

Item-Focused Trees for the Detection of Differential Item Functioning in Partial Credit Models

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Abstract

Various methods to detect differential item functioning (DIF) in item response models are available. However, most of these methods assume that the responses are binary, and so for ordered response categories available methods are scarce. In the present article, DIF in the widely used partial credit model is investigated. An item-focused tree is proposed that allows the detection of DIF items, which might affect the performance of the partial credit model. The method uses tree methodology, yielding a tree for each item that is detected as DIF item. The visualization as trees makes the results easily accessible, as the obtained trees show which variables induce DIF and in which way. In the present paper, the new method is compared with alternative approaches and simulations demonstrate the performance of the method.

Keywords

partial credit model, differential item functioning, recursive partitioning, item-focused trees

Introduction

In psychometric tests, it is generally assumed that measurement properties are stable across individuals, a property that is known as measurement invariance (Millsap, 2012). However, it might occur that different groups of people react differently to

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the same test, which threatens the validity of the measurements. Also, test fairness is violated if tests lead to different conclusions for distinct groups of people. Differential item functioning (DIF) means that measurement invariance is violated on the item level. More precisely, DIF is present if one or more items are significantly more difficult for one group than for the other after controlling for the underlying ability or trait. One can distinguish between uniform and nonuniform DIF. *Uniform DIF* means that the difference between the groups is constant across levels of the latent continuum of the individual. If it is dependent on the ability or trait of the person *non-uniform DIF* is present. DIF detection procedures can also be classified into item response theory (IRT) methods and non-IRT methods. The IRT methods, also called parametric methods, are those in which an IRT model is used for the detection of DIF. For an overview of IRT methods and non-IRT methods, see Holland and Wainer (1993), Magis, Bèland, Tuerlinckx, and De Boeck (2010) and Penfield and Camilli (2006).

The basic idea of traditional DIF detection procedures in both dichotomous and polytomous IRT models is to prespecify two groups of persons and then determine whether item parameter estimates differ between these groups. The first method that was used for the detection of DIF in IRT models was the likelihood ratio (LR) test (Andersen, 1973). An alternative approach that can be used for any kind of IRT models is *Lord's chi square test* (Lord, 1980). While this test is restricted to the comparison of two groups, its extension by Kim, Cohen, and Park (1995), the *generalized Lord test*, can be used for more than one focal group. A third approach is the *Raju method* (Raju, 1988), which is based on the idea that the difference between the shape of item response curves (IRCs) between two groups indicates DIF. Further test statistics for parameter differences between pre-specified groups were suggested by Thissen, Steinberg, and Wainer (1993) and Holland and Thayer (1988). These classical methods have in common that they are limited to few sub-groups that need to be pre-specified by the user. Moreover, it is hard to consider more than one DIF-inducing covariate at a time.

More recently, two strategies were proposed for the detection of DIF in Rasch models that is generated by multiple covariates and for which sub-groups do not have to be pre-specified. The first strategy uses regularization methods to handle the abundance of parameters in the model. Tutz and Schauburger (2015), Magis, Tuerlinckx, and De Boeck (2015) and Thissen et al. (1993) used penalized likelihood estimation, whereas Schauburger and Tutz (2016) proposed boosting methods to obtain regularized estimates. The second strategy is to use recursive partitioning techniques, often called tree methods. One needs to distinguish between two quite different forms of tree methods in DIF detection. In the method proposed by Strobl, Kopf, and Zeileis (2015), called *RaschTree*, the covariate space is recursively partitioned to identify regions of the covariate space in which item parameters differ. In the investigated regions, a parametric latent trait model that includes covariates is fitted. Regions are suspected to be relevant if their parameter estimates differ strongly. A disadvantage of the method is that it detects regions of the covariate space that are linked to DIF, but does not automatically detect the responsible items. The alternative recursive

partitioning method propagated by Tutz and Berger (2016) focuses on the detection of the items that are responsible for DIF. Unlike the RaschTree, it uses recursive partitioning on the item level, not on the global level. Since the method is able to flag DIF items it is referred to as item-focused trees.

For polytomous models, in particular the partial credit model, DIF detection methods are less common than for dichotomous models. Some methods are proposed in Penfield and Camilli (2006). For further concepts, see also Penfield and Algina (2003), Penfield (2007) and Penfield (2008). A method that competes with the approach considered here is the extension of the RaschTree to the partial credit model proposed by Komboz, Strobl, and Zeileis (2018). It will be considered in more detail in the section Comparison of Recursive Partitioning Methods.

The objective of the present paper is the development of item-focused trees for the partial credit model. In the next section, the notation and the basic model will be introduced. In addition, we present an illustrative example. The tree algorithm that is used is given in detail in the subsequent section. In the penultimate section, we show results of wider simulation studies. In the final section, we discuss further possible extensions of our modeling approach.

DIF in Partial Credit Models

In the following, we consider I items with ordered categories and P persons. For simplicity, we assume that the number of categories k is equal across items.

The Partial Credit Model

Let $Y_{pi} \in \{0, 1, \dots, k\}$, $p = 1, \dots, P$, $i = 1, \dots, I$, denote the ordinal response of person p on item i . The partial credit model (PCM), which was proposed by Masters (1982), assumes for the probabilities

$$P(Y_{pi} = r) = \frac{\exp(\sum_{l=1}^r \theta_p - \delta_{il})}{\sum_{s=0}^k \exp(\sum_{l=1}^s \theta_p - \delta_{il})}, \quad r = 1, \dots, k,$$

where θ_p is the person parameter and $(\delta_{i1}, \dots, \delta_{ik})$ are the item parameters of item i . For notational convenience, the definition of the model uses implicitly $\sum_{k=1}^0 \theta_p - \delta_{ik} = 0$. With this convention, an alternative form of the model is

$$P(Y_{pi} = r) = \frac{\exp(r\theta_p - \sum_{k=1}^r \delta_{ik})}{\sum_{s=0}^k \exp(\sum_{k=1}^s \theta_p - \delta_{ik})}.$$

The link to the binary Rasch model becomes obvious when one considers responses in adjacent categories. Given response categories r and $r - 1$, the presentation

$$\log\left(\frac{P(Y_{pi}=r)}{P(Y_{pi}=r-1)}\right) = \theta_p - \delta_{ir}, \quad r = 1, \dots, k, \tag{1}$$

shows that the model is locally a binary Rasch model with person parameter θ_p and item difficulty δ_{ir} . The properties of the model can be visualized by IRCs that show the probabilities of a response in category r as a function of the person parameter θ_p .

An example of the IRCs for one item with four categories is displayed in Figure 1. From the curves, it is immediately seen that for $\theta_p = \delta_{ir}$ the probabilities of adjacent categories are equal, that is, $P(Y_{pi}=r) = P(Y_{pi}=r-1)$. This means that the IRCs of adjacent categories intersect at $\theta_p = \delta_{ir}$. Therefore, the parameters δ_{ir} can be seen as thresholds between categories $r-1$ and r . In Figure 1, the thresholds are marked by dashed lines at the intersections of the curves. For example, $Y_{pi}=0$ means that category 0 was chosen and no threshold was exceeded. The score $Y_{pi}=2$ implies a response which exceeds thresholds 1 and 2 but fails threshold 3. For more details regarding the model see also Masters (1982), Masters and Wright (1984) and Andrich (1978, 2013, 2015).

