Cursed Consumers and the Effectiveness of Consumer Protection Policies

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Abstract

We model firms' quality disclosure and pricing in the presence of cursed consumers, who fail to be sufficiently skeptical about undisclosed quality. We show that cursed consumers are exploited in duopoly markets if firms are vertically differentiated, if there are few cursed consumers, and if average product quality is high. Three common consumer protection policies that work under monopoly, i.e. mandatory disclosure, third party disclosure and consumer education, may all increase exploitation and decrease welfare. Even where these policies improve overall welfare, they often lead to a reduction in consumer surplus. We show that our conclusions hold in extensions with endogenous quality choice and horizontal differentiation.

Keywords: naive, cursed, disclosure, consumer protection, labeling, competition

JEL classification: C72, D03, D82, D83

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1 Introduction

Firms generally have better information about the quality of their products and services than consumers. A food producer knows the nutritional content of its products, a hospital collects data on the effectiveness of its care, and a financial advisor is aware of her conflicts of interest. Under the right circumstances, firms' voluntary disclosure of verifiable information has the potential to eradicate any information asymmetry between firms and consumers (Grossman and Hart, 1980; Grossman, 1981; Milgrom, 1981): since high-quality firms will disclose in order to separate themselves from low-quality firms, consumers can infer that undisclosed quality is likely to be low. However, reliance on voluntary disclosure requires a high degree of consumer sophistication.

If consumers are cursed (Eyster and Rabin, 2005), then they fail to condition their quality perception on firms' strategies and remain too optimistic about quality in the face of non-disclosure. Cursedness can thus explain the failure of information transmission observed in many markets (see Fung, Graham and Weil 2007 and Dranove and Jin 2010 for surveys) and provides an apparent rationale for protecting consumers by means of mandatory disclosure laws (e.g. the US Nutrition Labeling and Education Act), third party disclosure (e.g. the Hospital Compare webpage) or consumer education (e.g. the EU financial literacy initiative).[1]

This paper analyzes the exploitation of cursed consumers and the effectiveness of common policy measures designed to protect them. A growing empirical literature documents firms' strategic non-disclosure of private information and consumers' misinference when they encounter non-disclosure. Our main contribution lies in demonstrating that neither observation necessarily implies a useful role for common consumer protection policies and in deriving the conditions under which such policies backfire.

We first model the interaction between privately informed firms and cursed consumers in the simplest possible setting and then enrich the model along various dimensions, both to investigate the scope and robustness of our results and to address important additional

[1]The EU financial literacy initiative educates consumers about the information they are entitled to demand from their financial service providers. For an example, see the EU regulation on key information documents for packaged retail and insurance-based investment products at http://data.europa.eu/eli/reg/2014/1286/oj.

questions. Our main model features two firms with identical marginal costs, each selling a single product with exogenous quality that may either be high or low. Firms simultaneously choose prices and whether to disclose quality. While quality cannot be misreported (e.g. for fear of litigation), it can be concealed. Consumers consist of both sophisticated and (fully) cursed types. Cursed consumers do not understand that a firm's disclosure and pricing strategy depends on the quality of its product. Thus, while they rationally take disclosed quality at face value, they believe that a firm that does not disclose has average quality.[2] Consumers are homogeneous in their tastes – though not necessarily in their subjective valuations – and efficiency demands that they all purchase the higher quality good. Exploitation takes the following form: a cursed consumer buys a low-quality product at a price that is higher than her objective valuation.

Exploitation can only occur when firms are vertically differentiated, i.e. when realized product qualities are heterogeneous. When qualities are the same, firms price at marginal cost irrespective of their quality level and the sophistication of consumers. In the case of heterogeneous qualities, the high-quality firm always discloses and the low-quality firm never discloses. But cursed consumers are only exploited if the high-quality firm is unwilling to attract them. This happens when cursed consumers are few and when they are optimistic about the undisclosed (low) quality, i.e. because average quality is high. If there are only a few cursed consumers, then the high-quality firm has little incentive to attract them by lowering prices on inframarginal sophisticated consumers. Similarly, if cursed consumers are optimistic about the undisclosed quality, they require too low of a price to buy high quality.

[2] Our analysis builds on the premise that some consumers may hold overoptimistic beliefs upon observing non-disclosure. Other forms of naivete, like analogy-based reasoning (Jehiel, 2005; Jehiel and Koessler, 2008), can also deliver this assumption. In the formalization of cursed equilibrium (Eyster and Rabin, 2005), each player is equally, but possibly only partially, cursed. While our framework can accommodate partially sophisticated consumers (see section 4.3.3), it is key that strategic sophistication is heterogeneous. The specific bimodal distribution of cursedness we consider is consistent with experimental evidence from disclosure games (Jin, Luca and Martin, 2021; Deversi, Ispano and Schwardmann, 2019), selection contexts (Enke, 2020), and second-price auctions (Turocy and Cason, 2015). Since these disclosure experiments feature no competition among senders and no pricing decisions, it is possible that a firm's pricing and/or its rival's behavior may have an impact on consumer sophistication. For example, even an otherwise fully cursed consumer may revise downward her perception of a silent firm's quality when its rival discloses high quality. All of our conclusions still apply provided that this downward revision is only partial.

Common remedies against exploitation aim at decreasing the proportion of consumers with a mistaken perception of product quality. Consumer education teaches some cursed consumers how to interpret non-disclosure; mandatory disclosure forces firms to make it easier for consumers to spot quality-relevant information on product labels and in contracts;[3] and third party disclosure (e.g. by an online platform) allows those consumers who are aware of the third party (e.g. the more tech-savy) to find out the exact quality of products. If policy measures are imperfect, then a small to medium-sized group of cursed consumers remains. It is precisely then that cursed consumers are exploited and the market equilibrium is inefficient.

Perfect policy measures eradicate all quality misperceptions and lead to efficiency. However, they may still erode consumer surplus through their equilibrium effect on prices. Because cursed consumers maintain too favorable an expectation of a silent (low-quality) firm's product, they reduce the price a high-quality firm charges. In this way their presence generates a positive externality for both sophisticated and other cursed consumers.

We also explore policies that leave the fraction of cursed consumers unaffected but reduce the size of the bias in their beliefs. We find that such policies can also have counterproductive effects on consumer welfare. However, they always weakly increase total welfare because consumers whose overestimation of the attractiveness of the low-quality product is less severe are also less likely to purchase it.

Market structure is an important determinant of the effect of consumer protection policy. In particular, the policy implications of the two-firm model differ from those we arrive at in monopoly, where consumer protection policies are always weakly beneficial for consumer surplus and welfare.

We analyze two extensions of our main model. First, we study endogenous quality choice to better understand the effect of product market interactions and policies on firms' incentive to innovate and invest in quality. Exploitation still occurs if the fraction of cursed consumers is too small. Moreover, by rewarding a low-quality firm, exploitation sometimes hampers

[3]Mandatory disclosure is imperfect if firms are forced to disclose information but fail to do so in a salient manner, leaving some consumers with mistaken beliefs. See Stango and Zinman (2011) for an example of non-salient disclosure and exploitation in consumer finance.

firms' incentives to invest in quality. As a result, the inefficiency associated with imperfect policy measures may be exacerbated. Conversely, perfect consumer protection always enhances welfare by simultaneously ensuring efficient purchasing decisions and stimulating firms' investment.

Second, we consider markets in which firms are not only vertically but also horizontally differentiated. Now, some consumers should rationally buy a low-quality product, e.g. because they are located near the firm selling it. We find that cursed consumers still exert a positive externality on both cursed and sophisticated consumers. Moreover, consumer protection may still backfire.

The next section discusses the related literature. We set up the main model in section 3 and analyze it in section 4. Section 5 features extensions to the model. In the conclusion, we cover general policy lessons, testable implications of our model, and avenues for future research. All proofs are in the appendix.

2 Related literature

Our paper is motivated by an empirical literature on quality disclosure (see Dranove and Jin 2010 and Fung, Graham and Weil 2007) that rarely finds the complete unraveling or full disclosure predicted by seminal theoretical work.[4] While incomplete disclosure can be explained by high disclosure costs, information being unavailable, or more complicated strategic considerations[5] consumer naivete or cursedness is likely to be a key driver. First, naivete on behalf of the uninformed party drives non-disclosure in experimental disclosure games, which can rule out rational explanations for non-disclosure.[6] Second, non-disclosure occurs

[4]When there are many quality levels and disclosure is free, all firms save for the firm with the lowest quality, have a strict incentive to disclose (Grossman and Hart 1980; Grossman 1981; Milgrom 1981). The firm with the highest quality discloses to separate from the pool of lower quality firms. This in turn provides the firm with the second highest quality with an incentive to disclose, etc.

[5]See for instance Matthews and Postlewaite (1985), Anderson and Renault (2006), Board (2009), Koessler and Renault (2012) and Janssen and Roy (2015).

[6]For evidence from the lab see Forsythe, Isaac and Palfrey (1989), King and Wallin (1991), Forsythe, Lundholm and Rietz (1999), Jin, Luca and Martin (2021), Hagenbach and Perez-Richet (2018) and Deversi, Ispano and Schwardmann (2019). Also see Wenner (2019) for evidence of cursedness in an experimental market game

in field settings with negligible disclosure costs. For example, Mathios (2000) finds that the producers of salad dressings do not disclose fat content on labels if the product in question is sufficiently fat. Third, consumers' difficulty with interpreting non-disclosure has also been documented in the field. Evidence in Brown, Camerer and Lovallo (2012, 2013) suggests that movie goers are systematically fooled into viewing bad movies that avoid certification from reviewers by being cold-opened.

Imperfect skepticism on behalf of receivers in disclosure games has received little attention in the theoretical literature since it was first studied by Milgrom and Roberts (1986).[7] In their main model, senders make no pricing decisions, each sender can disclose all the relevant information and all receivers are equally naive. As a result, competition neutralizes the effects of deception and no exploitation occurs in equilibrium. Going beyond this, we show that firms' pricing decisions and heterogeneity in consumers' sophistication are key to understanding exploitation and the effects of policy.

Our paper contributes to a behavioral industrial organization literature that studies the consequences of consumer naivete and policies aimed at protecting them (see Spiegler 2015 and Heidhues and Kőszegi 2018). We deviate from previous work by focusing on (microfounded) quality misperceptions that are driven by cursedness, a well-documented bias in strategic reasoning. Our asymmetric information game, in which firms can reveal their private quality to consumers before a once-off purchase, naturally captures different markets than papers following Gabaix and Laibson (2006) that focus on settings in which firms compete on up-front prices and may shroud add-on costs. Moreover, several of our conclusions differ from those of shrouded attribute models (e.g. Gabaix and Laibson 2006 and Heidhues, Kőszegi and Murooka 2017), which predict that deceptive equilibria are more likely when there are fewer sophisticated consumers and that mandatory disclosure of add-on costs generally makes con-

that does not feature disclosure.

[7]Exceptions include Hagenbach and Koessler (2017), who extend the model with a single sender in Milgrom and Roberts (1986) to the case in which the sender is sometimes uninformed and study the effect of the language at his disposal. Also, Fishman and Hagerty (2003) consider a monopolist facing consumers that are fully sophisticated but do not understand the content of disclosure. Hoffmann, Inderst and Ottaviani (2020) study a setting in which firms are constrained in the amount of information they can disclose and some consumers do not take into account firms' selective disclosure strategy. Their results generally support the idea that competition can limit deception.

sumers better off.[8]

Once firms' disclosure has occurred, the fact that some consumers fail to fully recognize a low-quality firm connects our model to the literature on hidden attributes or inferior products. In particular, in Armstrong and Chen (2009) a fraction of consumers cannot observe firms' quality choices and naively believe that all firms produce high quality. Since a high-quality firm cannot prove its superiority, equilibrium purchases always entail some inefficiency. Then, transparency sometimes harm consumers, as in our model, but always improves welfare, since fewer naive consumers imply lower incentives for firms to 'cheat' with low quality. Gamp and Krähmer (2018) obtain similar policy conclusions in a shrouded attribute model that also features search frictions and hence additional search cost externalities between naives and sophisticates. As we show in the extension with endogenous quality, in our setting firms' investment need not increase with transparency. Moreover, such an increase is not automatically desirable since, as cursed consumers correctly estimate a firm's average quality, their beliefs about a silent, low-quality firm, also become more biased.

