Physics Letters B 786 (2018) 171–179

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Supersymmetric AdS_7 and AdS_6 vacua and their minimal consistent truncations from exceptional field theory

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ARTICLE INFO

Article history:
Received 20 August 2018
Received in revised form 11 September 2018
Accepted 14 September 2018
Available online 22 September 2018
Editor: M. Cvetič

Keywords:
Flux vacua
AdS vacua
Consistent truncations

ABSTRACT

We show how to construct supersymmetric warped AdS_7 vacua of massive IIA and AdS_6 vacua of IIB supergravity, using "half-maximal structures" of exceptional field theory. We use this formalism to obtain the minimal consistent truncations around these AdS vacua.

1. Introduction

Many supersymmetric AdS vacua of 10- and 11-dimensional SUGRA are known and play a crucial role in the AdS/CFT correspondence. Yet, we do not currently have a systematic geometric understanding of such vacua. Unlike fluxless supersymmetric Minkowski vacua, which are described by integrable G-structures and thus special holonomy manifolds, supersymmetric AdS vacua require non-vanishing fluxes and thus are not described by integrable G-structures in Riemannian geometry. However, recently a description of supersymmetric AdS vacua in terms of generalised G-structures has appeared using Exceptional Field Theory (ExFT) [1–4], focusing on the case of half-maximal supersymmetry [5–7], and Exceptional Generalised Geometry (EGG) [8,9], focusing on 1/4-maximal supersymmetry [10–12] and more general analyses [13,14].[1] ExFT and EGG provide a unified description of metric and flux degrees of freedom of 10-/11-dimensional SUGRA by using enlarged geometric structures called generalised tangent bundles. It is on these enlarged bundles that the generalised G-structures can be defined.

This reformulation of SUGRA has already been successfully used to study flux vacua. For example, it has led to maximally supersymmetric consistent truncations [16–18] which uplift several interesting lower-dimensional gauged supergravities and their vacua to 10-/11-dimensional SUGRA [19–27]. Recently, [5,7] showed how to generalise this procedure to construct consistent truncations of 10-/11-dimensional SUGRA which break half the supersymmetry. Furthermore, [7] proved that for each warped half-maximally supersymmetric AdS_D vacuum of 10-/11-dimensional SUGRA, there exists a consistent truncation to D-dimensional half-maximal gSUGRA containing only the graviton supermultiplet, thereby proving the half-maximal case of the conjecture [28]. Such consistent truncations are particularly useful for the AdS/CFT correspondence where they allow us to study AdS vacua of 10-/11-dimensional SUGRA using lower-dimensional gauged SUGRAs. This can be used to study deformations of the AdS vacua, e.g. finding domain-wall solutions which are holographically dual to RG flow.

In this letter, we will show that generalised G-structures can be used to efficiently construct supersymmetric AdS vacua of 10-/11-dimensional SUGRA. We will focus on AdS_7 vacua of massive IIA and AdS_6 vacua of IIB SUGRA. A family of infinitely many such solutions have recently been found [29–34], parameterised by

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[1] [15] pointed out that there may be supersymmetric AdS vacua in even dimensions that are not described by generalised G-structures. However, these AdS vacua are unlikely to be complete and regular.

https://doi.org/10.1016/j.physletb.2018.09.037
0370-2693/

a cubic function in one variable in the case of AdS_7 and two holomorphic functions for AdS_6.[2] We will show how these solutions can easily be constructed in terms of a universal half-maximal structure underlying them.

A further benefit of our approach is that, as shown in [7], once we have described a supersymmetric AdS_D vacuum by generalised G-structures, we immediately obtain a consistent truncation of the higher-dimensional SUGRA around the AdS vacuum to the minimal D-dimensional gauged SUGRA containing only the graviton supermultiplet. Thus, we will rederive the consistent truncation around the AdS_7 vacua of [31] and construct the minimal consistent truncation around the AdS_6 vacua. The fact that these vacua are described by a universal half-maximal structure explains the universality of their truncation Ansatz, which takes the same form for the entire family of solutions.

The outline of the letter is as follows. In section 2, we review the description of half-maximally supersymmetric AdS vacua using generalised G-structures [7], and how to construct the minimal consistent truncations of these vacua. Next, we show how to calculate the generalised metric, which encode the SUGRA fields, from the half-maximal structures in section 3. We then show how to construct the family of infinitely-many AdS_7 vacua of mIIA and AdS_6 vacua of IIB using this method in sections 4 and 5. Finally, in section 6, we derive the minimal consistent truncation around these vacua. A more detailed version of these results including consistent truncations with additional matter multiplets will appear in [37].

Note added While finalising this manuscript, the paper [38] appeared which also constructs the minimal consistent truncation around supersymmetric AdS_6 vacua that we derive in section 6.2.

2. Half-maximal AdS vacua from ExFT

Supersymmetric AdS vacua can be naturally described in ExFT using the language of generalised G-structures, analogous to the description of special holonomy spaces in Riemannian geometry. In particular, as was shown in [5–7], flux geometries of type II or 11-dimensional SUGRA admitting a half-maximal set of spinors can be described by a set of generalised tensors satisfying certain algebraic conditions. Here we will consider 10/11-dimensional geometries consisting of warped products $M_D \times M_{int}$, where M_D denotes the external spacetime, M_{int} is the internal space and we will focus on $D = 6, 7$.

In order for M_D to be a half-maximal AdS vacuum, M_{int} must admit a set of nowhere-vanishing $d-1$ generalised vector fields $J_u \in \Gamma(\mathcal{R}_1)$, where $d = 11 - D$ and $u = 1, \ldots, d-1$, as well as a generalised tensor field $\hat{K} \in \Gamma(\mathcal{R}_{D-4})$ satisfying

$$J_u \wedge J_v - \frac{1}{d-1}\delta_{uv} J_w \wedge J^w = 0 , \qquad J_u \wedge J^u \wedge \hat{K} > 0 , \qquad \hat{K} \otimes \hat{K}|_{R_c} = 0 . \tag{2.1}$$

Here $\mathcal{R}_i$ are different generalised vector bundles whose fibre is the R_i representation of $E_{d(d)}$, listed in Table 1, where also the representation R_c is given. The $\wedge$ products map

$$\wedge : \mathcal{R}_i \otimes \mathcal{R}_j \longrightarrow \mathcal{R}_{i+j} \text{ when } i + j < D - 2 , \qquad \wedge : \mathcal{R}_i \otimes \mathcal{R}_{D-2-i} \longrightarrow \mathcal{S} , \tag{2.2}$$

where $\mathcal{S}$ denotes the space of scalar densities of weight 1 under generalised diffeomorphisms. The explicit expressions of $\wedge$ in terms of $E_{d(d)}$ invariant is given in [7] and summarised in Appendix A. It is worth noting that generalised tensor fields are always a formal sum of differential forms and, in the case of $\mathcal{R}_1$ also a vector field. We use the notation $\wedge$ because the product (2.2) reduces to the usual wedge-product of differential forms (and a contraction between vector fields and differential forms).[3]

Table 1
Different representations of the exceptional groups relevant to half-maximal structures.

D	$E_{d(d)}$	R_1	R_2	R_3	R_4	R_c
7	SL(5)	**10**	$\overline{\mathbf{5}}$	**5**	$\overline{\mathbf{10}}$	∅
6	Spin(5, 5)	**16**	**10**	$\overline{\mathbf{16}}$	N/R	**1**

Throughout, we will raise and lower the $u, v = 1 \ldots, d-1$ indices using δ_{uv}. A set of generalised tensors $\left(J_u, \hat{K}\right)$ as above are called a half-maximal structure and are stabilised by a $G_{half} = SO(d-1)$ subgroup of the exceptional group $E_{d(d)}$. The maximal commutant of $SO(d-1) \subset E_{d(d)}$ is $SO(d-1)_R$ and rotates the $d-1$ J_u's amongst each other.