Item-Focused Trees for the Partial Credit Model

In model representation (1), the linear predictor for person p and the r -th threshold of item i is given by

$$\eta_{pir} = \theta_p - \delta_{ir}.$$

In item-focused trees, the predictor is successively modified by allowing different predictors in different regions of the covariate space. In the simple case of a continuous variable x , one allows a split into the regions $\{x \leq c\}$ and $\{x > c\}$ at split-point c . A tree is grown by successive splitting of one of the available variables at one of the corresponding split-points. The root is the top node without splitting and the terminal nodes represent the identified partitioning of the covariate space.

For a more concise description, let $\mathbf{x}_p^T = (x_{p1}, \dots, x_{pV})$ denote a vector of measurements on person p . Starting from the root, the predictor that is fitted for one item i and all persons has the form

$$\eta_{pir} = \theta_p - [\gamma_{ir(1)}I(x_{pv} \leq c_v^{(i)}) + \gamma_{ir(2)}I(x_{pv} > c_v^{(i)})], \quad r = 1, \dots, k, \tag{2}$$

where $I(\cdot)$ denotes the indicator function with $I(a)=1$ if a is true and $I(a)=0$ otherwise. This means that item i shows DIF generated by the v th variable. The item has parameters $\gamma_{i1(1)}, \dots, \gamma_{ik(1)}$ in the left node $I(x_{pv} \leq c_v^{(i)})$ and parameters $\gamma_{i1(2)}, \dots, \gamma_{ik(2)}$ in the right node $I(x_{pv} > c_v^{(i)})$. The split-point $c_v^{(i)}$ defines the regions that are used for item i and has to be chosen in an appropriate way. Since an own tree is built for each item, the split-points typically vary over items.

Further splitting means that one of the nodes, for example the left node $I(x_{pv} \leq c_v^{(i)})$, is additionally split in variable s , yielding the partition into left and right node

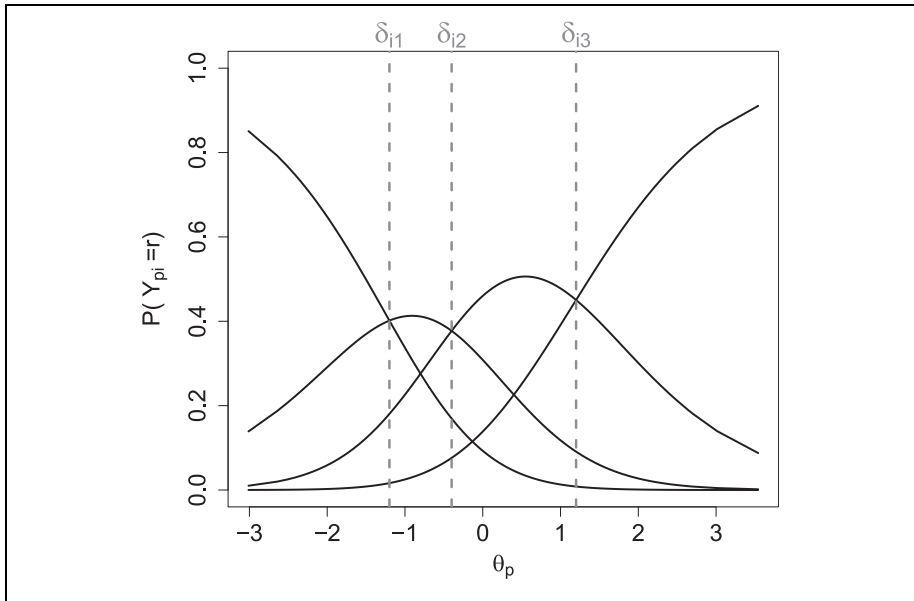


Figure 1. Item response curves (IRCs) for one item with four categories. The item parameters are marked by dashed lines.

$$I(x_{pv} \leq c_v^{(i)})I(x_{ps} \leq c_s^{(i)}) \quad \text{and} \quad I(x_{pv} \leq c_v^{(i)})I(x_{ps} > c_s^{(i)}),$$

where $c_s^{(i)}$ is a new split-point for variable x_{ps} and item i . For each region, one again obtains new parameters for the item. Naturally, only items should be split that carry DIF and the variables and their split-points have to be selected carefully.

In the following, the model abbreviation PCM-IFT will be used for item-focused trees based on the PCM.

An Illustrative Example

Before presenting the fitting procedure of the proposed model in detail (see the next section), we give an illustrative example. The example is used to demonstrate how a tree that is obtained by PCM-IFT looks like and how it can be interpreted. The data considered here are the responses of 1000 subjects on the 8 items of the subfacet *Fantasy* of the factor *Openness to experience* of the German version of the NEO personality inventory revised (NEO-PI-R; Ostendorf and Angleitner, 2004). The 1000 subjects were randomly drawn out of the 11,724 cases of the norm data set. The sample was taken to obtain standard values for the test manual. Each of the items has five categories (from *strongly disagree* to *strongly agree*). Additionally, the data set comprises the two variables age and gender. There are 382 males and 618 females between 16 and 81 years of age.

The major domain *Openness to Experience* is described in the manual as *the active seeking and appreciation of experiences for their own sake* and the subfacet *Fantasy* as *receptivity to the inner world of imagination* (Ostendorf & Angleitner, 2004).

Using PCM-IFT, two of the eight items were selected as DIF items. The two items with DIF are the following:

Item 3: I have an active and lively fantasy life.

Item 6: When I feel that my thoughts are drifting off into daydreams, I usually become busy and start to focus on a task or an activity. (R)

The (R) behind item 6 indicates that this item was reverse coded. This means that strong agreement to this question indicates a low level of fantasy. For simplicity, all reverse coded items have been recoded before the analysis. Therefore, for all analyses a high value on this item means the person disagreed with it.

Item 3 was only split once in covariate gender. The resulting tree is shown in Figure 2 (upper panel). At the terminal nodes, the four threshold parameters for the respective partition are given. It is seen that for both groups, thresholds are not ordered indicating that a higher latent trait is required for passing the second threshold than for passing the third threshold. This effect is slightly more extreme for males than for females. Also, for males an even higher latent trait is required to pass the fourth threshold.

Item 6 was split twice with regard to gender and age. The first split was found for variable gender and within the sub-group of females it is distinguished between younger women (age ≤ 40 years) and older women (age > 40 years). The resulting tree is shown in the lower panel of Figure 2. Similar to Item 3, in none of the terminal nodes the thresholds are ordered. The main difference between the three groups is the variation of the threshold parameter δ_{61} , which is highest for females with age ≤ 40 years and lowest for females with age > 40 years. For the latter, this threshold parameter was even below -4 and is therefore not visible in the figure that is truncated at -4 . Since the item is reverse coded, this means that for older females, the probability was particularly low to pass the last threshold from *agree* to *strongly agree* for this question. Looking at the answers, we see that in this group (terminal node 3) only 2 persons out of 133 had chosen the last category.

The illustration shows that items with DIF can simply be identified by using the proposed PCM-IFT. The resulting trees are easily interpretable and show which variables and split-points determine DIF.

Fitting Item-Focused Trees

In this section, a detailed description of the fitting procedure for the proposed PCM-IFT is given.

