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Spillovers in Vocational Training: An Analysis of Incentive Schemes

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Abstract

The German apprenticeship system is often considered a role model for vocational education. Its influence on economic growth and technological progress through the provision of human capital to the workforce is widely acknowledged. But recent declines in the number of apprenticeships have led to increasing unrest among policy makers. To counter this development, the government is considering to introduce a training levy scheme that collects training levies from non-training firms in order to subsidize apprenticeship training ("Ausbildungsplatzabgabe"). Such training levy schemes already exist in several industrialized countries and even in some sectors in Germany. Yet, economists differ greatly in opinion about this policy. More surprisingly, however, a general economic analysis of this policy instrument is still lacking. Recent contributions have relied on rather qualitative and partial analyses. This paper aims at closing this gap. Following the training literature, we use a simple oligopsonistic labor market model. Such a setting allows to explain why firms provide and (at least partially) finance general vocational training. Moreover, it can demonstrate that a positive externality arises as other firms benefit from vocational training through poaching. In principle, the Pigouvian prescription of a subsidy scheme financed by a non-distortionary tax could restore the social optimum. The proposed training levy scheme, by contrast, is a particular scheme that links subsidies and levies. This paper unveils that it basically corresponds to a uniform subsidy on apprenticeship training that is financed by a distortionary tax on labor. We show that introducing such a levy scheme can entail ambiguous repercussions on general welfare.

JEL classification: H23, I22, I28, J24
Keywords: Vocational Training, Frictional Labor Markets, Poaching, Levy-Grant-Scheme, Training levy

Zusammenfassung

Das duale Ausbildungssystem in Deutschland gilt vielen Ländern als Vorbild für die berufliche Bildung. Es dient einem großen Teil der Bevölkerung zum Aufbau von Humankapital. Der Einfluss auf Wirtschaftswachstum und technischen Fortschritt ist weithin anerkannt. Der beobachtbare Rückgang der Ausbildungsplätze und Ausbildungsbereitschaft gibt jedoch zunehmend Anlass zur Beunruhigung. Um dieser Entwicklung entgegenzuwirken, bestehen seit längerem Überlegungen für die Einführung einer Ausbildungsplatzabgabe. Sie soll einen finanziellen Ausgleich für Ausbildungsunternehmen schaffen und zugleich Nicht-Ausbildungsunternehmen zu eigener Ausbildung ermuntern. Anreizsysteme für betriebliche Bildung bestehen bereits in einer Reihe anderer Industriestaaten und sogar in einigen Sektoren in Deutschland. Unter Ökonomen ist das Vorhaben einer Ausbildungsplatzabgabe jedoch umstritten. Überraschenderweise besteht jedoch keine allgemeine Analyse dieses Politikvorschlags. Bestehende Analysen sind eher qualitativ und partialanalytisch. Dieser Beitrag versucht diese Lücke zu schließen. Im Anschluss an die jüngere Humankapitalliteratur wird hier ein einfaches Modell betrieblicher Ausbildung in friktionellen Arbeitsmärkten vorschlagen. In einem solchen Rahmen werden Unternehmen allgemeine Ausbildung bereitstellen und zumindest partiell finanzieren. Zugleich lässt sich ein positiver externer Effekt aufzeigen, der aufgrund von Abwerbung ("Poaching") von ausgebildeten Mitarbeitern entsteht. Grundsätzlich könnte eine ideale Pigou-Subvention das soziale Optimum wiederherstellen. Dagegen stellt die vorgeschlagene Ausbildungsabgabe ein bestimmtes Steuer-Subventions-Verfahren dar. Mit diesem Instrument erfährt Ausbildung eine Einheitssubvention, während aktuelle Beschäftigung besteuert wird. Es kann gezeigt werden, dass ungeachtet des positiven externen Effekts die Einführung einer Ausbildungsplatzabgabe nicht zwingend wohlfahrtssteigernd ist.

JEL Klassifizierung: H23, I22, I28, J24
Stichworte: Berufliche Bildung, Arbeitsmarktfriktionen, Abwerbung, Steuer-Subventions-Verfahren, Ausbildungsplatzabgabe

1 Introduction

The German apprenticeship system is often considered a role model for vocational education. It provides basic vocational training to a large share of the workforce. Roughly two thirds of the age-group from 15 to 24 attend vocational education through the apprenticeship system. Apprenticeship training is well-structured and leads to a certified qualification widely recognized among employers. Moreover, the system is renowned for its *duality* of simultaneous schooling and training on-the-job. For several years, however, the number of registered apprenticeships has declined and unemployment among the youth has risen simultaneously.[1] Considering that general unemployment is already high in general, this development increasingly raises concerns and provokes doubts about the apprenticeship system in general. It is questioned whether this decline is of cyclical and thus only transitional nature or, to the contrary, results from insufficient returns to training due to poaching of trained apprentices by other firms. The fact that the proportion of firms that actually provide training is small in absolute terms, is taken as an indication for the latter.[2]

Given the importance of the apprenticeship system for the provision of human capital and thus for economic growth and technological progress, remedies are eagerly sought. Government is considering to introduce a system that subsidizes apprenticeship training and collects training levies from non-training firms. Such *training levy schemes* already exist in several industrialized countries for vocational training and sometimes also for continuous training.[3] Economists differ greatly in opinion about this policy proposal.

[1] The number of registered apprenticeships has declined from 1,629,312 in 1991 to 1,581,629 in 2003. Since the peak in 2000 at 1,702,017 this is a decline of 7%. Similarly, the number of new apprenticeship contracts decreased in the same period from 571,206 to 564,493 peaking in 1999 with 635,559 (BMBF 2004, 9).

[2] In 2002, only 31.3% of all firms were actually providing apprenticeship training (BMBF 2004, 102).

[3] In Germany, such transfer systems have been introduced through collective agreements in the construction and chemical industry, but attempts for legislation at the federal level have failed so far. France is already using such schemes to fund training as well as continuous training. For an international overview of training schemes see in particular Gasskov (1994).

Some agree with the idea that it would internalize positive training spillovers and thereby provide additional incentives for training. Others accept that due to poaching, private and social returns from training diverge, but consider the instrument to be overly bureaucratic and not in accordance with the institutional order of a market economy. Still others question the existence of an externality arising from training altogether and therefore see little need for this economic policy. But given the importance of the question, it seems surprising that a theoretical analysis of this policy instrument is still lacking.[4]

This paper aims at closing this gap. First of all we will ask whether *poaching* indeed gives rise to a positive externality leading to an under-provision of apprenticeship training. We will then investigate whether the introduction of a levy-grant-system for vocational education would theoretically be a welfare-improving policy.

Following the recent training literature, we study a simple oligopsonistic labor market model. Such a setting allows to explain why firms provide and also finance (at least partially) general vocational training. At the same time, however, we can show that a positive externality from training exists as other firms benefit from poaching. In a subsequent policy analysis we can then demonstrate that in principle the Pigouvian prescription of an ideal subsidy scheme financed by a non-distortionary tax could restore the social optimum. A levy-grant-scheme, by contrast, is a particular scheme where subsidies and levies are interlinked. This paper unveils that it is equivalent to a uniform subsidy on apprenticeship training that is financed by a distortionary tax on labor. We show that introducing such a levy-grant-scheme is theoretically ambiguous to welfare even when informational and administrative costs are absent.

We will proceed as follows: In the next section we review the state of the literature on vocational training and positive spillovers. In accordance

[4]For an overview of the policy debate see, for example, the differing contributions in the special issue of ifo Schnelldienst, Vol. 57, No. 6. See Franz (1983) for a study on an earlier policy proposal for a levy-grant-scheme for vocational training in Germany. His analysis is however only partial and does not address the question of an externality.

with recent research we will then introduce a simple model of apprenticeship training. Based on this model we will thereafter study incentives schemes for training. We will conclude with an overview of our findings and draw some policy recommendations for vocational training.

2 Vocational Education and Positive Spillovers

Positive spillovers in vocational education have long ago attracted the interest of economists. Early contributions hinted at the problem of positive externalities accruing to society and proposed policy measures to improve welfare. Later contributions, quite to the contrary, questioned the existence of such positive spillovers and offered completely different policy conclusions. In light of these opposing views, a short review of the literature on vocational education and positive spillovers seems strongly mandated.[5]

2.1 Precursors and Initial Theoretic Approaches

Already classical economists noted the existence of educational spillovers. In his renowned contribution, Smith (1776) refers to the positive value of education for society in general. He sees in education not only a means of skill acquisition but also benefits for law, order and culture. Because of these beneficial social effects, he derives a role for the state in providing basic education and suggests partial public contributions (Smith 1776, 990).

While Smith's analysis was concerned with education in general and with incentives for instructors in particular, later writers dealt more specifically with the vocational training of the workforce by employing firms. Marshall (1920) distinguished between basic education and technical education. He considered basic education to be a public duty, but vocational training an employer obligation (Marshall 1920, IV, vi). Firms were seen to first and foremost benefit from the resulting gains in labor productivity. Therefore, they should also have an inherent self-interest to offer vocational training.

[5] For an extended overview on the history of educational economics see, for instance, Pfahler (2000).

Yet, in light of the industrial revolution, economists increasingly observed a lack of technical education. On the one hand, universal apprenticeship training declined as machines and assembly lines replaced manual techniques and pushed traditional trades and crafts aside. On the other hand, the expanding industrial sector offered only little on-the-job training. Industrial workers were left to execute simple and repeatable tasks, despite striking opportunities to enlarge their skills and qualifications (Smits & Stromback 2001).

Pigou (1912) is acknowledged for the first economic analysis of on-the-job training. He emphasized that a training firm may be inhibited from obtaining the full returns to its training efforts because, with some workers quitting for another firm, the new employer also participates in the returns to training. Thereby he identified *poaching* as the source of the discrepancy between private and social returns to training. The following quotation illustrates this idea.

> "It is, however, obvious that openings exist for investments by the tenant (i.e. the employer) in workpeople's capacity, which would yield considerable social net product. Under a slave economy, since the employer could secure for himself the whole result of increased efficiency in his workpeople and their families, the whole of the social net product of any unit of resources invested in the improvement of their quality would be represented in private net product. Under a free economy, however, since workpeople are liable to change employers, and so to deprive investing tenants of the fruits of their investment, the private net product is apt to fall considerably short of the social net product. Hence, socially profitable expenditure by employers in the training of their workpeople, in building up their health, and in defending them against accident does not carry a corresponding private profit." (Pigou 1912, 153)

Pigou observed that this positive spillover from training is lost to the investing firm, but not to society as a whole. From a social point of view, therefore, training will be undersupplied because private investment is only carried out up to the point where additional private returns equal the additional costs. With the extra social returns not taken into account by the training firm, welfare falls short of its optimal level.

Following this assessment of the economic situation, Pigou indentified a possible role for the state to improve overall welfare. He proposed to introduce fiscal incentives (or disincentives) to private activities that bring about additional returns (or costs) to society. With such a policy, the social externalities caused by an action can be attributed to an individual decision and will thereby lead to a true economic calculation. Modern economists now commonly refer to this policy instrument as ‘Pigouvian subsidy’ or ‘Pigouvian tax’.

> ”It is plain that divergences between private and social net product of the kind just considered cannot [...] be mitigated by a modification of the contractual relation between any two contracting parties, because the divergence arises out of a service or disservice rendered to persons other than the contracting parties. It is, however, possible for the State, if it so chooses, to remove the divergence in any field by ’extraordinary encouragements’ or ’extraordinary restraints’ upon investments in that field. The most obvious forms, which these encouragements and restraints may assume, are, of course, those of bounties and taxes.” (Pigou 1912, 164)

While a Pigouvian tax deliberately discourages the taxed activity and brings about some revenues, a Pigouvian subsidy, by contrast, requires additional financial resources in order to encourage socially beneficial activities. Public funds are however scarce. Thus, in case subsidies are introduced, either alternative public spending is to be discarded or additional funds must be collected. Pigou noted that raising tax revenue inflicts costs on society. Therefore, in case subsidies are introduced to encourage activities beneficial to social welfare, their benefits are to exceed their financial costs.

> "The raising of an additional £ of revenue ... inflicts indirect damage on the taxpayers as a body over and above the loss they suffer in actual money payment. Where there is indirect damage, it ought to be added to the direct loss of satisfaction involved in the withdrawal of the marginal unit of resources by taxation, before this is balanced against the satisfaction yielded by the marginal expenditure." (Pigou 1947, 33-34)

Economists widely accepted Pigou's conjecture of a poaching externality in vocational education. Not surprisingly, subsequent public policy towards vocational education supported a stronger public influence and proposed subsidies for worker training. In fact, several countries introduced financial incentives to vocational training.[6] However, Pigou's assessment of training as well as the role of public policy therein did not remain unchallenged.

2.2 Human Capital Theory

The emergence of human capital theory gave rise to a major revision to the economics of training. This literature drew attention to the shortcomings of neoclassical economics and criticized in particular the prevailing assumption of homogeneous labor. It pointed out that the growth in per capita incomes across countries cannot be explained by using a standard production function approach with stocks of land and capital and the size of the workforce. Therefore, early propopents of human capital theory (e.g. Schultz 1961, Becker 1962, Mincer 1974) urged to reconsider the representation of people in economic models. They emphasized to account for workers' skills and knowledge as key determinants of economic performance. In contrast to neoclassical economics, human capital theory endorsed a more general concept of capital that allowed in particular for human resources.

