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Economic Surplus and Derived Demand

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Economic Surplus and Derived Demand

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1 Introduction

Alfred Marshall (1920, v.vi.4) has pointed out that most demand – especially labor demand – is derived from the demand for some other product. He wrote: "To take another illustration, the direct demand for houses gives rise to a joint demand for the labour of all the various building trades, and for bricks, stone, wood, etc. which are factors of production of building work of all kinds, or as we may say for shortness, of new houses. The demand for any one of these, as for instance the labour of plasterers, is only an indirect or derived demand."

This note demonstrates that the usual analysis of economic rent, as typically explained for the case of consumers' surplus, carries over to the case of derived demand. The assertion comes as no surprise; rather such is typically presupposed in all kinds of cost-benefit considerations involving derived demand, yet there is, to the best of my knowledge, no demonstration to the be found in the texbooks or the literature. This note is intended to fill the gap.

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2 Derived Demand

Consider a market for a consumer good with the falling demand curve (inverse demand function)

$$p = p(x) \ , \ p' < 0 \tag{1}$$

where x denotes the quantity demanded at price p.

Assume that the commodity is produced by many firms by means of some input n (labor, for instance) according to a production function

$$x_i = f_i(n_i) \ , \ f_i' > 0 \, , \ f_i'' < 0.$$

where the index i refers to firm number i.[1]

Profits of firm i are $p_i f_i(n_i) - wn_i$, and the profit maximizing level of production is characterized by the marginal productivity condition

$$pf_i'(n_i) = w. \tag{2}$$

Denote by $n = \sum_i n_i$ the aggregate labor input and define the aggregate production function f as

$$f(n) := \max\left\{ \sum_i f_i(n_i) \middle| \sum_i n_i = n \right\}. \tag{3}$$

Assuming a unique interior maximum, we obtain from (2) and (3)

$$pf'(n) = w. \tag{4}$$

For any given product price p, this equation gives the ordinary (inverse) demand curve for the input (*e.g.* labor). Its slope is equal to $pf'' < 0$. The sum of profits accruing to all firms together is $\sum_i (pf_i(n_i) - wn_i)$ and hence

$$\pi = pf(n) - wn. \tag{5}$$

Any level of production x is, according to (1), uniquely related to the product price p that is necessary to clear the market. These interrelations can be incorporated within the analysis by inserting (1) and (3) into (4), yielding a relationship between

[1] In the simplest case, only one input (such as labor) is needed. For reasons of simplicity this is assumed in the following exposition. The more general case of many inputs can be covered by re-interpreting $f_i(n_i)$ as the maximum value added obtainable for firm i if factor input n_i is costlessly provided, *etc.*

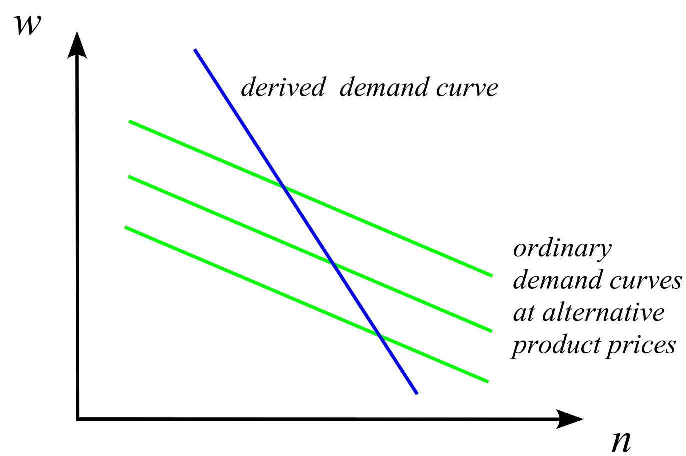

Figure 1: The derived input demand curve is steeper than the ordinary input demand curves.

factor input and input price: $p(f(n))\,f'(n) = w$. This gives rise to the *indirect input demand curve*

$$w(n) := p(f(n))\,f'(n)\,. \tag{6}$$

Its slope is $w' = pf'' + p'(f')^2 < pf''$. It is, therefore, steeper than the ordinary demand curve (Figure 1).[1]

Consider a wage reduction from w_0 to w_1 that goes along with an employment increase from n_0 to n_1, an output increase from x_0 to x_1, and a price reduction from p_0 to p_1. These quantities relate as follows:

$$\begin{aligned} x_0 &= f(n_0)\,, \quad & p_0 &= p(x_0) \\ x_1 &= f(n_1)\,, \quad & p_1 &= p(x_1)\,. \end{aligned} \tag{7}$$

Consider the area under the demand curve (1) between x_0 and output level $x = f(n)$ belonging to some employment level n:

$$P(n) = \int_{x_0}^{f(n)} p(x)\,dx. \tag{8}$$

Differentiation of (8) with respect to n yields

$$\frac{\partial P}{\partial n} = p(f(n))\,f'(n) = w(n)$$

[1] See Pindyck and Rubinfeld (2005, 521). Note that some authors use the concept in a different sense; see Varian (1996, 338), for instance.

