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Subsidising Education with Unionised Labour Markets

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Subsidising Education

with Unionised Labour Markets

Florian Wöhlbier #

It is well known from the literature that a unionisation of labour markets leads to an increase in wages and a decrease in employment. However, in these models human capital formation is usually taken as given. This paper internalises the education decision and shows that a unionisation of the labour market for unskilled workers will also lead to an inefficiently low education level. We discuss the effects of an education subsidy. It will turn out that both the way of financing and the reaction of the trade union to tax rate changes are crucial for the employment and welfare effects.

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1 Introduction

In most European countries the problem of high and persistent unemployment is one of the central issues in the political and scientific debate. Having a closer look at the structure of unemployment, one is struck by the high percentage of low-educated unemployed. On average, 71% of all unemployed in the European Union are low-educated according to the ECHP (European Community Household Panel). In Germany their share even amounts to 86% (see Salomäki (2001)). Therefore, a closer look at the correlation between an individual’s level of education and his unemployment probability is highly important.¹ In 1999 the unemployment rate among the low-educated men in the European Union was nearly 2.6 times as high as among the high-educated, in Germany the factor was even 4.0 (OECD (2001)).

Many reasons for this correlation are discussed in the literature. Teulings/Koopmanschap (1989) and Groot/Oosterbeek (1992) attribute this to the fact that higher educated individuals are more flexible and therefore able to accept a job requiring only a lower level of education. According to, e.g., Salomaki (2001) the benefit replacement rates are higher for less educated individuals making employment less attractive for them. Brunello (2001) believes that the empirical observation is due to the fact that the loss of human capital if unemployed is higher for highly educated individuals. Another reason can be seen in the organisation of labour markets. The rate of unionisation is higher in lower educated sectors leading to higher unemployment in this segment of the labour market.

However, human capital formation is usually taken as given in trade union models. Oswald (1985) shows in a model with exogenous human capital that a unionisation of the labour market leads to an increase in wages and, thus, a decrease in employment. Human capital formation with imperfect labour markets is already discussed in the literature, though. According to Acemoglu (1997), the main problem with respect to training in imperfect labour markets is rent sharing of the returns. Often, returns to general training can’t be monitored, as they are, somehow, firm specific. As a consequence, the return to training is shared between the worker and the firm. This inefficiency can be overcome if employment contracts can be

¹ A strong correlation was proven by Ashenfelter/Ham (1979) and Nickell (1979) and is confirmed, e.g., by Kiefer (1985) and Kettunen (1997).
written which specify both the training and wage level in advance. However, as also discussed by Acemoglu (1997), the so-called quitting externalities have to be considered, too. Training in non-competitive labour markets often benefits future employers. With imperfect labour markets workers are not paid their marginal product due to compressed wage structures. Therefore, the future employer receives part of the return of human capital formation. This externality can’t be internalised by a decentralised market. However, it might be overcome by a subsidy, which should be accompanied by a regulation of training. Fuest/Huber (2001) discuss the cause of rent sharing in a world with firm specific training and worker firm bargaining. They show that without a government intervention this may both lead to a too low level of training of the employed and a too low employment level compared to the first best. The literature mostly focuses on the aspect of training, though. The question of education hasn’t been at the centre of analysis up to now.

In this paper we will focus on the impact of a unionisation of the lower segments of the labour market on the individuals’ education decision. It will be shown that an increase in the union bargaining power will not only lead to a reduction in the level of employment but to a decrease in education. It will be shown that an education subsidy may lead to an increase in education, employment and welfare. So a further justification for the high level of education subsidies in addition to the ones already known will be brought forward.²

The remainder of the paper is organised as follows. Section 2 presents the theoretical model while in section 3 the effects of an education subsidy on the level of education, employment and welfare are discussed. Finally, section 4 concludes.

2 The Model

We consider a small open economy which is divided into two sectors of production. In both sectors a homogenous output good is produced. The price of the good is normalised to one. While the production in the first sector takes place using only unskilled labour, only skilled labour is required in the second sector. So there are two kinds of workers: skilled and unskilled ones. The number of firms is assumed to be the same in both sectors and normalised

² On the EU-average, 85% of the direct costs of education are born by the public even on the tertiary level (OECD (2000)).
to one. Before discussing the production functions in more detail, we will look at the sequence of decision making. In stage one the individuals decide on their skill formation, in stage two wage setting, employment and production take place.

![Fig. 1. Sequence of decision making.](image)

Turning to stage one, the individuals can get educated and thus skilled workers at a cost of $c$. Otherwise they can only supply unskilled work in the future. The individuals are heterogeneous with respect to their potential earnings in case of an education. In case of an education an individual is assumed to supply $(1+\delta_i)$ efficiency units of skilled labour. $\delta_i$ is the ability parameter of individual $i$ and is uniformly distributed in the zero-one interval. Making up his mind, each individual compares his expected utility in case of education with the non-education situation.

