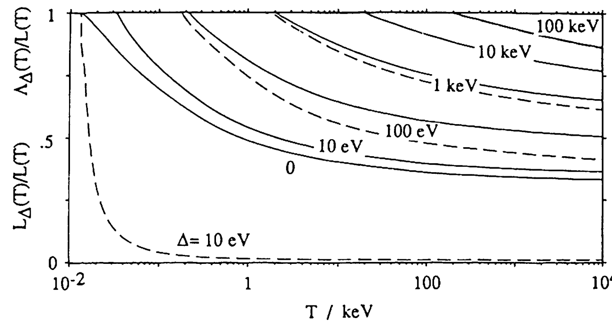

FIG. 5. Ratio, $\Lambda_\Delta(T)/L(T)$, of the reduced stopping power to the unrestricted stopping power of electrons in water (solid lines) and the corresponding ratio, $L_\Delta(T)/L(T)$, of restricted stopping power to unrestricted stopping power (broken lines).

The track-end term in the CSDA equals the product of Δ and the number per unit mass of electrons that go, in the course of their slowing down, through the energy value Δ. This number equals $n(\Delta) = \varphi(\Delta)L(\Delta)/\rho$. Accordingly one obtains the expression for reduced cema

$$C_\Delta = \frac{1}{\rho}\left(\int_\Delta^{T_{\max}} \varphi(T)\Lambda_\Delta(T)dT + \Delta\varphi(\Delta)L(\Delta)\right), \quad (7)$$

where $\varphi(T)dT$ is the (total) fluence of electrons between energy T and $T + dT$. Analogous expressions—but without the track-end term—obtain for ions.

C_Δ has the essential feature that it does not depend on the electron fluence below Δ. The magnitude of the fluence at energy Δ is, however, important because the track-end contribution, $n(\Delta)\Delta$, can be substantial, as is shown in Fig. 6 for electrons of different initial energies. Equation (7) does not account for primary electrons with initial energy below Δ. The missing source term can be included in the formula, but it will usually be unimportant.

[7] To simplify terminology "restricted linear collision stopping power" is replaced by "restricted stopping power." The term "reduced stopping power" is used in the present article to distinguish Λ_Δ from L_Δ. However, changing the definition of restricted stopping power to Λ_Δ will be proposed, and the new term will then not be required.

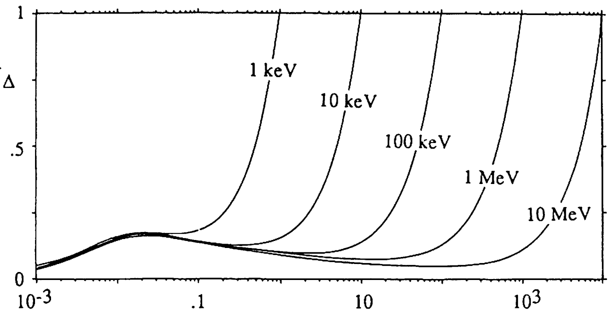

FIG. 6. The fractional contribution, f_Δ, of "track ends" to the energy imparted, C_Δ [i.e., the relative contribution of the last term in Eq. (7)], for electrons of specified initial energy that are absorbed in water.

Applicability of Reduced Cema

The considerations under *Applicability of Cema* also apply to reduced cema. In the same way as kerma and cema, the quantity has commonly been employed without formal definition as a convenient approximation in absorbed-dose calculations. Any such calculation employs a cutoff, Δ, in electron energy, and below this energy it disregards further energy transport. The resulting spatial resolution is roughly equal to the range, $r(\Delta)$, of an electron with energy Δ. At small values of the cutoff C_Δ becomes equal to D, and in this sense one can consider C_Δ a mere generalization of absorbed dose.

However, the variable cutoff is not the only generalization. C_Δ shares with kerma and with cema the added property that it can be specified at a point for a material that is not actually present. A somewhat intricate example—which is given here without explanations—is the relationship of C_Δ to cavity theory. When nonhomogeneous (usually air-filled) ionization chambers are calibrated in photon fields, it is common to employ an energy cutoff for electron fluence in the calculations, and the resulting approximations are largely equivalent to the use of reduced cema. The cavity theory of Spencer and Attix (*9*, *10*) can, in fact, be conveniently phrased in terms of reduced cema. Its central statement is that the measurement in the air cavity provides the value of reduced air cema, $C_{\Delta,\text{air}}$, in the wall. The cutoff Δ equals the energy of electrons with range comparable to the radius of the cavity. The conversion factor, f, in the Spencer and Attix theory is thus equal to $C_{\Delta,\text{air}}/C_{\Delta,\text{wall}}$. The two quantities $C_{\Delta,\text{air}}$ and $C_{\Delta,\text{wall}}$ are determined by Eq. (7), with the equilibrium electron fluence in the wall material, but with $\Lambda_{\Delta,\text{air}}$ in the expression for $C_{\Delta,\text{air}}$ and $\Lambda_{\Delta,\text{wall}}$ in the expression for $C_{\Delta,\text{wall}}$.