[6] The British Training Act of 1972 is exemplary for revisions of training policy to economic knowledge. Aiming to overcome market failure in training, it introduced levy-grant schemes for apprenticeship training at the sector level. These schemes operated until 1982 but were then abandoned in favor of more market-oriented solutions. See in particular Stevens (1999) for an account of vocational training policy in Britain.

In analogy to physical or financial capital, *human capital* is perceived as the *stock of knowledge, skills, health, or abilities* that is *embodied in a person* and that *can be put to productive work.* Moreover, human capital can be increased, alike investments in physical or financial capital, by training, schooling or health provisions (Becker 1962, 11).

In a pioneering contribution, Becker (1962) analyzes incentives for human capital investments and the emergence of positive spillovers. In particular, he focuses on motives for on-the-job training. In contrast to school training, on-the-job training is carried out at the workplace and through the firm. It raises the productivity of the workforce in the future, but involves costs to the training firm at present, such as time, effort, material, and equipment. Becker points out that resources spent for on-the-job training compete with alternative investment opportunities. By consequence, a firm incurring additional costs or lower revenues from training necessarily expects sufficiently larger revenues or fewer expenditures in the future.

Becker distinguishes between *general* and *specific* training. General training raises the worker's marginal product in all firms, regardless whether the worker remains with the firm or quits for another employer. Specific training, by contrast, gives rise to an increase in the marginal product of labor only in the training firm.

General training is equally valuable to all firms. This forces the training firm to pay workers the market wage. Otherwise, the worker could use the general skills and quit for another firm to obtain the prevailing market wage. If labor markets are competitive, the market wage equals the marginal product of labor. This implies that firms cannot draw any returns from training workers. All returns from the human capital investment are reflected in the higher wage and accrue fully to the worker. Receiving the full benefits, however, the worker has proper incentives to bear the investment costs. Becker concludes that firms might provide general training, but workers will effectively pay for it, either directly or indirectly via wage reductions in the investment period. Moreover, as all returns accrue to the worker, there are no spillovers from general training onto other firms. Thus, contrary to Pigou, training is efficient (Becker 1962, 17).

For *specific training*, Becker argues that firms will be able to recover training costs. Because these skills are of no use to other firms, the competitive market wage remains unaffected. The training firm will thus capture all returns from specific training through the increase in worker productivity. But since this investment is rewarding only as long as the worker stays with the firm, some of the returns from this investment will be shared between the firm and the worker in order to prevent workers from quitting. Yet again, as all returns are collected by the private parties there is no spillover to other firms by this type of investment (Becker 1962, 21).

In sum, standard human capital theory negates positive spillovers from vocational training. As private and social returns do not diverge, human capital investment will be at its socially optimal level. General training is paid for by the worker, specific training by both the employer and the worker. Becker acknowledges that insufficient general training may indeed arise if workers are exposed to credit constraints and/or are risk averse (Becker 1962, 41f.). In this case, however, training subsidies or public training provision are not warranted. Rather, educational policy should address the problems prevailing in credit and insurance markets that cause these training limitations.

2.3 Human Capital Investments and Imperfect Competition

Despite its theoretical appeal, standard human capital theory faces profound shortcomings. *Firstly*, it is (at least partially) contradicted by empirical evidence. In contrast to its central proposition, a large number of empirical studies confirm the existence of firm-provided *and* firm-financed general training. In fact, even despite the risk of quits, firms seem to incur substantial training costs to provide general skills to their workforce. Acemoglu & Pischke (1998) display various evidence for firm-financed general training. Several studies also document net training costs for apprenticeship training in

Germany. Harhoff & Kane (1997) estimate training costs by sector and firm size. The *gross* costs of apprenticeship training across sectors are displayed at $17,645 per apprentice and year with on average higher costs in larger firms and industrial sectors. Even after taking the apprentice's productivity into account, *net* costs to firms remain high at $10,657 per apprentice and year, or 60% of the gross costs.[7]

Secondly, human capital theory is unable to explain some common features of apprenticeship training, such as the fixed duration of training contracts or the restrictions on unilateral termination. Under the assumption of competitive labor markets with full information and perfect contracting, these particularities should be superfluous (Smits & Stromback 2001, 32).

The contradictions between human capital theory and empirical evidence have ignited new interest in the economics of training. Several recent publications investigate the economic rationales behind firm-financed general training. They explicitly consider imperfect competition. Even though Becker already noted the role of labor market conditions, he left oligopsonistic labor markets unconsidered and regarded the dichotomy of general and specific skills a useful simplification. Recent contributions question the assumption of a perfect labor market for general skills altogether and analyze training technology and labor market conditions separately.[8] In order to explain firm-financed general training these studies commonly feature labor market imperfections that give rise to a compressed wage structure. In such a setting, firms can extract a rent from employing skilled workers, as the market wage is below the marginal product of labor. General training increases the marginal product of labor. Because of the compressed wage structure, however, wages increase to a lesser degree. This turns technologically general skills into de facto firm-specific skills. Contrary to standard human capital theory it is therefore profitable for a firm to finance general training. In the optimum, a firm will provide a training level where the marginal increase in

[7]For detailed studies assessing the cost of apprenticeship training in Germany see in particular Bardeleben, Beicht & Fehér (1995) and Winkelmann (1997).

[8]See in particular Acemoglu & Pischke (1998) for a basic overview and survey of the literature.

the rent is equal to the marginal cost of training.

The literature has identified several mechanisms causing a compressed wage structure. They relate to production technology, market competition as well as informational and institutional conditions.

For instance, a compressed wage structure could result from *technological complementarities*. Capital and labor input may be complements. Such complementarities in the production technology increase the marginal product of labor. The value of a trained worker is therefore higher to the training firm than to outside firms (Acemoglu & Pischke 1998, 559). Complementarities may also arise in the training process when simultaneously instructing general and firm-specific skills is complementary (Franz & Soskice 1995, 219f.).

Competitive conditions on the output market could also account for wage compression. Hentschirsch (1999) analyzes training investments for product markets that are characterized by Cournot or Bertrand competition. Similarly, Gersbach & Schmutzler (2003) use a game-theoretic structure to show that firms will provide training when competition on the final market is sufficiently soft.

Another source of wage compression may be *informational asymmetries*. In a pioneering contribution, Katz & Ziderman (1990) show that when training is not verifiable to outside firms, workers will be unable to receive a wage reflecting their training level. The training firm has an informational advantage regarding training in comparison to other firms. Consequently, training will be worthwhile, as productivity increases more than wage payments. Likewise, a training firm may also dispose of superior knowledge about a worker's innate characteristics, i.e. abilities, talents and personal qualities. They are difficult to assess by other firms so that outside wage offers will fall short of a worker's marginal product (Acemoglu & Pischke 1998, 556f.).

Other labor market frictions impeding workers to quit instantaneously for outside firms may result from *mobility restrictions* and personal preferences. For instance, commuting or relocation costs render job turnover costly to workers. Other frictions are generally related to job search, such as *search costs*, search uncertainties and matching problems. Workers will abstain from perfect job turnover in regard of the costs associated with it. Again, the employing firm can appropriate a rent and wages are compressed, rendering general training investments profitable.

Finally, *labor market institutions* have been identified to cause wage compression, e.g., through minimum wages, collective bargaining, dismissal protection and unions (Acemoglu & Pischke 1998, Smits & Stromback 2001).

This review demonstrates that human capital investments under imperfect competition can explain why firms finance some general training even though workers are mobile and able to quit. But will human capital investment be carried out to a socially optimal extent? In fact, these models point out that training in imperfect labor markets is necessarily accompanied by a positive externality. Labor market frictions cause workers not to react instantaneously to wage differentials. They yield rents to firms and thereby provide training incentives. While frictions reduce turnover, they cannot inhibit quits entirely. Thus, positive spillovers arise because non-training firms benefit from training by employing workers trained in other firms without requiring a wage payment equal to the marginal product of labor. This is commonly referred to as *poaching*.[9] With an externality present, private incentives are insufficient and training will not be provided to an optimal extent (Stevens 1994).

In sum, labor market imperfections allow for an explanation of firm-financed vocational training, but they also assert the existence of a positive externality. This contrasts with conventional human capital theory and therefore invites to reconsider appropriate public policy. Comprehensive studies analyzing different policy instruments are lacking. The next section therefore introduces a simple model in order to put different policy instruments into analysis.

[9]More precisely, poaching may be active or passive in nature, i.e. attempts to recruit skilled workers may be systematic or resulting from general job turnover. However, we do not pursuit this distinction further.

3 Simple Model of Vocational Training

This section presents a simple model of apprenticeship training. By considering an oligopsonistic labor market with frictions, the model allows to explain why firms partially finance vocational training. However, at the same time, an externality can be shown to exist which leads to insufficient training. In later sections, the assumptions of this simple model will be relaxed and further complications will be added.

3.1 Model Structure and Assumptions

Consider an economy where firms rely on skilled workers to produce goods and services. Firms can obtain skilled workers in two ways: On the one hand, they can train unskilled workers by offering apprenticeship training. On the other hand, they can recruit skilled workers through the labor market by offering competitive wages.

Wage competition for skilled labor

Let firms compete for the services of skilled workers by posting wage offers. Workers arbitrage between firms' wage offers and choose to work for the firm with the highest wage. If labor markets were perfect, workers would instantaneously move to the highest paying firm. This forces firms to pay skilled workers a wage equal to their marginal product. In such a situation firms cannot obtain any return for general training expenses. They would therefore shift the costs of training onto trained workers or provide no general training at all. Essentially, this is the standard result of human capital theory according to Becker (1962).

In reality, labor markets are imperfect. Skilled individuals usually do not quit a firm instantaneously, although they could obtain a somewhat higher wage elsewhere. The literature review above has specified a number of labor market frictions that inhibit workers from perfect wage arbitrage, such as search and switching costs, information asymmetries, and institutional conditions. We abstain from modeling such frictions in full detail and use a reduced form instead. More precisely, we simply represent such frictions by assuming that workers favor high wages but also have some preferences over firms. We thereby bring about some attachment to firms even when wage differences are present. Workers will consequently only quit for another firm if the wage increase at least compensates for the lost attachment to the firm.[10]

Formally, assume workers to differ in location. We represent this by the differentiation parameter θ. Along the lines of Hotelling (1929) let there be a mass of homogeneous skilled workers N_t which are uniformly distributed along a street with length normalized to unity. There are two firms, $i = \{1, 2\}$, that produce at either end of the street. Firms make wage offers w_{it} in order to attract skilled workers. The higher a firm's wage offer in period t, the greater will be its share in recruiting the available skilled workforce. Workers, on the other hand, incur commuting costs in order to work for a firm. They will choose to work for the firm with the higher net wage $\tilde{w}_i$ where the net wage is simply the wage minus commuting costs.

$$\tilde{w}_{it} = w_{it} - T_i(\theta)$$

Let δ be the (time-invariant) linear rate of commuting expenses. A worker at θ faces travel costs $T_1(\theta) = \delta\theta$ to work with firm 1 and $T_2(\theta) = \delta(1-\theta)$ to work with firm 2.[11] Figure 1 graphically illustrates the allocation of the workforce depending on firms' wage offers and workers' residence locations. Workers located to the left of $\hat{\theta}$ will choose the wage offer of firm 1, workers to the right of $\hat{\theta}$ will choose to work for firm 2.

[10] As an example, firms could differ in their geographical location and workers incur commuting costs from their residence to the workplace. Therefore, when choosing among job offers, workers will trade off firms' wage offers against the costs of travelling to work. Alternatively, the quality of a job match could differ so that workers must decide between wages and good job matches. Similarly, affection to a firm could also be grounded to the liking of particular sectors, products and services, working conditions, workplaces, etc.

[11] It may reasonably be argued that commuting costs increase convex rather than linear with the distance from work. Assuming these costs to be quadratic, i.e. $T_1(\theta) = \delta\theta^2$ and $T_2(\theta) = \delta(1-\theta)^2$, leads to the same results. We use the linear set-up for simplicity.

