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The Poverty of Growth with Interdependent Utility Functions
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In this essay we explore the implications of economic growth for welfare in the
presence of interdependent utility functions with negative externalities in
consumption, i.e. envy. In conventional theory, the utility function of a "typical"
economic agent is generally assumed to be independent of the consumption of
others; hence more consumption invariably "...leads to more happiness" not only of
the individual, but in the aggregate as well. According to the common wisdom,
economic growth leads invariably to an increase in welfare (Layard and Walter,
1978, p. 5). Yet, experimental evidence refutes the notion that people invariably feel
better off with increased consumption (Easterlin, 1974; Abramovitz, 1979; p. 9).

Our paper assumes that "...the individual does not think and act in the same
way irrespective of whether he is or is not a member of a group" (Weisskopf, 1971,
p. 161; Katona, 1975, p. 49; Leibenstein, 1950: Frank 1985; McCracken, 1988, p. xi;
Konrad, 1990). "Reference groups" are important for providing standards for the
subjective evaluation of one's own well-being, and in formulating expectations and
aspirations on which satisfaction frequently depends. Thus, "...people's standards
of wealth are determined by comparisons with kin, peers, etc., and thus if they all
rise together [over time] the relative difference disappears, and so as people get
wealthier over time they do not become any more happy" (Furnham and Lewis,

This skepticism toward the received wisdom is shared by an increasing
number of researchers (Sen, 1977; Hammond, 1989; Konrad, 1992a), but
interdependent utility functions have been formulated ever since the publication of
Veblen's seminal theory of conspicuous consumption (1899; Mishan, 1961; Frank, March 1985; Bagwell and Bernheim, 1996; Postlewaite, 1998). Duesenberry "...specified utility of an individual as a function of his own income and that of the income of the other members of the society" (Duesenberry, 1949, p. 97).

Admittedly, consumption can also have positive externalities: for example, a wife's utility can enter the utility function of her spouse altruistically (Becker, 1981).

Empirical and theoretical investigations of the social nature of utility functions are important for several reasons: the "neglect of interdependence of welfare functions may substantially bias policy conclusions" (Kapteyn and Herwaarden, 1980, p. 395); it also has major implications for progressive taxation (Feldstein, 1976, p. 81; Ng, 1987), for income redistribution, for the formulation of growth policy, as well as for the justification of the existence of the welfare state in general (Freshtman, Murphy, Weiss, 1966; Hammond, 1991). We proceed in this vein, and explore some implications of economic growth for welfare in the presence of interdependent utility functions following Feldstein's formulation. We demonstrate that, contrary to standard theory, growth need not raise aggregate welfare, if its benefits accrue unevenly, and if the utility functions of the population are interdependent (Konrad, 1992b).

2. The Model

We begin with a model in which the society is assumed to be composed of two individuals A and B with interdependent Cobb-Douglas utility functions with one good, x, (i.e., income). The social welfare function is assumed to be additive and democratic:

\[ U_A = \frac{x_A^{\alpha_a}}{x_B^{\beta_a}} \quad ; \quad U_B = \frac{x_B^{\alpha_b}}{x_A^{\beta_b}}. \]

2) \[ U_T = U_A + U_B. \]
Suppose that national income increases exogenously, e.g. through the
discovery of new resources. Assume, that the increment in x accrues only to A. The
question is under what circumstances, if any, might economic growth lead to a
decline in total welfare, contrary to standard analysis (McAdams, 1992).

Noting that

$$\frac{\partial UT}{\partial x_A} = \alpha a \frac{U_A}{x_A} - \beta b \frac{U_B}{x_A}$$

we obtain that the aggregate utility of the society decreases if and only if

$$U_A/U_B < \frac{\beta b}{\alpha a}.$$.

The outcome depends on the relative magnitudes of the terms on the two
sides of this inequality. The right hand side is an index of the degree of
interdependence of the utility functions, while the left hand side measures the
inequality in the initial distribution of utility. The initial income distribution determines
$$U_A/U_B$$, and hence whether it is smaller than $$\frac{\beta b}{\alpha a}$$. The stronger is the
interdependence, i.e. the larger is the value of $$\frac{\beta b}{\alpha a}$$, the larger is the range of utility
values for which inequality (4) holds.

