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# Event Simultaneity in Cavities[1]

## Theory of the Distortions of Energy Deposition in Proportional Counters

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KELLERER, ALBRECHT M. Event Simultaneity in Cavities. Theory of the Distortions of Energy Deposition in Proportional Counters. *Radiat. Res.* **48,** 216–233 (1971).

The so-called wall effects are distortions of the statistics of energy deposition which occur in proportional counters even if the counting gas and the counter walls have the same atomic composition. The distortions are due to the density difference between walls and counting gas. They consist in the simultaneity of particle passages which in the case of uniform density would occur separately. For heavy particles the delta-ray effect is dominant, but the V-effect can be significant at very high energies of the primary particles. In the case of x-rays, $\gamma$-rays, and fast electrons the distortions are due to the reentry, or backscattering, effect. Numerical estimates for the extent of the different processes and the corresponding change of mean event size and of event frequency are given.

## INTRODUCTION

Evaluation of radiation quality has in the past been based on the concept of linear energy transfer (LET). This concept does not adequately describe the microscopic patterns of energy deposition and is therefore of limited applicability. In recent years the microdosimetric approach developed by Rossi and his coworkers (*1*, *2*) has become widely accepted; it is now increasingly used in attempts to correlate radiation quality and biological effect.

While technique and theory of microdosimetry have advanced there remains one unresolved aspect which limits the validity of microdosimetric measurements. This is the problem of the so-called wall effects. These effects occur if microscopic

[1] Based on work performed under Contract AT-(30-1)-2740 for the U.S. Atomic Energy Commission.

tissue regions are simulated by proportional counters. The counters are made of tissue equivalent plastic and are filled with tissue equivalent gas; they therefore represent what in the theory of energy deposition in cavities is called the "homogeneous case" (*3*). The density dependence of cross sections of charged particle interactions will be neglected in this paper. Accordingly, if all particle tracks were ideal straight lines without branches of secondary particles ($\delta$-rays) the density differences between the walls and the gas would not influence the track pattern in the sensitive volume of the counter. Particle tracks can, however, be curled and they do have branches of secondaries and tertiaries. This leads to distortions of the track patterns in the cavity because angular relations are not preserved when a particle track repeatedly passes the boundary between the gas and the solid walls (*4*). It is for this reason that experimental microdosimetric distributions are not strictly valid.

Attention has been given to this problem throughout the development of microdosimetry; particularly it has been discussed by Rossi (*5*) who has also introduced wall-less proportional counters. Consequently various such instruments have been constructed to eliminate wall effects in microdosimetric measurements (*6–8*). Though the problem has frequently been discussed, there is as yet no established basis for a systematic analysis.

It is the purpose of this paper to present certain definitions and a number of geometrical theorems which provide the conceptual framework for such an analysis.

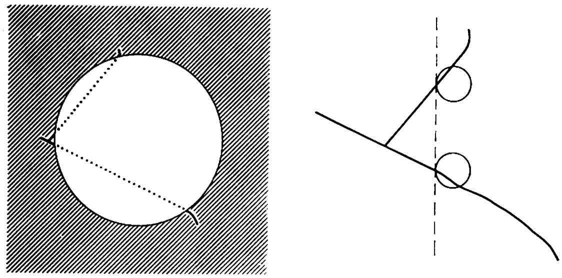
## CLASSIFICATION OF DISTORTED EVENTS

This section contains a survey of the principal types of effects which contribute to the wall effects. Detailed analysis of these processes will be given in the succeeding sections.

One simplification will be utilized throughout the discussion. This is the assumption that all particle tracks in the walls are short as compared to the dimensions of the cavity.[2] The boundary of the cavity can then be approximated by a plane whenever one considers only a single segment of a particle track in the wall. When seen from the inside the curvature of the boundary must be taken into account; when seen from the outside it is neglected. This "exterior flatness" of the cavity is used throughout this paper and is essential to its results.

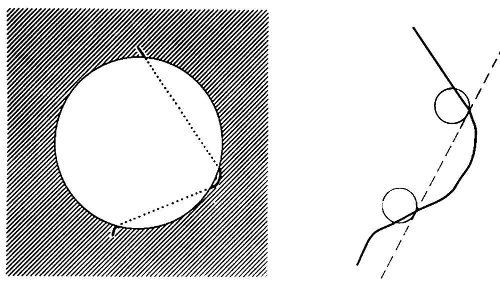
While the term *cavity case* designates simulation of a microscopic tissue region by a gas volume, the term *standard case* will be used to refer to the actual situation where the density is uniform throughout.

[2] With cavity radii larger than 1 cm the condition holds for electrons up to 1 MeV and for heavy particles of much higher energy. The discussion is restricted to these cases. Higher electron energies must be excluded also because the collision cross sections for electrons depend significantly on density above a few MeV.



a) b)

FIG. 1. Diagram of the δ-ray effect.



a) b)

FIG. 2. Diagram of the reentry effect.