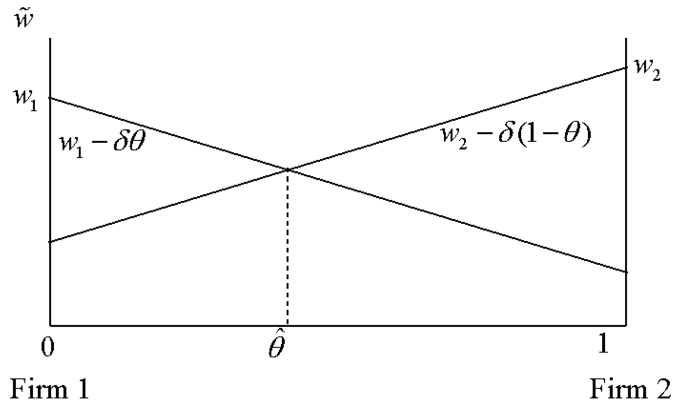

Figure 1: Hotelling street

Production

Let firms' production technology use skilled labor as the only input and possess constant returns to skilled labor. Equation (1) denotes the production function of firm i where v_{it} is a worker's (constant) marginal product and N_{it} is skilled labor employed by firm i in period t.

$$y_{it} = v_{it} N_{it} \tag{1}$$

Equation (1) implies that all skills are technologically general. This assumption is made to reflect that vocational education, at least in Germany, can safely be regarded as providing mainly general skills. Contents and schedule of apprenticeship training are regulated and training leads to a widely accepted training certificate. Moreover, the notion of specific training can be questioned to be of great practical relevance.[12]

Also notice that two interpretations of labor productivity can be given. The parameter v_{it} could simply indicate the firm's technological productivity. Alternatively, it could signify the firm's competitiveness in the output market

[12] Lazear (2003) forwards that few skills are limited only to a single firm and thereby doubts sizeable firm-specific skills to exist. Instead he argues that firms seek specific input-combinations of general skills, stressing the importance of labor markets rather than skill-types.

to be able to obtain large selling prices.

Labor supply

Firms offering apprenticeship training to unskilled workers affect future labor supply. They enlarge the skilled workforce that can potentially be recruited by all firms. Skilled workers available for employment in the future period N_{t+1} consist of the skilled workforce from the previous period N_t and the sum of newly trained apprentices A_{it}.

$$N_{t+1} = N_t + \sum_i A_{it} \tag{2}$$

This formulation abstracts from retirement and assumes all apprentices to enter the labor market. Also, it neglects any acquisition of general skills outside on-the-job training, for instance through schooling or universities. These are clearly simplifications which could be accounted for in a richer setting.[13]

Moreover, let labor supply in each period be an increasing function of workers' net wages, $N_t(\tilde{w}_t)$. By assuming labor supply to rise in wage income, $\frac{\partial N_t(\tilde{w}_t)}{\partial \tilde{w}_t} > 0$, alternative choices of workers' time outside of the labor market are taken into account. Such alternatives could be, for instance, the value of home production and/or unemployment benefits.

Apprentice's training decision

An unskilled worker faces the decision whether to undergo apprenticeship training and thereby become a skilled worker or alternatively to remain unskilled and obtain an income from simple labor, home production or welfare benefits. Let u_t denote the income available to an unskilled worker in period t. By contrast, apprentices usually receive some apprenticeship pay in the initial training period, w_t^a, which usually falls considerably short of the income for unskilled labor. However in the future period, upon completion of

[13] Equation (2) could easily be enriched by retirees exiting from the labor force and university graduates entering into it. For this model it is only important that apprenticeship training increases the future skilled workforce.

the vocational education, they can obtain the wage for skilled labor.

Similar to Becker (1962) the apprentice's training decision can be formalized by an investment problem. A rational, risk-neutral unskilled worker will choose apprenticeship training if the present value of the expected net income stream as a skilled worker exceeds the present value of the income stream from remaining unskilled. This is put formally in equation (3) where $\tilde{w}_t$ is the net wage of a skilled worker accounting for commuting costs and ρ denotes the discount factor at the market rate of interest r, $\rho = \frac{1}{1+r}$.

$$w_t^a + \sum_{t,t \neq 1} \rho^{t-1} E\left(\tilde{w}_t\right) \geq \sum_t \rho^{t-1} u_t \tag{3}$$

This formulation declines any non-monetary costs or benefits of apprenticeship training, such as joy or effort from learning. Note also that w_t^a can be negative. In this case, instead of receiving apprenticeship pay, apprentices transfer "tuition fees" to the firm in the training period.

For simplicity, normalize the present value of the income stream of an unskilled worker to zero, $\sum \rho^{t-1} u_t \equiv 0$. The participation constraint of an apprentice to become a skilled worker thus reduces to equation (4).

$$w_t^a + \sum_{t+1} \rho^{t-1} E\left(\tilde{w}_t\right) \geq 0 \tag{4}$$

Rewriting to

$$-w_t^a \leq \sum_{t+1} \rho^{t-1} E\left(\tilde{w}_t\right)$$

this essentially states that an apprentices is willing to initially forgo earnings and invest w_t^a into vocational training at most equal to the net increase in income in comparison to remaining remaining unskilled. With alternative income normalized to zero, forgone earnings may amount at most to the present value of future net income.

Notice that this participation constraint tacitly assumes workers to face no credit constraints. In particular, they are able to forego earnings and transfer the training investment w_t^a to the firm by borrowing against their future income. By constrast, if workers are credit constraint or have only partial access to capital markets, they are unable to carry out the investment as the wage in the training period is to exceed some threshold value m. In this case, the additional credit constraint (5) is to be taken into account.

$$w_t^a \geq m \tag{5}$$

Training costs

Apprenticeship training brings about costs to firms. It involves direct costs that are made up of remuneration to trainers and material expenses. Moreover, there are indirect costs that result from forgone production when staff or machines are used for training.

Let $c_i(A_{it})$ denote the training cost function of firm i where A_{it} is the number of apprentices trained by the firm in period t. We assume increasing marginal training costs in order to display limitations to the capacity of training facilities and training staff. Equation (6) summarizes these properties.

$$c_i(A_{it}) \text{ with } c_i' > 0,\ c_i'' > 0,\ c_i(0) = 0 \tag{6}$$

By presuming an explicit training cost function we tacitly assume that production and training technology can be separated. This assumption is often made in the literature to simplify the problem (e.g. Stevens 1994, Acemoglu & Pischke 1998). Of course, this assumption does not represent reality in small companies where older, experienced employees train apprentices and thereby necessarily incur opportunity costs from forgone production. By contrast, large companies often have special training departments and respective instructing staff so that this assumption is in line with reality in large firms.[14]

[14] Note that cost assessments of apprenticeship training often also include direct pay by the firm to apprentices. *Gross* training costs measure all expenses of firms and will therefore also take *apprentices' wages* into account. By contrast, *net* training costs take the contribution of apprentices to production into account (e.g. Bardeleben et al. 1995). Our formulation makes the simplifying assumption that there is no production value of apprentices during the training period. Any pay to apprentices therefore only constitutes a transfer mechanism between firms and workers that allows to shift the costs of training between both parties.

Time structure

To keep things simple, we restrict ourselves to a two-period model with a present and a future period, $t = \{t, t+1\}$. In the present period, firms compete for the currently existing skilled workforce. In addition, they may decide to train unskilled workers through an apprenticeship that will thereby become skilled workers in the future. We call this period the *training period*. In the future period, firms compete anew for skilled workers. With training being sufficiently general, this allows for poaching of the newly trained workers by other firms. We therefore refer to this period as the *poaching period*. Thus, the time structure of this two-period model can be summarized as follows:

- Training period t:
  Firms make wage offers w_{it} and take A_{it} apprentices for training.
  $N_t(.)$ workers choose their employer and production takes place.

- Poaching period $t+1$:
  Firms make wage offers $w_{i,t+1}$
  $N_{t+1}(.)$ workers choose their employer and production takes place.

3.2 Private Optimum

With this being the basic set-up let us now solve the model by backward induction. Consider first the poaching period. Firms compete for skilled workers by posting appropriate wage offers, $w_{1,t+1}$ and $w_{2,t+1}$. A proportion of the skilled workforce will take the offer at firm 1, while the remainder will take the offer at firm 2. We can infer the share of the skilled workforce to each firm from the worker located at $\hat{\theta}_{t+1}$, who is just indifferent between working for either firm. This is the case if net wages equal, or, put differently, if the arbitrage condition (7) is fulfilled.

$$w_{1,t+1} - \delta \cdot \hat{\theta}_{t+1} = w_{2,t+1} - \delta \cdot \left(1 - \hat{\theta}_{t+1}\right) \tag{7}$$

The share of workers employed by both firms can therefore be stated as a function of wages by

$$\hat{\theta}_{t+1} = \frac{w_{1,t+1} - w_{2,t+1} + \delta}{2\delta} \tag{8}$$

and likewise by

$$1 - \hat{\theta}_{t+1} = \frac{w_{2,t+1} - w_{1,t+1} + \delta}{2\delta} \tag{9}$$

Firms maximize profits by optimally choosing the wages they pay. Profits are simply $\pi_{i,t+1} = (v_{i,t+1} - w_{i,t+1})\, N_{i,t+1}$ and the objective functions of firms 1 and 2 are therefore

$$\max_{\{w_{1,t+1}\}} \left\{ \pi_{1,t+1} = (v_{1,t+1} - w_{1,t+1})\, \hat{\theta}_{t+1}\,(.)\, N_{t+1} \right\} \tag{10}$$

$$\max_{\{w_{2,t+1}\}} \left\{ \pi_{2,t+1} = (v_{2,t+1} - w_{2,t+1}) \left(1 - \hat{\theta}_{t+1}\,(.)\right) N_{t+1} \right\} \tag{11}$$

From the first order conditions

$$-2w_{1,t+1} + w_{2,t+1} - \delta + v_{1,t+1} = 0 \tag{12}$$

$$-2w_{2,t+1} + w_{1,t+1} - \delta + v_{2,t+1} = 0 \tag{13}$$

we can obtain the reaction functions, i.e. firm's optimal wage offer as a function of the other firm's wage offer.

$$w^*_{1,t+1}(w_{2,t+1}) = \frac{1}{2}\left(v_{1,t+1} + w_{2,t+1} - \delta\right) \tag{14}$$

$$w^*_{2,t+1}(w_{1,t+1}) = \frac{1}{2}\left(v_{2,t+1} + w_{1,t+1} - \delta\right) \tag{15}$$

The second order conditions for a local maximum are fulfilled with $\frac{\partial^2 \pi_{i,t+1}}{\partial^2 w_{i,t+1}} = -2 < 0$. From equating the reaction functions then follow the equilibrium wages $w^*_{1,t+1}$ and $w^*_{2,t+1}$.

$$w^*_{1,t+1} = \frac{2}{3} v_{1,t+1} + \frac{1}{3} v_{2,t+1} - \delta \tag{16}$$

$$w^*_{2,t+1} = \frac{1}{3}v_{1,t+1} + \frac{2}{3}v_{2,t+1} - \delta \tag{17}$$

Using equations (8) and (9), the equilibrium wages $w^*_{1,t+1}$ and $w^*_{2,t+1}$ determine the allocation of skilled workers across both firms in period $t+1$. A share $\hat{\theta}^*_{t+1}$ of the workforce is employed by firm 1 and a share $(1 - \theta^*_{t+1})$ by firm 2.

$$\hat{\theta}^*_{t+1} = \frac{w^*_{1,t+1} - w^*_{2,t+1} + \delta}{2\delta} = \frac{1}{2} + \frac{v_{1,t+1} - v_{2,t+1}}{6\delta} \tag{18}$$

$$1 - \hat{\theta}^*_{t+1} = \frac{w^*_{2,t+1} - w^*_{1,t+1} + \delta}{2\delta} = \frac{1}{2} - \frac{v_{1,t+1} - v_{2,t+1}}{6\delta} \tag{19}$$

Using $w^*_{1,t+1}$, $w^*_{2,t+1}$ and $\hat{\theta}^*_{t+1}$, equilibrium firm profits can be derived as

$$\pi^*_{1,t+1} = \left(v_{1,t+1} - w^*_{1,t+1}\right)\hat{\theta}^*_{t+1} N_{t+1} = \frac{\left(v_{1,t+1} - v_{2,t+1} + 3\delta\right)^2}{18\delta} N_{t+1} \tag{20}$$

$$\pi^*_{2,t+1} = \left(v_{2,t+1} - w^*_{2,t+1}\right)\left(1 - \hat{\theta}^*_{t+1}\right) N_{t+1} = \frac{\left(v_{2,t+1} - v_{1,t+1} + 3\delta\right)^2}{18\delta} N_{t+1} \tag{21}$$

These results can be summarized in a proposition.

Proposition 1 *When labor market frictions render job mobility costly, i.e. for $\delta > 0$, workers refrain from perfect wage arbitrage and show some attachment to a firm. This gives firms some wage-setting power. In equilibrium, firms post wages below the worker's marginal product, $w^*_{i,t+1} < v_{i,t+1}$. Firms therefore earn some employment rents and realize positive profits, $\pi^*_{i,t+1} > 0$.*

Proof. See appendix. ■

In order to easily understand these results, consider the case of *homogeneous technology* across firms, $v_{1,t+1} = v_{2,t+1} = v_{t+1}$. With identical production functions, firms will also post identical wage offers for skilled labor, $w^*_{1,t+1} = w^*_{2,t+1} = w^*_{t+1}$, and the equilibrium wage is simply $w^*_{t+1} = v_{t+1} - \delta$. This can be easily interpreted: A worker located at firm 1 would incur commuting costs δ to quit for an employment at firm 2. The same is true, vice versa, for the worker at firm 2. Firms can therefore safely reduce the wage from the marginal product v_{t+1} by an amount δ without risking to lose all workers. Thus, firms earn a rent of δ from employing a skilled worker. With identical wage offers, both firms will acquire half of the available workforce, $N^*_{1,t+1} = N^*_{2,t+1} = \frac{1}{2} N_{t+1}$, and earn profits from employment of $\pi^*_{1,t+1} = \pi^*_{2,t+1} = \frac{1}{2}\delta N_{t+1}$.