From the half-maximal structure one can also define the following generalised tensors that will be useful in the following:

$$J_u \wedge J_v = \delta_{uv} K , \qquad K \wedge \hat{K} = \kappa^{D-2} , \qquad \hat{J}_u = J_u \wedge \hat{K} , \tag{2.3}$$

where $K \in \Gamma(\mathcal{R}_2)$, κ is a scalar density of weight $\frac{1}{D-2}$ and $\hat{J}_u \in \Gamma(\mathcal{R}_{D-3})$. The explicit expressions for the above equations (2.1) and (2.3) in terms of $E_{d(d)}$-invariants can be found in [7].

Furthermore, the BPS equations for the AdS_D vacuum are equivalent to the following differential equations

$$\begin{aligned} &\mathcal{L}_{J_u} J_v = -\Lambda_{uvw} J^w , \\ &\mathcal{L}_{J_u} \hat{K} = 0 , \\ &d\hat{K} = \begin{cases} \frac{1}{3!\sqrt{2}} \epsilon^{uvw} \Lambda_{uvw} K , & \text{when } D = 7 , \\ \frac{1}{9} \epsilon_{uvwx} \Lambda^{uvw} J^x , & \text{when } D = 6 , \end{cases} \end{aligned} \tag{2.4}$$

where $\Lambda_{uvw} = \Lambda_{[uvw]}$ are completely antisymmetric, $\mathcal{L}$ denotes the generalised Lie derivative of ExFT [2,8,3] and $d : \Gamma(\mathcal{R}_{D-4}) \longrightarrow \Gamma(\mathcal{R}_{D-5})$ is a certain nilpotent operator as defined in [5,7] and which also appears in studies of the tensor hierarchy of ExFT [39–41]. We will give its explicit expression in Appendix A. We use the notation d because this operator is nothing but the exterior derivative of differential forms. Given that generalised tensors in $\mathcal{R}_i$, for $1 < i < d-2$ which are the only ones relevant for us here, are a formal sum of differential forms, d acts as the exterior derivative on each differential form.[4]

The differential conditions (2.4) encode the BPS conditions for the supersymmetric AdS_D vacuum in a geometric language. As we will see the algebraic and differential conditions (2.1) and (2.4) can easily be solved in a variety of different cases, providing an efficient way of constructing supersymmetric AdS vacua of 10/11-dimensional supergravity. Moreover, once we have the half-maximal structure for an AdS vacuum, we can immediately construct a consistent truncation around the AdS vacuum to the D-dimensional half-maximal gauged SUGRA containing only the graviton supermultiplet [5,7], as we review in section 2.1. We will

[2] Previous attempts at constructing these general classes of solutions include [35,36].

[3] Later in this paper, we will also use $\wedge$ to denote the usual wedge-product between differential forms. We do not anticipate any confusion: when $\wedge$ acts between generalised tensors it is the operator (2.2), acting as explained here and explicitly in [7] and Appendix A; when $\wedge$ acts between differential forms it is the usual wedge-product.

[4] In this paper, we will also use d to denote the exterior derivative acting on differential forms. It will always be clear from the object that d is acting which differential operator is meant.

use this method to find the minimal consistent truncations around supersymmetric AdS_6 vacua of IIB SUGRA, as well as derive the consistent truncations around supersymmetric AdS_7 vacua of massive IIA SUGRA, where our expressions agree with [31].

The half-maximal structure encodes the 11-dimensional/type II supergravity fields, just like a complex and Kähler structure encode the metric. In ExFT, the supergravity fields parameterise the generalised metric $\mathcal{M}_{MN}$ which lives in the coset space $\mathcal{M}_{MN} \in \frac{E_{d(d)}}{H_d}$, where H_d is the maximal compact subgroup of $E_{d(d)}$. As we will show in section 3, the generalised metric $\mathcal{M}_{MN}$ can be expressed in terms of an $SO(d-1)_R$-invariant combination of the half-maximal structure J_u and $\hat{K}$.

Before we proceed to discuss the $AdS_{6,7}$ cases in detail, we will first make some general observations about the differential conditions (2.4) and what they imply for J_u and $\hat{K}$. We see that the generalised Lie derivative of the J_u's generates an $SO(d-1)_R$ rotation under which the J_u's transform in the vector representation while $\hat{K}$ is invariant. However, as mentioned above, the generalised metric and hence the SUGRA fields are constructed from $SO(d-1)_R$-invariant combinations of the J_u's and $\hat{K}$. Therefore $\mathcal{L}_{J_u}\mathcal{M}_{MN} = 0$, and the J_u are generalised Killing vector fields. Generalised vector fields are a formal sum of spacetime vector fields plus certain differential forms. Therefore, for J_u to be generalised Killing, implies that either they consist of non-zero spacetime Killing vector fields with accompanying gauge transformations such that the gauge potentials are left invariant, or they have an identically vanishing spacetime vector field part and consist of trivial gauge transformations, i.e. exact differential forms. For such a "trivial" generalised Killing vector field V we would have $\mathcal{L}_V = 0$ acting on any tensor. We will make use of this general insight in sections 4 and 5 when constructing the AdS vacua.

2.1. Minimal consistent truncation

Once we have constructed the half-maximal structure J_u and $\hat{K}$ corresponding to a half-maximal AdS_D vacuum, we can immediately construct a consistent truncation around this vacuum to a minimal half-maximal D-dimensional SUGRA [7]. That such a consistent truncation should always exist for any warped supersymmetric AdS vacuum of 10-/11-dimensional SUGRA was conjectured in [28] and proven in the half-maximal case for $D \geq 4$ in [7].

The truncation Ansatz is linear on the half-maximal structure and given as follows. We denote by Y^M the internal coordinates on M_{int} and by x^μ the external coordinates on M_D. Then, the truncation Ansatz (of the scalar sector) is given by [5,7]

$$\mathcal{J}_u(x,Y) = X^{-1}(x)\, J_u(Y)\,, \qquad \hat{\mathcal{K}}(x,Y) = X^2(x)\,\hat{K}(Y)\,, \tag{2.5}$$

where $X(x)$ is the scalar field of the D-dimensional half-maximal SUGRA. The consistency of the truncation Ansatz is guaranteed by the differential conditions (2.4) satisfied by the J_u, $\hat{K}$ as shown in [7]. Upon truncation, $X(x)$ becomes the scalar field of the minimal half-maximal D-dimensional gauged SUGRA with embedding tensor given by Λ_{uvw} in (2.4).

The consistent truncation can easily be extended to the other fields of the D-dimensional SUGRA as explained in [7]. However, since non-vanishing vacuum expectation values of these fields will typically break Lorentz symmetry, we will not include them in this letter.

3. Generalised metric from the half-maximal structure

As we mentioned above, the half-maximal structure determines the supergravity fields which in ExFT are encoded in the generalised metric. We therefore need to find a way to compute the generalised metric from the half-maximal structure J_u, $\hat{K}$. The generalised metric parameterises the coset space $\mathcal{M}_{MN} \in \frac{E_{d(d)}}{H_d}$, and hence must be an $E_{d(d)}$ group element that is invariant under H_d. Since J_u and $\hat{K}$ are by construction invariant under $G_{half} = SO(d-1) \subset H_d$, we must construct $\mathcal{M}_{MN}$ using an $SO(d-1)_R$-invariant combination of J_u and $\hat{K}$.

