

Convex vacancy creation costs and on-the-job search in a global economy

Christian Holzner¹ | Mario Larch²

¹University of Munich, CESifo, Munich, Germany

²University of Bayreuth, CEPII, Ifo Institute, CESifo, and GEP, Bayreuth, Germany

Correspondence

Christian Holzner, University of Munich, CESifo, 80539 Munich, Germany.
Email: christian.holzner@econ.lmu.de

Abstract

Convex vacancy creation costs shape firms' responses to trade liberalisation. They induce capacity constraints by increasing firms' costs of production. A profit maximising firm will therefore not fully meet the increased foreign demand, but serve only a few export markets. More productive firms will export to more countries and profit more from trade liberalisation. To get an effect of trade liberalisation on wage inequality, we need on-the-job search and convex vacancy creation costs because with linear costs trade liberalisation affects all wages in equal proportion. Furthermore, with convex vacancy creation costs, not all firms export to all foreign markets even if trade is fully liberalised. This implies that wage inequality under free trade is always higher than under autarky.

KEYWORDS

convex vacancy creation costs, heterogeneous firms, international trade, monopolistic competition, on-the-job search

1 | INTRODUCTION

In response to the public's growing concern that trade liberalisation adversely affects unemployment and wage inequality, researchers have begun to integrate labour market models that generate unemployment and wage distributions into classical trade models. These models predict positive as well as negative effects of trade liberalisation on unemployment and find an inverse u-shape relationship between trade liberalisation and residual wage dispersion among workers with similar observed characteristics.

This is an open access article under the terms of the Creative Commons Attribution License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited.

© 2021 The Authors. *The World Economy* published by John Wiley & Sons Ltd.



The existing theoretical trade models, which are used to investigate the effect of trade liberalisation on residual wage inequality (i.e. wage inequality within an industry not explained by observables), distinguish only between non-exporting and exporting firms.¹ They disregard the richness of firms' export behaviour including the facts that most exporting firms sell to only one foreign market, that the number of firms that sell to multiple markets declines with the number of destinations and that the export strategies of observationally equivalent firms vary greatly across countries with similar characteristics.² Our theoretical model suggests that the effect of trade liberalisation on residual wage inequality depends on whether the underlying model generates only non-exporting and exporting firms, or is able to generate a trade pattern consistent with these empirical facts.

The result that trade liberalisation affects residual wage inequality only in a model which is able to generate these empirical findings can best be seen by comparing our on-the-job search framework with convex vacancy creation costs with an on-the-job search model with linear vacancy creation costs. With linear vacancy creation costs firms post-vacancies until their marginal revenues are equal to the constant cost of vacancy creation. This implies that all firms, irrespective of their export strategy, have the same marginal profit in equilibrium. As in Burdett and Mortensen (1998) firms still pay different wages, but wages will not depend on a firm's export strategy. They only depend on aggregate labour market tightness. Trade liberalisation, which increases profits (of exporting firms) and vacancy creation, therefore increases all wages proportionally. Convex vacancy creation costs induce capacity constraints by increasing firms' costs of expanding production. Therefore, not all exporting firms will export to all countries. More productive firms will export to more countries. Firms that export to more countries profit more from lower trade costs. They will therefore compete more for workers than firms that export to fewer countries if trade is liberalised. This increases wages of firms that export to more destinations more than wages of firms selling to fewer destinations (or even domestic firms) and leads to an additional effect (not present in models without labour market imperfections) on wage inequality that can not be studied in two-country models.

We introduce convex vacancy creation costs and on-the-job search like in Burdett and Mortensen (1998) into a Melitz (2003)-type new trade model with monopolistic competition and heterogeneous firms. This leads to a simple and analytically tractable multi-country trade model that is in line with many empirical facts about both the trade and labour market side. To obtain realistic trade patterns, we need convex vacancy creation costs because firms with convex vacancy creation costs and fixed entry costs for export markets find it optimal to sell only to a subset of foreign markets. This can explain why the number of export markets served by a firm increases with its productivity. To obtain a realistic wage distribution, we need on-the-job search, since without on-the-job search all firms would pay the reservation wage.³ While we could have assumed other types of convex costs to obtain similar results for the trade pattern, we assume convex vacancy creation costs because they are well in line with the empirical evidence on the shape of the vacancy creation cost function. Coşar et al. (2016) and Manning (2006) find direct evidence for a convex vacancy creation cost function using U.K. and Colombian data, respectively. Merz and Yashiv (2007) and Yashiv (2006) find a convex hiring cost function using U.S. data, and Blatter et al. (2012) find the same using Swiss data.

¹Akerman et al. (2013) provide nice empirical evidence for the importance of residual within-sector wage inequality and the role of the export status for wage payments.