*Delta-Ray Effect*

Figure 1a depicts a charged particle which enters the cavity simultaneously with one of its δ-rays. The distance between the two entrance points may be large enough that either the primary particle or the δ-ray but not both could enter the actual microscopic region. The two passages are then separate events in the standard case (see Fig. 1b). The distortion towards larger events in the cavity may be termed δ-ray effect. The term refers also to the analogous situation where the δ-ray is formed after the primary charged particle has left the cavity. It will be shown that the δ-ray effect is most important for heavy charged particles.

*Reentry Effect*

An electron may traverse the cavity, then due to its curled track it may reenter it (see Fig. 2a). The points of exit and reentrance can be far enough apart that the electron would not reenter the actual microscopic region. Figure 2b represents the two events which are statistically independent in the standard case. As in the δ-ray effect the distortion consists in a superposition of two events in the cavity. This reentry effect applies only to electrons because they are the only particles with significant curvature of their tracks.

*V-Effect*

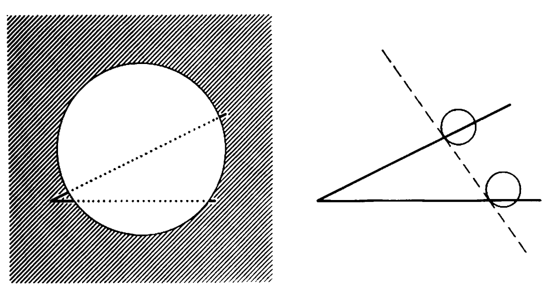
In a nonelastic neutron collision or in a nuclear collision of an energetic charged particle several charged particles can be set in motion simultaneously. Figure 3 depicts the case where two nuclear fragments originate outside the cavity and both traverse it (Fig. 3a). The two entrance points may again be far enough apart that in the standard case the corresponding passages would be separate events as indicated in Fig. 3b. This is therefore a third mechanism whereby for a cavity one obtains events which are distorted towards larger energy deposition. The process has been termed V-effect in view of the V-shaped tracks which are involved. It does, however, also apply to spallation where one may be dealing with a cluster of numerous nuclear fragments.

## DISTORTION AS SUPERPOSITION OF EVENTS

*The Concept of Segment Multiplicity*

One can summarize the preceding section and state that wall effects lead to an increase in event size by superposition of events. This general rule has been pointed out earlier (*5*, *9*). Wall effects consist in nothing but the simultaneous occurrence of energy deposition events which would otherwise occur independently and separately. Apart from this change in statistical correlation the track pattern in the region U remains unchanged. For a rigorous formulation of the statement one needs the notion of a track segment.

Figure 4 illustrates this notion. The region of interest contains the four segments A, B, C, and D of tracks. Each of these is called a *track segment*. A, B, C though being separate track segments belong to the same *event*, or *energy deposition event*,



a) b)

FIG. 3. Diagram of the V-effect.

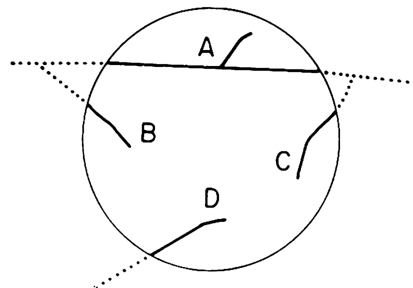



FIG. 4. Definition of the concept track segment.

because they are correlated i.e., due to the same primary process. The track segment D, on the other hand, forms a separate event.

With these concepts one can state the invariance of the segment spectrum in the following way: Assume that a region $U$ is exposed to a uniform field of charged particles. Then the relative frequency of track segments of various shapes is independent of the density of matter surrounding the region.

The relation follows directly from the fact that under the conditions for Fano's theorem (*10*) the fluence and fluence spectrum throughout the region $U$ and its surrounding is constant and independent of the density of the surrounding matter. The frequency of the entrance of particles of given energy is therefore independent of the density outside the region. After the entrance of the charged particle into the region, the shape of the track segment is determined by the collision cross sections of the particle in the region $U$. It is therefore independent of the density outside $U$. For particles originating inside $U$ the same considerations hold. The spectrum of track segments in $U$ is therefore independent of the density of the surrounding medium. According to this simple but important relation the wall effects merely change the degree of simultaneity; i.e., the statistical coupling of track segments differs with the density of the surrounding medium. As a measure of the extent of this coupling one can use the mean number of segments involved in an event. This quantity will be called *multiplicity*. It is the object of the following sections to derive the multiplicities in the standard case and in the cavity case. First the relations which connect event frequency and mean event size with the segment frequency and the multiplicity will be given, and the formulae for the absolute segment frequencies for different radiations and different region sizes will be derived.