3.1. Generalised metric in SL(5) ExFT

In SL(5) ExFT [1,2,42], the generalised metric is often used either in the $\overline{\mathbf{10}}$ (with inverse in $\mathbf{10}$) representation or in the fundamental representation, $\overline{\mathbf{5}}$ (with inverse in $\mathbf{5}$), of SL(5). The two are related by $\mathcal{M}_{ab,cd} = 2\mathcal{M}_{a[c}\mathcal{M}_{d]b}$, where $a, b = 1, \ldots, 5$ denote fundamental SL(5) indices [2].

The generalised metric and its inverse are given by

$$\begin{aligned}
\mathcal{M}_{ab,cd} &= 8\,\kappa^{-8}\,\hat{J}_{u\,ab}\,\hat{J}^{u}{}_{cd} - \kappa^{-3}\epsilon_{abcde}\hat{K}^e \\
&\quad - \frac{1}{6\sqrt{2}}\kappa^{-3}\epsilon^{uvw}\epsilon_{abefg}\epsilon_{cdhij}\,J_u{}^{ef}\,J_v{}^{hi}\,J_w{}^{gj}\,, \\
\mathcal{M}^{ab,cd} &= 2\,\kappa^{-2}\,J_u{}^{ab}\,J^{u,cd} - \kappa^{-2}\epsilon^{abcde}K_e \\
&\quad - \frac{2\sqrt{2}}{3}\kappa^{-12}\epsilon^{uvw}\epsilon^{abefg}\epsilon^{cdhij}\,\hat{J}_{u\,ef}\,\hat{J}_{v\,hi}\,\hat{J}_{w\,gj}\,,
\end{aligned} \tag{3.1}$$

where $\hat{J}_{u\,ab}$ and κ are defined as in (2.3) which here become

$$\hat{J}_{u\,ab} = \frac{1}{4}\epsilon_{abcde}\,J_u{}^{cd}\,\hat{K}^e\,, \qquad \kappa^5 = \frac{1}{12}\epsilon_{abcde}\,J_u{}^{ab}\,J^{u\,cd}\,\hat{K}^e\,, \tag{3.2}$$

where ϵ_{abcde} is the constant SL(5)-invariant tensor. Similarly, the generalised metric and its inverse in the $\mathbf{5}$ and $\overline{\mathbf{5}}$ representations of SL(5) are given by

$$\begin{aligned}
\mathcal{M}_{ab} &= \kappa^{-4}\left(K_aK_b + \frac{4\sqrt{2}}{3}\kappa^{-5}\,\epsilon^{uvw}\,\hat{J}_{u,ac}\,\hat{J}_{v,bd}\,J_w{}^{cd}\right), \\
\mathcal{M}^{ab} &= \kappa^{-6}\left(\hat{K}^a\hat{K}^b + \frac{2\sqrt{2}}{3}\,\epsilon^{uvw}\,J_u{}^{ac}\,J_v{}^{bd}\,\hat{J}_{w,cd}\right).
\end{aligned} \tag{3.3}$$

3.2. Generalised metric in SO(5, 5) ExFT

In SO(5, 5) ExFT [43,44], the generalised metric is often used either in the $\mathbf{16}$ and $\overline{\mathbf{16}}$ representation or in the fundamental representation, $\mathbf{10}$, of SO(5, 5). The two are related by

$$\mathcal{M}_{MP}\mathcal{M}_{NQ}\,(\gamma_I)^{MN}\,\mathcal{M}^{IJ} = \left(\gamma^J\right)_{PQ}\,, \tag{3.4}$$

where $M = 1, \ldots, 16$ label the $\mathbf{16}/\overline{\mathbf{16}}$ representations of SO(5, 5) when upstairs/downstairs respectively, $I = 1, \ldots, 10$ label the $\mathbf{10}$ representation of SO(5, 5) and $(\gamma_I)^{MN}$ and $(\gamma_I)_{MN}$ are the SO(5, 5) γ-matrices.

We thus find the generalised metric and its inverse in the $\overline{\mathbf{16}}/\mathbf{16}$ are given by

$$\begin{aligned}
\mathcal{M}_{MN} &= \frac{1}{\sqrt{2}}\Big(4\,\kappa^{-6}\,\hat{J}^u{}_M\,\hat{J}_{u\,N} - \kappa^{-2}\left(\gamma^I\right)_{MN}\hat{K}_I \\
&\quad - \frac{1}{4!}\,\kappa^{-6}\epsilon^{uvwx}\,(\gamma_I)_{MP}\,(\gamma_J)_{NQ} \\
&\quad \times \left(\gamma^{IJ}\right)^S{}_R\,J_u{}^P\,J_v{}^Q\,J_w{}^R\,\hat{J}_{x,S}\Big)\,, \\
\mathcal{M}^{MN} &= \frac{1}{\sqrt{2}}\Big(2\,\kappa^{-2}\,J_u{}^M\,J^{u\,N} - \kappa^{-2}\,(\gamma_I)^{MN}\,K^I \\
&\quad - \frac{2}{4!}\kappa^{-10}\epsilon_{uvwx}\,(\gamma_I)^{MP}\,(\gamma_J)^{NQ} \\
&\quad \times \left(\gamma^{IJ}\right)^S{}_R\,\hat{J}^u{}_P\,\hat{J}^v{}_Q\,J_w{}^R\,\hat{J}_{x,S}\Big)\,,
\end{aligned} \tag{3.5}$$

where $\hat{J}_{u\,M}$ and κ are defined in (2.3), and here given explicitly by

$$\hat{J}^u{}_M = \frac{1}{2}\left(\gamma^I\right)_{MN} \hat{K}_I J^{u\,N}\,, \qquad \kappa^4 = \frac{1}{8}\left(\gamma^I\right)_{MN} J_u{}^M J^{u\,N} \hat{K}_I\,. \tag{3.6}$$

Similarly, the generalised metric in the **10** is

$$\mathcal{M}_{IJ} = \left(\frac{1}{4!}\epsilon^{uvwx}\left(\gamma_{IK}\right)_M{}^N \left(\gamma_J{}^K\right)_P{}^Q J_u{}^M \hat{J}_{v,N} J_w{}^P \hat{J}_{x,Q} + \kappa^{-4} K_I K_J + \kappa^{-4}\hat{K}_I \hat{K}_J\right). \tag{3.7}$$

4. AdS$_7$ vacua from massive IIA supergravity

We will now show how to use this method to construct AdS$_7$ vacua of massive IIA SUGRA. First, we let $\Lambda_{uvw} = \sqrt{2}R^{-1}\epsilon_{uvw}$, where R is the AdS$_7$ radius, so that the differential conditions (2.4) become

$$\mathcal{L}_{J_u} J_v = -\frac{\sqrt{2}}{R}\epsilon_{uvw} J^w\,, \qquad \mathcal{L}_{J_u}\hat{K} = 0\,, \qquad d\hat{K} = \frac{1}{R}K\,. \tag{4.1}$$