²Arkolakis (2010) and Eaton et al. (2011) document these facts.

³Helpman et al. (2010) generate wage dispersion without on-the-job search based on sorting of workers into different firms.

If vacancy creation costs are sufficiently convex, then even if trade is fully liberalised not all firms will export. The trade pattern mentioned above continues to persist. Free trade in Helpman et al. (2010) and Melitz (2003) implies that all active firms in the economy serve the domestic and all export markets. In Helpman et al. (2010) starting from autarky, wage inequality first increases with trade liberalisation as those firms that start to export increase their wages. As the number of exporting firms increases, wage inequality decreases at the upper end of the wage distribution, since all exporting firms profit equally from exporting. If all firms export to all foreign countries, all firms increase their wages proportionally and wage inequality is the same as under autarky. This leads to an inverse u-shape relationship between trade liberalisation and wage inequality if firms face linear vacancy creation costs. In our framework with convex vacancy creation costs not all firms export to all countries even if trade is fully liberalised. In an open economy, wage inequality is therefore always higher than under autarky. Our theoretical model suggests that studies employing structural estimation to evaluate the extent to which trade liberalisation can explain the rise in wage inequality should not only capture firm-level employment and wage patterns but also the richness of firms' export behaviour.

The paper is structured as follows. In the next section, we provide a short overview of the related literature. In Section 3, we present the general framework that links Melitz's (2003)-type new trade theory model with the on-the-job search model by Burdett and Mortensen (1998). In Section 4, we present the equilibrium of our model, which we characterise and compare with the linear vacancy creation costs case. Section 5 contains our main results on the comparative statics of trade liberalisation on trade patterns, wages, and wage inequality for the different frameworks. Section 6 concludes.

2 | RELATED LITERATURE

Researchers have begun to integrate labour market models that generate unemployment into classical trade models. Brecher (1974) was the first to study minimum wages in the Heckscher–Ohlin model with two countries, two factors and two goods, and Davis (1998) generalised this model. Davidson et al. (1999) and Davidson and Matusz (2004) introduced search frictions and wage bargaining into multisector models of international trade governed by comparative advantage and Felbermayr et al. (2011a), Felbermayr et al. (2011b) and Helpman and Itskhoki (2010) did the same for the new trade theory based on the Melitz (2003) model. Cuñat and Melitz (2010, 2012) embed firing restrictions in a Ricardian setting. This variety of models predicts positive as well as negative effects of trade liberalisation on unemployment. The empirical literature finds evidence for positive (Dutt et al., 2009; Felbermayr et al., 2011a, 2011b; Hasan et al., 2012) as well as for hardly any or negative effects (Attanasio et al., 2004; Menezes-Filho & Muendler, 2011).

These models' silence on how trade liberalisation affects the wage distribution was the impetus for a new strand of literature. Helpman et al. (2010) use a matching framework to explain wage differences across observationally equivalent workers by the firm-level heterogeneity that arises if some firms hire only high productive workers while others hire all types of workers. Egger and Kreckemeier (2012) explain intra-group wage inequality among ex-ante identical workers using a fair wage effort mechanism. Amiti and Davis (2012) also assume a fair wage constraint and investigate the effects of input and output tariffs. All these models find that trade liberalisation increases the wage distribution of ex-ante identical workers. In our context, wage inequality is not the result of fair wage preferences or the result of monitoring or screening costs, but is instead the result of workers' continuous search for better-paid jobs, as introduced in Burdett and Mortensen (1998). Stijepic (2016) and Suverato (2014) consider interesting extensions; the first builds on an on-the-job search model with bargaining developed by Mortensen (2009) in order to characterise the transitional dynamics of the

wage distribution in response to trade liberalisation, and the second considers different skill groups in an on-the-job search model based on Holzner and Launov (2010) and shows that trade liberalisation increases the wage gap between skilled and unskilled workers. More recently Felbermayr et al. (2018) investigate in a symmetric two-country framework with directed search how heterogeneity in firm characteristics affects wage dispersion between and within firms. Based on German-linked employer–employee data, they attribute half of the surge in inequality to firm heterogeneity. Card et al. (2013) show that assortative matching and firm characteristics account for a substantial part of the overall dispersion of wages in German, respectively. Song et al. (2018) find for the US that one-third of the wage inequality occurred within firms, whereas two-thirds is due to a rise in the dispersion of average wages between firms. Bellon (2018) develops a two symmetric countries trade model of firm and worker heterogeneity with endogenous firm dynamics and shows that larger and more productive firms immediately export, increase employment, and screen more intensively at a faster rate after lowering trade costs. Smaller and less productive firms will not export, grow more slowly, shrink or exit after trade liberalisation.

