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Loss Aversion, Moral Hazard, and Stochastic Contracts

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Loss Aversion, Moral Hazard, and Stochastic Contracts

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I examine whether stochastic contracts benefit the principal in the setting of moral hazard and loss aversion. Incorporating that the agent is expectation-based loss averse and allowing the principal to add noise to performance signals, I find that stochastic contracts reduce the principal's implementation cost in comparison with deterministic contracts. Surprisingly, if performance signals are highly informative about the agent's action, stochastic contracts strictly dominate the optimal deterministic contract for almost any degree of loss aversion. The optimal stochastic contract pays a high wage whenever the principal observes good performance signals, while upon observing bad performance signals it adds a lottery that gives either the high wage or a low wage that serves as a harsh penalty to the agent. In the general case when the agent is both risk and loss averse, I show that if a penalty wage (i.e., a wage level at which the agent feels a substantial disutility) exists, the first best can be approximated closely but not attained. The findings have an important implication for designing contracts for loss-averse agents: the principal should insure the agent against wage uncertainty by employing stochastic contracts that increase the probability of a high wage.
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The interplay between risk aversion and incentives is central to the moral hazard literature, especially in designing an optimal contract. In this literature, one of the very few general results, as Bolton and Dewatripont (2005) argue, is the informativeness principle. This theory, going back to Holmstrom et al. (1979), Holmstrom (1982), and Grossman and Hart (1983), states that a wage contract should contain only informative signals about the agent's effort. Despite the well-established paradigm, many labor contracts are stochastic in that they include noise that does not provide any statistical information about the agent's effort.[1] This gap between theory and observed contracts suggests that a traditional approach focusing solely on risk aversion might give a partial and incomplete picture of the moral hazard problem.

Although loss aversion is a fundamental concept in behavior economics and is well-established with ample experimental and field evidence, the interplay between loss aversion and incentives remains understudied in the moral hazard literature. More recently, Camerer, Loewenstein and Rabin (2004) argue that loss aversion drives much of human behavior. "In a wide variety of domains", as Rabin (2004) puts forward, "people are significantly more averse to losses than they are attracted to same-sized gains". One prominent realm in which loss aversion plays an significant role is the domain of money and wealth (Tversky and Kahneman, 1991). It is thus important to incorporate loss aversion in the analysis of the optimal wage contract, and to better understand how loss aversion affects the tradeoff between insurance and incentives in the moral hazard model.

This paper analyzes the optimal wage contract in the setting of moral hazard and loss aversion, in which the agent is expectation-based loss averse and the principal can use stochastic contracts. The main result is that stochastic contracts reduce the principal's implementation cost in comparison with deterministic contracts that implement the same action. When performance signals are highly informative about the agent's effort, the dominance of stochastic contracts over deterministic contracts holds for almost any degree of loss aversion. Furthermore, I find that limited liability ensures the existence of the optimal con-

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[1]In workplaces, firms successfully adopt teams and team incentives (Che and Yoo, 2001; Lazear and Shaw, 2007; Bandiera, Barankay and Rasul, 2013) in which a team's performance depends not only on an employee's effort but also the effort exerted by other team members. In addition, non-executive employees increasingly receive payments in stock options (Core and Guay, 2001; Bergman and Jenter, 2007; Hochberg and Lindsey, 2010; Kim and Ouimet, 2014) whose valuation is influenced by external shocks in the financial sector.

tract, and that the optimal stochastic contract pays a high wage with certainty when a good signal is realized and with a positive probability when a bad signal is realized. Lastly, I show a condition under which stochastic contracts mitigate the inefficiencies arising from risk and loss aversion.

More specifically, I extend the simple principal-agent model under moral hazard, in which both the agent's actions and observable signals are binary, by making two assumptions. The first assumption is that the agent is expectation-based loss averse as defined in Kőszegi and Rabin (2006, 2007). In particular, the agent forms a reference point *after* taking an action, and thus his chosen action affects his reference point. The agent compares his realized wage to the stochastic reference point, and he feels a loss if the actual wage is smaller than the reference wage. The second assumption is that the principal can add noise to performance signals by employing stochastic contracts. In particular, the principal can add a lottery after observing the realized signal. Stochastic contracts thus serve as a tool for the principal to manipulate the signal distribution. A crucial feature of my model is that the principal can fully control the structure of the stochastic contract, i.e., the odds of the lottery.

I find that there exists a stochastic contract that strictly dominates deterministic contracts. Under the stochastic contract, the principal pays out a high wage whenever she observes a good signal, while upon observing a bad signal she adds a lottery that gives either the high wage or a low wage that serves as a harsh penalty to the agent for the bad signal. The advantages of this stochastic contract under loss aversion are twofold. First, the stochastic contract with this turning-a-blind-eye structure remedies an implementation problem associated with loss aversion. In deterministic contracts, this implementation problem is well-established, i.e., the agent may choose the stochastically dominated action when he is sufficiently loss averse (Herweg, Müller and Weinschenk, 2010). As a result, the principal may be unable to induce the agent to exert effort. In sharp contrast, by employing the stochastic contract, the principal can always implement the desired action for any degree of loss aversion.

Second, even if deterministic contracts do not face the implementation problem, the stochastic contract helps the principal lower the cost of implementing the desired action beyond what is achieved under the optimal deterministic contract. Note that the stochastic contract, as compared to deterministic contracts, has two countering effects on the principal's cost. On the one hand, the stochastic contract might increase the principal's cost, because the high wage is now paid out more often and a larger wage spread is required to incentivize the agent to work. On the other hand, the stochastic contract reduces the probability that the agent feels a loss, thus the principal might capitalize on this reduction in the agent's loss premium to achieve a lower cost. When the positive effect of reducing the loss premium outweighs the negative effect of increasing the expected bonus, the stochastic contract dominates deterministic contracts. Whether the stochastic contract is dominant depends on the agent's degree of loss aversion and the informativeness of performance signals.

Interestingly, as performance signals get more informative about the agent's action, the principal favors the stochastic contract under a wider range of the degree of loss aversion. When performance signals are highly uninformative, the principal is better off with the stochastic contract under a most restrictive condition, i.e., only when the agent feels losses at least twice as strongly as same-sized gains. This condition gets weaker if performance signals provide some information about the agent's action. When performance signals convey almost perfect information, the stochastic contract dominates deterministic contracts for almost any degree of loss aversion. Intuitively, when performance signals are highly informative, the principal can provide further wage certainty at a negligible cost. Thus, this finding has an important implication for designing contracts for loss-averse agents: the principal has an incentive to add noise after the bad signal to insure the agent against wage uncertainty.

Yet I show that the second-best optimal stochastic contract might not exist. In particular, the principal's cost strictly decreases as the probability of getting the high wage increases. This implies that the principal prefers to push the probability of the high wage close to one. However, the principal cannot provide wage certainty because of the incentive constraint, and hence the solution to the principal's problem is not well-defined. This existence problem differs from the above implementation problem under loss aversion in that the stochastic contract can always implement the desired action, but if used, the optimal stochastic contract does not exist. Given the wide range under which stochastic contracts dominate deterministic contracts, the existence problem appears more severe than previously thought.

In mitigating the non-existence problem, I find that limited liability helps restore the existence of the optimal stochastic contract. The optimal stochastic contract pays a bonus with certainty when the good signal is realized and with a positive probability when the bad signal is realized; otherwise, the agent receives a lowest possible wage, at which the limited liability constraint is binding. This finding highlights the importance of imposing limited liability in stochastic contracts to restrict the extent the principal can punish the agent in the event of the bad signal and to ensure that the second-best optimal contract exists.

Lastly, I consider the general case when the agent is both risk and loss averse, and show that if a penalty wage (i.e., a wage level at which the agent feels a substantial disutility) exists, the first best can be approximated closely but not attained. The benefit of the stochastic contract crucially depends on how effectively the principal can punish the agent in the event of bad signals, because the effective punishment is necessary for inducing the agent to work. If the principal uses the penalty wage to punish the agent, she can employ stochastic contracts to insure the agent against wage uncertainty to a very large extent and still create a strong marginal incentive to work. The finding that the first best can be closely approximated but not attained resembles the result in Mirrlees (1974) seminal work. In Mirrlees (1974), the approximation of the first best is driven by the signal's normal distribution which has the property that for extreme outcomes it becomes very informative about the agent's action. This assumption, however, does not play any role for my result.

While for the most part of the paper, I assume that a reference point is formed after the decision is taken, and allow for a stochastic reference point. In the Discussion section, I relax these assumptions and discuss alternative notions of loss aversion. In particular, the result holds under the forward-looking disappointment aversion according to Bell (1985), Loomes and Sugden (1986), or Gul (1991), in which the reference point is the recent expectation but does not allow for stochastic reference points. It also remains valid to the concept of *preferred personal equilibrium* by Kőszegi and Rabin (2007), which assumes that the reference point is formed before taking the decision and hence is taken as given. The robustness of the result suggests that noise should be generally added to performance signals in the optimal contract for loss-averse agents. When loss aversion plays a significant role in the agent's preferences, the principal can insure the agent against wage uncertainty by employing stochastic contracts.

The rest of the article is organized as follows. Section I summarizes the related literature. Section II outlines the model, and Section III specifies the principal's problem and derives the set of feasible contracts. Section IV presents the main results and discusses alternative notions of loss aversion. Section V concludes. All proofs of lemmas and propositions are relegated to the Appendix.

I. Related Literature

In this section, I provide an overview of the literature on behavioral contract theory, which is most related to this paper. I also refer to the literature that highlights the optimality of stochastic contracts and that provides explanations for the unresponsiveness of wages to performance.

My paper is most closely related and complementary to Herweg, Müller and Weinschenk (2010) who show that, in the setting of moral hazard and loss aversion, the optimal deterministic contract is a bonus contract. Complementary to their finding, my paper provides further insight into the characteristics of the optimal contract under loss aversion: the probability of getting a bonus is set as high as possible. Furthermore, while their paper proposes stochastic contracts as a remedy to the implementation problem of deterministic contracts, my paper highlights the optimality of stochastic contracts for almost any degree of loss aversion, and even when deterministic contracts are implementable.