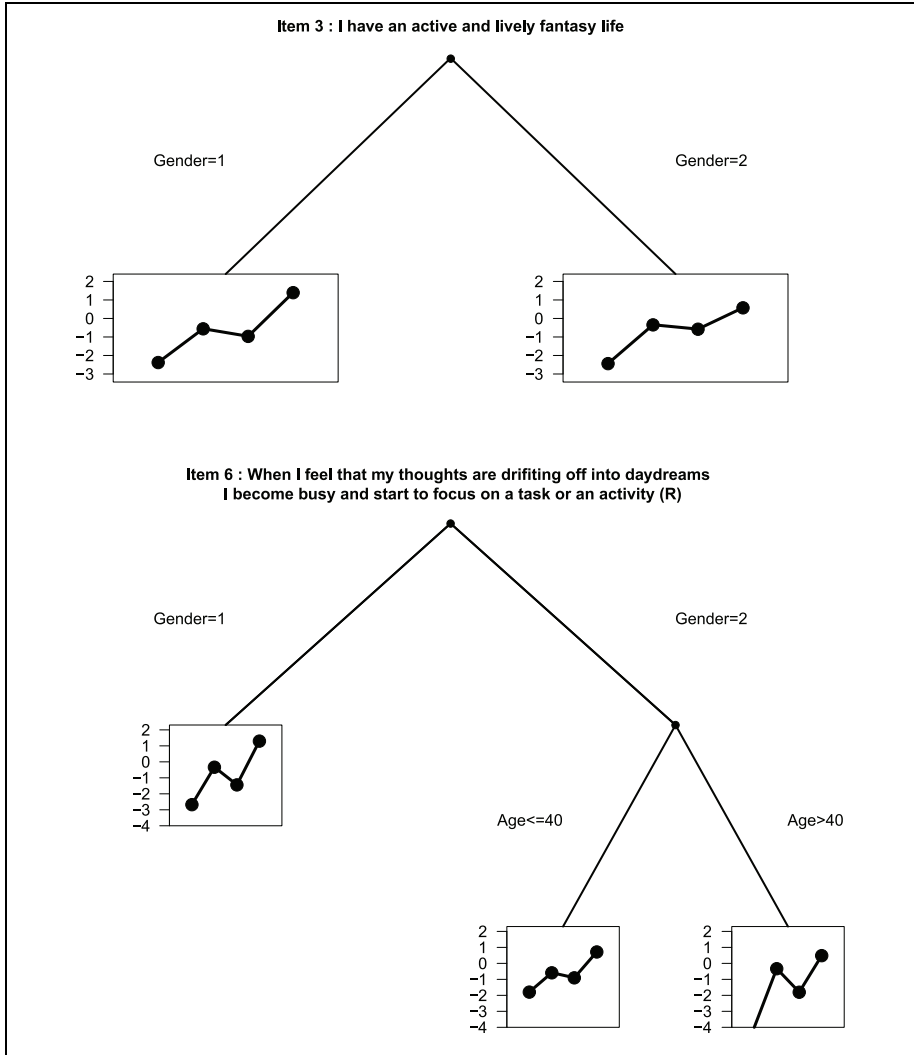


Figure 2. Trees for Item 3 and Item 6 of the subfacet *Fantasy* (NEO-PI-R).

The Partial Credit Model as a Generalized Linear Model

Under usual assumptions, the partial credit model can be embedded into the framework of multivariate generalized linear models (GLMs). Let the data be given by (Y_{pi}, \mathbf{x}_p) , $p = 1, \dots, P$, $i = 1, \dots, I$. For the item responses, one assumes a multinomial distribution $Y_{pi} | \mathbf{x}_p \sim M(1, \boldsymbol{\pi}_{pi})$, where $\boldsymbol{\pi}_{pi}^T = (\pi_{pi1}, \dots, \pi_{pik})$ with components $\pi_{pir} = P(Y_{pi} = r | \mathbf{x}_p)$. The link function of the GLM can be derived from model equation (1) and has the form

$$g(\pi_{pir}) = \eta_{pir} = \log\left(\frac{P(Y_{pi} = r)}{P(Y_{pi} = r - 1)}\right) = (\mathbf{1}_p^{(p)})^T \boldsymbol{\theta} - (\mathbf{1}_r^{(k)})^T \boldsymbol{\delta}_i, \tag{3}$$

where $\boldsymbol{\theta}^T = (\theta_1, \dots, \theta_p)$, $\boldsymbol{\delta}_i^T = (\delta_{i1}, \dots, \delta_{ik})$ and $\mathbf{1}_r^{(k)}$ denotes the unit vector of length k with a 1 in component r . To ensure the identifiability of model (3) one parameter has to be fixed. In the following we set $\theta_p = 0$. By defining the whole parameter vector $\boldsymbol{\beta}^T = (\boldsymbol{\theta}^T, \boldsymbol{\delta}_1^T, \dots, \boldsymbol{\delta}_I^T)$, the PCM can be written in the closed form

$$\eta_{pir} = \mathbf{z}_{pir} \boldsymbol{\beta},$$

where \mathbf{z}_{pir} is the design vector for person p , item i and threshold r that has to be specified accordingly.

Computation of Estimates

Estimates for Model (3) can be obtained by using the flexible R-package VGAM (Yee, 2010, 2014). Function `vglm()` allows the estimation of so-called vector GLMs (Yee and Wild, 1996). One simply needs to specify the design matrix as described above and estimation can easily be obtained. In addition, one can make use of the argument `parallel()` to specify category-specific item parameters. In the following algorithm, which yields the PCM-IFT, this estimation procedure serves as a building block in each iteration.

Fitting of Trees

When growing trees, two decisions have to be made in each step. One has to determine the best split due to an optimality criterion and has to decide if the split is relevant or not. In contrast to alternative approaches, the trees are not pruned to an adequate size after building an oversized tree. By early stopping, the size of the trees is controlled directly.

To determine the first split, one examines for all the items, all the variables and possible split-points the PCM with predictors

$$\eta_{pir} = \theta_p - [\gamma_{ir(1)} I(x_{pv} \leq c_v^{(i)}) + \gamma_{ir(2)} I(x_{pv} > c_v^{(i)})], \quad r = 1, \dots, k.$$

DIF occurs, if $\boldsymbol{\gamma}_{i(1)} \neq \boldsymbol{\gamma}_{i(2)}$, where $\boldsymbol{\gamma}_{i(\ell)}^T = (\gamma_{i1(\ell)}, \dots, \gamma_{ik(\ell)})$, $\ell \in \{1, 2\}$. The corresponding hypothesis $H_0 : \boldsymbol{\gamma}_{i(1)} - \boldsymbol{\gamma}_{i(2)} = \mathbf{0}$ can be tested by a LR test. One simply selects the combination of item, variable and split-point that yields the smallest p value, which is equivalent to selecting the model with minimal deviance. In later steps, the basic procedure is the same. One performs LR tests for the two parameter sets that are involved in the splitting and selects the combination that yields the smallest p value as the optimal one.

In order to determine the optimal size of the trees one has to decide in each step if the split should be performed or not. To obtain a decision, one investigates the dependence of the response and the selected variable. For fixed item i and variable v , let the

maximal value statistic $T_v = \max_{c_v} T_{vc_v}$, be defined as the maximum of all the LR test statistics T_{vc_v} , where c_v is from the set of possible split-points for item i . Typically, the test statistics T_{vc_v} are strongly correlated. The relevance of variable v is judged by the p value of the distribution of T_v , which is not influenced by the number of split-points, since it is already taken into account, see Hothorn and Lausen (2003), Shih (2004), Shih and Tsai (2004), and Strobl, Boulesteix, and Augustin (2007). A permutation test is used for the decision on the null hypothesis controlling for a given significance level α and thus distributional assumption are not necessary. The test statistic T_v is computed based on a data matrix in which variable v is randomly permuted. The maximal value statistic for a large number of permutations provides a distribution of T_v , under the assumption of the null hypothesis that variable v has no effect on item i . The derived p -value is used to make the splitting decision.

