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# Ten-dimensional origin of Minkowski vacua in $\mathcal{N}=8$ supergravity

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ABSTRACT

Maximal supergravity in four dimensions admits two inequivalent dyonic gaugings of the group SO(4)×SO(2,2)⋉$T^{16}$. Both admit a Minkowski vacuum with residual SO(4)×SO(2)$^2$ symmetry and identical spectrum. We explore these vacua and their deformations. Using exceptional field theory, we show that the four-dimensional theories arise as consistent truncations from IIA and IIB supergravity, respectively, around a $\text{Mink}_4 \times S^3 \times H^3$ geometry. The IIA/IIB truncations are efficiently related by an outer automorphism of SL(4) ⊂ $E_{7(7)}$. As an application, we give an explicit uplift of the moduli of the vacua into a 4-parameter family of ten-dimensional solutions.



## 1. Introduction: $D=4$ Minkowski vacua from ten dimensions

Maximal $\mathcal{N}=8$ gauged supergravity in four dimensions allows for a number of Minkowski vacua with various gauge groups and different degrees of supersymmetry, many of which have only been revealed and studied in recent years [1–5]. Their existence is often based on symplectic deformations of maximal supergravity [6,7] whose higher-dimensional origin in turn remains largely mysterious.

In an a priori unrelated development, new efficient tools for the higher-dimensional uplift of four-dimensional solutions and theories have emerged from the duality covariant reformulations of the higher-dimensional supergravity theories. In this framework, non-toroidal compactifications of supergravity are realized as generalized Scherk–Schwarz reductions on extended spacetimes [8–14]. In [15], these techniques were used to prove a conjecture from [19] that the NS–NS sector of ten-dimensional supergravity admits a consistent truncation based on a group manifold $G$ to a half-maximal supergravity retaining non-abelian gauge bosons associated with the full isometry group $G\times G$. The scalar fields of the lower-dimensional theory parametrize the coset space $\mathbb{R}^+ \times \text{SO}(d,d)/(\text{SO}(d)\times\text{SO}(d))$ with $d=\dim G$ and couple via the scalar potential

$$V = \frac{1}{12}\, e^{2\varphi(x)} X_{\mathcal{MN}}{}^{\mathcal{K}} X_{\mathcal{PQ}}{}^{\mathcal{R}} M^{\mathcal{MP}}(x) \times \left( M^{\mathcal{NQ}}(x) M_{\mathcal{KR}}(x) + 3\,\delta^{\mathcal{Q}}_{\mathcal{K}}\delta^{\mathcal{N}}_{\mathcal{R}} \right) . \tag{1}$$

Here, $M_{\mathcal{MN}}(x)$ is the SO$(d,d)$ valued matrix parametrizing the scalar target space, $\varphi(x)$ is the dilaton field, and the generalized structure constants $X_{\mathcal{MN}}{}^{\mathcal{K}}$ encode the structure constants $f_{kmn}$ of the group $G$, see (6) below.

It has further been observed in [15] that for non-compact groups $G$ the potential (1) admits a Minkowski vacuum if the number of compact and non-compact generators of $G$ are related by $n_{\rm cp} = 2\,n_{\rm non\text{-}cp}$. An interesting example of such a group which we shall further study in this paper is provided by

$$G = \text{SO}^*(4) \;\equiv\; \text{SO}(3)\times\text{SO}(2,1)\,, \tag{2}$$

which gives rise to a four-dimensional $\mathcal{N}=4$ supergravity with gauge group

$$\text{G}_{\rm gauge} = G\times G \;=\; \text{SO}(4)\times\text{SO}(2,2)\,, \tag{3}$$

embedded into the isometry group of the scalar target space $\text{SO}(6,6)/(\text{SO}(6)\times\text{SO}(6))$.

The associated Minkowski vacuum of the scalar potential (1) corresponds to a ten-dimensional solution of the type

$$\text{Mink}_4 \times S^3 \times H^3\,, \tag{4}$$

of a warped product of four-dimensional Minkowski space, a compact three-sphere, and the non-compact hyperboloid $H^3$ with

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isometry group SO(2, 2). At the vacuum, the gauge group (2) is broken down to its compact part, SO(4) × SO(2) × SO(2), and supersymmetry is completely broken. Yet the vacuum is classically stable at the quadratic level [3].

The aim of this letter is to further explore the Minkowski vacuum (4) and its deformations. We construct its embedding into $\mathcal{N}=8$ supergravity, i.e. into consistent truncations of IIA and IIB supergravity to inequivalent $\mathcal{N}=8$ gauged supergravities, both gauging the same non-semisimple group

$$\mathrm{SO}(4)\times\mathrm{SO}(2,2)\ltimes T^{16}\,, \tag{5}$$

extending (3). To show the inequivalence of the gaugings, we work out and compare their scalar potentials in a 10-scalar truncation. We construct the twist matrices that allow an explicit uplift of the four-dimensional theories into IIA and IIB supergravity, respectively, via a generalized Scherk–Schwarz reduction. Exceptional field theory is particularly useful for this because it captures both IIA and IIB supergravity in one formalism [16–18]. As a further application, we give an explicit uplift of the moduli of this vacuum into a 4-parameter family of ten-dimensional solutions. These deform the background geometry (4) such that only a $\mathrm{U}(1)^4$ subgroup of its isometries is preserved.

The rest of the letter is organized as follows. In section 2 we describe the inequivalent embeddings of the half-maximal supergravity with gauge group (3) into $\mathcal{N}=8$ supergravity. In section 3 we construct the twist matrices that describe the uplift into IIA and IIB supergravity via generalized Scherk–Schwarz reduction of exceptional field theory. We illustrate the inequivalence of the two resulting four-dimensional theories by comparing their potentials in a 10-scalar truncation in section 4. Finally, in section 5 we give an explicit uplift of the moduli of the Minkowski vacuum into a 4-parameter solution of $D=10$ supergravity.

## 2. Embedding into maximal supergravity

In this section we discuss the embedding of the $D=4$, $\mathcal{N}=4$ gauged supergravity with gauge group (3) obtained from compactification on (4) into $\mathcal{N}=8$ gauged supergravities describing consistent truncations of maximal IIA and IIB supergravity, respectively. We first discuss this embedding on the level of the four-dimensional supergravities in terms of the embedding tensor, enhancing the gauge group (3) to (5). The latter gaugings have been found and studied in [3–6]. We then review the consistent truncation of $D=10$, $\mathcal{N}=1$ supergravity around the solution (4) by virtue of a generalized Scherk–Schwarz reduction encoded in a properly chosen SO(6, 6) twist matrix $U$. Upon embedding of this twist matrix into $\mathrm{E}_{7(7)}$ we arrive at consistent truncations of IIA and IIB supergravity to the $\mathcal{N}=8$ gauged supergravities.

