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Complete one-loop renormalization of the Higgs-electroweak chiral Lagrangian

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Abstract

Employing background-field method and super-heat-kernel expansion, we compute the complete one-loop renormalization of the electroweak chiral Lagrangian with a light Higgs boson. Earlier results from purely scalar fluctuations are confirmed as a special case. We also recover the one-loop renormalization of the conventional Standard Model in the appropriate limit.

1. Introduction

Effective field theories (EFTs) for the electroweak interactions are nowadays part of the canonical set of techniques used at the LHC [1] in the search for new physics. Electroweak EFTs have unique features that make them especially suited as discovery tools at high-energy colliders: they factor out in a very efficient way the known infrared physics (particle content and symmetries at the electroweak scale) from unknown ultraviolet physics. The former determine

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the operators of the EFT expansion, while the presence of the latter only affects the operator coefficients. In the context of the current experimental situation, where no hints of new physics have been detected, EFTs are extremely useful: they provide the most general, model-independent, unbiased parametrization of new physics compatible with quantum field theory requirements.

The Higgs-electroweak chiral Lagrangian (HEWχL) [2–4] is an effective field theory[1] of the electroweak interactions especially suited to study the Higgs boson nature and interactions. It is a generalization of the Standard Model where the Higgs boson is not required to be a weak doublet. As such, it is the natural upgrade of Higgs characterization schemes commonly employed at the LHC (most prominently the κ formalism) into full-fledged quantum field theories [5]. A clear advantage of this is that radiative corrections can be readily implemented with known techniques, such that Higgs physics can be studied with increasing precision in a well-defined way.

A peculiar aspect of the HEWχL is that, as opposed to the Standard Model, it is nonrenormalizable: loop divergences are absorbed by counterterms, which introduce new operators. This is not a problem as long as those new operators are subleading in the EFT expansion. This can be achieved if the EFT is defined as a loop expansion, where the operators at a given order include the counterterms that absorb all the divergences up to that order. The paradigmatic example of such an EFT expansion is chiral perturbation theory (ChPT), the theory of pion dynamics [6]. The systematics associated with loop expansions has recently been revised [4] and generalized power-counting formulas have been provided that address the specific needs of an electroweak EFT [5].

Power counting defines the EFT expansion and is useful to find out the counterterms at each loop order, but the divergence structure of the EFT can only be determined by the explicit renormalization of the theory at the loop level. While calculations based on Feynman diagrams are useful for specific processes [7], when it comes to the full renormalization of HEWχL they are rather inefficient and a path integral approach is preferable [8]. Partial results in this direction already exist in the literature, where the divergent structure associated with the scalar fluctuations of HEWχL has been worked out [9–11].

In this paper we will extend those studies and evaluate the complete one-loop renormalization of HEWχL. We will integrate all the one-loop fluctuations in the path integral using the background field method together with the super-heat-kernel expansion. Specifically, in this paper we will determine the $1/\varepsilon$ poles in dimensional regularization from the second Seeley–DeWitt coefficient. In order to do so we will re-derive a master formula due to 't Hooft [12] with superspace methods [13,14], which are convenient when dealing with both bosonic and fermionic fluctuations. The required input of the master formula are the field fluctuations, which can be parametrized in different ways and need to be gauged-fixed. Certain choices can simplify the algebraic manipulations, but the final results should be independent of the manner the field fluctuations are parametrized. In order to cross-check our results, we have performed the calculation in five independent ways.

We will present our results such that the RG evolution of the coefficients of the NLO basis of HEWχL can be directly read off. We will not renormalize the SM parameters explicitly, albeit we provide all the ingredients to perform this final step. In this paper we will restrict ourselves to the formal aspects of the computation only. The full renormalization programme is in general needed when analyzing specific processes and will be carried out in a companion paper, with a focus on the phenomenological applications of our results.

[1] The expression Higgs effective field theory (HEFT) is also used by some authors.

As expected, we find that fluctuations of gauge bosons and fermions define the renormalizable sector of the EFT and therefore do not generate new counterterms. These stem from the nonrenormalizable sector, i.e. the pure scalar (Goldstone and Higgs) fluctuations, and the mixed loops between the renormalizable and nonrenormalizable sectors of the theory. Our results for the one-loop divergences of HEWχL confirm the partial results from the scalar sector presented in [9]. They also reproduce the renormalization of the Standard Model at one loop in the corresponding limit of parameters. In particular, all NLO counterterms vanish in that case. Finally, our results also show that chiral dimensions d_χ, as defined in [4], are the correct expansion parameter for HEWχL: we consistently find that $d_\chi[\mathcal{L}_{\rm NLO}] = 4$.

This paper is organized as follows: in section 2 we summarize HEWχL at leading order, mostly to set our notations. In section 3 we discuss the generic master formula that we employed to calculate the one-loop divergences. Specific details for HEWχL are given in section 4 and the final results for the divergences are presented in section 5. As a cross-check, in section 6 we show how the results contain the Standard Model renormalization as a particular case. We conclude in section 7.