A simpler example is the use of C_Δ to specify the dose-generating potential of a charged-particle radiation *in vacuo* or free in air. If one were to use no other quantity than absorbed dose, one could specify its mean value, $\overline{D}_r$, in a small receptor of specified size, shape, and material, e.g., a tissue or water sphere of radius r. The direct linkage to absorbed dose would be attractive and, at least with a sphere, the choice of a specific receptor geometry would not be too objectionable. However, impracticable computations would be required to derive this parameter from a known or measured fluence spectrum. The use of C_Δ is more convenient, and it happens to provide, as will be seen, nearly the same information as $\overline{D}_r$.

Figure 7 gives the quantities C_Δ and $\overline{D}_r$ for monoenergetic electrons of specified energies. The two quantities are plotted as a function of Δ and r. The scale of Δ is chosen in such a way that the associated range $r(\Delta)$ of an electron with energy Δ coincides with the radius, r, of the sphere. The range, $r(\Delta)$, that is chosen here equals the thickness of a layer of water that transmits 5% of the electrons normally incident with energy Δ. The mean absorbed doses are calculated by Monte Carlo methods; computational details are given in the Appendixes.

A notable result of the computations is the near equality of C_Δ and $\overline{D}_r$, which holds when r equals $r(\Delta)$ and when it is small compared to the range of the primary electron. Utilization of the radius to specify the size of the sphere is not critical; $\overline{D}_r$ varies so slowly that one may equally use the diameter. Approximate equality also holds for somewhat different receptor geometries; Fig. 8 gives the data for cubes, for long cylinders, and for infinite slabs exposed to normally incident electrons. C_Δ can therefore be used as an adequate approximation of the mean absorbed dose free in air to a receptor of characteristic dimension $r(\Delta)$.

CONCLUSION

Absorbed dose and the intermediate dosimetric quantities can be considered as variations of a generalized dose

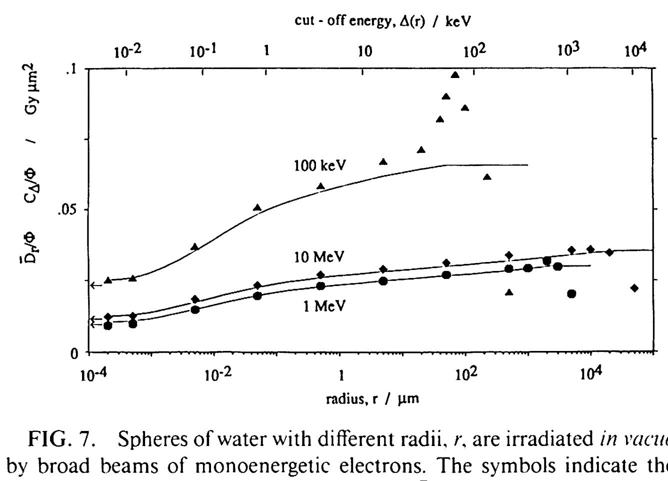

FIG. 7. Spheres of water with different radii, r, are irradiated *in vacuo* by broad beams of monoenergetic electrons. The symbols indicate the mean absorbed doses normalized to fluence, $\overline{D}_r/\Phi$, in the spheres, and the solid lines indicate the corresponding normalized reduced cema $C_{\Delta,\text{water}}/\Phi$ *in vacuo* (see text). The arrows at 0.1 nm indicate the limit $\Delta = 0$.

