LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
VOLKSWIRTSCHAFTLICHE FAKULTÄT

Maria Lehner:

Group versus Individual Lending in Microfinance

Munich Discussion Paper No. 2008-24

Department of Economics
University of Munich

Volkswirtschaftliche Fakultät
Ludwig-Maximilians-Universität München

Online at http://epub.ub.uni-muenchen.de/7486/

Group Lending versus Individual Lending in Microfinance

Maria Lehner*

University of Munich

September 2008

Abstract

Microfinance is typically associated with joint liability of group members. However, a large part of microfinance institutions rather offers individual instead of group loans. We analyze the incentive mechanisms in both individual and group contracts. Moreover, we show that microfinance institutions offer group loans when the loan size is rather large, refinancing costs are high, and competition between microfinance institutions is low. Otherwise, individual loans are offered. Interestingly, our analysis predicts that individual lending in microfinance will gain in importance in the future if microfinance institutions continue to get better access to capital markets and if competition further rises.

JEL classification: F37, G21, G34, L13, O16

Keywords: microfinance, group lending, individual lending

*University of Munich, Akademiestr. 1/III, 80799 Munich, Germany. Tel: +49 89 2180 2766. Email: maria.lehner@lrz.uni-muenchen.de

1 Introduction

In 2006, the Nobel Peace Prize was awarded to Mohammad Yunus. Since he founded the Grameen Bank in Bangladesh in the late 1970s, microfinance has experienced an impressive growth. This is largely due to the many positive effects attributed to microfinance programs. Microfinance schemes have been found to reduce poverty and to positively affect nutrition, health and education as well as gender empowerment (Littlefield et al. (2003)). In 2006, microfinance institutions reached around 130 million customers around the world (Daley-Harris (2007)).

Typically, microfinance is associated with joint liability lending. When borrowers form groups and are held liable for each other, lending to the poor can be profitable even if borrowers do not possess any collateral and lack a credit history. Interestingly, however, a large part of microfinance institutions does not offer group but individual loans. This gives rise to several questions: what are the incentive mechanisms that play a role in individual and group lending schemes and how do they differ? Under which circumstances do microfinance institutions offer group or individual loan contracts? What are the differences between individual lending programs of microfinance institutions and the individual lending technology applied by commercial banks?

According to Giné and Karlan (2006), the different features of group and individual lending schemes have not yet been studied in detail "despite being a question of first-order importance".[1] With the aim to contribute to a theoretical foundation of this topic, we set up a model of spatial competition between microfinance institutions. Microfinance institutions offer either group or individual loans and compete in the repayments they charge their clients. Borrowers differ in their success probabilities and lack pledgeable collateral. As borrowers have no documented credit history, they are unable to provide hard information.

Consequently, in contrast to commercial banks, microfinance institutions cannot screen borrowers and secure loans with collateral. Screening borrowers is feasible only when a relatively standardized evaluation procedure based on the analysis of hard information such as balance sheet data is applicable. In addition, any lending strategy pursued by microfinance institutions must ensure monitoring of borrowers in order to prevent the diversion of loans to urgent consumption needs.

When a microfinance institution opts for the group lending technology, it transfers the monitoring role to borrowers. Joint liability ensures strong incentives of group members

[1] Giné and Karlan (2006, p. 3)

to monitor each other in order to make their peers succeed. Furthermore, self-selection of borrowers into different credit contracts can be achieved.

In case it grants individual loans, the microfinance institution specializes in closely monitoring clients. Borrowers are offered a pooling contract. However, borrowers are exempt from negative effects of group lending schemes such as bearing additional risk, loss of privacy from disclosing their financial situation and investment projects to potential peers, or time spent on group meetings.

Our first focus of interest lies on how the decision of a microfinance institution to offer either group or individual loans depends on the size of a loan. Controversial arguments are brought forward in the so far rather descriptive literature on this topic. For instance, Kota (2007) and Harper (2007) state that microfinance institutions offer individual contracts if clients are in need for larger loans. In contrast, Giné and Karlan (2006) advocate precisely the reverse correlation. Our analysis aims to contribute to a theoretical foundation of this discussion.

Another major focus of our study is to investigate how the choice of lending technology depends on refinancing conditions and competitive pressure in the microfinance market. According to Isern and Porteous (2005) as well as Reddy and Rhyne (2006), the world of microfinance currently changes substantially in both these respects. The emergence of rating agencies specializing in the microfinance business and the growing awareness regarding the potential of the microfinance industry makes investors channel more and more funds into this market. Enhanced access to capital markets, in turn, implies improved refinancing conditions for microfinance institutions. In addition, competition among microfinance banks steadily intensifies. Our analysis provides a theoretical framework that allows us to study in detail how changes in refinancing conditions and competition affect a microfinance institution's lending strategy.

Interestingly, our results show that microfinance institutions decide to offer individual loans when the loan size is rather small. Moreover, microfinance institutions favor individual over group contracts when refinancing costs are low and when competition is intense. Hence, our analysis allows for some interesting predictions concerning the future shape of the microfinance industry. Given that access to capital markets continues to improve and competition between microfinance institutions rises further, our results imply that individual loan contracts in the microfinance market will gain in importance over the next years.

The remainder of this paper is organized as follows. The next section reviews the literature. Section 3 describes the set-up of the model. In section 4, we study the choice

of lending technology of microfinance institutions. We present our comparative statics analysis in section 5. Empirical hypotheses are stated in section 6. Section 7 concludes.

2 Related Literature

Although individual loans account for a large portion of microfinance loans, the literature is heavily biased towards an analysis of group loan contracts. Individual lending schemes have only very recently attracted the interest of researchers.

Numerous theoretical papers have addressed the positive effects of group lending mechanisms. Ghatak and Guinnane (1999), Ghatak (2000) as well as Van Tassel (1999) show that group lending achieves self-selection of borrowers and acts as a screening device. Armendáriz de Aghion and Gollier (2000) find that even if borrowers do not know each other's type, group lending may be feasible due to lower interest rates as a result of cross subsidization of borrowers. Stiglitz (1990) outlines the role of peer-monitoring in group lending schemes, which transfers the monitoring role from the bank to the borrowers and acts as an incentive device. Armendáriz de Aghion (1999) demonstrates that the benefits from peer monitoring are largest when risks are positively correlated among borrowers. Laffont and N'Guessan (2000) conclude that social connections facilitate the monitoring and enforcement of joint liability loan contracts. This result has been confirmed in an empirical study by Karlan (2007). Furthermore, Armendáriz de Aghion and Morduch (2000) point to a fall in transaction costs when - instead of individual visits of clients - group meetings are held. In addition, the contact with banks to which poor borrowers typically are not used to is facilitated.

However, certain drawbacks of group lending exist. Giné and Karlan (2006) state that the demand for credit within a group may change over time, forcing clients with small loans to be liable for larger loans of their peers. Furthermore, the growth of group lending programs may slow down when new borrowers with looser social ties enter and, consequently, the group lending technology loses some of its power. Besley and Coate (1995) stress negative welfare effects if the group as a whole defaults even if some members had repaid under individual lending. In a case study, Montgomery (1996) outlines the unnecessary social costs of repayment pressure. Stiglitz (1990) points to the higher risk borrowers assume when they are not only liable for themselves but also for their group partners.

The so far rather descriptive literature on individual lending schemes typically focuses on the crucial role of closely monitoring borrowers. Navajas et al. (2003), Armendáriz

de Aghion and Morduch (2005) as well as Giné et al. (2006) describe the problem that poor borrowers may divert a loan, at least partly, to urgent consumption needs. In order to ensure the use of the loan for the agreed upon investment project, Champagne et al. (2007) as well as Zeitinger (1996) stress the importance of regularly visiting clients. In a theoretical analysis of individual lending schemes by Gangopadhyay and Lensink (2007), the monitoring of borrowers by informal lenders plays a central role. Armendáriz de Aghion and Morduch (2000) as well as Dellien et al. (2005) also point to the importance of monitoring borrowers in individual lending schemes.

Only recently, researchers have been interested in comparing group lending programs to individual lending schemes. Giné and Karlan (2006) conduct a field experiment in the Philippines. They find that by offering individual loans, a microfinance institution can attract relatively more new clients. Yet, both lending schemes do not differ in repayment rates. In a recent empirical study, Ahlin and Townsend (2007) find a U-shaped relationship between joint liability contracts relative to individual contracts and a borrower's wealth. Furthermore, they conclude that higher correlation across projects makes group lending contracts more likely relative to individual contracts. In her theoretical analysis, Madajewicz (2008) shows that, in general, borrowers prefer individual loans the wealthier they are. Nevertheless, she demonstrates that for very low levels of borrower wealth, group loans are larger than individual loans. Moreover, she finds that businesses funded with individual loans grow more than those funded with group loans.

3 The Model

Consider a continuum of borrowers with mass 2 that is uniformly distributed along a straigt line of length 1. Each borrower can engage in one investment project that requires an initial outlay of i, $i > 0$. Borrowers are not endowed with any initial wealth and therefore need to apply for credit at a microfinance institution, the only source of finance in our model. Borrowers have either safe or risky projects. It is common knowledge that the fraction of borrowers with safe projects is γ and the fraction of borrowers with risky projects is $1 - \gamma$, $0 < \gamma < 1$. We assume that borrowers with safe and risky projects are distributed with density 2γ and $2(1 - \gamma)$ along the Hotelling line, respectively. As a result, two borrowers of the same type are located at a certain point of the Hotelling street. As will be explained in more detail later on, this assumption ensures costless formation of groups. Individual borrowers know about the type of their own and the other borrowers' investment projects. In case a project is successful it generates a return of $v > 0$ and zero otherwise. The success probability of safe and risky projects is given by p_S and p_R,

respectively, with $0 < p_R < p_S < 1$. The returns of the projects are observable and contractible. We assume that borrowers must be monitored closely in order to prevent the diversion of the loan to consumption needs.[2]

The financial sector serving the borrowers consists of two representative microfinance institutions A and B that are located at the two ends of the Hotelling line. In our model, both microfinance institutions are profitable and compete with each other. Note that the profitability of microfinance institutions has risen considerably over the last few years (Christen and Cook (2001)). Some microfinance institutions are now even listed at stock exchanges, such as Compartamos in Mexico or Equity Bank in Kenya. Furthermore, due to the immense growth of the microfinance industry, in many countries, there is now fierce competition between microfinance institutions (Fernando (2007), McIntosh et al. (2005), and Christen and Rhyne (1999)).

Microfinance institutions A and B compete in the repayments they simultaneously ask from borrowers. Microfinance institutions incur refinancing costs $c > 0$ per loan of size i. We take it as given that each microfinance institution disposes of enough funds to finance all borrowers applying for a loan. Microfinance institutions do not know whether borrowers have safe or risky projects.

A microfinance institution may choose to offer either group or individual loans. We abstract from the possibility that a bank offers both group and individual contracts. In fact, most microfinance institutions offer either one or the other type of loan as is pointed out in Ahlin and Townsend (2007), Giné and Karlan (2006) as well as Madajewicz (2008).