2. Leading-order chiral Lagrangian

To leading order, at chiral dimension 2, the effective Lagrangian is given by [3,4]

$$\mathcal{L} = -\frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{v^2}{4}\,\langle L_\mu L^\mu\rangle\,F(h) + \frac{1}{2}\partial_\mu h\partial^\mu h - V(h)$$
$$+\,\bar\psi i\not{D}\psi - \bar\psi m(h,U)\psi \qquad (1)$$

G, W and B are the gauge fields of $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$, respectively. The trace is denoted by $\langle\ldots\rangle$. h is the Higgs field. The electroweak Goldstone bosons are parametrized as $U = \exp(2i\varphi/v)$, where $\varphi = \varphi^a T^a$. T^a are the generators of $SU(2)$, normalized as $\langle T^a T^b\rangle = \delta^{ab}/2$, and $v = 246\,\text{GeV}$ is the electroweak scale. We define

$$L_\mu = iUD_\mu U^\dagger\,, \quad \text{where} \quad D_\mu U = \partial_\mu U + igW_\mu U - ig'B_\mu U T_3\,, \qquad \tau_L = UT_3U^\dagger \quad (2)$$

The Standard-Model (SM) fermions are collected in the field $\psi = (u_i, d_i, \nu_i, e_i)^T$. Here i is the generation index, u_i and d_i are color triplets, and the u_i, d_i, ν_i and e_i are Dirac spinors. The covariant derivative is

$$D_\mu\psi = \big(\partial_\mu + ig_s G_\mu + igW_\mu P_L + ig'B_\mu(Y_L P_L + Y_R P_R)\big)\,\psi \qquad (3)$$

P_L, P_R are the left and right chiral projectors. Weak hypercharge is described by the diagonal matrices

$$Y_L = \text{diag}(1/6, 1/6, -1/2, -1/2)\,, \qquad Y_R = \text{diag}(2/3, -1/3, 0, -1) \qquad (4)$$

The Yukawa term can be compactly expressed as

$$m(h,U) \equiv U\mathcal{M}(h)P_R + \mathcal{M}^\dagger(h)U^\dagger P_L \qquad (5)$$

with $\mathcal{M}$ the block-diagonal mass matrix, acting on ψ,

$$\mathcal{M} = \text{diag}(\mathcal{M}_u, \mathcal{M}_d, \mathcal{M}_\nu, \mathcal{M}_e) \qquad (6)$$

The entries $\mathcal{M}_f \equiv \mathcal{M}_f(h)$ are matrices in generation space and functions of h. It is understood that the right-handed neutrinos are absent when we assume SM particle content. Accordingly, we will take $\mathcal{M}_\nu = 0$ in our calculation.

The Higgs-dependent functions can be expanded as

$$F(h) = 1 + \sum_{n=1}^{\infty} F_n \left(\frac{h}{v}\right)^n, \qquad V(h) = v^4 \sum_{n=2}^{\infty} V_n \left(\frac{h}{v}\right)^n, \qquad \mathcal{M}(h) = \sum_{n=0}^{\infty} \mathcal{M}_n \left(\frac{h}{v}\right)^n \tag{7}$$

3. Master formula for one-loop divergences

In this section, we review the master formula giving the one-loop divergences of a general Lagrangian including both spin 0 and spin 1 bosons, as well as fermions. An equivalent result has been obtained a long time ago in [12] and in [15,16]. We re-derive it here using the super-heat-kernel framework of [13]. The formula will be the basis for the calculation of the one-loop renormalization of the electroweak chiral Lagrangian in (1). The discussion will also serve to fix our notation.

Starting from our general Lagrangian, we expand each field around a classical background configuration. The fluctuating parts of the various fields are denoted generically as ξ, ω_μ, and χ for the spin 0, spin 1 and spin 1/2 Dirac fields, respectively. Notice that all internal indices have been omitted, so that these fields denote in general multi-component objects. The bosonic fields are furthermore conveniently collected in a single multi-component object

$$\phi_i = (\xi, \omega_\mu), \quad \phi^i = (\xi, -\omega^\mu). \tag{8}$$

Assuming that the Lagrangian we started with is at most bilinear in the fermion fields, the part that is quadratic in the fluctuations takes, up to an irrelevant total derivative, the general form

$$\mathcal{L}_2 = -\frac{1}{2}\phi^i A_i^{\,j} \phi_j + \bar{\chi}\,(i\not{\partial} - G)\,\chi + \bar{\chi}\Gamma^i \phi_i + \phi^i \bar{\Gamma}_i \chi \tag{9}$$

with

$$A = (\partial^\mu + N^\mu)(\partial_\mu + N_\mu) + Y \tag{10}$$

For the fluctuating gauge fields, the Feynman gauge has to be used to ensure the canonical form of the kinetic term for the bosons in (9). The Dirac matrix G can be written as

$$G \equiv (r + \rho_\mu \gamma^\mu) P_R + (l + \lambda_\mu \gamma^\mu) P_L \tag{11}$$

The quantities Y, N, r, l, ρ, λ are bosonic, while Γ and $\bar{\Gamma}$ are Dirac spinors. They all depend on the background fields. Notice that in all generality one could also add a tensor contribution $\sigma_{\mu\nu} t^{\mu\nu}$ to G. Since such a term does not arise in the case we will study, we do not consider it.

The Gaussian integral over the bosonic and fermionic variables in

$$e^{iS_{\text{eff}}} \sim \int [d\phi_i \, d\chi \, d\bar{\chi}] e^{i \int d^D x \, \mathcal{L}_2(\phi_i, \chi, \bar{\chi})} \tag{12}$$

(gauge-fixing in the way described below is understood) leads to an expression for S_{eff} in terms of the fluctuation operator in $\mathcal{L}_2$. Keeping only the terms needed for the divergent part of S_{eff}, this expression can be written as [13]

$$S_{\text{eff}} = \frac{i}{2} \text{Str} \ln \Delta \tag{13}$$

where

$$\Delta \equiv \begin{pmatrix} A & \sqrt{2}\bar{\Gamma}\gamma_5 B\gamma_5 \\ -\sqrt{2}\Gamma & B\gamma_5 B\gamma_5 \end{pmatrix}, \qquad B \equiv i\partial\!\!\!/ - G \tag{14}$$

Here the supertrace str of a general supermatrix

$$M = \begin{pmatrix} a & \alpha \\ \beta & b \end{pmatrix} \tag{15}$$

with a, b bosonic and α, β fermionic sub-matrices, is defined by

$$\mathrm{str}\, M = \mathrm{tr}\, a - \mathrm{tr}\, b \tag{16}$$

The analogous trace operations Str and Tr include an integration over space-time.