Some papers have investigated the impact of cursed inference in financial markets (Vayanos, Eyster and Rabin 2018 and Kondor and Köszegi 2017) and the macro economy (Eyster, Madarász and Michaillat, 2021). In particular, Kondor and Köszegi (2017) study security design by better-informed issuers facing cursed investors. Providing cursed investors with information increases their confidence and the scope for issuers to profitably exploit their belief disagreement with investors. In our model, providing information backfires because of its effect on firms' pricing strategies. In Spiegler (2006) consumers mistakenly attribute higher quality to some sellers based on a coincidental correlation between their sampling of a product and their subsequent well-being. The resulting exploitation is not abated by any policy that does not correct the mistake in consumer's reasoning. In contrast, even correcting the mistaken inference for (some) consumers may backfire in our setting.

Finally, the mechanism through which consumer protection policy may backfire in our

[8]Nonetheless, Kosfeld and Schüwer (2017) show that if firms in a shrouded attribute model can price discriminate between different levels of sophistication, then educating naive consumers may increase exploitation, as it does in our model.

framework has some parallels with the search literature that demonstrates how the presence of informed consumers may harm uninformed ones by increasing firms' market power (Anderson and Renault, 2000; Armstrong, 2015). Within the behavioral industrial organization literature, related versions of this softening of competition effect arise in Johnen (2020) and Herweg and Rosato (2020). While these two papers consider rather different settings, they, like us, feature non-monotonicities of consumer and total welfare in the proportion of naive consumers.

3 Setup

Two firms produce substitute goods and compete for a mass one of consumers with unit demand. Consumers have homogeneous preferences and derive utility $q-p$ from purchasing a good, where p denotes the good's price and q its quality. Each firm's quality is independently drawn from a commonly known binary distribution. A firm's quality is high, i.e. equal to q_h, with probability $\theta \in (0,1)$ and low, i.e. equal to q_ℓ, with complementary probability. We assume that the low-quality product is socially useless, i.e. $q_\ell = 0$,[9] and normalize q_h to 1, so that a firm's average quality is equal to θ.

We will refer to a generic firm as i and to its competitor as j. Qualities q_i and q_j are observed by both firms and unobserved by consumers.[10] Each firm can credibly reveal the quality of its own product to consumers at no cost ($m_i = q_i$) or remain silent ($m_i = \emptyset$). A firm cannot reveal the quality of its rival. Marginal costs of production are normalized to zero.

[9] We relax this assumption in section 5.2, where we consider a setting that features vertical as well as horizontal differentiation. Also, as long as a firm's average quality is positive, our equilibrium analysis equally applies to settings in which the low-quality product is socially harmful. Welfare should then include an additional potential source of inefficiency, i.e. that consumers purchase the harmful product rather than no product at all when high quality is not available.

[10] Within the industrial organization literature on information disclosure in duopoly, this information structure appears in Board (2009), Anderson and Renault (2009) and Janssen and Teteryatnikova (2016). In particular, Board (2009) and Janssen and Teteryatnikova (2016) consider costless disclosure of vertical quality and pricing as in our framework. Other than for featuring rational consumers only, these two works differ in the timing, i.e. pricing following disclosure, and/or in the form of consumers' preferences, i.e. respectively a random taste for quality and horizontal quality differentiation. Instead, Cheong and Kim (2004), Levin, Peck and Ye (2009), Hotz and Xiao (2013) and Janssen and Roy (2015) consider frameworks in which each firm ignores the quality of its rival/s.

A fraction $\chi \in (0,1)$ of consumers is fully cursed (Eyster and Rabin, 2005) and the remaining fraction is rational. Because fully cursed consumers draw no inference about a firm's quality from its failure to disclose, nor from its pricing, their perception of a silent firm's quality is equal to their prior, i.e. average quality θ.

We consider the following timing:

- **t=0**: Firms observe q_i and q_j,
- **t=1**: Each firm simultaneously decides whether or not to disclose quality and posts a price $p_i \geq 0$, [11]
- **t=2**: Consumers observe firms' disclosure and pricing decisions and choose a product or an outside option of zero.

Our solution concept is the Perfect Bayesian Equilibrium with two natural adaptations to our setting. First, the beliefs of cursed consumers about undisclosed quality, which are exogenously specified as above, need not be correct. Second, if a firm discloses, then the beliefs of both rational and cursed consumers about its quality are equal to the disclosed quality. For simplicity and ease of exposition, we only consider deterministic disclosure decisions. Moreover, we restrict our attention to equilibria in which consumers' beliefs about a firm's quality depend only on this firm's strategy.[12] Finally, we adopt the convention that firms and consumers who are indifferent choose respectively to sell and buy. Henceforth, we simply refer to an equilibrium with these properties as an equilibrium and uniqueness of an equilibrium must be intended within this class. We will also often refer to consumers' beliefs upon nondisclosure without reference to the specific price/s charged since, as we will see, there is

[11]All equilibria we describe in this paper survives under the alternative timing in which pricing follows disclosure. This follows from disclosure being in pure strategies, a high-quality firm always disclosing, and there being no signalling through prices. For a given equilibrium disclosure strategy, pricing is then optimal - otherwise the original strategy would not represent an equilibrium under the simultaneous timing the first place. Likewise, a firm in the game with sequential timing cannot profit by deviating at the disclosure stage, as this would weakly lower its good's valuation of all consumers.

[12]This restriction only plays a role in the benchmark where all consumers are rational (see Section 4.2) and it selects as equilibrium outcome the limit of the equilibrium of our main model as the fraction of cursed consumers vanishes.

no scope for signaling through prices in our framework.[13]

We use our model to study the effects of three consumer protection policies, i.e. consumer education, mandatory disclosure, and third-party disclosure. Each of these policies has the effect of turning some consumers with cursed beliefs upon non-disclosure into consumers with accurate beliefs, either by making them rational or by assuring that information is disclosed to them. We will distinguish between perfect and imperfect consumer protection policies. The former results in a completely informed consumer base, which, as will become clear, is equivalent to assuming that $\chi = 0$. The latter merely leads to a reduction in the proportion of cursed types.

4 Analysis

Before we analyze the main model with two firms and heterogeneous strategic sophistication on behalf of consumers, we establish two important benchmarks. We study, in turn, the case of a monopoly firm and the case of competition over only rational consumers.

4.1 Benchmark: Monopoly with cursed and rational consumers

Suppose that there is a single firm. The following proposition characterizes the unique equilibrium outcome.

Proposition 1 (Monopoly)**.** *If quality is high, then the monopoly discloses, charges $p^* = 1$, and attracts all consumers. If quality is low, then the monopolist does not disclose, charges $p^* = \theta$, and attracts only cursed consumers.*

A monopolist with high quality discloses in order to separate from the low-quality type. Since rational consumers anticipate this disclosure strategy, a monopolist with low quality only sells to cursed consumers, who are exploited because they pay θ for a worthless product. Thus, consumer protection policies that reduce the fraction of cursed consumers are always

[13] The updating of cursed consumers is fixed by assumption. Instead, rational consumers will always perfectly infer firms' qualities from their disclosure strategies.

beneficial for consumer welfare and detrimental to the monopolist. Besides, as exploitation entails a pure transfer of surplus from consumers to the firm, these policies leave total welfare unaffected.[14]

4.2 Benchmark: Competition with only rational consumers

We now turn to the case of two firms competing over a homogeneous group of rational consumers, i.e. the case of $\chi = 0$. In this setting, consumers' belief that a high-quality firm always discloses is self-fulfilling in that it incentivizes a high-quality firm to disclose. Then, firms' private information is perfectly revealed, irrespective of the disclosure decision of a low-quality firm. Moreover, if a firm wants to sell its product, then it can at most charge its quality advantage over the other firm. Consumers' purchasing decisions are efficient.[15] More specifically, firms' simultaneous choice of disclosure and prices results in the following equilibrium allocation.

Proposition 2 (Competition with only rational consumers)**.** *In equilibrium, either firm discloses if it has high quality. If qualities are the same, then consumers buy from either firm at a price equal to zero. If the two qualities differ, then consumers buy from the high-quality firm at price* $p_h^* = 1$.

This equilibrium allocation mirrors the complete information outcome. As a result, policy interventions that seek to better inform consumers are unnecessary. In our framework, cursed consumers represent the sole potential barrier to full information revelation and efficiency. Therefore, their presence provides the sole rationale for consumer protection.

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[14]When $q_l \neq 0$, however, these policies (weakly) increase total welfare. This is obvious if $q_l < 0$. As for the case $q_l > 0$, denote the firm's average quality by $\mu \equiv \theta q_h + (1-\theta) q_l$. The equilibrium is as in Proposition 1 if $q_\ell < \chi\mu$. Instead, if $q_\ell \geq \chi\mu$ even the low-quality type would attract all consumers by charging $p^* = q_\ell$. Thus, a policy that reduces χ so that the latter case obtains improves welfare because rational consumers are no longer inefficiently excluded.

[15]Absent the restriction that consumers' beliefs about a firm's quality depend only on this firm's behavior, this allocation is not a unique equilibrium because even a high-quality firm may refrain from disclosing if, somewhat arbitrarily, doing so would also boost consumers' perception of its rival's quality (see Board (2009) and Janssen and Teteryatnikova (2016)).

4.3 Competition with cursed and rational consumers

4.3.1 Market equilibrium

This section features our main model with both cursed and rational consumers. Here, firms' simultaneous choice of disclosure and prices results in the following equilibrium allocation.

Proposition 3 (Competition with cursed and rational consumers)**.** *In equilibrium, a firm discloses if and only if its quality is high. If the two firms have the same quality, then they each charge a price of zero and make zero profits. If the two qualities differ and*

- $\chi \geq \theta$*, then the high-quality firm charges* $p_h^* = 1 - \theta$ *and attracts all consumers;*
- $\chi < \theta$*, then the high-quality firm randomizes in prices according to a distribution with support* $[1-\chi, 1]$*, the low-quality firm randomizes in prices according to a distribution with support* $[\theta - \chi, \theta]$*, rational consumers buy from the high-quality firm, and cursed consumers buy from either firm depending on realized prices.*

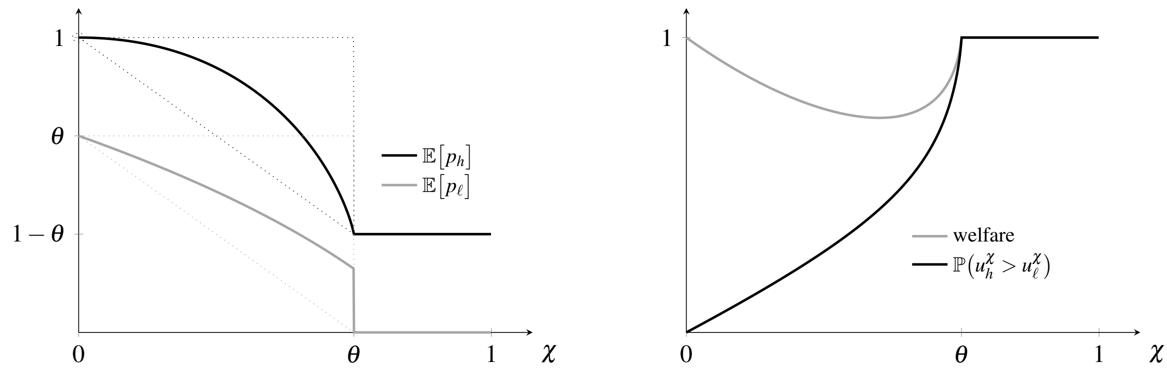
When firms have identical quality levels, competition implies zero profits regardless of the composition of consumer types. As a result, no exploitation takes place. This is true even when both firms have low-quality since, even if their strategies of nondisclosure effectively mislead cursed consumers, neither firm is ultimately perceived to have a quality advantage. Therefore, competition is at least partially successful at protecting cursed consumers, who would always be exploited by a monopolist selling a low-quality product (see Proposition 1).