If a microfinance institution opts for the group lending technology, loans are offered to groups consisting of two borrowers each. Note that limiting the group size to two borrowers is a standard assumption in the literature and greatly simplifies our analysis (see, for instance, Ghatak (2000) or Laffont and N'Guessan (2000)). Group contracts imply a transfer of the monitoring role to the group members. Due to joint liability, group members have a strong incentive to monitor each other in order to ensure the correct investment of the loan and to make their partners succeed. We assume that due to close social ties between group members, borrowers monitor each other at zero cost.[3] The size of a loan a group receives is $2i$ so that each borrower receives an amount i of the loan. In case both group members are successful, the loan is fully paid back with interest. If both partners fail, no repayments are made. In the case that only one of

[2] We could as well assume that borrowers divert a certain part of their loans for household needs if not monitored. As a result, borrowers could only afford low quality inputs for their investment projects implying returns too low to pay back their loans.

[3] One might argue that borrowers incur costs of monitoring. However, the monitoring costs should clearly be lower for borrowers than for banks due to strong social ties between borrowers (Karlan (2007)). By normalizing monitoring costs of borrowers to zero, our results remain qualitatively unaffected.

the two group members is successful, the successful borrower pays back her part of the loan plus interest and, in addition, the loan share of her peer with interest, weighed by a joint liability parameter $\lambda > 0$. The joint liability parameter expresses the degree of joint liability to which group members stand in for each other. Microfinance institutions compete both in interest rates and the joint liability parameters. Microfinance institutions can induce self-selection of borrowers by offering two different contracts. A contract with a low interest rate r_S and a high degree of joint liability λ_S will attract safe borrowers whereas risky borrowers prefer a contract with a high interest rate r_R and a low joint liability factor λ_R.[4] Borrowers incur some disutility d from group lending. The disutility d captures drawbacks of group loans such as time spent on finding a partner and group meetings (Armendáriz de Aghion and Morduch (2000)), the higher risk borrowers bear due to joint liability (Stiglitz (1990)) or social costs of repayment pressure (Montgomery (1996)). Borrowers may also suffer from reduced privacy when disclosing details of their investment project or their financial situation to their peers (Harper (2007)).

In the case a microfinance institution decides to offer individual loans, it has no mechanism at hand to assess a borrower's type. Note, first, that a collateralized contract cannot be offered since borrowers lack any pledgeable assets. Second, screening borrowers is not an option as potential clients are unable to provide hard information. Hence, microfinance institutions offer a pooling contract with repayment rate r_{SR} per credit of size i. In order to prevent the diversion of the loan for consumption needs once it is received, microfinance institutions need to closely monitor clients. This imposes a per borrower cost of k on the microfinance institution.[5] The crucial role of closely monitoring clients in individual lending programs has been stressed, for instance, by Champagne et al. (2007) as well as Zeitinger (1996). Armendáriz de Aghion and Morduch (2000) and Dellien et al. (2005) confirm the importance of regularly visiting clients in individual lending schemes.

Borrowers base their decision at which microfinance institution to apply for credit on the repayments r^j, $j = A, B$, and the joint liability factors λ^j asked by the microfinance institutions as well as on the transport costs they incur by travelling to a microfinance institution. We assume that transport costs tx are proportional to the distance x between the borrower and the microfinance institution. If borrowers apply for a group contract, transportation costs arise for both group members. Furthermore, we assume that the return of a project v is high enough so that the market is covered at equilibrium prices. Borrowers and microfinance institutions are risk neutral and maximize profits.

The time structure of the game is as follows. At stage 1, microfinance institutions

[4] See Ghatak (2000), Stiglitz (1990) or Van Tassel (1999) for a similar set-up.

[5] Evidence for larger costs per loan in case of individual compared to group lending schemes is provided by Giné and Karlan (2006). They find that credit officers spend more time per borrower when individual contracts are offered.

decide which lending technology to apply and simultaneously set interest rates and joint liability parameters. At stage 2, borrowers decide at which institution to apply for credit and form groups when applying for a group contract. At stage 3, returns realize and borrowers make repayments if they have been successful. We solve the game by backward induction.

4 Choice of Lending Technology

In this section, we derive the choice of lending technology of microfinance institutions. Both institutions can pursue the lending strategy "group loans" (G) or "individual loans" (I). In order to solve the game we compare the profit of a microfinance institution in case it offers group loans to the case it grants individual loans given that its competitor, firstly, offers group loans and, secondly, concedes individual loans.

Both Microfinance Institutions Offer Group Loans

If both microfinance institutions offer group contracts, borrowers form groups of two borrowers each in order to apply for a loan. Before we turn to the profits of microfinance institutions when they both offer group loans, we show that a borrower group always consists of borrowers of the same type.

Consider, first, a safe borrower forming a group with another safe borrower. Any contract with interest rate r and degree of joint liability λ gives the borrower a utility

$$U_{S,S} = i + p_S^2[v - (1+r)\,i] + p_S\,(1-p_S)\,[v - (1+r)\,i - \lambda\,(1+r)\,i] - tx - d. \quad (1)$$

That is, the safe borrower receives her part of the credit, i. With probability p_S^2 both group members are successful so that the borrower receives return v and pays back her part of the loan with interest $(1+r)\,i$. With probability $p_S\,(1-p_S)$ the borrower is successful but her group partner is not. Then, the borrower pays back her part of the loan with interest $(1+r)\,i$ and also stands in for her group partner with the amount $\lambda\,(1+r)\,i$. If the borrower is unsuccessful, her return is zero and she does not make any repayments to the microfinance institution. The borrower's utility is reduced by the costs for travelling to the microfinance institution tx and the disutility related to group contracts d.

When the safe borrower has a risky partner, her utility from any contract with interest rate r and degree of joint liability λ is given by

$$U_{S,R} = i + p_S p_R[v - (1+r)i] + p_S(1-p_R)[v - (1+r)i - \lambda(1+r)i] - tx - d. \quad (2)$$

Now, both group members are successful with probability $p_S p_R$. Then, the safe borrower receives return v and pays back her part of the loan with interest $(1+r)\,i$. With probability $p_S\,(1-p_R)$, the safe borrower is successful but her risky partner is not. In that case, the safe borrower pays back her part of the loan with interest $(1+r)\,i$ and, in addition, the amount $\lambda\,(1+r)\,i$ in lieu of her risky partner.

The difference in a safe borrower's utility stemming from the formation of a group with a safe versus a risky borrower is given by $U_{S,S} - U_{S,R} = \lambda p_S\,(p_S - p_R)\,(1+r)\,i$. This expression is clearly positive. Hence, a safe borrower always prefers to be part of a group with a borrower of her own type.

Second, let us look at the preferences of a risky borrower concerning her partner. The utility of a risky borrower when having a risky peer amounts to

$$U_{R,R} = i + p_R^2[v - (1+r)\,i] + p_R\,(1-p_R)\,[v - (1+r)\,i - \lambda\,(1+r)\,i] - tx - d. \qquad (3)$$

With probability p_R^2 both group members are successful. The borrower receives return v and pays back her part of the loan with interest $(1+r)\,i$. With probability $p_R\,(1-p_R)$ the borrower is successful but her partner is not. Then, the borrower pays back her part of the loan with interest $(1+r)\,i$ and, in addition, she stands in for her peer with the amount $\lambda\,(1+r)\,i$.

When a risky borrower forms a group with a safe borrower, she attains the utility level

$$U_{R,S} = i + p_R p_S[v - (1+r)i] + p_R(1-p_S)[v - (1+r)i - \lambda(1+r)i] - tx - d. \qquad (4)$$

Now, projects of both borrowers turn out to be successful with probability $p_R p_S$. In that case, the risky borrower pays back her part of the loan with interest $(1+r)\,i$. With probability $p_R\,(1-p_S)$, only the risky borrower is successful. Then, the risky borrower pays back her part of the loan with interest $(1+r)\,i$ and, in addition, the amount $\lambda\,(1+r)\,i$ for her partner.

The difference in the utility of a risky borrower when being part of a group with a risky versus a safe borrower amounts to $U_{R,R} - U_{R,S} = -\lambda p_R\,(p_S - p_R)\,(1+r)\,i$. As this expression is negative, a risky borrower clearly prefers to have a safe borrower as her partner. However, as safe borrowers prefer to form groups with safe borrowers, risky borrowers will not find a safe borrower willing to form a group with them. As a consequence, risky borrowers form groups with partners of their own type as well. Note that our assumption concerning the density of the borrowers' distribution along the Hotelling line ensures that two borrowers of the same type are located at a certain point of the Hotelling street. Since we have shown that borrowers form groups with borrowers of their own type, we

can abstract from costs related to the formation of groups such as costs of searching for a partner.

Let us now turn to the profits microfinance institutions achieve when they both offer group contracts. Microfinance institutions can induce self-selection of borrowers according to their types into two different kinds of contracts. Safe borrowers will accept a contract defined by a low interest rate r_S and a high degree of joint liability λ_S. In contrast, risky borrowers prefer a loan contract based on a high interest rate r_R and a low joint liability factor λ_R. Both interest rates and the joint liability factors are set endogenously.

When borrowers decide about where to apply for credit, they compare the utility they obtain when borrowing from microfinance institution A to the utility level they achieve when accepting a loan from microfinance institution B. The resulting marginal borrowers in the segment of safe and risky borrowers $x_S(G,G)$ and $x_R(G,G)$, respectively, determine the microfinance institutions' profits as given below. Note that throughout this paper, the first letter in brackets stands for the strategy pursued by microfinance institution A and the second letter for the strategy followed by microfinance institution B.

$$\begin{aligned}\pi^A(G,G) = {} & \gamma x_S(G,G)[2p_S^2(1+r_S^A) + 2p_S(1-p_S)(1+\lambda_S^A)(1+r_S^A) - 2(1+c)]i + \\ & (1-\gamma)x_R(G,G)[2p_R^2(1+r_R^A) + 2p_R(1-p_R)(1+\lambda_R^A)(1+r_R^A) - 2(1+c)]i \end{aligned} \tag{5}$$

$$\begin{aligned}\pi^B(G,G) = {} & \gamma[1-x_S(G,G)][2p_S^2(1+r_S^B) + 2p_S(1-p_S)(1+\lambda_S^B)(1+r_S^B) - 2(1+c)]i + \\ & (1-\gamma)[1-x_R(G,G)][2p_R^2(1+r_R^B) + 2p_R(1-p_R)(1+\lambda_R^B)(1+r_R^B) - 2(1+c)]i. \end{aligned} \tag{6}$$

Due to our assumption concerning the distribution of borrowers, in the case of group lending, a certain point on the Hotelling line represents a group consisting of two borrowers. Hence, microfinance institution A serves $\gamma x_S(G,G)$ safe and $(1-\gamma)\,x_R(G,G)$ risky clients. With probability p_S^2, both members of a group of safe borrowers succeed so that the microfinance institution receives $2\left(1+r_S^A\right)i$. With probability $2p_S(1-p_S)$, the project of only one group member turns out to be successful. Then, the successful borrower stands in for her partner and repays the amount $\left(1+\lambda_S^A\right)\left(1+r_S^A\right)i$. No repayments are made if both group members fail which happens with probability $(1-p_S)^2$. Microfinance institutions incur refinancing costs $2(1+c)\,i$ per group of borrowers. Similar considerations hold for the market share the microfinance institution holds in the segment of risky borrowers. The profit of microfinance institution B is derived analogously.

Both microfinance institutions maximize their profit with respect to the interest rates and the degree of joint liability they demand from the two types of borrowers. The resulting equilibrium profits are stated in Lemma 1.