### 2.1. Embedding $\mathcal{N}=4$ into $\mathcal{N}=8$ supergravity

The scalar potential (1) appears in a gauging of $D=4$, $\mathcal{N}=4$ supergravity [20,21] whose generalized structure constants $X_{\mathcal{M}\mathcal{N}}{}^{\mathcal{K}}$ are given in terms of the structure constants $f_{kmn}$ of the group $G=\mathrm{SO}^*(4)$ as

$$X_{\mathcal{M}\mathcal{N}}{}^{\mathcal{K}}:\; X_{kmn}=f_{kmn}\,,\quad X_k{}^{mn}=f_k{}^{mn}\,,\quad X^k{}_m{}^n=f^k{}_m{}^n\,,$$
$$X^{mn}{}_k=f^{mn}{}_k\,, \tag{6}$$

where SO(6, 6) indices $\mathcal{M},\mathcal{N}=1,\dots,12$, are decomposed as $\{V^{\mathcal{M}}\}\to\{V^m,V_m\}$ and raised/lowered with the SO(6, 6) invariant $\eta_{\mathcal{M}\mathcal{N}}$, and adjoint algebra indices $m,n=1,\dots,6$, are raised and lowered with the Cartan–Killing form $\kappa_{mn}$.

To describe the embedding of this half-maximal into maximal supergravity, we consider the decomposition of the symmetry group of ungauged $\mathcal{N}=8$ supergravity

$$\mathrm{E}_{7(7)}\longrightarrow\mathrm{SO}(6,6)\times\mathrm{SL}(2)\,, \tag{7}$$

such that vector fields and the adjoint representation decompose as

$$\mathbf{56}\longrightarrow(\mathbf{12},\mathbf{2})+(\mathbf{32_s},\mathbf{1})\,,$$
$$\mathbf{133}\longrightarrow(\mathbf{66},\mathbf{1})+(\mathbf{1},\mathbf{3})+(\mathbf{32_c},\mathbf{2})\,, \tag{8}$$

respectively, i.e. for $M=1,\dots,56$ and $\Sigma=1,\dots,133$,

$$A_\mu{}^M\longrightarrow\{A_\mu{}^{\mathcal{M}\alpha},A_\mu{}^{\mathcal{A}}\}\,,\qquad \mathcal{M}=1,\dots,12\,,\;\alpha=\pm\,,$$
$$T_\Sigma\longrightarrow\{t^{\mathcal{M}\mathcal{N}},t^{(\alpha\beta)},t^{\dot{\mathcal{A}}\alpha}\}\,,\qquad \mathcal{A},\dot{\mathcal{A}}=1,\dots,32\,. \tag{9}$$

The gauge couplings in the maximal theory are described by an embedding tensor $\Theta_M{}^\Sigma$ in the **912** of $\mathrm{E}_{7(7)}$ [22]

$$D_\mu=\partial_\mu+A_\mu{}^M\,\Theta_M{}^\Sigma\,T_\Sigma\,. \tag{10}$$

In our case, $\Theta_M{}^\Sigma$ is induced by the embedding tensor $X_{\mathcal{M}\mathcal{N}\mathcal{K}}$, (6), of the half-maximal $\mathcal{N}=4$ theory, living in the $(\mathbf{220},\mathbf{2})$ of SO(6, 6) × SL(2), see [23] for a detailed discussion of such embeddings. In particular, (6) satisfies the additional quadratic constraints [24,23]

$$X_{\mathcal{M}\mathcal{N}\mathcal{K}}\,X^{\mathcal{M}\mathcal{N}\mathcal{K}}=0\,, \tag{11}$$

required for an embedding into a maximal theory.[1] In the maximal theory, this induces gauge couplings

$$A_\mu{}^M\,\Theta_M{}^\Sigma\,T_\Sigma=A_\mu{}^{\mathcal{K}+}X_{\mathcal{K}\mathcal{M}\mathcal{N}}\,t^{\mathcal{M}\mathcal{N}}+A_\mu{}^{\mathcal{A}}X_{\mathcal{K}\mathcal{M}\mathcal{N}}\,\Gamma^{\mathcal{K}\mathcal{M}\mathcal{N}}_{\mathcal{A}\dot{\mathcal{A}}}\,t^{\dot{\mathcal{A}}+}\,. \tag{12}$$

The first term describes the gauging of the $\mathfrak{so}(4)\oplus\mathfrak{so}(2,2)$ generators within the algebra $\mathfrak{so}(6,6)=\langle t^{\mathcal{M}\mathcal{N}}\rangle$, i.e. reproduces the gauge group (3) of the $\mathcal{N}=4$ theory. The second term, which carries the SO(6, 6) gamma-matrices $\Gamma^{\mathcal{K}\mathcal{M}\mathcal{N}}_{\mathcal{A}\dot{\mathcal{A}}}$ describes the new generators that are gauged in the maximal theory. In our case, it is straightforward to see, that these correspond to 16 commuting generators of $\mathrm{E}_{7(7)}$ that transform as a bi-fundamental vector under the semi-simple part (3) of the gauge group. The full gauge group within the maximal theory then is given by

$$\mathrm{G}_{\text{gauge}}=(\mathrm{SO}(4)\times\mathrm{SO}(2,2))\ltimes T^{16}\,. \tag{13}$$

A closer analysis of the gauge couplings (12) shows that the gauging of the 16 nilpotent generators can be realized in two different ways depending on if the higher-dimensional origin corresponds to the IIA or the IIB theory. While the embedding of SO(6, 6) into $\mathrm{E}_{7(7)}$ according to (7) is unique, the subgroup GL(6) ⊂ SO(6, 6) can be embedded in two inequivalent ways, related by an exchange of the SO(6, 6) spinor representations, and corresponding to a IIA or IIB origin. Accordingly, there are two ways of embedding the $\mathcal{N}=4$ theory into an $\mathcal{N}=8$ gauging with gauge group (13). Specifically, the additional 32 vector fields in (9) transforming in the spinor representation of SO(6, 6) decompose as

$$\mathrm{IIA}:\left\{A_\mu{}^0\,,\;A_\mu{}^{mn}\,,\;A_{\mu\,mn}\,,\;A_{\mu\,0}\right\}\,,$$
$$\mathrm{IIB}:\left\{A_\mu{}^m\,,\;A_\mu{}^{kmn}\,,\;A_{\mu m}\right\}\,, \tag{14}$$

[1] This is a general property of $\mathcal{N}=4$ gaugings that descend from Scherk–Schwarz reductions respecting the section constraints [10].