The ratio $$\frac{\beta b}{\alpha a}$$ indicates the extent to which the consumption of x by A
decreases B’s utility as compared with the extent that it increases A’s own utility.
The size of the ratio is indicative of the degree to which members of society may be
characterized as "envious" ($$\beta$$) relative to being self-contained ($$\alpha$$). The larger is the
ratio, the stronger is the interdependence. Without interdependence $$\beta = 0$$ and the
ratio on the right hand side is zero, making it impossible for inequality (4) to ever
hold, since the left hand side is always positive. In an altruistic society $$\beta < 0$$, and
again the inequality becomes impossible. Consequently, in these two cases we have
the standard conclusion that growth always increases total welfare. If $$\beta > 0$$,
however, the inequality can hold provided the ratio of utility levels is less than $$\frac{\beta b}{\alpha a}$$. 
The case $\beta_b/\alpha_a > 1$ corresponds to the unrealistic case of a society in which enviousness dominates over self-centeredness.\textsuperscript{8} Hence, we assume that $0 < \beta_b/\alpha_a < 1$.

In this parameter range, growth can lead to a diminution in total social welfare in the two person case provided the gain accrues to the individual whose utility is initially lower by a sufficient ratio. This result becomes understandable by noting that in this model a part of the utility of the "wealthier" person, B, is derived from his relative wealth. If he is sufficiently envious, his utility can decrease more than the increase in A's utility.\textsuperscript{9}

The main conclusion from the above example is that there exists a region of parameter space where an exogenous increase in income which changes the distribution of income, decreases total welfare in a democratic aggregate utility function. One should note that this conclusion is dependent on the assumed form of the utility function. In particular, if we repeat the analysis with $U' = \ln U$, where $U$ is as in eq. 1., the conclusion changes and welfare increases in response to growth with any income distribution provided only that $\beta_b < \alpha_a$, as expected. On the other hand, replacing the Cobb-Douglas utility functions (eq. 1) with Stone-Geary, or, in fact, with separable utility functions of the form

\begin{equation} \text{5) } U_A = \frac{\hat{U}_A}{\hat{U}_B^{\delta_a}} ; \quad U_B = \frac{\hat{U}_B}{\hat{U}_A^{\delta_b}}. \end{equation}

where $\hat{U}$ indicates the utility of a consumer at some fixed level of utility for the other consumer,\textsuperscript{10} we obtain that total welfare can decrease provided\textsuperscript{11}

\begin{equation} \text{6) } U_A/U_B < \delta_b. \end{equation}

The parameter $\delta_b$ now plays the role of $\beta_b/\alpha_a$.\textsuperscript{11}
3. Conclusion

The above model explores the welfare implications of economic growth in a society composed of individuals with interdependent utility functions with negative externalities in consumption. We demonstrate that there do exist specifications of utility functions with regions of parameter space in which the welfare consequences of a rise in per-capita income are ambiguous, if the gains are distributed unequally. Hence, economic growth might bring about an increase as well as a decrease in aggregate social welfare depending on the relationship between the two indices in eq 4. One index measures the extent of envy in the society and the other the degree of income inequality as measured by the initial ratio of utilities. Total welfare is more likely to decrease if the gains from growth accrue disproportionately to the individual with the lower level of utility, the more envious is the individual who does not benefit from growth, the less selfish is the individual who gains from growth, and the more unequally income is distributed initially.¹²

These considerations are important, insofar as economic growth has been frequently associated with the skewing of the income distribution in favor of the upper income brackets. In an historical context, Simon Kuznets recognized this pattern for the early phases of modern economic growth (Kuznets, 1996, p. 212; Williamson and Lindert, 1980, pp. 62, 67; Lindert, 1991, p.. 216; Williamson, 1985, p. 18; Williamson, 1991, p. 15),¹³ and recent evidence confirms the increase in the Gini coefficient in Eastern Europe and the successor states of the Soviet Union during the recent transition to a market economy (Milanovic, 1996, p. 133). In many of these countries the Gini coefficient increased from 0.25 to 0.30 in just five years (United Nations Development, 1993, p. 17). In the United Kingdom a 2.1 percent growth rate in per capita GNP between 1965 and 1993 has been accompanied by a rise in the gini
coeffienient from 0.25 to 0.32. (The World Bank, 1993). In most countries of the world a wide divide separates the poor from the rich. In the United States, for instance, the richest 20 percent of the population earns 9 times as much as the poorest 20 percent.\textsuperscript{14} Hence, insofar as the benefits of economic growth often accrue unevenly, our theoretical considerations have practical policy implications.

Clearly, we analyzed a special case, and therefore, the result ought not be assumed to pertain generally. Nonetheless, our derivation does have far-reaching policy implications. One issue to explore is the extent to which the utility function of economic agents are interdependent. We have demonstrated that with gains of economic growth accruing unevenly, circumstances do exist such that a society could be better off either by choosing to forgo the opportunities for growth, or by government policy affecting the distribution of gains. Thus, there is room for government intervention. The reason for the existence of the welfare state is not only the amelioration of market failure, or the provision of comprehensive security, but also because of the existence of externalities in consumption – envy - imply that individual utility maximization does not automatically bring about utility maximization for the society. The society could be not only happier, but also more stable politically, with a more even distribution of the gains from growth, and a diminution in envy (Hammond, 1995). Our conclusion vividly redirects our focus from growth per se to its distributional consequences for aggregate welfare.\textsuperscript{15} In sum, the interdependent nature of a society’s utility function should not be neglected in the formulation economic policy, particularly with regard to such key issues as progressive taxation, redistribution of income, and the enactment of growth-inducing policy measures.\textsuperscript{16}
References