In the literature on behavioral contract theory, my paper also relates to Daido and Murooka (2016) who show that the principal may employ team incentives when the agents are loss averse. Similar to their paper, my paper stresses the role of limited liability in ensuring the existence of the optimal contract. However, their paper focuses on team incentives and takes a team structure as given, whereas I examine individual stochastic contracts and consider noise as one of the principal's variables.

My paper also relates to the extensive literature on reference-dependent preferences, starting out with the seminal work of Kahneman and Tversky (1979) where the agent's utility depends on a reference point and the agents feel losses more strongly than gains. Subsequently, as reviewed by Barberis (2013), several papers have contributed to theoretical extensions—covering reference-dependent models of both static (Bell, 1985; Loomes and Sugden, 1986; Munro and Sugden, 2003; Sugden, 2003; Kőszegi and Rabin, 2006; De Giorgi and Post, 2011) and dynamic nature (Kőszegi and Rabin, 2009; Barberis and Huang, 2001; Barberis, Huang and Santos, 2001)—and applications of reference-dependent preferences into real-life problems, such as in tournaments (Gill and Stone, 2010), saving decisions (Jofre, Moroni and Repetto, 2015), asset pricing (Pagel, 2016), life-cycle consumption (Pagel, 2017), intertemporal incentives (Macera, 2018), and portfolio choices (Pagel, 2018). My paper contributes to the literature strand that incorporates expectation-based reference-dependent preferences into moral hazard models, as summarized by Koszegi (2014), by providing the characteristics of the optimal stochastic contracts for loss-averse agents.

My results speak to a growing literature that highlights the optimality of noise in the contract. Haller (1985) finds that randomization benefits the principal when the agent faces an aspiration constraint of achieving certain income levels with certain probabilities. Strausz (2006) shows that stochastic mechanisms may be optimal in a screening context. Lang (2020) examines the optimal contract with subjective evaluations, and shows that stochastic contracts may increase the principal's profits and eliminate the requirement of a third-party payment. Ostrizek (2020) finds that the principal prefers to set wages contingent on a noisy information structure, because the agent remains uninformed about their match-specific ability and is cheaper to motivate. Contributing to this literature, I show that noise can serve as a tool to insure the agent against wage uncertainty.

By highlighting that the principal prefers to lump signals together into a bonus set, my findings also adds to the rich literature attempting to explain why wages are rigid relative to performance. Considering multiple tasks that are substitutes, Holmstrom and Milgrom (1991) shows that wages should not respond to performance because strong incentives for an observable task worsens the agent's performance on the other unmeasurable task. At large, several explanation for a fixed-wage contract have proposed, including monitoring cost (Lazear, 1986), relative performance and cooperation (Lazear, 1989), relational contracts (e.g., MacLeod and Malcomson, 1989; Levin, 2003), and reciprocal preferences (Englmaier and Leider, 2012).

II. The Model

I consider a principal-agent model in a moral hazard and loss aversion setting. The principal (she) offers an one-period employment contract to the agent (he), which the agent either accepts or rejects. If the agent rejects, he receives his reservation utility which is assumed to be zero.[2] If the agent accepts the contract,

[2]Assuming the reservation utility is zero is consistent with the "quitting" constraint. This assumption is made for the sake of simplicity of analysis. The main results would continue to hold when the reservation

he then makes a binary action $a \in \{a_H, a_L\}$, i.e., he either "works" ($a = a_H$) or "shirks" ($a = a_L$). The cost of working for the agent is $c(a_H) = c$, for $c > 0$, and the cost of shirking is normalized at zero $c(a_L) = 0$.

The action a is private information of the agent that the principal cannot observe. Instead, the principal is assumed to observe a contractible signal for the agent's action. The signal $s \in S = \{1, 2\}$ is good ($s = 2$) or bad ($s = 1$). The agent receives the good signal with probability q_H if he works and with probability q_L if he shirks, where $1 > q_H > q_L > 0$. The signal distribution is common knowledge.

The agent exhibits expectation-based loss aversion as defined in Kőszegi and Rabin (2006, 2007). The agent's utility has two additively separable components: the standard "consumption utility" and the reference-dependent "gain-loss utility". The agent's consumption utility, denoted by $u(\cdot)$, is assumed to be strictly increasing, (weakly) concave, and unbounded, i.e., $u'(\cdot) > 0$ and $u''(\cdot) \leq 0$. The second component comes from reference-dependent preferences: the agent compares a realized outcome to a stochastic reference point, and how his overall utility is affected depends on whether this comparison is perceived as a gain or a loss. The gain-loss function $\mu(\cdot)$ satisfies the assumptions on the "value function" by Tversky and Kahneman (1991). I assume that the gain-loss function is piecewise linear,

$$\mu(m) = \begin{cases} m & \text{for } m \geq 0 \\ \lambda m & \text{for } m < 0 \end{cases}$$

where $\lambda \geq 1$ represents the degree of loss aversion.

To determine the reference point, I apply the concept of choice-acclimating personal equilibrium (CPE) in the sense of Kőszegi and Rabin (2007), which makes two important assumptions. First, the agent forms the reference point, to which realized outcomes are evaluated, *after* making the decision, and thus his decision affects his reference point. As mentioned by Kőszegi and Rabin (2007), CPE considers outcomes that are resolved long after all decisions are made. Thus, the reference point is endogenously determined as the agent's rational expectation about the outcomes given his decision. Second, the reference point is stochastic if the decision's outcome is stochastic. To form a stochastic reference point, it is assumed that the agent knows the set of possible outcomes and its probability

utility is positive.

distribution conditional on his decisions. These two assumptions give rise to a crucial feature of CPE: a stochastic outcome is evaluated to a stochastic reference point by comparing outcome by outcome, where each comparison is weighted with the joint probability with which a certain outcome is realized and an alternative outcome is expected.

On the other hand, the principal is assumed to be risk and loss neutral. I assume that the agent's "work" generates sufficient profit to the principal that she strictly prefers to implement the high action a_H. Thus I focus on the principal's cost minimization problem, and inquire into the optimal contract design under moral hazard with loss aversion.

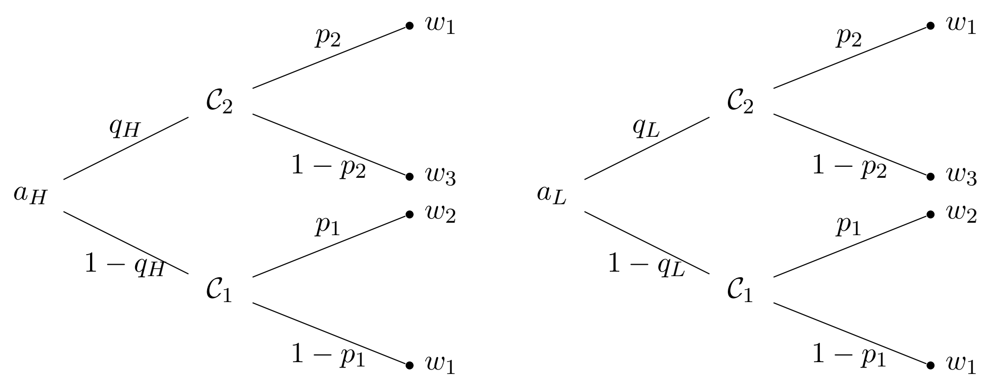
In designing the optimal contract, the principal can distort the outcome distribution by adding noise to the performance signals. Put differently, she can fully employ stochastic contracts to implement the desired action. A stochastic contract specifies wage payments contingent not only on the contractible signals but also on a stochastic device that does not depend on the agent's action. Formally, the principal offers the agent a state-contingent stochastic contract $(\mathcal{C}_s)_{s\in S}$, in which each $\mathcal{C}_s$ entails a stochastic device — uncorrelated with the agent's action — that specifies wage payments within the contract.

In the setting of two signals, the principal offers a stochastic contract $(\mathcal{C}_1, \mathcal{C}_2)$. If the principal observes the good signal $s = 2$, then the agent receives $\mathcal{C}_2$ that specifies a lottery $(p_2, 1 - p_2)$ over wage payments.[3] Analogously, $\mathcal{C}_1$ with a lottery $(p_1, 1-p_1)$ is realized if the bad signal $s = 1$ is observed. Importantly, the principal has full control over the design of these lotteries (p_1, p_2) that I refer to as the "stochastic structure".

As shown in Figure 1, the distribution over the outcomes $i \in \{1, \ldots, 4\}$ depends on both the agent's action and the principal's choice of stochastic structure. Figure 1 represents how the distribution over the wage payments $(w_i)_{i=1}^4$ depends on the agent's action $a \in \{a_H, a_L\}$ under the stochastic contract. By committing to the stochastic structure (p_1, p_2) in the contract, the principal makes the wage distribution common knowledge to the agent before he chooses his action. Thus, in the process of choosing an action, the agent incorporates the structure of the stochastic contract and forms a rational expectation about monetary outcomes.

[3]The assumption that a lottery specifies two outcomes is without loss of generality. Even when the lottery specifies more than two outcomes, the principal prefers to lump outcomes into two distinct sets. This is in line with the finding by Herweg, Müller and Weinschenk (2010) that the optimal contract specifies two levels of wages.

Figure 1. : Distribution over wage payments under stochastic contracts



Note: The left diagram depicts the distribution of wage payments conditional on the agent's high action a_H. The right diagram depicts the distribution of wage payments conditional on the agent's low action a_L.

More precisely, consider a particular case in which the agent chooses the high action a_H and that a certain outcome i is realized. The agent receives w_i and incurs effort cost c. Given that w_i is realized, he compares the realized outcome w_i to all alternative outcomes. Although w_i is realized, with some probability $f_j(a_H)$ he expects an alternative outcome $j \neq i$ to be observed. If $w_i > w_j$, the agent experiences a gain of $u(w_i) - u(w_j)$, whereas if $w_i < w_j$, the agent experiences a loss of $\lambda(u(w_i) - u(w_j))$. If $w_i = w_j$, there is no gain or loss involved. The agent's utility in this particular case is given by

$$u(w_i) + \sum_{j|w_i>w_j} f_j(a_H)(u(w_i) - u(w_j)) + \sum_{j|w_i<w_j} f_j(a_H)\lambda(u(w_i) - u(w_j)) - c$$

Notice that this particular comparison occurs with the probability $f_i(a_H)$ that outcome i is realized. When there is uncertainty in the decision's outcome, the agent's expected utility is obtained by averaging over all possible comparisons.