The operator Δ in (14) has the canonical form

$$\Delta \equiv (\partial^\mu + \Lambda^\mu)(\partial_\mu + \Lambda_\mu) + \Sigma \tag{17}$$

which defines the supermatrices Λ_μ and Σ. In Euclidian space, the differential operator Δ is elliptic, and the divergent part, in four dimensions, of the effective action is given by the second Seeley–DeWitt coefficient of the corresponding heat-kernel expansion. The computation of the second Seeley–DeWitt coefficient for an operator like Δ is described in [13]. The divergent part of the dimensionally regularized one-loop effective Lagrangian is then given, in Minkowski space, by

$$\mathcal{L}_{\mathrm{div}} = \frac{1}{32\pi^2\varepsilon}\, \mathrm{str}\left[\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2\right] \tag{18}$$

where $\varepsilon = 2 - d/2$ and

$$\Lambda_{\mu\nu} \equiv \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu + [\Lambda_\mu, \Lambda_\nu] \tag{19}$$

Extracting Λ_μ and Σ from (14) and (17), and performing the traces over Dirac matrices explicitly [17–21], one finally arrives at the master formula [12]

$$\mathcal{L}_{\mathrm{div}} = \frac{1}{32\pi^2\varepsilon}\Bigg(\mathrm{tr}\left[\frac{1}{12}N^{\mu\nu}N_{\mu\nu} + \frac{1}{2}Y^2 - \frac{1}{3}\left(\lambda^{\mu\nu}\lambda_{\mu\nu} + \rho^{\mu\nu}\rho_{\mu\nu}\right) + 2D^\mu l D_\mu r - 2lrlr\right] \\ + \bar{\Gamma}\left(i\partial\!\!\!/ + iN\!\!\!/ + \frac{1}{2}\gamma^\mu G\gamma_\mu\right)\Gamma\Bigg) \tag{20}$$

with

$$N_{\mu\nu} \equiv \partial_\mu N_\nu - \partial_\nu N_\mu + [N_\mu, N_\nu] \tag{21}$$

$$\lambda_{\mu\nu} \equiv \partial_\mu\lambda_\nu - \partial_\nu\lambda_\mu + i[\lambda_\mu, \lambda_\nu], \qquad \rho_{\mu\nu} \equiv \partial_\mu\rho_\nu - \partial_\nu\rho_\mu + i[\rho_\mu, \rho_\nu] \tag{22}$$

$$D_\mu l \equiv \partial_\mu l + i\rho_\mu l - il\lambda_\mu, \qquad D_\mu r \equiv \partial_\mu r + i\lambda_\mu r - ir\rho_\mu \tag{23}$$

In (20) the terms bilinear in $N_{\mu\nu}$, Y, in $\lambda_{\mu\nu}$, $\rho_{\mu\nu}$, l, r, and in Γ, $\bar{\Gamma}$, originate, respectively, from pure bosonic loops, pure fermionic loops, and mixed contributions with both bosons and fermions in the loop. The expression in (20) holds up to surface terms that we have dropped. The ghost contribution for non-abelian gauge fields has to be added and will be discussed in section 4.

4. Technical aspects of the calculation

In order to implement the background field method [22], all fields are split additively into background and quantum components except for the Goldstone boson matrix U, which is expanded in multiplicative form following [23,24]. This allows us to remove the background Goldstone fields from the Lagrangian using a generalization of the Stückelberg formalism for the background field method [25]. The fact that no background Goldstone fields are present simplifies intermediate steps of the calculation. The background Goldstone fields are then recovered at the end of the calculation by inverting the Stückelberg transformation [23,24].

In the presence of non-abelian gauge fields, one needs to add the contribution arising from ghost loops. Let us denote the fluctuating components of the B_μ, W_μ and G_μ fields by b_μ, ω_μ and ε_μ, respectively. Choosing background-covariant gauge conditions

$$\mathcal{L}_{\text{g.f.}} = -\frac{1}{2}(\partial^\mu b_\mu)^2 - \langle (D^\mu \omega_\mu)^2 \rangle - \langle (D^\mu \varepsilon_\mu)^2 \rangle \tag{24}$$

the additional contribution to the divergent part of the one-loop effective Lagrangian reads

$$\mathcal{L}_{\text{div;ghost}} = \frac{1}{32\pi^2\varepsilon}\left(\frac{1}{3}g^2 C_2^W \langle W_{\mu\nu}W^{\mu\nu}\rangle + \frac{1}{3}g_s^2 C_2^G \langle G_{\mu\nu}G^{\mu\nu}\rangle\right) \tag{25}$$

where $C_2^W = 2$ and $C_2^G = 3$ are the quadratic Casimirs for the $SU(2)$ and $SU(3)$ gauge groups, respectively.

Though, within Feynman gauge, the total result for the one-loop divergences will be independent of the gauge fixing choice (up to field redefinitions), the individual contributions to the master formula will depend in general on the gauge fixing term. We will report our results using the (electroweak) gauge fixing term

$$\mathcal{L}_{\text{g.f.}} = -\frac{1}{2}\left(\partial^\mu b_\mu + \frac{g' v}{2} F \varphi_3\right)^2 - \frac{1}{2}\left(D^\mu \omega_\mu^a - \frac{g v}{2} F \varphi^a\right)^2 \tag{26}$$

with φ^a the fluctuating Goldstone fields. This gives the following divergent contribution to the one-loop effective Lagrangian from the ghost sector

$$\mathcal{L}_{\text{div;ghost}} = \frac{1}{32\pi^2\varepsilon}\left(\frac{2}{3}g^2 \langle W_{\mu\nu}W^{\mu\nu}\rangle - (3g^4 + 2g^2 g'^2 + g'^4)\frac{v^4}{16}F^2\right) \tag{27}$$

The gauge fixing term (26) is invariant under background-gauge transformations and cancels the mixing between the Goldstone fields and the gauge fields arising from the Goldstone-boson kinetic term. A similar choice of gauge fixing was used in [24]. In our calculation we checked explicitly that, as expected, the total result for the divergent contributions to the one-loop effective Lagrangian is the same with the two gauge fixing terms specified above.