Lemma 1 *If both microfinance institutions offer group loan contracts, equilibrium profits of microfinance institutions are given by*

$$\pi^A(G,G) = \pi^B(G,G) = t. \tag{7}$$

Proof: see Appendix.

Both Microfinance Institutions Offer Individual Loans

We now analyze the situation in which both microfinance institutions offer individual loans. Borrowers compare the utility they achieve when borrowing from microfinance institution A to the utility they obtain when funded by microfinance institution B. The resulting marginal borrowers in the segment of safe and risky borrowers $x_S(I,I)$ and $x_R(I,I)$, respectively, determine the profits of microfinance institutions given as follows:

$$\begin{aligned}\pi^A(I,I) = {} & 2\gamma x_S(I,I)\left[p_S\left(1+r_{SR}^A\right)i-(1+c)i-k\right]+ \\ & 2(1-\gamma)x_R(I,I)\left[p_R\left(1+r_{SR}^A\right)i-(1+c)i-k\right]\end{aligned} \tag{8}$$

$$\begin{aligned}\pi^B(I,I) = {} & 2\gamma[1-x_S(I,I)][p_S\left(1+r_{SR}^B\right)i-(1+c)i-k]+ \\ & 2(1-\gamma)[1-x_R(I,I)][p_R\left(1+r_{SR}^B\right)i-(1+c)i-k].\end{aligned} \tag{9}$$

Due to our assumptions concerning the borrowers' distribution, a certain point on the Hotelling line represents two borrowers in the case of individual lending. Hence, microfinance institution A serves $2\gamma x_S(I,I)$ safe and $2(1-\gamma)x_R(I,I)$ risky borrowers. Microfinance institution A charges the pooled lending rate r_{SR}^A to both safe and risky clients. It receives the amount $\left(1+r_{SR}^A\right)i$ from safe borrowers with probability p_S and from risky borrowers with probability p_R. Monitoring clients amounts to a per borrower cost of k. Microfinance institutions incur refinancing costs of $(1+c)i$ per client. The profit of microfinance institution B is derived analogously.

Both microfinance institutions maximize their profit with respect to the interest rates they charge borrowers. Our results are stated in Lemma 2.

Lemma 2 *If both microfinance institutions offer individual loan contracts, equilibrium profits of microfinance institutions are given by*

$$\pi^A(I,I) = \pi^B(I,I) = \frac{t[\gamma p_S + p_R(1-\gamma)]^2 - \gamma(p_S-p_R)^2(1-\gamma)[k+(1+c)i]}{\gamma p_S^2 + (1-\gamma)p_R^2}. \tag{10}$$

Proof: see Appendix.

Microfinance Institution A Offers Individual Loans and Microfinance Institution B Offers Group Loans

Let us now turn to the situation in which microfinance institution A offers individual loan contracts and microfinance institution B offers group loans. The marginal borrowers in the safe and risky market segment $x_S(I,G)$ and $x_R(I,G)$, respectively, determine the profits of microfinance institutions. Analogous to our reasoning above, profits of both microfinance institutions are now given as follows:

$$\begin{aligned}\pi^A(I,G) &= 2\gamma x_S(I,G)[p_S(1+r^A_{SR})i-(1+c)i-k]+\\ &\quad 2(1-\gamma)x_R(I,G)[p_R(1+r^A_{SR})i-(1+c)i-k] \qquad (11)\end{aligned}$$

$$\begin{aligned}\pi^B(I,G) &= \gamma[1-x_S(I,G)][2p_S^2(1+r^B_S)+2p_S(1-p_S)(1+\lambda^B_S)(1+r^B_S)-2(1+c)]i+\\ &\quad (1-\gamma)[1-x_R(I,G)][2p_R^2(1+r^B_R)+2p_R(1-p_R)(1+\lambda^B_R)(1+r^B_R)-2(1+c)]i. \qquad (12)\end{aligned}$$

Both microfinance institutions set interest rates and microfinance institution B, in addition, the joint liability factors in order to maximize profit. The resulting equilibrium profits are stated in Lemma 3.

Lemma 3 *If microfinance institution A offers individual loan contracts and microfinance institution B offers group loans, equilibrium profits of both microfinance institutions are given by*

$$\pi^A(I,G) = \frac{2[\gamma p_S+p_R(1-\gamma)]^2(d-k+3t)^2-9\gamma(p_S-p_R)^2(1-\gamma)[k+(1+c)i][d+3t+(1+c)i]}{18t[\gamma p_S^2+(1-\gamma)p_R^2]} \qquad (13)$$

$$\pi^B(I,G) = \frac{4[\gamma p_S+p_R(1-\gamma)]^2(d-k-3t)^2+9\gamma(p_S-p_R)^2(1-\gamma)[d-t+(1+c)i]^2}{36t[\gamma p_S^2+(1-\gamma)p_R^2]}. \qquad (14)$$

Proof: see Appendix.

Microfinance Institution A Offers Group Loans and Microfinance Institution B Offers Individual Loans

Clearly, this case is symmetric to the situation described before. For the sake of completeness, the equilibrium profits of both microfinance institutions are stated in Lemma 4.

Lemma 4 *If microfinance institution A offers group contracts and microfinance institution B offers individual loans, equilibrium profits of both microfinance institutions are given by*

$$\pi^A(G,I) = \frac{4[\gamma p_S + p_R(1-\gamma)]^2(d-k-3t)^2 + 9\gamma(p_S-p_R)^2(1-\gamma)[d-t+(1+c)i]^2}{36t[\gamma p_S^2 + (1-\gamma)p_R^2]} \tag{15}$$

$$\pi^B(G,I) = \frac{2[\gamma p_S + p_R(1-\gamma)]^2(d-k+3t)^2 - 9\gamma(p_S-p_R)^2(1-\gamma)[k+(1+c)i][d+3t+(1+c)i]}{18t[\gamma p_S^2 + (1-\gamma)p_R^2]}. \tag{16}$$

Proof: analogous to proof of Lemma 3.

Nash Equilibrium

We now turn to the Nash equilibrium in this market. We determine microfinance institution A's best response both given that microfinance institution B offers group loans and individual contracts. With respect to microfinance institution B, we proceed analogously. Due to reasons of symmetry, we limit our exposition to the point of view of microfinance institution A. The matrix of the game is given in Figure 1.

		B	
		group loans	individual loans
A	group loans	$\pi^A(G,G), \pi^B(G,G)$	$\pi^A(G,I), \pi^B(G,I)$
	individual loans	$\pi^A(I,G), \pi^B(I,G)$	$\pi^A(I,I), \pi^B(I,I)$

Figure 1: Matrix of the Game

Given that microfinance institution B offers group loans, microfinance institution A offers individual contracts if $\pi^A(I,G) - \pi^A(G,G) > 0$ holds. Our results are stated in Proposition 1.

Proposition 1 *Given that microfinance institution B offers group loans, microfinance institution A offers individual loan contracts if $\pi^A(I,G) - \pi^A(G,G) > 0$ holds. That is, if*

$$9\gamma(p_S - p_R)^2(1-\gamma)[(k+i+ci)(d+i+ci) + (3k+2t+3i+3ci)t] - \\ 2(p_R + \gamma p_S - \gamma p_R)^2(d-k)(d-k+6t) < 0. \tag{17}$$

Proof: straight forward.

Given that microfinance institution B offers individual loans, microfinance institution A offers individual contracts if $\pi^A(I,I) - \pi^A(G,I) > 0$ holds. Our results are given in Proposition 2.

Proposition 2 *Given that microfinance institution B offers individual loans, microfinance institution A offers individual contracts if* $\pi^A(I,I) - \pi^A(G,I) > 0$ *holds. That is, if*

$$9\gamma\left(p_S - p_R\right)^2(1-\gamma)\left[(d+i+ci)^2 + (4k-2d+t+2i+2ci)\,t\right] - \\ 4\left(p_R + \gamma p_S - \gamma p_R\right)^2(6t-d+k)(d-k) < 0. \tag{18}$$

Proof: straight forward.

We cannot unambiguously determine whether the above two expressions are positive or negative. That is why we now turn to a comparative statics analysis. In doing so, we gain interesting insights in how the choice of lending technology is influenced by the size of credit, the refinancing conditions of microfinance institutions and the competitive pressure of the market environment.

5 Comparative Statics Analysis

The first focus of our comparative statics analysis lies on the impact of the loan size for a microfinance institution's decision to grant individual or group loans. Controversial arguments are brought forward in the so far rather descriptive literature on this topic. For instance, Kota (2007) and Harper (2007) state that microfinance institutions offer individual contracts if clients are in need for larger loans. In contrast, Giné and Karlan (2006) advocate precisely the reverse correlation. Our analysis aims to contribute to a theoretical foundation of this discussion.

We are further interested in how a microfinance institution's choice of lending technology depends on refinancing conditions and competitive pressure in the microfinance market. According to Isern and Porteous (2005) as well as Reddy and Rhyne (2006), the world of microfinance currently changes substantially in both these respects. Microfinance institutions get increasingly better access to capital markets which should transform into improved refinancing conditions. In addition, competition among microfinance banks steadily intensifies, in large part due to the enormous growth of the industry. Our analysis provides a theoretical framework that allows us to study in detail how changes in refinancing conditions and competition affect a microfinance institution's lending strategy.

5.1 Size of Credit

When we look at the role of the loan size, interestingly, we find that a microfinance institution prefers to offer individual contracts when the size of credit is rather small, irrespective of whether its competitor grants individual or group loans. Conversely, when a loan is relatively large, microfinance institutions favor the group lending technology. Our results are stated in Proposition 3.

Proposition 3 *Microfinance institutions offer individual contracts when a loan is rather small. Group contracts are preferred by microfinance institutions when a loan is rather large. That is,*

$$\pi^A(I,G) - \pi^A(G,G) > 0 \text{ if } i < i_1 \text{ and } \pi^A(I,G) - \pi^A(G,G) < 0 \text{ if } i > i_1 \tag{19}$$
$$\pi^A(I,I) - \pi^A(G,I) > 0 \text{ if } i < i_2 \text{ and } \pi^A(I,I) - \pi^A(G,I) < 0 \text{ if } i > i_2 \tag{20}$$
$$\pi^B(G,I) - \pi^B(G,G) > 0 \text{ if } i < i_1 \text{ and } \pi^B(G,I) - \pi^B(G,G) < 0 \text{ if } i > i_1 \tag{21}$$
$$\pi^B(I,I) - \pi^B(I,G) > 0 \text{ if } i < i_2 \text{ and } \pi^B(I,I) - \pi^B(I,G) < 0 \text{ if } i > i_2. \tag{22}$$

Proof: see Appendix.

The intuition for this result is as follows. Consider, first, the situation of microfinance institution A given that microfinance institution B offers individual loans. If microfinance institution A offers group contracts instead, it can charge lending rates according to the borrowers' types implying a relatively larger interest burden for risky than for safe borrowers. Since the interest repayments increase proportionally to the loan size, the advantage of safe relative to risky borrowers under group contracts is the larger, the larger the size of a loan is. Hence, the larger the loan, the more attractive group contracts of microfinance institution A become to safe borrowers and the less attractive such contracts get for risky borrowers. Put differently, an increasing loan size improves the quality of the borrower pool of microfinance institution A when it offers group contracts relative to microfinance institution B. Therefore, the larger a loan is, the more microfinance institution A favors the group over the individual lending technology.