In the case of vertical differentiation, the parameter space is partitioned into a no-exploitation region ($\chi \geq \theta$) and an exploitation region ($\chi < \theta$). In the no-exploitation region, all consumers buy the high-quality product and obtain positive utility. In the exploitation region, cursed consumers sometimes buy the low-quality, worthless, product at a positive price.

Because rational consumers can never be fooled into buying a low-quality product, whether exploitation arises ultimately depends on the incentives of the high-quality firm to attract cursed consumers. If cursed consumers are many or if they hold sufficiently pessimistic beliefs about undisclosed quality ($\chi \geq \theta$), then they represent a profitable segment of the market and the high-quality firm chooses to attract them by charging a relatively low price. If the proportion of cursed consumers is low or their inflated assessment of a silent firm's quality is high ($\chi < \theta$), then they are less profitable and the high-quality firm will not pursue an aggressive pricing strategy to capture them. Since the distribution of perceived valuations for the low-quality product is binary in the population, firms then necessarily share the cursed segment of the market probabilistically, i.e. pricing is in mixed strategies. Firms' indifference obtains from the trade-off that a higher price results in higher profits if a firm succeeds in capturing cursed consumers but also in a lower probability of doing so. In particular, the high-quality firm is indifferent between attracting only rational consumers at $p_h^* = 1$ or all consumers at $p_h^* = 1 - \chi$. Likewise, the low-quality firm is indifferent between attracting cursed consumers for sure at $p_l^* = \theta - \chi$ or only with probability $(\theta - \chi)/\theta$ at $p_l^* = \theta$. This probability corresponds to the atom that the pricing distribution of the high-quality firm has at $p_h^* = 1$, which guarantees that the expected demand of the low-quality firm is positive even for p_l^* close or equal to θ. Also, in the limit as χ converges to zero, the equilibrium converges to the full-information deterministic outcome with only rational consumer described at proposition 2, in which the high-quality firm attracts all consumers at $p_h^* = 1$.

4.3.2 Lifting the curse: consumer protection policies and welfare

Since consumers always appropriate all gains from trade when qualities are homogeneous, our assessment of consumer protection policies focuses on the case of vertical differentiation. In that case, cursed consumers' inflated perception of a silent firm's quality strengthens competition. As a result, expected prices are decreasing in the proportion of cursed consumers (see Figure 1a). Simultaneously, because the high-quality firm competes more aggressively for cursed consumers the more there are, the likelihood that cursed consumers buy high quality, denoted by $\mathbb{P}\left(u_h^\chi > u_\ell^\chi\right)$, is increasing (see Figure 1b). Thus, while an individual cursed consumer may well be hurt by her naivete if it causes her to buy the inferior product, she exerts a positive externality on all other consumers. In contrast to the monopoly setting, interventions aimed at limiting consumers' naivete therefore have double-edged effects.



(a) Expected prices (with dashed supports)

(b) Welfare and the probability that cursed consumers buy the high-quality product

Figure 1 Equilibrium prices and welfare in the case of vertical differentiation

Proposition 4 (Imperfect consumer protection policies)**.** *If no exploitation occurs in equilibrium ($\chi \geq \theta$), then a reduction in the proportion of cursed consumers weakly increases firms' total profits and weakly decreases consumer surplus and welfare.*

If exploitation occurs in equilibrium ($\chi < \theta$), then a reduction in the proportion of cursed consumers increases firms' total profits and decreases the surplus of rational consumers and of consumers who remain cursed. Its effect on consumers who become rational, on overall consumer surplus, and on welfare is generally ambiguous, but positive if χ is sufficiently small.

The comparative statics with respect to the proportion of cursed consumers χ that give rise to Proposition 4 are depicted in Figures 1b, 2a and 2b. Consider first welfare, i.e. the sum of firms' total expected profits and consumers' expected surplus, which coincides with the expected fraction of consumers who purchase the high-quality good

$$W \equiv (1-\chi) + \chi \mathbb{P}\big(u_h^\chi > u_\ell^\chi\big). \tag{1}$$

As Figure 1b illustrates, welfare is maximal for $\chi = 0$ and $\chi \geq \theta$, where all consumers buy the efficient high-quality product. Otherwise, welfare is u-shaped and in particular, after computing $\mathbb{P}\big(u_h^\chi > u_\ell^\chi\big)$ as per equation (5) in the appendix and replacing in equation (1), it is equal to

$$W = (1-\chi) + \chi \frac{(1-\chi)\left((1-\theta)\chi - (\theta-\chi)\log\left(\frac{\theta(1-\chi)}{\theta-\chi}\right)\right)}{(1-\theta)^2\chi}.$$

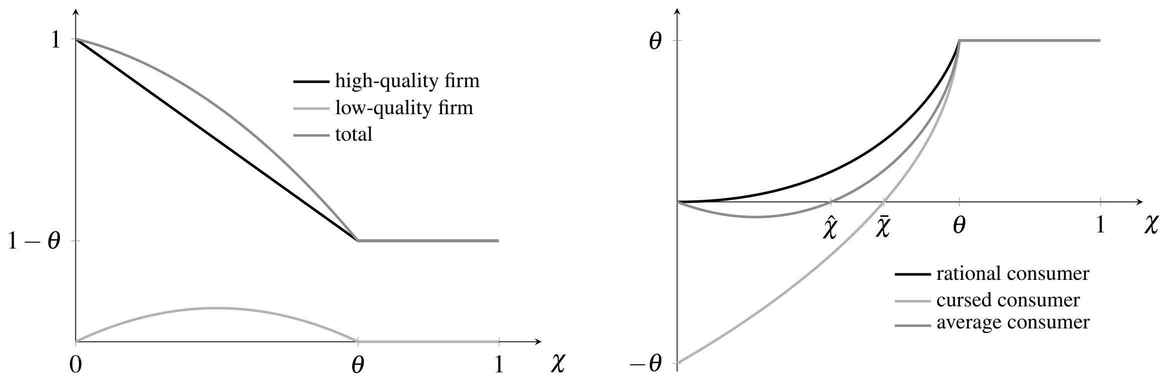
Differentiating equation (1) with respect to χ clarifies how an increase in the fraction of cursed consumers has both a negative *composition effect* and a positive *equilibrium effect* on welfare:

$$W' = \underbrace{-(1-\mathbb{P}(u_h^\chi > u_\ell^\chi))}_{\textit{composition effect}} + \underbrace{\chi\mathbb{P}'(u_h^\chi > u_\ell^\chi)}_{\textit{equilibrium effect}}. \tag{2}$$

The negative composition effect arises because with positive probability an additional cursed consumer makes an inefficient purchasing decision that costs society one util. The positive equilibrium effect arises because the probability with which the cursed segment makes the inefficient decision decreases. The non-monotonicity of welfare is due to the fact that the composition effect is large when there are few cursed consumers, because the additional cursed consumer then makes the inefficient purchasing decision with probability close to one, while the equilibrium effect is by construction large when the cursed segment is sizable.

The u-shape implies that imperfect consumer protection may decrease welfare when there are many cursed consumers but increase welfare at an intermediate proportion of cursed consumers. Moreover, consumer protection policies are most likely to be socially desirable if exploitation and the resulting buyers' remorse occur very frequently in equilibrium, i.e. at the point where welfare is at its lowest. Therefore, a high prevalence of exploitation, not cursedness, provides a sound rationale for imperfect consumer protection.

Figure 2a depicts firms' profits. In the exploitation region, aggregate profits decrease as the fraction of cursed consumers increases. While the profits of the high-quality firm always decrease in χ, the profits of the low-quality firm are hill-shaped in χ. As χ increases, the low-quality firm's pool of potential customers increases, but prices and the probability of attracting an individual cursed consumer decrease. There is a region in which both types of firm benefit from a reduction in χ through imperfect consumer protection policies. However, since the



(a) Expected profits **(b)** Consumer surplus

Figure 2 Equilibrium profits and consumer surplus

low-quality firm's profits are equal to "expected" exploitation, a policy that has the support of the low-quality firm is always detrimental to consumer surplus. Note also that the high-quality firm would always have an incentive to engage in comparative advertising if this has the effect to debias cursed consumers' beliefs.

Figure 2b considers the consumers' perspective. A rational consumer's utility always decreases as the fraction of cursed consumers shrinks, because the price of the high-quality product increases. The same is true for a cursed consumer, not only because the prices of both products increase, but also because the likelihood of her purchasing the inferior product increases. Below some threshold $\bar{\chi}$, a cursed consumer would be better off by staying out of the market all together, i.e. her expected utility from a purchase becomes negative, which is a necessary condition for policy interventions to be beneficial for consumers as a whole. In particular, while for any given χ a cursed consumer is naturally always worse-off than a rational one, starting from $\chi > \bar{\chi}$ a policy initiative of large enough scale even hurts previously cursed consumers who become rational. Indeed, as a result of the policy they now always buy the high-quality good but at a much higher expected price. Instead, when the fraction of cursed consumers is small enough, then even imperfect consumer protection policies are an effective measure to enhance consumer surplus, both because exploitation is severe and because competition is weak.[16]

[16]In the limit, as χ approaches zero, not only are rational consumers held close to their reservation utility, but

From Figure 1 and 2, the effect of perfect consumer protection measures is also apparent.

Proposition 5 (Perfect consumer protection policies)**.** *If no exploitation occurs ($\chi \geq \theta$), then perfect consumer protection preserves efficiency and redistributes surplus from both types of consumers to firms.*

If exploitation occurs ($\chi < \theta$), then perfect consumer protection restores efficiency, increases firms' total profits, reduces the surplus of rational consumers, and increases the surplus of cursed consumers and overall consumer surplus if and only if χ is sufficiently low.

A perfect consumer protection policy results in all consumers behaving as if they were rational. Since all consumers will then buy the high-quality product, efficiency is assured. However, the high-quality firm becomes a de-facto monopolist and extracts all gains from trade. In terms of consumer surplus, the negative effect of the increase in the high-quality firm's market power may outweigh the benefits of a superior allocation (when $\chi > \hat{\chi}$ in Figure 2b).

4.3.3 Changes in the degree of naivete

Suppose that cursed consumers' naivete is only partial. More specifically, given prior average quality θ, their belief about a silent firm's quality is $\gamma\theta$, with $\gamma \in (0,1]$ measuring their degree of naivete. Variations in γ may capture different market characteristics, e.g. products for which the lack of disclosure is more or less salient for consumers, but could also result from consumer protection, e.g. an information campaign that has the effect to reduce, but not eliminate, the bias in cursed consumers' beliefs. The equilibrium is completely unaffected by the degree of naivete γ when firms have homogeneous qualities. As a result, the effect of a change in γ can be directly deduced from comparative statics with respect to θ in the case of vertical differentiation.

Proposition 6 (Changes in the degree of naivete)**.** *Fix χ. If $\theta \leq \chi$, then consumer surplus is strictly increasing, firms' total profits are strictly decreasing and welfare is constant in the*

the marginal increase in prices due to the policy initiative is negligible. This can be seen in Figure 1a, by looking at the flat slope of $\mathbb{E}[p_h]$ when $\chi = 0$.

degree of naivete of cursed consumers γ. If $\theta > \chi$, then consumer surplus and firms' total profits are non-monotone and welfare is (weakly) decreasing in γ.