5. One-loop divergences

In this section we give the explicit expressions for the complete one-loop divergences of the Higgs-electroweak chiral Lagrangian. They provide the counterterms that renormalize the theory at this order. These formulas are the main result of our paper. The divergences can be separated into the contributions from the electroweak sector and those from QCD, which we present in turn.

5.1. Electroweak sector

We define

$$\eta \equiv \frac{h}{v}, \qquad \kappa \equiv \frac{1}{2} F' F^{-1/2}, \qquad \mathcal{B} \equiv -F^{-1/2}\kappa' = \frac{F'^2}{4F^2} - \frac{F''}{2F} \tag{28}$$

Here and in the following, a prime on η-dependent functions denotes differentiation with respect to this variable.

For the contributions to the master formula (20) we finally obtain

$$\begin{aligned}
\mathrm{tr}\left(\frac{1}{12} N^{\mu\nu} N_{\mu\nu} + \frac{1}{2} Y^2\right) &= \frac{22}{3} g^2 \langle W^{\mu\nu} W_{\mu\nu}\rangle \\
&- (g'^2 + g^2(\kappa^2+2))\frac{v^2}{2} F\, \langle L^\mu L_\mu\rangle + g'^2 v^2 (1-\kappa^2) F\, \langle \tau_L L^\mu\rangle \langle \tau_L L_\mu\rangle \\
&- (3g^2 + g'^2) v^2 \kappa^2\, \partial^\mu \eta \partial_\mu \eta + (3g^4 + 2g^2 g'^2 + g'^4)\frac{v^4}{16} F^2
\end{aligned} \tag{29}$$

$$\begin{aligned}
&+ (3g^4 + 2g^2 g'^2 + g'^4)\frac{v^4}{32} F^2 + \frac{3g^2 + g'^2}{8} F' V' + \frac{3}{8}\left(\frac{F' V'}{F v^2}\right)^2 + \frac{1}{2}\left(\frac{V''}{v^2}\right)^2 \\
&+ \left((3g^2 + g'^2)\frac{v^2}{4} F + \frac{3}{2}\frac{F' V'}{F v^2}\right) \mathcal{B}\, \partial^\mu \eta \partial_\mu \eta \\
&- \left[(\kappa^2 - 1)\left((2g^2 + g'^2)\frac{v^2}{8} F + \frac{F' V'}{2 F v^2}\right) - \frac{V'' F}{2 v^2}\mathcal{B}\right] \langle L^\mu L_\mu\rangle \\
&+ \left((3g^2 + g'^2)\frac{v^2}{4} F + \frac{3}{2}\frac{F' V'}{F v^2}\right) \frac{F^{-1}}{v^2}\left(\bar{\psi}_L U \left(\frac{F'}{2}\mathcal{M}' - \mathcal{M}\right)\psi_R + \text{h.c.}\right) \\
&+ \frac{V''}{v^4}\left(\bar{\psi}_L U \mathcal{M}'' \psi_R + \text{h.c.}\right) - \frac{\kappa^2+1}{24}\left(2g^2 \langle W^{\mu\nu} W_{\mu\nu}\rangle + g'^2 B^{\mu\nu} B_{\mu\nu}\right) \\
&+ \frac{\kappa^2 - 1}{6} g g' \langle \tau_L W^{\mu\nu}\rangle B_{\mu\nu} - \frac{\kappa^2-1}{12}\left(ig\langle W^{\mu\nu}[L_\mu, L_\nu]\rangle + i g' B^{\mu\nu}\langle \tau_L [L_\mu, L_\nu]\rangle\right) \\
&- \frac{\kappa\kappa'}{3}\partial^\mu \eta \left(g\langle W_{\mu\nu} L^\nu\rangle - g' B_{\mu\nu}\langle \tau_L L^\nu\rangle\right) + \frac{1}{4} g'^2 v^2 (\kappa^2 - 1) F \langle \tau_L L^\mu\rangle \langle \tau_L L_\mu\rangle \\
&+ \frac{(\kappa^2-1)^2}{6}\langle L_\mu L_\nu\rangle^2 + \left(\frac{(\kappa^2-1)^2}{12} + \frac{F^2 \mathcal{B}^2}{8}\right)\langle L^\mu L_\mu\rangle^2 + \frac{2}{3}\kappa'^2 \langle L_\mu L_\nu\rangle \partial^\mu \eta \partial^\nu \eta \\
&- \left((\kappa^2 - 1)\mathcal{B} + \frac{\kappa'^2}{6}\right)\langle L^\mu L_\mu\rangle \partial^\nu \eta \partial_\nu \eta + \frac{3}{2}\mathcal{B}^2 (\partial^\mu \eta \partial_\mu \eta)^2 \\
&+ \langle L^\mu L_\mu\rangle \left[\frac{F\mathcal{B}}{2v^2}\bar{\psi}_L U \mathcal{M}'' \psi_R - \frac{\kappa^2 - 1}{F v^2}\bar{\psi}_L U \left(\frac{F'}{2}\mathcal{M}' - \mathcal{M}\right)\psi_R + \text{h.c.}\right] \\
&+ \frac{2\kappa'}{v^2}\partial^\mu \eta \left(i \bar{\psi}_L L_\mu U (F^{-1/2}\mathcal{M})' \psi_R + \text{h.c.}\right) \\
&+ \frac{3\mathcal{B}}{F v^2}\partial^\mu \eta \partial_\mu \eta \left(\bar{\psi}_L U \left(\frac{F'}{2}\mathcal{M}' - \mathcal{M}\right)\psi_R + \text{h.c.}\right)
\end{aligned} \tag{30}$$