*Formulae for Event Frequency and Segment Frequency*

As will be shown in a later section one obtains maximum multiplicity $M^*$ in the cavity case and under the condition of a high density ratio which has been discussed earlier. An asterisk will be used throughout the following text to indicate values which belong to this cavity case. The other limit is the *free space case*, i.e., the case where a convex volume $U$ is positioned *in vacuo*. In this case the multiplicity has the value 1. The multiplicity $M$ in the standard case is between 1 and $M^*$. In a later section it will be seen that for small diameters of the region $U$ the multiplicity $M$ tends towards 1.

If the *segment frequency* per rad in the region $U$ is designated by $\Phi_s$ and the *event frequency* by $\Phi$, then one has the relations:

$$\Phi = \frac{\Phi_s}{M} \quad \text{and} \quad \Phi^* = \frac{\Phi_s}{M^*}. \tag{1}$$

The mean energy deposition $\bar{\epsilon}_s$ per segment and the mean energy deposition per event are inversely proportional to the frequencies $\Phi_s$ and $\Phi$. One therefore obtains:

$$\bar{\epsilon} = M\bar{\epsilon}_s \quad \text{and} \quad \overline{\epsilon^*} = M^*\bar{\epsilon}_s. \tag{2}$$

In view of the invariance of the segment spectrum the asterisk is omitted in $\Phi_s$ and in $\bar{\epsilon}_s$. One concludes that the event frequencies change inversely to the multiplicities:

$$\Phi^*/\Phi = M/M^*. \tag{3}$$

While in the cavity the event frequency is decreased due to the wall effects, the mean energy $\bar{\epsilon}$ deposited in an event is correspondingly increased by the factor $M^*/M$:

$$\bar{\epsilon}^*/\bar{\epsilon} = \Phi/\Phi^* = M^*/M. \tag{4}$$

The formula for the segment frequency can be derived for regions exposed to isotropic uniform fields of ionizing radiation. One can apply to this purpose a theorem on the mean segment length in a region randomly traversed by particle tracks. If the region has the volume $V$ and the surface area $S$ then its mean chord length is given by the relation:

$$\bar{l} = 4V/S. \tag{5}$$

This is the so-called theorem of Cauchy which holds for straight infinite tracks. The theorem can be generalized to tracks of finite length which may be curled and branched (*11*). The term track is used for the line traversed by the primary charged particle together with the lines traversed by all its secondaries. The length of a track is the total length of these lines. In the same way the term segment length is used to denote the total length of a track segment in the region $U$ (see Fig. 4). One then has the following relation for the mean segment length $\bar{s}$:

$$\bar{s} = (1/\bar{l} + 1/\bar{r})^{-1} \tag{6}$$

where $\bar{r}$ is the mean track length and $\bar{l}$ is defined according to Eq. (5). This is the generalization of Cauchy's theorem. If $\bar{E}_0$ is the mean initial energy of the charged particles then one has the following mean energy deposition $\bar{\epsilon}_s$ due to a segment:

$$\bar{\epsilon}_s = \bar{E}_0 \frac{\bar{s}}{\bar{r}} = \frac{\bar{E}_0}{1 + \bar{r}/\bar{l}} \quad \text{with:} \quad \bar{l} = 4V/S. \tag{7}$$

The frequency of segments per rad is therefore:

$$\Phi_s = 0.0624\ V/\bar{\epsilon}_s = 0.0624\ (1 + \bar{r}/\bar{l})V/\bar{E}_0\,. \tag{8}$$

The numerical constants in this and in the following equations reflect the choice of units; $\bar{E}_0$ is measured in keV and the volume $V$ of the region is measured in $\mu m^3$. It is assumed that the simulated medium has unit density. One should note that relations (6) to (8) hold for any uniform isotropic radiation field and for regions of arbitrary and even nonconvex shape. In the special case where the tracks are long as compared to the dimensions of the region, $\bar{r} \gg \bar{l}$, one has according to Eqs. (5) and (7):

$$\bar{\epsilon}_s = \frac{\bar{E}_0 4V}{\bar{r}S} \tag{9}$$

and similarly:

$$\Phi_s = 0.0156 \frac{S\bar{r}}{\bar{E}_0}. \tag{10}$$

The event frequencies $\Phi$ and $\Phi^*$ in the standard case and for the cavity are equal to the segment frequency divided by the multiplicity.