For *heterogeneous technology*, in contrast to the case of homogeneous technology, there is wage dispersion and differing firm sizes in equilibrium. Wages increase in own, but decrease in foreign productivity, $\frac{\partial w^*_{i,t+1}}{\partial v_{i,t+1}} \geq 0$ and $\frac{\partial w^*_{i,t+1}}{\partial v_{j,t+1}} \leq 0$. These results conform to empirical estimates of the labor market and also reproduce the outcomes from other frictional labor market models (e.g. Montgomery 1991, Lang 1991).

With this being the situation in the poaching period let us now turn to the training period. In this period, firms compete for the (initial) skilled workforce N_t and decide whether to provide costly training or not. Firms' objective functions consist of profits from employing skilled workers in the present and the future period reduced by pay to apprentices and training costs, $\rho\pi^*_{i,t+1} + \pi_{it} - w^a_t A_{it} - c_i(A_{it})$. We can thus characterize firms' decision problem by

$$\underset{\{w_{it}, A_{it}\}}{\arg\max} \left\{ \rho\pi^*_{i,t+1} + (v_{it} - w_{it}) N_{it} - w^a_t A_{it} - c_i(A_{it}) \right\} \tag{22}$$

In this period, just as in the poaching period, firms' shares of the available workforce are determined using the indifferent worker $\hat{\theta}_t$. Deriving an indifferent worker analoguous to equation (7) then allows to write the optimization problem for firm 1

$$\underset{\{w_{1t}, A_{1t}\}}{\arg\max} \left\{ \rho R^*_{1,t+1} N_{t+1} + (v_{1t} - w_{1t}) \hat{\theta}_t N_t - w^a_t A_{1t} - c_1(A_{1t}) \right\} \tag{23}$$

and for firm 2

$$\underset{\{w_{2t}, A_{2t}\}}{\arg\max} \left\{ \rho R^*_{2,t+1} N_{t+1} + (v_{2t} - w_{2t}) \left(1 - \hat{\theta}_t\right) N_t - w^a_t A_{2t} - c_2(A_{2t}) \right\} \tag{24}$$

The first order conditions with respect to the wage offers w_{1t} and w_{2t} are given by (25) and (26) which, as before, represent a pair of reaction functions $w_{1t}^{*}(w_{2t})$ and $w_{2t}^{*}(w_{1t})$.

$$-2w_{1t} + w_{2t} - \delta + v_{1t} = 0 \tag{25}$$

$$-2w_{2t} + w_{1t} - \delta + v_{2t} = 0 \tag{26}$$

Setting equal then again allows to solve for the equilibrium wages.

$$w_{1t}^{*} = \frac{2}{3}v_{1t} + \frac{1}{3}v_{2t} - \delta \tag{27}$$

$$w_{2t}^{*} = \frac{1}{3}v_{1t} + \frac{2}{3}v_{2t} - \delta \tag{28}$$

Equations (27) and (28) state, analoguous to before, that wages for skilled workers in period t are below the marginal product of labor. Firms' respective shares of the workforce in period t are $\hat{\theta}_t^*$ and $1 - \hat{\theta}_t^*$.

Now take the first order conditions with respect to the number of apprentices by the firm. The second order conditions are satisfied with $\frac{\partial^2}{\partial A_{it}} = -c_i'' < 0$. Thus, equations (29) and (30) display the conditions for optimal firm provision of apprenticeship training.

$$\rho R_{1,t+1}^{*} - w_{1t}^{a} - c_1'(A_{1t}) = 0 \tag{29}$$

$$\rho R_{2,t+1}^{*} - w_{2t}^{a} - c_2'(A_{2t}) = 0 \tag{30}$$

When firms possess all bargaining power, the pay to apprentices in the training period can be inferred from the participation constraint (4), i.e. $w_t^a = -\rho E(\tilde{w}_{t+1})$. Inserting into the first order conditions then gives the optimality conditions for vocational training.

$$\rho R_{1,t+1}^{*} + \rho E(\tilde{w}_{t+1}) = c_1'(A_{1t}^{*}) \tag{31}$$

$$\rho R_{2,t+1}^{*} + \rho E(\tilde{w}_{t+1}) = c_2'(A_{2t}^{*}) \tag{32}$$

From these conditions we can deduce proposition (2).

Proposition 2 *In the private optimum, vocational training is carried out such that marginal costs and returns of an additional apprentice are equal. Firms provide A^*_{1t} and A^*_{2t} apprenticeships. Costs and returns from apprenticeship training are shared by the apprentice and the firm in the training contract.*

Proof. Obvious from (31) and (32). ■

The intuition for this result is straightforward: Apprenticeship training is beneficial to both parties in the training contract. Both are therefore also willing to bear some training costs. The apprentice, on the one hand, is turned from an unskilled worker to a skilled worker. She is willing to invest in vocational training as she can thereby expect an increase in her future net income. The firm, on the other hand, earns some returns from training, too. It obtains a rent from employing former apprentices as skilled workers. This rent occurs because mobility frictions allow for wages below the marginal product of labor. It can be used to finance costly general training.

The private optimum for vocational training is disturbed when apprentices face financial constraints. Then, instead of the participation constraint, apprenticeship pay is subject to the credit constraint condition (5). Apprentices can only participate in the training costs as long as $w^a_t \geq m$. Under binding credit restrictions, the optimality conditions for vocational training are

$$\rho\pi^*_{1,t+1} - m = c'_1\left(A^{cc}_{1t}\right) \tag{33}$$

$$\rho\pi^*_{2,t+1} - m = c'_2\left(A^{cc}_{2t}\right) \tag{34}$$

Proposition 3 *Credit constraints reduce the private optimum for vocational training to A^{cc}_{it}, where $A^{cc}_{it} < A^*_{it}$.*

Proof. The credit constraint is binding for $w^a_t = m$ where $m > -\rho E\left(\tilde{w}_{t+1}\right)$. Rewriting to $-m < \rho E\left(\tilde{w}_{t+1}\right)$ and inserting allows to compare the training conditions under credit constraints (33) and (34) to the normal case (31) and

(32). With the left-hand side smaller under credit constraints, it becomes immediately clear that $A_{it}^{cc} < A_{it}^{*}$. ■

Due to capital market imperfections and informational restraints, the apprentice cannot borrow against her future net income that would return from vocational training. Being credit constraint, the apprentice thus cannot make the necessary training investments. Training will therefore be lower than it would otherwise be. This result essentially replicates earlier results from human capital theory in the context of apprenticeship training (e.g. Becker 1962, Smits & Stromback 2001).

From comparative-static analysis some further results can be derived that characterize the private optimum. They are summarized in the following propositions:

Proposition 4 *Apprenticeship training*
a) increases in firms with higher productivity, $\frac{\partial A_{it}^{*}}{\partial v_{it}} > 0$,
b) decreases in the opponents productivity, $\frac{\partial A_{it}^{*}}{\partial v_{jt}} < 0$,
c) decreases in the interest rate, $\frac{\partial A_{it}^{*}}{\partial r} < 0$,
d) decreases in the training costs, $\frac{\partial A_{i}^{*}}{\partial c_{i}} < 0$.

Proof. Apply the implicit function theorem on the optimal training conditions (29) and (30). ■

A higher productivity increases the value of a skilled worker to a firm and thus stimulates to train more. By contrast, a higher productivity of the opponent decreases the ability to retain skilled workers. Larger training costs and a higher interest rate obviously diminish training.

Proposition 5 *Labor market frictions increase firms' share in the training costs, but decrease overall, apprenticeship training,* $\frac{\partial A_{it}^{*}}{\partial \delta} < 0$.

Proof. Apply the implicit function theorem on conditions (31) and (32) and obtain $\frac{\partial A_{it}^{*}}{\partial \delta} < 0$. For detailed formal proof see appendix. ■

This result can be explained quite intuitively. Rising labor market frictions dampen wage arbitrage by workers and increase firms' monopsony power. The wage for skilled workers decreases, thereby obviously diminishing the incentives for unskilled workers to seek training. By contrast, a lower wage allows firms to obtain larger rents from employing workers which improves firms' training incentives. Overall, however, private incentives decrease in the costs of labor market frictions as the training firm cannot collect all rents from training.

Proposition 6 *Firms differ in training quotas α_{it}. In particular, a firm's training quota increases in own productivity, $\frac{\partial \alpha_{it}}{\partial v_{it}} > 0$, and decreases in costs to apprentices, $\frac{\partial \alpha_{it}}{\partial c_i} < 0$.*

Proof. Let α_{it} define the training quota of firm i where $\alpha_{it} = \frac{A_{it}}{N_{it}}$. Inserting the values from the individual firm's optimum and applying the implicit function theorem then leads to the proposition. ■

Clearly, differences in apprenticeship training will also be reflected in training quotas, i.e. in the proportion of apprentices to employees. The model thereby allows to account for empirical observations of the German apprenticeship system. In reality, training quotas differ strongly within a sector as well as across sectors.

3.3 Social optimum

The analysis so far showed that firms provide apprenticeship training and the training costs are shared by the firm and the apprentice. It is now of interest whether the actions of the contracting parties are also socially efficient. We will therefore compare the private optimum with the social optimum.

In this simple two-firm two-period model, social welfare can be denoted by the net output produced by skilled workers in both periods. Training costs are subtracted because resources put to vocational training have alternative uses.

$$W = \sum_{t=1,2} \rho^{t-1} W_t - \sum_{i=1,2} c_i \left(A_{it} \right) \tag{35}$$

In period t, social welfare W_t consists of firms' profits and net wages.

$$W_t = \int_0^{\hat{\theta}_t} \left(v_{1t} - w_{1t} + w_{1t} - \delta\theta \right) N_t d\theta + \int_{\hat{\theta}_t}^1 \left(v_{2t} - w_{2t} + w_{2t} - \delta \left(1 - \theta \right) \right) N_t d\theta \tag{36}$$

The first term denotes profits and net wages from employment at firm 1 and while the second term does so for firm 2. Simplifying and rearranging leads to equation (37). Intuitively, it states that social welfare is total production net of commuting and training costs.

$$W = \sum_{t=1,2} \rho^{t-1} N_t \left(\int_0^{\hat{\theta}_t^*} (v_{1t} - \delta\theta) d\theta + \int_{\hat{\theta}_t^*}^1 (v_{2t} - \delta \left(1 - \theta \right)) d\theta \right) - \sum_{i=1,2} c_i \left(A_{it} \right) \tag{37}$$

Solving the integral, inserting for $\hat{\theta}_t^*$ and some algebraic rearrangements then gives equation (38). See the appendix for detailed derivation.

$$W = \sum_{t=1,2} \rho^{t-1} N_t \left(\frac{v_{1t} + v_{2t}}{2} + \frac{5}{36\delta} \left(v_{1t} - v_{2t} \right)^2 - \frac{1}{4}\delta \right) - \sum_{i=1,2} c_i \left(A_{it} \right) \tag{38}$$

The social planner's problem is to optimally choose the number of apprentices to train.

$$\max_{\{A_{1t}, A_{2t}\}} W \left(A_{1t}, A_{2t} \right) \tag{39}$$

From the first order conditions then obtain the conditions for socially optimal apprenticeship training A_{it}° in each firm.

$$\rho\left(\frac{v_{1t}+v_{2t}}{2}+\frac{5}{36\delta}\left(v_{1t}-v_{2t}\right)^2-\frac{1}{4}\delta\right)-c_i'\left(A_{it}^{\circ}\right)=0\ \ \forall i=\{1,2\} \tag{40}$$

This allows to state proposition (7).

Proposition 7 *In the private optimum, apprenticeship training in each firm is lower than socially desirable, i.e.* $A_{it}^{*}<A_{it}^{\circ}$.

Proof. Compare the condition for the social optimum (40) with the conditions for the private optimum (29) and (30) to derive that too few apprentices receive training, $A_{it}^{*}<A_{it}^{\circ}$. See the appendix for detailed proof. ■

The intuition for this result can be put as follows: Apprenticeship training is socially inefficient as the social returns of apprenticeship training exceed the private returns collected by the apprentices and the training firm.

To pinpoint this result, look at the returns from vocational training separately. For the *apprentice* the return from vocational training is the present value of the expected net wage as a skilled worker.

$$R_w=\rho E\left(\tilde{w}_{t+1}^{*}\right)$$

The expected net wage can be calculated using $w_{1,t+1}^{*}$ and $w_{2,t+1}^{*}$.

$$R_w=\rho\left(\frac{v_{1,t+1}+v_{2,t+1}}{2}+\frac{\left(v_{1,t+1}-v_{2,t+1}\right)^2}{6\delta}-\frac{5}{4}\delta\right)$$

See the appendix for detailed algebraic derivation.