$$
+\frac{3F^{-2}}{2v^4}\left(\bar\psi_L U\left(\frac{F'}{2}\mathcal{M}'-\mathcal{M}\right)\psi_R+\text{h.c.}\right)^2+\frac{1}{2v^4}\left(\bar\psi_L U\mathcal{M}''\psi_R+\text{h.c.}\right)^2
+\frac{4}{v^4}\left(i\bar\psi_L UT^a\left(F^{-1/2}\mathcal{M}\right)'\psi_R+\text{h.c.}\right)^2 \tag{31}
$$

The terms in (29) arise from loops with gauge fields and include the ghost contribution. The remaining ones come from loops with scalars. Operators in (30) have the form of terms in the leading-order Lagrangian, with the exception of $\langle\tau_L L^\mu\rangle\langle\tau_L L_\mu\rangle$ and the gauge kinetic terms $\langle W^{\mu\nu}W_{\mu\nu}\rangle$, $B^{\mu\nu}B_{\mu\nu}$ multiplied by powers of the Higgs field h^n, $n\geq 1$. All the operators in (31) arise only at next-to-leading order.

$$
-\frac{1}{3}\,\text{tr}(\lambda^{\mu\nu}\lambda_{\mu\nu}+\rho^{\mu\nu}\rho_{\mu\nu})=
-\frac{1}{2}\langle W^{\mu\nu}W_{\mu\nu}\rangle\frac{2}{3}(N_c+1)fg^2-\frac{1}{4}B^{\mu\nu}B_{\mu\nu}\left(\frac{22N_c}{27}+2\right)fg'^2 \tag{32}
$$

where N_c is the number of colors and f the number of fermion generations.

$$
2\,\text{tr}\left(D^\mu l D_\mu r-lrlr\right)= \tag{33}
$$
$$
2\langle\partial^\mu\mathcal{M}^\dagger\partial_\mu\mathcal{M}\rangle+4i\langle(\partial^\mu\mathcal{M}^\dagger\mathcal{M}-\mathcal{M}^\dagger\partial^\mu\mathcal{M})T_3\rangle\langle\tau_L L_\mu\rangle
+\langle\mathcal{M}^\dagger\mathcal{M}\rangle\langle L^\mu L_\mu\rangle-2\langle(\mathcal{M}^\dagger\mathcal{M})^2\rangle
$$

$$
\begin{aligned}
&\bar\Gamma\left(i\not\partial+i\not N+\frac{1}{2}\gamma^\mu G\gamma_\mu\right)\Gamma=\frac{4}{v^2}\bar\psi_L UT^a\mathcal{M}F^{-1/2}i\not\partial\left(\mathcal{M}^\dagger F^{-1/2}\right)T^aU^\dagger\psi_L\\
&+\frac{4}{v^2}\bar\psi_L UT^a\mathcal{M}\mathcal{M}^\dagger F^{-1}T^aU^\dagger i\not D\psi_L+\frac{1}{v^2}\bar\psi_L\not L U\mathcal{M}\mathcal{M}^\dagger U^\dagger F^{-1}\psi_L\\
&+\frac{1}{v^2}\bar\psi_L U\mathcal{M}'i\not\partial\mathcal{M}'^\dagger U^\dagger\psi_L+\frac{1}{v^2}\bar\psi_L U\mathcal{M}'\mathcal{M}'^\dagger U^\dagger(i\not D+\not L)\psi_L\\
&-\frac{\kappa}{v^2}F^{-1/2}\left(\bar\psi_L U\mathcal{M}'\mathcal{M}^\dagger U^\dagger\not L\psi_L+\text{h.c.}\right)\\
&+\frac{3}{v^2}\bar\psi_R\mathcal{M}^\dagger F^{-1/2}i\not D\left(\mathcal{M}F^{-1/2}\psi_R\right)+\frac{1}{v^2}\bar\psi_R\mathcal{M}'^\dagger i\not D\left(\mathcal{M}'\psi_R\right)\\
&-\frac{F^{-1}}{v^2}\bar\psi_R\mathcal{M}^\dagger U^\dagger\not L U\mathcal{M}\psi_R-\frac{1}{v^2}\bar\psi_R\mathcal{M}'^\dagger U^\dagger\not L U\mathcal{M}'\psi_R\\
&+\frac{\kappa}{v^2}F^{-1/2}\left(\bar\psi_R\mathcal{M}^\dagger U^\dagger\not L U\mathcal{M}'\psi_R+\text{h.c.}\right)\\
&-\frac{8}{v^2}F^{-1}\left(\bar\psi_L UT^a\mathcal{M}\mathcal{M}^\dagger T^a\mathcal{M}\psi_R+\text{h.c.}\right)+\frac{2}{v^2}\left(\bar\psi_L U\mathcal{M}'\mathcal{M}^\dagger\mathcal{M}'\psi_R+\text{h.c.}\right)\\
&+\bar\psi_L\left(\frac{3}{2}g^2+2g'^2Y_L^2\right)i\not D\psi_L+\bar\psi_R\,2g'^2Y_R^2 i\not D\psi_R-8g'^2\left(\bar\psi_L Y_L U\mathcal{M}Y_R\psi_R+\text{h.c.}\right)
\end{aligned} \tag{34}
$$