The corresponding equations for the microscopic quantities mean specific energy per event, $\bar{z}_F = \Phi^{-1}$, and mean lineal energy, $\bar{y}_F$, are[3]:

$$\bar{z}_F = 16.02 \frac{\bar{\epsilon}}{V} = 16.02 \frac{\bar{E}_0}{(1 + \bar{r}/\bar{l})V} M \tag{11}$$

and

$$\bar{y}_F = \frac{\bar{\epsilon}}{\bar{l}} = \frac{\bar{E}_0}{\bar{l} + \bar{r}} M. \tag{12}$$

In the special case where the tracks are long as compared to the region of interest

$$\bar{z}_F = 64.08 \frac{\bar{E}_0}{\bar{r}S} M \tag{13}$$

[3] For the definition of these microdosimetric quantities which measure the mean event size see Refs. (*12*) and (*13*). The quantities used here refer to all events including those where a charged particle traverses $U$ without energy transfer; these "zero events" are excluded in the usual definitions. The differences are significant only for small equivalent diameters of $U$.

and

$$\bar{y}_F = \frac{\bar{E}_0}{\bar{r}} M. \quad (14)$$

In the free space case the factor $M$ is equal to 1; as discussed in the next section, it is slightly larger than 1 in the standard case. For the cavity case one must use $M^*$ instead of $M$ in Eqs. (11) to (14). The values $M^*$ which represent the distortions due to wall effects will be discussed in a later section.

## MULTIPLICITY IN THE STANDARD CASE

This section deals with the multiplicity $M$ in the standard case. Numerical solutions are complicated and are outside the scope of this study. The discussion will be limited to a geometrical theorem which indicates how solutions can in principle be obtained. This will suffice to visualize the relative importance of the three factors $\delta$-rays, track curvature, and V-shaped tracks.

Consider a spherical region $U$ of diameter $d$ and define the associated volume of a track in analogy to the definition given by Lea (*14*). As indicated in Fig. 5 the associated volume $V_a$ is the volume of the region of all the points which are not more than a distance $d/2$ away from the track. This quantity has to be averaged over all particle tracks. With this definition the theorem which will be used can be formulated as follows:

The mean number $M$ of segments in an event is equal to $V_a'/\bar{V}_a$, where $\bar{V}_a$ is the mean associated volume of the tracks and $V_a'$ is the associated volume which results if each track is transformed into a straight line of length equal to the track length including the secondaries. $V_a$ is represented in Fig. 5a and $V_a'$ is represented in Fig. 5b.

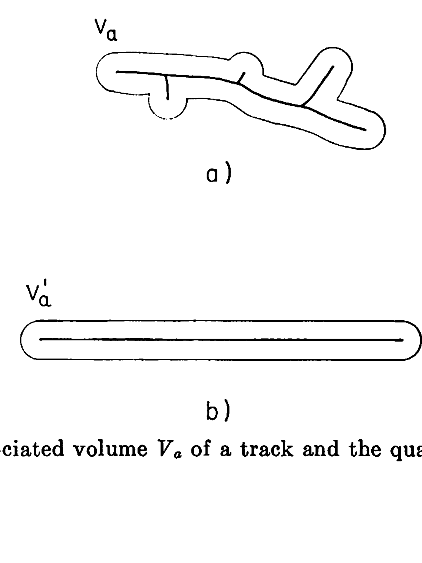



FIG. 5. The associated volume $V_a$ of a track and the quantity $V_a'$.

To demonstrate the relation one must compare the segment frequency with the event frequency in the spherical region. Assume a field such that in the mean one has one track originating per unit volume. The mean total track length in the spherical region of volume $V$ is then $V\bar{r}$. According to Eq. (6) the mean segment length in the region is $(\bar{r}^{-1} + S/4V)^{-1}$. Thus the segment frequency is equal to:

$$V\bar{r}(\bar{r}^{-1} + S/4V) = V + \bar{r}\frac{d^2}{4}\pi. \quad (15)$$

This is equal to the associated volume $V_a'$ indicated in Fig. 5b. The event frequency, on the other hand, is equal to the actual associated volume $V_a$. The multiplicity, being equal to the ratio of segment frequency and event frequency, is therefore $V_a'/V_a$. This proves the theorem.

The relation could in principle be used for a numerical derivation of the multiplicities, if the associated volume $V_a$ were obtained by Monte Carlo calculations. In the present context, however, very approximate estimates are sufficient.

Concerning the $\delta$-rays one may make the rough assumption that those $\delta$-rays whose range exceeds $d/2$ contribute fully to $V_a$, while the $\delta$-rays with range less than $d/2$ contribute not at all. The multiplicity $M$ in the standard case is then equal to the full track length divided by the track length excluding all $\delta$-rays shorter than $d/2$. In the next section it will be shown that the multiplicity $M^*$ in the cavity case is obtained if one excludes the $\delta$-rays altogether. Numerical data for the contribution of $\delta$-rays of different range to the track length will be given in the next section. At present it may suffice to remark that for electrons the resulting multiplicity is below 1.05, while for protons it can reach the value 1.2.

As far as the curled tracks of the primary electrons are concerned one can assume that the associated volume $V_a$ is nearly unaffected by the track curvature, if the energy of the electron exceeds about 20 keV because the track curvature is then small as compared to the typical site diameters of the order of 1 $\mu$m. Except for low electron energies the reentry of electrons does therefore not significantly increase the multiplicity in the standard case.