In the second stage wages and employment are determined and production takes place. We will now focus on the production functions.

$$Y^u = L^\gamma$$ with $0 < \gamma < 1$ \hspace{1cm} (1)

$$Y^s = H^\beta$$ with $0 < \beta < 1$ \hspace{1cm} (2)

The first sector is the so-called unskilled sector. The homogenous good is produced employing $L$ unskilled workers. All unskilled workers supply one unit of labour. The output of the representative firm in the sector is $Y^u$. The second sector, the skilled sector, uses $H$ units of skilled labour. $H$ is not the number of skilled workers employed but the input of efficiency units of labour of the employed heterogeneous skilled workers. The output of the

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3 This assumption is made only for simplicity.
representative firm in the skilled sector is $Y^s$. $\gamma$ and $\beta$ are assumed to lie between zero and one. Both sectors thus have decreasing returns to scale. This implies that there exists a fixed factor of production, e.g. land, which is not explicitly modelled.\(^4\) The overall output in the economy is

$$Y = Y^u + Y^s.$$  \hfill (3)

2.1 Welfare properties of outcome with perfect labour markets

In this section, we will have a look at the reference case of perfect labour markets in both sectors and show that the market outcome is efficient. The model has to be solved using backward induction. So we will first turn to the wage determination in stage two. The wage rates are given by

$$w_u = \gamma L^{\gamma-1} \geq b$$  \hfill (4)

$$w_s = \beta H^{\beta-1}$$  \hfill (5)

The wage rate in the unskilled sector ($w_u$) has to be at least as high as the alternative utility $b$, as otherwise no unskilled would be willing to work. $b$ is the utility from leisure or illicit work. $w_s$ is the wage rate per efficiency unit of labour in the skilled sector. Whereas some unskilled might be voluntarily unemployed if $w_u = b$, all skilled workers will be employed. The employment level is determined by the labour demand functions in both sectors.

Now we turn to the stage of the education decision. We assume rational individuals that are fully aware of the wage and employment determination in stage two. An individual $i$ decides to get educated if his utility in the case of education is at least as high as if he chooses not to get educated. If he gets educated his income in stage two equals $(1 + \delta_i)$ times the wage rate per efficiency unit of labour, $w_s$. However, he has to bear the education costs $c$. If he doesn’t

\(^4\) An introduction of capital with a fixed interest rate as a further factor of production wouldn’t change the results and is therefore omitted.
get educated he earns $w_u$. There is no discounting with respect to future earnings, so it is assumed that the individuals can instantly acquire education. The decision rule is:

\[(1 + \delta_i)w_s - c \geq w_u \geq b\]  

(6)

Due to the heterogeneity there exists a marginal individual which is indifferent between getting skilled or staying unskilled. This individual is named $\delta^*$. All individuals with a higher earnings potential than $\delta^*$ in case of an education get skilled, the other individuals decide not to educate. Therefore a proportion of $(1-\delta^*)$ individuals gets educated whereas a share of $\delta^*$ individuals decides not to get educated. An interior solution for $\delta^*$ is guaranteed due to the properties of the production functions. The supply of efficiency units of labour in the skilled sector $(H)$ equals the sum over the supply of all $1-\delta^*$ educated individuals:

\[H = \int_{\delta^*}^{1} (1 + \delta)d\delta = \left[\delta + \frac{1}{2}\delta^2\right]_{\delta^*}^{1} = \frac{3}{2} - \left(\delta^* + \frac{1}{2}\delta^*^2\right)\]  

(7)

Out of the group of unskilled workers $\delta^* - L$ will be employed and the rest will be voluntarily unemployed.

We will turn to the welfare properties of the market outcome. In order to do so, we use a rather simple welfare function:

\[W = L^\gamma + H^\beta - (1 - \delta^*)c + (\delta^* - L)b\]  

(8)

Welfare in the economy is defined as the output in the two sectors minus the costs of education plus, eventually, utility of voluntarily unemployed unskilled workers. Skilled workers won’t be voluntarily unemployed as there wage income always exceeds the wage income of an unskilled and, thus, the alternative utility.
It is easy to show that the maximisation of the welfare function with respect to the number of educated individuals leads to the same results as the market outcome.\(^5\)

\[
\frac{\partial W}{\partial \delta^*} = \gamma L^{\gamma-1} - \beta H^{\beta-1}(1 + \delta^*) + c = 0
\]  

(9)

Therefore, the market equilibrium is efficient in the case of perfect labour markets. This result doesn’t hinge upon the existence or non-existence of voluntary unemployment.

2.2 Unionised labour market in the unskilled sector

Having looked at the reference case of perfect labour markets, we now turn to the situation with a unionisation of labour markets. We assume the labour market for the unskilled to be unionised, while the skilled labour market is assumed to stay competitive. This is a simplified representation of the fact that the unionisation of labour markets and, thus, the wage rigidity are decreasing with the skill level.

Turning to the wage setting in the unionised labour market, we consider a right-to-manage model.\(^6\) Before looking at the wage bargaining in detail, we have to specify the union objective function:

\[
V = L^\nu \cdot (w_u - b) \quad \nu \geq 0
\]  

(10)

The objective function \((V)\) of the union depends on the employment level \(L\) of the unskilled and on the difference between the unskilled wage rate and the utility of an unemployed. The union thus has two objectives: A wage objective and an employment objective. However, the goals are competing since a higher wage rate will lead to a reduction in the firm’s labour demand. \(\nu\) is the relative weight the trade unions puts on its employment objective. The union objective function implies that the individuals are risk neutral.