For the *training firm* the return consists of the rent that is earned in the future on skilled employment from apprentices that remain with the firm. It arises as the worker's marginal product exceeds the wage due to frictions. Note that rents differ across firms according to labor productivity.

$$R_{1,f}=\rho\frac{\left(v_{1,t+1}-v_{2,t+1}+3\delta\right)^2}{18\delta}$$

$$R_{2,f} = \rho \frac{(v_{2,t+1} - v_{1,t+1} + 3\delta)^2}{18\delta}$$

Private returns collected by the parties in the training contract are

$$R_{1,priv} = R_w + R_{1,f} = \frac{5v_{1,t+1} + v_{2,t+1}}{6} + \frac{4\,(v_{1,t+1} - v_{2,t+1})^2}{18\delta} - \frac{3}{4}\delta$$

$$R_{2,priv} = R_w + R_{2,f} = \frac{v_{1,t+1} + 5v_{2,t+1}}{6} + \frac{4\,(v_{1,t+1} - v_{2,t+1})^2}{18\delta} - \frac{3}{4}\delta$$

Social returns are given by

$$R_{social} = \rho \left(\frac{v_{1,t+1} + v_{2,t+1}}{2} + \frac{5}{36\delta}\,(v_{1,t+1} - v_{2,t+1})^2 - \frac{1}{4}\delta \right)$$

Finally, an *externality* of vocational education can be easily derived from calculating the difference between social and private returns.

$$X_i = R_{social} - R_{priv}$$

Thus, the positive externality associated with an additional apprenticeship by each firm is given by

$$X_1 = \frac{1}{2}\delta - \frac{(v_1 - v_2)}{12\delta}\,(v_1 - v_2 + 4\delta)$$

$$X_2 = \frac{1}{2}\delta - \frac{(v_2 - v_1)}{12\delta}\,(v_2 - v_1 + 4\delta)$$

Proposition 8 *Vocational training brings about a positive externality onto other firms. The externality decreases in firm's productivity, but increases in opponents productivity and labor market frictions.*

Proof. From comparative-static analysis follows $\frac{\partial X_i}{\partial v_{i,t+1}} < 0$, $\frac{\partial X_i}{\partial v_{j,t+1}} > 0$, and $\frac{\partial X_i}{\partial \delta} > 0$ for $i, j = \{1, 2\}\,; i \neq j$. ■

The intuition for this result is straightforward. Because of wage competition among firms, only a share of apprentices remain with the training firm. Put differently, due to poaching of skilled workers, vocational training carries a positive externality onto other firms.

As an illustration, consider the case of *homogeneous technology* as before. By setting $v_{1,t+1} = v_{2,t+1} = v_{t+1}$, the expected return from apprenticeship training to an unskilled worker is simply

$$R_w = \rho \left(v_{t+1} - \frac{5}{4}\delta \right)$$

Likewise, firms' returns from training an apprentice are

$$R_f = R_{1f} = R_{2f} = \rho\frac{\delta}{2}$$

Private returns of the apprentice and the firm are therefore

$$R_{priv} = R_w + R_f = \rho \left(v_{t+1} - \frac{3}{4}\delta \right)$$

Social returns are however

$$R_{social} = v_{t+1} - \frac{1}{4}\delta$$

which clearly indicates a positive externality from training an apprentice.

$$X = X_1 = X_2 = \rho\frac{\delta}{2}$$

Intuitively, there is a return to vocational training that can neither be obtained by the apprentice, nor the training firm, but rather accrues to the rival firm. The opponent obtains a rent which arises because it can poach some apprentices without being required to pay these skilled workers their full marginal product. In the symmetric case this rent amounts to $\rho\frac{\delta}{2}$. With some returns from training remaining unconsidered by the private parties, vocational training will be short of its socially efficient level, $A^*_{it} < A^\circ_{it}$. Figure (2) summarizes this situation for the symmetric case with homogeneous technology.

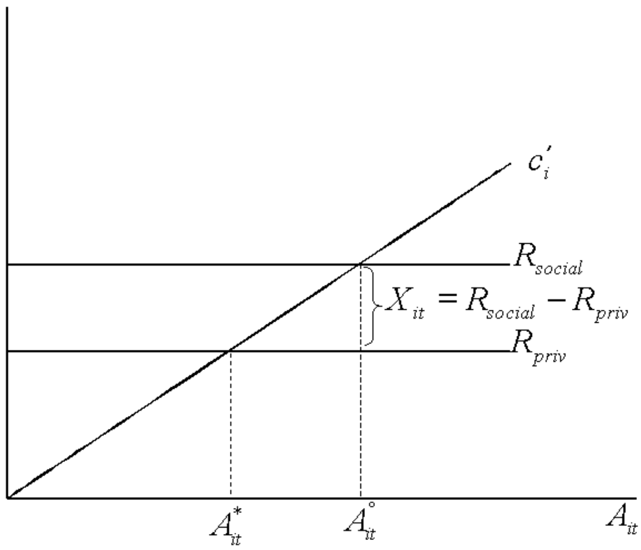

Figure 2: Social and private returns to education

3.4 Discussion

In this simple model, due to costly labor market frictions, skilled workers are inhibited from perfect wage arbitrage. There is some attachment for a skilled individual to work for firm i although a higher wage could be earned elsewhere. These labor market imperfections allow firms to obtain some rents from employment. They induce firms not only to provide but also to partially finance apprenticeship training providing general skills. Thereby the net costs of training carried by firms can be explained.

Moreover, the model also proposes the existence of an externality from vocational training. Offering apprenticeships allows for positive spillovers on rival firms that result from the possibility to poach workers. While frictions cause some attachment of workers to the firm, some fraction still quits for another employer, ridding the training firm from its investment. With training efforts distorted, from a social point of view too few apprenticeships are provided.

This model drew on a number of simplifying assumptions. Firstly, the model is limited to two periods only. Secondly, apprenticeship training consists of perfectly general skills with no firm-specific element. Thirdly, apprentices and workers stay within the sector and there is no retirement. These assumptions can be relaxed, altering the results quantitatively, but not qualitatively.

In the following section, emphasis will be put on the question whether public policy possesses instruments to internalize the externality and thereby increase social welfare.

4 Incentive Schemes

It was noted above that already Pigou (1912) named poaching as the cause of underprovision of vocational education. With training possessing positive social returns, he proposed to introduce a system of "bounties and taxes" as a means to provide additional training incentives to firms. This section will take up this proposal. It will analyze in further detail several *incentive schemes* for vocational education. Other policy instruments, such as the regulation of training contracts, although also conceivable, will not be considered here. They will be the focus of another paper.

4.1 Pigouvian Subsidies

For theoretical reference consider at first the case of ideal Pigouvian subsidies. Under this scheme, firm i receives a subsidy z_i per apprentice it takes for training. The total subsidy payment β_i to firm i is therefore

$$\beta_i = z_i A_i \tag{41}$$

The subsidy payment alters firms' objective functions in the training period by an additional term. The optimization problems for firm 1 and 2 thus become

$$\underset{\{w_{1t},A_{1t}\}}{\arg\max} \left\{\rho R_{1,t+1} N_{t+1} + (v_{1t} - w_{1t})\, N_{1t} - w_t^a A_{1t} - c_1\,(A_{1t}) + z_1 A_{1t}\right\} \tag{42}$$

$$\underset{\{w_{2t},A_{2t}\}}{\arg\max} \left\{\rho R_{2,t+1} N_{t+1} + (v_{2t} - w_{2t})\, N_{2t} - w_t^a A_{2t} - c_2\,(A_{2t}) + z_2 A_{2t}\right\} \tag{43}$$

The first order conditions with respect to firms' wage offers remain unaffected. Equilibrium wages in the training period are therefore again given by equations (27) and (28). However, the subsidy payment modifies firms' optimality conditions for the number of apprentices.

$$\rho R_{1,t+1} - w_t^a + z_1 = c_1'(A_{1t}^{Pigou}) \tag{44}$$

$$\rho R_{2,t+1} - w_t^a + z_2 = c_2'(A_{2t}^{Pigou}) \tag{45}$$

Equations (44) and (45) state, similar to before, that firms will offer vocational training such that the marginal returns from an additional apprentice equal the marginal costs. Naturally, with the subsidy payment raising the marginal returns, the optimal number of apprenticeships in each firm increases. Using the inverse of the cost function, the optimal number of apprenticeships provided by each firm can be deduced.

$$A_1^{Pigou} = c_1'^{-1}(.) = \phi_1(z_1, v_{1,t+1}, v_{2,t+1}, \rho, \delta) \tag{46}$$

$$A_2^{Pigou} = c_2'^{-1}(.) = \phi_2(z_1, v_{1,t+1}, v_{2,t+1}, \rho, \delta) \tag{47}$$

A benevolent social planner would strive to set the subsidy such that the private training conditions equal the social training condition. Thereby we can state the next proposition:

Proposition 9 *An ideal Pigouvian subsidy scheme allows to restore efficiency. Optimal subsidy rates for each firm are*

$$z_1 = \rho\left(\tfrac{1}{2}\delta - \tfrac{(v_1 - v_2)^2}{12\delta} - \tfrac{(v_1 - v_2)}{3}\right) \equiv X_1 \tag{48}$$

$$z_2 = \rho\left(\tfrac{1}{2}\delta - \tfrac{(v_2 - v_1)^2}{12\delta} - \tfrac{(v_2 - v_1)}{3}\right) \equiv X_2 \tag{49}$$

Proof. Equate conditions (44) and (40) as well as (45) and (40) and easily obtain z_1 and z_2. ■

The intuition to this result is common. By providing firms with a subsidy per apprentice that equals the marginal externality, private and social incentives to training are aligned. The positive spillovers from apprenticeship training are internalized and first best training levels can be achieved.

In analogy with the externality, comparative-static analysis shows that the optimal subsidy rate z_i decreases in firm's own productivity $v_{i,t+1}$, but increases in the opponent's productivity $v_{j,t+1}$ and the friction parameter δ.

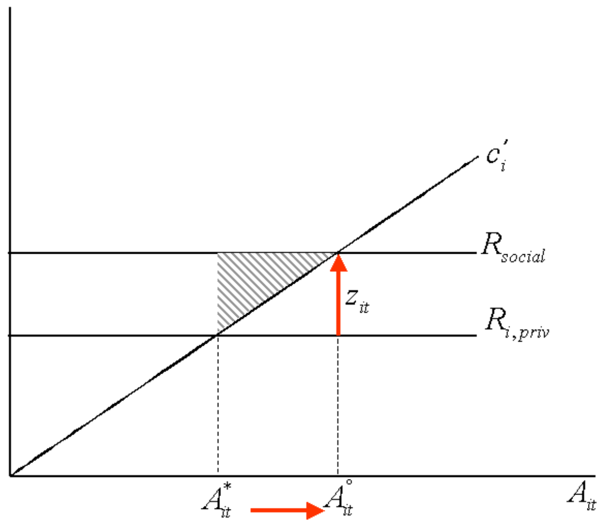

Figure 3: Pigouvian subsidy

Also note that the ideal subsidy does not depend at all on firm's training cost function.

Figure (3) depicts this situation. In the private optimum, firm i equalizes the marginal private returns of an additional apprenticeship $R_{i,priv}$ to its marginal returns c_i'. An additional subsidy per apprentice z_i increases the marginal private returns such that the social optimum A_i° can be achieved. The Pigouvian subsidy thereby brings about a welfare gain that is equivalent to the striped triangle.

Although an ideal Pigouvian subsidy possesses a strong theoretical appeal for its ability to correct for the positive spillovers from vocational training, its premises are highly unrealistic. In particular, the instrument requires to calculate subsidy rates at the firm level. As the externality arises because of poaching, the specific subsidy must be determined from firms' *future* labor productivities. This demands to gather and process very accurate and detailed information on production as well as output market parameters, which would already cause severe difficulties in practice. Additionally, and making matters even worse, the interests of firms and government to disclose the information may strongly diverge. While the government seeks to obtain precise information to assess the size of the externality, firms have an interest to falsely state their productivity levels in order to obtain larger subsidy payments.[15]

Clearly, with only limited and inaccurate information available, the possibility to calculate firm-specific subsidies is strongly reduced. Alternatively, one may therefore consider to introduce a uniform subsidy that provides additional training incentives *on average* for the economy or a particular sector. Although this departs from the theoretical ideal, such an instrument could be much simpler to determine and administer. It will now be analyzed.

4.2 Uniform Subsidies

Under a uniform subsidy scheme, government is constraint to a single subsidy rate z. The total payment to firm i thus becomes

$$\beta_i = zA_i \tag{50}$$

As before, include the uniform subsidy in firms' decision problem. The optimality conditions for apprenticeship training are now

$$\rho R_{1,t+1} - w_t^a + z = c_1'(A_1^{uni}) \tag{51}$$

$$\rho R_{2,t+1} - w_t^a + z = c_2'(A_2^{uni}) \tag{52}$$

Firms' optimal training levels A_1^{uni} and A_2^{uni} increase in the subsidy rate. They can again be determined according to

$$A_1^{uni} = c_1'^{-1}(.) = \phi_1(z, v_{1,t+1}, v_{2,t+1}, \rho, \delta) \tag{53}$$

$$A_2^{uni} = c_2'^{-1}(.) = \phi_2(z, v_{1,t+1}, v_{2,t+1}, \rho, \delta) \tag{54}$$

When introducing a uniform subsidy scheme, government faces the task to

[15] More precisely, government faces asymmetric information with respect to firms' productivity which cause an *adverse selection* problem to arise.

determine the appropriate subsidy rate z such that social welfare $W\left(A_1^{uni}(z), A_2^{uni}(z)\right)$ is maximized. After some simplications, the government's optimization problem can be stated by

$$\arg\max_{\{z\}} \left\{ \sum_{i=1,2} A_i^{uni} \cdot \rho\left(\tfrac{v_1+v_2}{2} + \tfrac{5}{36\delta}\left(v_1 - v_2\right)^2 - \tfrac{1}{4}\delta\right) - \sum_{i=1,2} c_i'\left(A_i^{uni}\right) \right\} \quad (55)$$

The first order condition (56) determines the optimal subsidy rate z°.

$$\sum_{i=1,2} \tfrac{\partial A_i^{uni}}{\partial z} \cdot \rho\left(\tfrac{v_1+v_2}{2} + \tfrac{5}{36\delta}\left(v_1 - v_2\right)^2 - \tfrac{1}{4}\delta\right) - \sum_{i=1,2} c_i'\left(A_i^{uni}\right) \tfrac{\partial A_i^{uni}}{\partial z} = 0 \quad (56)$$

Equation (56) states that, for the optimal subsidy rate, the sum of the marginal social returns from an increase in the subsidy rate equals the sum of the marginal increases in the training costs. Moreover, it allows to deduce the next proposition.