III. The Principal's Problem

Denote $u_i = u(w_i)$. With this notation, the agent's expected utility from choosing action $a \in \{a_H, a_L\}$ is given by

$$EU(a) = \sum_i f_i(a)u_i - (\lambda - 1)\sum_i \sum_{j|u_i>u_j} f_i(a)f_j(a)(u_i - u_j) - c(a)$$

The first term captures the agent's expected consumption utility. For $\lambda = 1$, we are back in the standard case without loss aversion. The second term captures the gain-loss utility. While the agent expects a high wage u_i to come up with probability $f_i(a)$, with probability $f_j(a)$ he receives a low wage u_j and experiences a loss of $\lambda(u_i - u_j)$. On the other hand, if the agent expects the low wage with probability $f_j(a)$, with probability $f_i(a)$ he receives the high wage and experiences a gain of $u_i - u_j$. Since losses loom larger than gains of equal size ($\lambda \geq 1$), the gain-loss utility is always negative in expectation. Following Herweg, Müller and Weinschenk (2010), I refer to this expected net loss as the agent's "loss premium". For an agent with a higher degree of loss aversion, the principal has to pay a higher loss premium in a given contract.

Let $h(\cdot) := u^{-1}(\cdot)$ be the wage that the principal offers the agent to obtain utility u_i, i.e., $h(u_i) = w_i$. Due to the assumptions on $u(\cdot)$, $h(\cdot)$ is strictly increasing and (weakly) convex. Following Grossman and Hart (1983), I regard $\mathbf{u} = (u_1, \ldots, u_4)$ as the principal's control variables in her cost minimization problem. The principal specifies a wage payment w_i for each outcome i in the employment contract, equivalently an utility level u_i.

The key assumption is that, besides the wage payments, the principal controls the stochastic structure $\mathbf{p} = (p_1, p_2)$. In sharp contrast to deterministic contracts, stochastic contracts allows the principal to manipulate the outcome distribution. Her problem is thus to minimize the expected wage payment that implements a_H subject to the participation and incentive compatibility constraints.

$$\min_{\mathbf{u},\mathbf{p}} \; E(h(u_i))$$

$$\text{(PC)} \qquad \text{subject to} \quad EU(a_H) \geq 0$$

$$\text{(IC)} \qquad EU(a_H) \geq EU(a_L)$$

In deterministic contracts, it is well-established that if the agent is sufficiently loss averse, i.e. $\lambda > 2$, then the agent might choose the stochastically dominated action, and the principal, facing a severe implementation problem, might be unable to induce the high action (Herweg, Müller and Weinschenk, 2010). I now examine if there are incentive-compatible wage payments under stochastic contracts to implement a_H and show that, in sharp contrast to deterministic contracts, stochastic contracts do not suffer from the implementation problem.[4]

LEMMA 1: *Suppose $u''(\cdot) \leq 0$ and $\lambda \geq 1$. For every λ, there exists a stochastic contract such that the action a_H can be implemented.*

Lemma 1 states that given any degree of loss aversion there are incentive-compatible wages and a stochastic structure such that the agent accepts the stochastic contract and chooses the high action. In particular, the principal pays out a high wage whenever she observes a good signal, while after observing a bad signal she adds a lottery that gives either the high wage or a low wage. This means, in the stochastic contract, the principal turns a blind eye on the agent's receiving a bad signal and insures the agent against wage uncertainty. The stochastic contract circumvents the implementation problem of deterministic contracts, because, by increasing the probability of getting the high wage, the principal simultaneously reduces the agent's expected net loss when he works and increases his expected net loss when he shirks. For a sufficiently loss-averse agent, whose primary concern is to minimize the expected net loss, the stochastic contract makes working more attractive than shirking.

So far it is established that the constraint set of the principal's cost minimization problem is non-empty for the high action a_H given any degree of loss aversion. I restrict attention to the stochastic contract of the turning-a-blind-eye structure for the following analysis.[5]

IV. The Optimal Contract

In this section, I examine the existence and the characteristics of the optimal contract. First, I focus on the case of a loss-averse but risk-neutral agent. I will show that under a weak condition there exists a stochastic contract that strictly dominates deterministic contracts. The principal can lower the cost of implementing the desired action by employing stochastic contracts rather than deterministic contracts. Surprisingly, this holds true even when deterministic contracts do not face the implementation problem. The dominance of stochastic

[4]All proofs of lemmas and propositions are provided in the Appendix.

[5]The strategy "turing a blind eye" was first discussed in Herweg, Müller and Weinschenk (2010), who show that indeed when facing an implementation problem, the principal can still implement the desired action by stochastically ignoring the agent's bad performance. In this paper, I focus more on the situations in which the implementation problem does not prevail and the principal can use deterministic contracts to induce the agent to work.

contracts, however, implies that for many cases the second-best optimal stochastic contract does not exist. With agents being expectation-based loss averse, an existence problem, which does not prevail in the standard model, arises. Second, I examine whether limited liability mitigates the non-existence issue of stochastic contracts and characterize the second-best optimal stochastic contract. Third, I consider the general case of a risk- and loss-averse agent and show that the first-best can be approximated closely, but not attained, by stochastic contracts that provide the bonus almost certainly.

A. *Strict Dominance of Stochastic Contracts*

Consider an agent who is risk neutral in the standard notion, $u''(\cdot) = 0$, but exhibits loss aversion $\lambda > 1$.

If the principal is restricted to offer deterministic contracts, with two possible signals $s \in \{1, 2\}$, the deterministic contract takes the form of a bonus contract: the agent is paid a base wage $\underline{w}$ if the bad signal is realized, and he is paid the base wage $\underline{w}$ plus a bonus $b > 0$ if the good signal is realized.

Under this deterministic contract, the agent prefers the high action a_H over the low action a_L if his utility from the high action exceeds his utility from the low action. This is the case if and only if

$$\begin{aligned} & \underline{w} + q_H b - (\lambda - 1) q_H (1 - q_H) b - c \geq \underline{w} + q_L b - (\lambda - 1) q_L (1 - q_L) b \\ \text{(IC-D)} \quad \Leftrightarrow \; & (q_H - q_L) b - (\lambda - 1) \left[q_H (1 - q_H) - q_L (1 - q_L) \right] b \geq c \end{aligned}$$

Because both the participation and incentive constraints are binding, the principal's cost minimization problem is equivalent to minimizing the agent's loss premium conditional on a_H subject to the incentive constraint. I examine whether there exists a stochastic contract that satisfies the incentive constraint and at the same time reduces the loss premium that the principal has to pay.

Assuming that the principal can employ stochastic contracts, I consider the stochastic contract that takes the turning-a-blind-eye structure: the principal pays a high wage with probability 1 if she observes the good signal, while if she observes the bad signal she stochastically ignores it by paying the high wage with probability p_1 and paying a low wage with probability $1 - p_1$. It follows directly from Lemma 1 that the stochastic contract satisfies the incentive constraint and implements the high action. I examine whether the stochastic contract benefits the principal from a cost perspective in the following proposition.

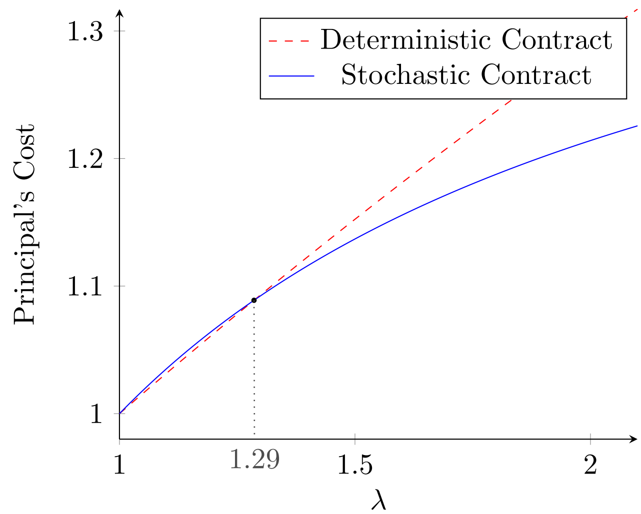
PROPOSITION 1: *Suppose* $u''(\cdot) = 0$ *and* $\lambda - 1 > \frac{1-q_H}{1-q_L}$. *Then, there exists a stochastic contract with the wage structure* $w_1 < w_2 = w_3 = w_4$ *that strictly dominates the optimal deterministic contract.*

Besides remedying the implementation problem, the stochastic contract benefits the principal from a cost perspective: the principal pays a lower loss premium to the agent in the stochastic contract. To see the intuition for Proposition 1, note first that the agent's loss premium depends on two variables: (i) the bonus size b and (ii) the probability with which the agent feels a loss when a deviation from his reference point occurs $q_H(1-q_H)$, which following Herweg, Müller and Weinschenk (2010) I refer to as "loss probabiliy". The loss probability is an inverted U-shaped function; it reaches its maximum when getting a bonus is completely random, i.e. $q_H = 1/2$, and it reaches its minimum of zero as the bonus probability moves to the extremes, i.e. $q_H = 0$ or $q_H = 1$. By employing the stochastic contract that pays the low wage only if the worst outcome ($i = 1$) is realized and pays the high wage for all other outcomes, the principal increases the bonus probability closer to one and thereby reduces the associated loss probability closer to zero.

Although the stochastic contract decreases the probability that the agent feels a loss, it increases the bonus size b required to incentivize the agent to work. As the probability of getting a bonus increases, the outcome distribution under the high action resembles that under the low action. Thus, to satisfy the incentive constraint, the principal needs a higher bonus. Put together, the stochastic contract has two opposing effects on the loss probability and the bonus size. While the insurance against wage uncertainty may come at the cost of a larger expected bonus required to induce the agent to work, the positive effect of the reduced loss probability outweighs the negative effect of the increased bonus size if the agent is sufficiently loss averse.

