The Demand for Enhanced Annuities

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Abstract

In enhanced annuities, the annuity payment depends on one's state of health at some contracted date while in "standard annuities", it does not. The focus of this paper is on an annuity market where "standard" and enhanced annuities are offered simultaneously. When all insured know equally well on their future health status either enhanced annuities drive standard annuities out of the market or vice versa. Both annuity types can exist simultaneously when the insured know varying exactly on their risk type. In the case of the existence of such an "interior" solution, its is derived that this solution must be unique in the case of risk averse insured and that it Pareto-dominates the corner solution. Finally, it is shown that in all cases where at least part of the insured buy enhanced annuities social welfare is reduced.

JEL-Classification: D14, D61, G11, G23

1 Introduction

Annuitizing a part of wealth in order to secure a constant consumption level[1] over the whole lifetime seems like a reasonable thing to do for a risk-averse individual. In particularly, annuities include protection against the loguevity risk, i.e. the risk of outliving the financial assets of an individual. This risks is not covered by any other financial investment possibilities. Yet, the theoretical results on a functioning annuity market (i.e. the advantageousness of annuities over conventional bonds, see Yaari (1965), Brown et al. (2005)) significantly differ from empirical

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[1]This result holds true when the subjective discount is equal the interest rate.

evidence of annuity demand (see e.g. Mitchell et al. (1999) and Dushi/Webb (2004)). Many different explanations have been provided in order to solve the so-called "annuity puzzle". Brown (2001) identifies money's worth[2], adverse selection, bequest-motives[3], financial inflexibility of annuities to secure against unsecure future expenses[4] and the fact that risk sharing might also be possible within families[5] as the most meaningful approaches to the annuity puzzle. However, a concluding clarification on the relevance of the different approaches has not been provided yet.

Concerning adverse selection, it is often argued that a potential insured has superior knowledge of his/her life expectancy compared to the insurance company at the conclusion of an annuity contract. As Akerlof (1970) shows, adverse selection can lead to a complete market failure. Rothschild/Stiglitz (1976) and Wilson (1977) examine the impact of adverse selection on insurance markets and discussed potential equilibria. Doherty/Thistle (1996) show that superior knowledge can have positive private value even though the social value is negative. Hence, individuals might be willing to acquire and use private information even though the overall social effects might be non- desirable.

Adverse selection on annuity markets has repeatedly been in the focus of many papers. Finkelstein/Poterba (2004) find empirical confirmation that adverse selection plays an important role in the annuity market and Brown/Orszag (2006) outline that the mortality rates of annuitants in different annuity markets differ from the overall mortality rates. Palmon/Spivak (2007) measure the social effects of adverse selection in the annuity market by using numerical analysis. Eichenbaum/Peled (1987) show that in annuities markets that suffer from adverse selection, some individuals will choose to accumulate capital privately even though annuities offer higher return rates.

Enhanced (or impaired) annuities were designed to address the problem of adverse selection in the annuity market. In a standard annuity, the annual payment does not depend on the state of health of the potential insured while in enhanced annu-

[2]See Mitchell et al. and James/Song (2001).

[3]See Abel/Warshawsky (1988) and Friedman/Warshawsky (1990).

[4]See Strawczynski (1999).

[5]See Kotlikoff/Spivak (1981) and Brown/Poterba (1999).

ities it does depend on the health status. There are different designs of enhanced annuities possible. For example, they can be conditioned on the health status at the contract conclusion or at the retirement commencement or at any other point of time.
Enhanced annuities are hence particularly attractive to persons with impaired health when they are contracted before the health status becomes observable to the insurance company. Being a low risk in the annuity market implies having a shorter life expectancy than other individuals. Therefore, risk classification is done by allowing a higher pension payment for characteristis of an individual that normally shorten the life expectancy. Yet, these factors usually also include things which are difficult to verify. One example is smoking. Since smoking significantly reduces life expectancy it would make sense to give smokers a higher annuity payment if all other are things equal. But it is difficult to consider smoking for pricing, since it is mutable (for a detailed analysis of the implications of mutable risk classification characteristics, see Bond/Crocker (1991)) and hard to verify.