under GL(6), with $m, n = 1, \ldots, 6$. In terms of the structure constants (6), the couplings (12) of these fields organize according to

$$\begin{aligned}
\text{IIA} \; : \; & A_\mu{}^{\mathcal{A}} X_{\mathcal{KMN}} \Gamma^{\mathcal{KMN}}_{\mathcal{A}\dot{\mathcal{A}}} t^{\dot{\mathcal{A}}+} \\
&= A_{\mu\,0}\, f_{kmn}\, t^{kmn} \\
&\quad + A_{\mu\,kl} \left( f_{mnp}\, \varepsilon^{klmnpq}\, t_q + f^{kl}{}_m\, t^m \right) \\
&\quad + A_\mu{}^{mn} f^{kl}{}_p\, \varepsilon_{klmnrs}\, t^{prs} \\
&\equiv A_{\mu\,0}\, \mathcal{X}^0 + A_{\mu\,kl}\, \mathcal{X}^{kl} + A_\mu{}^{mn}\, \mathcal{X}_{mn} , \\
\text{IIB} \; : \; & A_\mu{}^{\mathcal{A}} X_{\mathcal{KMN}} \Gamma^{\mathcal{KMN}}_{\mathcal{A}\dot{\mathcal{A}}} t^{\dot{\mathcal{A}}+} \\
&= A_\mu{}^{kmn} \left( f_{kmn}\, t_0 + f^{pq}{}_k\, \varepsilon_{pqrsmn}\, t^{rs} \right) \\
&\quad + A_{\mu q}\, f_{mnp}\, \varepsilon^{klmnpq}\, t_{kl} + A_\mu{}^m f^{kl}{}_m\, t_{kl} \\
&\equiv A_\mu{}^{kmn}\, \mathcal{X}_{kmn} + A_{\mu q}\, \mathcal{X}^q + A_\mu{}^m \mathcal{X}_m ,
\end{aligned} \tag{15}$$

where the generators $t^{\dot{\mathcal{A}}+}$ decompose as (14) (with IIA and IIB interchanged).

Although both expression seem to formally involve more than 16 vector fields and generators, both, the IIA and the IIB connection can be shown to contain precisely 16 independent vector fields. For example, the generators $\mathcal{X}^{kl}$ and $\mathcal{X}_{mn}$, etc., contracting the vector fields, are not independent, but constrained by

$$\begin{aligned}
&\mathcal{X}_{mn}\, \mathcal{X}^{mn} = 0 , \\
&\varepsilon^{klmnpq}\, \mathcal{X}_{klm}\, \mathcal{X}_{npq} = 0 = \mathcal{X}^m\, \mathcal{X}_m ,
\end{aligned} \tag{16}$$

as follows from the Jacobi identities of the structure constants $f_{mn}{}^k$. As a result, for both cases in (15), the resulting gauge algebra is identical to (13), yet the two gaugings are inequivalent as we shall explicitly confirm below by comparing their scalar potentials.

In [3], two gaugings of maximal supergravity with gauge group (13) have been identified, constructed in the SL(8) frame and in the SU*(8) frame of $E_{7(7)}$, respectively. We will establish the link in section 3, with the former one describing the IIB embedding and the latter one describing the IIA embedding of the $\mathcal{N} = 4$ theory.

### 2.2. Uplift of $\mathcal{N} = 4$ supergravity

We have described the embedding of the $\mathcal{N} = 4$ theory with embedding tensor (6) into maximal $\mathcal{N} = 8$ supergravity. The half-maximal theory can be obtained as a consistent truncation from ten-dimensional supergravity. This is most conveniently described by a Scherk–Schwarz reduction in a double field theory (DFT) reformulation [25–27] of ten-dimensional supergravity, in terms of an SO(6, 6) twist matrix $U$ given by [15]

$$U_M{}^{\underline{K}} = \left\{ -\kappa^{\underline{KL}} \mathcal{K}_{\underline{L}\,m} + \eta^{\underline{KL}} \mathcal{K}_{\underline{L}}{}^n\, \tilde{C}_{nm} \, , \; \eta^{\underline{KL}} \mathcal{K}_{\underline{L}}{}^m \right\} , \tag{17}$$

in terms of the Killing vectors

$$\mathcal{K}_{\underline{K}}{}^m \equiv \{ L_{\underline{k}}{}^m + R_{\underline{k}}{}^m, L^{\underline{k}\,m} - R^{\underline{k}\,m} \} , \tag{18}$$

of left and right $G \times G$ isometries, the Cartan–Killing form $\kappa^{\underline{KL}}$, the SO(6, 6) invariant tensor $\eta^{\underline{KL}}$ and the two-form gauge potential $\tilde{C}_{mn}$ of the three-form flux on the group manifold $G$ defined by

$$\begin{aligned}
3\, \partial_{[k} \tilde{C}_{mn]} = \tilde{H}_{kmn} \; &\equiv \; -16\, f^{\underline{kmn}}\, L_{\underline{k}\,k} L_{\underline{m}\,m} L_{\underline{n}\,n} \\
&= -16\, f^{\underline{kmn}}\, R_{\underline{k}\,k} R_{\underline{m}\,m} R_{\underline{n}\,n} \, .
\end{aligned} \tag{19}$$

We refer to [15] for details.

The construction applies to arbitrary groups $G$. The fact that the relevant group (2) factorizes into two three-dimensional groups implies that the twist matrix $U$ only lives in the subgroup

$$\mathrm{SL}(4) \times \mathrm{SL}(4) \simeq \mathrm{SO}(3,3) \times \mathrm{SO}(3,3) \, \subset \, \mathrm{SO}(6,6) \, . \tag{20}$$

An equivalent presentation of the twist matrix (17) can be given in terms of the explicit SL(4)-valued twist matrices for $S^3$ and $H^3$ from [12,13].

The twist matrix together with a generalized Scherk–Schwarz Ansatz allow us to derive the explicit uplift formulae of the four-dimensional $\mathcal{N} = 4$ supergravity up to ten dimensions [15]. As an example, we can use these formulae to derive the ten-dimensional origin of the four-dimensional Minkowski vacuum carried by the scalar potential (1) at the scalar origin. This ten-dimensional background is conveniently described by embedding the six-dimensional internal space into $\mathbb{R}^8$ via the coordinates

$$\begin{aligned}
&\{U^a, Y^a\} , \quad a = 1, \ldots, 4 , \\
\text{with} \quad &U^a U^a = 1 = Y^a \eta_{ab} Y^b ,
\end{aligned} \tag{21}$$

in terms of the SO(2, 2) invariant metric $\eta_{ab} = \mathrm{diag}\{-1, -1, 1, 1\}$. The $D = 10$ dilaton and metric then take the following form

$$\begin{aligned}
&e^\phi = \frac{1}{\sqrt{1 + 2\, y^2}} , \qquad y^2 \equiv (Y^1)^2 + (Y^2)^2 , \\
&ds^2 = e^{-\phi/2} \eta_{\mu\nu}\, dx^\mu\, dx^\nu + 2\, e^{-\phi/2} dU^a\, dU^a + 2\, e^{3\phi/2} dY^a\, dY^a ,
\end{aligned} \tag{22}$$

with the four-dimensional Minkowski metric $\eta_{\mu\nu}$. The geometry is a warped product (4) with manifest isometry group $\mathrm{SO}(1,3) \times \mathrm{SO}(4) \times \mathrm{SO}(2)^2$. The three-form flux takes the form

$$H_3 = 24 \left( \omega_S + e^{4\phi}\, \omega_H \right) , \tag{23}$$

in terms of the canonical volume forms $\omega_S$ and $\omega_H$, of $S^3$ and $H^3$ given by

$$\begin{aligned}
\omega_S &= \frac{1}{3!}\, \varepsilon_{abcd}\, U^a\, dU^b \wedge dU^c \wedge dU^d , \\
\omega_H &= \frac{1}{3!}\, \varepsilon_{abcd}\, Y^a\, dY^b \wedge dY^c \wedge dY^d ,
\end{aligned} \tag{24}$$

respectively.