\(^5\) In case of \(L < \delta^*\) the output in the unskilled sector is independent of the number of unskilled. In case of \(L = \delta^*\) the last term in the welfare function vanishes.

\(^6\) See Manning (1987) for a detailed discussion of this approach.
The wage rate is determined by Nash-bargaining.\(^7\) The wage is set in order to maximise the Nash-maximand

\[
\Omega = \mu \ln V + (1 - \mu) \ln (\Pi - \Pi_0). \quad \mu \in [0,1]
\]

\(\mu\) is the bargaining power of the trade union, whereas \(1 - \mu\) is the bargaining power of the firm. With \(\mu\) being zero we have the competitive labour market outcome. With \(\mu\) equalling one the trade union can set the wage rate unilaterally. The objective of the firm is to maximise the surplus of its profit \((\Pi)\) with respect to a fall back profit of \(\Pi_0\), whereas the union tries to maximise the objective function \(V\). Maximising the Nash-maximand we get:

\[
\Omega_{w_u} = \mu \left( \frac{v L_w}{L} + \frac{1}{w_u - b} \right) - (1 - \mu) \frac{L}{\Pi - \Pi_0} = 0
\]

(12)

Thus, the wage rate depends on the distribution of the bargaining power, the union's weight on employment, the labour demand elasticity, the alternative utility \(b\) and the fall back profit of the firm:

\[
w_u = w_u(\mu, v, \gamma, b, \Pi_0)
\]

(13)

It is independent of the number of unskilled workers or the level of unemployment.\(^8\) After the wage negotiations the firm will determine the employment level \(L\). \(\delta - L\) unskilled will be unemployed. In the skilled sector full employment will prevail.

Looking at the education decision in stage one, the individuals face a different situation now. If they don't get educated they get a job at the wage rate \(w_u\) with the probability \(L/\delta^*\) and with the inverse probability they will be involuntarily unemployed and get the reservation

\(^7\) A detailed discussion of Nash-Bargaining can be found in Nash (1950) and Binmore et al. (1986).

\(^8\) For a detailed discussion of this result see Ulph/Ulph (1990) and Bean et al. (1986). The latter shows empirically that the level of unemployment has limited if any impact on the wage rate if the society is not corporatistic.
utility b. They compare the expected utility with the certain income in the case of education less the cost of education. The marginal individual $\delta^*$ is given by

$$\frac{L}{\delta^*} w_u + \frac{\delta^* - L}{\delta^*} b = (1 + \delta^*) w_s - c. \quad (14)$$

We will now look at the effects of a unionised labour market on the number of educated individuals, overall employment and welfare. We will do so, by analysing the effects of an increase in the union’s bargaining power. We will show that the following Proposition holds:

**Proposition 1** An increase in the union bargaining power in the unskilled sector leads to a wage increase and an employment reduction in the unionised sector. If

a) $\nu \geq 1$ the number of educated will decrease. Therefore, overall employment and welfare will decrease.

b) $\nu < 1$ the number of educated will decrease starting at low levels of $\mu$, thus overall employment and welfare will decrease. However, at higher levels of the union bargaining power a further increase in $\mu$ will lead to a higher level of education. Overall, compared to a situation with competitive labour markets, a unionisation of labour markets will lead to an unambiguous increase in unemployment and decrease in education and welfare, though.

In Appendix 1, the familiar effect of an increase in the union bargaining power is proven for the general case of $\nu \geq 0$. The resulting wage increase leads to a reduction in employment in the unskilled sector. The impact on the level of education is not obvious, though. Differentiating equation (14) with respect to the wage rate we get:

$$\frac{\partial \delta^*}{\partial w_u} = -\frac{1}{\delta^*} \left( \frac{\partial L}{\partial w_u} (w_u - b) + L \right) \begin{cases} > \\ - \frac{L}{\delta^*} (w_u - b) - w_s - (1 + \delta^*) \frac{\partial w_s}{\partial \delta^*} \begin{cases} > \\ < \end{cases} 0 \end{cases} \quad (15)$$

for

$$\frac{\partial L}{\partial w_u} (w_u - b) + L \begin{cases} > \\ = 0 \end{cases} \text{ or } w_u \begin{cases} < \\ > \gamma \end{cases}.$$
An increase in the union bargaining power leads to an increase in the union's rent and, thus, in the expected utility of an unskilled for a given level of education if $w_u < b/\gamma$. Therefore, the education threshold $\delta^*$ will increase and a smaller fraction of individuals will get educated. This is true for all levels of $\mu$ if $\nu > 1$ (case a)).

Turning to the welfare function an increase in the bargaining power has two effects:

$$
\frac{\partial W}{\partial w_u} \frac{\partial w_u}{\partial \mu} = \left[ \frac{\partial L}{\partial w_u} \left( \gamma L^{\gamma-1} - b \right) + \frac{\partial \delta^*}{\partial w_u} \left( c + b - \beta H^{\beta-1} (1 + \delta^*) \right) \right] \frac{\partial w_u}{\partial \mu} < 0
$$

if $(\partial L/\partial w_u)(w_u - b + L) > 0$

First of all, of course, employment in the unionised sector decreases which has the well know negative welfare effects. Moreover, education and thus employment in the skilled sector will decrease as well, leading to a second welfare loss. The second effect hasn't been dealt with in the literature up to now. Overall employment $(B = L + (1 - \delta^*))$ and welfare will decrease.