As long as the degree of naivete is such that no exploitation takes place, which is always the case when $\theta \leq \chi$, more biased beliefs on behalf of cursed consumer simply induce the high-quality firm to price more aggressively to retain them, redistributing rents to both types of consumers. Instead, when $\theta > \chi$, a sufficiently high bias entails a shift from the no exploitation to the exploitation region, given that cursed consumers find the low-quality firm too attractive. Within the exploitation region, the benefits of the softening of competition resulting from the increase in the bias are reaped entirely by the low-quality firm, who attracts cursed consumers with a higher probability and at a higher price, at the expenses of consumer and total welfare.

Parameter γ could also be thought of as a strategic obfuscation choice of the low-quality firm, $\gamma = 0$ corresponding to full disclosure, $\gamma \in (0,1)$ to partial disclosure and $\gamma = 1$ to nondisclosure. The previous observation clarifies that the low-quality firm has always an incentive to choose nondisclosure, i.e. to maximally obfuscate its quality to induce the most optimistic belief in cursed consumers. Conversely, while this case is not covered explicitly, it is clear that the high-quality firm would always favor full disclosure, i.e. $\gamma = 0$ when cursed beliefs about its quality are defined as $1 - \gamma(1-\theta)$.

5 Extensions

5.1 Endogenous quality

In this section, we consider firms' incentives to invest in product quality. The game is as before, except that each firm i rather than nature chooses its own quality $q_i \in \{0,1\}$ at an initial stage. The cost of quality for each firm is independent of the quantity supplied and it is zero for low quality and $c \in (0,1)$ for high quality. To adapt the equilibrium concept to this setting, we focus on symmetric equilibria in which each firm chooses high quality with probability θ, to be determined endogenously, and cursed consumers' belief about the quality of a silent firm is exactly θ.[17] Thus, in equilibrium, while the belief of cursed consumers about a firm's average quality is correct, a firm cannot affect this belief by deviating. As in our main model, cursed consumers still correctly infer the quality of a firm that discloses.

Proposition 7 (Endogenous quality)**.** *There is a unique symmetric equilibrium in which each firm chooses high quality with probability $\theta^*(\chi,c) \in (0,1)$ and the continuation is as in Proposition 3. No exploitation occurs if and only if $\chi \geq \tilde{\chi}(c) \in (0,1)$. There is a parameter region in which imperfect consumer protection unambiguously reduces welfare as it not only generates exploitation but also an inefficient reduction in firms' investment θ^*. Instead, perfect consumer protection always increases welfare.*

As in our main model, in the case of vertical differentiation, no exploitation occurs when the fraction of cursed consumers is sufficiently large ($\chi \geq \tilde{\chi}$), so that the high-quality firm finds it profitable to attract all consumers. The threshold $\tilde{\chi}$ is decreasing in c, implying the condition for no exploitation becomes more easily satisfied for higher costs. Since the investment of each firm θ^* decreases, cursed consumers overestimate the value of the low-quality product by less.

Similar to our main model, an imperfect consumer protection policy that has the effect to move χ below the exploitation threshold $\tilde{\chi}$ introduces allocative inefficiency. However, the welfare assessment of this policy must now also include considerations on equilibrium investment θ^*, which is always inefficiently low as a firm benefits from investing only in the event that its rival does not. Provided c is sufficiently small and χ does not move too far below $\tilde{\chi}$, investment decreases, so that the policy is then surely harmful. The reason behind this disincentive effect on investment is that, due to the softening of competition discussed in the main model, not only high-quality but also low-quality becomes more profitable. Indeed, initially, the profits of the low-quality increase, and possibly at a faster rate. Thus, θ^* has to decrease to preserve firms' indifference with respect to their investment strategy.

Consider now the effect of consumer protection in the exploitation region ($\chi < \tilde{\chi}$) starting

[17] While there is no notion of player types in this game, our modeling choice is in the spirit of the original formulation of cursed equilibrium, according to which a fully cursed player believes that each type of each given other player plays the same mixed action profile that corresponds to this player's average distribution of actions.

from a sufficiently small χ. Then, investment θ^* necessarily increases as χ decreases since high quality and low quality become respectively more and less profitable. In our main model, it was also the case that allocative efficiency increased. In this setting of endogenous quality, this might not necessarily be the case because, as shown in section 4.3.3, in the exploitation region, a higher cursed belief decreases allocative efficiency. However, perfect consumer protection is still unambiguously desirable as it eliminates allocative inefficiency and simultaneously moves investment to its constrained optimum (given that firms must invest with the same probability).

5.2 Horizontal differentiation

In our main model, it is socially optimal for every consumer to purchase the high-quality product. To capture situations in which this is not the case, we now allow for consumers having a taste for the product of a given firm that is independent of the vertical quality dimension: some consumers may live closer to one firm than another or be attached to a particular brand.

Consider two firms that are located on opposite ends of a unit interval along which rational and cursed consumers are uniformly distributed (see Figure 3). The measure of rational and cursed consumers is given by $1-\chi$ and χ respectively. A consumer's location is given by t, which represents both the distance to the firm located at the left of the interval and the transport cost associated with purchasing from that firm. Similarly, purchasing the product on the right implies transport costs of $1-t$. In addition to this horizontal differentiation, product qualities may still differ along an independent vertical dimension and firms still decide whether to disclose their own quality. Similarly to the main model, each firm's quality is equal to $q_h > 0$ with probability $\theta \in (0,1)$ and equal to $q_\ell = 0$ with complementary probability. Each consumer has unit demand and derives a net utility of $v + q_i$ from purchasing a product of quality $q_i \in \{0, q_h\}$. We assume that v is large enough to ensure that each consumer makes a purchase in equilibrium. We also assume that $q_h < 1$, so that, when firms are vertically differentiated, efficiency now dictates that some consumers should purchase the low-quality product.

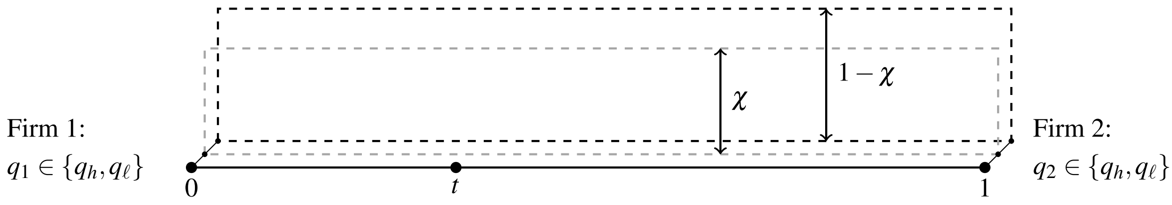


Figure 3 Cursed and rational consumers along a Hotelling line

Firms' incentives to reveal quality are as in our main model. A firm discloses if and only if its quality is high. To assess the effect of consumer protection policies we can therefore again focus on the case of vertical differentiation.[18] In the equilibrium we construct, both the high-quality and the low-quality firm now serve some rational and some cursed consumers. Nonetheless, similar trade-offs as in the main model arise.

Proposition 8 (Horizontal differentiation)**.** *There exists an equilibrium in which firms' profits are decreasing in χ, while the average welfare of rational consumers and consumers that remain cursed is increasing. Consumer surplus is increasing in χ when θ is sufficiently low and u-shaped otherwise. Welfare is decreasing in χ when θ is sufficiently low and u-shaped otherwise.*

Due to its competitive advantage, the high-quality firm covers more of the market and lowering prices implies greater losses on inframarginal consumers. As a result, the price of the high-quality product is inefficiently high and the low-quality firm draws more demand than is socially optimal.

As in the main model, an increase in the fraction of cursed consumers χ yields a composition and an equilibrium effect on overall welfare. The composition effect is negative: when a consumer turns cursed, she will make weakly more inefficient purchasing decisions. Indeed, she is more likely than a rational consumer at the same location to purchase the low-quality product, which is already overconsumed. The equilibrium effect on welfare is positive: an

[18] When both firms have high quality, it is as if there existed only a homogeneous group of consumers and, due to the lack of vertical differentiation, firms share the market equally by charging identical prices that are independent of χ. This is also the case when both firms have low quality, since neither group of consumers perceive one firm to have an advantage over the other (although cursed consumers overestimate the net utility that either product delivers).

increase in the fraction of cursed consumers exerts competitive pressure on the high-quality firm. As a result, the high-quality firm is induced to decrease its price, whereas the low-quality firm increases its price. This in turn leads to some consumers making more efficient purchasing decisions.

Following a similar logic as in the main model, the composition effect dominates the equilibrium effect for low levels of χ, whereas the latter may eventually dominate the former for higher χ. Even in the case of horizontal differentiation it may therefore be the case that imperfect consumer protection policies decrease welfare when there are many cursed consumers (see section A.7.2 for more detailed comparative statics and intuitions).

6 Conclusion

The intuition that policies that help consumers draw better inferences about firms' private information lead to more desirable outcomes is compelling. From a partial equilibrium perspective, information unambiguously improves an individual's decision making. And since information nudges do not restrict consumers' choice sets, they qualify as the sort of soft paternalism that should not invite strong ideological opposition. This paper cautions policy makers that it is crucial to consider the equilibrium effects of information-based consumer protection.

In vertically differentiated markets with a socially desirable high-quality good, consumer protection may decrease consumer and overall welfare. If the majority of consumers is naive, then the high-quality firm has every incentive to attract them with low prices. However, when the group of naive consumers is small and unprofitable, it will be left to buy the exploitative and inefficient product. Consequently, the most socially inefficient outcomes obtain when well-intentioned policy leaves behind a small to medium-sized group of naive consumers.

Our analysis identifies a novel channel for why society may sometimes be better off without mandatory disclosure, third party disclosure or consumer education. Also, our results suggest that, if these policies are nonetheless deemed desirable, then they should be implemented wholeheartedly and comprehensively. Since an inefficient firm's profits are highest for intermediate levels of cursedness, policy makers should be weary of industry representatives who condone mandatory disclosure but are eager to put bounds on the salience or informativeness of the disclosed information.

As a rule of thumb, imperfect policy measures are more likely to have their intended effect if there is a high incidence of exploitation in equilibrium. Policy should therefore be predicated on a high level of observed exploitation or high profits from the sale of undesirable products. However, note that exploitation is distinct from consumer misperception or cursedness. A high incidence of misperception need not imply a useful role for consumer protection. High levels of consumer misperception in combination with low levels of consumer exploitation may reflect a socially desirable equilibrium. Therefore, it is not advisable to base consumer protection measures on observed misperception alone.

Both misperception and exploitation can be measured. Misperception can be elicited in surveys of people's subjective quality expectations of products that do not disclose quality.[19] Exploitation can be identified from a change in consumers' purchasing behavior in response to exogenously provided information, absent firms' strategic response in prices. This identification can be delivered by a field experiment that is sufficiently small to be ignored by firms.

Further testable implications of our model can be investigated by means of policy experiments that are large enough to solicit a strategic response from firms. For example, an unexpected mandatory disclosure law should lead to higher average prices in vertically differentiated duopolies and to lower prices under monopoly. The policy has an ambiguous effect on exploitation and the market share of low-quality products in duopoly and decreases exploitation under monopoly.

Since the impact of consumer protection policy depends on market structure, it would be important to study more comprehensively the effects of entry and increased competition.

[19] Similar elicitations are already employed by policy makers. For example, the EU regulation on key information documents for packaged retail and insurance-based investment products (downloadable at http://data.europa.eu/eli/reg/2014/1286/oj) explicitly takes into account "existing and ongoing research into consumer behaviour, including results from testing the effectiveness of different ways of presenting information with consumers." [Paragraph 17].