We see that the J_u's generate SU(2)$_R$ rotations via the generalised Lie derivative. J_u form triplets of SU(2)$_R$, while $\hat{K}$ are invariant. As we discussed in section 2 this implies that J_u are generalised Killing vector fields. Since $\mathcal{L}_{J_u} \neq 0$, none of the J_u are trivial generalised Killing vector fields and hence must contain spacetime Killing vectors. From (4.1) we see that these spacetime Killing vectors must generate an SU(2)$_R$ algebra and hence are related to an S^2 geometry. Therefore, we will consider the internal space $M_{int} = S^2 \times I$, where I is an interval with coordinate z, where in principle we allow off-diagonal metrics between the $S^2 \times I$ (although we will see that supersymmetry does not allow these off-diagonal terms). We will parameterise S^2 by the three functions y_u, $u = 1, \ldots, 3$ satisfying $y_u y^u = 1$. In terms of these functions, the round metric on S^2 and its volume form are given by

$$ds^2_{S^2} = dy_u dy^u\,, \qquad vol_{S^2} = \frac{1}{2}\epsilon_{uvw} y^u dy^v \wedge dy^w\,. \tag{4.2}$$

The Killing vectors of the round S^2 are given by

$$v_u^i = g^{ij}\epsilon_{uvw} y^v \partial_j y^w\,, \tag{4.3}$$

where $i, j = 1, 2$ denote a local coordinate basis and g^{ij} is the inverse metric of the round S^2. We also make repeated use of the 1-forms

$$\theta_u = \epsilon_{uvw} y^v dy^w\,. \tag{4.4}$$

4.1. SL(5) *ExFT and IIA SUGRA*

Supersymmetric AdS$_7$ vacua are characterised by three generalised vector fields $J_u \in \Gamma(\mathcal{R}_1)$ and a generalised tensor field $\hat{K} \in \Gamma(\mathcal{R}_3)$. In IIA SUGRA these become formal sums of spacetime vector fields and differential forms,

$$J_u = V_u + \lambda_u + \sigma_u + \phi_u\,, \qquad \hat{K} = \omega_{(0)} + \omega_{(2)} + \omega_{(3)}\,, \tag{4.5}$$

where V_u, λ_u, σ_u and ϕ_u are the vector, 1-form, 2-form and scalar parts of J_u, while $\omega_{(p)}$ are the p-forms appearing in $\hat{K}$. The generalised tensor $K = \bar{\omega}_{(0)} + \bar{\omega}_{(1)} + \bar{\omega}_{(3)} \in \Gamma(\mathcal{R}_2)$ consists of p-forms $\bar{\omega}_{(p)}$.

In IIA SUGRA, the wedge products appearing in the algebraic conditions (2.1) become

$$\begin{aligned} J_u \wedge J_v &= 2\iota_{V_{(u}}\lambda_{v)} - 2\left(\lambda_{(u}\phi_{v)} + \iota_{V_{(u}}\sigma_{v)}\right) - 2\lambda_{(u}\wedge\sigma_{v)}\,, \\ \hat{K}\wedge K &= \omega_{(0)}\,\bar{\omega}_{(3)} + \omega_{(1)}\wedge\bar{\omega}_{(2)} + \bar{\omega}_{(0)}\,\omega_{(3)}\,. \end{aligned} \tag{4.6}$$

The quadratic algebraic constraint on $\hat{K}$ is automatically fulfilled for SL(5) ExFT [7]. The differential operators appearing in the differential conditions (4.1) become

$$\begin{aligned} \mathcal{L}_{J_u} J_v &= L_{V_u} V_v + L_{V_u}\lambda_v + L_{V_u}\sigma_v + L_{V_u}\phi_v \\ &\quad + \iota_{V_v}\left(m\lambda_u - d\phi_u\right) - \iota_{V_v}\left(d\lambda_u\right) - \iota_{V_v}\left(d\sigma_u\right) \\ &\quad + \phi_v\left(d\lambda_u\right) + \lambda_v \wedge \left(m\lambda_u - d\phi_u\right)\,, \\ \mathcal{L}_{J_u}\hat{K} &= L_{V_u}\omega_{(0)} + L_{V_u}\omega_{(2)} + L_{V_u}\omega_{(3)} \\ &\quad - \omega_{(0)}\left(d\lambda_u\right) - \omega_{(0)}\left(d\sigma_u\right) - \omega_{(2)}\wedge\left(m\lambda_u - d\phi_u\right)\,, \\ d\hat{K} &= -d\omega_{(0)} + d\omega_{(2)}\,, \end{aligned} \tag{4.7}$$

where we have included the Roman's mass m as in [24,45]. As explained above, the $\wedge$-product and d operator on the left-hand side are those of ExFT, while the $\wedge$-product and d operator on the right-hand side are the usual ones acting on differential forms.

4.2. *Half-maximal structure*

Before continuing, we need to discuss the possible gauge potentials living in M_{int}. In IIA SUGRA, we need to consider a 2-form and 3-form field strength in M_{int} which must form SU(2)$_R$-symmetry singlets. The 2-form gauge potential can always be chosen to be an SU(2)$_R$-symmetry singlet. However, the 1-form gauge potential A will necessarily violate the SU(2)$_R$-symmetry. As we will use R-symmetry as a guiding principle, we will have to include the 1-form gauge potential A by hand as a "twist term", as e.g. in [19]. This implies that we take $\phi_u = \hat{\phi}_u + \iota_{V_u}A$ and $\sigma_u = \hat{\sigma}_u + \lambda_u \wedge A$ and $\omega_{(3)} = \hat{\omega}_{(3)} + \omega_{(2)}\wedge A$. On the other hand, our Ansatz below will naturally incorporate the 2-form potential.

The most general J_u we can construct that is compatible with the SU(2)$_R$ symmetry and that satisfies the algebraic conditions (2.1) is, up to generalised diffeomorphisms (i.e. gauge transformations and diffeomorphisms), given by

$$\begin{aligned} J_u &= \frac{2\sqrt{2}}{R}v_u + \frac{R}{4}\left(g(z)\,dy_u - \frac{h(z)}{q(z)}y_u\,dz\right) - \frac{R}{2}q(z)\,y_u \\ &\quad + \frac{R^3}{16\sqrt{2}}\left(q(z)\,g(z)\,y_u\,vol_{S^2} + h(z)\,\theta_u\wedge dz\right) + \frac{2\sqrt{2}}{R}\iota_{v_u}A \\ &\quad + \frac{R}{4}\left(g(z)\,dy_u - \frac{h(z)}{q(z)}y_u\,dz\right)\wedge A\,, \end{aligned} \tag{4.8}$$

where v_u are Killing vectors and θ_u the 1-forms (4.4) on S^2, and $g(z)$, $q(z)$, $h(z)$ are so far arbitrary functions of z. Furthermore, the most general $\hat{K}$ constructed from R-symmetry singlets is, up to generalised diffeomorphisms, given by

$$\begin{aligned} \hat{K} &= \frac{R}{2}s(z) + \frac{R^3}{16\sqrt{2}}\left(g(z)\,s(z) - t(z)\right)vol_{S^2} \\ &\quad + \frac{R^3}{16\sqrt{2}}\left(g(z)\,s(z) - t(z)\right)vol_{S^2}\wedge A\,. \end{aligned} \tag{4.9}$$

Allowing for the S^2 to shrink at the boundary of the interval parameterised by z, the algebraic conditions $J_u \wedge J^u \wedge \hat{K} > 0$ now becomes $h(z)\,t(z) \geq 0$, with equality at the boundary of I. Finally, as discussed R-symmetry implies that $dA = R^2 l(z)\,vol_2$, for some $l(z)$. With (4.8), the differential conditions (4.7) reduce to

$$
\begin{aligned}
& m\lambda_u + \iota_{V_u} dA - d\hat{\phi}_u = d\hat{\sigma}_u - \lambda_u \wedge dA = d\lambda_u = 0\,, \\
& d\omega_{(0)} = -\frac{2}{3R}\lambda_u \hat{\phi}^u\,, \\
& d\omega_{(2)} = -\frac{2}{3R}\left(\iota_{V_u}\hat{\sigma}^u + \lambda_u \wedge \hat{\sigma}^u\right)\,.
\end{aligned} \tag{4.10}
$$