### 2.3. Embedding DFT into ExFT

The construction can be extended to maximal supergravity by embedding the ten-dimensional supergravity into $E_{7(7)}$ exceptional field theory (ExFT) [18]. This is the duality covariant formulation of maximal supergravity in which the fields are reorganised into $E_{7(7)}$ covariant objects living on an extended space of 56 coordinates $\{Y^M\}$ constrained by the strong section condition

$$(t_\Sigma)^{MN}\, \partial_M \otimes \partial_N = 0 , \tag{25}$$

with the $E_{7(7)}$ generators $(t_\Sigma)^{MN}$. There are two inequivalent solutions to this condition which correspond to selecting within the $\{Y^M\}$ six internal coordinates corresponding to either IIA or IIB supergravity [18]. Only the former set of coordinates may be extended by a seventh coordinate without violating (25), corresponding to $D = 11$ supergravity.

Consistent truncations to maximal supergravities are described in exceptional field theory by generalized Scherk–Schwarz reductions in terms of $E_{7(7)}$ valued twist matrices $U$. Upon embedding the SO(6, 6) twist matrix (17) into $E_{7(7)}$, we thus obtain an embedding of the four-dimensional maximal supergravities discussed in section 2.1 into ten dimensions. Although the embedding of

SO(6, 6) into $E_{7(7)}$ is unique, the two inequivalent ways of identifying coordinates (corresponding to the inequivalent embeddings of GL(6) into $E_{7(7)}$) result in two inequivalent ten-dimensional uplifts, into IIA and IIB supergravity, respectively. The corresponding coordinates are identified within the 56 internal coordinates of $E_{7(7)}$ ExFT as

$$
\begin{aligned}
\text{IIA} \,:\, \mathbf{56} \longrightarrow\, & \boxed{\mathbf{6}'_{-4}} + \mathbf{1}_{-3} + \mathbf{6}_{-2} + \mathbf{15}_{-1} + \mathbf{15}'_{+1} \\
& + \mathbf{6}'_{+2} + 1_{+3} + \mathbf{6}_{+4}\,, \\
\text{IIB} \,:\, \mathbf{56} \longrightarrow\, & \boxed{(\mathbf{6}', \mathbf{1})_{-4}} + (\mathbf{6}, \mathbf{2})_{-2} + (\mathbf{20}, \mathbf{1})_0 \\
& + (\mathbf{6}', \mathbf{2})_{+2} + (\mathbf{6}, \mathbf{1})_{+4}\,.
\end{aligned}
\tag{26}
$$

While this construction provides a neat and compact proof for the existence of consistent uplifts of these four-dimensional supergravities, in practice the embedding of the twist matrix (17) into $E_{7(7)}$ requires its evaluation in the spinor representations of the group SO(6, 6) according to the decomposition of (8) which is a somewhat cumbersome exercise. In the next section we thus give an alternative direct derivation of the full $E_{7(7)}$ twist matrices.

## 3. The IIA/IIB twist matrices

In [28], twist matrices for the uplift of certain dyonic $\mathcal{N} = 8$ gaugings have been constructed after decomposing the 56 coordinates in the SL(8) frame into what we will refer to as 'electric' and 'magnetic' coordinates

$$\left\{Y^M\right\} = \left\{Y^{[AB]}, Y_{[AB]}\right\}\,, \qquad A, B = 1, \ldots, 8\,. \tag{27}$$

In these coordinates the section condition (25) takes the form

$$
\begin{aligned}
\partial_{AC} \otimes \partial^{BC} + \partial^{BC} \otimes \partial_{AC} &= \frac{1}{8}\,\delta^B_A \left(\partial_{CD} \otimes \partial^{CD} + \partial^{CD} \otimes \partial_{CD}\right)\,, \\
\partial_{[AB} \otimes \partial_{CD]} &= \frac{1}{24}\,\varepsilon_{ABCDEFGH}\,\partial^{EF} \otimes \partial^{GH}\,,
\end{aligned}
\tag{28}
$$

and twist matrices are constructed as products of matrices depending on electric and on magnetic coordinates, respectively.

### 3.1. IIB twist matrix

Choosing physical coordinates as

$$\left\{y^i \equiv Y^{i8}\,,\quad \tilde{y}_a \equiv Y_{a7}\right\}\,, \qquad i, j \in \{1, 2, 3\}\,,\ a, b \in \{4, 5, 6\}\,, \tag{29}$$

among (27), it is straightforward to verify that restricting the dependence of fields to these coordinates solves the section condition (28) and that (29) cannot be extended by any of the other 50 internal coordinates without violating the section constraint. ExFT evaluated on these coordinates thus describes IIB supergravity. Specifically, the $GL(1)_{\text{IIB}}$, which provides the geometric grading of coordinates (26) and fields, is generated by

$$
\begin{aligned}
GL(1)_{\text{IIB}} = \Big\langle & \frac{3}{4}\left(T_8{}^8 - T_7{}^7\right) \\
& + \frac{1}{4}\left(T_1{}^1 + T_2{}^2 + T_3{}^3 - T_4{}^4 - T_5{}^5 - T_6{}^6\right)\Big\rangle\,,
\end{aligned}
\tag{30}
$$

resulting in the charges

$$
\begin{aligned}
&\{Y^{i8}, Y_{a7}\} : -4\,, \quad \{Y^{a8}, Y^{ij}, Y_{i7}, Y_{ab}\} : -2\,, \\
&\{Y^{ia}, Y^{78}, Y_{78}, Y_{ia}\} : 0\,, \quad \ldots\,,
\end{aligned}
\tag{31}
$$

for the coordinates, in accordance with (26). The twist matrices considered in [28] are of the form

$$U(y^i, \tilde{y}_a) \equiv \mathring{U}(\tilde{y}_a)\,\hat{U}(y^i)\,, \tag{32}$$

with the two commuting factors $\mathring{U}$ and $\hat{U}$ given by the sphere/hyperboloid solutions from [13]. They describe the embedding of maximal four-dimensional gaugings with a dyonic embedding tensor given by