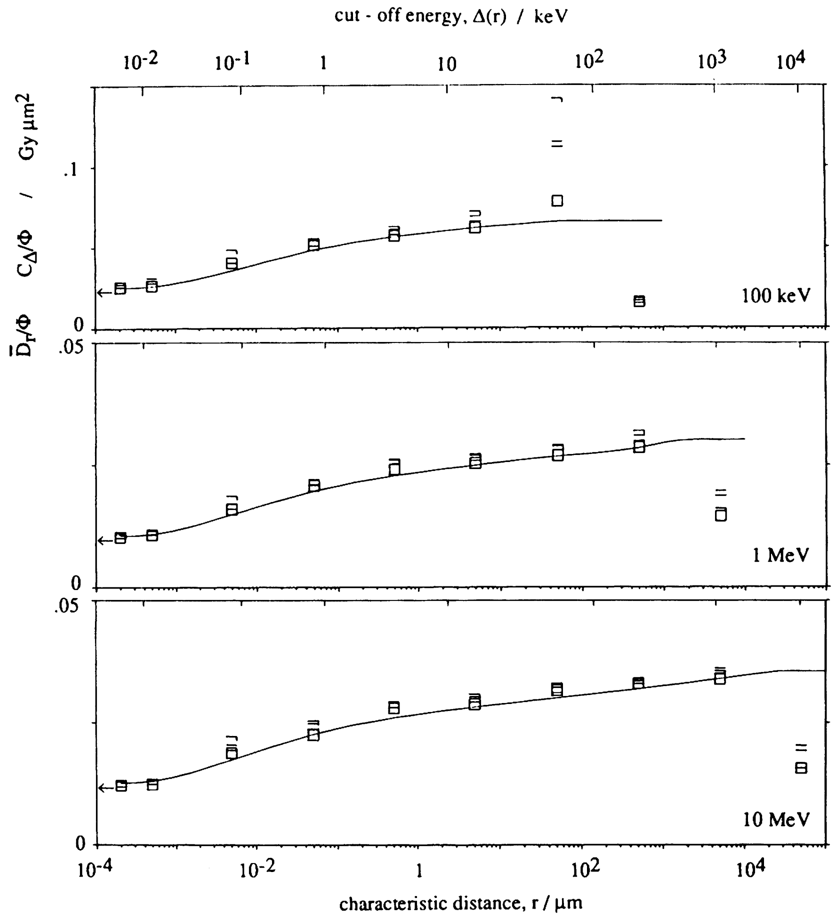

FIG. 8. Diagrams analogous to Fig. 7, but for cubes (□), for long cylinders (=), and for infinite slabs (—); as in case of the sphere, the characteristic distance equals the half thickness of the target. The beam is taken to be normal to a face of the cube or the slab or to the axis of the cylinder.

concept. From this unifying point of view—which is not further explored here—they differ merely by the degree of exclusion of certain radiation components from the field. The excluded components are treated as if they were absorbed on the spot, i.e., their contribution to the energy transport is disregarded. Kerma excludes charged particles, cema excludes secondary electrons, and reduced cema excludes all electrons below a specified cutoff.

The intimate connection between absorbed dose and the intermediate quantities exhibits itself in the fact that these are used variously as substitutes or approximations for absorbed dose, often without formal distinction from absorbed dose. The intermediate quantities are, however, more than mere approximations to absorbed dose; they can be used as meaningful parameters even in free-field situations where it is meaningless to specify a value of absorbed dose.

Kerma and the related quantity exposure are routinely employed in standardization and calibration of devices for the measurement of uncharged particles. They have also been commonly applied in evaluating radiation environments for purposes of radiation protection. Cema can serve analogous purposes for charged particles.

ICRU Report 39 (*13*) recommends operational quantities that are appropriate in radiation protection and are related to a basic phantom, the ICRU sphere. But a simple, and often sufficiently accurate, approach for free-field measurements is to determine $Q_u K$ and $Q_c C$, where Q_u and Q_c are the quality factors for the uncharged and the (primary) charged particles. In most cases the sum of the two terms provides an overestimate of $H^*(10)$ and $H'(0.07)$, the *ambient dose equivalent* (at 10 mm depth) and the *directional dose equivalent* (at 0.07 m depth), which are the quantities recommended in ICRU Report 39. This is so not only because the maximum values, rather than those under a fixed depth, are involved, but also because partial equilibrium may exist even under free-field conditions between the uncharged particles and the charged particles that they have released. In the case in which uncharged particles appear in substantial equilibrium with charged particles, their contribution to the maximum absorbed dose in a phantom could be exaggerated by a factor of about two.