Finally, one has to address the problem of multiple testing. In DIF detection, one typically controls for the type I error, that is the item-wise significance level. A Bonferroni adjustment is applied, in order to ensure that the proposed procedure also controls this level. The local significance level for one permutation test is set to α/V for fixed item and variable, where V is the number of variables. Using this adaption, the probability of a false DIF result or the probability of falsely identifying at least one variable as responsible for DIF is controlled by α . Of course, the adjustment is only applied when several variables are available. If in later steps a variable is no longer available because all possible splits were already performed, the adaption is changed to $V - 1$ in all further nodes. All the results presented in this article are based on significance level $\alpha = .05$ and 1,000 permutations. This ensures that the p values can be determined with sufficient accuracy.

A second criterion that is used to define the size of the trees is the minimal sample size in each node. In order to provide a sufficient basis for parameter estimation in each node, splitting is stopped in a node if the number of observations in one of the children nodes falls below a predefined threshold. Komboz et al. (2016) suggested a minimal node size of 10 times the number of thresholds per item. In the present simulations and applications, the corresponding node size is 20 for the cases in which $k = 2$ and 40 for $k = 4$. Based on this rule, 30 observations were chosen as the minimal node size for the illustration in sections An Illustrative Example and Comparisons of Recursive Partitioning Methods and the simulations in the Simulation Studies section.

If no further significant effect is found or splitting is stopped due to minimal node sizes constraint, the algorithm stops. After several splits, each node can be represented by a product of B indicator functions, namely

$$node^{(i)}(\mathbf{x}_p) = \prod_{b=1}^B I(x_{pj_b} > c_{j_b}^{(i)})^{a_b} I(x_{pj_b} \leq c_{j_b}^{(i)})^{1-a_b}, \tag{4}$$

where B is the total number of indicator functions or branches, $c_{j_b}^{(i)}$ is the selected split-point in variable j_b for item i and $a_b \in \{0, 1\}$ indicates which of the indicator functions, below or above the threshold, is involved. Since the trees and thus all the

nodes are item specific, the node is labeled by superscript (i) for the item. Using this definition, the final model of an item i that has been split can be represented by

$$\eta_{pir} = \theta_p - tr_r^{(i)}(\mathbf{x}_p) = \theta_p - \sum_{\ell=1}^{L_i} \gamma_{ir(\ell)} node_{\ell}^{(i)}(\mathbf{x}_p), \quad r = 1, \dots, k,$$

where $tr_r^{(i)}(\mathbf{x}_p)$ is the tree component for item i containing subgroup-specific threshold parameters γ_{ir} and $\ell = 1, \dots, L_i$ denote the terminal nodes of the tree. Please note that the number of terminal nodes depends on the item. If an item is never chosen for splitting, it is assumed to be free of DIF and thus $L_i = 1$ and the constant $tr_r^{(i)}(\mathbf{x}_p) = \delta_{ir}$ (corresponding to the threshold of the simple PCM) is fitted.

A concise description of the basic algorithm is given in the following.

Basic Algorithm - PCM-IFT

Step 1 (Initialization)

Set counter $m = 1$

(a) Estimation

For all items $i = 1, \dots, I$, fit all the candidate PCMs, that fulfill the minimal node size constraint, with predictors

$$\eta_{piv} = \theta_p - [\gamma_{ir(1)} I(x_{pv} \leq c_{vj}^{(i)}) + \gamma_{ir(2)} I(x_{pv} > c_{vj}^{(i)})],$$

$$v = 1, \dots, V, \quad j = 1, \dots, J_v$$

(b) Selection

Select the model that has the best fit. Let $c_{v_1, j_1}^{(i_1)}$ denote the best split, which is found for item i_1 and variable x_{v_1} .

(c) Splitting decision

Select the item and variable with the largest value of T_v . Carry out a permutation test for this combination with significance level α/V . If significant, fit the selected model yielding estimates $\hat{\theta}_p, \hat{\gamma}_{i_1, 1}, \hat{\gamma}_{i_1, 2}$ and nodes $node_{j_1}^{(i_1)}, node_{j_2}^{(i_1)}$, set $m = 2$. If not, stop, no DIF detected.

Step 2 (Iteration)

(a) Estimation:

For all items $i = 1, \dots, I$ and already built nodes $\ell = 1, \dots, L_{im}$, fit all the candidate PCMs, that fulfill the minimal node size constraint, with new intercepts

$$\gamma_{i, L_{im}+1} node_{\ell}^{(i)} I(x_{pv} \leq c_{vj}) + \gamma_{i, L_{im}+2} node_{\ell}^{(i)} I(x_{pv} > c_{vj})$$

for all v and remaining, possible split-points $c_{vj}^{(i)}$.

(b) Selection

Select the model that has the best fit yielding the split-point $c_{v_m, j_m}^{(i_m)}$, which is found for item i_m in node $node_{\ell_m}^{(i_m)}$ and variable x_{v_m} .

(c) Splitting decision

Select the node and variable with the largest value of T_v . Carry out a permutation test for this combination with significance level α/V . If significant, fit the selected model yielding the additional estimates $\hat{\gamma}_{i_m, L_{im, m}+1}, \hat{\gamma}_{i_m, L_{im, m}+2}$, set $m = m + 1$. If not, stop.

Comparison of Recursive Partitioning Methods

Recently, Komboz et al. (2016) proposed an extension of the RaschTree, a competing recursive partitioning method for the detection of DIF in polytomous items. It will be abbreviated by TREE-PCM. Even though both techniques use tree methods for identifying splits in the predictor space, the approaches are quite different. These differences are outlined in the following.

In item-focused trees (PCM-IFT), one tries to identify variables that generate splits in specific items. Thus, recursive partitioning methods are used on the item level to find those items that suffer from DIF. TREE-PCM, however, follows a quite different strategy. Splits are global, if a specific split is considered, the partial credit model is fitted in each region separately.

More concrete, in PCM-IFT the generation of a split uses the form (as already given in Equation 2)

$$\eta_{pir} = \theta_p - [\gamma_{ir(1)}I(x_{pv} \leq c_v^{(i)}) + \gamma_{ir(2)}I(x_{pv} > c_v^{(i)})], \quad r = 1, \dots, k,$$

where $c_v^{(i)}$ is the split-point in variable v for item i . Thus, a PCM is fitted with a split in item i , variable v at split-point $c_v^{(i)}$. PCM-IFT examines the corresponding models for all items $i = 1, \dots, I$, all variables $v = 1, \dots, V$, and possible splits $j = 1, \dots, J_v$.

TREE-PCM uses the form

$$\eta_{pir} = \theta_p - [\gamma_{ir(1)}I(x_{pv} \leq c_v) + \gamma_{ir(2)}I(x_{pv} > c_v)], \quad r = 1, \dots, k,$$

to generate splits. In contrast to PCM-IFT, the split-point c_v in variable v is the same for all items. Fitting of a PCM given a global split at c_v is equivalent to fitting two PCMs, one for the data, for which $x_{pv} \leq c_v$ and one for the data for which $x_{pv} > c_v$ holds. One obtains different item parameter estimates in the two regions $\{x_{pv} \leq c_v\}$ and $\{x_{pv} > c_v\}$ for *all* items.