At the threshold of $\nu = 1$ and $\mu = 1$ the numerator of equation (15) is zero. The second term in equation (16) is zero, too. Assuming $\nu < 1$ (case b)) an increase in the union bargaining power will still lead to an increase in the wage rate for all levels of $\mu$. At higher levels of $\mu$ this may, however, lead to a reduction in the expected income of a union member, i.e. the numerator of equation (15) will become negative. Education will increase. The welfare loss from a too low level of employment in the unskilled sector will increase, while the second welfare will decrease. Irrespective of the level of $\nu$, a unionisation of the labour markets will always lead to an increase in the expected income of an unskilled, a reduction of the education level and, thus, a second welfare loss compared to the situation with an exogenous number of unskilled workers, though.

Summing up we can say that the unionisation of the labour market has two negative effects on welfare, compared to only one in the case with exogenous human capital formation. Therefore, the question arises whether there do exist government policies in order to counteract the welfare losses.
3 Subsidising education

Government policies to reduce the first welfare loss from a decrease in employment in the unskilled sector have been widely discussed in recent years, one possible measure, e.g., discussed by Lockwood/Manning (1993), being a progressive tax structure in order to reduce union wage demands. Therefore, the focus here lies on the second welfare loss, resulting from an inefficiently low level of education.

The easiest way to influence the education decision is the introduction of a subsidy (s) on education. This leads to a reduction in the private costs of education. Doing so we will assume different ways of financing. First of all, we will look at the consequences of a lump-sum financed subsidy before turning to the more realistic case of a labour income tax financing. We exclude a profit taxation by assuming that profits are either already fully taxed or not taxable.

3.1 Effects of a lump-sum financed subsidy

As a reference point we will look at the case of a lump-sum financing. We will do so in order to better analyse the isolated effects of a subsidy.

**Proposition 2:** The introduction of a lump-sum financed education subsidy increases the level of education and employment in the skilled sector. It has no influence on wages or employment in the unskilled sector. The level of welfare increases.

The subsidy leads to a reduction in the education costs, whereas the lump-sum tax doesn’t have any influence on the individuals’ education decision. It has to be paid irrespective of the education level and therefore doesn’t appear in the calculation of the marginal individual. The decision of the marginal individual is given by:

$$\frac{L}{\delta^*} w_u + \frac{\delta^* - L}{\delta^*} b = (1 + \delta^*) w_s - (c - s)$$

(17)

It is easy to show that an increase in the subsidy leads to an increase in education:
\[ \frac{\partial \delta^*}{\partial s} = \frac{1}{-\frac{L}{\delta^*} (w_u - b) - w_s - (1 + \delta^*) \frac{\partial w_s}{\partial \delta^*}} < 0 \] (18)

An increase in the subsidy has the following welfare implications:

\[ \frac{\partial W}{\partial \delta^*} \frac{\partial \delta^*}{\partial s} = \left[ c + b - (1 + \delta^*) \beta H^{\beta-1} \right] \frac{\partial \delta^*}{\partial s} > 0 \] (19)

The sign in equation (19) is positive, since we get \((1 + \delta^*) \beta H^{\beta-1} - c > b\) from equation (14).

The subsidy has a positive effect since the individual and the social maximisation don’t coincide in case of imperfect labour markets. Deciding on his education an individual compares the expected utility of an uneducated with his earnings potential in the case of education less education costs, whereas the welfare function compares the potential earning in case of education less the costs of education with the welfare of an unemployed unskilled. This is due to the fact that in this kind of model the number of unskilled worker doesn’t have an impact on employment in the unionised sector, which is true for most kinds of trade union models. Thus, the subsidy helps to increase the number of skilled by the same amount as the number of unemployed unskilled is reduced.

3.2 Financing by a tax on skilled labour income

After looking at the case of lump-sum financing we will turn to a subsidy that is financed by a labour income tax. Before turning to a general labour income tax, however, we will have a look at the effects of a tax on skilled labour incomes only. Thus, both the subsidy and the necessary increase in the tax rate only have a direct effect on the income in the case of education. This leads to the following condition for the marginal individual:

\[ \frac{L}{\delta^*} w_u + \frac{\delta^* - L}{\delta^*} b = (1 - \tau_s)(1 + \delta^*) w_s - (1 - \alpha \tau_s) c + s \] (20)

\(\tau_s\) is the tax rate on the labour income in the skilled sector, \(\alpha\) shows which part of the education cost is tax deductible. We will assume that the costs of education \((1 - \delta^*)\) are fully
tax deductible, i.e. $\alpha = 1$. The subsidy is not subject to taxation. This leads to the following government budget constraint:

$$\bar{R} = 0 = \tau_s \left[ w_s H - c (1 - \delta^*) \right] - (1 - \delta^*) s \quad (21)$$

We assume that the tax revenue is only used to finance the subsidy. The government revenue requirement ($\bar{R}$) is zero. In the following we will show that the Proposition 3 holds.