Proposition 10 *A **uniform** Pigouvian subsidy scheme, in contrast to an **ideal** Pigouvian subsidy scheme, cannot achieve Pareto-optimality. At the optimal subsidy rate z there will be over- and undertraining .*

Proof. Rearrange the first order condition (56) and obtain

$$\sum_{i=1,2} \tfrac{\partial A_i^{uni}}{\partial z} \left(\rho\left(\tfrac{v_1+v_2}{2} + \tfrac{5}{36\delta}\left(v_1 - v_2\right)^2 - \tfrac{1}{4}\delta\right) - c_i'\left(A_i^{uni}\right)\right) = 0$$

Note that the term in the round brackets displays the social training condition. In Pareto-optimum, this term should equal zero for both firms. However, unless firms were identical, the training cost functions and also the number of apprentices vary across firms. But with $c_i'(A_i^{uni}) \neq c_j'(A_j^{uni})$, the social training condition cannot be fulfilled and training therefore cannot be efficient. Moreover, because $\frac{\partial A_i^{uni}}{\partial z} > 0$, the whole condition equalizes to zero only if the bracket term is positive for one firm, and negative for the other. Thereby, it is clearly implied that the optimal subsidy causes overtraining in one firm and undertraining in the other. ■

The intuition for this result is straightforward. Subsidization leads firms to increase training, but government is constraint to set a single subsidy rate. Because the size of the training externality varies across firms, the optimal subsidy will exceed the ideal amount for the low externality firm and fall short of the ideal amount at the high externality firm. Hence, the Pareto optimal allocation cannot be achieved.

This can be demonstrated graphically in figure (4). Without a subsidy scheme, firms take A^*_{1t} and A^*_{2t} apprentices for training which equates the private marginal returns $R_{1,priv}$ and $R_{2,priv}$ to the marginal training costs c'. The social optimal number of apprentices at each firm, A°_{1t} and A°_{2t}, follows from the intersection with R_{social}. An uniform subsidy z now raises the marginal returns to $R_{1,priv} + z$ and $R_{2,priv} + z$. Vocational training thereby increases to A^{uni}_{1t} and A^{uni}_{2t}. This leads to undertraining in firm 1, $A^{uni}_{1t} < A^{\circ}_{1t}$, and overtraining in firm 2, $A^{uni}_{2t} < A^{\circ}_{2t}$.

Welfare effects can also be displayed within the graph. The uniform subsidy brings about welfare gains from increasing training above the private level (striped areas). However, welfare losses from overtraining must be substracted (shaded area). The net welfare effect is clearly positive. Moreover, for the optimal uniform subsidy, the marginal social loss from overtraining equals the marginal social gain from larger subsidization. Or, put differently, this is equivalent to $R_{social} - (R_{1,priv} + z) = -(R_{social} - (R_{2,priv} + z))$.

Because a uniform subsidy scheme cannot achieve first best training, this instrument cannot restore *Pareto-efficiency*. However, its introduction can be shown to be *Pareto-improving*.[16]

Proposition 11 *The introduction of a small uniform subsidy leads to a Pareto-improvement.*

Proof. At a subsidy rate of zero, the welfare function is increasing in z. Or, formally, $\frac{\partial W}{\partial z}|_{z=0} > 0$. ■

Despite the constraint to use a uniform subsidy rate, introducing a subsidy scheme is welfare-improving. A small positive subsidy increases training

[16] For the distinction between Pareto-optimality and Pareto-improvement see Atkinson & Stiglitz (1980).

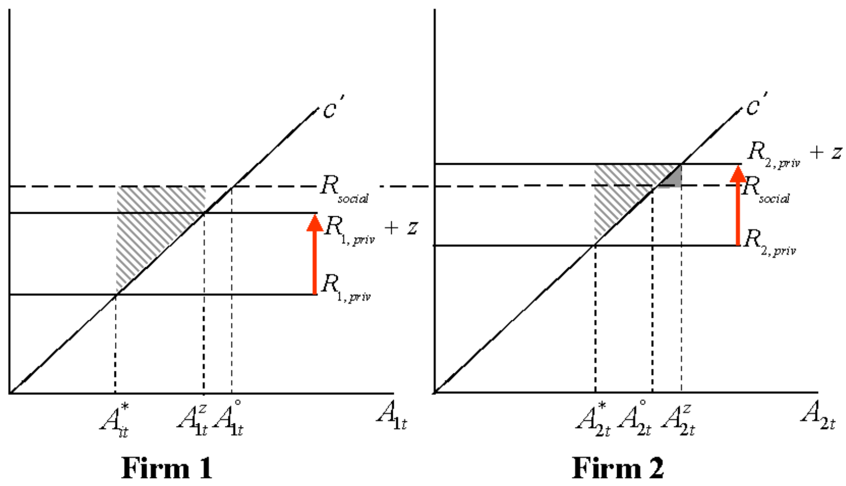

Figure 4: Uniform subsidy

at the margin. It thereby slightly internalizes the externality which is beneficial to welfare.

Note, however, that the analysis so far did not spend any thoughts on the financing of the subsidy payments. In fact, it implicitly assumed costless funding from the general budget. Yet, the government budget itself is constrained because public funds are limited and subject to alternative uses. Moreover, non-distortionary revenues from lump-sum taxes are not feasible. The financing of the subsidy payments may therefore require a tax increase or the introduction of a new distortionary tax. Therefore, without taking the funding of the subsidy payments into account, the present results are to be treated with strict caution.

4.3 Training levies

Up to now the analysis focussed on subsidy schemes to internalize positive spillovers from training. However, training levies are often proposed, too. In fact, this policy proposal has recently received much attention in Germany. This section will therefore take this instrument to a rigorous formal analysis.

The basic idea of any training levy scheme is to penalize firms that do not train at all or only insufficiently. These firms are considered to benefit from positive spillovers and to free-ride on the efforts of training firms. Presumably, by imposing training levies, non-training firms will also participate in the training costs and receive additional incentives for their own training efforts.

Training levy schemes exist in several European countries, such as Britain, Denmark, or France. However, their design varies considerably. They can, for instance, possess the form of revenue-generating levy schemes, levy-exemption-schemes, or levy-grant-systems. Although all schemes impose training levies, differences occur in the collection and spending of the funds raised. Thus, despite their close similarity, these policies will differ somewhat in their economic consequences and call for a detailed analysis. This task, however, would reach beyond the scope of this work.[17] In what follows, focus will be put on the training levy scheme that has recently been proposed for Germany.

The training levy scheme under consideration in Germany aims to penalize firms that train not at all or insufficiently while simultaneously rewarding firms that train exceedingly. Although the bill entered the formal legislative readings, it has been put on hold prior to the final vote in the federal parliament.[18] The issue whether to pass the law or not remains at present still heavily debated. If it were enacted, the proposed federal bill would enable the federal government to collect levies from firms that train less than a certain standard and redistribute the revenues to firms that exceed this standard. The benchmark is a mandatory training quota that has been set to 7% of the employed workforce. If a firm falls short of this prescribed quota, it is to pay a training levy, and if it exceeds the quota, it is eligible to additional funds. For details to the law proposal see the draft in Deutscher Bundestag (2004), in particular §§9-11 BerASichG.

[17] For an overview and international comparison on various policies towards financing training see in particular Gasskov (1994). Greenhalgh (1999), Stevens (1999) and Bosch (2004) provide some account of training levies throughout the world.

[18] For the legislative status, the precise text of the law and an explanatory statement to the legislative intentions see in particular Deutscher Bundestag (2004).

The amounts of levies or grants to a firm increase with the deviation from the benchmark level. The scheme thereby intends to discriminate between varying training efforts across firms. More precisely, firms are entitled to a subsidy for every apprentice exceeding the training quota.[19] By contrast, firms are levied a certain penalty per employee if the training quota is not attained.[20]

In constrast to the subsidy rates and the mandatory training quota, the law proposal leaves the penalty rate unspecified. Instead, it is to be determined such that the whole scheme is self-financing, i.e. that subsidies and administrative expenses can be financed.

In order to allow for a detailed economic analysis, we will now formulate this proposal. Let β_{it} again be the payment to firm i resulting from the scheme in the training period. Moreover, let α_{it} define the training quota of firm i, i.e. the firm's ratio of apprentices to skilled employees.

$$\alpha_{it} = \frac{A_{it}}{N_{it}} \tag{57}$$

Define $\hat{\alpha}$ to be the mandatory training quota that specifies the desired training standard. Now, if a firm falls short of the mandatory level, $\alpha_{it} < \hat{\alpha}$, the firm has to pay a penalty p for every skilled worker it employs. Because the penalty wishes to reflect the differences in training across firms, the total levy decreases for every apprentice the firm trains. Accordingly, the workforce that would be equivalent to the actual number of apprentices at the mandatory training quota, i.e. $\hat{N} = \frac{1}{\hat{\alpha}} A_{it}$, is deducted from the levy base.

[19] The subsidy amounts under discussion range from €580 and €1240 per apprentice per month depending on the trained profession. The variation in the training subsidies is aimed to reflect trainee wages that differ strongly across occupations.

[20] The law defines apprentices as those trainees employed by a firm that fall under the conditions of the apprenticeship training law. Employees are full-time workers that are subject to social insurance contributions. Thus, the law excludes interns, student apprentices, freelance workers, temporary workers and so-called minor employments. Thereby some substitution between different trainee and worker categories may arise resulting in distortions from the levy-grant scheme. These issues are however neglected here.

Training levies to firm i are thus calculated according to

$$levy_{it} = -\left(N_{it} - \frac{1}{\hat{\alpha}} A_{it}\right) \cdot p$$

By constrast, if a firm exceeds the mandatory level, $\alpha_{it} > \hat{\alpha}$, the firm receives a grant z per apprentice it trains in addition to the mandatory training level. The training grant to firm i is thus

$$grant_{it} = (A_{it} - \hat{\alpha} N_{it}) \cdot z$$

Equation (58) summarizes the payment to firm i resulting from the proposed training levy scheme.

$$\beta_{it} = \left\{ \begin{array}{ccc} -\left(N_{it} - \frac{1}{\hat{\alpha}} A_{it}\right) \cdot p & & \alpha_{it} < \hat{\alpha} \\ (A_{it} - \hat{\alpha} N_{it}) \cdot z & \text{if} & \alpha_{it} > \hat{\alpha} \\ 0 & & \alpha_{it} = \hat{\alpha} \end{array} \right\} \quad (58)$$

Factoring out N_{it}, using (57) to replace for firm's training quota and rearranging then gives

$$\beta_{it} = \left\{ \begin{array}{ccc} (\alpha_{it} - \hat{\alpha}) \cdot \frac{p}{\hat{\alpha}} N_{it} & & \alpha_{it} < \hat{\alpha} \\ (\alpha_{it} - \hat{\alpha}) \cdot z N_{it} & \text{if} & \alpha_{it} > \hat{\alpha} \\ 0 & & \alpha_{it} = \hat{\alpha} \end{array} \right\} \quad (59)$$

Let us assume firms are given the same financial incentive for an additional apprentice, regardless of whether the firm exceeds or undershoots the training quota. Then, as follows from (59), penalty and subsidy must obey the relation $\frac{p}{\hat{\alpha}} = z$ or $p = z\hat{\alpha}$.[21] The net payment thereby simplifies to

$$\beta_{it} = (\alpha_{it} - \hat{\alpha}) \, z N_{it} \quad (60)$$

Note the close similarity of equation (60) to an incentive scheme introduced

[21] This assumption could be justified on efficiency grounds because otherwise an additional redistributive element would be introduced into the scheme.

by Falkinger (1996) for the *private* provision of public goods which proposes to punish or reward deviations from the average contribution level.

In equation (60) replace for the training quota using (57), and the scheme can be split up into two parts.

$$\beta_{it} = zA_{it} - z\hat{\alpha}N_{it} \tag{61}$$

One term depends on the number of apprentices, the other on the number of employees. For notational ease, set $\tau = z\hat{\alpha}$. The payment term simplifies to $\beta_{it} = zA_{it} - \tau N_{it}$ whereby it becomes evident that firms receive a uniform subsidy z per apprentice and pay a tax τ per employee. This directly leads to the following conclusion.