The focus of this paper is on deferred annuities where the contract inception precedes the retirement entry age. This implies that the annuitization is not fixed when the contract is signed but depends on the health state at the entry age. This assumes that the insurance companies will do a more precise analysis of one's health at the retirement date. The superior knowledge of the potential insured diminishes at this point.

Most of the existing literature on enhanced annuities employs an actuarial science approach. Weinert (2006), e.g, discusses underwriting approaches to enhanced annuities while Ainslie (2000) analyzes the potential market for enhanced annuities and covers different actuarial aspects such as reserving, modelling mortality and other pricing features. Richards/Jones (2004) focus on the impact of mortality changes on enhanced annuities.
Recently though, Hoermann/Russ (2007) examined the potential economic relevance of enhanced annuities in an insurance market using Monte-Carlo simulations. They find that those companies who do not offer enhanced but offer standard annuities will end up with group of insured with a higher life expectnacy on average.

Thus, they their profit will be reduced or they need to charge higher prices.
In the model introduced in this paper, results are derived for markets where enhanced and standard annuities are offered simultaneously. It expands the informational setting of Hoermann/Russ (2007) by allowing different forms of superior knowledge of the insured and adds an analytical approach to the numerical work of Hoermann/Russ(2007).
This approach distinguishes between the case where all insured have the same knowledge on their health status,[6] i.e. they have homogenous knowledge, and the case, where some insured know more than others, i.e. heterogenous knowledge. When all insured know equally well on their future health status either enhanced annuities drive standard annuities out of the market or vice versa. Both annuity types can exist simultaneously in the case of heterogeneous knowledge. In the case of the existence of such an "interior" solution, its is derived that this solution must be unique in the case of risk averse insured and that it Pareto-dominates the corner solution. Finally, it is shown that in all cases where at least part of the insured buy enhanced annuities social welfare is reduced.

After this short introduction the remaining paper is structured as follows. In section two the model background is illustrated. The implications of homogeneous superior knowledge are presented in section three while, in section four, heterogeneous superior knowledge is introduced. Finally, section five contains some concluding remarks.

2 Model Background

So far, there is not a capacious amount of economic literature on enhanced annuities. Most of the existing literature on enhanced annuities employs an actuarial science approach. Weinert (2006), e.g., gives a market survey and discusses underwriting approaches to enhanced annuities while Ainslie (2000) analyzes the potential market for enhanced annuities and covers different actuarial aspects such as reserving, modelling mortality and other pricing features. Richards/Jones (2004)

[6]This assumption does not imply that all insured are the same risk. They only know the same on how their risk type will develop.

focus on the impact of mortality changes on enhanced annuities.

Recently though, Hoermann/Russ (2008) first examine the economic relevance of enhanced annuities in an insurance market using Monte-Carlo simulations. In their model, the insured have perfect knowledge regarding their life expectancy and will choose a different insurer if the premium difference exceeds a certain threshold. They find that those companies who do not offer enhanced but standard annuities suffer from adverse selection. The model introduced in this paper expands the informational setting of Hoermann/Russ (2008) by allowing different forms of superior knowledge of the insured and adds an analytical approach to the numerical work of Hoermann/Russ (2008).

In this paper, a market is analyzed where standard and enhanced annuities are offered simultanously. A standard annuity does not contract upon the health status of a potential insured while enhanced annuities do so. This adresses the problem that the health status of the insured influences the mortality rate and, thus, determines the advantageousness of an annuity contract with a fixed premium and a fixed payment for the insured. Hence, if the insured has superior knowledge regarding his/her life expectancy, adverse selection threatens. Low risks could be driven out of the market. Yet, if enhanced annuities can take all factors correlated with one's life expectancy into account, adverse selection problems are eliminated.

In the approach presented here, the focus is on deferred annuities. Thus, the retirement payments begin considerably later than the contract inception. It is assumed that the insurance companies could do a more precise underwriting at the retirement commencement than at the contract conclusion and that informational asymmetries might diminish between those two points of time as argued in the introduction.