**Proposition 3:** The introduction of an education subsidy, financed by a linear tax on skilled labour income, leads to an increase in education and employment in the skilled sector. Wages and employment in the unskilled sector are not affected. Welfare increases.

Proof. Totally differentiation equations (20) and (21) with respect to $\tau_s$, $s$ and $\delta^*$ leads to:

$$
\begin{bmatrix}
- \frac{L}{\delta^*} (w_u - b) - (1 - \tau_s) w_s \\
-(1 - \tau_s)(1 + \delta^*) \frac{\partial w_s}{\partial \delta^*} \\
\frac{\partial w_s H}{\partial \delta^*} \tau_s + \alpha \tau_s c + s
\end{bmatrix}
\begin{bmatrix}
(1 - \delta^*) w_s - \alpha c \\
(1 - \delta^*) \tau_s + \alpha \tau_s c + s \\
(1 - \delta^*)
\end{bmatrix}
\begin{bmatrix}
d\delta^* \\
d\tau_s \\
ds
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
1 - \delta^*
\end{bmatrix} ds \quad (22)
$$

We can now calculate the effects of an increase in the subsidy on the level of education and the tax rate:

$$
\frac{d\delta^*}{ds} = \frac{w_s \frac{1}{2} (1 - \delta^*)^2}{(-)} < 0 \quad (23)
$$

$$
\frac{d\tau_s}{ds} = (1 - \delta^*) n - \frac{\frac{\partial w_s H}{\partial \delta^*} \tau_s + \alpha \tau_s c + s}{(-)} \quad (24)
$$

However, the following results also go through if this were not the case.
with: \[ n = -\frac{L}{\delta^2} (w_u - b) - (1 - \tau_s) w_s - (1 - \tau_s) (1 + \delta^*) \frac{\partial w_s}{\partial \delta^*} < 0 \]

In the following we will always assume that an increase in the subsidy has to be financed by an increase in the tax rate, equation (24) is positive. We assume to be in the increasing part of the Laffer-Curve as it would otherwise be possible to increase welfare by just reducing the tax rate. The increase in the subsidy unambiguously has a positive effect on the level of education. All skilled individuals get the same subsidy regardless of their potential income after education. However, the tax burden is unequally distributed among the skilled worker. It depends on their income which is increasing with an individual earnings potential. The marginal individual has the lowest earning potential of all skilled workers. Therefore it only has to finance a relatively small part of the subsidy. The marginal individual’s overall tax burden including the subsidy is negative. So, the subsidy is partly financed by taxing intramarginal individuals and, thus, rents. The number of educated individuals increases. The effect on education is the bigger, the more heterogeneous the educated individuals are, i.e. the smaller the share of the marginal individual in financing the subsidy is.

The welfare effect of the subsidy is equivalent to the lump-sum financing case. An increase in education without a decrease in unskilled employment increases overall employment and welfare.

3.3 Financing by a general labour income tax

We now turn to the case, where the subsidy is financed by a general labour income tax. Before turning to the subsidy we will briefly analyse the isolated effects of an increase in the general labour tax rate. It is well known from the literature that in the case of perfect labour markets with full employment \((w_u > b)\) an increase in the labour tax rate has no effects on education or employment if all costs of education are tax deductible. Otherwise it will reduce education and welfare.\(^{10}\) Assuming a unionised labour market the effect depends on the union’s objective function. In order to simplify the analysis it is again assumed, that the costs of education are fully tax deductible. Considering a taxation of labour income the union’s objective function takes the form

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\(^{10}\) See., e.g., Heckman (1976).
\[ V = L' \left( (1 - \tau)w_u - b \right) \nu \geq 0. \]  \hspace{1cm} (25)

\( \tau \) is the general labour income tax rate. The union is now focusing on the difference between the after tax wage rate and the alternative utility. It is assumed that the alternative utility is not subject to taxation as it is, e.g., impossible to tax leisure. The education decision of the marginal individual transforms to:

\[ (1 - \tau) \frac{L}{\delta^*}w_u + \frac{\delta^*}{\delta^*}L - b = (1 - \tau)(1 + \delta^*)w_s - (1 - \tau)c \]  \hspace{1cm} (26)

The government budget constraint is given as:

\[ \bar{R} = \tau \left[ (w_uL + w_sH) - c(1 - \delta^*) \right]. \]  \hspace{1cm} (27)

Following, e.g. Fuest/Huber (2000) we now focus on two polar cases with respect to the incidence of taxation. In the first case the union's weight on the employment and the wage objectives are assumed to be the same (\( \nu = 1 \)). We get the so-called rent maximising union model. The other polar case is the one we get setting the weight on employment to zero, the union only has a wage objective.