Figure 2 illustrates the dominance of the stochastic contract for a simple example with $q_H = 0.8, q_L = 0.3$, and $p_1 = 0.75$. The dashed line in Figure 2 shows the principal's implementation cost under the optimal deterministic contract, and the solid line shows the minimum cost under the stochastic contract with the wage structure $w_1 < w_2 = w_3 = w_4$. Given $q_H = 0.8$ and $q_L = 0.3$, the condition $\lambda - 1 > \frac{1-q_H}{1-q_L}$ in Proposition 1 translates to $\lambda > 1.29$. As shown in Figure 2, for $\lambda \in [1, 1.29]$, the optimal deterministic contract yields a lower cost

Figure 2. : Principal's cost under stochastic contracts versus deterministic contracts



Note: The figure shows an illustration of the principal's cost under stochastic contracts and deterministic contracts for $q_H = 0.8, q_L = 0.3$, $p_1 = 0.75$, $p_2 = 1$, and $c = 1$. The dashed line shows the principal's implementation cost in the optimal deterministic contract. The solid line shows the the principal's minimum cost in the stochastic contract with the wage structure $w_1 < w_2 = w_3 = w_4$.

for the principal, while for $\lambda > 1.29$, the stochastic contract strictly dominates the optimal deterministic contract. The higher the degree of loss aversion, the larger the relative benefit of using the stochastic contract for the principal.

Interestingly, the condition on the degree of loss aversion in Proposition 1 is much weaker than that previously established in the literature. Herweg, Müller and Weinschenk (2010) establish that turning a blind eye enables the principal to achieve a lower cost if and only if $\lambda > 2$.[6] Notice that in Proposition 1 the condition $\frac{1-q_H}{1-q_L} + 1$ is strictly smaller than 2; this would imply a larger set of degrees of loss aversion than previously thought under which stochastic contracts strictly dominate deterministic contracts.

A second interesting observation is that as the performance signals become more informative about the agent's action, the principal favors the stochastic contract under a wider range of the degree of loss aversion. Let us consider two extreme

[6]In particular, Herweg, Müller and Weinschenk (2010) assume an incomplete contracting environment, which implies that performance measures are inherently noisy. Thus, this limits the extent to which the principal can add noise in the optimal contract as compared to the complete contracting setting in my model.

cases. If the signals are highly uninformative, i.e. $\frac{1-q_H}{1-q_L} \to 1$, then the most restrictive condition under which the stochastic contract dominates deterministic contracts becomes $\lambda > 2$, which coincides with the well-established condition in the literature. The condition on the degree of loss aversion, however, gets weaker as the performance signals provide more information about the agent's action. At the other extreme, if the signals are highly informative, i.e. $\frac{1-q_H}{1-q_L} \to 0$, then the condition becomes $\lambda > 1$. This means if the signals provide almost precise information about the agent's action, then the principal benefits from using the stochastic contract almost all the time. The logic is that when the given signals are very informative, the principal provides further wage certainty at a negligible cost and prefers to do so to a large extent. Put differently, in the limit the stochastic contract strictly dominates deterministic contracts for almost any degree of loss aversion.

B. Non-Existence of The Second-Best Optimal Contract

In this part, I focus on the cases where stochastic contracts strictly dominate deterministic contracts, and attempt to characterize the second-best optimal stochastic contract, assuming for now that the solution exists. Formally, I assume that $u''(\cdot) = 0$ and $\lambda - 1 > \frac{1-q_H}{1-q_L}$.

Similar to the finding by Herweg, Müller and Weinschenk (2010), a first important observation is that the optimal stochastic contract should take the form of a bonus contract. When an agent is risk neutral but loss averse, it is optimal for the principal to pool as many informative signals as possible into a bonus set and pay a high wage only if the realized signal lies in this bonus set. The logic is that when facing the risk-neutral agent, the principal cannot capitalize on a higher degree of wage differentiation. On the other hand, pooling wages together helps the loss-averse agent avoid unfavourable comparisons and yields him a higher expected utility. To satisfy the incentive constraint, the optimal contract requires a minimum degree of wage differentiation in that the principal offers two wage levels – a base wage and a bonus – no matter how rich the signal space is.

It remains to determine which outcomes $i \in \{1, \dots, 4\}$ should be included in the bonus set. Given any contract $(\hat{w}_i)_{i=1}^4$ that the principal offers, I can relabel the outcomes i such that this contract is equivalent to a contract $(w_i)_{i=1}^4$ of an (weakly) increasing wage profile with $w_{i-1} \leq w_i$ for all $i \in \{2, 3, 4\}$. Thus the bonus set can be one of the three options: (i) the bonus set includes only the highest outcome $\{w_4\}$, or (ii) the bonus set includes two highest outcomes $\{w_4, w_3\}$, or (iii) the bonus set includes all but the lowest outcome $\{w_4, w_3, w_2\}$. I examine the option (i) in the following lemma.

LEMMA 2: *Suppose $u''(\cdot) = 0$ and $\lambda > 1$. Then, any stochastic contract with the wage structure $w_1 = w_2 = w_3 < w_4$ is weakly dominated by the optimal deterministic contract.*

A stochastic contract that rewards only the highest outcome reduces the probability of getting a bonus; a slim chance of getting a bonus in turn simultaneously increases the agent's expected net loss when he works and decreases his expected net loss when he shirks. Because the agent cares sufficiently about minimizing the expected loss, this implies that the stochastic contract of the wage structure $w_1 = w_2 = w_3 < w_4$ worsens the implementation problem under loss aversion. Moreover, the principal requires a substantially higher bonus to motivate the agent to work. Due to the worsened implementation problem, the negative effects of an increased bonus outweighs the positive effects of a reduced loss probability, leading to that the principal's implementation cost actually increases with such a stochastic contract.

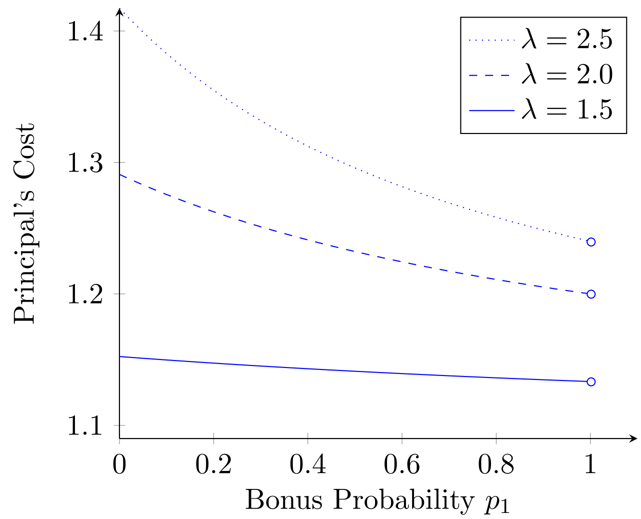
Note that the option (ii) coincides with the deterministic contract. As in Proposition 1, the optimal deterministic contract is strictly dominated by the stochastic contract with the wage structure $w_1 < w_2 = w_3 = w_4$. Taken these two observations together, it is thus optimal to include all but the worst outcome in the bonus set.

With the bonus set including all except for the worst outcome $i = 1$, I derive the principal's implementation cost for a given stochastic structure. The comparative statics of the principal's implementation cost with respect to the probability of getting a bonus p_1 reveals an insight about the existence of the second-best optimal stochastic contract, which is covered in the following proposition.

PROPOSITION 2: *Suppose $u''(\cdot) = 0$ and $\lambda - 1 > \frac{1-q_H}{1-q_L}$. Then, the second-best optimal stochastic contract does not exist.*

The solution to the principal's problem with the above stochastic contract is not well-defined. The reason is that the principal can always achieve a lower cost by further increasing the probability of getting a bonus p_1 close to one and rendering the penalty harsher in the event of the bad signal. However, p_1 cannot

Figure 3. : Principal's cost as a function of the bonus probability



Note: The figure shows an illustration of the principal's cost under the stochastic contract of the wage structure $w_1 < w_2 = w_3 = w_4$ for $q_H = 0.8, q_L = 0.3$, $p_2 = 1$, and $c = 1$.

reach the value of one, as the contract then becomes a fixed wage contract that does not satisfy the incentive constraint. In the limit, the principal's cost C_r in the stochastic contract is given by

$$\lim_{p_1 \to 1} C_r = c + \frac{\lambda - 1}{\lambda} \cdot \frac{(1 - q_H)c}{(q_H - q_L)}$$

Figure 3 illustrates how the principal's implementation cost changes with respect to the probability of getting a bonus p_1 for a simple example with $q_H = 0.8$ and $q_L = 0.3$. The solid, dashed, and dotted lines correspond to the principal's cost for $\lambda = 1.5$, $\lambda = 2$, and $\lambda = 2.5$ respectively. All the lines exhibit a downward trend, implying that the principal's cost decreases as p_1 increases. However, there is a discontinuity, depicted as empty circles, at $p_1 = 1$. If $p_1 = 1$, the principal cannot induce the agent to work, her implementation cost becomes prohibitively high.

C. Limited Liability

The non-existence of the second-best optimal stochastic contract hinges on the principal's desire to insure the agent against wage uncertainty to the largest possible extent, and thereby to further reduce her cost, if the agent is sufficiently loss averse. On the other hand, to motivate the agent to work in the face of such insurance, the principal punishes the agent indefinitely when the worst outcome is realized. If the punishment for the worst outcome is, however, limited, the principal faces an upper bound of how much wage certainty she can provide to the agent. In this part, I show that the second-best optimal stochastic contract exists if the principal faces a limited liability constraint, and characterize the second-best optimal contract.