In my model, it is supposed that there is a homogenous risk group willing to buy an annuity by paying a lump-sum P in t_0. The annuity payment begins at date t_1, i.e. a deferred annuity is contracted. Due to some factors like life style and genetic disposition, the homogenous risk group divides into two groups between t_0 and t_1: One with a high frailty rate and one with a low frailty rate which directly affects the survival probabilities within each risk group. As illustrated in figure 1, the probability of becoming a high risk in t_1 is $1 - \eta$ where $0 \leq \eta \leq 1$ and, hence, the probability of becoming a low risk is η.

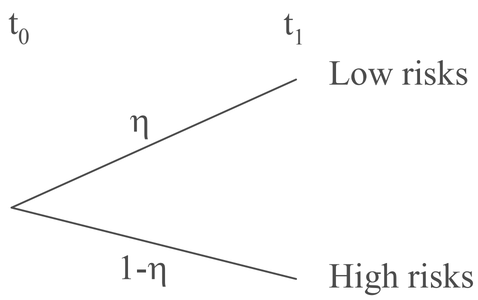

Figure 1: Development of risks

The annuity buyers can choose in t_0 whether they buy a standard annuity with an annual payment of b or an enhanced annuity where the annual payment depends on the health status in t_1. Hence, having chosen the enhanced annuity, the insured either receive a payment of b^L if they turn out to be a low risk or b^H if they are a high risk in t_1. Corrspondingly, it holds that $b^H \leq b \leq b^L$.

The assumption that all individuals invest a fixed lump-sum P in annuity insurance reagardless of the annuity product chosen might be quite restrictive. An individual faces a higher income risk during retirement if he/she purchases an enhanced annuity since the payouts depend on the realisation of the risk type. Thus, one can argue that individuals that decide for enhanced annuities and face uncertainty on their risk type will rather invest more or accumulate some capital privately to cover the additonal risk. Correspondingly, it would be sensible to allow a modelling of the annuity demand depending on the income and income risk before and after retirement. However, it is empirically shown e.g. by Skinner (1988) that the capital saved for retirement was not dependent on the riskiness of the income. Thus, the simplifications of a fixed investment in annuities can be motivated by this empirical fact.

Assumptions about how individuals value the utility of future annuity payments have to be made. Folllowing Yaari, an intertemporally separable utility function for the vector $\mathbf{b}$ of payment flow $v(\mathbf{b})$ is assumed such that $u(b)$ denotes the utility function of the annual payment of b at any point of time. Suppose that the agent is risk-averse and hence, we have $u' > 0$ and $u'' < 0$. In addition, it is assumed that an individual will decide for the annuity product that offers the higher expected utility to him/her. Concerning the market on which annuities are sold, it is assumed that there is perfect competition. Thus, insurance companies will not make any profits but offer insurance at fair prices.

3 Homogeneous Knowledge of the Insured

In this section, it is assumed that all insured have the same knowledge on their life expectancy. This does not mean that all insured have the same risk type, it is simply assumed that there are not any insured that know better than others. There are only two possible types of this homogeneous knowledge: Either all insured know only the proportion of high and low risks in t_0 and, thus, they have identical knowledge as the insurance companies. The alternative is that all insured are already informed in t_0 about which risk type they will be in t_1, i.e. they have superior knowledge compared to the insurance company.[7]

If the insured have no superior knowledge, the following proposition holds:

Proposition 1 *If risk-averse individuals have no further information on their risk status in t_1 except than the general probability of becoming a high or a low risk, they will prefer the standard annuity to the enhanced annuity.*

Proof:

This holds true because the standard annuity and the enhanced annuity have the same expected payment. Yet, if an insured choose the enhanced annuity, he/she also bears the risk of either becoming a high or low risk which is covered by the standard annuity. Thus, a risk-averse individual prefers a standard annuity.