5.2. QCD sector

At one-loop order, QCD and electroweak renormalization can be treated separately. To obtain the one-loop divergences from QCD, we expand the Lagrangian in (1) to second order in fluctuations of the quark and gluon fields, treating gauge fixing and ghosts in the usual way. We follow again the procedure outlined in section 3. For the divergent part of the one-loop effective Lagrangian we find

$$\mathcal{L}_{\rm div,QCD} \equiv \frac{1}{32\pi^2\varepsilon} L_{\rm div,QCD} \tag{35}$$

$$L_{\rm div,QCD} = \frac{22N_c - 4N_f}{6} g_s^2 \langle G^{\mu\nu} G_{\mu\nu}\rangle + 2g_s^2 C_F\, \bar{q}\left(i\not{D} - 4(U\mathcal{M}_q P_R + \mathcal{M}_q^\dagger U^\dagger P_L)\right) q \tag{36}$$

Here $C_F = (N_c^2 - 1)/(2N_c)$ and N_f is the number of quark flavors. In analogy to section 2 we take $q = (u, d)^T$ and $\mathcal{M}_q = \mathrm{diag}(\mathcal{M}_u, \mathcal{M}_d)$.

5.3. Renormalization

The divergences displayed in eq. (31) are absorbed by the effective Lagrangian at NLO, whose structure has been systematically analyzed in [3],

$$\mathcal{L}_{\rm NLO} = \sum_i \frac{v^{6-d_i}}{\Lambda^2} F_i(h) \mathcal{O}_i, \tag{37}$$

with $\Lambda = 4\pi v$. A complete basis of operators $\mathcal{O}_i$ is also provided in [3]. Upon minimal subtraction of the one-loop divergences displayed in (31), the functions $F_i(h)$ will depend on the renormalization scale μ, with

$$F_i(h;\mu) = F_i(h;\mu_0) + \beta_i(h)\ln(\mu/\mu_0). \tag{38}$$

As announced in the Introduction, we will not give the explicit expressions of the beta functions $\beta_i(h)$ corresponding to the complete basis of operators $\mathcal{O}_i$ here, leaving this last step for future work. At this stage, let us just make a remark concerning the divergences given in eqs. (29), (30), which correspond to terms already present in the lowest-order effective Lagrangian $\mathcal{L}$ in (1). The form of the latter is the most general up to field redefinitions of $h(x)$. The latter have been used in order to (see, for instance, the discussion in appendix A of [3]):

i) remove any arbitrary functions of h in front of the kinetic terms of the Higgs field and of the fermion fields;
ii) remove a linear term in the Higgs potential, i.e. imposing $V'(0) = 0$.

These features are modified by the structure of the one-loop divergences given in (29), (30). One thus needs to perform the appropriate field redefinition of $h(x)$ in order to restore them.

6. Standard-Model limit

The Higgs dynamics in the chiral Lagrangian (1) is encoded in the functions $F(h)$, $V(h)$ and $\mathcal{M}(h)$. The renormalizable Standard Model (SM) is recovered in the limit ($\eta \equiv h/v$)

$$F = (1+\eta)^2, \qquad V = \frac{m_h^2 v^2}{8}\left(-2(1+\eta)^2 + (1+\eta)^4\right), \qquad \mathcal{M} = \mathcal{M}_0\,(1+\eta) \tag{39}$$

In this limit, all nonrenormalizable operators disappear from the divergent part of the effective Lagrangian given in section 5. The remaining expressions give the one-loop divergences of the

Standard Model, from which the well-known one-loop beta functions of the SM couplings can be obtained. We find agreement with the beta functions compiled in [26]. This serves as an important check of our results.

It will be useful to write the scalar fields in terms of the usual Higgs doublet Φ and $\tilde{\Phi}$, where $\tilde{\Phi}_i = \epsilon_{ij}\Phi^*_j$. The relation to the chiral coordinates is given by

$$(\tilde{\Phi}, \Phi) = \frac{v}{\sqrt{2}}(1+\eta)U \tag{40}$$

and we have

$$D^\mu\Phi^\dagger D_\mu\Phi = \frac{1}{2}\partial^\mu h\partial_\mu h + \frac{v^2}{4}\langle L^\mu L_\mu\rangle(1+\eta)^2\,, \qquad \Phi^\dagger\Phi = \frac{v^2}{2}(1+\eta)^2 \tag{41}$$

We will also use the SM relations

$$M_W^2 = \frac{1}{4}g^2v^2\,, \quad M_Z^2 = \frac{1}{4}(g^2+g'^2)v^2\,, \quad m_h^2 = 2\mu^2 = \lambda v^2\,, \quad m_t^2 = \frac{1}{2}y_t^2v^2 \tag{42}$$

This defines the parameters μ^2 and λ of the Higgs potential and the top-quark Yukawa coupling y_t. In general, the Yukawa matrix $\mathcal{Y}$ and the mass matrix $\mathcal{M}_0$ are related through

$$\mathcal{M}_0 = \frac{v}{\sqrt{2}}\mathcal{Y}\,, \qquad \text{where} \quad \mathcal{Y} = \text{diag}(\mathcal{Y}_u, \mathcal{Y}_d, \mathcal{Y}_\nu, \mathcal{Y}_e) \tag{43}$$

such that

$$U\mathcal{M}_0(1+\eta) = (\tilde{\Phi}, \Phi)\,\mathcal{Y} \tag{44}$$

and we have

$$\langle\mathcal{M}_0^\dagger\mathcal{M}_0\rangle = N_c(m_t^2+m_c^2+m_u^2+m_b^2+m_s^2+m_d^2)+m_\tau^2+m_\mu^2+m_e^2 \approx N_c m_t^2 \tag{45}$$

Here the trace is over color, family and isospin indices, and includes quarks and leptons. Below we will sometimes retain only the top-quark part to simplify expressions. Similarly, $\langle(\mathcal{M}_0^\dagger\mathcal{M}_0)^2\rangle \approx N_c m_t^4$.