Analogous to the previous analysis, it can be shown that the optimal bonus set consists of all but the worst outcome. I thus restrict my attention to stochastic contracts of the wage structure $w_1 < w_2 = w_3 = w_4$. Let f_H and f_L be the probability of getting a bonus conditional on the agent's high and low action respectively, i.e., $f_H = P[i > 1|a_H] = q_H + p_1(1 - q_H)$ and $f_L = P[i > 1|a_L] = q_L + p_1(1 - q_L)$. The principal's problem becomes

$$\min_{\underline{w},b,p_1} \ \underline{w} + f_H b$$

subject to

$$\text{(PC)} \qquad \underline{w} + f_H b - (\lambda - 1) b f_H (1 - f_H) \geq c$$

$$\text{(IC)} \qquad b(f_H - f_L) - (\lambda - 1) b [f_H (1 - f_H) - f_L (1 - f_L)] \geq c$$

$$\text{(LL)} \qquad \underline{w} \geq 0$$

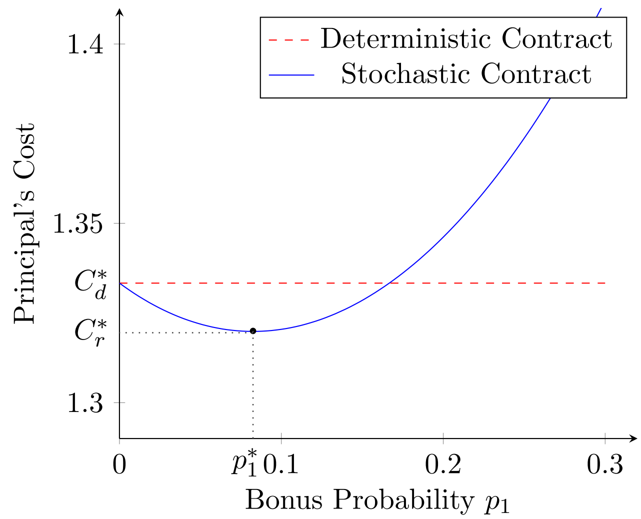
Because the (IC) binds at optimum (else, the principal can reduce b by a small amount), the optimal bonus size can be written as a function of p_1:

$$b^*(p_1) = \frac{c}{(q_H - q_L)(1 - p_1)[1 - (\lambda - 1)(1 - q_H - q_L - p_1(2 - q_H - q_L))]}$$

The (LL) constraint is also binding at optimum. Else, by reducing $\underline{w}$ by a small amount, the principal decreases the expected payment without changing (IC) or violating (LL). Thus, the principal's cost in the stochastic contract is given by $C_r(p_1) = f_H b^*$. Note that at $p_1 = 0$, the stochastic contract coincides with the deterministic contract such that the principal's minimum cost remains unchanged. The principal reduces her implementation cost by using the stochastic contract if the following assumption holds.

ASSUMPTION 1 (A1): $(\lambda - 1)(1 - q_H - q_L + q_H(2 - q_H - q_L)) > 1$

Figure 4. : Principal's cost under limited liability



Note: The figure shows an illustration of the principal's cost under stochastic contracts and deterministic contracts under limited liability for $q_H = 0.8, q_L = 0.3$, $p_2 = 1$, $c = 1$ and $\lambda = 3$. The dashed line shows the principal's implementation cost in the optimal deterministic contract. The solid line shows the the principal's minimum cost in the stochastic contract with the wage structure $w_1 < w_2 = w_3 = w_4$.

Assumption (A1) is a sufficient and necessary condition for the principal's minimum cost function to be locally decreasing at $p_1 = 0$. Given (A1), there exists a stochastic contract that strictly dominates the optimal deterministic contract under limited liability. Solving for the optimal p_1^* that minimizes $C_r(p_1)$, I characterize the second-best optimal stochastic contract in the following proposition.

PROPOSITION 3: *Suppose (A1) holds, $u''(\cdot) = 0$, and $w \geq 0$. Then, the second-best optimal stochastic contract exists. The optimal stochastic contract pays $b^*(p_1^*)$ with probability one when the good signal is realized and with probability p_1^* when the bad signal is realized. The optimal p_1^* is given by*

$$p_1^* = \frac{1}{1-q_H}\left(\sqrt{1 - \frac{\lambda}{\lambda - 1} \cdot \frac{1-q_H}{2-q_H-q_L}} - q_H\right)$$

Figure 4 illustrates the second-best optimal stochastic contract under limited liability with a simple example of $q_H = 0.8$, $q_L = 0.3$ and $\lambda = 3$. With this parameter specification, the principal can implement the desired action with a deterministic contract that reaches the lowest cost of $C_d^* = 1.33$. The Assumption

(A1), translating to $\lambda > 2.61$, is satisfied under the specification of $\lambda = 3$. The second-best optimal stochastic contract pays $b^*(p_1^*) = 1.62$ with probability one if the principal observes the good signal $s = 2$ and with probability $p_1^* = 0.08$ if she observes the bad signal $s = 1$. Thus, the principal yields the optimal cost of $C_r^* = 1.32$, which is strictly lower than C_d^*.

If the agent is subject to limited liability, the solution of the principal's problem is well-defined. Intuitively, limited liability limits the extent to which the principal can punish the agent in the event of bad outcomes, and in turn her ability to insure the agent against wage uncertainty. Put differently, the principal does not benefit from increasing the bonus probability p_1 close to 1 under the limited liability constraint. As the base wage $\underline{w}$ is bounded by zero, in order to motivate the agent to work, the bonus b becomes substantially large after a certain level of wage certainty.

D. First Best Approximation

Loss aversion imposes an extra cost for the principal: the principal has to pay the loss premium to the agent for him to accept the contract and choose to work. So far I show that stochastic contracts help the principal reduce her implementation cost as compared to deterministic contracts if the agent is risk neutral but loss averse. In this section, I examine whether stochastic contracts reduce the principal's cost in a general case, i.e., the agent is both risk- and loss-averse.

In particular, I assume that the agent is both risk averse, $u''(\cdot) < 0$, and loss averse, $\lambda \geq 1$. Suppose that there exists a penalty wage $(w = 0)$ at which the agent feels a substantial disutility, i.e., $\lim u(w) = -\infty$ as $w \to 0$. This assumption is standard in the contract theory literature (Mirrlees, 1999), which can be interpreted as, for example, a firing decision. If the agent gets fired and receives a wage close to zero, he feels a substantial disutility. The following proposition characterizes the principal's cost for the general case.

PROPOSITION 4: *Assume $u''(\cdot) < 0$ and $\lambda \geq 1$. Suppose $\lim_{w \to 0} u(w) = -\infty$, then the first-best can be approximated arbitrarily closely, but not attained, by stochastic contracts that provide a bonus almost certainly.*

To understand Proposition 4, recall the intuition from Proposition 2: stochastic contracts reduce the principal's cost by providing the wage certainty to the loss-averse agent, but this certainty comes at the cost of a larger bonus required to induce the high action. Put differently, the dominance of stochastic contracts depends on how effectively the principal can punish the agent if the worst outcome is realized. If the principal can employ a penalty wage to punish the agent effectively—for example, firing decisions in dismissal contracts—she can create a strong marginal work incentive with an arbitrarily small effect on her cost. This, in turn, allows the principal to insure the agent with the wage certainty to the largest possible extent. Yet in the light of Proposition 2 the first best can only be approximated closely, but not attained, because the second-best optimal stochastic contract does not exist. In the limit, as the bonus probability $p_1 \to 1$, the efficiency loss associated with moral hazard and loss aversion becomes negligible.

E. Alternative Notions of Loss Aversion

The notion of loss aversion crucially depends on how the reference point is conceptualized. In my model the reference point has two important features. First, it allows for stochastic reference points; the agent compares a realized outcome with all possible outcomes. This pairwise comparison implies a possibility of "mixed feeling", i.e., the same realized outcome can be perceived as both a gain and a loss at the same time, depending on which possible outcomes the agent expects. Second, the reference point is formed after the decision is made, and hence is influenced by the chosen decision. Thus, the reference point is endogenously determined by recent expectations.

A related notion to the CPE concept is the forward-looking disappointment aversion according to Bell (1985), Loomes and Sugden (1986), or Gul (1991). Under the disappointment aversion model, the reference point is also formed after the decision is made, but the reference point takes the form of certainty equivalent of the prospect, and hence it admits only static reference points. The certainty equivalent of the prospect is a point estimate and does not allow for mixed feelings; the agent feels a gain if the realized outcome is above it, and vice versa. As it turns out, even in this case, stochastic contracts help the principal reduce the implementation cost beyond what is achieved under deterministic contracts. Again, stochastic contracts add noise after the worst outcome to insure the agent that he is more likely to receive the high wage.[7]

[7]De Meza and Webb (2007) examine the concept of Gul (1991), which is closely related to Bell

PROPOSITION 5: *Suppose the agent exhibits disappointment aversion according to Bell (1985),* $u''(\cdot) = 0$, *and* $\lambda - 1 > \frac{1-q_H}{1-q_L}$. *Consider two actions and two signals. Then, there exists a stochastic contract with the wage structure* $w_1 < w_2 = w_3 = w_4$ *that strictly dominates the optimal deterministic contract.*

The forward-looking disappointment according to Bell (1985) implies that the agent first calculates an expected outcome, and then compares the realized outcome with his expectation. Under a deterministic contract, if a bonus is realized, the agent feels elated as the realized outcome is higher than the expected one. While, if a bonus is not realized, the agent instead feels disappointed as the realized outcome is lower than the expected one. By increasing the bonus probability in the stochastic contract, the principal simultaneously increases the probability that the agent feels elated and reduces the probability that he feels disappointed. Because the agent prioritizes minimizing the feeling of disappointment, if he is sufficiently disappointment averse, the principal can capitalize on the stochastic contract to reduce her implementation cost.

An alternative specification of the reference point is that it is given exogenously and does not internalize the effect of the decision, namely the *preferred personal equilibrium* (PPE) notion. In PPE, the agent can choose his optimal action only from the actions he knows he will follow through, whereas in CPE he can commit to the action. The analysis of the optimal contract is very similar and gives rise to the similar result. However, it is known that the distaste for the risk is stronger when the decision is made up front, as in CPE, than when the decision is made later, as in PPE. The principal benefits from stochastic contracts that insure the agent with wage certainty to a lesser extent.

PROPOSITION 6: *Suppose the agent exhibits the PPE loss aversion,* $u''(\cdot) = 0$, $q_H + 2q_L \leq 2$, *and* $\lambda - 1 > \frac{1-q_H}{1-q_L}$. *Consider two actions and two signals. Then, there exists a stochastic contract with the wage structure* $w_1 < w_2 = w_3 = w_4$ *that strictly dominates the optimal deterministic contract.*

The robustness of the dominance of stochastic contracts suggests that noise should be generally added to performance measures in the optimal contract for loss averse agents. Put differently, loss aversion implies a first-order aversion

(1985), and finds that the optimal contracts have intermediate intervals in which wages are insensitive to performance.

to wage uncertainty, and this creates incentives for the principal to insure the agent against this uncertainty. By employing stochastic contracts, the principal manipulates the outcome distribution to her favor and provides the agent a higher wage certainty. When loss aversion plays a role, the principal capitalizes on this reduction in uncertainty and achieves a lower cost.