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Footnotes
Admittedly, in the short run "...one might feel happier by a recent increase in the standard of living, [but] one soon adapts to this and the positive relationship disappears." Hence, over time "...no strong positive relationship [has been found] either cross-sectionally or longitudinally between money [income] and happiness" (Furnham and Lewis, 1986, p. 109). We are not substantially happier than our ancestors even though we consume much more than they did (Scitovsky, 1976).

As Duesenberry pointed out a generation ago, "the utility derived from one's own house or car depends on a friend's house or car" (1949, p. 27).

Its main insight fits well the post-World War II U.S. private consumption experience (Bassman, Molina, and Slottje, 1988).

Furthermore, the price of the consumed good can be included in the utility function, thus generating a "snob appeal" effect (Basman, Molina, and Slottje, 1983; Haavelmo, 1970; Fisher, 1977; Spiegel and Templeman, 1985; Frank 1987).

For instance, Spiegel and Templeman have shown "...that a 'compulsory bundle' which a group... accepts as binding upon all its members might be superior to any other consumption pattern from the point of view of each and every member of the group" (1985, p. 315). Moreover, employees care about relative income within a firm, not only about their own absolute salaries (Frank 1984).

In Feldstein's analysis initially $U_A = U_B$, and $\alpha = \beta$ (notation as in eq. 1). He finds that an increment accruing unequally between two members of a society raises social welfare.

The presence of additional goods complicates the notation while leaving the analysis and conclusions unchanged.

If the distribution of income is egalitarian to begin with (i.e. $U_B = U_A$), then social welfare is likely to increase. This follows, since in this case inequality (4) holds only if $\beta_b/\alpha_a > 1$.

We can extend the analysis of the model in eq. (1) to n individuals. We treat only the simplest case of a society with two types of individuals A and B. We assume that all type A (or type B) individuals have identical utility functions and are at identical levels of utility. If there are $n_A$ individuals of type A and $n_B$ individuals of type B, then the total democratic welfare function becomes $U_T = n_A U_A + n_B U_B$. The
constants \( n_A \) and \( n_B \) are carried through the derivation, yielding a modified form of inequality (4):

\[
n_A U_A / n_B U_B < \beta_b / \alpha_a.
\]

The larger is the number of individuals who do not partake in the benefits of growth relative to the number of individuals who do benefit, the smaller \( \beta_b / \alpha_a \) can be and still allow the last inequality to be satisfied, implying a decline in welfare as a consequence of growth.

Note that similar inequalities also follow for non-democratic welfare functions if one reinterprets the coefficients \( n_A \) and \( n_B \) as the degrees to which society values the utility that accrues to persons A and B. The n person case also follows if the coefficients are again reinterpreted as the number of individuals times the degree to which the utility of such individuals is valued. Again, if the total initial welfare of the subgroup to which the gain accrues is sufficiently small as compared with the total welfare of the subgroup not enjoying gains, the total combined welfare of the two subgroups will decrease. Thus, there exist ways to apportion additional income to a population so as to decrease its total welfare.

10 We require that the indifference curves of each consumer be unaffected by the consumption of the other.

11 Once again, using a democratic welfare function and assuming that the gains accrue entirely to A as an increase in \( x_A \), we have

\[
U_T = U_A + \delta \frac{U_B}{U_A}.
\]

and

\[
\frac{U_T}{U_A} = \frac{U_A}{U_B} \left( 1 - \delta \frac{U_B}{U_A} \right).
\]

which is negative provided \( \frac{(\hat{U}_A)_{\delta_b+1}}{(\hat{U}_B)_{\delta_b+1}} < \delta_b \). If we note that \( \frac{U_A}{U_B} = \frac{(\hat{U}_A)_{\delta_b+1}}{(\hat{U}_B)_{\delta_b+1}} \) the inequality reduces to

\[
U_A / U_B < \delta_b.
\]

12 In the n person case the outcome depends also on the number of people who benefit from the gains from growth relative to the number who do not.

13This is the case partly because during economic growth gainers never compensate the losers whose marketable skills or investments became devalued.
The comparable figure for Brazil is 32 (United Nations Development, 1993, p. 17).

In addition, envy can have an influence on the growth process itself (Cole, Maliath, and Postelwaite, 1992).

"If growth is less important for welfare than we have so far supposed, other goals would rise in the scale of social priorities" (Abramovitz, 1979, p. 20).