Proposition 12 *The proposed training levy scheme is essentially a tax-subsidy-system. It subsidizes apprenticeship training through an additional employment tax.*

Proof. Obvious. ■

Despite their apparent similarity, there are nevertheless notable differences between tax-subsidy-systems and levy-grant-systems. Under a tax-subsidy-system, an economic agent is paying taxes as well as receiving subsidies for providing a beneficial activity. All monetary flows pass through the public budget. Under a levy-grant-system, by contrast, the agent only exchanges a net payment with the public budget, i.e. a levy net of any subsidies. The volume of the fiscal transfers is therefore much smaller. Essentially, levy-grant-systems possess the feature of central *clearing* while tax-subsidy-systems do not. This can bring about some savings in administration, collection, and payment transactions.

The proposed levy scheme is self-financing only if the budget balances. This is the case when all levies raised equal grants and administrative expenses. Assume the scheme to work without any cost.[22] The budget con-

[22] Of course, this is not an innocuous assumption. It is made here for expositional purposes. Further below the analysis will reconsider the issue of administrative costs more explicitly.

straint is thus given by

$$\sum_i \beta_{it} = \sum_i (\alpha_{it} - \hat{\alpha}) \cdot zN_{it} = \sum_i zA_{it} - \sum_i \tau N_{it} = 0 \quad \forall t \tag{62}$$

Proposition 13 *The budget of the training levy scheme balances if the mandatory training quota is set to equal the average training quota.*

Proof. From (62) follows that the budget balances for $\sum_i (\alpha_{it} - \hat{\alpha}) \cdot zN_{it} = 0$. This can be rearranged to $\sum_i \alpha_{it} N_{it} = \hat{\alpha} \sum_i N_{it}$ which then leads to $\hat{\alpha} = \frac{\sum_i A_{it}}{\sum_i N_{it}} \equiv \bar{\alpha}$. ■

Intuitively, by setting the mandatory training quota to equal the average quota, the budget will automatically be equalized as training below and above this standard will reciprocally balance.

With this being some general remarks on the proposed training levy scheme, now investigate how firms' decisions are affected. The optimization problems for firm 1 and 2 are again altered. They now become

$$\underset{\{w_{1t}, A_{1t}\}}{\arg\max} \left\{ \rho R_{1,t+1} N_{t+1} + (v_{1t} - w_{1t} - \tau) N_{1t} - w_t^a A_{1t} - c_1 (A_{1t}) + zA_{1t} \right\} \tag{63}$$

$$\underset{\{w_{2t}, A_{2t}\}}{\arg\max} \left\{ \rho R_{2,t+1} N_{t+1} + (v_{2t} - w_{2t} - \tau) N_{2t} - w_t^a A_{2t} - c_2 (A_{2t}) + zA_{2t} \right\} \tag{64}$$

The first order conditions with respect to firms' wage offers w_{1t} and w_{2t} in the training period are given by (65) and (66) which, as before, represent a pair of reaction functions.

$$-2w_{1t} + w_{2t} - \tau - \delta + v_{1t} = 0 \tag{65}$$

$$-2w_{2t} + w_{1t} - \tau - \delta + v_{2t} = 0 \tag{66}$$

Setting equal again allows to solve for the equilibrium wages w_{1t}^{levy} and w_{2t}^{levy}.

$$w_{1t}^{levy} = \frac{2}{3} v_{1t} + \frac{1}{3} v_{2t} - \tau - \delta \tag{67}$$

$$w_{2t}^{levy} = \frac{1}{3} v_{1t} + \frac{2}{3} v_{2t} - \tau - \delta \tag{68}$$

Proposition 14 *The employee tax to finance the training subsidies is fully shifted onto workers.*

Proof. This follows from (67) and (68) with $\frac{\partial w_{it}^{levy}}{\partial \tau} = -1$. ■

The intuition for this result can be put as follows: The employee tax decreases the value product that a firm can earn from employing skilled workers. Firms will therefore reduce their wage offer by this amount. The training levy scheme thus burdens skilled workers employed in the training period and leaves firms' profits unaffected.

It must be noted, however, that this proposition, at least partially, results from the constant labor productivity that is exogeneous to the firm. Essentially, this assumes the output market to be perfectly competitive, such that the employee tax cannot be shifted onto consumers by increasing prices.

So far, it has become evident that the training levy scheme subsidizes additional training by a uniform subsidy z that is financed by a tax $\tau = z\hat{\alpha}$ levied on employment. It has two immediate effects. Firstly, the subsidy encourages additional apprenticeship training. Similar to the uniform subsidy scheme, marginal training returns increase for all firms. Secondly, the employment tax reduces the wage offers to the present skilled workforce. In comparison to alternative income sources, skilled work becomes relatively less attractive. In the training period, labor supply will decrease and production output will shrink.

Thus, when introducing the levy scheme, a benevolent government will trade off the welfare gains from additional training against the welfare losses for the present workforce. Additionally, it is constraint to a balanced budget. Aiming to determine the optimal subsidy and tax rates, the government's decision problem can be displayed by

$$\begin{aligned} \max_{\{z,\hat{\alpha}\}} W &= \sum_{t=1,2} \rho^{t-1} W_t - \sum_{i=1,2} c_i(A_i^{levy}) \\ \text{s.t.} & \quad \sum_{i=1,2} \left(z A_{it}^{levy} - z\hat{\alpha} N_{it} \right) = 0 \end{aligned}$$

This constrained maximization problem can be solved using Lagrange's common method. The Lagrangian problem is given by (69).

$$\max_{\{z,\hat{\alpha}\}}\mathcal{L} = \sum_{t=1,2}\rho^{t-1}W_t - \sum_{i=1,2}c_i(A_{it}^{levy}) - \lambda\sum_{i=1,2}\left(zA_{it}^{levy} - z\hat{\alpha}N_{it}\right) \tag{69}$$

The first order conditions are

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial z} &= \sum_{t=1,2}\rho^{t-1}\frac{\partial W_t}{\partial z} - \sum_{i=1,2}c_i'\frac{\partial A_{it}^{levy}}{\partial z} - \lambda\sum_{i=1,2}\left(z\frac{\partial A_{it}^{levy}}{\partial z} - z\hat{\alpha}\frac{\partial N_{it}}{\partial z}\right) = 0\\
\frac{\partial\mathcal{L}}{\partial\hat{\alpha}} &= \sum_{t=1,2}\rho^{t-1}\frac{\partial W_t}{\partial\hat{\alpha}} - \lambda\sum_{i=1,2}\left(-zN_{it} - z\hat{\alpha}\frac{\partial N_{it}}{\partial\hat{\alpha}}\right) = 0\\
\frac{\partial\mathcal{L}}{\partial\lambda} &= \sum_{i=1,2}\left(zA_{it}^{levy} - z\hat{\alpha}N_{it}\right) = 0
\end{aligned}$$

Using these equations, the socially optimal values for z^* and $\hat{\alpha}^*$ can be solved for.

$$\sum_{t=1,2}\rho^{t-1}\left(\frac{\partial W_t}{\partial z} + \frac{\partial W_t}{\partial\hat{\alpha}}\frac{\sum_{i=1,2}\left(\frac{\partial A_{it}^{levy}}{\partial z} - \hat{\alpha}\frac{\partial N_{it}}{\partial z}\right)}{\sum_{i=1,2}\left(N_{it} + \hat{\alpha}\frac{\partial N_{it}}{\partial\hat{\alpha}}\right)}\right) = \sum_{i=1,2}c_i'\frac{\partial A_{it}^{levy}}{\partial z}$$

Similar to before, address the question whether the introduction of the training levy scheme would be welfare-improving. From the balanced budget constraint (62) follows that the mandatory training quota cannot be set independently, but depends positively on the subsidy rate chosen. Or formally $\hat{\alpha}(z)$ with $\hat{\alpha}' > 0$. Now using this budget constraint, the welfare problem can be rewritten as a function $W(z, \hat{\alpha}(z))$. Take the total differential with respect to the subsidy rate, $\frac{dW}{dz} = \frac{\partial W}{\partial z} + \frac{\partial W}{\partial\hat{\alpha}}\frac{\partial\hat{\alpha}}{\partial z}$, which is stated in (70).

$$\frac{dW}{dz} = \sum_{t=1,2}\rho^{t-1}\left(\frac{\partial W_t}{\partial z} + \frac{\partial W_t}{\partial\hat{\alpha}}\frac{\partial\hat{\alpha}}{\partial z}\right) - \sum_{i=1,2}c_i'\frac{\partial A_{it}^{levy}}{\partial z} \gtreqless 0 \tag{70}$$

$$\frac{dW}{dz} = \rho\frac{\partial W_{t+1}}{\partial z} + \frac{\partial W_t}{\partial z} + \frac{\partial W_t}{\partial\hat{\alpha}}\frac{\partial\hat{\alpha}}{\partial z} - \sum_{i=1,2}c_i'\frac{\partial A_{it}^{levy}}{\partial z} \gtreqless 0 \tag{71}$$

Proposition 15 *The introduction of a levy-grant-scheme can be ambiguous to welfare.*

Proof. If the total differential were positive for $z = 0$, $\frac{dW}{dz}|_{z=0} > 0$, the introduction of the levy scheme would be welfare-improving. Equation (70) consists of four terms. The first term represents the discounted welfare gain of training brought about by a subsidy. The second and third term depict the welfare loss resulting from the levy in the training period. Both terms are negative negative because $\frac{\partial W_t}{\partial z} < 0$, $\frac{\partial W_t}{\partial \hat{\alpha}} < 0$ and $\frac{\partial \hat{\alpha}}{\partial z} > 0$. Lastly, the fourth term states the marginal cost of additional training that is to be deducted from welfare when training incentives are introduced. Without specific functional forms, however, the total differential cannot be signed. ■

The proposition essentially states that the welfare effects from introducing a levy scheme for apprenticeship training cannot be determined *theoretically*. Whether this scheme would be beneficial or detrimental to the economy's welfare critically depends on firms' training with the subsidy and workers' changed labor supply. An *empirical* assessment of the training and labor supply elasticities would therefore be required to determine net welfare.

4.4 Discussion

In the previous section it could be demonstrated that apprenticeship training exhibits positive spillovers to other firms. Public policies addressing this problem and increasing training efforts are therefore of interest. This section analyzed various instruments introducing financial incentives for vocational training.

It was shown that an ideal Pigouvian subsidy could, at least in theory, restore the social optimum. For this to be the case the subsidy should depend on productivity levels in the economy. In reality, however, this information may not be available. Thus, the use of a uniform subsidy may be required. In comparison to the ideal Pigouvian scheme it will bring about additional distortions. Because a uniform subsidy rate cannot account for differences between firms, such a training subsidy scheme may lead to undertraining in some firms and overtraining in others. Nevertheless, it was pointed out that introducing a small uniform subsidy would be welfare-improving if non-distortionary funding were available.

This section also investigated the widespread proposal of training levies. It was unveiled that such schemes are a particular form of tax-subsidy-systems. A tax is levied on employment and a subsidy is paid per apprenticeship. The analysis furthermore showed this tax, although paid by the firm, to be fully shifted onto workers in the form of lower wages. This outcome, however, can (partially) be attributed to the model. In reality, some shifting onto consumers may occur if output markets are not perfectly competitive.

Whether the tax is fully shifted onto workers or only partially so, if labor supply is elastic, the levy scheme will decrease labor supply and output. A training levy thus brings about welfare gains from subsidizing apprenticeship training while causing welfare losses from lower output due to reduced labor supply. The net welfare effect was shown to be theoretically ambiguous, requiring an empirical estimation.

Nevertheless, this analysis requires some caution. So far, no administrative costs were taken into account. Yet, there are costs to administer, monitor and verify the subsidy scheme, that arise both for the government as well as the firms. In particular, non-negligible costs are associated with the collection and distribution of funds or with the gathering and processing of the necessary information. Including these costs would additionally reduce welfare.

5 Summary and Conclusions

The aim of this paper was threefold. Firstly, it sought to understand why firms partially finance vocational education although it is mostly general in nature and thereby of use to many firms. Secondly, it aimed to determine whether externalities arise from firms providing costly apprenticeship training which would lead to socially inefficient training. Thirdly, it was asked whether incentive schemes, in particular levy-grant-schemes, would be a welfare-improving policy.

Following recent developments in the training literature, a simple frictional labor market model was introduced. In this setting, firms can be shown to rationally provide apprenticeship training and partially incur some of its costs although training is mostly general and workers are subject to poaching by other firms. At the same time, however, it can also be demonstrated that vocational training brings about positive spillovers on other firms. With "poaching" therefore causing an externality, apprenticeship training will fall short of the socially optimal level. This raises interest for public policies internalizing the externality and increasing training incentives.

At first an ideal Pigouvian subsidy was analyzed as a reference case. In principle, this scheme could restore the social optimum by providing the training firm with a subsidy equal to the value of the positive spillover. However, in order to calculate firm-specific subsidies, such an instrument raises exceptional informational requirements. Alternatively, a uniform subsidy could be set *on average* for all firms. Yet, as it does not distinguish between firms, it could lead to under- and overtraining. Nevertheless, introducing a marginal uniform subsidy could be shown to be welfare-improving if non-distortionary sources of funding were available.