V. Conclusion

This paper studies the optimal contract design under moral hazard and loss aversion, and finds that the optimal contract adds noise in the event of bad outcomes to insure the loss-averse agent against wage uncertainty. To reach this finding, I modify the standard moral hazard model with two departures: the agent is expectation-based loss averse, and the principal can add noise in the contract to manipulate the outcome distribution in her favor. Importantly, the principal fully controls where to add noise and how to structure noise in the contract, i.e., the structure of stochastic contracts.

There are three key takeaways from this paper. First, the principal is strictly better off with stochastic contracts, as compared to deterministic contracts, in implementing the desired action if the agent is loss averse. This result relates to the literature on behavioral contract theory, which has pointed out that if deterministic contracts face an implementation problem, turning a blind eye (Herweg, Müller and Weinschenk, 2010) or team incentives (Daido and Murooka, 2016) help the principal induce the agent to work. Contributing to this literature strand, I find that even if deterministic contracts do not face the implementation problem, the principal can still reduce her cost by employing stochastic contracts. In fact, if the signals are highly informative about the agent's action, stochastic contracts strictly dominate deterministic contracts for almost any degree of loss aversion. Thus, this finding has an important implication for designing contracts for loss-averse agents: the principal has an incentive to add noise after the bad signal is realized to insure the agent against wage uncertainty.

Second, limited liability mitigates the non-existence problem of the second-best optimal stochastic contract. Instead of the implementation problem, stochastic contracts face a non-existence problem that the optimal contract does not exist, because the principal has an incentive to insure the agent to the largest possible extent. Given a wide range of loss aversion over which stochastic contracts dominates deterministic contracts, the non-existence problem proves to be severe. To solve the non-existence problem, I find that limited liability helps restore the existence of the second-best optimal contract. This finding highlights the importance of limited liability in stochastic contracts to ensure that the second-best optimal contract exists.

Third, in a general case when the agent is both loss and risk averse, the principal can closely approximate the first best scenario by employing a stochastic contract that provides a bonus almost certainly but punishes the agent sternly in the event of the worst outcome—mimicking dismissal contracts. From the standard perspective that risk aversion increases the cost of stochastic contracts, this finding is theoretically interesting: stochastic contracts that insure the agent against wage uncertainty help the principal reduce both risk premium and loss premium that she has to pay. This finding also provides an explanation for why fixed wage contracts with firing threats remain the most commonly used contract in practice.

Given that loss aversion is an important and well-established behavioural trait, this paper helps explain the relevance of stochastic contracts (e.g., dismissal contracts) in the real world. Going forward, it would be interesting to examine the interaction of loss aversion with other behavioural or cognitive biases, such as overconfidence, that may induce the agent to have an incorrect model of the world. The interaction of these biases and their implications on the optimal contract design is an exciting research topic.

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Mathematical Appendix

Proofs of Propositions and Lemmas

PROOF OF LEMMA 1:

Suppose $u''(\cdot) \leq 0$ and $\lambda \geq 1$. For every λ, there exists a stochastic contract such that the action a_H can be implemented.

Without loss of generality, assume $1 > p_1 \geq 1/2$. Consider a contract of the form

$$u_i = \begin{cases} \underline{u} + b & \text{for } i > 1 \\ \underline{u} & \text{for } i = 1 \end{cases}$$

where $b > 0$.

Let f_1^H and f_1^L be the probability of getting a bonus conditional on the agent's high and low action respectively, i.e., $f_1^H = P[i > 1|a_H] = q_H + p_1(1 - q_H)$ and $f_1^L = P[i > 1|a_L] = q_L + p_1(1 - q_L)$. Under this contractual form, (IC) is given by

$$\text{(IC)} \qquad b(f_1^H - f_1^L) - (\lambda - 1)b[f_1^H(1 - f_1^H) - f_1^L(1 - f_1^L)] = c$$

which can be rewritten as

$$\text{(IC')} \qquad b\{(f_1^H - f_1^L)[1 - (\lambda - 1)(1 - f_1^H - f_1^L)]\} = c$$

Under this stochastic contract, $f_1^H = q_H + p_1(1 - q_H)$ and $f_1^L = q_L + p_1(1 - q_L)$. It is straight-forward to see that $f_1^H > f_1^L$ as $q_H > q_L$ and $p_1 < 1$.
Consider

$$\begin{aligned} 1 - f_1^H - f_1^L &= 1 - (q_H + p_1(1 - q_H)) - (q_L + p_1(1 - q_L)) \\ &= 1 - q_H - q_L - p_1(2 - q_H - q_L) \end{aligned}$$

Notice for $p_1 \geq 1/2$, this above term is strictly negative. This implies the term in curly brackets in (IC') is strictly positive for $1 > p_1 \geq 1/2$. Hence, with $c > 0$, b can always be chosen such that (IC) is met.
The binding participation constraint can be written as follows

$$\underline{u} + bf_1^H - (\lambda - 1)bf_1^H(1 - f_1^H) = c$$

(PC) is satisfied whenever $\underline{u}$ is chosen as above.

PROOF OF PROPOSITION 1:

Suppose $u''(\cdot) = 0$ and $\lambda - 1 > \frac{1-q_H}{1-q_L}$. Then, there exists a stochastic contract with the wage structure $w_1 < w_2 = w_3 = w_4$ that strictly dominates the optimal deterministic contract.

Without loss of generality, assume $1 > p_1 \geq 1/2$. Consider a stochastic contract of the form

$$w_i = \begin{cases} \underline{w} + b & \text{for } i > 1 \\ \underline{w} & \text{for } i = 1 \end{cases}$$

where $b > 0$. The non-emptiness of the constraint set follows from Lemma 1.

Let f_1^H and f_1^L be the probability of getting a bonus conditional on the agent's high and low action respectively, i.e., $f_1^H = P[i > 1|a_H] = q_H + p_1(1 - q_H)$ and $f_1^L = P[i > 1|a_L] = q_L + p_1(1 - q_L)$.
Consider any $p_1 \in [\frac{1}{2}, 1)$. The principal's problem becomes

$$\min_{\underline{w},b} \; \underline{w} + f_1^H b$$

subject to

$$\text{(PC)} \qquad \underline{w} + f_1^H b - (\lambda - 1) b f_1^H (1 - f_1^H) = c$$

$$\text{(IC)} \qquad b(f_1^H - f_1^L) - (\lambda - 1) b [f_1^H (1 - f_1^H) - f_1^L (1 - f_1^L)] = c$$

From (IC), the optimal bonus size is given by

$$b = \frac{c}{(f_1^H - f_1^L)[1 - (\lambda - 1)(1 - f_1^H - f_1^L)]}$$

Recall that $f_1^H = q_H + p_1(1 - q_H)$ and $f_1^L = q_L + p_1(1 - q_L)$. Under the stochastic contract of this form, the principal's cost, $C_r = c + (\lambda - 1) f_1^H (1 - f_1^H) b$, is given by

$$C_r = c + \frac{(\lambda - 1)[q_H + p_1(1 - q_H)](1 - q_H)(1 - p_1)c}{(q_H - q_L)(1 - p_1)[1 - (\lambda - 1)(1 - q_H - q_L - p_1(2 - q_H - q_L))]}$$

Suppose that the optimal deterministic contract exists.[8] Then the principal's cost under the optimal deterministic contract (i.e., $p_1 = 0$) is given by

$$C_d = c + \frac{(\lambda - 1)q_H(1 - q_H)c}{(q_H - q_L)[1 - (\lambda - 1)(1 - q_H - q_L)]}$$

The stochastic contract reduces the principal's cost if and only if $C_d \geq C_r$.

$$\Leftrightarrow \frac{q_H}{1 - (\lambda - 1)(1 - q_H - q_L)} \geq \frac{q_H + p_1(1 - q_H)}{1 - (\lambda - 1)(1 - q_H - q_L - p_1(2 - q_H - q_L))}$$

Since the solution exists for both deterministic and stochastic contracts, both denominators are positive. Cross multiply the inequalities.
Notice the term $q_H[1-(\lambda-1)(1-q_H-q_L)]$ is present on both sides. The inequality is reduced to

$$\Leftrightarrow q_H(\lambda - 1)p_1(2 - q_H - q_L) \geq p_1(1 - q_H)[1 - (\lambda - 1)(1 - q_H - q_L)]$$
$$\Leftrightarrow q_H(\lambda - 1)(2 - q_H - q_L) \geq (1 - q_H)[1 - (\lambda - 1)(1 - q_H - q_L)]$$

Removing the term $q_H(\lambda - 1)(1 - q_H - q_L)$ on both sides, I have

$$\Leftrightarrow q_H(\lambda - 1) \geq 1 - (\lambda - 1)(1 - q_H - q_L) - q_H$$
$$\Leftrightarrow 0 \geq 1 - q_H - (\lambda - 1)(1 - q_L)$$
$$\Leftrightarrow \lambda - 1 \geq \frac{1 - q_H}{1 - q_L}$$

Since $\lambda - 1 > \frac{1-q_H}{1-q_L}$, $C_r < C_d$. This completes the proof.

PROOF OF LEMMA 2:

Suppose $u''(\cdot) = 0$ and $\lambda \geq 1$. Then, any stochastic contract with the wage structure $w_1 = w_2 = w_3 < w_4$ is weakly dominated by the optimal deterministic contract.

[8]If the principal's constraint set is empty under deterministic contracts, then it is assumed that the principal's cost becomes prohibitively high. It follows directly that stochastic contracts, which enable the principal to implement the desired action, strictly dominate deterministic contracts.