This result is similiar to the results of Bruigivani (1993) and Shesinski (2007). Bruigivani shows that individuals should always contract annuities immediately before their own knowledge of their risk status increases. Shesinski models the demand for annuities in dependance of the life income and retirement age. He shows

[7] Because of the constraint that η is the proportion of low risks in t_0 there are no other possibilities where the knowledge of the insured is homogeneous.

as well that uniformed insured should refrain from signing an annuity where the future payments depend on their health status and prefer an uniform annuity for all risk types with a fixed retriement entry age.
The proposition shows that, ex-ante, the standard annuity is beneficiary for an uninformed, risk-averse insured. The insurance companies cannot offer an enhanced annuity with higher expected utility as, by assumption, all insured invest P. Therefrom, insurance companies can only attract customers by increasing the payouts. As payouts can only be increased for one risk type by decreasing the payouts for the other risk types - otherwise the insurance company will default - such a contract will not attract both risk types and, thus, make negative profits. The insurance companies cannot as well offer more beneficiary standard annuity as only a standard annuity with higher payouts can attract customers. Such a contract cannot create non-negative profit. Therefore, the standard annuity must be an Nash-equilibrium in the Rothschild/Stiglitz-sense ex-ante.
Yet, it has to be examined whether this situation will also be an equilibrium ex-post. Ex-post, when the risk type is revealed, offering an enhanced annuity then and letting the insured contract out by disbursing P, all low risks would contract out and choose the enhanced annuity now. Hence, the standard annuity providers must reduce payout for the standard annuity from b to b^H, if the standard annuity provider cannot prevent the contracting out. In that case, the standard annuity does not differ anymore from the enhanced annuity. A prevention against this ex-post contracting out can be achived by reducing the disbursement sufficiently.
If instead it is assumed that the insured already know their future risk status, then it holds that:

Proposition 2 *If the potential insured have perfect knowledge on their future risk status, all individuals will receive annuity insurance according to their risk status in* t_1.

Proof:
In t_0, the low risk can either choose the standard annuity with certain annual payment of b or the enhanced annuity with the payment of b^L which is certain in that case as well. Since $b \leq b^L$ holds, the low risk type will always decide to purchase the enhanced annuity. High risks will prefer the standard annuity and, thus, they can be identified because of buying the standard annuity. Therefore,

the standard annuity will be priced accordingly and a seperating equilibrium will be implemented.[8]

As a part of the first best optimum, all insured receive a contract according to their risk type. A self-selection contract which is traditionally offered in situation with asymmetric information is strictly dominated by this solution since high risks are offered the same contract as in a self-selection design but low risks are better off. This welfare gain compared to the traditional asymmetric information situation occurs due to fact that the offered contract incorporates the elimination of asymmetric information in t_1.

4 Heterogeneous Knowledge of the Insured

Both cases of section 3 (the insured have either perfect knowledge regarding their life epectancy in t_0 or no superior knowledge compared to the insurance companies in t_0 at all) might not apply to all potential insured. It might be more realistic to expect some people to know perfectly while others only have a vague idea of how their life expectancy will develop. In terms of the smoking and unhealthy nutrition example an individual that smokes and eats in an unhealthy way might be quite sure that he/she will turn out to have a short life expectancy and, thus, be a low risk. Meanwhile someone who has developed "average" healthy habits might face a higher insecurity about his/her future risk type due to e.g. genetic disposition.

In this spirit, a signal is introduced to model heterogeneous superior knowledge. It is assumed that all potential insured receive a signal z in t_0 that is a realisation of the random variable Z. Z is continuously distributed on $[0, 1]$ with probability density function $g(z)$ such that

$$\mathrm{E}(Z|z < z^*) = c \cdot z^* \ \forall z^* \in [0, 1]. \tag{1}$$

[8]If annuities are only offered at a fixed premium P this seperating equlibrium will always be a Nash-Equilibrium in the Rothschild/Stiglitz-sense because a pooling contract (standard annuity) will contain a lower annual payment b if $\eta < 1$.