Since the approaches use quite different predictors also fitting methods and outcomes differ. Main differences are the following:

- When deciding if a variable is involved in DIF, TREE-PCM uses a structural change test from econometrics that is based on score contributions (for more details, see Komboz et al., 2016). PCM-IFT uses a permutation test with the maximal value statistic $T_v = \max_{c_v} T_{vc_v}$, which is defined as the maximum of LR test statistics. In contrast to the structural change test, which selects variables, the maximal value statistic selects variables and items.
- For the estimation of model parameters, TREE-PCM uses a conditional maximum likelihood approach, which is possible since two separate models are fitted to different samples obtained by the split. In contrast PCM-IFT is based on joint maximum likelihood estimation.
- TREE-PCM yields one tree, for which each node of the fitted tree can be represented by a product of B indicator functions

$$node(\mathbf{x}_p) = \prod_{b=1}^B I(x_{pjb} > c_{jb})^{a_b} I(x_{pjb} \leq c_{jb})^{1-a_b}.$$

The nodes are the same for each item. Typically, all the estimates of item parameters differ for different nodes, which makes it hard to identify which item is carrying DIF. In contrast, PCM-IFT yields several trees, one for each item that is identified as DIF item. The nodes for item i have the form (4) with item-specific split-points. The single trees show which variables induce DIF for the corresponding item. The size of the trees varies across items.

For the use in practice, important differences are the following:

- PCM-IFT is able to detect single items that show DIF while the other items are still compatible with the PCM. This enables users to reword or remove crucial items in order to construct a DIF-free questionnaire. If TREE-PCM performs a split, all the estimated item parameters differ.
- TREE-PCM potentially is a useful tool if one wants to investigate if DIF is generally present in a test. However, it is hard to see which items are responsible. If one wants to know which items are DIF items, PCM-IFT is more appropriate.
- The item-focused tree approach provides an item-specific interpretation of DIF.
- The TREE-PCM approach is based on the conditional likelihood instead of the joint likelihood, which reduces computational costs.

The described differences are now illustrated by applying the TREE-PCM approach to the same data set that was used in the section An Illustrative Example. The resulting models for the sub-facet *Fantasy* when using TREE-PCM (significance level $\alpha = .05$) are presented in Figure 3. Only one split is performed for the variable age at 43 years of age. Unlike PCM-IFT, TREE-PCM yields ordered thresholds for items 3 and 6. Nevertheless, these two items do reveal strong differences in the effect plots between the two groups. However, from this plot it is not easy to identify the items that are responsible for DIF in this subfacet because almost all items show light to strong differences in the plots between the two groups.

The two methods agree on age being a DIF-inducing covariate for this facet. However, only PCM-IFT also identifies gender as DIF-inducing variable. After a split into age groups, the overall differences of further splits are not strong enough for TREE-PCM to warrant further splits. In contrast, for the two items, identified by PCM-IFT, the differences were strong enough concerning gender groups. By construction, PCM-IFT is more sensitive to DIF in only a few items while TREE-PCM is more sensitive to DIF in multiple items. Therefore, it seems sensible that TREE-PCM is prone to find more splits than PCM-IFT when small parameter

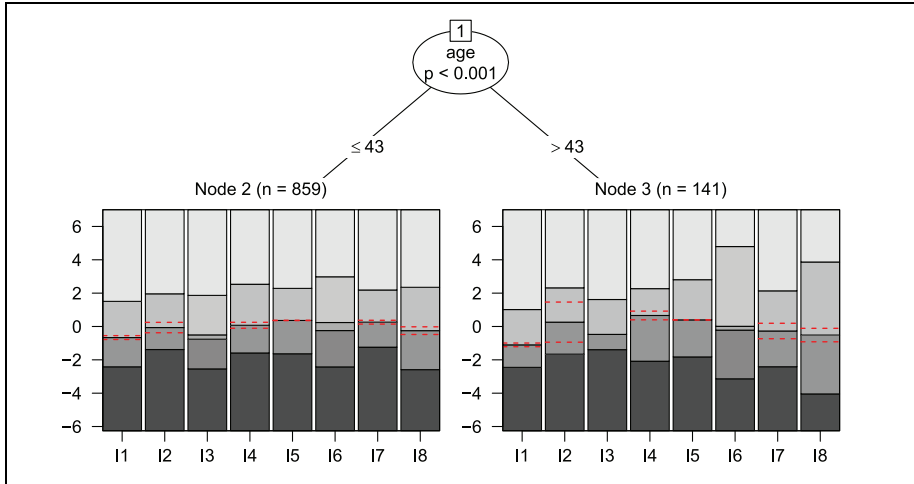


Figure 3. Estimated tree for TREE-PCM of the subsfacet *Fantasy* (NEO-PI-R).

differences are present in many of the items while PCM-IFT performs more splits than TREE-PCM when only one or two items have DIF but none of the others.

Simulation Studies

In this section, we examine the performance of the new PCM-IFT approach that was introduced in the previous sections. More precisely, we evaluate the procedure’s ability to detect items that show DIF and to estimate the item difficulty parameters in each node in three simulation studies. In addition, the performance is compared with TREE-PCM proposed by Komboz et al. (2016).

In Simulation I (One Binary Covariate), a simple model with only one binary covariate will be considered. In Simulation II (Three Different Covariates), a more complex model with three different covariates (binary, ordinal and numeric) will be the data generating model. Finally, in Simulation III nonhomogeneous DIF will be considered in a simulation with one binary covariate.

Evaluation Criteria and Experimental Design

For the evaluation of simulation results, true positive rates (TPRs) and false positive rates (FPRs) are reported in each simulation scenario.

Let each item be characterized by a vector $\epsilon_i^T = (\epsilon_{i1}, \dots, \epsilon_{iV})$ with $\epsilon_{iv} = 1$ if item i has DIF in variable v and $\epsilon_{iv} = 0$ otherwise. An item is a non DIF item if $\epsilon_i^T = (0, \dots, 0)$. As soon as one of the components is 1, it is a DIF item. In addition, each variable can be characterized by a vector $\epsilon_v^T = (\epsilon_{v1}, \dots, \epsilon_{vI})$, where $\epsilon_{vi} = 1$ if variable v induces DIF in item i and $\epsilon_{vi} = 0$ otherwise. With $\hat{\epsilon}_i^T = (\hat{\epsilon}_{i1}, \dots, \hat{\epsilon}_{iV})$

denoting the corresponding estimated indicator vector, the indicator function $I(\cdot)$ and the zero vector $\mathbf{0}$, the following criteria are used:

1. TPR and FPR on the item level (items correctly identified as DIF items and non-DIF items incorrectly identified as DIF items):

$$TPR_I = \frac{1}{\#\{i : \epsilon_i \neq \mathbf{0}\}} \sum_{i:\epsilon_i \neq \mathbf{0}} I(\hat{\epsilon}_i \neq \mathbf{0})$$

$$FPR_I = \frac{1}{\#\{i : \epsilon_i = \mathbf{0}\}} \sum_{i:\epsilon_i = \mathbf{0}} I(\hat{\epsilon}_i \neq \mathbf{0})$$

2. TPR and FPR for the combination of item and variable:

$$TPR_{IV} = \frac{1}{\#\{i, v : \epsilon_{iv} \neq \mathbf{0}\}} \sum_{i, v: \epsilon_{iv} \neq \mathbf{0}} I(\hat{\epsilon}_{iv} \neq \mathbf{0})$$

$$FPR_{IV} = \frac{1}{\#\{i, v : \epsilon_{iv} = \mathbf{0}\}} \sum_{i, v: \epsilon_{iv} = \mathbf{0}} I(\hat{\epsilon}_{iv} \neq \mathbf{0})$$

3. TPR and FPR on the variable level (variables correctly identified as DIF inducing variables and variables incorrectly identified as DIF inducing variables):

$$TPR_V = \frac{1}{\#\{v : \epsilon_v \neq \mathbf{0}\}} \sum_{v:\epsilon_v \neq \mathbf{0}} I(\hat{\epsilon}_v \neq \mathbf{0})$$

$$FPR_V = \frac{1}{\#\{v : \epsilon_v = \mathbf{0}\}} \sum_{v:\epsilon_v = \mathbf{0}} I(\hat{\epsilon}_v \neq \mathbf{0})$$

Each rate is reported as the average over all repetitions. All simulation scenarios were replicated 100 times.