Consider a stochastic contract of the form

$$w_i = \begin{cases} \underline{w} + b & \text{for } i = 4 \\ \underline{w} & \text{for } i < 4 \end{cases}$$

where $b > 0$. Let f_4^H and f_4^L be the probability of getting a bonus conditional on the agent's high and low action respectively, i.e., $f_4^H = P[i = 4|a_H] = p_2 q_H$ and $f_4^L = P[i = 4|a_L] = p_2 q_L$.
The principal's problem becomes

$$\min_{\underline{w}, b} \ \underline{w} + f_4^H b$$

subject to

$$\text{(PC)} \qquad \underline{w} + f_4^H b - (\lambda - 1) b f_4^H (1 - f_4^H) = c$$

$$\text{(IC)} \qquad b(f_4^H - f_4^L) - (\lambda - 1) b [f_4^H (1 - f_4^H) - f_4^L (1 - f_4^L)] = c$$

Suppose that the above constraint set is non-empty, the optimal bonus size is given by

$$b = \frac{c}{(f_4^H - f_4^L)[1 - (\lambda - 1)(1 - f_4^H - f_4^L)]}$$

Recall that $f_4^H = p_2 q_H$ and $f_4^L = p_2 q_L$. Under the stochastic contract of this form, the principal's cost, $C = c + (\lambda - 1) f_4^H (1 - f_4^H) b$, is given by

$$C = c + \frac{(\lambda - 1) p_2 q_H (1 - p_2 q_H) c}{p_2 (q_H - q_L)[1 - (\lambda - 1)(1 - p_2 q_H - p_2 q_L)]}$$

Note that if the constraint set for the above stochastic contract is non-empty, then the constraint set for the deterministic contract is also non-empty. Thus, the principal's cost under the optimal deterministic contract (i.e., $p_2 = 1$) is given by

$$C_d = c + \frac{(\lambda - 1) q_H (1 - q_H) c}{(q_H - q_L)[1 - (\lambda - 1)(1 - q_H - q_L)]}$$

It is straight-forward to see that $C \geq C_d$ for any $1 \geq p_2 > 0$.

PROOF OF PROPOSITION 2:

Suppose $u''(\cdot) = 0$ *and* $\lambda - 1 > \frac{1-q_H}{1-q_L}$. *Then, the second-best optimal stochastic contract does not exist.*

Suppose, by contradiction, the solution for the principal's problem exists.

I decompose the principal's problem into two subproblems. First, for a given stochastic structure (p_1, p_2), I derive the optimal wage payments that implement a_H. Second, I choose the stochastic structure to achieve the lowest cost.

Step 1: Given any contract $(\hat{w}_i)_{i=1}^4$ the principal offers, I can relabel the states such that this contract is equivalent to a contract $(w_i)_{i=1}^4$ of an (weakly) increasing wage profile with $w_{i-1} \leq w_i$ for all $i \in \{2,3,4\}$. Let $b_i = w_i - w_{i-1} \geq 0$ for all $i \in \{2,3,4\}$. Let f_i^H and f_i^L be the probability that state i is realized conditional on a_H and a_L respectively.

The principal's problem can be rewritten as

$$\min_{b_2,\ldots,b_4} (\lambda - 1) \sum_{i=2}^{4} b_i \sum_{\tau=i}^{4} f_\tau^H \sum_{t=1}^{i-1} f_t^H$$

subject to

$$\text{(IC)} \qquad \sum_{i=2}^{4} b_i \beta_i = c$$

$$b_i \geq 0 \;\; \forall i \in \{2,3,4\}$$

where

$$\beta_i := \left(\sum_{\tau=i}^{4} (f_\tau^H - f_\tau^L) \right) - (\lambda - 1) \left(\sum_{\tau=i}^{4} f_\tau^H \sum_{t=1}^{i-1} f_t^H - \sum_{\tau=i}^{4} f_\tau^L \sum_{t=1}^{i-1} f_t^L \right)$$

The principal's problem is a linear programming problem. It is well known that if a linear programming has a solution, this (unique) solution is an extreme point of the constraint set. All extreme points of the constraint set are characterised by the following property: $b_i > 0$ for exactly one state $i \in \{2,3,4\}$ and $b_t = 0$ for all $t \neq i, t \in \{2,3,4\}$.

It remains to determine for which state $i \in \{2,3,4\}$ the bonus is set strictly positive. From Lemma 2 and Proposition 1 if $\lambda - 1 > \frac{1-q_H}{1-q_L}$, the second-best optimal stochastic contract has the optimal wage structure $w_1 < w_2 = w_3 = w_4$.

Step 2: I now consider the optimal stochastic structure p_1 to achieve the lowest cost. Recall that under the stochastic contract with the wage structure $w_1 < w_2 = w_3 = w_4$, the principal's cost is given by

$$C_r = c + \frac{(\lambda-1)[q_H + p_1(1-q_H)][1-q_H]c}{(q_H - q_L)[1-(\lambda-1)(1-q_H-q_L-p_1(2-q_H-q_L))]}$$

Differentiation of C_r with respect to p_1 yields

$$\frac{\partial C_r}{\partial p_1} = \frac{c(\lambda-1)(1-q_H)[2-q_H-q_L-\lambda(1-q_L)]}{(q_H-q_L)[1-(\lambda-1)(1-q_H-q_L-p_1(2-q_H-q_L))]^2}$$

Obviously, $\partial C_r/\partial p_1 < 0$ for all p_1 as $\lambda > \frac{2-q_H-q_L}{1-q_L}$. The principal can always achieve a lower cost by increasing p_1 close to 1, i.e., the probability of bonus is almost 1. However, p_1 can not reach 1 due to the incentive constraint. Hence, the second-best optimal stochastic contract does not exist.

PROOF OF PROPOSITION 3:

Suppose (A1) holds, $u''(\cdot) = 0$, and $w \geq 0$. Then, the second-best optimal stochastic contract exists. The optimal stochastic contract pays $b^*(p_1^*)$ with probability one when the good signal is realized and with probability p_1^* when the bad signal is realized. The optimal p_1^* is given by

$$p_1^* = \frac{1}{1-q_H}\left(\sqrt{1 - \frac{\lambda}{\lambda-1}\cdot\frac{1-q_H}{2-q_H-q_L}} - q_H\right)$$

Consider a stochastic contract of the form

$$w_i = \begin{cases} \underline{w} + b & \text{for } i > 1 \\ \underline{w} & \text{for } i = 1 \end{cases}$$

where $b > 0$. Let f_1^H and f_1^L be the probability of getting a bonus conditional on the agent's high and low action respectively, i.e., $f_1^H = P[i > 1|a_H] = q_H + p_1(1-q_H)$ and $f_1^L = P[i > 1|a_L] = q_L + p_1(1-q_L)$.

The principal's problem becomes

$$\min_{\underline{w},b} \ \underline{w} + f_1^H b$$

subject to

$$\text{(PC)} \qquad \underline{w} + f_1^H b - (\lambda-1) b f_1^H (1-f_1^H) \geq c$$

$$\text{(IC)} \qquad b(f_1^H - f_1^L) - (\lambda-1) b [f_1^H(1-f_1^H) - f_1^L(1-f_1^L)] \geq c$$

$$\text{(LL)} \qquad \underline{w} \geq 0$$

Notice first that the (LL) constraint is binding. Suppose, by contradiction, $\underline{w} > 0$ is the optimal wage scheme. Reducing $\underline{w}$ by a small amount ϵ, the principal decreases the expected payment without changing (IC) or violating (LL) constraint. Thus, $\underline{w}^* = 0$.

Notice also that the (IC) constraint is binding. Suppose, by contradiction, (IC) is slack. Reducing b by a small amount ϵ, the principal decreases the expected payment without changing (LL) or violating (IC) constraint.

Assume that the optimal deterministic contract exists, then the constraint set for the above stochastic contract is non-empty.[9] Thus, at optimum, the bonus is given by

$$b^* = \frac{c}{(f_1^H - f_1^L)[1-(\lambda-1)(1-f_1^H-f_1^L)]}$$

Under the stochastic contract of this form, the principal's cost, $C_r = \underline{w}^* + f_1^H b^* = f_1^H b^*$, is given by

$$C_r = \frac{(q_H + p_1(1-q_H))c}{(q_H-q_L)(1-p_1)[1-(\lambda-1)(1-q_H-q_L-p_1(2-q_H-q_L))]}$$

Analogously, the principal's cost under the optimal deterministic contract with limited liability is given by

$$C_d = \frac{q_H c}{(q_H-q_L)[1-(\lambda-1)(1-q_H-q_L)]}$$

Note that if $p_1 = 0$, then $C_r = C_d$

[9] If the deterministic contract has no solution, the dominance of the stochastic contract is trivial. The reason is that the principal can always implement a_H under the stochastic contract by setting $p_1 \in [1/2, 1)$ (Lemma 1). On the other hand, if the optimal deterministic contract exists, i.e., $(\lambda-1)(1-q_H-q_L) < 1$, it follows that $(\lambda-1)(1-q_H-q_L-p_1(2-q_H-q_L)) < 1$. Thus, the constraint set under the stochastic contract is non-empty for all p_1.

Differentiating C_r with respect to p_1 yields

$$\frac{\partial C_r}{\partial p_1} = \frac{c}{q_H - q_L} \frac{1-(\lambda-1)(1-q_H-q_L+q_H(2-q_H-q_L))+p_1 2q_H(\lambda-1)(2-q_H-q_L)+p_1^2(1-q_H)(\lambda-1)(2-q_H-q_L)}{(1-p_1)^2[1-(\lambda-1)(1-q_H-q_L-p_1(2-q_H-q_L))]^2}$$

The stochastic contract reduces the principal's cost, i.e., $C_d > C_r$ if

$$\text{(A1)} \qquad \left.\frac{\partial C_r}{\partial p_1}\right|_{p_1=0} < 0 \Leftrightarrow (\lambda-1)(1-q_H-q_L+q_H(2-q_H-q_L)) > 1$$

Provided that (A1) holds, there exists a stochastic contract of the wage structure $w_1 < w_2 = w_3 = w_4$ that strictly dominates the optimal deterministic contract.[10] Solving for the first order condition, I obtain the optimal p_1^*

$$p_1^* = \frac{1}{1-q_H}\left(\sqrt{1 - \frac{\lambda}{\lambda-1} \cdot \frac{1-q_H}{2-q_H-q_L}} - q_H\right)$$

The second-best optimal stochastic contract is characterized by $\underline{w}^* = 0$, $b^*(p_1^*)$, and p_1^*. This completes the proof.