In order to simplify the analysis we will assume the special case where the union has all the bargaining power.\(^{11}\) First we will turn to the case of a rent maximising union. As the union has all the bargaining power, we get a so-called monopoly union. In order to simplify analysis we set \( \Pi_0 \) equal to zero. The alternative utility has to be strictly positive due to the production function. The wage rate is a mark-up on the alternative utility:

\[ w_u = \phi b \quad \text{with} \quad \phi = \frac{1 + \mu(1 - \gamma)}{\gamma (1 - \tau)} \]  \hspace{1cm} (28)

\(^{11}\) However, the results would also go through if we assumed \( \mu \) to be smaller than one.
An increase in the tax rate leads to an increase in the gross wage rate \((\partial w_u/\partial \tau = w_u/(1-\tau) > 0)\) such that the net wage rate stays constant. We therefore have the special case of a net wage rigidity. All the economic incidence of the income tax lies with the firm as the labour costs increase. A tax increase thus leads to a reduction in employment in the unskilled sector. Next we will analyse the effect on education:

\[
\frac{\partial \delta^*}{\partial \tau} = -\frac{\delta^* - L \frac{b}{(1-\tau)} + L \frac{w_u}{\delta^*(1-\tau)}}{-\frac{L}{\delta^*} \left( w_u - \frac{b}{(1-\tau)} \right)} > 0
\]

Here we have to consider two aspects. First of all, the tax rate leads to a proportional reduction in the income of skilled workers minus costs of education. For a given wage rate the expected income of an unskilled worker is only partly reduced, though. This is due to the fact that only the income of employed workers is taxed, the utility from leisure is not subject to taxation. The labour income tax is biased against education. The second effect stems from the increase in the unskilled wage rate. We have just seen that the union increases the wage rate which leads to an increase in the expected income of unskilled individuals and thus, to a further reduction in education. However, the rent can’t be increased to the pre-tax level. The effect on overall employment and welfare is unambiguous as employment in both sectors decreases.

After looking at the effects with a rent maximising union we will turn to a seniority model. So the weight on the employment objective is set to zero. We now assume the fall back profit of the firm to be strictly positive in order to avoid boundary solutions. For simplification the alternative utility is set to zero. Using the Nash-maximum and the union’s objective function we get:

\[
w_u = \frac{\gamma \Pi}{\mu - \gamma L}
\]

Given a union bargaining power of one, the wage rate is independent of the tax rate. The union maximises the wage rate given the constraint of the minimum profit \(\Pi_u\). It can be easily seen that the wage rate is independent of the tax level, we have the special case of a
fixed gross wage \((\partial w_u/\partial \tau = 0)\). All the economic incidence of the tax increase lies with the workers, the firm’s profit doesn’t change. Moreover, it doesn’t have an impact on education either if the alternative income is assumed to be zero or subject or taxation.

Having considered the polar cases, the empirical situation is of importance. Although an unambiguous prediction is not possible, a fairly general statement can be made. Labour costs seem to increase following a tax increase in the short run and medium term (see Layard et al. (1991, p. 209f)). In the long run, the tax rate has to be born by the employees, though, according to an OECD study (see OECD (1990, p. 176f) and Newell/Symons (1987).

We will now turn to the effects of a tax financed increase in the subsidy where we will again distinguish the two polar cases. The education decision of the marginal individual is given by:

\[
(1 - \tau) \frac{L}{\delta^*} w_u + \frac{\delta^*}{\delta^*} b = (1 - \tau) [(1 + \delta^*) w_s - c] + s
\]  

whereas the government budget constraint turns to:

\[
\bar{R} = \tau [(w_u L + w_s H) - c(1 - \delta^*)] - (1 - \delta^*) s
\]  

We will show that the following proposition holds:

**Proposition 4:** The introduction of an education subsidy, financed by a linear labour income tax, unambiguously leads to an increase in education. In the case of a

a) rigid gross wage \((\nu = 0, \Pi_u > 0, b = 0)\) in the unskilled sector the tax increase has no effect on wages and employment in the unskilled sector. Overall employment and welfare increase unambiguously.

b) rigid net wage \((\nu = 1, \Pi_u = 0, b > 0)\) in the unskilled sector the tax increase leads to a wage increase and employment decrease in the unionised sector. The overall employment and welfare effects are ambiguous. An increase in overall employment is a necessary but not a sufficient condition for an increase in welfare.
Proof. The derivations can be found in appendix 2. First we will have a look at the case of a fixed gross wage (case a)) derived from the seniority wage model. An increase in the subsidy has the following effect on the education level:

\[
\frac{d\delta}{ds} = \frac{1}{2} w_s (1-\delta_*)^2 + \frac{L}{\delta_*} w_u < 0
\]  

(33)

The subsidy unambiguously increases education and welfare. For a given subsidy rate the marginal individual now has to pay a smaller part of the financing compared to the case with a tax on skilled labour income only. This is due to a positive tax revenue from taxing the unskilled workers. Thus, the measure helps to reduced the second welfare loss of the union wage setting without having an effect on the first welfare loss due to too low employment in the unskilled sector.\(^{12}\)

Finally, we will turn to case b). Regarding a fixed net wage, the effect of the financing side has already been discussed. This had a negative impact on education. The isolated subsidy, however, had a positive effect on education. Therefore there are two countervailing effects. As shown in appendix 2 the government measure has the following impact on education:

\[
\frac{d\delta}{ds} = \frac{1}{2} w_s (1-\delta_*)^2 + \frac{1-\tau-\gamma + \tau\gamma (1-\delta_*)}{(1-\tau)(1-\gamma)\delta_*} w_u L
\]  