In agreement with formulations developed by Spencer and by Alm Carlsson one can express absorbed dose as an integral over electron fluence times the linear rate of energy conversion (see Appendixes) for a small cutoff, Δ, which leads to reduced cema. As Δ, which characterizes a particle as ionizing, might be close to zero, one can even approximate absorbed dose by an integral [see Eq. (6)] over electron fluence times the completely reduced stopping power. However, this is an abstract concept, because both quantities depend critically on the behavior of electrons at low energies, which is difficult to measure and also to quantify theoretically. To obtain more stable and easier-to-use quantities, one must therefore choose a cutoff energy, Δ, that is high enough to exclude the details of low-energy electron degradation.

The integrals over fluence that determine reduced cema require a modified definition of restricted stopping power, and to avoid confusion with the present convention a different symbol, Λ_Δ, and a different name, reduced stopping power, have been used here for the modified quantity. Λ_Δ is the energy-loss rate of a charged particle excluding the kinetic energy of the secondary electrons released with kinetic energy in excess of Δ. In the familiar definition of L_Δ one excludes the kinetic energy of the secondary electrons as well as the binding energy, when their sum exceeds Δ; a cutoff $\Delta = 0$ is then meaningless. With the modified definition one can choose zero cutoff energy, and Λ_0 then appears, as stated above, in the integral over fluence that approximates absorbed dose. While a distinction has been made here between L_Δ and Λ_Δ, it will be preferable to change the definition of L_Δ and to make it equal to the reduced stopping power, Λ_Δ; the symbol L_Δ can then be retained. In fact, there appear to be few, if any, applications that require the present definition rather than the modified convention.

APPENDIX A

Exact Formulae for Reduced Cema

Using the same approximation as the simplest form of the Spencer and Attix theory [(*9*), Eq. (3)] one could write the equation for reduced cema in the form

$$C_\Delta \approx \frac{1}{\rho}\int_\Delta^{T_{\max}} \varphi(T)L_\Delta(T)dT, \qquad \text{(A.1)}$$

where Δ is the cutoff energy, $L_\Delta(T)$ is the restricted stopping power of an electron of energy T, and $\varphi(T)dT$ is the (total) fluence of electrons with energies between T and $T + dT$.

However, according to the definition of restricted stopping power (*1, 10*), this equation excludes the energy expended against binding energy in all collisions with energy loss in excess of Δ. It disregards, furthermore, the energy of "track ends," i.e., of primary electrons or "fast" secondary electrons after falling below Δ. The energy of these track ends is, in the same way as that of low-energy secondaries, to be treated as if it were dissipated on the spot, but it is not contained in the integral of Eq. (A.1). In the cavity theory the first inaccuracy has not been critical, because comparatively large values of Δ, substantially in excess of the binding energies, were employed which were equated—in the simplest initial treatment—to the energy of secondary electrons just sufficient to span the cavity (*9*). The exclusion of the binding energy in the production of the fast electrons is then insignificant (see Fig. 5), and this is reflected in the current, somewhat arbitrary definition of restricted stopping power. The second inaccuracy, too, is of comparatively minor influence in cavity theory, because it affects equally the two terms in a ratio, i.e., the energy densities in the gas and in the wall material. However, Spencer and Attix have, even in their initial calculations (*9*), used modified formulations to account for the influence of track ends.

In the present, more general context a rigorous formulation of reduced cema is required. Disregarding electrons with initial energy less than Δ—an approximation that will be retained subsequently, to simplify the formulae—reduced cema is given by the equation

$$C_\Delta = \frac{1}{\rho}\int_\Delta^{T_{\max}} \varphi(T)\lambda_\Delta(T)dT, \qquad \text{(A.2)}$$

where $\lambda_\Delta(T)$ is, for an electron of energy T, the linear rate of energy conversion to slow electrons, i.e., to electrons with kinetic energy less than Δ, and to binding energy. For large values of Δ and for $T > 2\Delta$ the quantity $\lambda_\Delta(T)$ is only slightly larger than $L_\Delta(T)$. But substantial differences can occur for smaller values of T or Δ, and it is therefore necessary to consider $\lambda_\Delta(T)$ in detail.