Consider now the situation of microfinance institution A given that microfinance institution B offers group contracts. If microfinance institution A offers individual loans, its pool of borrowers worsens with an increasing loan size as microfinance institution B becomes relatively more attractive for safe and less attractive for risky borrowers analogous to the reasoning above. Hence, the larger a loan is, the more microfinance institution A prefers to offer group contracts.

Consequently, irrespective of whether microfinance institution B offers group or individual loans, the group lending technology becomes more attractive for microfinance

institution A with an increasing loan size. Analogous arguments hold for microfinance institution B. Accordingly, a Nash equilibrium in which both microfinance institutions offer group contracts tends to emerge when the size of a credit is rather large. With a rather small loan size, an equilibrium in which both microfinance institutions offer individual loans is more likely to result.

Our findings contradict the point of view of authors such as Kota (2007) and Harper (2007). In a theoretical analysis, Madajewicz (2008) shows that individual loans tend to be larger than group loans. However, her result only holds for borrowers that already have accumulated a certain level of wealth. For low levels of individual wealth, she demonstrates that group loans are larger than individual loans. In line with our results, Giné and Karlan (2006) conclude from their empirical study that the loan size is smaller for individual than for group contracts. However, their argument is somewhat different. They state that when credit officers concede individual loans, they alone assume the monitoring role and, thus, bear a higher responsibility. This is why they argue that credit officers may be stricter on the size of individual loans.

5.2 Refinancing Conditions

We now turn to the impact of refinancing conditions on a microfinance institution's choice of lending technology. We find that when refinancing costs are relatively low, a microfinance institution favors individual over group contracts, irrespective of the behavior of its competitor. Conversely, group loans tend to be preferred when refinancing costs are rather high. Our results are stated in Proposition 4.

Proposition 4 *Microfinance institutions offer individual contracts when refinancing costs are rather low. Group loans are preferred in the presence of rather high refinancing costs. That is,*

$$\pi^A(I,G) - \pi^A(G,G) > 0 \text{ if } c < c_1 \text{ and } \pi^A(I,G) - \pi^A(G,G) < 0 \text{ if } c > c_1 \tag{23}$$
$$\pi^A(I,I) - \pi^A(G,I) > 0 \text{ if } c < c_2 \text{ and } \pi^A(I,I) - \pi^A(G,I) < 0 \text{ if } c > c_2 \tag{24}$$
$$\pi^B(G,I) - \pi^B(G,G) > 0 \text{ if } c < c_1 \text{ and } \pi^B(G,I) - \pi^B(G,G) < 0 \text{ if } c > c_1 \tag{25}$$
$$\pi^B(I,I) - \pi^B(I,G) > 0 \text{ if } c < c_2 \text{ and } \pi^B(I,I) - \pi^B(I,G) < 0 \text{ if } c > c_2. \tag{26}$$

Proof: see Appendix.

The intuition for this result is as follows. Consider, first, the situation of microfinance institution A given that microfinance institution B offers individual loans. If microfinance

institution A offers group contracts instead, it can adjust lending rates to the borrowers' types. When refinancing costs increase, lending rates of both microfinance institutions go up. However, rising refinancing costs drive up the lending rate that microfinance institution A charges risky borrowers more than the rate it offers to safe borrowers. This makes microfinance institution A relatively less attractive for risky and relatively more attractive for safe borrowers relative to the pooled rate charged by microfinance institution B. It follows that increasing refinancing costs improve the quality of microfinance institution A's pool of borrowers when it offers group contracts. Hence, the higher the refinancing costs are, the more microfinance institution A prefers to offer group loans.

Consider now the situation of microfinance institution A given that microfinance institution B offers group contracts. If microfinance institution A offers individual loans, its pool of borrowers worsens with increasing refinancing costs as microfinance institution B becomes relatively more attractive for safe and less attractive for risky borrowers. Hence, the higher the refinancing costs are, the more microfinance institution A favors to offer group loan contracts.

Consequently, irrespective of whether microfinance institution B offers group or individual loans, the group lending technology becomes more attractive for microfinance institution A when refinancing costs increase. Clearly, the same arguments apply to microfinance institution B. Thus, a Nash equilibrium in which both microfinance institutions offer group contracts tends to emerge in the presence of rather high refinancing costs. When refinancing costs are rather low, an equilibrium in which both microfinance institutions offer individual loans is more likely to result.

The emergence of rating agencies specialized in the evaluation of microfinance institutions and the growing awareness of the industry's potential makes investors channel more and more funds into this market. By now, some microfinance institutions are listed at stock exchanges, such as Compartamos in Mexico or Equity Bank in Kenya. Clearly, enhanced access to capital markets implies reduced refinancing costs. Given a continuation of this trend, interestingly, our model predicts that individual lending schemes in microfinance will gain in importance in the future.

5.3 Competitive Pressure

Finally, we analyze the influence of competition on a microfinance institution's choice of lending technology. The competitive pressure of the market environment can be expressed by the inverse of transportation cost, $\frac{1}{t}$. Note that the larger the transportation cost parameter t and the more costly it becomes for borrowers to travel to a microfinance

institution, the less intense price competition will be between microfinance institutions. Conversely, the lower t is, the stronger is competition.

We find that with increasing competitive pressure, a microfinance institution prefers to offer individual contracts, irrespective of whether its competitor grants individual or group loans. Conversely, the less intense competition is, the more attractive group lending becomes for microfinance institutions. Our results are stated in Proposition 5.

Proposition 5 *If the market environment is rather competitive, microfinance institutions prefer to grant individual loans. Group loans are offered if competitive pressure is rather low. That is,*

$$\pi^A(I,G) - \pi^A(G,G) > 0 \text{ if } t < t_1 \text{ and } \pi^A(I,G) - \pi^A(G,G) < 0 \text{ if } t > t_1 \tag{27}$$
$$\pi^A(I,I) - \pi^A(G,I) > 0 \text{ if } t < t_2 \text{ and } \pi^A(I,I) - \pi^A(G,I) < 0 \text{ if } t > t_2 \tag{28}$$
$$\pi^B(G,I) - \pi^B(G,G) > 0 \text{ if } t < t_1 \text{ and } \pi^B(G,I) - \pi^B(G,G) < 0 \text{ if } t > t_1 \tag{29}$$
$$\pi^B(I,I) - \pi^B(I,G) > 0 \text{ if } t < t_2 \text{ and } \pi^B(I,I) - \pi^B(I,G) < 0 \text{ if } t > t_2. \tag{30}$$

Proof: see Appendix.

The intuition for this result is as follows. Consider, first, the situation of microfinance institution A given that microfinance institution B offers group contracts. If microfinance institution A offers individual loans, the pooled interest rate it charges decreases with increasing competition. The repayments asked by microfinance institution B also decline when competition becomes stronger. However, the fall in the interest rate is more pronounced for risky than for safe borrowers. Hence, microfinance institution B loses in attractiveness concerning the segment of risky borrowers whereas microfinance institution A becomes relatively more attractive for safe borrowers. Consequently, the more competitive the market environment is, the more microfinance institution A prefers to offer individual loans instead of group contracts.

The intuition is similar when we analyze the situation of microfinance institution A given that microfinance institution B offers individual loans. If microfinance institution A offers group loans instead, it charges interest rates according to the borrowers' types. Both repayment rates decline with increasing competition. However, interest rates fall relatively more for risky than for safe borrowers. Now, with rising competition, microfinance institution A becomes relatively more attractive for risky and less attractive for safe borrowers if it offers group loans and microfinance institution B offers individual contracts. Hence, the more competitive the market is, the more microfinance institution A prefers the individual over the group lending technology.

Consequently, irrespective of whether microfinance institution B offers group or individual loans, the individual lending technology becomes more attractive for microfinance institution A when competition toughens. Analogous considerations hold for microfinance institution B. Hence, a Nash equilibrium in which both microfinance institutions offer individual contracts tends to emerge when competition is intense. In contrast, in markets characterized by rather low competitive pressure, an equilibrium in which both microfinance institutions offer group loans is more likely to result.

According to Fernando (2007), McIntosh et al. (2005) and Christen and Rhyne (1999), markets for microfinance are often no more characterized by local monopolies of microfinance banks. Instead, due to the immense growth of the microfinance industry, there is now fierce competition between microfinance institutions in many countries. Given a continuation of this trend, interestingly, our analysis again predicts that individual lending techniques will play a more important role in the future. This hypothesis is in line with Dellien et al. (2005) who argue that due to rising competition, individual lending schemes already gained in importance over the last few years.

Summing up our findings from the comparative statics analysis, we conclude that a Nash equilibrium in which both microfinance institutions apply the group lending technology is the more likely to emerge when loans are rather large, refinancing costs are relatively high and competitive pressure is rather low. Otherwise, microfinance institutions favor individual loan contracts. Our results predict that when refinancing conditions continue to improve and competition rises further, individual lending schemes in microfinance will become more important in the future.

6 Empirical Hypotheses

Our model gives rise to several testable hypotheses concerning a microfinance institution's choice of lending technology.

We found that the smaller the loan size, the more likely it is that microfinance institutions offer individual loans. Hence, our first hypothesis is stated as follows.

Hypothesis 1 *Microfinance institutions are more likely to grant individual loans the smaller the size of a loan. The larger the amount of credit is, the more likely it is that microfinance institutions offer group loans.*

Next, we demonstrated that the lower refinancing costs are, the more microfinance institutions prefer to offer individual loans. This gives rise to our second hypothesis.

Hypothesis 2 *The higher refinancing costs are, the more likely it is that microfinance institutions offer group contracts. The lower refinancing costs are, the more likely microfinance institutions are to grant individual loans.*

Third, we showed that the more intense competition is, the more microfinance institutions tend to offer individual instead of group contracts. Based on this result, we formulate our third testable prediction.

Hypothesis 3 *Microfinance institutions are more likely to offer individual loans the stronger competition is. The lower the competitive pressure, the more microfinance institutions tend to offer group contracts.*

Data best suited for testing our hypotheses concerning a microfinance institution's lending strategy are cross country data. In that case, cultural effects that may influence a microfinance institution's choice of lending technology could be controlled for. Furthermore, panel data would render possible an analysis of how the relative importance of group and individual loans altered following past changes in refinancing conditions and competitive pressure in the market for microfinance.

7 Conclusions

In this paper, we have set up a model of competition between microfinance institutions in order to study a microfinance bank's choice of lending technology. We found that microfinance institutions tend to prefer individual loans over group loans when the size of a loan is small, refinancing costs are low, and competition is intense.

Currently, microfinance institutions obtain increasingly better access to capital markets. Moreover, competition among microfinance institutions increases steadily. Given a continuation of these trends, our analysis predicts that individual lending schemes will become more important in the microfinance industry in the future.

Interestingly, when we interpret our results in the context of a recent trend in microfinance, namely upscaling and downscaling, we can give further predictions about future trends in the market for microfinance. On the one hand, microfinance institutions increasingly start to invest in traditional banking technologies such as screening techniques, a process called upscaling. On the other hand, commercial banks begin to downscale, i.e. to invest in microfinance technologies.