6.1. Electroweak sector – bosonic part

We start with the bosonic part of the electroweak sector collected in (29)–(33). Using the relations above we find in the SM limit

$$\begin{aligned}
&32\pi^2\varepsilon\,\mathcal{L}^{\text{SM}}_{\text{div,EWb}} = \\
&-\frac{1}{2}\langle W^{\mu\nu}W_{\mu\nu}\rangle\left(-\frac{44}{3}+\frac{2}{3}(N_c+1)f+\frac{1}{3}\right)g^2 - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}\left(\left(\frac{22N_c}{27}+2\right)f+\frac{1}{3}\right)g'^2 \\
&+D^\mu\Phi^\dagger D_\mu\Phi\left(-6g^2-2g'^2+2N_c y_t^2\right)+\mu^2\Phi^\dagger\Phi\left(-\frac{3}{2}g^2-\frac{1}{2}g'^2-6\lambda\right) \\
&-\frac{\lambda}{2}(\Phi^\dagger\Phi)^2\left(-3g^2-g'^2-12\lambda-\frac{3}{4\lambda}(3g^4+2g^2g'^2+g'^4)+\frac{4N_c}{\lambda}y_t^4\right)
\end{aligned} \tag{46}$$

For the gauge-kinetic terms the contributions from gauge fields, fermions ($\sim f$) and scalars ($+1/3$) have been written separately.

Renormalizing fields

$$W^{\mu}_{(0)} = W^{\mu} Z_W^{1/2}, \quad B^{\mu}_{(0)} = B^{\mu} Z_B^{1/2}, \quad \Phi_{(0)} = \Phi Z_\Phi^{1/2} \tag{47}$$

and couplings

$$g_{(0)} = g\,\nu^{\varepsilon} Z_g\,, \quad g'_{(0)} = g'\,\nu^{\varepsilon} Z_{g'}\,, \quad \lambda_{(0)} = \lambda\,\nu^{2\varepsilon} Z_\lambda\,, \quad \mu^2_{(0)} = \mu^2 Z_m \tag{48}$$

where ν is a renormalization scale, we obtain from the leading-order SM Lagrangian the counterterms

$$\begin{aligned}\mathcal{L}^{\rm SM}_{\rm CT,EWb} = & -\frac{1}{2}\langle W^{\mu\nu} W_{\mu\nu}\rangle (Z_W - 1) - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}(Z_B - 1) + D^{\mu}\Phi^\dagger D_\mu \Phi (Z_\phi - 1) \\ & + \mu^2 \Phi^\dagger \Phi (Z_\Phi Z_m - 1) - \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 (Z_\Phi^2 Z_\lambda - 1)\end{aligned} \tag{49}$$

In the background field method we are using, the renormalization factors of gauge couplings and gauge fields are related through

$$Z_g = Z_W^{-1/2}, \qquad Z_{g'} = Z_B^{-1/2} \tag{50}$$

To one-loop order the renormalization factor Z_X for quantity X can be written as

$$Z_X = 1 + \frac{A_X}{16\pi^2 \varepsilon} \tag{51}$$

where A_X only depends on couplings. The beta function for parameter X, defined by [26]

$$\frac{dX}{dt} \equiv \frac{1}{16\pi^2}\beta_X\,, \qquad t = \ln\nu \tag{52}$$

is then given by

$$\beta_X = -X\,\frac{1}{\varepsilon}\frac{dA_X}{dt} \tag{53}$$

and can be evaluated using

$$\frac{d}{dt}\left(g^{n_g} g'^{n_{g'}} \lambda^{n_\lambda} y_t^{n_y}\right) = -(n_g + n_{g'} + 2n_\lambda + n_y)\varepsilon\; g^{n_g} g'^{n_{g'}} \lambda^{n_\lambda} y_t^{n_y} + \mathcal{O}\left(\frac{1}{16\pi^2}\right) \tag{54}$$

for the scaling of products of couplings in dimensional regularization.

Requiring that the $1/\varepsilon$ poles of $\mathcal{L}^{\rm SM}_{\rm div,EWb}$ in (46) are cancelled by adding the counterterms in (49) fixes the renormalization factors Z_Φ, Z_g, $Z_{g'}$, Z_λ and Z_m. Using (51), (53) and (54), we recover the one-loop beta functions of the SM couplings g, g', λ and μ^2:

$$\beta_g = -\left(\frac{22}{3} - \frac{N_c + 1}{3} f - \frac{1}{6}\right) g^3 = -\frac{19}{6} g^3 \tag{55}$$

$$\beta_{g'} = \left(\left(\frac{11 N_c}{27} + 1\right) f + \frac{1}{6}\right) g'^3 = \frac{41}{6} g'^3 \tag{56}$$

$$\beta_\lambda = -3(3g^2 + g'^2)\lambda + 12\lambda^2 + \frac{3}{4}(3g^4 + 2g^2 g'^2 + g'^4) + 4N_c \lambda y_t^2 - 4N_c y_t^4 \tag{57}$$

$$\beta_{\mu^2} = \mu^2 \left(-\frac{9}{2} g^2 - \frac{3}{2} g'^2 + 6\lambda + 2N_c y_t^2\right) \tag{58}$$

6.2. Electroweak sector – fermionic part

We next turn to the fermionic part of the electroweak sector given in (34). Taking the SM limit, the one-loop divergent terms from this sector become

$$
\begin{aligned}
&32\pi^2\varepsilon\,\mathcal{L}^{\rm SM}_{\rm div,EWf} = \\
&\bar\psi_L\left(\frac{3}{2}g^2+2g'^2Y_L^2\right)i\not{D}\psi_L+\bar\psi_R\,2g'^2Y_R^2 i\not{D}\psi_R-8g'^2\left(\bar\psi_L U Y_L\mathcal{M}Y_R\psi_R+\text{h.c.}\right)\\
&+\frac{2}{v^2}\bar\psi_L\langle\mathcal{M}_0\mathcal{M}_0^\dagger\rangle_I\,i\not{D}\psi_L+\frac{4}{v^2}\bar\psi_R\mathcal{M}_0^\dagger\mathcal{M}_0\,i\not{D}\psi_R\\
&-\frac{4}{v^2}\left(\bar\psi_L U(\langle\mathcal{M}_0\mathcal{M}_0^\dagger\rangle_I-\mathcal{M}_0\mathcal{M}_0^\dagger)\mathcal{M}\psi_R+\text{h.c.}\right)
\end{aligned}
\tag{59}
$$

Here $\langle\ldots\rangle_I$ denotes a trace over isospin indices only.