Person Parameters. The number of persons in all simulations is 500. However, all persons are excluded from the analyses, who have answers in only one category. As a result, the actual number of persons P in most of the scenarios is slightly less than 500. The person parameters are simulated from a standard normal distribution, $\theta_p \sim N(0, 1)$.

Number of Items. In most scenarios, the number of items is $I = 8$, and one of these items is simulated to have DIF. This makes our simulations comparable to the illustration in the section An Illustrative Example, where each unidimensional subfacet consists of eight items. Also, Komboz et al. (2016) used eight items in their simulation studies. In order to examine how the performance of our method changes with increasing number of items, we conduct one scenario with $I = 20$ and three DIF items in Simulation I.

Item Parameters. In most scenarios, we simulate data with three response categories ($k = 2$). In addition, in Simulation I, one scenario is included with five response categories ($k = 4$). In a first step, the threshold parameters for item i are drawn from the following normal distribution:

$$k = 2 : \delta_i \sim N_3(\boldsymbol{\mu}_3, \boldsymbol{\Sigma}_3 = \mathbf{I}_3), \quad \boldsymbol{\mu}_3 = (-0.50, 0.50)^T$$

$$k = 4 : \delta_i \sim N_5(\boldsymbol{\mu}_5, \boldsymbol{\Sigma}_5 = \mathbf{I}_5), \quad \boldsymbol{\mu}_5 = (-1.50, -0.50, 0.50, 1.50)^T$$

If item i is simulated to have DIF, the corresponding item parameters are subsequently transformed by step functions.

Structure of DIF. To simulate DIF in item i , the item parameters are shifted for one sub-group (the focal group) corresponding to a prespecified split-point c_{vj} in covariate x_v . There is always one split in each DIF item.

For each scenario, three different strengths of DIF are defined: weak, medium, and strong. The strength is determined by an additional parameter λ . In the *weak* condition, the mean vector of the focal group is shifted by $\lambda = 0.25$, in the *medium* condition by $\lambda = 0.5$ and in the *strong* condition by $\lambda = 1$ in relation to the values in the reference group. Additionally, we add one condition in which *no DIF* is present (the item parameters for both groups are drawn from the same distribution). Further details are given in the respective sections.

The methods considered in the simulations are

- The proposed *item-focused tree approach* (PCM-IFT).
- The *partial credit tree approach* (TREE-PCM) proposed by Komboz et al. (2016).

During estimation, each permutation test is based on 1000 permutations and the global significance level $\alpha = .05$.

A reviewer suggested to also compare the methods to more conventional testing approaches. Therefore, we additionally consider the performance of an itemwise LR test in simulations in which it can be applied. This self-implemented test is in accordance with Lord’s chi-square test or Wald test as described in Magis et al. (2010) for dichotomous items. When testing the difference between two groups in the PCM, all the threshold parameters $\delta_{ir}, r = 1, \dots, k$, are tested simultaneously. This approach is

Table 1. Number of Items (I), Number of Response Categories (k), and Number of DIF Items (I_{DIF}) for the Three Scenarios of Simulation I.

Simulation I	I	k	I_{DIF}
Scenario 1	8	3	1
Scenario 2	20	3	3
Scenario 3	8	5	1

designed for the comparison of two or several groups, only. Hence, it is only applied in Simulation I.

Simulation I: One Binary Covariate

In the first simulation study, the data set contains only one binary covariate $x \in \{0, 1\}$. Therefore, the simple LR test can also be used for DIF detection. Covariate x induces DIF in one or three items. The item parameters for the two groups defined by x are

$$\gamma_{ir(2)} = \gamma_{ir(1)} + \lambda \cdot I(x_p = 1), \quad r = 1, \dots, k.$$

All thresholds of the DIF items are shifted in the same direction by the same value λ depending on the strength of DIF. For the settings with no DIF λ is set to 0.

Three scenarios are considered that differ with regard to the number of items (I) the number of response categories (k) and the number of DIF items (I_{DIF}). A detailed overview is given in Table 1.

Results. The evaluated criteria of the first simulation are shown in Tables 2 and 3. When no DIF is present, only FPRs are available. In both tables, results are first shown for eight items with three categories, second for 20 items with three categories, and third for eight items with five categories. In the case of one single covariate the covariate vector ϵ_i only has one element, so TPRs and FPRs for the combination of item and variable for PCM-IFT and LR test correspond to those on the item level (see Table 2). TREE-PCM does not test single items for DIF and therefore we only obtain detection rates on the variable level. In the no DIF scenario we get a FPR_V and in all other scenarios a TPR_V . They are reported for both methods in Table 3.

It can be seen from Table 2 that the proposed PCM-IFT approximately keeps the given significance level. In contrast, the LR test shows strongly inflated Type I error rates, in particular for strong DIF. In this simple case, the difference between PCM-IFT and LR test is the testing procedure. The LR test approach tests each item separately given all other items are free of DIF. The PCM-IFT pursues a sequential strategy that accounts for DIF found in previous steps. As was to be expected, TPRs on

Table 2. Average True Positive and False Positive Rates for PCM-IFT and the LR Test Based on 100 Replications (Simulation I).

	DIF strength	PCM-IFT		LR test	
		TPR _i	FPR _i	TPR _i	FPR _i
Scenario 1	No DIF	—	0.068	—	0.090
	Weak	0.270	0.057	0.340	0.093
	Medium	0.820	0.061	0.890	0.110
	Strong	1.000	0.058	1.000	0.163
Scenario 2	No DIF	—	0.053	—	0.057
	Weak	0.290	0.053	0.280	0.093
	Medium	0.836	0.048	0.800	0.171
	Strong	0.996	0.051	1.000	0.250
Scenario 3	No DIF	—	0.055	—	0.081
	Weak	0.350	0.055	0.430	0.081
	Medium	0.940	0.056	0.970	0.098
	Strong	0.980	0.057	1.000	0.177

Note. DIF = differential item functioning; TPR = true positive rate; FPR = false positive rate; PCM-IFT = item-focused trees based on the partial credit model; LR = likelihood ratio.

the item level increase with increasing strength of DIF for both methods. They are also slightly higher for the third scenario with five categories.

It should be noted that inflated type I error rates on the item level are also observed, when using the itemwise Wald test implemented in the function `Waldtest()` of the R package `eRm` (Mair and Hatzinger, 2007). We prefer to report the results of the self-implemented LR-test, because the function `Waldtest()` uses separate tests for each threshold.