As mentioned earlier, microfinance banks typically offer either group or individual loans. Even more so, very often either group or individual loans dominate the market

for microfinance in a given country or region (Madajewicz (2008)). Let us first consider an environment in which microfinance banks primarily offer group loans. Then, a microfinance bank would only have an incentive to invest in screening if this technique were better in terms of assessing a borrower's type than the group lending technology. Analogously, commercial banks would have an incentive to invest in group lending only if this technology would ensure a better evaluation of a borrower's type. Hence, in such a setting, upscaling and downscaling would constitute a form of substitutes.

Second, consider an environment characterized by microfinance banks granting individual loans. In such a situation, upscaling would allow a microfinance institution to (more or less effectively) assess a borrower's type through screening in addition to the realization of high repayment rates by using the microfinance monitoring technology. Similarly, a commercial bank would gain from downscaling since in addition to assessing a borrower's type via screening, it can ensure higher repayment rates due to the microfinance monitoring technology. Hence, in such a setting, upscaling and downscaling tend to work as a form of complements. As a consequence, the gains from upscaling and downscaling should be much higher in an environment where individual instead of group lending dominates the market for microfinance.

Coming back to our model, if due to rising competition and better access to capital markets individual loan contracts in microfinance will become more important in the future, this development may at the same time boost upscaling of microfinance institutions and downscaling of commercial banks.

8 References

Ahlin C, Townsend R. Selection Into and Across Credit Contracts: Theory and Field Research. Journal of Econometrics 2007;136; 665-698

Armendáriz de Aghion B. On the Design of a Credit Agreement with Peer Monitoring. Journal of Development Economics 1999;60; 79-104

Armendáriz de Aghion B, Gollier C. Peer Group Formation in an Adverse Selection Model. The Economic Journal 2000;110; 632-643

Armendáriz de Aghion B, Murdoch J. Microfinance beyond Group Lending. Economics of Transition 2000;8; 401-420

Armendáriz de Aghion B, Morduch J. The Economics of Microfinance. MIT Press: Cambridge; 2005

Besley T, Coate S. Group Lending, Repayment Incentives and Social Collateral. Journal of Development Economics 1995;46; 1-18

Champagne P, Tjossem P, Ohman T, Pikholz L, Cracknell D. Toolkit for Individual Lending for Credit Managers. ShoreCap Exchange Corporation; 2007

Christen R, Cook T. Commercialization and Mission Drift: The Transformation of Microfinance in Latin America. CGAP Occasional Paper 2001;5

Christen R, Rhyne E. Microfinance Enters the Marketplace. USAID Microenterprise Publications: Washington; 1999

Daley-Harris S. State of the Microcredit Summit Campaign Report 2007. Microcredit Summit Campaign: Washington; 2007

Dellien H, Burnett J, Gincherman A, Lynch E. Product Diversification in Microfinance: Introducing Individual Lending. Women's World Banking Report: New York; 2005

Fernando N. The Asian Microfinance Industry. Some Changes and Continuities. ADB Finance for the Poor 2007;8; 1-11

Gangopadhyay S, Lensink R. Delegated Monitoring and Moral Hazard in Underdeveloped Credit Markets. Working Paper: University of Groningen; 2007

Ghatak M. Screening by the Company You Keep: Joint Liability Lending and the Peer Selection Effect. The Economic Journal 2000;110; 601-631

Ghatak M, Guinnane T. The Economics of Lending with Joint Liability: Theory and Practice. Journal of Development Economics 1999;60; 195-228

Giné X, Karlan D. Group versus Individual Liability – A Field Experiment in the Philippines. World Bank Policy Research Working Paper 4008: Washington; 2006

Giné X, Jakiela P. Karlan, D., Murdoch, J., Microfinance Games. World Bank Policy Research Working Paper 3959: Washington; 2006

Harper M 2007. What's Wrong with Groups? In: Dichter T, Harper M (Ed.), What's Wrong with Microfinance? Practical Action Publishing: Eastbourne;1990. p. 35-48

Isern J, Porteus D. Commercial Banks and Microfinance: Evolving Models of Success. CGAP Focus Note 2005;28

Karlan D. Social Connections and Group Banking. The Economic Journal 2007;117; F52-F84

Kota I. Microfinance: Banking for the Poor. Finance & Development 2007;6; 44-45

Laffont J, N'Guessan T. Regulation and Development – Group Lending with Adverse Selection. European Economic Review 2000;44; 773-784

Littlefield E, Murdok J, Hashemi, S. Is Microfinance an Effective Strategy to Reach the Millennium Development Goals? CGAP FocusNote 2003;24

Madajewicz M. Joint Liability versus Individual Liability in Credit Contracts. Forthcoming in Journal of Economic Behavior & Organization 2008

McIntosh C, de Janvry A, Sadoulet E. How Rising Competition among Microfinance Institutions Affects Incumbent Lenders. The Economic Journal 2005;115; 987-1004

Montgomery R. Disciplining or Protecting the Poor? Avoiding the Social Costs of Peer Pressure in Micro-credit Schemes. Journal of International Development 1996;8; 289-305

Navajas S, Conning J, Gonzalez-Vega C. Lending Technologies, Competition and Consolidation in the Markets for Microfinance in Bolivia. Journal of International Development 2003;15; 747-770

Reddy R, Rhyne E. How Will Buy Our Paper? Microfinance Cracking the Capital Markets? Accion Insight 2006;18

Stiglitz J. Peer Monitoring and Credit Markets. The World Bank Economic Review 1990;4; 351-366

Van Tassel E. Group Lending under Asymmetric Information. Journal of Development Economics 1999;60; 3-25

Zeitinger C-P 1996. Micro-lending in the Russian Federation. In: Levitsky J (Ed.), Small Business in Transition Economies. ITDG Publishing: London;1996. p. 85-94

9 Appendix

Proof of Lemma 1:

The utility of a safe borrower if she receives a loan from microfinance institution A is given by

$$U_S^A = i + p_S^2[v - \left(1 + r_S^A\right) i] + p_S\left(1 - p_S\right)[v - \left(1 + r_S^A\right) i - \lambda_S^A \left(1 + r_S^A\right) i] - tx - d.$$

The utility of a safe borrower if she receives a loan from microfinance institution B is given by

$$U_S^B = i + p_S^2[v - \left(1 + r_S^B\right) i] + p_S\left(1 - p_S\right)[v - \left(1 + r_S^B\right) i - \lambda_S^B \left(1 + r_S^B\right) i] - t\left(1 - x\right) - d.$$

Hence, the marginal borrower in the segment of safe borrowers is given by

$$x_S\left(G, G\right) = \frac{t - ip_S[\lambda_S^A(1-p_S)\left(1+r_S^A\right) + r_S^A - \lambda_S^B(1-p_S)\left(1+r_S^B\right) - r_S^B]}{2t}.$$

The utility of a risky borrower if she receives a loan from microfinance institution A is given by

$$U_R^A = i + p_R^2[v - \left(1 + r_R^A\right) i] + p_R\left(1 - p_R\right)[v - \left(1 + r_R^A\right) i - \lambda_R^A \left(1 + r_R^A\right) i] - tx - d.$$

The utility of a risky borrower if she receives a loan from microfinance institution B is given by

$$U_R^B = i + p_R^2[v - \left(1 + r_R^B\right) i] + p_R\left(1 - p_R\right)[v - \left(1 + r_R^B\right) i - \lambda_R^B \left(1 + r_R^B\right) i] - t\left(1 - x\right) - d.$$

Hence, the marginal borrower in the segment of risky borrowers is given by

$$x_R\left(G, G\right) = \frac{t - ip_R[\lambda_R^A(1-p_R)\left(1+r_R^A\right) + r_R^A - \lambda_R^B(1-p_R)\left(1+r_R^B\right) - r_R^B]}{2t}.$$

Profits of microfinance institutions are given as follows:

$$\begin{aligned}\pi^A(G,G) = {} & \gamma x_S(G,G)[2p_S^2(1+r_S^A) + 2p_S(1-p_S)(1+\lambda_S^A)(1+r_S^A) - 2(1+c)]i + \\ & (1-\gamma)x_R(G,G)[2p_R^2(1+r_R^A) + 2p_R(1-p_R)(1+\lambda_R^A)(1+r_R^A) - 2(1+c)]i\end{aligned}$$

$$\begin{aligned}\pi^B(G,G) = {} & \gamma[1 - x_S(G,G)][2p_S^2(1+r_S^B) + 2p_S(1-p_S)(1+\lambda_S^B)(1+r_S^B) - 2(1+c)]i + \\ & (1-\gamma)[1 - x_R(G,G)][2p_R^2(1+r_R^B) + 2p_R(1-p_R)(1+\lambda_R^B)(1+r_R^B) - 2(1+c)]i.\end{aligned}$$

Microfinance institutions maximize their profit with respect to interest rates and the joint liability factors. Note that the following relationships hold:

$\frac{d\left(\pi^j(G,G)\right)}{dr_S^j} \stackrel{!}{=} 0$ is equivalent to $\frac{d\left(\pi^j(G,G)\right)}{d\lambda_S^j} \stackrel{!}{=} 0$

$\frac{d\left(\pi^j(G,G)\right)}{dr_R^j} \stackrel{!}{=} 0$ is equivalent to $\frac{d\left(\pi^j(G,G)\right)}{d\lambda_R^j} \stackrel{!}{=} 0$.

The first order conditions imply the following equilibrium interest rates dependent on the joint liability parameters:

$$r_S^j(G,G) = \frac{t+(1+c)i-p_S[1+\lambda_S^j(1-p_S)]i}{p_S[1+\lambda_S^j(1-p_S)]i}$$
$$r_R^j(G,G) = \frac{t+(1+c)i-p_R[1+\lambda_R^j(1-p_R)]i}{p_R[1+\lambda_R^j(1-p_R)]i}.$$

The resulting market shares and profits are

$$x_S^j(G,G) = x_R^j(G,G) = \tfrac{1}{2}$$

$$\pi^A(G,G) = \pi^B(G,G) = t.$$

Proof of Self-Selection of Borrowers:

In order to induce self-selection of borrowers into different contracts offered, the incentive constraints for both safe and risky borrowers must be fulfilled:

If a group of risky borrowers truly reveals its type, the utility of a group member is given by

$$\begin{aligned} U_R(R) = {} & i + p_R^2[v - \left(1 + r_R^j(G,G)\right) i] + \\ & p_R(1-p_R)\left[v - \left(1 + r_R^j(G,G)\right) i - \lambda_R^j\left(1 + r_R^j(G,G)\right) i\right] - tx - d. \end{aligned}$$

If a group of risky borrowers pretends to be of the safe type, the utility of a group member is given by

$$\begin{aligned} U_R(S) = {} & i + p_R^2[v - \left(1 + r_S^j(G,G)\right) i] + \\ & p_R(1-p_R)\left[v - \left(1 + r_S^j(G,G)\right) i - \lambda_S^j\left(1 + r_S^j(G,G)\right) i\right] - tx - d. \end{aligned}$$

Note that $U_R(R) - U_R(S) > 0$ is equivalent to $\lambda_S^j > \frac{1}{p_R+p_S-1}$.[6]

If a group of safe borrowers truly reveals its type, the utility of a group member is given by

$$\begin{aligned} U_S(S) = {} & i + p_S^2[v - \left(1 + r_S^j(G,G)\right) i] + \\ & p_S(1-p_S)\left[v - \left(1 + r_S^j(G,G)\right) i - \lambda_S^j\left(1 + r_S^j(G,G)\right) i\right] - tx - d. \end{aligned}$$

If a group of safe borrowers pretends to be of the risky type, the utility of a group member is given by

$$U_S(R) = i + p_S^2[v - \left(1 + r_R^j(G,G)\right) i] +$$

[6] We will assume throughout our analysis that $p_R + p_S - 1 > 0$ holds.

$$p_S\left(1-p_S\right)\left[v-\left(1+r_R^j\left(G,G\right)\right)i-\lambda_R^j\left(1+r_R^j\left(G,G\right)\right)i\right]-tx-d.$$

Note that $U_S\left(S\right)-U_S\left(R\right)>0$ is equivalent to $\lambda_R^j<\frac{1}{p_S+p_R-1}$.