For instance, in the context of our main model under vertical differentiation, the entry of an additional high-quality firm would suffice to prevent exploitation. However, the entry of an additional low-quality firm can exacerbate it since cursed consumers may then always buy only the low-quality product.

Besides, in our model, a firm's information is exogenous. In many markets, firms' private information is the product of search and certification. Understanding the incentives of certifiers in the presence of cursed consumers is a promising topic for future research. Furthermore, the complexity of the information firms disclose is likely to impact on how both disclosure and non-disclosure, as well as partial or vague disclosure, are interpreted and on how much information is transmitted to consumers. Exploring the link between informational complexity and naivete theoretically and experimentally can inform how regulators should design standardized labels and independent ratings to facilitate information transmission and efficiency.

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A Appendix

A.1 Proof of proposition 1

If the monopolist discloses its quality $q \in \{q_\ell, q_h\}$, then both rational and cursed consumers are willing to pay up to q. Therefore, the optimal price and the resulting profits are $p = q$ and $\pi = q$

respectively. If the firm does not disclose and faces only cursed consumers, its optimal price is $p = \theta$, which yields profits $\pi = \chi\theta$. It is then easy to verify that the strategies described in the proposition are an equilibrium, since upon non-disclosure rational consumers infer that $\mathbb{E}[q|\emptyset] = 0$ and, at $p = \theta$, they refuse to buy. To establish uniqueness, note that in equilibrium type h must necessarily earn 1. Otherwise it could profitably deviate by disclosing and charging $p = 1$. But type h can only earn 1 if it discloses, since otherwise cursed consumers' willingness to pay is lower than 1 by construction. Uniqueness does not hinge on the presence of cursed consumers. However, in the limit case, with $\chi = 0$, the argument for uniqueness is slightly different. Type h must fully separate in order to earn 1. Then, in any candidate fully separating equilibrium strategy in which type h does not disclose, type l must earn less than 1 and would therefore have an incentive to mimic type h.

A.2 Proof of proposition 2

We will repeatedly use a Bertrand competition argument, which we summarize in the following lemma. Throughout, we maintain for ease of exposition that pricing is in pure strategy, even though there may also exist outcome equivalent equilibria in which the inactive firm randomizes.[20]

Lemma 1 (Bertrand competition)**.** *Fix consumers' expectations about the quality levels of firm i and j and denote these expectations by $\tilde{q}_i$ and $\tilde{q}_j$ respectively. Without loss of generality, assume that $\tilde{q}_i \geq \tilde{q}_j \geq 0$. In equilibrium,*

- *if $\tilde{q}_i = \tilde{q}_j$, then consumers buy the product at a price o zero; moreover, if $\tilde{q}_i > 0$, then $p_i^* = 0 = p_j^*$, whereas if $\tilde{q}_i = 0$, then one of the two firms' prices, say p_i^*, is equal to zero while the other can take any value;*
- *if $\tilde{q}_i > \tilde{q}_j$, then consumers buy the product from firm i at a positive price; if $\tilde{q}_j > 0$, then $p_i^* = \tilde{q}_i - \tilde{q}_j$ and $p_j^* = 0$, while if $\tilde{q}_j = 0$, then $p_i^* = \tilde{q}_i$ and p_j^* can take any value.*

Proof. Define $u_i = \tilde{q}_i - p_i$ as a consumer's perceived utility of buying from firm i and $\pi_i = p_i D_i$ as firm i's profits, where D_i denotes the firm's demand.

Suppose first that $\tilde{q}_i = \tilde{q}_j$. Then, an equilibrium in which one firm, say firm i, makes positive profits cannot exist. For any price pair such that $u_i \geq u_j$ and $\pi_i > 0$, it is the case that $\pi_j < p_i$ and firm j would

[20] See Blume (2003).

profit from charging $p_j = p_i - \varepsilon$ for $\varepsilon > 0$ arbitrarily small and attracting all consumers. Thus, if a firm is active, say firm i, $p_i^* = 0$. Unless $\tilde{q}_i = \tilde{q}_j = 0$, it must also be the case that $u_i = u_j$, i.e. that $p_j^* = 0$, or otherwise firm i could make positive profits by charging $p_j^* - \varepsilon$ for $\varepsilon > 0$ arbitrarily small.

Now suppose that $\tilde{q}_i > \tilde{q}_j$. Then, firm j cannot sell in equilibrium, since for any price pair such that $u_i \leq u_j$, $u_j \geq 0$ and $D_j > 0$, firm i would profit from reducing p_i and attracting all consumers. Unless $\tilde{q}_j = 0$, it must also be that $u_i = u_j$, i.e. that $p_i^* = \tilde{q}_i - \tilde{q}_j - p_j^*$, and that $p_j^* = 0$. Otherwise firm i and j could profitably deviate by respectively increasing and decreasing their prices. □

Existence. If one replaces expected qualities with actual qualities, then Lemma 1 pins down firms' pricing and profits in the complete information outcome. Given consumers' belief that a silent firm has low quality, we can then verify that the equilibrium in Proposition 2 exists. Regardless of its disclosure decision and the quality of its rival, a low-quality firm cannot attract consumers at a price above zero. Similarly, a high-quality firm always benefits from disclosing, strictly so when its rival has low quality and weakly otherwise. Therefore, consumers' beliefs are consistent.

Uniqueness. Suppose by contradiction that there exists an equilibrium in which firms' qualities are not perfectly revealed. That is, if we denote by $\tilde{q}_i$ and $\tilde{q}_j$ consumers' expectation of firms' qualities, there is an on-the-equilibrium-path history in which for at least a firm, say firm i, $\tilde{q}_i \in (0,1)$. Clearly, firm i cannot disclose in this history. We will consider all possible messages of firm j.

- Suppose that $m_j = 0$. Given the restriction that a firm's behavior does not affect consumers' belief about the quality of its competitor and that disclosure decisions entail no randomization, Lemma 1 describes the pricing and profits of firm j in this history. It also describes the maximum price and profits of firm i, namely $\tilde{q}_i$. In principle, these could also be lower, since a price raise by firm i could now be deterred by a decrease in $\tilde{q}_i$ off-the-equilibrium-path. Therefore, unless $\tilde{q}_i = 1$, when $q_i = 1$ firm i could profitably deviate by disclosing and charging $p_i = 1 - \varepsilon$ for $\varepsilon > 0$ arbitrarily small. Hence, either $\tilde{q}_i = 1$ or $\tilde{q}_i = 0$.

- Suppose that $m_j = 1$. Lemma 1 implies that firm i makes zero profits, while the price and profits of firm j are now at least $p_j \geq 1 - \tilde{q}_i$. Similar to the previous case, p_i and hence p_j can in principle be higher if a price cut by firm i is deterred by a decrease in $\tilde{q}_i$ off-the-equilibrium-path. Therefore, unless $\tilde{q}_i = 1$, when $q_i = 1$ firm i could profitable deviate by disclosing and charging $p_i = 1 - \tilde{q}_i - \varepsilon$

for $\varepsilon > 0$ arbitrarily small. Again, either $\tilde{q}_i = 1$ or $\tilde{q}_i = 0$.

- Suppose that $m_j = \emptyset$ and assume without loss of generality that $\tilde{q}_i \geq \tilde{q}_j$ (and therefore that $\tilde{q}_i > 0$ and $\tilde{q}_j < 1$, otherwise firms' qualities would be perfectly revealed). Firm i can now set a price and make profits of at most $\tilde{q}_i$. Indeed, as $\tilde{q}_j \geq 0$, by Lemma 1 this bound is attained if $\tilde{q}_j = 0$. In principle, it can also be attained if $\tilde{q}_j > 0$ and $p_j > 0$ (then in particular p_j must be such that $p_i = \tilde{q}_i = \tilde{q}_j - p_j$) because a price cut from firm j is deterred by the off-the-equilibrium-path belief that $\tilde{q}_j = 0$. Therefore, unless $\tilde{q}_i = 1$, when $q_i = 1$ firm i could profitable deviate by disclosing and increasing p_i. It must then be the case that $\tilde{q}_i = 1$ and that firm i is selling at $p_i \geq 1 - \tilde{q}_j$. But then, it must also be the case that $\tilde{q}_j = 0$, since whenever $q_j = 1$ firm j would have an incentive to disclose and charge $p_j = p_i - \varepsilon$ for $\varepsilon > 0$ arbitrarily small.

Given that firms' qualities are perfectly revealed, the only possibility for the equilibrium outcome not to be as in Lemma 1 with actual qualities replacing conjectured ones would be that at least one firm, say firm i, does not disclose and pricing differs because a deviation in p_i is discouraged by an off-the-equilibrium-path adverse inference on q_i. This is clearly not possible if $q_i = 0$. It is also not possible when $q_i = 1$, since firm i could simultaneously deviate in pricing and disclosure, so that all deviations by a high-quality firm in the proof of Lemma 1 remain profitable.

A.3 Proof of proposition 3

Existence. Considering all cases, we will show that firms have no profitable deviations and, along the way, we will provide the elements of the equilibrium left unspecified in the proposition and the derivation of firms' pricing strategies. Throughout, $\tilde{q}_i$ and $\tilde{q}_i^\chi$ represent the belief about firm i's quality of rational and cursed consumers respectively, and u_i and u_i^χ their perceived utility from buying from firm i.

- Suppose that both firms have low quality. Upon non-disclosure by both firms $\tilde{q}_i = \tilde{q}_j = 0$, so that the willingness to pay of rational consumers is zero. Hence, the two firms compete only for cursed consumers (whose $\tilde{q}_i^\chi = \tilde{q}_j^\chi = \theta$) and, by Lemma 1, they charge zero prices and make zero profits. If firm i deviates by disclosing, it attracts no consumer regardless of the positive price it charges.
- Suppose that both firms have high quality. Since both firms disclose, Lemma 1 implies that they

must charge zero prices and make zero profits. If a firm, say firm i, deviates by not disclosing, for any $p_i > 0$ it does not attract any consumers, since $\tilde{q}_i(\emptyset) = 0$ and $\tilde{q}_i^{\chi}(\emptyset) = \theta < 1$.

- Suppose that the qualities of the two firms differ and let the subscript h and ℓ refer to the high- and low-quality firm respectively. Given firms' disclosure strategies, we have $\tilde{q}_\ell = 0$, $\tilde{q}_\ell^{\chi} = \theta$ and $\tilde{q}_h = \tilde{q}_h^{\chi} = 1$. We will distinguish two sub-cases.

  – Suppose first that $\chi \geq \theta$. When $p_h^* = 1 - \theta$ and $p_\ell^* = 0$, firm h attracts all consumers. If firm ℓ deviates by disclosing or by charging a positive price, it keeps attracting no consumer. If firm h deviates by not disclosing, it attracts no consumer for any $p_h > 0$. If firm h deviates in prices, its best deviation is $p_h = 1$, which attracts only rational consumers (because $u_\ell^{\chi} = \theta > u_h^{\chi} = 0$) and hence yields $1 - \chi$. This deviation is not profitable if and only if $1 - \theta \geq 1 - \chi$, that is, if and only if $\chi \geq \theta$.