PROOF OF PROPOSITION 4:

Assume $u''(\cdot) < 0$ and $\lambda > 1$. Suppose $\lim_{w\to 0} u(w) = -\infty$, then the first-best can be approximated arbitrarily closely, but not attained, by stochastic contracts that provide a bonus almost certainly.

Consider the stochastic contract of the form

$$w_i = \begin{cases} w_H & \text{for } i > 1 \\ \delta w_H & \text{for } i = 1 \end{cases}$$

where $\delta \in (0,1)$ remains to be chosen.

I define w_H such that $u(w_H) = u(w_{FB}) + \epsilon$, for $\epsilon > 0$. Thus, $u(w_H) = c + \epsilon$, equivalently $w_H = h(c+\epsilon)$. Let $f_1^H = P[i > 1|a_H] = q_H + p_1(1-q_H)$ and $f_1^L = P[i > 1|a_L] = q_L + p_1(1-q_L)$.

[10] Analogous to the proof of Lemma 2, it can be shown that under limited liability, adding noise to the good outcome is weakly dominated by the optimal deterministic contract. The cost of a stochastic contract with the wage structure $w_1 = w_2 = w_3 < w_4$ under limited liability is given by $C = \frac{q_H c}{(q_H-q_L)(1-(\lambda-1)(1-p_2 q_H - p_2 q_L))}$, which is weakly larger than C_d – the cost under the optimal deterministic contract – for all $p_2 \in [0,1]$. Thus, the second-best optimal stochastic contract has the wage structure of $w_1 < w_2 = w_3 = w_4$.

I will show that this above stochastic contract satisfies both the participation and incentive constraint. First, I check that the incentive constraint (IC) holds under w_i. To see this, note that for $\delta = 1$, this incentive scheme becomes a fixed wage. Thus, for $\delta = 1$, the left-hand side of (IC) equals to zero, and the incentive constraint (IC) in consequence is not satisfied. As $\delta \to 0$, by assumption $\lim_{w \to 0} u(w) = -\infty$, the left-hand side of (IC) tending to infinity. Since the left-hand side of (IC) is continuous in δ under the wage scheme w_i, by intermediate-value theorem, there exists a (unique) $\hat{\delta} \in (0, 1)$ such that (IC) is satisfied with equality.

Next, consider (PC). The left-hand side of (PC) under the wage scheme w_i with $\delta = \hat{\delta}$ amounts to

$$\text{(PC)} \qquad u(w_H)[f_1^H - (\lambda - 1)\rho_1^H] + u(\hat{\delta} w_H)[1 - f_1^H + (\lambda - 1)\rho_1^H] \geq c$$

where $\rho_1^H = f_1^H(1 - f_1^H)$.

Note that there exists a stochastic structure p_1 such that $(f_1^H - (\lambda - 1)\rho_1^H) > 0$. I restrict the attention to this set of stochastic structure. Since $u(\cdot)$ is strictly increasing, the left-hand side of (PC) is strictly increasing in ϵ. Note that when $\epsilon = 0$, (PC) is not satisfied. Since $u(\cdot)$ is an unbounded and continuous function, there exists $\hat{\epsilon} > 0$ such that (PC) is satisfied with equality.

Now consider the principal's cost of providing incentive. The principal's second-best cost is given as follows

$$C_{SB}(a_H) = f_1^H h(c + \hat{\epsilon}) + (1 - f_1^H)(\hat{\delta} h(c + \hat{\epsilon}))$$

Remember that $f_1^H = q_H + p_1(1 - q_H)$ and $\rho_1^H = f_1^H(1 - f_1^H)$. As $p_1 \to 1$, we have $f_1^H \to 1$. From the binding (PC), this implies that $\hat{\epsilon} \to 0$. Hence, $\lim_{p_1 \to 1} C_{SB}(a_H) = h(c) = C_{FB}(a_H)$.

PROOF OF PROPOSITION 5:

Suppose the agent exhibits disappointment aversion according to Bell (1985), $u''(\cdot) = 0$, *and* $\lambda - 1 > \frac{1 - q_H}{1 - q_L}$. *Consider two actions and two signals. Then, there exists a stochastic contract with the wage structure* $w_1 < w_2 = w_3 = w_4$ *that strictly dominates the optimal deterministic contract.*

The proof of Proposition 4 closely follows the proof of Proposition 1. I first show that the principal's problem remains the same regardless of whether the agent exhibits disappointment aversion (Bell, 1985) or loss aversion (Kőszegi and

Rabin, 2006, 2007).

Consider a stochastic contract of the form

$$w_i = \begin{cases} \underline{w} + b & \text{for } i > 1 \\ \underline{w} & \text{for } i = 1 \end{cases}$$

where $b > 0$. Let f_1^H and f_1^L be the probability of getting a bonus conditional on the agent's high and low action respectively, i.e., $f_1^H = P[i > 1 | a_H] = q_H + p_1(1 - q_H)$ and $f_1^L = P[i > 1 | a_L] = q_L + p_1(1 - q_L)$.

Under the disappointment aversion, the agent compares a realized outcome to the certainty equivalence of the prospect, which is given by $\text{CE}_r(a_H) = \underline{w} + f_1^H b$. With probability f_1^H a bonus is realized, the agent feels elated by receiving $(1 - f_1^H)b$ more than the certainty equivalence. With probability $(1 - f_1^H)$ a bonus is not realized, the agent feels disappointed by receiving $f_1^H b$ less than the certainty equivalence. The agent's utility from choosing a_H is given by

$$\underline{w} + f_1^H b + f_1^H (1 - f_1^H) b - \lambda (1 - f_1^H) f_1^H b = \underline{w} + f_1^H b - (\lambda - 1) f_1^H (1 - f_1^H) b$$

The (IC) constraint is given by

$$b(f_1^H - f_1^L) - (\lambda - 1) b [f_1^H (1 - f_1^H) - f_1^L (1 - f_1^L)] = c$$

Notice that the above (PC) and (IC) constraints coincide with the principal's constraints under CPE loss aversion.

Assume w.l.o.g. $1 > p_1 \geq 1/2$, the non-emptiness of the constraint set follows from Lemma 1, and the dominance of the stochastic contract analogously follows from Proposition 1.

PROOF OF PROPOSITION 6:

Suppose the agent exhibits the PPE loss aversion, $u''(\cdot) = 0$, $q_H + 2q_L \leq 2$ *and* $\lambda - 1 > \frac{1 - q_H}{1 - q_L}$. *Consider two actions and two signals. Then, there exists a stochastic contract with the wage structure* $w_1 < w_2 = w_3 = w_4$ *that strictly dominates the optimal deterministic contract.*

Consider a stochastic contract of the form

$$w_i = \begin{cases} \underline{w} + b & \text{for } i > 1 \\ \underline{w} & \text{for } i = 1 \end{cases}$$

where $b > 0$. Let f_1^H and f_1^L be the probability of getting a bonus conditional on the agent's high and low action respectively, i.e., $f_1^H = P[i > 1|a_H] = q_H + p_1(1 - q_H)$ and $f_1^L = P[i > 1|a_L] = q_L + p_1(1 - q_L)$.

Under PPE loss aversion, the agent identifies (i) the set of personal equilibrium (PE) that includes all actions the agent can follow through, and (ii) the preferred action among the set of personal equilibrium (PPE).

$$a \in \text{PE} \Leftrightarrow EU(a|a) \geq EU(a'|a)\ \forall a' \neq a$$
$$a \in \text{PPE} \Leftrightarrow EU(a|a) \geq EU(a'|a')\ \forall a' \in \text{PE}$$

For $a_H \in$ PE, $EU(a_H|a_H) \geq EU(a_L|a_H)$, the latter refers to the expected utility when the agent expects to choose a_H but actually chooses a_L, is given by

$$\underline{w} + f_1^H b - (\lambda - 1) f_1^H (1 - f_1^H) b - c \geq \underline{w} + f_1^L b + f_1^L (1 - f_1^H) b - \lambda (1 - f_1^L) f_1^H b + c$$

This is equivalent to

$$(a_H\text{-PE}) \qquad b \geq \frac{2c}{(f_1^H - f_1^L)[2 + f_1^H(\lambda - 1)]} := \underline{b}$$

Analogously, for $a_L \in$ PE

$$(a_L\text{-PE}) \qquad b \leq \frac{(\lambda + 1)c}{(f_1^H - f_1^L)[2 + f_1^L(\lambda - 1)]} := \bar{b}$$

Note that $\bar{b} > \underline{b}$ for all $\lambda \geq 1$.

The principal's problem becomes

$$\min_{\underline{w}, b} \ \underline{w} + f_1^H b$$

subject to

$$(\text{PC}) \qquad \underline{w} + f_1^H b - (\lambda - 1) b f_1^H (1 - f_1^H) = c$$

$$(a_H\text{-PPE}) \qquad b \geq \frac{c}{(f_1^H - f_1^L)[1 - (\lambda - 1)(1 - f_1^H - f_1^L)]} := \tilde{b}$$

$$(a_H\text{-PE}) \qquad b \geq \underline{b}$$

Assume that the optimal deterministic contract exists, it follows that the principal's constraint set for the stochastic contract is non-empty. There exists

$p_1 \in [0, 1)$ such that $\tilde{b} \geq \underline{b}$. Consider a relaxed problem without (a_H-PE) constraint. The relaxed problem coincides with the principal's problem of CPE loss aversion and, from Proposition 1, the cost is given by

$$C_r = c + \frac{(\lambda - 1)[q_H + p_1(1 - q_H)][1 - q_H]c}{(q_H - q_L)[1 - (\lambda - 1)(1 - q_H - q_L - p_1(2 - q_H - q_L))]}$$

Following the above analysis analogously, if $q_H + 2q_L \leq 2$, then the principal's cost under the optimal deterministic contract is given by

$$C_d = c + \frac{(\lambda - 1)q_H(1 - q_H)c}{(q_H - q_L)[1 - (\lambda - 1)(1 - q_H - q_L)]}$$

Since $\lambda - 1 > \frac{1 - q_H}{1 - q_L}$, $C_r < C_d$. This completes the proof.