$$
\begin{aligned}
X_{AB,CD}{}^{EF} &= \eta_{A[C}\delta_{D]B}{}^{EF} - \eta_{B[C}\delta_{D]A}{}^{EF}\,, \\
X^{AB}{}_{CD}{}^{EF} &= -\tilde{\eta}^{A[E}\delta_{CD}{}^{F]B} + \tilde{\eta}^{B[E}\delta_{CD}{}^{F]A}\,,
\end{aligned}
\tag{33}
$$

with

$$
\begin{aligned}
\eta_{AB} &= \text{diag}(\overbrace{1, \ldots, 1}^{p}, \overbrace{-1, \ldots, -1}^{4-p}, 0, \ldots\ldots, 0)\,, \\
\tilde{\eta}^{AB} &= \text{diag}(0, \ldots\ldots, 0, \underbrace{1, \ldots, 1}_{q}, \underbrace{-1, \ldots, -1}_{4-q})\,,
\end{aligned}
\tag{34}
$$

as constructed in [3]. For this paper, we are interested in the case $p = 4$, $q = 2$, corresponding to the gauge group (13). The gauge algebra thus is a subalgebra of $\mathfrak{sl}(8) = \langle T_A{}^B\rangle$ with the gauge connection given by

$$D_\mu = \partial_\mu - \left(A_\mu{}^{AB}\,\eta_{BC} - A_{\mu\,CB}\,\tilde{\eta}^{BA}\right) T_A{}^C\,, \tag{35}$$

corresponding to the IIB couplings of (15).

### 3.2. IIA twist matrix

We note that the above IIB Ansatz defines a natural embedding $SL(4) \times SL(4) \subset SL(8)$, with each of the twist matrices $\mathring{U}$ and $\hat{U}$ an element of one of the two SL(4) factors. Using these two SL(4) subgroups we can slightly generalise the above Ansatz for the physical coordinates, by embedding them into

$$\left\{Y^{IJ}\,,\ Y_{\dot{I}\dot{J}}\right\}\,, \qquad I, J = \{1, 2, 3, 8\}\,, \quad \dot{I}, \dot{J} = \{4, 5, 6, 7\}\,, \tag{36}$$

with $\hat{U}(Y^{IJ})$ and $\mathring{U}(Y_{\dot{I}\dot{J}})$. The section condition (28) then becomes

$$\varepsilon^{IJKL}\partial_{IJ} \otimes \partial_{KL} = \varepsilon_{\dot{I}\dot{J}\dot{K}\dot{L}}\partial^{\dot{I}\dot{J}} \otimes \partial^{\dot{K}\dot{L}}\,. \tag{37}$$

Here, we further restrict to the case

$$\varepsilon^{IJKL}\partial_{IJ} \otimes \partial_{KL} = \varepsilon_{\dot{I}\dot{J}\dot{K}\dot{L}}\partial^{\dot{I}\dot{J}} \otimes \partial^{\dot{K}\dot{L}} = 0\,. \tag{38}$$

We can now follow [29] and apply the outer automorphism of the SL(4) factor defined by $\dot{I}, \dot{J} = \{4, 5, 6, 7\}$. This takes

$$\partial_{\dot{I}\dot{J}} \longrightarrow \frac{1}{2}\varepsilon^{\dot{I}\dot{J}\dot{K}\dot{L}}\partial_{\dot{K}\dot{L}}\,, \qquad \mathring{U} \longrightarrow \mathring{U}^{-T}\,, \tag{39}$$

and one can easily show that it satisfies the conditions in [28], ensuring a consistent truncation. The new full set of physical coordinates is now given by

$$
\begin{aligned}
&\left\{y^i \equiv Y^{i8}\,,\quad \tilde{y}^a \equiv \frac{1}{2}\varepsilon^{abc}\,Y_{bc}\right\}\,, \\
&\qquad i, j \in \{1, 2, 3\}\,,\ a, b \in \{4, 5, 6\}\,,
\end{aligned}
\tag{40}
$$

with twist matrix

$$U(y^i, \tilde{y}^a) \equiv \mathring{U}(\tilde{y}^a)^{-T}\,\hat{U}(y^i)\,. \tag{41}$$

It is straightforward to verify that the new set of coordinates can be extended by a seventh coordinate $Y^{78}$, while still satisfying the section constraints (27). The resulting theory is thus type

IIA supergravity (with possible $D = 11$ embedding). The $\mathrm{GL}(1)_{\mathrm{IIA}}$, which provides the geometric grading of coordinates (26) and fields, is generated by

$$\mathrm{GL}(1)_{\mathrm{IIA}} = \left\langle \frac{1}{3}\left(T_1{}^1 + T_2{}^2 + T_3{}^3\right) - \frac{2}{3}\left(T_4{}^4 + T_5{}^5 + T_6{}^6\right) + T_8{}^8 \right\rangle , \tag{42}$$

giving charges ($i, j = 1, \dots, 3$, $a, b = 4, \dots, 6$)

$$\begin{aligned} &\{Y^{i8}, Y_{ab}\} : -4 , \quad \{Y^{78}\} : -3 , \quad \{Y^{ij}, Y_{a7}\} : -2 , \\ &\{Y^{i7}, Y^{a8}, Y_{ia}\} : -1 , \quad \dots , \end{aligned} \tag{43}$$

for the coordinates, in accordance with (26).

The new embedding tensor corresponding to this IIA reductions is given by

$$\begin{aligned} X_{AB,CD}{}^{EF} &= \eta_{A[C}\delta_{D]B}{}^{EF} - \eta_{B[C}\delta_{D]A}{}^{EF} , \\ X^{AB}{}_{CD}{}^{EF} &= -\Sigma^{AB[E}_{[C}\,\delta_{D]}{}^{F]} , \end{aligned} \tag{44}$$

where

$$\Sigma^{ABC}_{D} = \omega^{ABCE}\,\tilde{\eta}_{DE} , \tag{45}$$

and the only non-vanishing components of $\omega^{ABCD}$ are

$$\omega^{\dot{I}\dot{J}\dot{K}\dot{L}} = \varepsilon^{\dot{I}\dot{J}\dot{K}\dot{L}} , \tag{46}$$

and $\tilde{\eta}_{AB} = \tilde{\eta}^{AB}$ of (34) with $p = 4$ and $q = 2$. The quadratic constraint for this type of embedding tensor are given by

$$\begin{aligned} \Sigma_{(A}{}^{CDE}\eta_{B)E} &= 0 , \\ \Sigma_A{}^{BC[D}\Sigma_C{}^{EFG]} &= 0 , \end{aligned} \tag{47}$$

which are satisfied in the given case.