  – Suppose instead that $\chi < \theta$. We will construct mixed pricing strategies such that firm h randomizes according to $G_h(p_h)$ over $\left[\underline{p}_h, \bar{p}_h\right]$, firm ℓ randomizes according to $G_\ell(p_\ell)$ over $\left[\underline{p}_\ell, \bar{p}_\ell\right]$, rational consumers always buy from firm h and cursed consumers buy with positive probability from either firm (and from firm ℓ whenever indifferent - equivalently, the tie-breaking rule can be arbitrary and the support of $G_\ell(p_\ell)$ be right-open). As supports, we guess $\underline{p}_h = 1 - \chi$, $\bar{p}_h = 1$, $\underline{p}_\ell = \theta - \chi$ and $\bar{p}_\ell = \theta$, so that $u_h^{\chi}(\bar{p}_h) = u_\ell^{\chi}(\bar{p}_\ell) = 0$ and $u_h^{\chi}(\underline{p}_h) = u_\ell^{\chi}(\underline{p}_\ell) = \chi$. Note that $\underline{p}_\ell$ is positive if and only if $\chi < \theta$. Given these supports, rational consumers always prefer to buy from firm h. Fix $G_\ell(p_\ell)$ and assume it is atomless. The expected profits of firm h for $p_h = \bar{p}_h$ are $\pi_h(\bar{p}_h) = 1 - \chi$, while for any other p_h in the candidate support

$$\pi_h(p_h) = p_h\Big(\underbrace{1 - G_\ell(p_h - (1-\theta))}_{\mathbb{P}\left(u_h^{\chi} > u_\ell^{\chi}\right)}\Big) + (1-\chi)p_h \underbrace{G_\ell(p_h - (1-\theta))}_{\mathbb{P}\left(u_h^{\chi} \leq u_\ell^{\chi}\right)}.$$

  Solving $\pi_h(p_h) = \pi_h(\bar{p}_h)$ yields $G_\ell(p_h - (1-\theta)) = \frac{p_h - (1-\chi)}{\chi p_h}$ and, after the change of variable $p_h = p_\ell + 1 - \theta$, $G_\ell^*(p_\ell) = \frac{p_\ell - (\theta - \chi)}{\chi(p_\ell + 1 - \theta)}$. Note that indeed $G_\ell^*(\underline{p}_\ell) = 0$ and $G_\ell^*(\bar{p}_\ell) = 1$. Therefore, when firm ℓ randomizes according to $G_\ell^*(\cdot)$, firm h is indifferent to any p_h in the candidate support. Any p_h above $\bar{p}_h$ would yield $\pi_h = 0$, while any $p_h < \underline{p}_h$ would yield $\pi_h = p_h < 1 - \chi$.

  Now fix $G_h(p_h)$ and assume it is atomless except possibly at $\bar{p}_h$. The expected profits of firm ℓ

from $p_\ell = \underline{p}_\ell$ are $\pi_\ell(\underline{p}_\ell) = \chi(\theta - \chi)$, while for any other p_ℓ in the candidate support

$$\pi_\ell(p_\ell) = \chi p_\ell \Big(\underbrace{1 - G_h(p_\ell + 1 - \theta)}_{\mathbb{P}(u_\ell^\chi \geq u_h^\chi)} \Big).$$

Solving $\pi_\ell(p_\ell) = \pi_\ell(\underline{p}_\ell)$ yields $G_h(p_h - (1-\theta)) = \frac{p_\ell - (\theta - \chi)}{p_\ell}$ and, after the change of variable $p_\ell = p_h - (1-\theta)$, $G_h^*(p_h) = \frac{p_h - (1-\chi)}{p_h - (1-\theta)}$. Note that $G_h^*(\underline{p}_h) = 0$ and $G_h^*(\bar{p}_h) = \frac{\chi}{\theta} < 1$, which means that $G_h^*(\cdot)$ has an atom of size $\alpha_h^* \equiv \frac{\theta - \chi}{\theta}$ at $\bar{p}_h$. Therefore, if firm h randomizes according to $G_h^*(\cdot)$, then firm ℓ is indifferent to any p_ℓ in the candidate support. Any p_ℓ above $\bar{p}_\ell$ would yield $\pi_\ell = 0$, while any $p_\ell \in (0, \underline{p}_\ell)$ would yield $\chi p_\ell < \chi \underline{p}_\ell$. As for deviations in disclosure strategies, if firm ℓ deviates by disclosing it attracts no consumer for any $p_l > 0$. If firm h deviates by not disclosing, it never attracts rational consumers for any $p_h > 0$ and, as it also lowers the valuation of cursed consumers, it cannot increase its profits.

Uniqueness. Note that there cannot exist an equilibrium in which a firm discloses when its quality is $q_i = 0$. Indeed, if this was the case, we would have $\tilde{q}_i = \tilde{q}_i^\chi = 0 = \pi_i = 0$ and firm j would behave as a monopolist (Proposition 1). As cursed consumers would obtain their perceived reservation utility, firm i could profitably deviate by not disclosing and attracting them with a $p_i > 0$. Thus, the valuation of rational consumers when a firm is silent must satisfy $\tilde{q}_i(\emptyset) < 1$. Then, no matter the quality of its rival, starting from any candidate equilibrium history in which firm i has high quality but remains silent, disclosure is a strictly profitable deviation for firm i since it raises the valuation of all consumers.

If disclosure decisions are as in the proposition, then the equilibrium pricing behavior characterized above is unique. In particular, in the case of vertical differentiation, when $\chi \geq \theta$, for any candidate equilibrium of the pricing game in which firm h does not attract cursed consumers with probability one, it would have an incentive to decrease p_h to ensures that it does. When $\chi < \theta$, instead, no pure strategy equilibrium can exist. Indeed, as shown above, given the unique candidate equilibrium prices for which firm h attracts cursed consumers with probability one, it would want to deviate. Similarly, there cannot exist a candidate equilibrium in which firm ℓ attracts cursed consumers with probability one, as at candidate equilibrium prices it should be that

$$u_h^\chi \equiv 1 - p_h^* = \theta - p_\ell^* \equiv u_\ell^\chi$$

and firm h would profit from slightly decreasing p_h. Uniqueness of the mixed-strategy equilibrium follows from noting that, because of standard arguments, price supports cannot have interior atoms or holes and that, unless $\bar{p}_h = 1$, firm h would profit from charging $p_h > \bar{p}_h$.

A.4 Proof of propositions 4 and 5

The two propositions follow directly from the comparative statics of the equilibrium of Proposition 3 with respect to the fraction of cursed consumer χ and their limit as χ converges to zero from above, i.e. the allocation at Proposition 2. Since the equilibrium is unaffected by changes in χ when $\chi \geq \theta$, we study comparative statics in the exploitation region. Throughout, $g_\ell^*(p_\ell)$ and $g_h^*(p_h)$ denote the derivatives of the equilibrium cumulative distributions of prices $G_\ell^*(p_\ell)$ and $G_h^*(p_h)$. Also, all derivatives below are taken with respect to χ.

Comparative Static A.4.1 (Prices)**.** Expected prices are decreasing.

We have

$$\mathbb{E}[p_h] = \int_{1-\chi}^{1} g_h^*(p_h) p_h \, \mathrm{d}p_h + \frac{\theta - \chi}{\theta} = 1 - \chi - (\theta - \chi)\log\left(1 - \frac{\chi}{\theta}\right) \tag{3}$$

$$\mathbb{E}[p_\ell] = \int_{\theta-\chi}^{\theta} g_\ell^*(p_\ell) p_\ell \, \mathrm{d}p_\ell = \frac{-\chi(1-\theta) - (1-\chi)\log(1-\chi)}{\chi}. \tag{4}$$

Thus, $\mathbb{E}'[p_h] = \log\left(1 - \frac{\chi}{\theta}\right) < 0$ and $\mathbb{E}'[p_\ell] = \frac{\chi + \log(1-\chi)}{\chi^2} < 0$, where the second inequality follows from the fact that $y < -\log(1-y)$ for any $y \in (0,1)$.

Comparative Static A.4.2 (Profits)**.** Expected total profits and expected profits of the high-quality firm are decreasing, while the expected profits of the low-quality firm are hill-shaped.

The expected profits of the high-quality and low-quality firm are respectively $\pi_h = 1 - \chi$ and $\pi_\ell = \chi(\theta - \chi)$, so that firms' total expected profits are $\pi = \pi_h + \pi_\ell = (1 - \chi(1 - \theta + \chi))$. While $\pi_h' < 0$ and $\pi' < 0$, π_ℓ is concave, with $\pi_\ell'(\chi) < 0$ if and only if $\chi > \frac{\theta}{2}$.

Comparative Static A.4.3 (Probability of buying high-quality)**.** The probability of cursed consumers buying the high-quality good is increasing.

We have

$$\mathbb{P}\big(u_h^\chi > u_\ell^\chi\big) = \mathbb{P}\big(p_h < p_\ell + (1-\theta)\big) = \int_{1-\chi}^{1}\int_{p_h-(1-\theta)}^{\theta} g_\ell^*(p_\ell) g_h^*(p_h)\,\mathrm{d}p_\ell \mathrm{d}p_h$$
$$= \frac{(1-\chi)\Big((1-\theta)\chi - (\theta-\chi)\log\Big(\frac{\theta(1-\chi)}{\theta-\chi}\Big)\Big)}{(1-\theta)^2\chi}. \tag{5}$$

Thus,

$$\mathbb{P}'\big(u_h^\chi > u_\ell^\chi\big) = \frac{\big(\theta-\chi^2\big)\log\Big(\frac{\theta(1-\chi)}{\theta-\chi}\Big) - (1-\theta)\chi(1+\chi)}{(1-\theta)^2\chi^2}.$$

The expression has the same sign as its numerator, denoted by $N(\chi)$, which is positive because $N(0) = 0 = N'(0)$, $N''(0) = \frac{(1-\theta)^2}{\theta} > 0$ and $N''' = \frac{2(1-\theta)^3\left(\theta-\chi^2\right)}{(\theta-\chi)^3(1-\chi)^3} > 0$.

Comparative Static A.4.4 (Utility of the two types of consumers)**.** The expected utility of a rational consumer and a cursed consumer are increasing. Moreover, the latter is positive if and only if χ is greater than some cutoff $\bar{\chi} \in (0,\theta)$.

The expected utility of a rational consumer is $U = 1 - \mathbb{E}\big[p_h\big]$, which is increasing because of comparative static A.4.1. The expected utility of a cursed consumer is

$$U_\chi = -\underbrace{\mathbb{P}\big(u_\ell^\chi \geq u_h^\chi\big)\mathbb{E}\big[p_\ell \big| u_\ell^\chi \geq u_h^\chi\big]}_{-\frac{\pi_\ell}{\chi}} + \mathbb{P}\big(u_h^\chi > u_\ell^\chi\big)\big(1 - \mathbb{E}\big[p_h \big| u_h^\chi > u_\ell^\chi\big]\big)$$
$$= \chi - \theta + \frac{(1-\chi)\left((1-\theta)\theta\chi - (\theta-\chi)\log(1-\chi) + (2-\theta)\theta(\theta-\chi)\log\left(1-\frac{\chi}{\theta}\right)\right)}{(1-\theta)^2\chi}.$$ [21]

Naturally, $\lim_{\chi\to 0} U_\chi = -\theta$ and $\lim_{\chi\to\theta} U_\chi = \theta$. Furthermore, differentiating U_χ with respect to χ yields

$$U_\chi' = 1 + \mathbb{P}'\big(u_h^\chi > u_\ell^\chi\big)\big(1 - \mathbb{E}\big[p_h \big| u_h^\chi > u_\ell^\chi\big]\big) - \mathbb{P}\big(u_h^\chi > u_\ell^\chi\big)\mathbb{E}'\big[p_h \big| u_h^\chi > u_\ell^\chi\big].$$

[21] We used

$$\mathbb{E}\big[p_h \big| u_h^\chi > u_\ell^\chi\big] = \frac{1}{\mathbb{P}\big(u_h^\chi > u_\ell^\chi\big)}\int_{1-\chi}^{1}\int_{p_h-(1-\theta)}^{\theta} p_h g_h^*(p_h) g_\ell^*(p_\ell)\mathrm{d}p_\ell \mathrm{d}p_h = \frac{(1-\theta)^2\Big((\theta-\chi)\log\Big(\frac{\theta}{\theta-\chi}\Big) - \chi\Big)}{(1-\theta)\chi - (\theta-\chi)\log\Big(\frac{\theta(1-\chi)}{\theta-\chi}\Big)}.$$

which is positive because $\mathbb{P}'\big(u_h^\chi > u_\ell^\chi\big) > 0$ by comparative static A.4.3 and $\mathbb{E}'\big[p_h \big| u_h^\chi > u_\ell^\chi\big] < 0$.[22]

Comparative Static A.4.5 (Consumer surplus)**.** Consumer surplus is u-shaped, equal to zero at the left limit of the exploitation region and positive at the right limit. Therefore, it is positive if and only if χ is greater than some cutoff $\hat{\chi} \in (0, \theta)$.