Ultimately, the focus turned to analyze levy-grant-schemes which have recently been proposed for Germany and which are in fact already existing in several industrialized countries. Such schemes were identified to be a particular type of tax-subsidy-systems. Effectively, a tax is levied on employment and paid out per apprentice. The analysis revealed further that the tax burden of this instrument will be fully shifted onto workers. A levy-grant-scheme for vocational education can therefore be characterized as a "pay-as-you-go"-financing of vocational education where present workers pay for the vocational education of future workers.

Moreover, it could be pointed out that the introduction of a levy-grant-scheme can be ambiguous to welfare. On the one hand, it provides additional incentives for apprenticeship training and thereby increases welfare. On the other hand, it reduces wages of the skilled workforce. Thus, labor supply will shrink and thereby reduce production and welfare. Ultimately, net welfare will depend on the size of both effects.

What can be concluded from this exercise for the policy debate on incentive schemes for vocational training? Essentially, proponents and opponents of a levy-grant-scheme are both somewhat right. Firms offering costly apprenticeship training providing mostly general training induce positive spillovers on other firms. Because not all returns accrue to the training firm, this may indeed lead to under-provision of training. There could thus in principle be a case for training policy to internalize the externality and provide additional training incentives. A policy using the instrument of a levy-grant-scheme may nevertheless not be warranted. It subsidizes socially beneficial apprenticeships but it also brings along economic distortions that may actually exceed the benefits from additional training.

From this analysis, what advise is to be given towards the proposal of introducing a levy-grant-scheme for apprenticeship training in Germany? If such an incentive scheme is indeed intended, less distortionary means of financing and subsidizing should be sought. In principle, funding from general tax receipts would be desirable, but given the state of the public budget at present this seems not a feasible choice. When additional sources of financing are therefore needed, a levy depending on payroll, i.e. firm's total wage bill, should be given preference over the present scheme proposal which refers only to the number of employees. Otherwise, firms may strive to substitute low-skilled workers for fewer high-skilled workers bringing along an additional distortion from taxation.

Overall, however, the case for a levy-grant-scheme must be considered as rather weak. Apart from its informational demands, high administrative costs are to be expected for government and firms as well. Moreover, this scheme brings along significant distortions arising from uniform subsidization as well as from raising funds. Allowing apprentices and training firms to sign reimbursement clauses seems an attractive alternative to provide increased training incentives (Alewell & Richter 2001, 162). In perfect labor markets, an agreement on reimbursing for training expenses essentially equals a training loan. By contrast, an analysis of reimbursement clauses within frictional labor markets with monopsony power is lacking at present. This will be the focus of another paper.

A Appendix

Proof of proposition (1)

Firms' wage offers vary with different valuations for skilled workers. Thus, the problem arises whether the respective wage offers allow both firms to recruit skilled workers. This requires $\theta^*_{t+1} > 0$ and also $1 - \theta^*_{t+1} > 0$. Put differently, conditions $v_{1,t+1} - v_{2,t+1} + 3\delta \geq 0$ and $v_{2,t+1} - v_{1,t+1} + 3\delta \geq 0$ must be met. Some rearranging of both inequalities gives

$$v_{1,t+1} - v_{2,t+1} \geq -3\delta \tag{72}$$

and

$$-\left(v_{1,t+1} - v_{2,t+1}\right) \geq -3\delta \iff 3\delta \geq v_{1,t+1} - v_{2,t+1} \tag{73}$$

One of these conditions will always be fulfilled. (The firm with the higher labor productivity will always be able to acquire skilled workers from the labor market because it can offer higher wages.) For the other condition to hold (and also the second firm to recruit workers from the labor market) the productivity difference is not to exceed the threefold of the commuting cost rate. We assume this to be the case because we wish to analyze the duopson case. From straightforward inspection of (16), (17), (20), and (21) using conditions (72) and (73) then directly follows proposition (1).

If both conditions were not met, the market for skilled labor would be a monopson. Naturally, poaching cannot arise in a monopson. However, it is a standard result that the monopson wage falls below the marginal product, too.

Expected future net wage

The present value of a skilled workers future net income is represented by the discounted expected net wage $\rho E\left(\tilde{w}_{t+1}\right)$ when working for firm 1 or 2.

$$E\left(\tilde{w}_{t+1}\right) = \int_0^{\hat{\theta}^*_{t+1}} \left(w^*_{1,t+1} - \delta\theta\right) d\theta + \int_{\hat{\theta}^*_{t+1}}^1 \left(w^*_{2,t+1} - \delta\left(1-\theta\right)\right) d\theta$$

Dissolving the integral yields

$$E\left(\tilde{w}_{t+1}\right)=\left[w_{1,t+1}^{*}\theta-\frac{\delta}{2}\theta^{2}\right]_{0}^{\hat{\theta}_{t+1}^{*}}+\left[w_{2,t+1}^{*}\theta-\delta\theta+\frac{\delta}{2}\theta^{2}\right]_{\hat{\theta}_{t+1}^{*}}^{1}$$

or

$$E\left(\tilde{w}_{t+1}\right)=w_{1,t+1}^{*}\hat{\theta}_{t+1}^{*}-\frac{\delta}{2}\hat{\theta}_{t+1}^{*2}+w_{2,t+1}^{*}-\delta+\frac{\delta}{2}-w_{2,t+1}^{*}\hat{\theta}_{t+1}^{*}+\delta\hat{\theta}_{t+1}^{*}-\frac{\delta}{2}\hat{\theta}_{t+1}^{*2}$$

which can be simplified to

$$E\left(\tilde{w}_{t+1}\right)=\left(w_{1,t+1}^{*}-w_{2,t+1}^{*}\right)\hat{\theta}_{t+1}^{*}+\delta\hat{\theta}_{t+1}^{*}\left(1-\hat{\theta}_{t+1}^{*}\right)+w_{2,t+1}^{*}-\frac{\delta}{2}$$

Inserting for $\hat{\theta}_{t+1}^{*}$, $w_{1,t+1}^{*}$, $w_{2,t+1}^{*}$ and a couple of algebraic rearrangements then yields

$$E\left(\tilde{w}_{t+1}\right)=\frac{v_{1,t+1}+v_{2,t+1}}{2}+\frac{\left(v_{1,t+1}-v_{2,t+1}\right)^{2}}{6\delta}-\frac{5}{4}\delta$$

Proof of proposition (5)

Firm's return

$$R_{i,f}=\rho R_{i,t+1}=\rho\frac{\left(v_{i,t+1}-v_{j,t+1}+3\delta\right)^{2}}{18\delta}$$

$$\frac{\partial}{\partial\delta}:=\frac{-\left(v_{i,t+1}-v_{j,t+1}\right)^{2}+9\delta^{2}}{18\delta^{2}}$$

$$\begin{aligned}9\delta^{2}&>\left(v_{i,t+1}-v_{j,t+1}\right)^{2}\\3\delta&>\left(v_{i,t+1}-v_{j,t+1}\right)\\\frac{\partial R_{i,t+1}}{\partial\delta}&>0\end{aligned}$$

Worker's share

$$R_{w}=\rho E\left(\tilde{w}_{t+1}\right)=\frac{v_{1,t+1}+v_{2,t+1}}{2}+\frac{\left(v_{1,t+1}-v_{2,t+1}\right)^{2}}{6\delta}-\frac{5}{4}\delta$$

$$\frac{\partial}{\partial \delta} := -\frac{6\left(v_{1,t+1} - v_{2,t+1}\right)^2}{6\delta} - \frac{5}{4} < 0$$

Private returns

$$R_{i,priv} = \rho\left(\frac{5v_{i,t+1} + v_{j,t+1}}{6} + \frac{4}{18\delta}\left(v_{i,t+1} - v_{j,t+1}\right)^2 - \frac{3}{4}\delta\right)$$

$$\frac{\partial}{\partial \delta} := -\frac{72\left(v_{i,t+1} - v_{j,t+1}\right)^2}{\left(18\delta\right)^2} - \frac{3}{4} < 0$$

Derivation of equation (38)

Welfare in this economy consists of the profits and wages net of commuting frictions.

$$W_t = \int_0^{\hat{\theta}_t} \left(v_{1t} - w_{1t} + w_{1t} - \delta\theta\right) N_t d\theta + \int_{\hat{\theta}_t}^1 \left(v_{2t} - w_{2t} + w_{2t} - \delta\left(1-\theta\right)\right) N_t d\theta$$

Put simpler, welfare is net production.

$$W_t = \int_0^{\hat{\theta}_t} (v_{1t} - \delta\theta) N_t d\theta + \int_{\hat{\theta}_t}^1 (v_{2t} - \delta\left(1-\theta\right)) N_t d\theta$$

By dissolving the integral

$$W_t = N_t \left[v_1\theta - \frac{\delta}{2}\theta^2\right]_0^{\hat{\theta}_t} + N_t \left[v_2\theta - \delta\theta + \frac{\delta}{2}\theta^2\right]_{\hat{\theta}_t}^1$$

obtain

$$W_t = N_t \left(v_1\hat{\theta}_t - \frac{\delta}{2}\hat{\theta}_t^2 + v_2 - \delta + \frac{\delta}{2} - v_2\hat{\theta}_t + \delta\hat{\theta}_t - \frac{\delta}{2}\hat{\theta}_t^2\right)$$

which can be simplified to

$$W_t = N_t \left(v_1\hat{\theta}_t - v_2\hat{\theta}_t + \delta\hat{\theta}_t + v_2 - \frac{\delta}{2} - \delta\hat{\theta}_t^2\right)$$

$$W_t = N_t \left(\hat{\theta}_t \left(v_1 - v_2 + \delta - \delta\hat{\theta}_t\right) + v_2 - \frac{\delta}{2}\right)$$

Inserting for $\hat{\theta}_t^*$

$$W_t = N_t \left(\frac{v_1 - v_2 + 3\delta}{6\delta} \left(v_1 - v_2 + \delta - \delta \frac{v_1 - v_2 + 3\delta}{6\delta} \right) + v_2 - \frac{\delta}{2} \right)$$

$$W_t = N_t \left(\frac{v_1 - v_2 + 3\delta}{6\delta} \cdot \frac{5v_1 - 5v_2 + 3\delta}{6} + v_2 - \frac{\delta}{2} \right)$$

$$W_t = N_t \left(\frac{1}{36\delta} \left(18\delta v_1 - 10 v_1 v_2 + 5v_1^2 + 5v_2^2 + 18\delta v_2 - 9\delta^2 \right) \right)$$

then yields

$$W_t = N_t \left(\frac{v_1 + v_2}{2} + \frac{5}{36\delta} \left(v_1 - v_2 \right)^2 - \frac{1}{4}\delta \right)$$

From

$$W = \sum_{t=1,2} \rho^{t-1} W_t - \sum_{i=1,2} c_i \left(A_{it} \right)$$

then follows equation (38).

Proof of proposition (7)

In order to show that social returns to apprenticeship training exceed the private returns, compare the condition for the social optimum (40)

$$\rho \left(\frac{v_1 + v_2}{2} + \frac{5}{36\delta} \left(v_1 - v_2 \right)^2 - \frac{1}{4}\delta \right) - c_i' \left(A_i \right) = 0 \quad \forall i = \{1, 2\}$$

with the conditions for the private optimum (29) and (30)

$$\rho \frac{\left(v_1 - v_2 + 3\delta \right)^2}{18\delta} - c_1' \left(A_1 \right) = 0$$

and

$$\rho \frac{\left(v_2 - v_1 + 3\delta \right)^2}{18\delta} - c_1' \left(A_1 \right) = 0$$

Consider first apprenticeship training at firm 1. Social returns to training will be above private returns if

$$\frac{v_1 + v_2}{2} + \frac{5}{36\delta} \left(v_1 - v_2 \right)^2 - \frac{1}{4}\delta > \frac{\left(v_1 - v_2 + 3\delta \right)^2}{18\delta}$$

This simplifies to

$$-9\delta^2 + 2\delta v_1 + 10\delta v_2 + (v_1 - v_2)^2 > 0$$

This condition essentially states combinations of δ, v_1 and v_2 for which training will cause a positive or negative externality to arise.

From above follows that both firms are in the market if conditions (72) and (73) are fulfilled. Inserting for $(v_1 - v_2)$ we can rewrite to

$$-9\delta^2 + 2\delta v_1 + 10\delta v_2 + 9\delta^2 > 0$$

which then rearranges to

$$2\delta v_1 + 10\delta v_2 > 0$$

This condition is always fulfilled as long as commuting costs are positive.

Proceed likewise for apprenticeship training at firm 2.

$$\frac{v_1 + v_2}{2} + \frac{5}{36\delta}(v_1 - v_2)^2 - \frac{1}{4}\delta > \frac{(v_2 - v_1 + 3\delta)^2}{18\delta}$$

which can be simplified to

$$10\delta v_1 + 2\delta v_2 + (v_1 - v_2)^2 - 9\delta^2 > 0$$

Again inserting for $(v_1 - v_2)$ this condition is also respected.

$$10\delta v_1 + 2\delta v_2 > 0$$

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