While approximate formulae were used in the main text, we will give here the exact formulae in a general form, without specific assumptions on the cross sections. Let E denote the energy of the liberated secondary electrons and W the corresponding energy loss, then the energy $(W - E)$ is expended against binding energy. The probability of an energy loss between W and $W + dW$ of an electron while traversing dx is defined as $\mu(W,T)dW\,dx$ while $\mu'(E,T)dE\,dx$ is the analogous quantity for the energy E of the secondary electron. The linear rate of energy conversion, $\lambda_\Delta(T)$, is then defined as

$$\lambda_\Delta(T) = \int_0^T W\mu(W, T)dW - \int_\Delta^{T/2} E\mu'(E, T)dE + \int_{T-\Delta}^{T} (T - W)\mu(W, T)dW, \qquad \text{(A.3)}$$

The first integral is the total linear collision stopping power, $L(T)$, of the electron, i.e., its total energy loss in collisions

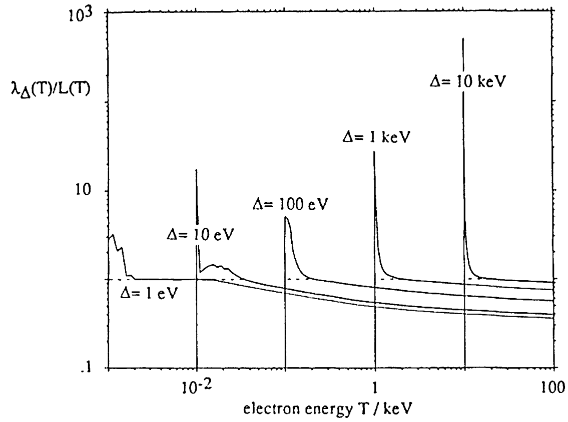

FIG. 9. The ratio, $\lambda_\Delta(T)/L(T)$, for electrons in water for selected cutoff energies, Δ. The peaks above the broken lines correspond to the track-end term in Eq. (A.3). The calculated values include all ionization and excitation orbits of the water molecules.

per unit path length. The second integral represents that part of the energy loss that reappears as kinetic energy of fast secondary electrons.[5] The difference of the two terms equals the reduced stopping power, $\Lambda_\Delta(T)$.

The last integral in Eq. (A.3) determines the *track-end term* in C_Δ (see under *Reduced Cema*); it refers to collisions in which the energy of the primary electron falls below Δ and is added because the remaining kinetic energy of the electron is treated as dissipated on the spot. This term vanishes at energies T larger than $2\Delta + b_{max}$; the electron then cannot lose enough energy in a collision to fall below energy Δ. When T becomes smaller than $2\Delta + b_{max}$ and approaches Δ, the energy of the scattered primary electron is contained with increasing probability in the track-end term. Figure 9 shows the ratio $\lambda_\Delta(T)/L(T)$ for different cutoff values, Δ, and electron energies, T (see Appendix B for computational details).

Instead of the rigorous solution one can use, in agreement with the treatment under *Reduced Cema*, the continuous slowing-down approximation for the last term in Eq. (A.3). This term is then equal to $\Delta L(\Delta)\delta(T - \Delta)$, and Eq. (A.2) takes the form of Eq. (7). In the diagram of Fig. 9 the extended peaks are replaced by the Dirac function at $T = \Delta$. The simplified formula will be an acceptable approximation in most dosimetric computations. In Monte Carlo simulations, however, the exact formula can be the most straightforward approach.

APPENDIX B

Details of the Calculations

This second appendix identifies the main physical data that are used in the calculations. We use the semiempirical differential ionization cross sections for electron scattering in water (vapor) that are defined by Rudd (*14*) according to the essentials of the Bethe theory. These cross sections cover the energy range up to 10 keV. To extend their applicability to higher energies, we extrapolated the total cross sections of Rudd by the relativistic asymptotic Bethe formula given in Eq. (4.55) by Inokuti (*15*) and we renormalized the corresponding differential cross sections by the asymptotic expressions. A similar procedure was performed with the excitation cross sections, which are taken from the data set of a Monte Carlo program written by Zaider, Brenner, and Wilson (*16*).

We were encouraged to use this set of cross sections for calculations up to 10 MeV for essentially two reasons: First, these cross sections reproduce the collision stopping power (*17*) up to 10 MeV with good precision. Second, the results of our calculations are rather insensitive to finer details of the secondary electron distributions; this was shown in preliminary calculations where even in the case of ionization the (renormalized) cross sections used by Zaider *et al.*, were applied [see also (*18*) for further examples].

Numerically, two techniques were used: In Figs. 2, 4, and 6 the electron fluence spectrum was derived by solving the electron transport equation via CSDA including, in an average way, the creation of secondary electrons. The mean doses in Figs. 7 and 8 were calculated by Monte Carlo methods. A Monte Carlo program, written originally by Zaider *et al.*, was modified to include geometric boundary conditions and the ionization and excitation cross sections that are referred to above.