The FPRs on the variable level for PCM-IFT (Table 3) seem surprisingly high. However, bearing in mind that FPRs were controlled on the item and not on the variable level, the results make sense. If the probability of one item to be falsely classified as DIF item is 0.05, then the probability that one or more out of 8 items are falsely classified as DIF item is: $1 - (0.95^8) = 1 - 0.663 = 0.337$. For 20 items: $1 - (0.95^{20}) = 1 - 0.358 = 0.642$. Of course, this only holds for simulation I in which there is only one covariate and each split is automatically made for this covariate. Consequently, FPRs on the variable level are much higher compared with the TREE-PCM procedure, in which they are controlled on the variable level, and therefore the significance level is mostly respected. It can further be seen, that also TPRs are much higher for PCM-IFT than for TREE-PCM. A TPR of 100 in Scenario 3 with weak DIF means that only in 10 % of the cases the present DIF is found. The reason might be that the ratio of DIF items to non-DIF items is very small in Scenarios 1 and 3. Therefore, for the detection of single items the power is much higher. Accordingly, in Scenario 2, where the ratio of DIF items to non-DIF items is higher) TREE-PCM performs better.

Table 3. Average TPR_V and FPR_V for TREE-PCM and PCM-IFT Based on 100 Replications (Simulation I).

	DIF strength	TREE-PCM		PCM-IFT	
		TPR_V	FPR_V	TPR_V	FPR_V
Scenario 1	No DIF	—	0.080	—	0.410
	Weak	0.100	—	0.490	—
	Medium	0.280	—	0.880	—
	Strong	0.930	—	1.000	—
Scenario 2	No DIF	—	0.050	—	0.680
	Weak	0.140	—	0.850	—
	Medium	0.610	—	1.000	—
	Strong	1.000	—	1.000	—
Scenario 3	No DIF	—	0.040	—	0.390
	Weak	0.100	—	0.590	—
	Medium	0.460	—	0.950	—
	Strong	1.000	—	0.980	—

Note. DIF = differential item functioning; TPR = true positive rate; FPR = false positive rate; PCM-IFT = item-focused trees based on the partial credit model; TREE-PCM = RaschTree partial credit model.

Simulation II: Three Different Covariates

In the second simulation study, we investigate how well the proposed method is able to detect the right DIF inducing covariate out of multiple present covariates. We consider scenarios with $I = 8$, $k = 3$ and $I_{DIF} = 1$. Now, there are three different covariates that possibly induce DIF—one binary variable $x_1 \in \{0, 1\}$, one ordered factor $x_2 \in \{1, 2, 3, 4\}$, and one numeric covariate $x_3 \in \{20, \dots, 50\}$. Variable x_3 could, for example, represent the variable age. In each of the following scenarios, exactly one of these covariates induces DIF in one item. Again, all thresholds of one item are shifted in the same direction. There is one split-point c_{vj} per item at $c_{vj} = x_{v_{med}}$. The threshold parameters of the two subgroups are given by

$$\gamma_{ir(2)} = \gamma_{ir(1)} + \lambda \cdot I(x_{pv} > x_{v_{med}}), \quad r = 1, 2.$$

To obtain weak, medium, and strong DIF, parameters λ are chosen in the same way as in the previous simulation.

Results. Figure 4 shows one estimated tree for Item 5 (the item with DIF), for the three different scenarios of Simulation II with strong DIF, respectively. In the chosen examples, the true underlying DIF structure was detected. In Scenario 1, DIF is induced by x_1 , in Scenario 2 by x_2 , and in Scenario 3 by x_3 . In these examples also, the true simulated split-points (2, 2 and 34), are correctly identified. In each scenario, the true item parameters are $\gamma_{5(1)} = (-0.5, 0.5)^T$ in the left node and $\gamma_{5(2)} = (0.5, 1.5)^T$ in the right node. It can be seen from the graphical representations

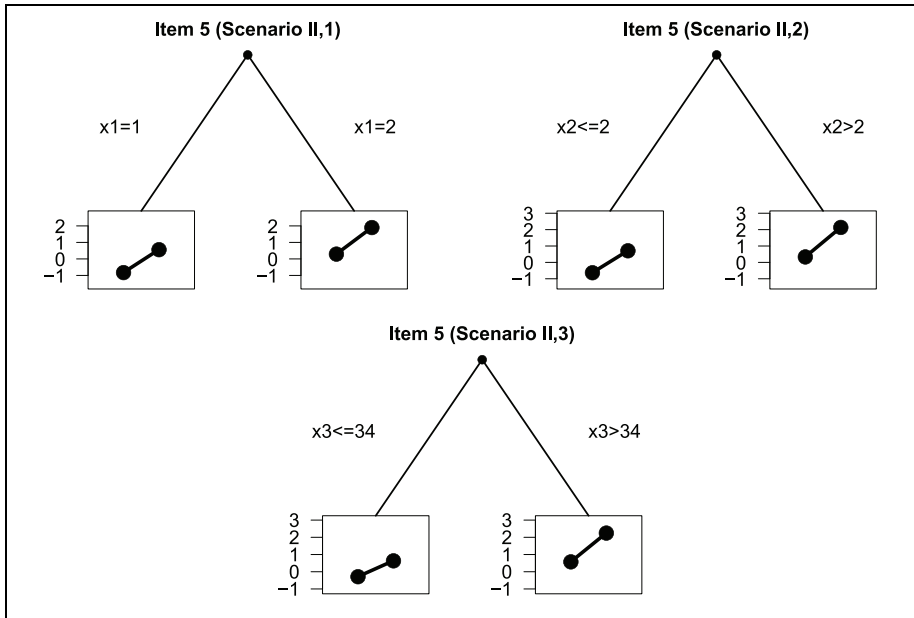


Figure 4. Estimation results for one example of the three scenarios of Simulation II with three covariates and strong differential item functioning (DIF). The estimated item parameters $\gamma_{5r(1)}$ and $\gamma_{5r(2)}$ are visualized in each leaf of the trees.

of the parameters in the leaves of the trees that the estimated parameters are quite close to the true ones.

To account for the multiple covariates in the model, the significance level at each node is divided by the number of covariates available at this node: $\alpha = 0.05/V$. Tables 4 and Table 5 give an overview of the TPRs and FPRs based on 100 replications for Simulation II.

In Table 4, it can be seen that FPRs are always close to the given significance level demonstrating that the alpha level correction works quite well. For the combination of items and variables they are necessarily smaller. From the first and the third columns in Table 4, one can conclude that almost in all cases where a split was performed, also the right variable was selected. On variable level (Table 5), FPRs again are higher than 0.05 but not as high as in simulation I. It is noteworthy that TREE-PCM is very conservative in this simulation which results in very small true and FPRs. Similar to simulation I, TPRs of PCM-IFT are much higher than those of TREE-PCM.

Simulation III: Nonhomogeneous DIF

In the third simulation, nonhomogeneous DIF is simulated in the settings with $I = 8$, $k = 3$, and $I_{DIF} = 1$ with regard to one binary DIF-inducing covariate. Unlike in the

Table 4. Average True Positive and False Positive Rates for PCM-IFT Based on 100 Replications (Simulation II).

	DIF strength	TPR_j	FPR_j	TPR_{jV}	FPR_{jV}
Scenario 1	No DIF	—	0.041	—	0.014
	Weak	0.124	0.039	0.072	0.014
	Medium	0.597	0.045	0.576	0.015
	Strong	0.988	0.038	0.988	0.012
Scenario 2	No DIF	—	0.035	—	0.011
	Weak	0.112	0.041	0.071	0.014
	Medium	0.368	0.051	0.357	0.016
	Strong	0.977	0.044	0.977	0.013
Scenario 3	No DIF	—	0.039	—	0.013
	Weak	0.096	0.042	0.085	0.013
	Medium	0.543	0.042	0.533	0.013
	Strong	1.000	0.044	1.000	0.013

Note. DIF = differential item functioning; TPR = true positive rate; FPR = false positive rate; PCM-IFT = item-focused trees based on the partial credit model.