Consumer surplus is defined as

$$S \equiv \chi U_\chi + (1-\chi)U.$$

Naturally, $S(0) = 0$ and $\lim_{\chi \to \theta} S = \theta$. Besides,

$$S' = \chi U_\chi'(\chi) + U_\chi(\chi) - U(\chi) + (1-\chi)U'(\chi)$$

As χ goes to zero, all terms in S' go to zero except $U_\chi < 0$, so that $S'(0) < 0$. Moreover,

$$S'' = \frac{(1-\theta)\left(\frac{1}{1-\chi} + \frac{1}{\theta-\chi} - 2\theta\right) + 2\log\left(\frac{\theta-\chi}{\theta(1-\chi)}\right)}{(1-\theta)^2},$$

which is positive since $S''(0) = \frac{2\theta+1}{\theta} > 0$ and $S''' = \frac{1-\theta}{(1-\chi)^2(\theta-\chi)^2} > 0$. Finally, note that $\hat{\chi} < \bar{\chi}$, i.e. S becomes positive for a lower χ than U_χ, since when $U_\chi = 0$ we have that $S > 0$.

Comparative Static A.4.6 (Welfare)**.** Welfare is u-shaped and maximal only at either limit of the exploitation region.

Welfare W is defined in equation (1) and its derivative W' in equation (2). Naturally, $W(0) = 1$ and $\lim_{\chi \to \theta} W = 1$. Besides,

$$W'' = \chi \mathbb{P}''\big(u_h^\chi > u_\ell^\chi\big) + 2\mathbb{P}'\big(u_h^\chi > u_\ell^\chi\big).$$

[22]Differentiating $\mathbb{E}\big[p_h \big| u_h^\chi > u_\ell^\chi\big]$ yields

$$\mathbb{E}'\big[p_h \big| u_h^\chi > u_\ell^\chi\big] = -\frac{(1-\theta)^2\chi \overbrace{\left(\log(1-\chi) + (2-\theta)\theta\log\left(\frac{\theta}{\theta-\chi}\right) - \chi\left(1-\theta+\log\left(\frac{\theta(1-\chi)}{\theta-\chi}\right)\right)\right)}^{A(\chi)}}{(1-\chi)\left(\chi - \theta\chi - (\theta-\chi)\log\left(\frac{\theta(1-\chi)}{\theta-\chi}\right)\right)^2}.$$

The expression has the opposite sign of $A(\chi)$, which is positive because $A(0) = 0 = A'(0) = A''(0)$ and $A''' = \frac{(1-\theta)^2\left(\theta+\chi-2\chi^2\right)}{(\theta-\chi)^3(1-\chi)^2} > 0$. The last inequality follows from the fact that the term $\theta + \chi - 2\chi^2$ is concave in χ and positive in $\chi = 0$ and $\chi = \theta$.

Since $\mathbb{P}'\left(u_h^\chi > u_\ell^\chi\right) > 0$ by comparative static A.4.3, a sufficient condition for W to be convex is that $\mathbb{P}''\left(u_h^\chi > u_\ell^\chi\right) \geq 0$. Differentiating $\mathbb{P}'\left(u_h^\chi > u_\ell^\chi\right)$ yields

$$\mathbb{P}''\left(u_h^\chi > u_\ell^\chi\right) = \frac{\frac{(1-\theta)\chi(\theta(2-\chi)-\chi)}{(\theta-\chi)(1-\chi)} - 2\theta \log\left(\frac{\theta(1-\chi)}{\theta-\chi}\right)}{(1-\theta)^2\chi^3}.$$

The expression has the same sign as its numerator, denoted by $M(\chi)$, which is positive because $M(0) = 0 = M'(0)$ and $M'' = \frac{2(1-\theta)^3\left(\chi\theta-\chi^3\right)}{(1-\chi)^3(\theta-\chi)^3} > 0$.

A.5 Proof of proposition 6

The equilibrium analysis of section 4.3.1 is affected only in the case of vertical differentiation, where $\gamma\theta$ replaces θ. When $\theta \leq \chi$, the results follow directly from the fact that, for any γ, all consumers buy the high-quality product at $p_h^* = 1-\gamma\theta$. When $\theta > \chi$, the same comparative statics hold as long as $\theta\gamma \leq \chi$. Consider hence the region $\chi < \gamma\theta \leq \theta$. While the expected profits of the high-quality firm $\pi_h = 1-\chi$ are constant, the expected profits of the low-quality firm $\pi_\ell = \chi(\gamma\theta-\chi)$ are strictly increasing in γ, which proves the statement about firms' profits. Total welfare is now determined only by the probability that a cursed consumer buys the high-quality good, which is defined in equation (5) (for $\gamma = 1$). Differentiating with respect to θ yields

$$\mathbb{P}'\left(u_h^\chi > u_\ell^\chi\right) = \frac{(1-\chi)\left(\left(1-\theta^2\right)\chi - \theta(\theta-2\chi+1)\log\left(\frac{\theta(1-\chi)}{\theta-\chi}\right)\right)}{(1-\theta)^3\theta\chi}.$$

The sign of the expression is determined by the sign of the numerator, denoted $N(\theta)$, which is negative because $N(1) = 0 = N'(1) = N''(1)$ and $N''' = \frac{(1-\chi)\chi^2\left(2\chi^2+3\theta-4\theta\chi-\chi\right)}{\theta^2(\theta-\chi)^3} > 0$, where the last inequality follows from the fact that $\left(2\chi^2+3\theta-4\theta\chi-\chi\right)$ is decreasing in χ and positive for $\chi = \theta$. As $\mathbb{P}\left(u_h^\chi > u_\ell^\chi\right)$ is decreasing in θ, it is also decreasing in γ in the region $\chi < \gamma\theta \leq \theta$, proving the statement on welfare. Finally, consider consumer surplus. The utility of a rational consumer decreases in γ in the whole region $\chi < \gamma\theta \leq \theta$ because the expected price of the high-quality good $\mathbb{E}\left[p_h\right]$, which is defined in equation (3) increases (continuously from $1-\gamma\theta$, which obtains for γ such that $\gamma\theta = \chi$). Indeed, differentiating with respect to θ, we have $\mathbb{E}'\left[p_h\right] = -\frac{\chi+\theta\log\left(1-\frac{\chi}{\theta}\right)}{\theta}$, which is positive in $\theta = 1$ ($\mathbb{E}'\left[p_h\right]|_{\theta=1} = -\chi-\log(1-\chi) > 0$, from the fact that $y < -\log(1-y)$ for any $y \in (0,1)$), and hence everywhere since $\mathbb{E}''\left[p_h\right] = -\frac{\chi^2}{(\theta-\chi)\theta^2} < 0$. The utility of a cursed consumer decreases in γ in the region

$\chi < \gamma\theta \leq \theta$ at least for $\gamma\theta$ sufficiently close to χ because, as just shown above, $\mathbb{P}\left(u_h^\chi > u_\ell^\chi\right)$ decreases and $\mathbb{E}\left[p_h\right]$ increases, while it is apparent from equation (4) that the expected price of the low-quality good $\mathbb{E}\left[p_\ell\right]$ also increases in θ, and hence in γ.

A.6 Proof of proposition 7

Consider a candidate symmetric equilibrium in which both firms choose high quality with probability $\theta \in (0,1)$, so that the belief of a cursed consumer about undisclosed quality is θ. Once firms' investment is determined accordingly, the possible sub-games are equivalent to the ones in our main model, so that Proposition 3 still describes the continuation equilibrium of the game.

Suppose first that $\chi \geq \theta$, so that no exploitation occurs. Then, to find it optimal to stick to its investment strategy each firm must be indifferent between the two quality levels. As a firm makes positive sales profits, equal to $p_h = 1-\theta$, only if it chooses high-quality and its rival turns out to have low quality, it must be that

$$(1-\theta)(1-\theta) = c.$$

Solving for θ yields the unique solution $\theta^* = \tilde{\chi} \equiv 1-\sqrt{c}$. Since it must be that $\chi \geq \tilde{\chi}$, only then this equilibrium exists.

Suppose that $\chi < \theta$, instead, so that exploitation occurs. The two firms make positive profits only in the case of vertical differentiation. In this case, using the results of section A.4, $\pi_h = (1-\chi)$ and $\pi_\ell = \chi(\theta-\chi)$. For each firm to be indifferent between the two quality levels it must be that

$$(1-\theta)(1-\chi) - \theta\chi(\theta-\chi) = c. \tag{6}$$

Denote the LHS of equation (6), seen as a function of θ, by $LHS(\theta)$. We have that $LHS'(\theta) = -2\theta\chi + \chi^2 + \chi - 1$, which is negative whenever $\chi < \theta$. Also, $LHS(\chi) = (1-\chi)^2$, which is strictly greater than c if and only if $\chi < \tilde{\chi}$, and $LHS(1) = -(1-\chi)\chi$. Thus, when $\chi < \tilde{\chi}$, by continuity equation (6) has a unique solution $\theta_e^* \in (\chi,1)$ and this candidate equilibrium exists. When $\chi > \tilde{\chi}$, this candidate equilibrium does not exist, while if $\chi = \tilde{\chi}$, $\theta_e^* = \tilde{\chi}$, i.e. the two equilibrium configurations coincide.

Hence, the two equilibrium configuration span the whole parameter space and do not overlap, which concludes the first part of the proposition, with $\theta^* = \tilde{\chi}$ if $\chi \geq \tilde{\chi}$ and $\theta^* = \theta_e^*$ if $\chi < \tilde{\chi}$ (note the proof allowed $\theta = 0$ or $\theta = 1$ and hence rules out symmetric pure investment strategies). To see that θ^* is decreasing in c, as mentioned in the main body, note first of all that θ^* is continuous and it is apparent that $\tilde{\chi}$ decreases with c. It hence suffices to show that also θ_e^* is decreasing. Replacing $\theta_e^*(c)$ in equation (6), implicitly differentiating both sides with respect to c and rearranging yields

$$\frac{d\theta_e^*}{dc} = \frac{2}{-1+\chi+\chi^2-2\chi\theta_e^*},$$

which is indeed negative whenever $\theta_e^* > \chi$.

Consider now the second part of the proposition on the effect of policy. Assuming that consumers' purchasing decisions are efficient, welfare for any given symmetric probability θ at which each firm invests in high-quality, as per the equilibrium concept, is given by

$$W(\theta) = \theta^2(1-2c) + 2\theta(1-\theta)(1-c).$$

The first term corresponds to the case in which both firms invest in high quality, while the second term to the case in which only one firm does so. $W(\theta)$ is concave, and uniquely maximized for the interior value $\theta_o \equiv 1-c$, which obtains as the solution to $W'(\theta) = 2-2c-2\theta = 0$. Since $\tilde{\chi} < \theta_o$, in the equilibrium without exploitation welfare is inefficiently low, and it is strictly increasing in firms' investment θ. Thus, suppose that $\chi \geq \tilde{\chi}$ and consider a consumer policy that reduces χ below but sufficiently close to $\tilde{\chi}$. This policy has a first negative effect on welfare since some consumers will now buy the low quality good. To show that this policy is sometimes unambiguously harmful, it hence suffices to show that there are cases in which $\theta_e^*(\chi)$ is increasing sufficiently close to $\tilde{\chi}$ (where it converges continuously to $\tilde{\chi}$). After replacing $\theta_e^*(\chi)$ in equation (6), implicitly differentiating both sides with respect to χ and rearranging, one obtains

$$\frac{d\theta_e^*}{d\chi} = \frac{1-\theta_e^*(\chi)-2\chi\theta_e^*(\chi)+(\theta_e^*(\chi))^2}{-1+\chi+\chi^2-2\chi\theta_e^*(\chi)}.$$

Replacing $\chi = \tilde{\chi}$ and $\theta_e^*(\chi) = \tilde{\chi}$ in the expression yields

$$\frac{d\theta_e^*(\chi)}{d\chi}\Big|_{\chi=\tilde{\chi}} = \frac{c-3\sqrt{c}+1}{c-\sqrt{c}+1}$$

which is strictly positive for $c < \frac{1}{2}\left(7-3\sqrt{5}\right)$.