In the case of V-shaped tracks of heavy particles and under the assumption that the orientation of the two legs of the track is uncorrelated one may estimate that $V_a'/V_a$ exceeds 1 by a value of the order of $d/l$, where $l$ is the combined length of the track. The multiplicity due to the $V$-effect can therefore be neglected if the track length greatly exceeds the dimensions of the region $U$.

## THE EXTENT OF WALL EFFECTS

### *Multiplicity in the Case of Cavities*

The fact that one obtains maximum multiplicity in the limit of a high density ratio between the walls and the interior of the cavity can be understood from the scheme of Fig. 6.

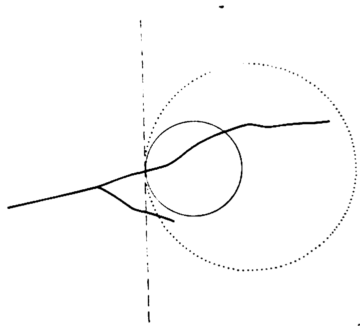


FIG. 6. Event simultaneity and density ratio.

A part of a particle track enters the region $U$ which is represented by the solid circle. Segment multiplicity is due to the possibility that another part of the track also enters the region. This is not the case in the example of Fig. 6. If the region is simulated by a sphere of half the density and therefore double the diameter one has to consider the surface represented by the dotted line. The probability for the entrance of a second part of the track is then increased, and it is further increased if the density ratio reaches such high values that the surface of the cavity in the vicinity of the track must be considered as a plane. This latter case is indicated by the broken line, it corresponds to maximum multiplicity.

One notes that the scheme of Fig. 6 is based on the assumption that the region $U$ is convex. The argument is reversed for a region which is concave. One may then obtain minimum multiplicity in the cavity case.

In the standard case multiplicities can be derived by computing associated volumes as described in the last section. In the cavity case the geometrical problem is different; according to the "exterior flatness" of the cavity boundary one deals with particle tracks randomly intersecting a plane. This aspect will be discussed in the following sections. The solution for $\delta$-rays which are short as compared to the primary track is simple. Whenever a $\delta$-ray traverses the plane the probability is high that the primary particle also traverses the plane. Therefore $\delta$-ray influx is nearly always coupled with direct traversal of the cavity by the primary. For the curved tracks of primary electrons one finds that the multiplicity is equal to $1/(1 - R)$, where $R$ is the Albedo for an isotropic field and a plane, i.e., the reentry probability of the electron into a plane after traversing it. Alternatively one can show that the multiplicity is equal to the mean number of intersection points which result if the track of the primary particle randomly intercepts the plane. This latter rule will also be applied to the case of V-shaped tracks.

### *Delta-Ray Effect*

A charged particle can produce an energy deposition event in a region $U$ in two different ways. It can traverse or partially traverse $U$; this will be termed a *direct*

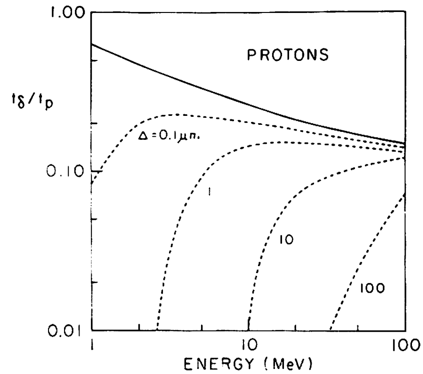



FIG. 7. Ratio of $\delta$-ray track length $t_\delta$ to primary track length $t_p$ for protons at energy $E$ in water ($I$ = 65 eV). The solid line includes all $\delta$-rays over 100 eV; the other curves include $\delta$-rays with ranges exceeding 0.1, 1, and 10 $\mu$m. Electron ranges and proton stopping powers from Ref. (*16*).

*event.* It can also pass outside $U$ and merely inject one or several of its $\delta$-rays; one can then speak of an *indirect event.*

The considerations will first refer to heavy charged particles. In this case all $\delta$-rays are short as compared to the primary track and it is therefore unlikely that a $\delta$-ray enters the cavity while the primary track does not cross its surface. The latter occurs only in the few cases where the primary track is nearly parallel to the surface of the cavity. All indirect events can therefore be disregarded and the number of events is equal to the number of primary particles entering $U$, if one neglects those events[4] where the primary charged particles originate inside $U$. The event frequency $\Phi^*$ is therefore proportional to the total primary track length, $t_p$. The segment frequency $\Phi_s$ on the other hand, is equal to the total number of particles, including $\delta$-rays, which enter $U$. It is thus proportional to the sum of $t_p$ and the total $\delta$-ray track length, $t_\delta$. The multiplicity, i.e., the ratio between segment frequency and event frequency, is therefore $1 + t_\delta/t_p$.