We can always redefine the coordinate z to make $h(z)$ any functions we choose. A particular convenient choice is to take $h(z) = q(z)$, whereupon the differential conditions (4.10) become

$$
\begin{aligned}
& \dot{g} = -1\,, \qquad \dot{q} = \frac{m}{2}\,, \qquad \dot{s} = q\,, \qquad \dot{t} = -s\,, \\
& l(z) = -\frac{q}{4\sqrt{2}} - \frac{m\,g}{8\sqrt{2}}\,.
\end{aligned} \tag{4.11}
$$

Without loss of generality we can integrate $\dot{g} = -1$ to $g = -z$, absorbing any constant of integration by shifting z. Furthermore, we can express s and q in terms of derivatives of t which must satisfy

$$
\dddot{t} = -\frac{m}{2}\,, \qquad t \geq 0 \text{ with equality at } \partial I\,. \tag{4.12}
$$

Altogether the half-maximal structure then becomes

$$
\begin{aligned}
J_u = & \frac{2\sqrt{2}}{R} v_u - \frac{R}{4} d\left(y_u z\right) + \frac{R}{2}\ddot{t}\, y_u \\
& + \frac{R^3}{16\sqrt{2}}\ddot{t}\left(d\left(z\theta_u\right) - z\, y_u\, vol_2\right) \\
& + \frac{2\sqrt{2}}{R}\iota_{v_u} A - \frac{R}{4} d\left(y_u z\right) \wedge A\,, \\
\hat{K} = & -\frac{R}{2}\dot{t} + \frac{R^3}{16\sqrt{2}}\left(z\dot{t} - t\right) vol_{S^2} \\
& + \frac{R^3}{16\sqrt{2}}\left(z\dot{t} - t\right) vol_{S^2} \wedge A\,,
\end{aligned} \tag{4.13}
$$

determined entirely by $t(z)$ and where $dA = \frac{R^2}{4\sqrt{2}}\left(\ddot{t} + \frac{m}{2}z\right) vol_{S^2}$.

4.3. The AdS_7 vacua

The SUGRA fields with legs on $M_{int} = S^2 \times I$ can be read off from the generalised metric constructed from J_u and $\hat{K}$ in (4.13). For this we use the parameterisation of generalised metric by IIA SUGRA fields given in [21]. The warp factor the AdS_7 metric is given by [5,7]

$$
w_7 = |g|^{1/5} \kappa^{-2} e^{-4\psi/5}\,, \tag{4.14}
$$

where $|g|$ is the determinant of the internal space in string frame and ψ is the IIA dilaton. Thus, we find the infinite family of supersymmetric AdS_7 vacua determined by the function $t(z)$ satisfying (4.12).

$$
\begin{aligned}
ds^2_{10} = & R^2\sqrt{-\frac{t}{\ddot{t}}}\, ds^2_{AdS_7} \\
& + \frac{R^2}{8}\sqrt{-\frac{\ddot{t}}{t}}\left(\frac{t^2}{\dot{t}^2 - 2\ddot{t}\,t} ds^2_{S^2} + dz^2\right)\,, \\
e^{\psi} = & \frac{2}{R}\left(-\frac{t}{\ddot{t}}\right)^{3/4} \frac{1}{\sqrt{\dot{t}^2 - 2\ddot{t}\,t}}\,, \\
B_2 = & \frac{R^2}{8\sqrt{2}}\left(z - \frac{\dot{t}\,t}{\dot{t}^2 - 2\ddot{t}\,t}\right) vol_2\,, \\
F_2 = & \frac{R^2}{8\sqrt{2}}\left(2\ddot{t} + \frac{m\,\dot{t}\,t}{\dot{t}^2 - 2\ddot{t}\,t}\right) vol_2\,,
\end{aligned} \tag{4.15}
$$

where the metric is expressed in string frame and $F_2 = dA - m\,B_2$ is the RR 2-form field strength of mIIA SUGRA. This is the family of AdS_7 solutions found in [29] in the coordinate choice of [46], where our variables are related to theirs by the rescaling $t = \frac{4\sqrt{2}}{81}\alpha$, $z = 2\sqrt{2}\pi Z$.

5. AdS_6 vacua from IIB supergravity

We next consider supersymmetric AdS_6 in IIB SUGRA. We begin by rewriting the differential conditions (2.4) by introducing the $SO(4)_R$ vector Λ_u defined as $\Lambda_{uvw} = \frac{3}{2^{1/4}}\epsilon_{uvwx}\Lambda^x$. Then the differential conditions (2.4) become

$$
\begin{aligned}
& \mathcal{L}_{J_u} J_v = -\frac{3}{2^{1/4}}\epsilon_{uvwx} J^w \Lambda^x\,, \qquad \mathcal{L}_{J_u}\hat{K} = 0\,, \\
& d\hat{K} = 2^{3/4}\Lambda^u J_u\,.
\end{aligned} \tag{5.1}
$$

The $SO(4)_R$ vector Λ_u encodes the AdS_6 radius and hence cosmological constant as follows. We can use a $SO(4)_R$ rotation to write, without loss of generality $\Lambda_u = \left(0, 0, 0, R^{-1}\right)$, with R the AdS_6 radius. This breaks the $SO(4)_R$ to the $SO(3)_R$ R-symmetry of AdS_6 vacua. Let us therefore write $u = (A, 4)$ with $A = 1, 2, 3$ labelling the vector representation of $SO(3)_R$. With respect to $(A, 4)$ the differential conditions become

$$
\begin{aligned}
& \mathcal{L}_{J_A} J_B = -\frac{3}{2^{1/4} R}\epsilon_{ABC} J^C\,, \qquad \mathcal{L}_{J_A} J_4 = 0\,, \\
& \mathcal{L}_{J_A}\hat{K} = 0\,, \qquad d\hat{K} = \frac{2^{3/4}}{R} J_4\,.
\end{aligned} \tag{5.2}
$$

Note that the conditions $\mathcal{L}_{J_4} J_u = 0$ and $\mathcal{L}_{J_4}\hat{K} = 0$ are automatically satisfied by $J_4 \propto d\hat{K}$ [41,7].

As we discussed in section 2, the J_u are generalised Killing vector fields since $\mathcal{L}_{J_u}\mathcal{M}_{MN} = 0$. However, in contrast to the AdS_7 case, the equation $J_4 \propto d\hat{K}$ implies that J_4 is a trivial generalised Killing vector field, containing no spacetime vector field but only exact differential forms. It therefore generates trivial gauge transformations of the gauge potentials. On the other hand, the three generalised vector fields J_A necessarily contain non-vanishing spacetime vector field components, since $\mathcal{L}_{J_A} \neq 0$. These spacetime vector fields must be Killing vector fields that generate the $SU(2)_R$ algebra and hence are related to an S^2 geometry. Therefore, we will consider the internal space $M_{int} = S^2 \times \Sigma$, where Σ is a Riemann surface with coordinates x^α, $\alpha = 1, 2$. We will, in principle, allow for metrics on M_{int} with off-diagonal components between S^2 and Σ although we will see that supersymmetry forbids such components.

The $SL(2)_S$ of the IIB S-duality acts naturally on the Riemann surface and we will raise/lower all $SL(2)_S$ indices using the alternating symbols $\epsilon_{\alpha\beta} = \epsilon^{\alpha\beta} = \pm 1$ with $\epsilon^{\alpha\gamma}\epsilon_{\beta\gamma} = \delta^\alpha_\beta$, and following a Northwest–Southeast convention.