Table 5. Average TPR_V and FPR_V for TREE-PCM and PCM-IFT Based on 100 Replications (Simulation II).

	DIF strength	TREE-PCM		PCM-IFT	
		TPR_V	FPR_V	TPR_V	FPR_V
Scenario 1	no DIF	—	0.013	—	0.110
	weak	0.030	0.025	0.154	0.113
	medium	0.160	0.035	0.630	0.097
	strong	0.870	0.025	0.988	0.075
Scenario 2	no DIF	—	0.013	—	0.093
	weak	0.060	0.015	0.153	0.122
	medium	0.110	0.015	0.410	0.131
	strong	0.700	0.025	0.977	0.102
Scenario 3	no DIF	—	0.013	—	0.104
	weak	0.040	0.015	0.202	0.090
	medium	0.180	0.030	0.619	0.076
	strong	0.970	0.025	1.000	0.096

Note. DIF = differential item functioning; TPR = true positive rate; FPR = false positive rate; PCM-IFT = item-focused trees based on the partial credit model; TREE-PCM = RaschTree partial credit model.

previous simulations, threshold parameters now are not all shifted by an equal amount from the reference to the focal group, but half of the parameters is shifted to the left and the other half to the right. More precisely, since only the case of two threshold parameters per item was considered, the first threshold parameter is shifted to the left and the second to the right. As a result, the difference between threshold parameters,

Table 6. Average True Positive and False Positive Rates for PCM-IFT Based on 100 Replications (Simulation III).

DIF strength	TPR_i	FPR_i
No DIF	—	0.057
Weak	0.190	0.058
Medium	0.590	0.057
Strong	0.990	0.056

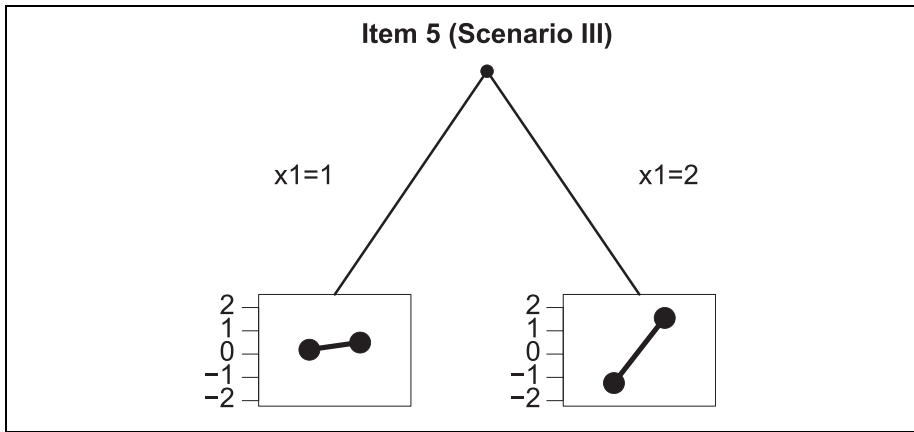


Figure 5. Estimation result for one example of Simulation III with one covariate and nonhomogeneous differential item functioning (DIF) (strong setting). The estimated item parameters $\gamma_{5r(1)}$ and $\gamma_{5r(2)}$ are visualized in each leaf of the tree.

i.e. the category width changes from the reference to the focal group. The two threshold parameters are then given through:

$$\gamma_{i1(2)} = \gamma_{i1(1)} - \lambda \cdot I(x_p = 1)$$

$$\gamma_{i2(2)} = \gamma_{i2(1)} + \lambda \cdot I(x_p = 1).$$

Results. Table 6 displays true and FPRs on the item level for PCM-IFT. Both, FPRs and TPRs are satisfactory and very similar to those in Simulation I, Scenario 1 where the same number of items and categories were used. Figure 5 shows one estimated tree for Item 5 (the item with DIF) for the setting with strong DIF of Simulation III, where the true underlying DIF structure was detected. In this scenario, with nonhomogeneous DIF, the true item parameters are $\gamma_{5(1)} = (-0.5, 0.5)^T$ in the left node and $\gamma_{5(2)} = (-1.5, 1.5)^T$ in the right node. From the graphical representations of the

parameters in the leaves of the trees, it can be seen that the underlying nonhomogeneous DIF structure is detected by the algorithm.

Concluding Remarks

We propose an approach to detect DIF in ordinal item response based on the PCM. By item-focused recursive partitioning, the proposed method allows for simultaneous detection of items and variables that are responsible for DIF. The results are small trees for each item that is not compatible with the PCM. Graphical representations of the threshold parameters in each terminal node enable an easy interpretation of the estimated effects and the differences between the detected groups. The simulations demonstrate that the proposed procedure works well, in particular, in settings where only few DIF items are present (which is usually the case in applications). The existing TREE-PCM, however, is more suitable in cases where users assume most of the items to have DIF or simply want to test for DIF in the questionnaire without flagging the individual DIF items. Surprisingly, a comparison to the item wise LR-test reveals that this conventional method does not work well in the simple simulation scenario since it shows strongly inflated type I error rates. We can therefore conclude that the proposed PCM-IFT shows advantages over conventional methods (multiple covariates, no prespecification of subgroups) and has the additional benefit that it does not only tests for DIF but also automatically detects DIF items.

The proposed model explicitly tests DIF on the *item level*. That means in each step the whole parameter vector ($H_0 : \boldsymbol{\gamma}_{i(1)} - \boldsymbol{\gamma}_{i(2)} = \mathbf{0}$) is tested and if a split is performed, all the threshold parameters are estimated in both nodes without any restrictions. An alternative strategy would be to test for DIF in single thresholds. Then for fixed item and variable in each step one tests the hypotheses $H_0 : \gamma_{ir(1)} - \gamma_{ir(2)} = 0$, $r = 1, \dots, k$, and selects the threshold that has the best fit. Accordingly, in each step only one threshold δ_{ir} changes for one group. In future research, one might also consider a *homogeneous modeling approach*, in which again all thresholds are shifted but now all in the same direction by an item-specific constant γ_i . Then, for example, after the first split the item parameters in region $\{x_{pv} > c_v\}$ are defined by $\delta_{i1} + \gamma_i, \dots, \delta_{ik} + \gamma_i$. Both strategies are certainly worth investigating but the adoption of the existing procedure needs further research.

We restricted consideration to the widely used partial credit model. However, the basic concept can also be used to model DIF in alternative ordinal item response models, for example in the rating scale model (Andrich, 1978). In the rating scale model, the predictor has the form $\theta_p - (\beta_i + \tau_r)$, with item location parameter β_i and threshold parameter τ_r . With item-focused trees, the location parameter β_i can be replaced by $\gamma_{i(1)}I(x_{pv} \leq c_v) + \gamma_{i(2)}I(x_{pv} > c_v)$, the threshold parameter τ_r can be replaced by $\alpha_{r(1)}I(x_{pv} \leq c_v) + \alpha_{r(2)}I(x_{pv} > c_v)$ or both parameters can be modified simultaneously. Fitting of corresponding models requires the development of tailored testing strategies and appropriate estimation tools, which is beyond the scope of this article.

All the results presented in this article were obtained by means of an R program (as described in the Computation of Estimates section), which is available from the authors and will soon be available in an extended version of the R add-on package DIFtree (Berger, 2016) on CRAN.

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