Another strand of the literature, spurred by the emergence of firm-level datasets, documents and explains firms' export behaviour. Arkolakis (2010) and Eaton et al. (2011) document that most exporting firms sell only to one foreign market, that the number of firms that sell to multiple markets declines with the number of destinations and that the export strategies of observationally equivalent firms vary greatly across countries with similar characteristics. To explain the first two empirical observations, Arkolakis (2010) and Eaton et al. (2011) introduce not only market but also firm-specific heterogeneity in entry costs and market size into the Melitz (2003) model. Convex vacancy creation costs can also explain the third finding. Convex costs of vacancies and hence production imply that exporting firms do not find it optimal to adjust their output so as to serve all foreign markets. Thus, even if two export markets are similar and firms are indifferent between them, they might export to just one country and not the other, because they find it too costly to increase production to serve both countries. Fajgelbaum (2020) also uses an on-the-job search equilibrium model based on Postel-Vinay and Robin (2002) and argues that this trade pattern arises because young firms need time to grow. Other explanations of the observed trade pattern are based on the heterogeneity among firms arising from information frictions. For example, Chaney (2014) proposes a dynamic model in which firms only export to markets where they have a contact, and Morales et al. (2019) argue that firms are more likely to export to destinations that are close to previous export destinations. A different approach is taken by Eaton et al. (2015) who focus on firm-to-firm trade and explain the trade pattern by heterogeneity among firms and search frictions.

3 | FRAMEWORK

3.1 | Labour market and workers' search strategy

The model has an infinite horizon, is set in continuous time and concentrates on steady states. The measure of firms M in the economy will be endogenously determined in the product market. Firms have to decide on the wage w they offer, the number of export destinations j they serve and the number of vacancies v they create. Each vacancy is assumed to be contacted by a worker at the endogenous rate $\eta(M)$. To keep the model analytically tractable, we assume that firms face zero vacancy creation costs for $v \leq \bar{v}$ and infinitely high costs for $v > \bar{v}$. This implies that all firms create \bar{v} vacancies. Hence, the total number of contacts made by firms is given by $\eta(M)M\bar{v}$. To highlight the role of convex

vacancy creation costs, in Section 4.4 we consider the case where firms can post-vacancies given the linear cost function cv like in Mortensen (2003).⁴

Workers' life-time is exponentially distributed with parameter ϕ . They are risk neutral and, for the sake of simplicity but without loss of generality, do not discount the future. Since we normalise the measure of workers to one, ϕ also describes the in- and outflow of workers in the labour market. Workers are either unemployed and receive an unemployment benefit z or they are employed and receive a wage w . Both unemployed and employed workers search for a job with the same intensity. Following Burdett and Mortensen (1998), the probability of a worker meeting a firm follows a Poisson process with rate λ . Since aggregation requires that the total number of firm contacts equals the total number of worker contacts, we obtain,

$$\lambda = \eta(M)M\bar{v}. \quad (1)$$

Since all workers, regardless of whether they are employed or unemployed, search with the same intensity, we know from Burdett and Mortensen (1998) that unemployed workers' reservation wage equals the unemployment benefit and that employed workers will accept any wage above their current wage. The wage offer distribution is denoted by $F(w)$.

3.2 | Product market and new product ideas market

In each of the N countries, there is a single final output good Y , which is produced under perfect competition. Good Y is assembled from a continuum of intermediate inputs, which may be produced domestically or imported and a numeraire good q_0 . We assume that each country has a fixed supply of the numeraire good q_0 . The numeraire good q_0 in the production function in Equation (2) absorbs all changes in aggregate demand and is assumed to be costlessly tradable, balancing trade between countries. The quantity of an input ω from country j in country i is denoted by $q_{ij}(\omega)$. The production function is given by,

$$Y_i = \frac{1}{\rho} \sum_{j \in N} \int_{\omega \in \Omega_{ij}} q_{ij}(\omega)^\rho d\omega + \sum_{j \in N} q_{ij} q_0, \quad (2)$$

where $0 < \rho < 1$ and Ω_{ij} equals the mass of varieties imported from country j in country i .⁵ Each variety ω is produced by a single firm in a monopolistic competitive market. The final good producers' maximisation problem leads to the following demand for a variety ω from country j ,

$$q_{ij}(\omega) = p_{ij}(\omega)^{-\frac{1}{1-\rho}}. \quad (3)$$

⁴In a previous working paper version (Holzner & Larch, 2011), we also consider a more general convex vacancy creation cost function $(c/\alpha)v^\alpha$ with $\alpha > 1$.

⁵Fixed cost of production and free entry of final good producers ensure that they make zero profits in equilibrium.