5.1. $SO(5,5)$ ExFT and IIB SUGRA

Supersymmetric AdS_6 vacua are characterised by four generalised vector fields $J_u \in \Gamma(\mathcal{R}_1)$ and a generalised tensor $\hat{K} \in \Gamma(\mathcal{R}_2)$. In IIB SUGRA these become formal sums of spacetime vector fields and differential forms as follows

$$
J_u = V_u + \lambda_u{}^\alpha + \sigma_u\,, \qquad \hat{K} = \omega^\alpha_{(0)} + \omega_{(2)} + \omega^\alpha_{(4)}\,, \tag{5.3}
$$

where V_u, $\lambda_u{}^\alpha$ and σ_u denote the vector, 1-form and 3-form parts of J_u, while $\omega_{(p)}$ are p-forms appearing in $\hat{K}$.

The wedge products and tensor products appearing in the algebraic conditions (2.1) are

$$
\begin{aligned}
J_u \wedge J_v &= \sqrt{2}\Big(\imath_{V_{(u}}\lambda^\alpha_{v)} + \lambda^\alpha_{(u} \wedge \sigma_{v)} \\
&\quad + \Big(-\imath_{V_{(u}}\sigma_{v)} - \frac{1}{2}\epsilon_{\alpha\beta}\lambda^\alpha_u \wedge \lambda^\beta_v\Big)\Big), \\
\hat{K} \otimes \hat{K}|_{R_c} &= \omega_{(2)} \wedge \omega_{(2)} + 2\,\epsilon_{\alpha\beta}\,\omega^\alpha_{(0)}\,\omega^\beta_{(4)}, \\
\hat{K} \wedge K &= \omega_{(2)} \wedge \bar{\omega}_{(2)} + \epsilon_{\alpha\beta}\,\omega^\alpha_{(0)}\,\bar{\omega}^\beta_{(4)} + \epsilon_{\alpha\beta}\,\bar{\omega}^\alpha_{(0)}\,\omega^\beta_{(4)},
\end{aligned} \tag{5.4}
$$

where we defined $K = \frac{1}{4} J_u \wedge J^u = \bar{\omega}^\alpha_{(0)} + \bar{\omega}_{(2)} + \bar{\omega}^\alpha_{(4)}$. Moreover, the differential operators appearing in the differential conditions (5.1) become

$$
\begin{aligned}
\mathcal{L}_{J_u} J_v &= L_{V_u} V_v + L_{V_u}\sigma_v + L_{V_u}\lambda_v{}^\alpha \\
&\quad - \imath_{V_v} d\lambda^\alpha_u - \imath_{V_v} d\sigma_u - \epsilon_{\alpha\beta}\,\lambda^\alpha_v \wedge d\lambda^\beta_u, \\
\mathcal{L}_{J_u}\hat{K} &= L_{V_u}\omega^\alpha_{(0)} + L_{V_u}\omega_{(2)} + L_{V_u}\omega^\alpha_{(4)} \\
&\quad + \epsilon_{\alpha\beta}\,\omega^\alpha_{(0)}\,d\lambda^\beta_u - \omega^\alpha_{(0)}\,d\sigma_u - \omega_{(2)} \wedge d\lambda^\alpha_u, \\
d\hat{K} &= -\sqrt{2}\,d\omega_{(2)} + \sqrt{2}\,d\omega^\alpha_{(0)}.
\end{aligned} \tag{5.5}
$$

5.2. Half-maximal structure

In contrast to AdS_7 vacua of mIIA, the gauge potentials of IIB SUGRA on $M_{int} = S^2 \times \Sigma$ can always be chosen to respect the $SU(2)_R$ symmetry. Therefore, they will automatically be included in the half-maximal structures we construct here.

The most general J_A we can construct from $SU(2)_R$-triplets that satisfies the algebraic conditions (2.1) is, up to generalised diffeomorphisms,

$$
\begin{aligned}
J_A &= \frac{1}{2^{1/4}}\Big(\frac{3}{R}\,v_A + 4\,R\left(y_A m^\alpha + k^\alpha dy_A\right) \\
&\quad + \frac{16\,R^3}{3}\left(|m|\,\theta_A \wedge vol_\Sigma - y_A k^\beta m_\beta \wedge vol_{S^2}\right)\Big),
\end{aligned} \tag{5.6}
$$

where v_A are Killing vectors and θ_A the 1-forms (4.4) on S^2, $m^\alpha = m^\alpha{}_\beta dx^\beta$ are 1-forms on Σ, where $m^\alpha{}_\beta$ depends only on the coordinates of Σ, k^α are functions on Σ, and we defined $|m| = \frac{1}{2} m_{\alpha\beta} m^{\alpha\beta}$.

Next, we construct $\hat{K}$ such that is an $SU(2)_R$-invariant and satisfies $\hat{K} \otimes \hat{K}|_{R_c} = 0$ and $J_A \wedge J^A \wedge \hat{K} > 0$. We find the unique combination, up to generalised diffeomorphisms,

$$
\hat{K} = 4\,p_\alpha - \frac{16\,R^2}{3}\left(r + p_\beta k^\beta\right) vol_2\,, \tag{5.7}
$$

and hence J_4

$$
J_4 = \frac{R}{2^{3/4}} d\hat{K} = \frac{1}{2^{1/4}}\left(4\,R\,dp^\alpha - \frac{16\,R^3}{3} d\left(r + p_\beta k^\beta\right) \wedge vol_{S^2}\right). \tag{5.8}
$$

The algebraic condition $J_A \wedge J^A \wedge \hat{K} > 0$ is satisfied iff $r\,|m| \geq 0$ with equality at the boundary of Σ, while the algebraic conditions for J_4 impose

$$
m_\alpha \wedge dp^\alpha = 0\,, \qquad m^\alpha \wedge m^\beta = dp^\alpha \wedge dp^\beta\,, \qquad dr + p_\alpha dk^\alpha = 0\,. \tag{5.9}
$$

Note that the final condition can be used to simplify the expression of J_4

$$
J_4 = \frac{1}{2^{1/4}}\left(4\,R\,dp^\alpha - \frac{16\,R^3}{3} k_\beta\,dp^\beta \wedge vol_{S^2}\right). \tag{5.10}
$$

Finally, we are left to solve the differential conditions (5.5). Here these simplify to $d\lambda^\alpha_A = d\sigma_A = 0$, which implies $m^\alpha = -dk^\alpha$.

Thus, we find that

$$
\begin{aligned}
J_A &= \frac{1}{2^{1/4}}\left(\frac{3}{R} v_A + 4\,R\,d\left(k^\alpha\,y_A\right) + \frac{8\,R^3}{3}\rho\,d\left(k^\alpha \theta_A \wedge dk_\alpha\right)\right), \\
J_4 &= \frac{1}{2^{1/4}}\left(4\,R\,dp^\alpha - \frac{16\,R^3}{3} k_\beta\,dp^\beta \wedge vol_{S^2}\right), \\
\hat{K} &= 4\,p_\alpha - \frac{16\,R^2}{3}\left(r + p_\beta k^\beta\right) vol_{S^2}\,,
\end{aligned} \tag{5.11}
$$

determined entirely by the two SL(2)-doublets of real functions k^α and p^α on Σ, which satisfy the differential conditions

$$
dk^\alpha \wedge dk^\beta = dp^\alpha \wedge dp^\beta\,, \qquad dk^\alpha \wedge dp_\alpha = 0\,, \tag{5.12}
$$

and positivity condition

$$
r|dk| \geq 0 \text{ with equality at } \partial\Sigma\,, \tag{5.13}
$$

where $|dk| = \partial_\alpha k_\beta \partial^\alpha k^\beta$, and r is defined up to an integration constant by $dr = -p_\alpha\,dk^\alpha$.