At this stage it is natural to ask whether the 4-dimensional gauged SUGRAs we obtained from IIA and IIB are different. In 7 dimensions, the IIA / IIB truncations related by an outer automorphism of SL(4) are clearly inequivalent because the resulting embedding tensor belongs to different irreducible representations under the global symmetry group SL(5) [29]. Here, this is much harder to assess because in both cases the embedding tensor belong to the **912** representation under $\mathrm{E}_{7(7)}$. Under $\mathrm{SL}(8) \subset \mathrm{E}_{7(7)}$, a difference emerges: the IIA embedding tensor corresponds to gaugings in the **36** and $\overline{\mathbf{420}}$ of SL(8) while the IIB one to gaugings in the **36** and $\overline{\mathbf{36}}$ of SL(8). However, the two embedding tensors also couple to different sets of vector fields so that this direct comparison is meaningless. Nonetheless, the IIA embedding tensor takes the same form as in (33) in the $\mathrm{SU}^*(8)$ frame, with gaugings in the **36** and $\overline{\mathbf{36}}$. This suggests that the IIA and IIB reductions yield different gauged SUGRAs, as one would have expected and as we will explicitly confirm in the next section.

## 4. Gaugings and potentials

So far we have shown that IIA and IIB supergravity compactified around (22)–(23) give rise to maximal $D = 4$ supergravities which share the same gauge group (13) but embedded in inequivalent ways within $\mathrm{E}_{7(7)}$. Around this background the two theories exhibit the same spectrum as can be confirmed by expanding the resulting scalar potentials to quadratic order.

In order to confirm explicitly that the two gaugings represent inequivalent four-dimensional theories, we will compute and compare a truncation of their full scalar potentials. To this end, we consider their respective truncations to singlets under the compact subgroup

$$\begin{aligned} G_0 \equiv \mathrm{SO}(3)_{\mathrm{D}} \times \mathrm{SO}(2)_{\mathrm{D}} &\subset (\mathrm{SO}(3) \times \mathrm{SO}(3)) \times (\mathrm{SO}(2) \times \mathrm{SO}(2)) \\ &\subset \mathrm{SO}(4) \times \mathrm{SO}(2,2) , \end{aligned} \tag{48}$$

of the gauge group. Within $\mathrm{E}_{7(7)}$ this group commutes with a $\mathrm{GL}(4) \times \mathrm{SO}(2)$, i.e. the scalar coset $\mathrm{E}_{7(7)}/\mathrm{SU}(8)$ contains 10 singlets under $G_0$ with the resulting kinetic term given by

$$\mathcal{L}_{\mathrm{scal}} = \frac{1}{4}\, D_\mu M_{uv} D^\mu M^{uv} + 3\, D_\mu\lambda\, D^\mu\lambda , \tag{49}$$

in terms of a symmetric SL(4) matrix $M_{uv}$, $u, v = 1, \dots, 4$ and a scalar $\lambda$. Under reduction to the common NS–NS sector, the GL(4) further breaks down to $\mathrm{GL}(2) \times \mathrm{GL}(2)$, in particular the SL(4) matrix $M_{uv}$ breaks down to an $\mathrm{SL}(2) \times \mathrm{SL}(2) \times \mathrm{GL}(1)$ matrix of block-diagonal form

$$M_{uv} = \begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix} . \tag{50}$$

For a general gauging, the embedding tensor $\Theta_M{}^\Sigma$ in the **912** representation of $\mathrm{E}_{7(7)}$ contains 64 singlets under $G_0$ which organise into SL(4) tensors according to

$$\begin{aligned} &+3 : X_{[uv]}{}^w , Y^{[uvw]} , Z^{[uvw]} , \\ &+1 : f_{[uvw]} = \varepsilon_{uvwx}\,\tilde{f}^x , \\ &-1 : f^{[uvw]} = \varepsilon^{uvwx}\,\tilde{f}_x , \\ &-3 : X^{[uv]}{}_w , Y_{[uvw]} , Z_{[uvw]} , \end{aligned} \tag{51}$$

with the grading referring to GL(1). The associated scalar potential is computed by applying the truncation to $G_0$ singlets to the general $\mathcal{N} = 8$ potential from [30], resulting in

$$\begin{aligned} V = {}& \frac{1}{16}\, e^{-3\lambda} \left( X_{uv}{}^w X_{yz}{}^x M_{wx} M^{uy} M^{vz} + 2\, X_{uw}{}^x X_{vx}{}^w M^{uv} \right) \\ & - \frac{3}{8}\, e^{-\lambda} \left( \tilde{f}^u \tilde{f}^v + f^{uwx} X_{wx}{}^v \right) M_{uv} \\ & + \frac{1}{4} \left( X_{uv}{}^y Z_{ywx} + X^{uv}{}_y Z^{ywx} \right) M^{uw} M^{vx} \\ & - \frac{3}{8}\, e^{\lambda} \left( \tilde{f}_u \tilde{f}_v + f_{wxu} X^{wx}{}_v \right) M^{uv} \\ & + \frac{1}{16}\, e^{3\lambda} \left( X^{uv}{}_w X^{yz}{}_x M^{wx} M_{uy} M_{vz} + 2\, X^{uw}{}_x X^{vx}{}_w M_{uv} \right) . \end{aligned} \tag{52}$$

For the IIA and IIB embedding tensors given in the last section, truncation to $G_0$ singlets yields

$$\begin{aligned} \text{IIA:}\quad &+3 : X_{uv}{}^w \;:\; \{X_{23}{}^4 = X_{42}{}^3 = X_{34}{}^1 = 1\} , \\ &-1 : f^{uvw} \;:\; \{f^{134} = 1\} , \\ &-3 : Z_{uvw} \;:\; \{Z_{234} = 1\} ; \end{aligned} \tag{53}$$

$$\begin{aligned} \text{IIB:}\quad &+3 : X_{uv}{}^w \;:\; \{X_{23}{}^4 = X_{42}{}^3 = 1\} , \\ &+1 : f_{uvw} \;:\; \{f_{234} = 1\} , \\ &-3 : X^{uv}{}_w \;:\; \{X^{34}{}_2 = 1\} , \quad Z_{mnk} \;:\; \{Z_{234} = 1\} , \end{aligned} \tag{54}$$

when written in the basis (51). In particular, in this truncation, only an $\mathrm{SO}(2)^2 \ltimes T^2$ subgroup of the gauge group (13) survives in both cases. The inequivalence of the two resulting gaugings now becomes manifest from the different forms the general scalar potential (52) takes for (53) and (54), respectively:

$$
\begin{aligned}
V_{\rm IIA} = {} & \frac{1}{16}\, e^{-3\lambda} \left( X_{uv}{}^{k} X_{yz}{}^{x}\, M_{wx} M^{uy} M^{vz} - 4\, M^{22} \right) - \frac{3}{4}\, e^{-\lambda}\, M_{11} \\
& + \frac{1}{4}\, X_{uv}{}^{y} Z_{ywx}\, M^{uw} M^{vx} - \frac{3}{8}\, e^{\lambda}\, M^{22}\,, \\
V_{\rm IIB} = {} & \frac{1}{16}\, e^{-3\lambda} \left( X_{uv}{}^{w} X_{yz}{}^{x}\, M_{wx} M^{uy} M^{vz} - 4\, M^{22} \right) - \frac{3}{8}\, e^{-\lambda}\, M_{11} \\
& + \frac{1}{4}\, X_{uv}{}^{y} Z_{ywx}\, M^{uw} M^{vx} - \frac{3}{4}\, e^{\lambda}\, M^{22} \\
& + \frac{1}{8}\, e^{3\lambda}\, M^{22} \left( M_{33} M_{44} - M_{34} M_{43} \right)\,. \qquad (55)
\end{aligned}
$$

In particular, in the IIA potential the $e^{3\lambda}$ term vanishes identically, showing the inequivalent asymptotic behaviour of the two potentials.