As a consequence, if $\lambda_R^j<\frac{1}{p_S+p_R-1}<\lambda_S^j$ is ensured, self-selection of borrowers into the different contracts can be achieved when interest rates are set accordingly.

We now show that $r_S^j\left(G,G\right)<r_R^j\left(G,G\right)$ holds for contracts that achieve self-selection of borrowers. This expression is equivalent to $p_S[1+\lambda_S^j\left(1-p_S\right)]-p_R[1+\lambda_R^j\left(1-p_R\right)]>0$. We now define $\lambda_R^j\equiv\alpha\frac{1}{p_S+p_R-1}$ with $0<\alpha<1$. We can then rewrite the expression as $p_S\left(1-p_S\right)\left[\lambda_S^j\left(p_S+p_R-1\right)-1\right]+p_R\left(1-p_R\right)\left(1-\alpha\right)>0$. Note that $\lambda_S^j\left(p_S+p_R-1\right)-1>0$ is equivalent to $\lambda_S^j>\frac{1}{p_S+p_R-1}$ which holds when the incentive constraint of the safe borrowers holds.

Hence, we have shown that if $\lambda_R^j<\frac{1}{p_S+p_R-1}<\lambda_S^j$ and $r_S^j\left(G,G\right)<r_R^j\left(G,G\right)$ holds, self-selection of borrowers can be achieved.

Proof of Lemma 2:

The utility of a safe borrower if she receives a loan from microfinance institution A is given by

$$U_S^A=i+p_S[v-\left(1+r_{SR}^A\right)i]-tx.$$

The utility of a safe borrower if she receives a loan from microfinance institution B is given by

$$U_S^B=i+p_S[v-\left(1+r_{SR}^B\right)i]-t\left(1-x\right).$$

Hence, the marginal borrower in the segment of safe borrowers is given by

$$x_S\left(I,I\right)=\frac{t-p_S\left(r_{RS}^A-r_{RS}^B\right)i}{2t}.$$

The utility of a risky borrower if she receives a loan from microfinance institution A is given by

$$U_R^A=i+p_R[v-\left(1+r_{SR}^A\right)i]-tx.$$

The utility of a risky borrower if she receives a loan from microfinance institution B is given by

$$U_R^B=i+p_R[v-\left(1+r_{SR}^B\right)i]-t\left(1-x\right).$$

Hence, the marginal borrower in the segment of risky borrowers is given by

$x_R(I,I) = \frac{t - p_R\left(r_{SR}^A - r_{SR}^B\right)i}{2t}$.

Profits of microfinance institutions are given as follows:

$$\pi^A(I,I) = 2\gamma x_S(I,I)\left[p_S\left(1+r_{SR}^A\right)i - (1+c)i - k\right] + 2(1-\gamma)x_R(I,I)\left[p_R\left(1+r_{SR}^A\right)i - (1+c)i - k\right]$$

$$\pi^B(I,I) = 2\gamma[1-x_S(I,I)]\left[p_S\left(1+r_{SR}^B\right)i - (1+c)i - k\right] + 2(1-\gamma)[1-x_R(I,I)]\left[p_R\left(1+r_{SR}^B\right)i - (1+c)i - k\right].$$

Microfinance institutions maximize their profit with respect to interest rates which results in the following equilibrium interest rates, market shares and profits:

$$r_{SR}^A(I,I) = r_{SR}^B(I,I) \equiv r_{SR}(I,I) = \frac{[\gamma p_S + p_R(1-\gamma)][k+t+(1+c)i] - [\gamma p_S^2 + (1-\gamma)p_R^2]i}{[\gamma p_S^2 + (1-\gamma)p_R^2]i}$$

$$x_S(I,I) = x_R(I,I) = \frac{1}{2}$$

$$\pi^A(I,I) = \pi^B(I,I) = \frac{t[\gamma p_S + p_R(1-\gamma)]^2 - \gamma(p_S - p_R)^2(1-\gamma)[k+(1+c)i]}{\gamma p_S^2 + (1-\gamma)p_R^2}.$$

Proof of Lemma 3:

The utility of a safe borrower if she receives a loan from microfinance institution A is given by

$$U_S^A = i + p_S\left[v - \left(1+r_{SR}^A\right)i\right] - tx.$$

The utility of a safe borrower if she receives a loan from microfinance institution B is given by

$$U_S^B = i + p_S^2\left[v - \left(1+r_S^B\right)i\right] + p_S(1-p_S)\left[v - \left(1+r_S^B\right)i - \lambda_S^B\left(1+r_S^B\right)i\right] - t(1-x) - d.$$

Hence, the marginal borrower in the segment of safe borrowers is given by

$$x_S(I,G) = \frac{t + d + p_S\left[r_S^B - r_{RS}^A + \lambda_S^B(1-p_S)\left(1+r_S^B\right)\right]i}{2t}.$$

The utility of a risky borrower if she receives a loan from microfinance institution A is given by

$$U_R^A = i + p_R\left[v - \left(1+r_{SR}^A\right)i\right] - tx.$$

The utility of a risky borrower if she receives a loan from microfinance institution B is given by

$$U_R^B = i + p_R^2\left[v - \left(1+r_R^B\right)i\right] + p_R(1-p_R)\left[v - \left(1+r_R^B\right)i - \lambda_R^B\left(1+r_R^B\right)i\right] - t(1-x) - d.$$

Hence, the marginal borrower in the segment of risky borrowers is given by

$$x_R\left(I,G\right)=\frac{t+d+p_R[r_R^B-r_{SR}^A+\lambda_R^B(1-p_R)\left(1+r_R^B\right)]i}{2t}.$$

Profits of microfinance institutions are given as follows:

$$\begin{aligned}\pi^A\left(I,G\right)=&\ 2\gamma x_S\left(I,G\right)\left[p_S\left(1+r_{SR}^A\right)i-\left(1+c\right)i-k\right]+\\&2\left(1-\gamma\right)x_R\left(I,G\right)\left[p_R\left(1+r_{SR}^A\right)i-\left(1+c\right)i-k\right]\end{aligned}$$

$$\begin{aligned}\pi^B(I,G)=&\ \gamma[1-x_S(I,G)][2p_S^2(1+r_S^B)+2p_S(1-p_S)(1+\lambda_S^B)(1+r_S^B)-2(1+c)]i+\\&(1-\gamma)[1-x_R(I,G)][2p_R^2(1+r_R^B)+2p_R(1-p_R)(1+\lambda_R^B)(1+r_R^B)-2(1+c)]i.\end{aligned}$$

Microfinance bank A chooses repayment rates and microfinance B both interest rates and joint liability factors to maximize profit. This results in the following equilibrium interest rates, market shares and profits:

$$\begin{aligned}r_{SR}^A\left(I,G\right)&=\frac{[\gamma p_S+p_R(1-\gamma)](d+2k+3t+3(1+c)i)-3[\gamma p_S^2+(1-\gamma)p_R^2]i}{3[\gamma p_S^2+(1-\gamma)p_R^2]i}\\r_S^B\left(I,G\right)&=\frac{p_S[\gamma p_S+p_R(1-\gamma)](d+2k+3t+3(1+c)i)+[\gamma p_S^2+(1-\gamma)p_R^2]\{3t-3d+3(1+c)i-6p_S[1+\lambda_S^B(1-p_S)]i\}}{6p_S[\gamma p_S^2+(1-\gamma)p_R^2][1+\lambda_S^B(1-p_S)]i}\\r_R^B\left(I,G\right)&=\frac{p_R[\gamma p_S+p_R(1-\gamma)][d+2k+3t+3(1+c)i]+[\gamma p_S^2+(1-\gamma)p_R^2]\{3t-3d+3(1+c)i-6p_R[1+\lambda_R^B(1-p_R)]i\}}{6p_R[\gamma p_S^2+(1-\gamma)p_R^2][1+\lambda_R^B(1-p_R)]i}\end{aligned}$$

$$\begin{aligned}x_S\left(I,G\right)&=\frac{2[p_R^2+\gamma\left(p_S^2-p_R^2\right)](d-k+3t)-(1-\gamma)p_R(p_S-p_R)[3(1+c)i+d+2k+3t]}{12t[p_R^2+\gamma\left(p_S^2-p_R^2\right)]}\\x_R\left(I,G\right)&=\frac{2[p_R^2+\gamma\left(p_S^2-p_R^2\right)](d-k+3t)+\gamma p_S(p_S-p_R)[3(1+c)i+d+2k+3t]}{12t[p_R^2+\gamma\left(p_S^2-p_R^2\right)]}\end{aligned}$$

$$\begin{aligned}\pi^A\left(I,G\right)&=\frac{2[\gamma p_S+p_R(1-\gamma)]^2(d-k+3t)^2-9\gamma(p_S-p_R)^2(1-\gamma)[k+(1+c)i][d+3t+(1+c)i]}{18t[\gamma p_S^2+(1-\gamma)p_R^2]}\\\pi^B\left(I,G\right)&=\frac{4[\gamma p_S+p_R(1-\gamma)]^2(d-k-3t)^2+9\gamma(p_S-p_R)^2(1-\gamma)[d-t+(1+c)i]^2}{36t[\gamma p_S^2+(1-\gamma)p_R^2]}.\end{aligned}$$

Proof of Proposition 3:

Note that

$$\begin{aligned}\pi^A(I,G)-\pi^A(G,G)=&\ \frac{1}{18t[p_R^2+\gamma\left(p_S^2-p_R^2\right)]}\{2[p_R+\gamma\left(p_S-p_R\right)]^2\left(d-k\right)\left(d-k+6t\right)-\\&9\gamma\left(1-\gamma\right)\left(p_S-p_R\right)^2\left[\left(1+c\right)^2i^2+\left(d+k+3t\right)\left(1+c\right)i+\right.\\&\left.dk+3kt+2t^2\right]\}.\end{aligned}$$

Solving $\pi^A(I,G)-\pi^A(G,G)=0$ for i, we arrive at

$$i=-\frac{d+k+3t}{2(1+c)}\pm\frac{1}{2(c+1)}\sqrt{d^2+k^2+t^2-2\left(dk-3dt+3kt\right)+\frac{8[p_R+\gamma(p_S-p_R)]^2(d-k)(d-k+6t)}{9\gamma(p_S-p_R)^2(1-\gamma)}}.$$

As we only look at positive values of i, only the larger one of both thresholds is relevant for our analysis. We define

$$i_1\equiv-\frac{d+k+3t}{2(1+c)}+\frac{1}{2(c+1)}\sqrt{d^2+k^2+t^2-2\left(dk-3dt+3kt\right)+\frac{8[p_R+\gamma(p_S-p_R)]^2(d-k)(d-k+6t)}{9\gamma(p_S-p_R)^2(1-\gamma)}}.$$

Furthermore,

$$\frac{d\left(\pi^A(I,G)-\pi^A(G,G)\right)}{di} = -\frac{\gamma(1-\gamma)(p_S-p_R)^2(1+c)(d+k+3t+2i+2ci)}{2t[p_R^2+\gamma\left(p_S^2-p_R^2\right)]} < 0 \text{ and}$$

$$\frac{d^2\left(\pi^A(I,G)-\pi^A(G,G)\right)}{di^2} = -\frac{\gamma(1-\gamma)(p_S-p_R)^2(1+c)^2}{t[p_R^2+\gamma\left(p_S^2-p_R^2\right)]} < 0.$$

Hence, $\pi^A(I,G)-\pi^A(G,G)$ describes a parabola with its maximum at $i = -\frac{d+k+3t}{2(1+c)}$.