At this stage, one might wonder how the quadratic differential conditions (5.12) can underly supersymmetric AdS vacua, which ought to be described by a first-order BPS equation. The answer is that we still have residual diffeomorphism symmetry on the Riemann surface Σ that can be used to turn (5.12) into first-order differential equations. We will show how to do this after calculating the supergravity fields from the structures.

5.3. The AdS_6 vacua

We will now compute the supergravity background corresponding to the half-maximal structures (5.11). The supergravity fields are encoded in the generalised metric (3.5), (3.7). Moreover, the 6-D metric is warped by the factor $w_6 = |g|^{-1/4}\kappa^2$, [7] where $|g|$ is the determinant of the internal four-dimensional space. From this, we find the following background in Einstein frame

$$
\begin{aligned}
ds^2 &= \frac{\sqrt{2}\,r^{5/4}\,\Delta^{1/4}\,R^2}{3^{3/4}|dk|^{1/2}}\left[\frac{12}{r}ds^2_{AdS_6} + \frac{|dk|^2}{\Delta}ds^2_{S^2} + \frac{4}{r^2}dk^\alpha \otimes dp_\alpha\right], \\
C_{(2)}{}^\alpha &= -\frac{4\,R^2}{3}\,vol_{S^2}\left(k^\alpha + \frac{r\,p_\gamma\,\partial^\beta k^\gamma\,\partial_\beta p^\alpha}{2\Delta}|dk|\right), \\
C_{(4)} &= 0\,, \\
H_{\alpha\beta} &= \frac{1}{2\sqrt{3}\,\Delta}\left(\frac{|dk|}{\sqrt{r}}\,p_\alpha p_\beta + 6\sqrt{r}\,\partial_\gamma k_\alpha \partial^\gamma p_\beta\right),
\end{aligned} \tag{5.14}
$$

where

$$
\Delta = \frac{3}{4} r\,|dk|^2 + \frac{1}{2}|dk|\,p_\gamma\,p_\delta\,\partial_\sigma k^\gamma\,\partial^\sigma\,p^\delta\,, \qquad |dk| = \partial_\alpha k_\beta \partial^\alpha k^\beta\,, \tag{5.15}
$$

and $H_{\alpha\beta}$ is the SL(2) matrix parameterised by the axio-dilaton $\tau = e^\psi + i\,C_0$ as

$$
H_{\alpha\beta} = \frac{1}{\operatorname{Im}\tau}\begin{pmatrix} |\tau|^2 & \operatorname{Re}\tau \\ \operatorname{Re}\tau & 1 \end{pmatrix}. \tag{5.16}
$$

The solutions are completely determined by p^α and k^α, which are any pair of real SL(2)-doublet functions on Σ satisfying (5.12) and (5.13) with r defined by $dr = -p_\alpha dk^\alpha$.

As we mentioned previously, we can use diffeomorphisms to turn the differential equations for k^α and p^α into first-order PDEs. In particular, we can always use diffeomorphisms to make the metric on Σ conformally flat. From (5.14) we see that this would impose

$$\partial_1 k^\alpha \partial_1 p_\alpha = \partial_2 k^\alpha \partial_2 p_\alpha \,, \qquad \partial_1 k^\alpha \partial_2 p_\alpha = 0 \,. \tag{5.17}$$

Together with (5.12), and requiring (5.13) the differential conditions become the Cauchy–Riemann equations

$$dk^\alpha = I \cdot dp^\alpha \,, \tag{5.18}$$

where $I_\alpha^\beta = \delta_{\alpha\gamma}\epsilon^{\gamma\beta}$ is a complex structure on Σ. This implies that p^α and k^α are the real and imaginary parts of two holomorphic functions $f^\alpha = -p^\alpha + i k^\alpha$ on Σ. We now immediately see that our solutions match those of [32] upon identifying our holomorphic functions with the $\mathcal{A}_\pm$ of [32] as follows

$$\mathcal{A}_\pm = i f^1 \pm f^2 \,, \tag{5.19}$$

whereupon our R^2 is identified with the coefficient c of [32]. These solutions can be extended to globally regular solutions by including a boundary of the Riemann surface on which the holomorphic functions f^α have poles as discussed in [33,34].

6. Minimal consistent truncations

As shown in [7], we immediately obtain the minimal consistent truncation around the supersymmetric AdS vacua we constructed here. The truncation Ansatz for the scalar fields is given in (2.5), while that for the remaining fields can be found in [7].

6.1. AdS_7

By computing the generalised metric corresponding to the $\mathcal{J}_u(x,Y)$ and $\hat{\mathcal{K}}(x,Y)$ in (2.5) we find the truncation Ansatz for the IIA SUGRA fields in string frame

$$\begin{aligned}
ds^2_{10} &= \frac{R^2}{4}\sqrt{-\frac{t}{\ddot t}}X^{1/2}ds^2_7 \\
&\quad + \frac{R^2}{4}\sqrt{-\frac{\ddot t}{t}}\left[X^{-5/2}dz^2 + X^{5/2}\frac{t^2}{\dot t^2 X^5 - 2t\ddot t}ds^2_{S^2}\right], \\
e^\psi &= \frac{2}{R}X^{5/4}\left(-\frac{t}{\ddot t}\right)^{3/4}\frac{1}{\sqrt{X^5\dot t^2 - 2t\ddot t}}, \\
H_3 &= -\frac{R^2}{4}X^{-5/4}\ddot t\left(-\frac{\ddot t}{t}\right)^{1/4}\left[3 - \frac{t}{\ddot t}\frac{m\dot t}{\dot t^2X^5 - 2t\ddot t}\right]vol_{\tilde M_3} \\
&\quad - \frac{R^2}{4}X^{-5/4}\ddot t\left(-\frac{\ddot t}{t}\right)^{1/4}\left(1 - X^5\right) \\
&\quad \times\left[1 + \frac{4t\ddot t}{\dot t^2X^5 - 2t\ddot t} + \frac{t}{\ddot t}\frac{m\dot t}{\dot t^2X^5 - 2t\ddot t}\right]vol_{\tilde M_3}, \\
F_2 &= \frac{R^2}{8\sqrt{2}}\left(2\ddot t + X^5\frac{m\dot t t}{\dot t^2X^5 - 2t\ddot t}\right)vol_2,
\end{aligned} \tag{6.1}$$

with the 2-form potential

$$B_2 = \frac{R^2}{8\sqrt{2}}\left(z - \frac{\dot t t X^5}{\dot t^2X^5 - 2t\ddot t}\right)vol_2 \,, \tag{6.2}$$

and where $vol_{\tilde M_3}$ is the volume form on the internal space of (6.1). The truncation Ansatz is completely determined by the functions $t(z)$ satisfying (4.12), and corresponds to the truncation Ansatz found in [31] in the coordinates of [46]. Upon truncation, X becomes the scalar field of the minimal 7-dimensional gauged SUGRA [47].