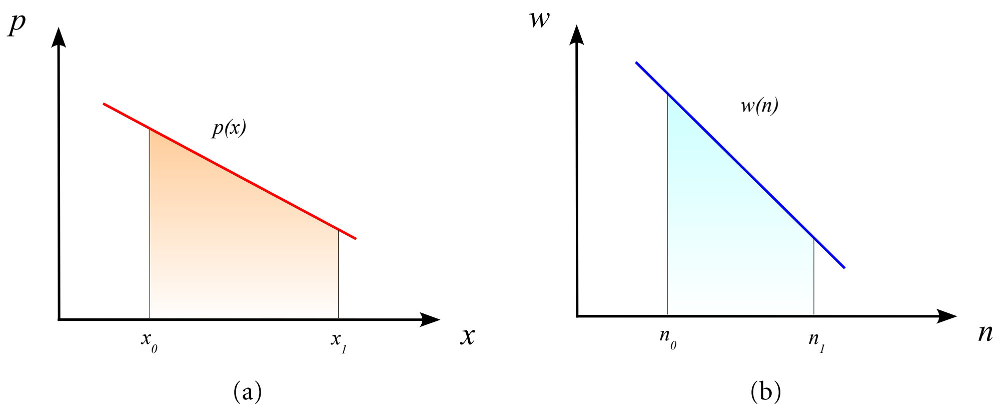

Figure 2: The area under the demand curve for output (a) and the area under the derived demand curve for input (b) between corresponding values of input and output are identical.

which implies

$$P(n_1) = \int_{n_0}^{n_1} w(n)\, dn.$$

This is the just the area under the derived input demand curve. Hence the corresponding areas under the product demand curve and the indirect input demand curve are identical in size (Figure 2).

3 Derived Surplus

Consider now the change in consumer's surplus that results from a price change from p_0 to p_1. It is

$$\Delta cs = p_0 x_0 - p_1 x_1 + \int_{x_0}^{x_1} p(x)\, dx \tag{9}$$

and is depicted in Figure 3. According to the previous argument, the integral over the demand curve equals the corresponding integral over the indirect demand curve and we have $\int_{x_0}^{x_1} p(x)\, dx = \int_{n_0}^{n_1} w(n)\, dn$. The profits associated with the different levels of production are

$$\begin{aligned} \pi_0 &= p_0 x_0 - w_0 n_0 \\ \pi_1 &= p_1 x_1 - w_1 n_1 \end{aligned}$$

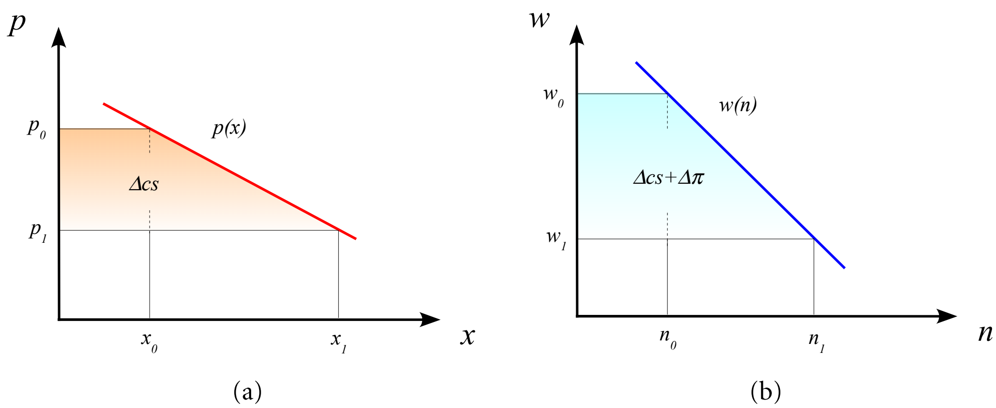

Figure 3: The change in consumers' surplus Δcs is given by the shaded area left of the final demand curve (a). The analogous area left of the derived demand curve (b) gives the change in economic rent as the sum of the changes in consumers' surplus Δcs and profits $\Delta\pi$.

and the change in profits is

$$\Delta\pi = \pi_1 - \pi_0.$$

Hence (9) can be written as

$$\Delta cs + \Delta\pi = w_0 n_0 - w_1 n_1 + \int_{n_0}^{n_1} w\left(n\right) dn. \tag{10}$$

The left-hand side gives the change in economic rent as the sum of the changes in consumers' surplus Δcs and profits $\Delta\pi$. The right-hand side gives the surplus area in the derived demand diagram, which is, thus, a measure for the increase in economic rent induced by a price decrease of input from w_0 to w_1 and an entailed price decrease of the final product from p_0 to p_1. (Figure 3).

References

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