Figure 7 gives the values $t_\delta/t_p$ for protons of different energy. These values refer to a particular energy, they are not averaged over the total proton track. The data have been computed on the basis of the inverse square law for the frequency of $\delta$-rays of different energy, and with the electron ranges given in Ref. (*16*). The solid line results if one includes all $\delta$-rays over 100 eV; this corresponds to the value $M^* - 1$. In fact one is, however, not interested in the total fraction of double events. Those which involve very short ranged $\delta$-rays occur also in the standard case, and they, therefore, do not reflect wall effects. As mentioned in the discus-

[4] If "starters" and "insiders" [see (*15*)] are included the number of direct events is larger by the factor $1 + \bar{l}/\bar{r}$, where $\bar{l}$ is the mean chord length of the region $U$ and $\bar{r}$ is the mean range of the primary particles [see (*11*)].

sion of the multiplicity in the standard case one may assume the approximation that only those δ-rays whose range exceeds the equivalent radius of the region $U$ are involved in wall effects. The broken lines in Fig. 7 give the values $t_\delta/t_p$ for δ-rays which exceed 0.1, 1, and 10 μm.

One concludes that for proton energies below a few MeV the δ-ray effects are insignificant. For higher energies the fraction of double events due to wall effects is of the order of magnitude of 15%. A corresponding number of indirect events occurs in the standard case, but is suppressed in the cavity case. The event frequency in the cavity case is decreased by about 15%, while the mean event size goes up correspondingly.

The fraction of absorbed dose involved in the suppressed indirect events has been calculated in an earlier work (*17*); those calculations have also covered the case of monoenergetic neutron fields. The fraction of energy involved in the δ-ray effects is smaller than the fraction of events involved in the δ-ray effect. Experimental data obtained with so-called wall-less proportional counters (*6*, *7*) are still too incomplete for an exact comparison with the theory but they appear to be consistent with the results given here and in the previous work.

For heavier particles of the same energy per nucleon one can also use the curves of Fig. 7. One must, however, multiply the values $t_\delta/t_p$ by $Z^2$, where $Z$ is the atomic number of the particles. This reflects the fact that for a given energy $E$ per nucleon (equal velocity) the δ-ray spectrum is equal for all nuclei, while the frequency of δ-rays per unit length of the particle track is proportional to $Z^2$. One concludes that the δ-ray effect can be quite significant for heavier particles; if the energy per nucleon exceeds several MeV most of the events are distorted due to wall effects.

For electrons one can apply the same considerations. The only difference is that the δ-rays are not all short as compared to the primary track, and the longest δ-rays can thus produce indirect events even in the cavity case. One therefore obtains a certain overestimate of the δ-ray effect from the values $t_\delta/t_p$ for electrons. On the other hand, one finds even with this conservative estimate that the δ-ray effect for electrons is relatively small. Figure 8 gives the values of $t_\delta/t_p$, i.e., the mean δ-ray track length per unit length of the primary track, for electrons. The data are computed on the basis of the Møller cross sections and of the electron ranges given in Ref. (*16*). It follows that the fraction of distorted events is less than 6% for electrons between 10 keV and 1 MeV. In the following section it will be shown that for electrons and therefore also for $x$- and $\gamma$-rays the reentry effect is dominant.

### *Reentry Effect*

In this section only primary electron tracks will be considered. The δ-rays are not taken into account; their contribution to the wall effects has been discussed in the preceding section.

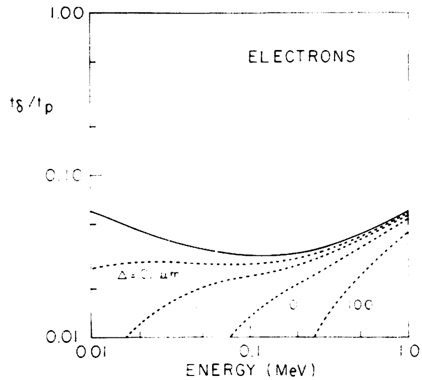



FIG. 8. Ratio of $\delta$-ray track length $t_\delta$ to primary track length $t_p$ for electrons at energy $E$ in water ($I$ = 65 eV). The solid line includes all $\delta$-rays over 100 eV; the other curves include $\delta$-rays with ranges exceeding 0.1, 1, 10, and 100 $\mu$m. Electron ranges and stopping powers from Ref. (*16*); $\delta$-ray spectrum according to the Møller cross sections.

It will further be assumed that all electron tracks terminating inside the cavity can be neglected.[5] Under this condition the frequency $\Phi_s$ of track segments in the cavity is equal to the number of electrons leaving it. The number of electrons leaving the cavity without return is equal to the event frequency $\Phi^*$. The multiplicity is then:

$$M^* = \Phi_s/\Phi^* \tag{16}$$

and the reentrance probability of the electron is:

$$R = \frac{\Phi_s - \Phi^*}{\Phi_s} = 1 - 1/M^*. \tag{17}$$

As discussed earlier the outer surface of the cavity can be treated as a plane. The reentrance probability of an electron is therefore equal to the retraversing probability of electrons passing a plane; this is the Albedo or reflection coefficient for an isotropic field and a plane. The quantity is related to the mean number of intersection points which result if an electron track randomly intercepts a plane. In the following it will be shown that this latter quantity is, in turn, related to the ratio of integrated range to effective range of the electron.