6.2. AdS_6

We can similarly use (2.5) to find the minimal consistent truncation corresponding to the supersymmetric AdS_6 vacua of IIB SUGRA we described here and which were previously constructed in [32]. We find in Einstein frame

$$\begin{aligned}
ds^2 &= \frac{\sqrt{2}r^{5/4}\bar\Delta^{1/4}R^2}{3^{3/4}|dk|^{1/2}}\left[\frac{12}{r}ds^2_6 + \frac{X^2|dk|^2}{\bar\Delta}ds^2_{S^2}\right. \\
&\quad \left. + \frac{4}{X^2r^2}dk^\alpha \otimes dp_\alpha\right], \\
C_{(2)}{}^\alpha &= -\frac{4R^2}{3}vol_{S^2}\left(k^\alpha + \frac{X^4 r p_\gamma \partial^\beta k^\gamma \partial_\beta p^\alpha}{2\bar\Delta}|dk|\right), \\
C_{(4)} &= 0, \\
H_{\alpha\beta} &= \frac{1}{2\sqrt{3}\bar\Delta}\left(\frac{X^4|dk|}{\sqrt{r}}p_\alpha p_\beta + 6\sqrt{r}\partial_\gamma k_\alpha \partial^\gamma p_\beta\right),
\end{aligned} \tag{6.3}$$

where

$$\bar\Delta = \frac{3}{4}r|dk|^2 + \frac{1}{2}X^4|dk|p_\gamma p_\delta \partial_\sigma k^\gamma \partial^\sigma p^\delta \,, \tag{6.4}$$

and k^α, p^α satisfy (5.12), (5.13). Upon truncation, X becomes the scalar field of the minimal 6-dimensional gauged SUGRA, the so-called $F(4)$ gauged SUGRA [48]. All these AdS vacua correspond to the same vacuum of the 6-dimensional gauged SUGRA. Our truncation Ansatz includes the previously-found consistent truncation of a particular AdS_6 vacuum in this family [49] as a particular example.

Acknowledgements

The authors thank Alex Arvanitakis, Davide Cassani, Yolanda Lozano, Hagen Triendl, Alessandro Tomasiello, Christoph Uhlemann, Oscar Varela and Daniel Waldram for helpful discussions. EM would also like to thank the organisers of the "2018 USU Workshop on Strings and Black Holes" for hospitality while part of this work was completed. EM is supported by the ERC Advanced Grant "Strings and Gravity" (Grant No. 320045).

Appendix A. Conventions of half-maximal structure

Here we summarise the algebraic and differential conditions for the half-maximal AdS vacua and give their explicit expressions in terms of $E_{d(d)}$ invariants.

The $d-1$ generalised vector fields J_u and the generalised tensor $\hat K$ together are stabilised by an $SO(d-1) \subset E_{d(d)}$ structure group, if and only if, they satisfy the algebraic conditions

$$\begin{aligned}
&J_u \wedge J_v - \frac{1}{d-1}\delta_{uv}J_w \wedge J^w = 0\,, \qquad J_u \wedge J^w \wedge \hat K > 0\,, \\
&\hat K \otimes \hat K|_{R_c} = 0\,.
\end{aligned} \tag{A.1}$$

The differential conditions are

$$\begin{aligned}
&\mathcal{L}_{J_u}J_v = -\Lambda_{uvw}J^w\,, \\
&\mathcal{L}_{J_u}\hat K = 0\,, \\
&d\hat K = \begin{cases} \frac{1}{3!\sqrt{2}}\epsilon^{uvw}\Lambda_{uvw}K\,, & \text{when } D = 7\,, \\ \frac{1}{9}\epsilon_{uvwx}\Lambda^{uvw}J^x\,, & \text{when } D = 6\,. \end{cases}
\end{aligned} \tag{A.2}$$

These conditions can be derived by writing J_u and $\hat{K}$ as spinor bilinears and then rewriting the supersymmetry variations of ExFT in terms of J_u and $\hat{K}$. Alternatively, one can use the results of half-maximal gSUGRA [50,51] to read off the embedding tensor that would be obtained after a consistent truncation. This can then be mapped into the differential equations for J_u and $\hat{K}$. Finally, the Λ_{uvw} corresponds to a non-zero singlet in the intrinsic torsion of the Spin$(d-1)$ structure, as further explained in [7], similar to the BPS equations for other amounts of supersymmetry [11,14,15].

A.1. SL(5) *conventions*

For SL(5) ExFT we use the conventions where $a, b = 1, \ldots, 5$ are fundamental SL(5) indices. Then following [7], the algebraic conditions (2.1) become

$$\begin{aligned} \frac{1}{4}\epsilon_{abcde}\left(J_u{}^{ab}J_v{}^{cd} - \frac{1}{3}\delta_{uv}J_w{}^{ab}J^{w\,cd}\right) &= 0\,, \\ \frac{1}{4}\epsilon_{abcde}J_u{}^{ab}J_v{}^{cd}\hat{K}^e &> 0\,. \end{aligned} \tag{A.3}$$

The differential conditions (2.4) involve the generalised Lie derivative and nilpotent derivative d whose explicit expressions are

$$\begin{aligned} \mathcal{L}_{J_u}J_v{}^{ab} &= \frac{1}{2}J_u{}^{cd}\partial_{cd}J_u{}^{ab} - 2J_v{}^{c[b}\partial_{cd}J_u{}^{a]d} + \frac{1}{2}J_v{}^{ab}\partial_{cd}J_u{}^{cd} \\ &= -\Lambda_{uvw}J^{w\,ab}\,, \\ \mathcal{L}_{J_u}\hat{K}^a &= \frac{1}{2}J_u{}^{bc}\partial_{bc}\hat{K}^a - \hat{K}^b\partial_{bc}J_u{}^{ac} + \frac{1}{2}\hat{K}^a\partial_{bc}J_u{}^{bc} = 0\,, \\ d\hat{K}_a &= \partial_{ba}\hat{K}^b = \frac{1}{3!\sqrt{2}}\epsilon^{uvw}\Lambda_{uvw}K_a\,. \end{aligned} \tag{A.4}$$

A.2. SO(5, 5) *conventions*

For SO(5, 5) ExFT we use the indices $M = 1, \ldots, 16$ and $I = 1, \ldots, 10$ for the spinor and fundamental representations of SO(5, 5), respectively. We raise/lower the fundamental indices by the constant SO(5, 5)-invariant metric η_{IJ}. Using the conventions of [7], the algebraic conditions (2.1) become

$$\begin{aligned} \frac{1}{2}\left(\gamma^I\right)_{MN}\left(J_u{}^MJ_v{}^N - \frac{1}{4}\delta_{uv}J_w{}^MJ^{wN}\right) &= 0\,, \\ \hat{K}_I\hat{K}^I &= 0\,, \\ \frac{1}{2}\left(\gamma^I\right)_{MN}J_u{}^MJ^{u\,N}\hat{K}_I &> 0\,. \end{aligned} \tag{A.5}$$

The differential conditions (2.4) involve the generalised Lie derivative and the nilpotent derivative d whose explicit expressions are

$$\begin{aligned} \mathcal{L}_{J_u}J_v{}^M &= J_v^N\partial_NJ_u^M - J_u^N\partial_NJ_v^M + \frac{1}{2}(\gamma_I)^{MN}\left(\gamma^I\right)_{PQ}J_u^P\partial_NJ_v^Q \\ &= -\Lambda_{uvw}J^{w\,M}\,, \\ \mathcal{L}_{J_u}\hat{K}^I &= J_u{}^M\partial_M\hat{K}^I + \frac{1}{2}\left(\gamma_J\gamma^I\right)_M{}^N\hat{K}^J\partial_NJ_u{}^M = 0\,, \\ d\hat{K}^M &= (\gamma_I)^{MN}\partial_N\hat{K}^I = \frac{1}{9}\epsilon_{uvwx}\Lambda^{uvw}J^x\,. \end{aligned} \tag{A.6}$$

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