(34)

Despite the countervailing effects, it is possible to show, that the overall impact on education is positive under one assumption. The effect of the tax increase on the tax revenue from the unskilled must be positive, which is true for low tax rates. This condition is, however, sufficient and not necessary. The result can be explained as follows. Although the union now reacts to a tax increase by increasing the wage rate, it can’t increase the rent to the pre-tax level. The expected after tax income of an unskilled worker decreases, whereas we showed that the marginal workers income was increased by the subsidy. The effect on education is

\(^{12}\) In the case of a bargaining power of \(\mu = 1\) this result is independent of the assumption of \(b = 0\).
again stronger than in the case with a tax on skilled labour income only. It is smaller than in
the seniority wage model, though.

The employment effect is more difficult to determine. The increase in education has a positive
effect on employment in the skilled sector, whereas employment in the unskilled sector
decreases:

\[
\frac{\partial B}{\partial s} = -\frac{\partial \delta^*}{\partial s} + \frac{\partial L}{\partial w_u} \frac{\partial \tau}{\partial \delta} = -\frac{\partial \delta^*}{\partial s} - \frac{1}{1-\gamma} \frac{L}{(1-\tau)} \frac{\partial \tau}{\partial s} \begin{cases} > \ 0 \\ < \end{cases}
\]

The same is true for the welfare effect. It is ambiguous. The first welfare loss due to a too
high wage rate in the unskilled sector increases while the second welfare loss from a too low
level of education is reduced. It can only be said that a positive employment effect is a
necessary but not a sufficient condition for a welfare increase. The welfare effect is given as:

\[
\frac{\partial W}{\partial s} = \frac{\partial L}{\partial w_u} \frac{\partial \tau}{\partial \delta} \left( \gamma L^{\gamma-1} - b \right) + \frac{\partial \delta^*}{\partial s} \left( \beta H^{\beta-1} \frac{\partial H}{\partial \delta^*} + c + b \right)
\]

\[
= -\frac{1}{1-\gamma} \frac{L}{(1-\tau)} (w_u - b) \frac{\partial \tau}{\partial s} = -\frac{\partial \delta^*}{\partial s} \left( w_s (1+\delta^*) - c - b \right)
\]

If overall employment would stay constant this would imply an output decrease in the
unskilled sector and an increase in the skilled sector. The education costs would increase. As
the income of an unskilled exceeds the income of the marginal skilled less his costs of
education in the equilibrium without taxation, welfare would decrease.

4 Conclusion

It was shown that a unionisation of labour markets not only has the well known negative
employment effect in the unionised sector but leads to a decrease in education and, thus,
overall employment. We then analysed whether an education subsidy could help to mitigate
the welfare loss resulting from the distortion of the education decision. It turned out that the
effect can be positive but that it very much depends on both the financing of the subsidy and
the union wage setting behaviour.
A positive effect can be achieved if the subsidy is financed lump-sum or by a tax on skilled labour income. If the subsidy is financed by a general labour income tax the effect depends on the union’s objective function. In the special case of a rigid gross wage it was shown to be positive, whereas we found the effect with a net wage rigidity to be ambiguous.

A positive effect was due to the influence of the subsidy on the education decision of the marginal individual. So, the model provides an argument why, like in the case of Germany, universities should not subsidise primarily the elite student but rather the “marginal” students in order to increase the overall education level. Moreover, the financing burden of the subsidy should be born by the individuals that benefit directly from its provision.

**Appendix 1:**

In this appendix we show for the general case $\nu \geq 0$ that the wage rate increases with the union’s bargaining power. For simplicity we assume $\Pi_u = 0$. Implicitly differentiating the first-order condition

$$\Omega_{w_u} = \mu \left( \frac{L_u}{L} + \frac{1}{w_u - b} \right) - (1 - \mu) \frac{L}{\Pi} = 0 \quad (A.1)$$

we get:

$$\frac{\partial w_u}{\partial \mu} = - \frac{\Omega_{w_u \mu}}{\Omega_{w_u w_u}} = - \frac{1}{w_u - b} - \frac{\nu}{(1 - \gamma)w_u} + \frac{L}{\Pi} \quad (-) \quad (A.2)$$

The denominator of equation (A.2) is negative, as the second order condition of the bargaining solution implies $\Omega_{w_u w_u} < 0$. In order to get $\frac{\partial w_u}{\partial \mu} > 0$ the nominator of equation (A.2) must be positive. It can be written as

$$\frac{1}{w_u - b} - \frac{\nu}{(1 - \gamma)w_u} + \frac{L}{\Pi} = \frac{w_u - \gamma w_u - \nu w_u + \nu b}{(w_u - b)(1 - \gamma)w_u} + \frac{L}{\Pi} \quad (A.3)$$

Using the firm’s profit function $\Pi = L^b - L w_u$ we get:
\[ \frac{w_u - vw_u + vb - \gamma b}{(w_u - b)\gamma(1-\gamma)w_u} \] (A.4)