Figure 9 is a schematic diagram of an electron track traversing a plane $P$. The

[5] This assumption is justified whenever the mean electron range is large as compared to the equivalent diameter of the region. The term *mean range* is used here in the connotation of statistical mean over the spectrum of initial electron energies. The term *range* or *integrated range* $r$ is used for the full length of the electron track; in the literature this latter quantity is frequently called csda range. An *effective range*, $r_0$, defined as the distance between the starting point and the end point of the electron track will also be used in this section.

integrated range is $r$, and $r_0$ is an effective range defined as the distance between the starting point $A$ and the end point $B$ of the electron.

Consider the closed figure $F$ formed by the curved track and the straight line connecting $A$ and $B$ and assume that the fluence is such that per unit interval of line length one has on the average one intersection with the randomly oriented plane $P$. The term interception will be used to denote passage of the track, the line $AB$, or the closed curve $F$ through the plane. An interception may be simple or multiple, i.e., it may produce one intersection point or more than one. Whenever the line $AB$ is intercepted the track is also intercepted. The line $AB$ of length $r_0$ is intercepted $r_0$ times because all its interceptions are simple. For the track the total number $I$ of interceptions must therefore be at least $r_0$. On the other hand one has $r + r_0$ intersection points on $F$, and because on $F$ each interception must at least produce two intersection points there cannot be more than $(r + r_0)/2$ interceptions of $F$, and $I$ must be less than $(r + r_0)/2$:

$$\frac{r + r_0}{2} \geqq I \geqq r_0. \tag{18}$$

The mean number $m$ of intersection points on the track per interception is $r/I$ and one therefore has:

$$\frac{2r}{r + r_0} \leqq m \leqq \frac{r}{r_0}. \tag{19}$$

If the track passes $m$ times through the plane, then it reenters after $m - 1$ of these $m$ passages and the reentry probability is therefore:

$$R = \frac{m - 1}{m} = 1 - 1/m \tag{20}$$

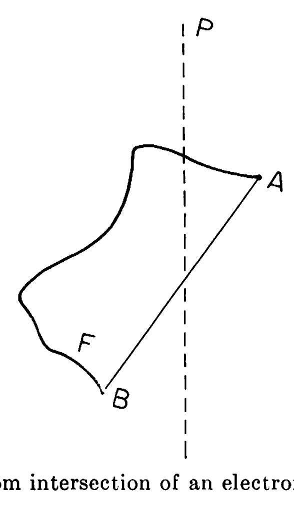



FIG. 9. Random intersection of an electron track and a plane $P$.

Comparison with Eq. (17) shows that the mean number $m$ of crossings through a plane is equal to the multiplicity $M^*$ of track segments in the cavity. Therefore one obtains the relations:

$$\frac{2r}{(r + r_0)} \leqq M^* \leqq \frac{r}{r_0} \tag{21}$$

$$\frac{r - r_0}{2r} \leqq R \leqq \frac{r - r_0}{r} \tag{22}$$

The lower limits are reached when the track, supplemented by the straight connection of its terminal points, is a planar and convex figure.[6] The upper limit is reached when the track is closely curled around the straight connection of its terminal points. Neither of these conditions is fulfilled for an electron track, and one therefore estimates that $M^*$ and $R$ lie near the middle of the intervals determined by Eqs. (21) and (22). The ratio $r_0/r$ could be obtained by the method developed by Spencer (*18*). In the absence of numerical results one may set $r_0$ to 0.75 $r$ for a wide range of energies. This is in approximate agreement with assumptions made earlier by Charlton and Cormack for tissue-equivalent material (*19*). With this value one concludes that the probability $R$ of double events due to the reentrance of electrons is about 0.19. This is only a rough estimate, but it is in substantial agreement with a more direct calculation. M. J. Berger[7] has performed Monte Carlo calculations for air which give reentrance probabilities, i.e., Albedo values $R$ for an isotropic field and a plane, which are 0.25 for 2 keV electrons and decrease to 0.20 for 1 MeV.

In the standard case reentrance of electrons is unlikely except for energies low enough that the electron range is comparable to the dimension of the region $U$. One therefore concludes that for electrons, and therefore also for $x$- and $\gamma$-rays, the wall effects are mainly due to reentrance of electrons. About 40 % of the events are misrepresented in the cavity case due to this effect; this results in about 20 % double events. The event frequency in the cavity is therefore decreased by 20 %, and the mean event size is increased by 20 %. According to the higher Albedo values the distortions should be somewhat larger for soft x-rays than for $\gamma$-rays. One should note that these theoretical predictions are not as yet verified by experimental findings. They are, however, in general agreement with results obtained with $^{14}C$ beta rays (*20*).