This means that the expected value conditioned on $z < z^*$ is some linear function of z^* as for uniform(0,1)-distributions. However, other probability density function are possible with this property.
A signal $z = 1$ implies that an individual will be a low risk in t_1 with certainty while $z = 0$ connotes that the individual will certainly be a high risk. Furthermore, it is assumed that $\mathrm{E}(Z) = \int z \cdot g(z)dz = \eta$. As it holds by (1) that

$$\mathrm{E}(Z) = \mathrm{E}(Z|z < 1) = c \cdot 1 = c$$

it can be followed that $c = \eta$.
It is assumed that all values of $z \in (0,1)$ can be interpreted as the probability of becoming a low risk. The heterogenous superior knowledge of the potential insured is hence modelled in such a way, that the individuals who receive a signal z close to 0 or close to 1 have significantly more knowledge than the insurance company because they know quite exactly about their future risk type. Meanwhile those individuals who received a signal close to η only have some knowledge on their furture risk type which is comparable to the knowledge of the insurance company.[9]

The individuals are now facing a different decision problem than in section 3 since they will take the additional information into account for their decision in t_0. The standard annuity has the expected utility of

$$v(\mathbf{b}) = \sum_{k=0}^{\omega-x} \alpha(k) \cdot u(b) \cdot {}_kp_x = u(b) \cdot e \qquad (2)$$

$$\text{with } e = \sum_{k=0}^{\omega-x} \alpha(k) \cdot {}_kp_x \text{ and}$$

the following denotations: ω is the maximum age, x is the pension entry age, $\alpha(k)$ is a subjective discount factor in the k^{th} period and ${}_kp_x$ is the survival probability. e is a factor that includes the sum of the subjective discount rate times the survival probabilties. (2) implies that $v(\mathbf{b})$ is an increasing and concave function.
The model assumes that b and e are dependent of the risk group buying the standard annuity and that both will adapt immediately to changes in the risk

[9] It is assumed that the insurance companies know about the average proportion of high and low risks in t_0.

group. Thus, it is supposed that insurance companies will anticipate which risks they will attract with a standard annuity at a certain price.
For an individual with the signal z, the enhanced annuity yields the expected utility of

$$z \cdot v(\mathbf{b^L}) + (1-z) \cdot v(\mathbf{b^H}) = z \cdot u(b^L)e^L + (1-z) \cdot u(b^H)e^H \tag{3}$$

where e^L is the subjectively discounted life expectancy for the low risks and e^H for the high risks, respectively.

By assumption, every individual compares the expected utility of the standard annuity and the enhanced annuity in t_1. For a consideration of a market equilibrium, the cutoff individual, i.e. the individual that has received the cut-off signal z^* and is indifferent between both annuity types, has to be examined. This implies that all individuals having received a signal $z > z^*$ will choose the enhanced annuity and all individuals having received a signal $z < z^*$ will choose the standard annuity (For a graphical illustration, please refer to figure 2). Prerequisite is the assumption that $u(b^H) \cdot e^H \leq u(b^L) \cdot e^L$ with $b^H \leq b^L$ and $b^H \cdot e^H = b^L \cdot e^L$. This assumption implies that an individual will rather be a low risk and receive higher annuity payouts.
Under these assumption, the following necessary condition must be fullfilled in an equilibrium:

$$\begin{aligned} u(\overbrace{\eta \cdot z^* \cdot b^L + (1-\eta \cdot z^*) \cdot b^H}^{\text{aligned payment of standard annuity}}) \cdot \overbrace{(z^* \cdot e^L + (1-z^*) \cdot e^H)}^{\text{aligned factor } e} &= \\ \underbrace{z^* \cdot u(b^L) \cdot e^L + (1-z^*) \cdot u(b^H) \cdot e^H}_{\text{utility of enhanced annuity}} & \end{aligned} \tag{4}$$

where $\eta \cdot z^*$ is - as defined in section 2 - the expected probability of becoming a low risk among the standard annuity buyers. Because of the alignment of the standard annuity, e and b depend on $\eta \cdot z^*$. (4) is a necessary conditon as in an equilibrium none should be in the position to increase his or her expected utility by changing the contract. All individuals having received a lower signal than z^* are more likley to become a high risk and, thus, gain more expected utility from

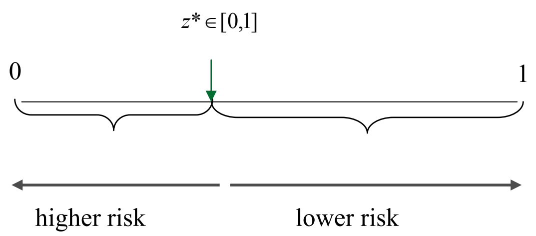

Figure 2: The cutoff signal z^*

the standard annuity. Those with an signal greater than z^* are "better" risks and they will choose the enhanced annuity. Only, the cut-off individual is indifferent between both annuities. Hence, nobody will be induced to substitute his or her product. In this setting, the following proposition holds:

Proposition 3 $z^* = 0$ *solves the equilibrium equation.*

Proof: Inserting $z^* = 0$ in (4) and using the assumption on the conditional expectation of Z yields

$$\begin{aligned} u(\eta \cdot 0 \cdot b^L + (1 - \eta \cdot 0)b^H) \cdot (0 \cdot e^L + (1 - 0) \cdot e^H) &= \\ 0 \cdot u(b^L) \cdot e^L + (1 - 0) \cdot u(b^H) \cdot e^H &\Leftrightarrow \quad u(b^H) \cdot e^H = u(b^H) \cdot e^H \end{aligned}$$

which proves the proposition.

At this potential equilibrium with $z^* = 0$, only an individual that will certainly turn out to be a high risk is indifferent between the standard and the enhanced annuity. Thus, the standard annuity will be priced accordingly. Therefrom, the pension payment of the standard annuity will be equal to the the payment for the high risk that choose the enhanced annuity, i.e. $b = b^H$. Altogether, one could argue that in this case the standard annuity is driven out of the market and only enhanced annuities are still offered. Hence, insurance against becoming a high or a low risk in t_1 is not available anymore since this protection is only included in standard annuities.
For the further analysis the following Lemma is required:

Lemma 1 *$z = 1$ is cannot be an equilibrium.*

Proof: This can be seen by inserting $z = 1$ in (4).

$$u(\eta \cdot 1 b^L + (1 - \eta \cdot 1) b^H) \cdot (1 \cdot e^L + (1-1) \cdot e^H) = u(\eta \cdot 1 b^L + (1 - \eta \cdot 1) b^H) \cdot e^L < u(b^L) \cdot e^L.$$

Correspondingly, it can be ruled out that only the standard annuity will be bought and enhanced annuities will not have any purchasers. In the following, possible interior solutions are examined. At an interior solution, part of the insured prefer the standard annuity while others purchase the enhanced annuity. As said before, the cutoff individual must be indifferent between the standard annuity and the enhanced annuity and, thus, value the protection against becoming a high or a low risk that is included in the standard annuity equally to the unfairness of the average priced standard annuity. The following property of an interior solution can be shown:

Proposition 4 *If an interior solution of (4) exists and the insured are risk-averse, then there cannot be another interior solution.*

Proof: This can be shown by examing the function

$$f(z^*) = z^* \cdot e^L \cdot [u(b^*) - u(b^L)] + (1 - z^*) \cdot e^H \cdot [u(b^*) - u(b^H)] \qquad (5)$$

where the expected utility of the the enhanced annuity is subtracted from the expected utility of the standard annuity and b^* is the aligned payout of the standard annuity for the cut-off signal z^*. Proposition 3 implies that $f(0) = 0$ while it can be deduced by applying Lemma 1 that $f(1) < 0$. For two or more interior solutions (5) has to have at least one inflection point.[10] Necessary condition for an inflection point is that the second derivative of a function equals 0 at this point. Thus, the first two derivatives of (5) have to be examined.

$$f'(z^*) = u(b^*) \cdot [e^L - e^H] + u(b^H) \cdot e^H - u(b^L) \cdot e^L + u'(b^*) \cdot (\eta \cdot b^L - \eta \cdot b^H) \cdot [z^* \cdot e^L + (1 - z^*) e^H]$$

[10]Note that an inflection point is not necessary for a single interior solution.