## 5. Uplift of the moduli

Around the Minkowski vacuum, the four-dimensional theories have a six-dimensional moduli space [4] which can be identified within the NS–NS sector. Apart from the trivial $\frac{\mathrm{SL}(2)}{\mathrm{SO}(2)}$ factor from the ten-dimensional dilaton and Kalb–Ramond field, the remaining four moduli $\{\varphi_i, \chi_i\}$, $i = 1, 2$, form an $\left(\frac{\mathrm{SL}(2)}{\mathrm{SO}(2)}\right)^2 \subset \mathrm{SO}(6,6)$ embedded according to the scalar moduli matrix $M_{AB}$

$$
M_{AB} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & s_1 s_2 & 0 & 0 & -s_1 \chi_2 & 0 & 0 & \chi_1 \chi_2 & 0 & 0 & s_2 \chi_1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -s_1 \chi_2 & 0 & 0 & s_1 e^{-\varphi_2} & 0 & 0 & -e^{-\varphi_2} \chi_1 & 0 & 0 & -\chi_1 \chi_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & \chi_1 \chi_2 & 0 & 0 & -e^{-\varphi_2} \chi_1 & 0 & 0 & e^{-\varphi_1 - \varphi_2} & 0 & 0 & e^{-\varphi_1} \chi_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & s_2 \chi_1 & 0 & 0 & -\chi_1 \chi_2 & 0 & 0 & e^{-\varphi_1} \chi_2 & 0 & 0 & s_2 e^{-\varphi_1}
\end{pmatrix}, \qquad (56)
$$

with $s_i = e^{\varphi_i}\left(1 + \chi_i^2\right)$. Their kinetic term in the four-dimensional theory is given by

$$
\begin{aligned}
\mathcal{L}_{\rm kin} = \frac{1}{2} \Big( & \partial_\mu \varphi_1\, \partial^\mu \varphi_1 + e^{-2\varphi_1}\, \partial_\mu \tilde{\chi}_1\, \partial^\mu \tilde{\chi}_1 + \partial_\mu \varphi_2\, \partial^\mu \varphi_2 \\
& + e^{-2\varphi_2}\, \partial_\mu \tilde{\chi}_2\, \partial^\mu \tilde{\chi}_2 \Big)\,,
\end{aligned} \qquad (57)
$$

with $\tilde{\chi}_i = e^{\varphi_i} \chi_i$. Using the formulae from [15] these moduli can be uplifted to $D = 10$ dimensions and we will work out the explicit uplift here.

The ten-dimensional background corresponding to these massless deformations preserves a set of $U(1)^4$ isometries and is therefore most conveniently described in terms of the following functions on the six-dimensional internal space

$$
\begin{aligned}
& \{u^\alpha, v^\alpha, y^\alpha, z^\alpha\}\,, \quad \alpha = 1, 2\,, \\
& \text{with} \quad u^\alpha u^\alpha + v^\alpha v^\alpha \; = \; 1 \; = \; z^\alpha z^\alpha - y^\alpha y^\alpha\,, \qquad (58)
\end{aligned}
$$

so that the $U(1)^4$ isometries are realised as rotations on the $\{u^\alpha, v^\alpha, y^\alpha, z^\alpha\}$, respectively. These functions are in fact the usual coordinates on $\mathbb{R}^8$ in which the six-dimensional manifold is embedded via (58). As a result these functions are *globally* well-defined on the internal space and allow us to give global expressions for the metric and form fields on the internal space, rather than local coordinate expressions.

To this end we introduce the $U(1)^4$ invariant one-forms

$$
\begin{aligned}
& \sigma_0 \equiv u^\alpha du^\alpha\,, \qquad \sigma_1 \;\equiv\; \varepsilon_{\alpha\beta}\, u^\alpha du^\beta\,, \qquad \sigma_2 \;\equiv\; \varepsilon_{\alpha\beta}\, v^\alpha dv^\beta\,, \\
& \tau_0 \equiv y^\alpha dy^\alpha\,, \qquad \tau_1 \;\equiv\; \varepsilon_{\alpha\beta}\, y^\alpha dy^\beta\,, \qquad \tau_2 \;\equiv\; \varepsilon_{\alpha\beta}\, z^\alpha dz^\beta\,, \quad (59)
\end{aligned}
$$

and functions $u^2 \equiv u^\alpha u^\alpha$, $y^2 \equiv y^\alpha y^\alpha$. In terms of these forms, the volume forms, $\omega_{S/H}$, of the undeformed $S^3/H^3$ are given by

$$
\begin{aligned}
\omega_S &= \frac{1}{2}\left(\sigma_1 \wedge d\sigma_2 + \sigma_2 \wedge d\sigma_1\right)\,, \\
\omega_H &= \frac{1}{2}\left(\tau_1 \wedge d\tau_2 + \tau_2 \wedge d\tau_1\right)\,,
\end{aligned} \qquad (60)
$$

as can, for example, be seen from [12]. We will moreover define the moduli-dependent functions

$$
\begin{aligned}
f_1(u) &\equiv e^{-\varphi_1}\left(1 - u^2\right) + e^{\varphi_2}\, u^2 \left(1 + \chi_2^2\right)\,, \\
f_2(u) &\equiv e^{-\varphi_2}\left(1 - u^2\right) + e^{\varphi_1}\, u^2 \left(1 + \chi_1^2\right)\,, \\
g_1(y) &\equiv e^{-\varphi_2}\, y^2 + e^{-\varphi_1}\left(1 + y^2\right)\,, \\
g_2(y) &\equiv e^{\varphi_1}\, y^2 \left(1 + \chi_1^2\right) + e^{\varphi_2}\left(1 + y^2\right)\left(1 + \chi_2^2\right)\,, \qquad (61)
\end{aligned}
$$

and note that for finite values of the moduli, these functions are given by a sum of two positive terms. Furthermore, those two terms do not both vanish at the same locations, and thus the functions $f_i$, $g_i$ are positive-definite for finite values of the moduli.