It follows from the analysis above that

$\pi^A(I,G)-\pi^A(G,G) > 0$ if $i < i_1$ and $\pi^A(I,G)-\pi^A(G,G) < 0$ if $i > i_1$.

Note that

$$\begin{aligned}\pi^A(I,I)-\pi^A(G,I) = \frac{1}{36t[p_R^2+\gamma\left(p_S^2-p_R^2\right)]}\{&4[p_R+\gamma\left(p_S-p_R\right)]^2\left(k-d\right)\left(d-k-6t\right)-\\ &9\gamma\left(1-\gamma\right)\left(p_S-p_R\right)^2\left[\left(c+1\right)^2 i^2+2\left(d+t\right)\left(1+c\right)i+d^2+\right.\\ &\left.t^2+4kt-2dt\right]\}.\end{aligned}$$

Solving $\pi^A(I,I)-\pi^A(G,I)=0$ for i, we arrive at

$$i = -\frac{d+t}{1+c} \pm \frac{1}{(1+c)}\sqrt{4t\left(d-k\right)-\frac{4[p_R+\gamma(p_S-p_R)]^2(d-k-6t)(d-k)}{9\gamma(p_S-p_R)^2(1-\gamma)}}.$$

As we only look at positive values of i, only the larger one of both thresholds is relevant for our analysis. We define

$$i_2 \equiv -\frac{d+t}{1+c} + \frac{1}{(1+c)}\sqrt{4t\left(d-k\right)-\frac{4[p_R+\gamma(p_S-p_R)]^2(d-k-6t)(d-k)}{9\gamma(p_S-p_R)^2(1-\gamma)}}.$$

Furthermore,

$$\frac{d\left(\pi^A(I,G)-\pi^A(G,G)\right)}{di} = -\frac{\gamma(1-\gamma)(p_S-p_R)^2(1+c)[d+t+i(1+c)]}{2t[p_R^2+\gamma\left(p_S^2-p_R^2\right)]} < 0 \text{ and}$$

$$\frac{d^2\left(\pi^A(I,G)-\pi^A(G,G)\right)}{di^2} = -\frac{\gamma(1-\gamma)(p_S-p_R)^2(1+c)^2}{2t[p_R^2+\gamma\left(p_S^2-p_R^2\right)]} < 0.$$

Hence, $\pi^A(I,I)-\pi^A(G,I)$ describes a parabola with its maximum at $i = -\frac{d+t}{1+c}$.

It follows from the analysis above that

$\pi^A(I,I)-\pi^A(G,I) > 0$ if $i < i_2$ and $\pi^A(I,I)-\pi^A(G,I) < 0$ if $i > i_2$.

Proof of Proposition 4:

Solving $\pi^A(I,G)-\pi^A(G,G)=0$ for c, we arrive at

$$c = -\frac{d+k+3t+2i}{2i} \pm \frac{1}{2i}\sqrt{d^2+k^2+t^2-2\left(dk-3dt+3kt\right)+\frac{8[p_R+\gamma(p_S-p_R)]^2(d-k)(d-k+6t)}{9\gamma(1-\gamma)(p_S-p_R)^2}}.$$

As we only look at positive values of i, only the larger one of both thresholds is relevant for our analysis. We define

$$c_1 \equiv -\frac{d+k+3t+2i}{2i} + \frac{1}{2i}\sqrt{d^2+k^2+t^2-2\left(dk-3dt+3kt\right)+\frac{8[p_R+\gamma(p_S-p_R)]^2(d-k)(d-k+6t)}{9\gamma(1-\gamma)(p_S-p_R)^2}}.$$

Furthermore,

$$\frac{d\left(\pi^A(I,G)-\pi^A(G,G)\right)}{dc} = -\frac{\gamma(1-\gamma)(p_S-p_R)^2[d+k+3t+2i(1+c)]i}{2t[p_R^2+\gamma\left(p_S^2-p_R^2\right)]} < 0 \text{ and}$$

$$\frac{d^2\left(\pi^A(I,G)-\pi^A(G,G)\right)}{dc^2} = -\frac{\gamma(1-\gamma)(p_S-p_R)^2 i^2}{t[p_R^2+\gamma\left(p_S^2-p_R^2\right)]} < 0.$$

Hence, $\pi^A(I,G)-\pi^A(G,G)$ describes a parabola with its maximum at $c=-\frac{d+k+3t+2i}{2i}$.

It follows from the analysis above that

$\pi^A(I,G)-\pi^A(G,G)>0$ if $c<c_1$ and $\pi^A(I,G)-\pi^A(G,G)<0$ if $c>c_1$.

Solving $\pi^A(I,I)-\pi^A(G,I)=0$ for c, we arrive at

$$c = -\frac{d+t+i}{i} \pm \frac{2}{i}\sqrt{(d-k)\,t-\frac{[p_R+\gamma(p_S-p_R)]^2(d-k-6t)(d-k)}{9\gamma(1-\gamma)(p_S-p_R)^2}}.$$

As we only look at positive values of i, only the larger one of both thresholds is relevant for our analysis. We define

$$c_2 \equiv -\frac{d+t+i}{i} + \frac{2}{i}\sqrt{(d-k)\,t-\frac{[p_R+\gamma(p_S-p_R)]^2(d-k-6t)(d-k)}{9\gamma(1-\gamma)(p_S-p_R)^2}}.$$

Furthermore,

$$\frac{d\left(\pi^A(I,G)-\pi^A(G,G)\right)}{dc} = -\frac{\gamma(1-\gamma)(p_S-p_R)^2[d+t+(1+c)i]i}{2t[p_R^2+\gamma\left(p_S^2-p_R^2\right)]} < 0 \text{ and}$$

$$\frac{d^2\left(\pi^A(I,G)-\pi^A(G,G)\right)}{dc^2} = -\frac{\gamma(1-\gamma)(p_S-p_R)^2 i^2}{2t[p_R^2+\gamma\left(p_S^2-p_R^2\right)]} < 0.$$

Hence, $\pi^A(I,I)-\pi^A(G,I)$ describes a parabola with its maximum at $c=-\frac{d+t+i}{i}$.

It follows from the analysis above that

$\pi^A(I,I)-\pi^A(G,I)>0$ if $c<c_2$ and $\pi^A(I,I)-\pi^A(G,I)<0$ if $c>c_2$.

Proof of Proposition 5:

(1) Shape of $\pi^A(G,G)$

$\pi^A(G,G)=t$

$\frac{d\left(\pi^A(G,G)\right)}{dt} = 1$

$\frac{d^2\left(\pi^A(G,G)\right)}{dt^2} = 0.$

(2) Shape of $\pi^A(I,G)$

$\pi^A(I,G) = \frac{2[\gamma p_S + p_R(1-\gamma)]^2(d-k+3t)^2 - 9\gamma(p_S-p_R)^2(1-\gamma)[k+(1+c)i][d+3t+(1+c)i]}{18t[\gamma p_S^2+(1-\gamma)p_R^2]}$

Note, first, that $\pi^A(I,G)$ is not defined at $t=0$.

Note, second, that

$\frac{d\left(\pi^A(I,G)\right)}{dt} = \frac{[p_R+\gamma(p_S-p_R)]^2}{p_R^2+\gamma\left(p_S^2-p_R^2\right)} - \frac{[p_R+\gamma(p_S-p_R)]^2(d-k)^2}{9[p_R^2+\gamma\left(p_S^2-p_R^2\right)]t^2} + \frac{\gamma(1-\gamma)(p_S-p_R)^2[d+(1+c)i][k+(1+c)i]}{2[p_R^2+\gamma\left(p_S^2-p_R^2\right)]t^2}.$

If we solve $\frac{d\left(\pi^A(I,G)\right)}{dt} = 0$ for t, we arrive at

$t = \pm\frac{\sqrt{[p_R+\gamma(p_S-p_R)]^2(d-k)^2 - \frac{9}{2}\gamma(1-\gamma)(p_S-p_R)^2[d+(1+c)i][k+(1+c)i]}}{3[p_R+\gamma(p_S-p_R)]}.$

In order for the above expression to be defined, we assume $[p_R+\gamma\left(p_S-p_R\right)]^2\left(d-k\right)^2 - \frac{9}{2}\gamma\left(1-\gamma\right)\left(p_S-p_R\right)^2\left[d+\left(1+c\right)i\right]\left[k+\left(1+c\right)i\right] > 0$. In what follows, we will refer to this assumption as *Condition (1)*.

Note, third, that

$\frac{d^2\left(\pi^A(I,G)\right)}{dt^2} = \frac{2}{9t^3[p_R^2+\gamma\left(p_S^2-p_R^2\right)]}\{[p_R+\gamma\left(p_S-p_R\right)]^2\left(d-k\right)^2 - \frac{9}{2}\gamma\left(1-\gamma\right)\left(p_S-p_R\right)^2$
$\left[d+\left(1+c\right)i\right]\left[k+\left(1+c\right)i\right]\} > 0$ due to *Condition (1)*.

Since we only consider $t>0$, $\pi^A(I,G)$ must be a parabola with a minimum at $t = \frac{\sqrt{[p_R+\gamma(p_S-p_R)]^2(d-k)^2 - \frac{9}{2}\gamma(1-\gamma)(p_S-p_R)^2[d+(1+c)i][k+(1+c)i]}}{3[p_R+\gamma(p_S-p_R)]}$.

Furthermore, it holds that $\lim_{t\to\infty}\frac{d\left(\pi^A(I,G)\right)}{dt} = \frac{[p_R+\gamma(p_S-p_R)]^2}{p_R^2+\gamma\left(p_S^2-p_R^2\right)}$.

Finally, note that $\frac{[p_R+\gamma(p_S-p_R)]^2}{p_R^2+\gamma\left(p_S^2-p_R^2\right)} < 1$ since this expression is equivalent to $-\gamma\left(p_S-p_R\right)^2\left(1-\gamma\right) < 0$. Hence, in the limit, the first order condition of $\pi^A(I,G)$ approaches $\frac{[p_R+\gamma(p_S-p_R)]^2}{p_R^2+\gamma\left(p_S^2-p_R^2\right)}$, a value that is smaller than the constant first order condition of $\pi^A(G,G)$ which is equal to 1. As a consequence, it must be true that there is exactly one intersection of $\pi^A(I,G)$ and $\pi^A(G,G)$. We now calculate the exact intersection point.