### *V-Effect*

Some theoretical considerations concerning the V-effect have been given earlier.[8] Monte Carlo calculations (*21*) indicate that the effect is insignificant for 6 MeV

[6] A special case of this is a track which consists of two straight parts. This configuration will be discussed in the following section in connection with the V-effect.

[7] Private communication.

[8] B. J. Biavati, Frequency of Multiple Tracks in Equivalent Cavities of Varying Density, 137–140, AEC-NYO-2740-3 (1966).

neutrons. This agrees with the fact that cross sections for nonelastic nuclear collisions are small for neutrons below 10 MeV. The cross sections are negligible for charged particles of considerably higher energies.

At sufficiently high energies, however, tracks of two or more nuclear fragments can contribute significantly to the absorbed dose. V-shaped tracks can also occur due to nuclear recoils produced by charged particles. The present discussion will not be concerned with an analysis of the relative contribution of the different processes,[9] nor will it be concerned with the distribution of lengths of V-shaped tracks or with the distribution of the angles in these tracks. The probability for the occurrence of simultaneous traversals will, however, be given as function of the shape of a V-track. This requires no formal derivation because it is a special case of the relations obtained in the preceding paragraph.

If one considers all those events where the cavity is affected by a V-shaped track formed outside the cavity, then according to Eq. (21) and its interpretation given in footnote 6 one has the following multiplicity:

$$M^* = \frac{2L}{L + c}, \tag{23}$$

where $L$ is the combined length of both arms of the $V$, and $c$ is the separation of its two end points. This means that the fraction of events which are double events is:

$$f = \frac{L - c}{L + c} \tag{24}$$

In an actual case one must integrate the values over the spectrum of track lengths and of angles between the two particles.

Pair production is analogous to the V-effect. Due to their curled tracks both electrons can simultaneously traverse the cavity even if both particles start out in the opposite direction. For $\gamma$-rays and x-rays of sufficient energy the multiplicity may therefore exceed the values derived in the preceding section.

Finally it should be remarked that there is a type of wall effect which has not been included in the preceding discussion. This is the simultaneous occurrence of two charged particles *separately* set in motion by the same uncharged particle. The charged particles can either be electrons liberated by x-rays or $\gamma$-rays, or they can be nuclear recoils from neutrons. In both cases the range of charged particles is much less than the mean free path of the uncharged particles. This means that the probability for simultaneous occurrence is small. For electrons the correlation can be completely neglected because the reentry effect dominates. For nuclear recoils the effect may not be completely insignificant though the absolute frequency is small (*21*).

[9] P. G. Steward, Results of Computations of Depth Dose in Tissue Irradiated by Protons, Report UCRL-16154. National Bureau of Standards (1965).

## CONCLUSION

One can make the following summarizing statements on the distortion of energy deposition spectra in cavities: **(1)** When a microscopic tissue region is simulated by a cavity, certain particle passages occur simultaneously which in a medium of uniform density are statistically independent. **(2)** One can distinguish three processes which are mainly responsible for the increase in multiplicity:

The $\delta$-ray effect consists in the suppression of indirect events, i.e., of those events where the primary charged particle does not actually enter the region, but merely injects its $\delta$-rays.

The reentry effect is significant only for electrons; it is due to the backscattering of the electron into the cavity.

The V-effect is the simultaneous passage of two or more heavy charged particles, specifically those produced in a nonelastic neutron collision, through the cavity. **(3)** For heavy particles the $\delta$-ray effect is dominant. For protons of energy above 5 MeV the frequency of double events is in the vicinity of 15%. For heavier nuclei of more than 5 MeV per nucleon this value has to be multiplied by $Z^2$. The suppressed indirect events are smaller than the direct events, and the fraction of energy involved in wall effects is therefore smaller than the fraction of events involved. For larger energies, such as of cosmic radiation, the V-effect can be significant. **(4)** For electrons up to 1 MeV the fraction of total track length represented by $\delta$-rays is smaller than for heavy particles and the $\delta$-ray effect is therefore insignificant. About 20% of all events should, however, be double events due to the reentry of the primary electron. **(5)** Event frequencies and mean event sizes are obtained by inserting the appropriate values of the multiplicity into formulae (9) to (14). Whenever the V-effect is insignificant one can take approximations which make it possible to drop the factor $M$ or $M^*$ in these equations. Event frequency and mean event size for the homogeneous medium are derived by excluding from $\bar{r}$ all $\delta$-rays with range less than the radius of the region. In the cavity case and for heavy particles the correct values are obtained by excluding all $\delta$-rays.

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