The $D = 10$ metric yields a deformation of (22)

$$
ds^2 = \Delta^{-1}\, \eta_{\mu\nu}\, dx^\mu\, dx^\nu + 2\, e^{4\varphi_0}\, \Delta^3\, d\hat{s}_6^2\,, \qquad (62)
$$

with warp factor

$$
\begin{aligned}
\Delta^{-4} &= e^{6\varphi_0}\left(y^2 f_2(u) + \left(1 + y^2\right) f_1(u)\right) \\
&= e^{6\varphi_0}\left(\left(1 - u^2\right) g_1(y) + u^2\, g_2(y)\right)\,, \qquad (63)
\end{aligned}
$$

and an internal six-dimensional metric $d\hat{s}_6^2$

$$
\begin{aligned}
d\hat{s}_6^2 = {} & g_1(y)\, du^\alpha\, du^\alpha + g_2(y)\, dv^\alpha\, dv^\alpha + f_1(u)\, dy^\alpha\, dy^\alpha \\
& + f_2(u)\, dz^\alpha\, dz^\alpha + 2\, \chi_2\left(\sigma_1\, \tau_1 + \sigma_2\, \tau_2\right) \\
& + 2\, \chi_1\left(\sigma_1\, \tau_2 + \sigma_2\, \tau_1\right)\,. \qquad (64)
\end{aligned}
$$

The $D = 10$ dilaton is given by

$$
e^{\phi/2} = \Delta\, e^{2\varphi_0}\,, \qquad (65)
$$

and the Kalb–Ramond form

$$
\begin{aligned}
B = {} & \tilde{B} + \iota_v\left(\omega_S + \omega_H\right) \\
& + 2\Delta^4\, e^{6\varphi_0}\left[\chi_1\left(\sigma_1 \wedge \tau_1 + \sigma_2 \wedge \tau_2\right)\right. \\
& \left. + \; \chi_2\left(\sigma_1 \wedge \tau_2 + \sigma_2 \wedge \tau_1\right)\right]\,, \qquad (66)
\end{aligned}
$$

where

$$
\tilde{H}_3 = d\tilde{B} = 4\left(\omega_S + \omega_H\right)\,, \qquad v^i = 8\Delta^{-1} \tilde{g}^{ij} \partial_j \Delta\,, \qquad (67)
$$

and $\tilde{g}_{ij}$ is an auxiliary pseudo-Riemannian metric with line element

$$
d\tilde{s}_6^2 = du^\alpha du^\alpha + dv^\alpha dv^\alpha - dy^\alpha dy^\alpha + dz^\alpha dz^\alpha\,. \qquad (68)
$$

The Kalb–Ramond three-form flux takes the form

$$H_3 = 12\, e^{12\,\varphi_0}\, \Delta^8\, \hat{H}_3 ,$$

$$\begin{aligned} \text{with} \quad \hat{H}_3 = {} & 2 g_1(y)\, g_2(y)\, \omega_S + 2 f_1(u)\, f_2(u)\, \omega_H \\ & + g_1(y)\, d\sigma_1 \wedge (\chi_2\, \tau_2 + \chi_1\, \tau_1) \\ & + g_2(y)\, d\sigma_2 \wedge (\chi_2\, \tau_1 + \chi_2\, \tau_1) \\ & - f_1(u) d\tau_1 \wedge (\chi_2\, \sigma_2 + \chi_1\, \sigma_1) \\ & + f_2(u)\, d\tau_2 \wedge (\chi_2\, \sigma_1 + \chi_1\, \sigma_2) \\ & + 2\left(\chi_1^2 - \chi_2^2\right) (\sigma_1 \wedge \sigma_2 \wedge \tau_0 - \tau_1 \wedge \tau_2 \wedge \sigma_0) \ , \quad (69) \end{aligned}$$

describing a four-parameter deformation of (23).

We have thus completed the uplift of the four moduli to the parameters of a solution of $D = 10$ supergravity. Let us note that in the truncation $\chi_i = 0$ and upon normalization $2\varphi_0 = -\varphi_1 - \varphi_2$, the remaining moduli $\{\varphi_i\}$ translate according to

$$x \equiv e^{-\varphi_1}\ , \quad y \equiv e^{-\varphi_2}\ , \tag{70}$$

into the notation of [4] whose mass spectrum we reproduce. Finally, as discussed in [5], when the moduli approach the boundary of the moduli space, e.g. $\varphi_{1,2} \to \pm\infty$, we obtain a different $\mathcal{N} = 8$ gauged SUGRA. In particular, these limits can be understood as contractions of the gauge group to $\mathrm{SO}(2) \times \mathrm{SO}(2) \ltimes T^{26}$.

## 6. Conclusions

In this paper we used exceptional field theory to find the $D = 10$ uplift of two inequivalent four-dimensional $\mathcal{N} = 8$ gauged SUGRAs with the same dyonic gauge group $\mathrm{SO}(4) \times \mathrm{SO}(2,2) \ltimes T^{16}$, and which admit a non-supersymmetric Minkowski vacuum. We showed that the inequivalent four-dimensional theories come from truncating IIA or IIB around the same $\mathrm{Mink}_4 \times S^3 \times H^3$ background, with the IIA / IIB gaugings naturally arising in the $\mathrm{SU}^*(8)$ and $\mathrm{SL}(8)$ frames, respectively. The two consistent truncations are related by an outer automorphism of SL(4) which can be taken to act on the $S^3$, or $H^3$, using the techniques outlined in [29].

The common $\mathcal{N} = 4$ sector of these theories falls within the class considered in [15]. By studying the $\mathcal{N} = 4$ scalar potential we identified the four moduli which lie in the common NS–NS sector and parameterise the coset space $(\mathrm{SL}(2)/\mathrm{SO}(2))^2$. Using [15] we uplifted these moduli to obtain a four-parameter family of Minkowski vacua in 10 dimensions which preserve a $\mathrm{U}(1)^4$ subgroup of the $\mathrm{SO}(4) \times \mathrm{SO}(2,2)$ isometries of the round $S^3 \times H^3$ background. Taking these scalar fields to the boundary of the moduli space results in new $\mathcal{N} = 8$ gauged SUGRAs with gauge group $\mathrm{SO}(2) \times \mathrm{SO}(2) \ltimes \mathrm{T}^{26}$.

Gauged SUGRAs with dyonic gaugings are particularly interesting because of their rich vacuum structure, but have only recently been uplifted to 10-/11-dimensional SUGRA [31–33,28]. For example, half-maximal AdS vacua of type II and 11-dimensional SUGRA must have non-zero de Roo–Wagemans angles [34,35]. We hope that the techniques developed here will be useful in those applications.

Another interesting question raised by this work is whether our IIA uplift can be obtained directly in the $\mathrm{SU}^*(8)$ frame, as opposed to the commonly used SL(8) frame. This might lead to a generalisation of the IIA uplift, just as the IIB twist matrix is a particular example of a family of truncations obtained in [28]. We leave these and other open questions for further work.

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