In Fig. 7 a range–energy relationship is employed to link the radius of the receptor, via the electron range, to the cutoff energy of reduced cema. The range here equals a 5% transmission range (see main text) for electrons with energies down to 0.1 keV. This range was obtained as the CSDA range divided by a detour factor given by Paretzke.[8] Below 0.1 keV experimental values given in ICRU Report 16 (*6*) were used. Suitable energy cutoff values, Δ, were employed in the calculation of the mean absorbed dose $\bar{D}_r$ i.e., $\bar{D}_r$ was approximated by the mean reduced cema $\bar{C}_\Delta$ in the receptor, with Δ sufficiently small with regard to the receptor radius, but sufficiently large to avoid unnecessary computations.

ACKNOWLEDGMENTS

This work was supported by the Federal Ministry for Environmental Protection and Reactor Safety of the Federal Republic of Germany, under

[8] H. G. Paretzke, Simulation von Elektronenspuren im Energiebereich .01–10 keV in Wasserdampf, GSF-Bericht 24/88, Gesellschaft für Strahlen- und Umweltforschung München–Neuherberg, 1988.

Contract St. Sch. 4009. The opinions expressed in this article reflect those of the authors and not necessarily those of the Ministry.

RECEIVED: October 9, 1991; ACCEPTED: November 4, 1991.

REFERENCES

1. ICRU, *Radiation Quantities and Units.* Report 33, International Commission on Radiation Units and Measurements, Washington, DC, 1980.
2. W. C. Roesch, Dose for nonelectronic equilibrium conditions. *Radiat. Res.* **9**, 399–410 (1958).
3. G. Alm Carlsson, Theoretical basis for dosimetry. In *The Dosimetry of Ionizing Radiation* (K. R. Kase *et al.*, Eds.), Vol. 1, pp. 1–75. Academic Press, London, 1985.
4. G. Alm Carlsson, Absorbed dose equations. On the derivation of a general absorbed dose equation and equations valid for different kinds of radiation equilibrium. *Radiat. Res.* **85**, 219–237 (1981).
5. H. H. Rossi and W. C. Roesch, Field equations in dosimetry. *Radiat. Res.* **16**, 783–795 (1962).
6. ICRU, *Linear Energy Transfer.* Report 16, International Commission on Radiation Units and Measurements, Bethesda, MD, 1970.
7. ICRP, *1990 Recommendations of the International Commission on Radiological Protection.* ICRP Publication 60, Vol. 21/1-3, International Commission on Radiological Protection, Oxford, 1991.
8. G. C. Laurence, The measurement of extra hard X-ray and gamma rays in roentgens. *Can. J. Res. A* **15**, 67–78 (1937).
9. L. V. Spencer and F. H. Attix, A theory of cavity ionization. *Radiat. Res.* **3**, 239–254 (1955).
10. NCRP, *Stopping Powers for Use with Cavity Chambers.* Handbook 79, National Council on Radiation Protection and Measurements, National Bureau of Standards, Washington, DC, 1961.
11. L. V. Spencer, Remarks on the theory of energy deposition in cavities. *Acta Radiol. Ther. Phys. Biol.* **10**, 1–20 (1971).
12. E. R. Bartels and D. Harder, The microdosimetric regularities of nanometer regions. *Radiat. Prot. Dosim.* **31**, 211–215 (1990).
13. ICRU, *Determination of Dose Equivalents Resulting from External Radiation Sources.* Report 39, International Commission on Radiation Units and Measurements, Washington, DC, 1985.
14. M. E. Rudd, Cross sections for production of secondary electrons by charged particles. *Radiat. Prot. Dosim.* **31**, 17–22 (1990).
15. M. Inokuti, Inelastic collisions of fast charged particles with atoms and molecules—The Bethe theory revisited. *Rev. Mod. Phys.* **43/3**, 297–347 (1971).
16. M. Zaider, D. J. Brenner, and W. E. Wilson, The applications of track calculations to radiobiology I. Monte Carlo simulation of proton tracks. *Radiat. Res.* **95**, 231–247 (1983).
17. ICRU, *Stopping Powers for Electrons and Positrons.* Report 37, International Commission on Radiation Units and Measurements, Washington, DC, 1984.
18. K. Hahn, H. Roos, and A. M. Kellerer, On the use of LET for a revised quality factor for photons. *Radiat. Prot. Dosim.* **31**, 367–370 (1990).