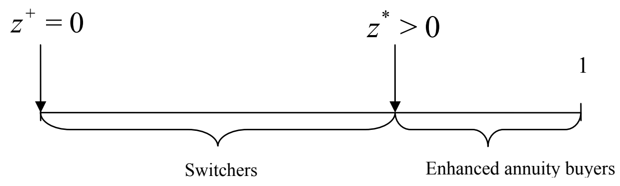

Figure 3: The two cutoffs $z^{+} = 0$ and $z^{*} > 0$.

and

$$f''(z^*) = 2 \cdot u'(b^L) \cdot [e^L - e^H] \cdot (\eta \cdot b^L - \eta \cdot b^H) + u''(b^*) \cdot (c \cdot b^L - c \cdot b^H)^2 \cdot [z^* \cdot e^L + (1 - z^*) e^H] \tag{6}$$

Setting (6) equal to 0 and rearranging it yields the following necessary condition for an inflection point where $r_A(b)$ denotes the absolute risk aversion coefficient for a premium b:

$$r_A(b^*) = -\frac{2 \cdot (e^H - e^L)}{\eta \cdot (b^L - b^H)[z^* \cdot e^L + (1 - z^*) \cdot e^H]}. \tag{7}$$

As $e^H - e^L > 0$ and $b^L - b^H > 0$ it follows that a necessary condition for a second interior solution is that $r_A(b) < 0$ which cannot be true for a risk averse individual.

Now, it remains to examine which solution will be reached in the case of the existence of an interior solution. From a welfare economics point of view it holds true that

Proposition 5 *The interior cutoff Pareto-dominates the corner solution with* $z^* = 0$.

Proof: The proposition can be derived by a case differentiation. Firstly, assume two possible cutoffs $z^+ = 0$ and $z^* > 0$. All individuals can be divided in two different groups. The first group purchases the enhanced annuity with both cutoffs, therefore the expected utility does not change with the cutoff for this group. The second group switches from the enhanced annuity in case of cutoff $z^+ = 0$ to the standard annuity in case of cutoff $z^* > 0$. Figure 3 demonstrates the distribution of the two sets.
The proposition follows from the fact that the switchers increase their expected

utility as the expected utility of the enhanced annuity does not change with a variation of the cutoff. If they prefer the standard annuity at an interior solution, they must be better off than with the enhanced annuity at $z^* = 0$.

Accordingly, it follows that the interior cutoff will be reached if existing. In cases, where no interior solutions exist, the corner solution where all individuals buy the enhanced annuity is the only possible solution for an equilibrium. Concludingly, it can be followed that enhanced annuities will definitely attract some purchasers. Examining the steadiness of such a cutoff reveals that a standard annuity with a lower payout cannot induce any purchaser to contract out as explained above. The corner solution $z^* = 1$ is not possible if there are different risk types. Any other standard annuity with a payout $\hat{b}$ such as $\bar{z}(z^*) \cdot b^L + (1 - \bar{z}(z^*)) \cdot b^H < \hat{b} < \eta \cdot b^L + (1 - \eta) \cdot b^H$ will attract the former standard annuity buyers but does not attract sufficient former enhanced annuity purchasers in order to generate a non-negative expected profit.[11] Therefore, the offered contracts will be purchased and generate non-negative profits. In addition, there are no other contracts that attract customers and generate non-negative profit. Correspondingly, the highest interior solution will be a Nash-equilibrium.

Considering social welfare, it holds true that:

Proposition 6 *From an utilitarian point of view, the additional offer of enhanced annuities will induce a loss of social welfare.*

Proof: Consider the utilitarian welfare function $w = \sum_I \mathrm{E}(v(\mathbf{b}))$ where I denotes the set of individuals. With no enhanced annuities available, this term can be reduced to $\sum_I \mathrm{E}(v(\mathbf{b})) = \sum_I v(\mathbf{b})$. When there are only enhanced annuities, the welfare funtion changes to

$$w = \sum_I [(1 - z_i) \cdot v(\mathbf{b^H}) + z_i \cdot v(\mathbf{b^L})] \tag{8}$$

where z_i denotes the signal of the i$^{\text{th}}$ individual. By Jensen's inequality and as

[11] This follows from the non-existence of a higher interior cutoff.

$v(\mathbf{b})$ is a concave function, it holds that

$$\sum_I [(1 - z_i) \cdot v(\mathbf{b^H}) + z_i \cdot v(\mathbf{b}^L)] \leq \sum_I v[(1 - z_i) \cdot \mathbf{b^H} + z_i \cdot \mathbf{b}^L]. \qquad (9)$$

The right hand side of (9) must be less than $\sum_I v(\mathbf{b})$ as v is a concave function and it holds that $\sum_I \mathbf{b} = \sum_I [(1 - z_i) \cdot \mathbf{b}^H + z_i \cdot \mathbf{b}^L]$. Thus, the exclusive offer of enhanced annuities reduces social welfare from an utilitarian point of view.