Finally, to see that perfect consumer protection (i.e. $\chi = 0$) increases welfare, note that it is also the case that $\theta_e^* \leq \theta_o = 1 - c$, with equality if and only if $\chi = 0$. Indeed, evaluating the LHS of (6) in $\theta = 1 - c$ yields $-c^2\chi - (1-\chi)\chi + c\left(1 + \chi - \chi^2\right) \leq c$, with strict inequality unless $\chi = 0$. Perfect consumer protection hence ensures allocative efficiency and raises investment to its most efficient level. The fact that $\theta_e^*(\chi)$ is continuous in χ and the previous observations also imply that, as mentioned in the main body, $\theta_e^*(\chi)$ necessarily increases as a result of consumer protection for χ sufficiently small.

A.7 Proof of proposition 8

Without loss of generality, suppose that the high-quality good is located at the left end of the unit interval, so that $t_{1-\chi}$ and t_χ denote both the locations of the rational and cursed marginal consumer respectively, and the fractions of rational and cursed consumers that firm h serves. Given that the market will always be covered in equilibrium, the corresponding fractions served by firm ℓ are $1 - t_{1-\chi}$ and $1 - t_\chi$.

A.7.1 Equilibrium

We focus on a candidate equilibrium in which $t_{1-\chi} \in (0,1)$ and $t_\chi \in (0,1)$. Then, it must be that $t_{1-\chi} = \frac{1}{2}(1 - p_h + p_\ell + q_h)$ and $t_\chi = \frac{1}{2}(1 - p_h + p_\ell + (1-\theta)q_h)$. Thus, the demands faced by firms h and ℓ are respectively $D_h = (1-\chi)t_{1-\chi} + \chi t_\chi$ and $D_\ell = 1 - D_h$, and their profits are $\pi_h = p_h D_h$ and $\pi_\ell = p_\ell D_\ell$. Equilibrium prices must then necessarily satisfy first order conditions, which yield $p_h^* = \frac{1}{3}(3 + q_h - \theta\chi q_h)$ and $p_\ell^* = \frac{1}{3}(3 - q_h + \theta\chi q_h)$, so that $t_{1-\chi}^* = \frac{1}{6}(3 + q_h(2\theta\chi + 1))$, $t_\chi^* = \frac{1}{6}(3 - (\theta(3-2\chi) - 1)q_h)$, $D_h^* = \frac{1}{6}(3 + q_h(1-\theta\chi))$, $D_\ell^* = \frac{1}{6}(3 - q_h(1-\theta\chi))$, $\pi_h^* = \frac{1}{18}(3 + q_h(1-\theta\chi))^2$ and $\pi_\ell^* = \frac{1}{18}(3 - q_h(1-\theta\chi))^2$. Notice that indeed $t_{1-\chi}^* \in (0,1)$ and $t_\chi^* \in (0,1)$.

While profits in the relevant range are concave, so that second order conditions are necessarily satisfied, for this to be an equilibrium, it should also be the case that no firm profits from foregoing a segment of the market, typically the cursed segment for firm h and the rational segment for firm ℓ. Once we allow for $t_{1-\chi} = \{0,1\}$ or $t_\chi = \{0,1\}$, a firm i's 'global" demand is continuous in p_i (although not necessarily differentiable in these points), and so are its global profits $\Pi_i(p_i)$. And since Π_i is concave in the relevant deviation range, if we denote by ∂_+ the right derivative, two sufficient conditions for the two deviations not to be profitable are

$$\partial_+\Pi_h(p_h)\Big|_{p_h=1-\theta q_h+p_\ell^*+q_h} = -\frac{1}{6}(1-\chi)\,(6+(2-\theta(6-\chi))q_h) < 0$$

and

$$\partial_+\Pi_\ell(p_\ell)\Big|_{p_\ell=p_h^*-q_h+1} = -\frac{1}{6}\chi\,(6-(2+\theta(3+\chi))q_h) < 0.$$

The two conditions are indeed satisfied when $q_h < 1$.

A.7.2 Comparative statics

While the effects of an increase in χ described in the proposition can be derived by simple inspection of the objects of interest, we will disentangle the different economic forces behind each comparative statics, in particular for welfare. Throughout, we drop the superscript * for ease of notation and the subscript $k \in \{\chi, 1-\chi\}$ refers respectively to the cursed and the rational segment of the market. Also, each derivative is taken with respect to χ.

Comparative Static A.7.1 (Prices, Demands and Profits)**.** The price and demand of the high-quality firm are decreasing while the price and demand of the low-quality firm are increasing. Total profits are decreasing.

Note that $0 > p_h' = -p_l'$, that $0 < t_\chi' = t_{1-\chi}'$ (we can hence drop the subscript from t_k') and that $D_h' = -D_l' = t' + t_\chi - t_{1-\chi} < 0$. We may thus write the derivative of total profits $\Pi \equiv \pi_h + \pi_\ell$ as

$$\Pi' = p_l'(D_l - D_h) + D_l'(p_l - p_h) < 0.$$

Thus, total profits decrease because variations in demands and prices of the two firms have opposite sign but the same magnitude and the high-quality firm is serving more consumers.

Comparative Static A.7.2 (Allocative efficiency of rationals and cursed)**.** For both the rational and the cursed segment average allocative efficiency is increasing.

The average allocative efficiency of the purchasing decisions of each consumer segment $k \in \{\chi, 1-\chi\}$ is

$$W_k = \int_0^{t_k^*} (v + q_h - t)\,\mathrm{d}t + \int_{t_k^*}^1 (v - 1 + t)\,\mathrm{d}t.$$

Thus, we may write

$$W_k' = 2(t^{fb} - t_k)t' > 0,$$

where $t^{fb} \equiv (1+q_h)/2$ is the efficient location of the marginal consumer. Average welfare on each segment increases with χ as the marginal consumer gets closer to its efficient location.

Comparative Static A.7.3 (Welfare)**.** Welfare is convex, decreasing if $\theta \leq \frac{4}{7}$ and u-shaped otherwise. It is maximal when $\chi = 0$.

Welfare is $W = (1-\chi)W_{1-\chi} + \chi W_\chi$. Thus, the variation in total welfare can be decomposed as

$$W' = \underbrace{W_\chi - W_{1-\chi}}_{CE_W<0} + \underbrace{2t'(t^{fb} - D_h)}_{EE_W>0} = \frac{1}{36}\theta(\theta(16\chi - 9) - 4)q_h^2.$$

The composition effect (CE_W) is negative since cursed consumer take on average less efficient decisions. The equilibrium effect (EE_W) is positive since the demand of the high quality firm gets closer to its efficient level. The overall effect is positive if and only if $\chi > \frac{4+9\theta}{16\theta}$, which in particular requires $\theta > \frac{4}{7}$. It is in this case that reducing the fraction of cursed consumers by means of imperfect consumer protection may decrease welfare. Differentiating a second time and noticing that $t'' = 0$ yields

$$W'' = \underbrace{2t'(t_{1-\chi} - t_\chi)}_{CE_W'>0} + \underbrace{2t'(-D_h')}_{EE_W'>0} = \frac{4}{9}\theta^2 q_h^2,$$

where $CE_W' > 0$ since $t_{1-\chi} > t_\chi$ and $EE_W' > 0$ since $D_h' < 0$. The last statement in the comparative statics simply follows from $W|_{\chi=0} - W|_{\chi=1} = \frac{1}{36}\theta(4+\theta)q_h^2 > 0$.

Thus, for low levels of χ, CE_W dominates EE_W. But because CE_W gets smaller in absolute value as χ increases and EE_W gets larger, EE_W may eventually dominate. The reason CE_W gets less negative as χ increases is that CE_W is driven by those rational consumers located where they would make different decisions if they were cursed. Because of EE_W, this group lives closer and closer to the efficient allocation t^{fb} as χ increases and therefore incurs smaller and smaller efficiency losses from turning cursed. Instead, EE_W is larger the smaller $t_{1-\chi}$ and t_χ, that is the further the two marginal consumers that react to price changes by switching to the high-quality product live from the low-quality firm. Because $t_{1-\chi} > t_\chi$, more cursedness implies larger efficiency gains from equilibrium price effects.

Comparative Static A.7.4 (Utility of rationals and cursed)**.** The average utility of rational and cursed consumers is increasing.

The average utility for each consumer segment $k \in \{\chi, 1-\chi\}$ is

$$U_k = \int_0^{t_k^*} (v + q_h - p_h^* - t)\,\mathrm{d}t + \int_{t_k^*}^1 (v - p_\ell^* - 1 + t)\,\mathrm{d}t.$$

Thus, after noticing that $p_l' = t'$ and $p_h' = -t'$, we obtain

$$U_k' = (q_h - (p_h - p_l))\,t' > 0,$$

and we can hence drop the subscript from U_k'. This equation clarifies that the assessment of the effect of a variation in χ on average surplus of each segment does not incorporate considerations on transportation costs. Intuitively, this occurs because the overall utility variation of consumers who switch their purchasing decision as a result of the change in χ is zero. Indeed, for any consumer that gains by switching to firm h, there is a consumer who suffers a proportional loss by having to shift to firm l and vice-versa (and in particular, the utility of the previous and new marginal consumer is identical since for the latter the variation in price is completely offset by the variation in transportation costs). Thus, the effect is positive since more consumers now buy the high-quality good, which yields a higher net utility than the low-quality good as consumers always appropriate some of the benefits of competition (i.e. $p_h - p_l < q_h$).

Comparative Static A.7.5 (Consumer surplus)**.** Consumer surplus is convex, increasing if $\theta \leq 4/9$ and u-shaped otherwise. It is maximal in $\chi = 1$ if $\theta < 4/5$ and in $\chi = 0$ otherwise.

Consumer surplus is $S = (1-\chi)U_{1-\chi} + (\chi)U_\chi$. The variation in total surplus can then again be decomposed in a negative composition effect (CE_S) and a positive equilibrium effect (EE_S)

$$S' = \underbrace{U_\chi - U_{1-\chi}}_{CE_S<0} + \underbrace{U'}_{EE_S>0} = \frac{1}{36}\theta(4 - \theta(9-8\chi))q_h^2.$$

The total effect is positive if and only if $\chi > \frac{9\theta-4}{8\theta}$, which is always satisfied if $\theta < \frac{4}{9}$. Differentiating a second time

$$S'' = \underbrace{0}_{CE_S'} + \underbrace{(-2p_h')t'}_{EE_S'>0} = \frac{4}{9}\theta^2 q_h^2 > 0.$$

Since the composition effect is independent of χ, the convexity of U_χ is driven exclusively by the equilibrium effect, which increases with χ as the high-quality price premium $p_h - p_l$ decreases. The last statement in the comparative static simply follows from $S|_{\chi=0} - S|_{\chi=1} = \frac{1}{36}\theta(5\theta - 4)q_h^2$.