Calculation of the Intersection Point

Note that $\pi^A(I,G) - \pi^A(G,G) = 0$ is equivalent to

$$t^2 + \frac{9\gamma(1-\gamma)(p_S-p_R)^2[k+(1+c)i]-4[p_R+\gamma(p_S-p_R)]^2(d-k)}{6\gamma(1-\gamma)(p_S-p_R)^2}t+$$
$$\frac{9\gamma(1-\gamma)(p_S-p_R)^2[(1+c)^2i^2+dk+(d+k)(1+c)i]-2[p_R+\gamma(p_S-p_R)]^2(d-k)^2}{18\gamma(1-\gamma)(p_S-p_R)^2} = 0$$

We define

$$B_1 \equiv \frac{9\gamma(1-\gamma)(p_S-p_R)^2[k+(1+c)i]-4[p_R+\gamma(p_S-p_R)]^2(d-k)}{6\gamma(1-\gamma)(p_S-p_R)^2}$$

$$C_1 \equiv \frac{9\gamma(1-\gamma)(p_S-p_R)^2[(1+c)^2i^2+dk+(d+k)(1+c)i]-2[p_R+\gamma(p_S-p_R)]^2(d-k)^2}{18\gamma(1-\gamma)(p_S-p_R)^2}$$

Solving the above expression for t, we arrive at

$$t = -\tfrac{1}{2}B_1 \pm \tfrac{1}{2}\sqrt{B_1^2 - 4C_1}.$$

Due to the above analysis, the larger one of both thresholds is the one that is relevant for our analysis. We define

$$t_1 \equiv -\tfrac{1}{2}B_1 + \tfrac{1}{2}\sqrt{B_1^2 - 4C_1}.$$

It follows from the analysis above that $\pi^A(I,G) > \pi^A(G,G)$ for $t < t_1$ and that $\pi^A(I,G) < \pi^A(G,G)$ for $t > t_1$.

(3) Shape of $\pi^A(I,I)$

$$\pi^A(I,I) = \frac{t[\gamma p_S+p_R(1-\gamma)]^2-\gamma(p_S-p_R)^2(1-\gamma)[k+(1+c)i]}{\gamma p_S^2+(1-\gamma)p_R^2}$$

$$\frac{d\left(\pi^A(I,I)\right)}{dt} = \frac{[\gamma p_S+p_R(1-\gamma)]^2}{\gamma p_S^2+(1-\gamma)p_R^2}$$

$$\frac{d^2\left(\pi^A(I,I)\right)}{dt^2} = 0.$$

(4) Shape of $\pi^A(G,I)$

$$\pi^A(G,I) = \frac{4[\gamma p_S+p_R(1-\gamma)]^2(d-k-3t)^2+9\gamma(p_S-p_R)^2(1-\gamma)[d-t+(1+c)i]^2}{36t[\gamma p_S^2+(1-\gamma)p_R^2]}$$

Note, that this is equivalent to

$$\pi^A(G,I) = \frac{\gamma}{36}\frac{\{2[p_R^2+\gamma\left(p_S^2-p_R^2\right)](d-k-3t)-p_R(p_S-p_R)(1-\gamma)(d+2k+3i+3ci+3t)\}^2}{t[p_R^2+\gamma\left(p_S^2-p_R^2\right)]^2}+$$
$$\frac{1-\gamma}{36}\frac{\{2[p_R^2+\gamma\left(p_S^2-p_R^2\right)](d-k-3t)+\gamma p_S(p_S-p_R)(d+2k+3t+3i+3ci)\}^2}{t[p_R^2+\gamma\left(p_S^2-p_R^2\right)]^2}$$

Note, first, that $\pi^A(G,I)$ is not defined for $t = 0$.

Second, note that

$$\frac{d\left(\pi^A(G,I)\right)}{dt} = -\frac{\gamma}{36t^2[p_R^2+\gamma\left(p_S^2-p_R^2\right)]^2}\{2[p_R^2 + \gamma\left(p_S^2 - p_R^2\right)]\left(d - k + 3t\right) - p_R\left(p_S - p_R\right)\left(1 - \gamma\right)$$

$$(d+2k+3i+3ci-3t)\}\{2[p_R^2+\gamma\left(p_S^2-p_R^2\right)]\left(d-k-3t\right)-p_R\left(p_S-p_R\right)$$
$$(1-\gamma)\left(d+2k+3i+3ci+3t\right)\}$$
$$-\frac{(1-\gamma)}{36t^2[p_R^2+\gamma\left(p_S^2-p_R^2\right)]^2}\{2[p_R^2+\gamma\left(p_S^2-p_R^2\right)]\left(d-k-3t\right)+\gamma p_S\left(p_S-p_R\right)$$
$$\left(d+2k+3i+3ci+3t\right)\}\{3\gamma p_S[\left(1+c\right)i+d+t]\left(p_S-p_R\right)+2p_R$$
$$[p_R+\gamma\left(p_S-p_R\right)]\left(d-k+3t\right)\}.$$

We now show that $\frac{d\left(\pi^A(G,I)\right)}{dt}>0$ holds. Therefore, we use the following four conditions:

Condition (2) follows from $x_S\left(I,G\right)>0$ and is given by

$$2[p_R^2+\gamma\left(p_S^2-p_R^2\right)]\left(d-k+3t\right)-p_R\left(p_S-p_R\right)\left(1-\gamma\right)\left(d+2k+3t+3i+3ci\right)>0.$$

From *Condition (2)* we get *Condition (3)* which is given by

$$d-k+3t>0.$$

Condition (4) follows from $x_S\left(I,G\right)<1$ and is given by

$$2[p_R^2+\gamma\left(p_S^2-p_R^2\right)]\left(d-k-3t\right)-p_R\left(p_S-p_R\right)\left(1-\gamma\right)\left(d+2k+3i+3ci+3t\right)<0.$$

Condition (5) follows from $x_R\left(I,G\right)<1$ and is given by

$$2[p_R^2+\gamma\left(p_S^2-p_R^2\right)]\left(d-k-3t\right)+\gamma p_S\left(p_S-p_R\right)\left(d+2k+3i+3ci+3t\right)<0.$$

It can be easily seen that these four conditions ensure that $\frac{d\left(\pi^A(G,I)\right)}{dt}>0$.

Third, note that

$$\frac{d^2\left(\pi^A(G,I)\right)}{dt^2}=\frac{4[p_R+\gamma(p_S-p_R)]^2(d-k)^2+9\gamma(1-\gamma)(p_S-p_R)^2(d+i+ci)^2}{18t^3[p_R^2+\gamma\left(p_S^2-p_R^2\right)]}>0.$$

Hence, it follows from the above analysis, that $\pi^A(G,I)$ is a parabola with a minimum where only the increasing part of the parabola is of interest for us.

Note, further, that $\frac{d\left(\pi^A(G,I)\right)}{dt}$ can be written as

$$\frac{d\left(\pi^A(G,I)\right)}{dt}=\frac{3[p_R+\gamma(p_S-p_R)]^2+[p_R^2+\gamma\left(p_S^2-p_R^2\right)]}{4[p_R^2+\gamma\left(p_S^2-p_R^2\right)]}-\frac{[p_R+\gamma(p_S-p_R)]^2(d-k)^2}{9[p_R^2+\gamma\left(p_S^2-p_R^2\right)]t^2}-\frac{\gamma(p_S-p_R)^2(d+i+ci)^2(1-\gamma)}{4[p_R^2+\gamma\left(p_S^2-p_R^2\right)]t^2}.$$

Hence, it holds that $\lim_{t\to\infty}\frac{d\left(\pi^A(G,I)\right)}{dt}=\frac{3[p_R+\gamma(p_S-p_R)]^2+[p_R^2+\gamma\left(p_S^2-p_R^2\right)]}{4[p_R^2+\gamma\left(p_S^2-p_R^2\right)]}$.

Note, further, that $\frac{3[p_R+\gamma(p_S-p_R)]^2+[p_R^2+\gamma\left(p_S^2-p_R^2\right)]}{4[p_R^2+\gamma\left(p_S^2-p_R^2\right)]}>\frac{[\gamma p_S+p_R(1-\gamma)]^2}{\gamma p_S^2+(1-\gamma)p_R^2}$ is equivalent to $2\gamma\left(p_S-p_R\right)^2\left(1-\gamma\right)[p_R^2+\gamma\left(p_S^2-p_R^2\right)]>0$. Hence, in the limit, the first order condition of $\pi^A(G,I)$ approaches $\frac{3[p_R+\gamma(p_S-p_R)]^2+[p_R^2+\gamma\left(p_S^2-p_R^2\right)]}{4[p_R^2+\gamma\left(p_S^2-p_R^2\right)]}$, a value that is larger than the constant first order condition of $\pi^A(I,I)$ which is equal to $\frac{[\gamma p_S+p_R(1-\gamma)]^2}{\gamma p_S^2+(1-\gamma)p_R^2}$. As a consequence, there must be exactly one

intersection point of $\pi^A(G,I)$ and $\pi^A(I,I)$ that is interesting for our analysis. To the left of this intersection, it must hold that $\pi^A(I,I) > \pi^A(G,I)$ and to the right of this threshold, it must hold that $\pi^A(I,I) < \pi^A(G,I)$. We now calculate the exact intersection point.

Calculation of the Intersection Point

$\pi^A(I,I) - \pi^A(G,I) = 0$ is equivalent to

$$t^2 + 2\frac{4[p_R+\gamma(p_S-p_R)]^2(k-d)+3\gamma(1-\gamma)(p_S-p_R)^2[(1+c)i+2k-d]}{3\gamma(1-\gamma)(p_S-p_R)^2}t+ \frac{4[p_R+\gamma(p_S-p_R)]^2(d-k)^2+9\gamma(1-\gamma)(p_S-p_R)^2[d^2+(1+c)^2i^2+2d(1+c)i]}{9\gamma(1-\gamma)(p_S-p_R)^2} = 0.$$

We define

$$B_2 \equiv 2\frac{4[p_R+\gamma(p_S-p_R)]^2(k-d)+3\gamma(1-\gamma)(p_S-p_R)^2[(1+c)i+2k-d]}{3\gamma(1-\gamma)(p_S-p_R)^2}$$

$$C_2 \equiv \frac{4[p_R+\gamma(p_S-p_R)]^2(d-k)^2+9\gamma(1-\gamma)(p_S-p_R)^2[d^2+(1+c)^2i^2+2d(1+c)i]}{9\gamma(1-\gamma)(p_S-p_R)^2}$$

Solving the above expression for t, we arrive at

$$t = -\tfrac{1}{2}B_2 \pm \tfrac{1}{2}\sqrt{B_2^2 - 4C_2}.$$

Due to the above analysis, the larger one of both thresholds is the one that is relevant for our analysis. We define

$$t_2 \equiv -\tfrac{1}{2}B_2 + \tfrac{1}{2}\sqrt{B_2^2 - 4C_2}.$$

It follows from the analysis above that $\pi^A(I,I) > \pi^A(G,I)$ for $t < t_2$ and that $\pi^A(I,I) < \pi^A(G,I)$ for $t > t_2$.