Yet, the analysis of the market reaction to the introduction of enhanced annuities has shown that there might also be an equilibrium where part of the insured buy standard and some others enhanced annuities. Analogously, it holds that such a solution is also welfare reducing compared to the only offer of standard annuities. In this case, the utilitarian welfare function changes to $\sum_{I^s} v(\mathbf{b}^*) + \sum_{I^e} (1 - z_i) \cdot v(\mathbf{b}^H) + z_i \cdot v(\mathbf{b}^L)$ where I^s is the set of individuals that buy the standard annuities and I^e the set of the enhanced annuity buyers. By Jensen's inequality, this expression is less or equal to $\sum_{I^s} v(\mathbf{b}^*) + \sum_{I^e} v((1 - z_i) \cdot \mathbf{b}^H + z_i \cdot \mathbf{b}^L)$. In addition, it must still be less than the social welfare when there are only standard annuities with the same arguments than before and because it hold that $\sum_{I^s} \mathbf{b}^* + \sum_{I^e} [(1 - z_i) \cdot \mathbf{b}^H + z_i \cdot \mathbf{b}^H] = \sum_{I^s} \mathbf{b}$.

Intuitively, this holds true as the sum of the annuity payouts over the whole set of annuity buyers is equal with both annuity types when annuities are priced fairly. Yet, with the enhanced annuity the payouts vary more in total. Thus, roughly speaking, the enhanced annuity contains more risk for the whole set of annuity buyers. Nevertheless, this result cannot be solely held liable for regulating on the annuity market as adverse selection was not modelled in this approach. Adverse selection could induce some market participants with a reduced life expectancy rather to accumulate capital privately as to invest in annuities. These individuals could be regained to the annuity market when enhanced annuities are offered. Thus, a overall welfare gain could be induced.

5 Conclusion

Enhanced annuities allow a more precise underwriting of risks in annuity insurance and thus are designed to reduce informational asymmetries between insurer and insured. This paper has investigated the demand reactions on a market where enhanced and standard annuities are offered simultaneously. A special focus was laid on different types of superior knowledge of the insured. It was shown that in the case of homogeneous knowledge, there were two possible results. If all insured know about their future risk type, standard annuities will be driven out of the market. On the opposite, only standard annuities will establish on the market, if the insured do not have any superior knowledge. In the case of heterogeneous superior knowledge, it was shown that either only enhanced annuities are purchased or both products exist simultaneously. The second solution when existent must be unique if the insured are risk averse and Pareto-dominates the case where only the enhanced annuities are bought. Finally, social welfare was examined. It is shown in the chosen setting that the introduction of enhanced annuities on a market with only standard annuities before will always reduce social welfare from an utilitarian point of view.

A limitations of the approach presented here is the fact that adverse selection was not modelled. A result is that the introduction of enhanced annuities induces a welfare loss due to the additional risk as no reduction of welfare losses due to adverse selection are considered. Concerning further research, modelling insurance demand in dependance of income and income risk will be interesting to investigate as adverse selection effects could be shown by that. In that case, the welfare implications are not clear as enhanced annuities, if pricing can be done properly and accordingly to different risk types, addresses adverse selection problems. Yet, they do not insure against the risk of becoming a high or low risk at the pension entry age which reduces welfare.
A characteristic of enhanced annuitiues is that they offer a higher pension payment in the case of a short life expectancy. Since a reduced life expectancy is mostly positively correlated with a impaired health, enhanced annuities also provide higher payments in cases with higher expected health expenditures. This feature can increase the attractiveness of enhanced annuities and may be modelled via state-dependent utility function which might be productive to examine as well. Kifmann (2008) has modelled the demand for annuities with state dependant utility functions and shows that in this context, the offer of annuities depending on the risk

status of the insured can be welfare enhancing. Yet, he does not model the case of heterogeneous knowledge of the insured.

6 Literature

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