The denominator of equation (A.4) is surely positive, therefore we have to show that this is true for the nominator as well in order to get \( \partial w_u / \partial \mu > 0 \). In order to get the sign, we consider two extreme cases for \( v \). Assuming \( v = 0 \) we get \( w_u - \gamma b \), which is positive. The other borderline case leads to \( \lim_{v \to \infty} w_u = b \), the nominator is again positive. In order to proof that it is always positive in the interval \( v = [0, \infty) \), we have to show, that the nominator is strictly decreasing in \( v \). Forming the first order condition we get:

\[ \partial \left( w_u - vw_u + vb - \gamma b \right) / \partial v = b - w_u + (1-v) \frac{\partial w_u}{\partial v} < 0 \] (A.5)

The derivation is strictly negative, as \( \partial w_u / \partial v < 0 \). This can be shown by implicitly differentiating equation (A.1). Summing up, equations (A.4) and (A.2) are unambiguously positive and we get: \( \partial w_u / \partial \mu > 0 \).

**Appendix 2**

In this appendix, we analyse the effects of a labour tax financed increase in the subsidy on the level of education. Totally differentiating equations (31) and (32) with respect to \( \delta^* \), \( \tau \) and \( s \), we get:

\[
\left\{ \begin{array}{c}
(1-\tau)m \\
\frac{\partial w_H}{\partial \delta^*} \tau + \tau c + s \\
\frac{\partial w_H}{\partial \delta^*} + \frac{\partial w_u L}{\partial \tau} + w_u L + w_s H - c (1-\delta^*) \end{array} \right\} \left\{ \begin{array}{c}
\frac{d \delta^*}{d \tau} = \left( \begin{array}{c} 1 \\
1-\delta^* \end{array} \right) ds \\
\frac{d \delta^*}{d \tau} = \left( \begin{array}{c} 1 \\
1-\delta^* \end{array} \right) ds \\
\frac{d \delta^*}{d \tau} = \left( \begin{array}{c} 1 \\
1-\delta^* \end{array} \right) ds \end{array} \right\} = \left( \begin{array}{c} 1 \\
1-\delta^* \end{array} \right) ds \\
(\delta^* \tau + \tau c + s)
\end{array}
\]

with: \( m = -\frac{L}{\delta^* s^2} \left( w_u - \frac{b}{(1-\tau)} \right) \)

\[ w_s - (1+\delta^*) \frac{\partial w_s}{\partial \delta^*} < 0 \]

We first discuss the case with fixed gross wages \( v = 0 \) before turning to the case with fixed net wages \( v = 1 \).
The case with $\nu = 0$

If the gross wages are rigid ($\partial w_u/\partial \tau = 0$) we can calculate the effect of an increase in the subsidy on the level of education:

$$\frac{d\delta^*}{ds} = \frac{w_u L + w_u H - (1 - \delta^*)c - (1 - \delta^*)\left(\frac{L}{\delta^*}w_u + (1 + \delta^*)w_s - c\right)}{(-)}$$  \hspace{1cm} (A.7)

Restructuring equation (A.7) we get:

$$\frac{d\delta^*}{ds} = \frac{1}{2}w_s(1 - \delta^*)^2 + \frac{L}{\delta^*}w_u < 0$$ \hspace{1cm} (A.8)

The case with $\nu = 1$

Suppose that the net wages are fixed ($\partial w_u/\partial \tau = w_u/(1 - \tau)$) Increasing the subsidy leads to:

$$\frac{d\delta^*}{ds} = \left(\frac{\partial w_u L}{\partial \tau} + w_u L + w_u H - c(1 - \delta^*)\right)(1 - \delta^*)\left(\frac{L}{\delta^*}w_u - c + (1 - \delta^*)w_s\right)$$

$$= \frac{w_u H - (1 - \delta^*)(1 + \delta^*)w_s + \frac{L}{\delta^*}w_u + \tau \frac{\partial w_u L}{\partial \tau}}{(-)}$$

$$= \frac{1}{2}w_u(1 - \delta^*)^2 + \frac{L}{\delta^*}w_u - \frac{\tau \gamma}{(1 - \tau)(1 - \gamma)}w_u L$$ \hspace{1cm} (A.9)

Restructuring (A.9) using

$$\frac{\partial (\tau L w_u)}{\partial \tau} = \frac{1 - \gamma - \tau}{(1 - \tau)(1 - \gamma)}L w_u > 0$$ \hspace{1cm} (A.10)
we get equation (34). Calculating the effect of an increase in the subsidy on the tax rate leads to:

\[
\frac{d\tau}{ds} = \frac{(1 - \delta^*) (1 - \tau) m - \left( \frac{\partial w}{\partial \delta^*} \tau + \tau + s \right)}{(-)}
\]

with \( m = -\frac{L}{\delta^*} \left( w_u - \frac{b}{(1 - \tau)} \right) - w_i - (1 + \delta^*) \frac{\partial w}{\partial \delta^*} < 0 \).

5 Literature


Binmore, Ken; Rubinstein, Ariel and Ascher Wolinsky (1986), The Nash Bargaining Solution in Economic Modelling, Rand Journal of Economics 17, 176-188.