Again, the nonrenormalizable operators in (34) have disappeared in the SM limit (59). The remaining divergences renormalize the fermionic part of the SM Lagrangian, which can be written as

$$
\mathcal{L}^{\rm SM}_{\rm f}=\bar\psi_L i\not{D}\psi_L+\bar\psi_R i\not{D}\psi_R-\left(\bar\psi_L(\tilde\Phi,\Phi)\mathcal{Y}\psi_R+\text{h.c.}\right)
\tag{60}
$$

We take

$$
(\tilde\Phi,\Phi)_{(0)}=Z_\Phi^{1/2}(\tilde\Phi,\Phi)\,,\qquad \psi^{(0)}_{L,R}=Z_{L,R}\psi_{L,R}\,,\qquad \mathcal{Y}_{(0)}=\nu^\varepsilon(\mathcal{Y}+\Delta\mathcal{Y})
\tag{61}
$$

where $Z_{L,R}=Z^\dagger_{L,R}$ are flavor matrices, which can be chosen to be hermitean. Z_Φ is determined from the bosonic sector discussed above. From the definition of $\Delta\mathcal{Y}$ we find for the running of the Yukawa matrix

$$
\frac{d\mathcal{Y}}{dt}=\frac{1}{16\pi^2}\beta_{\mathcal{Y}}=-\varepsilon\mathcal{Y}-\varepsilon\,\Delta\mathcal{Y}-\frac{d}{dt}\Delta\mathcal{Y}+\mathcal{O}\left(\frac{1}{(16\pi^2)^2}\right)
\tag{62}
$$

Inserting (61) into the (unrenormalized) Lagrangian (60), and using (43), (44) and $\Delta Z_X=Z_X-1$, we find the counterterms

$$
\begin{aligned}
\mathcal{L}^{\rm SM}_{\rm CT,EWf}=&\,\bar\psi_L\,2\Delta Z_L\,i\not{D}\psi_L+\bar\psi_R\,2\Delta Z_R\,i\not{D}\psi_R\\
&-(1+\eta)\left(\bar\psi_L U\left[\Delta Z_L\mathcal{M}_0+\mathcal{M}_0\Delta Z_R+\frac{1}{2}\Delta Z_\Phi\mathcal{M}_0+\Delta\mathcal{M}_0\right]\psi_R+\text{h.c.}\right)
\end{aligned}
\tag{63}
$$

Requiring (63) to cancel the divergences of $\mathcal{L}^{\rm SM}_{\rm div,EWf}$ in (59), we obtain $\Delta Z_{L,R}$ and $\Delta\mathcal{M}_0\equiv v\Delta\mathcal{Y}/\sqrt{2}$. We find

$$
\Delta\mathcal{Y}=-\frac{1}{32\pi^2\varepsilon}\left[\left(\frac{9}{4}g^2+\left(\frac{3}{4}+6Y_LY_R\right)g'^2-N_cy_t^2\right)\mathcal{Y}+\frac{3}{2}\left(\langle\mathcal{Y}\mathcal{Y}^\dagger\rangle_I-2\mathcal{Y}\mathcal{Y}^\dagger\right)\mathcal{Y}\right]
\tag{64}
$$

From (62) the beta function becomes

$$
\beta_{\mathcal{Y}}=\frac{3}{2}\left(2\mathcal{Y}\mathcal{Y}^\dagger-\langle\mathcal{Y}\mathcal{Y}^\dagger\rangle_I\right)\mathcal{Y}-\left(\frac{9}{4}g^2+\left(\frac{3}{4}+6Y_LY_R\right)g'^2-N_cy_t^2\right)\mathcal{Y}
\tag{65}
$$

with $3/4+6Y_LY_R=\text{diag}(17/12,5/12,3/4,15/4)$, in agreement with [26].

7. Conclusions

The main results of this paper are:

- We computed for the first time the complete one-loop renormalization of the electroweak chiral Lagrangian including a light Higgs. All the divergent structures that we found either renormalize the LO Lagrangian or correspond to counterterms of the NLO Lagrangian according to the chiral counting of [3]. This result therefore corroborates that the chiral counting proposed in [3,4] governs the divergence structure of the electroweak chiral Lagrangian.
- We used the background-field method [22] to ensure explicit gauge invariance of background fields in all steps of the computation. The divergent contributions to the one-loop effective Lagrangian were extracted using the super-heat-kernel formalism [13]. As intermediate result, we rederived the 't Hooft master formula [12].
- To cross-check the full result among ourselves, we have carried out independent calculations using different gauge fixing terms. We find full agreement in the final result.
- We considered the SM limit as explicit cross-check of our result. We reproduce all the one-loop beta-functions in this limit. Considering only scalar (Goldstone and Higgs) fluctuations, we further reproduce [9], which was also cross-checked later by [10].

Note added

After the present paper had been made public on arXiv, the article [27] appeared, in which essentially the same topic is addressed, and which includes the formulation of renormalization-group equations.

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