⁶Our symmetric country setup ensures that the numeraire good q_0 is always used in production of all countries. The proof is available upon request. where $p(\omega)$ is the respective price of intermediate good ω .⁶

Assuming symmetric countries (which allows us to drop country indices henceforth), intermediate goods producers face the same demand curve in each export market as they face in the domestic market. Serving an export market involves some proportional shipping costs τ per unit exported. Thus, if the quantity $q_x(\omega)$ leaves the factory gate $q_x(\omega)/\tau$ reaches the export market. A firm that decides to serve $j \leq n$ export markets will choose the output sold in each of the j export markets $q_x(\omega)$ and the domestic market $q_d(\omega)$ such that profits are maximised (see derivation in Appendix A),

$$q_d(\omega) = \frac{1}{1 + j\tau^{\rho/(\rho-1)}} q(\omega), \text{ and} \tag{4}$$

$$q_x(\omega) = \frac{\tau^{\rho/(\rho-1)}}{1 + j\tau^{\rho/(\rho-1)}} q(\omega). \tag{5}$$

Producers use only labour $l(\omega)$ in production. As in Melitz (2003), firms differ in labour productivity such that the output of a firm that produces good ω is given by $q(\omega) = \varphi(\omega) l(\omega)$, where $\varphi(\omega)$ denotes the labour productivity of goods producer ω . As is standard in the literature, we use φ to index intermediate goods producers.⁷ Due to the monopsony power in the labour market in the Burdett–Mortensen model, the size $l(w)$ of the labour force employed by a firm depends on its wage w .

Goods producers are risk-neutral and live forever.⁸ Each firm is concerned about its steady-state profit flow. We assume that a goods producers’ demand totally breaks down at the Poisson rate δ , reflecting the end of a specific variety’s product cycle.⁹ The rate δ acts as discount rate for firms. Thus, a firm that pays a wage w and serves j export markets has a steady-state profit flow equal to the per period revenues from serving the domestic and j export markets minus the wage bill, production fixed costs f and exporting fixed costs f_x for each foreign market, that is

$$\delta\Pi(w, j|\varphi) = Y(j) [\varphi l(w)]^\rho - wl(w) - f - jf_x, \tag{6}$$

where

$$Y(j) = \left[1 + j\tau^{\frac{\rho}{\rho-1}} \right]^{(1-\rho)}.$$

A firm with productivity φ maximises profits by deciding on the wage offer w and on the number j of export destinations. Denote by $w(\varphi)$ and $j(\varphi)$ the wage offer and the number of export destinations that maximise Equation (6) for a firm with productivity φ , respectively. Furthermore, denote by $\Pi(\varphi)$ the maximum discounted profits that a firm with a product idea φ can make, that is

$$\Pi(\varphi) = \max_{w, j} \Pi(w, j|\varphi).$$

⁷We can think of φ as labour productivity or as quality of a product idea. With the given form of product differentiation, these two interpretations are isomorphic (see Melitz, 2003, page 1699 and Footnote 7). We use quality and productivity interchangeably, since firms only care about profit per unit of labour.

⁸This standard assumption in the Burdett–Mortensen model allows us to refrain from considering the effects of firm growth.

⁹Note, that we do not aim at modelling the life cycle of the product itself. See Klepper (1996) and Vernon (1966) for the idea of product life cycles.

If demand for a specific variety breaks down at rate δ , a firm will acquire a new product idea in the market for product ideas.¹⁰ Product ideas are sold in a perfectly competitive market. Existing goods producers that compete for new product ideas differ in their stock of labour. In the Burdett–Mortensen model, the stock of workers $l(w(\varphi))$ that a firm employs depends on the wage $w(\varphi)$ that the firm commits itself to pay to all its workers for their entire employment spell. A firm with labour force $l(w(\varphi))$ that pays the wage $w(\varphi)$ is thus willing to bid up to $\Pi(\varphi)$ for a product idea φ . Since there are always several firms with a labour force arbitrarily close to $l(w(\varphi))$, the price for the product idea is bid up to $\Pi(\varphi)$.

Inventors of new product ideas have to invest in research and development at cost f_e before they will know the quality of the idea. As it is common in the literature, we assume that product idea qualities φ are drawn from a Pareto distribution denoted by $\Gamma(\varphi)$. Since existing firms are only willing to buy profitable product ideas, only products with quality $\varphi \in [\varphi^*, \infty)$ will be available in the market, where φ^* is defined as the cut-off productivity, that is

$$\Pi(\varphi^*) = 0. \quad (7)$$

The distribution of qualities sold in the market is therefore given by $\tilde{\Gamma}(\varphi) \equiv [\Gamma(\varphi) - \Gamma(\varphi^*)] / [1 - \Gamma(\varphi^*)] = 1 - (\varphi^*/\varphi)^\gamma$ with $\gamma > 1$. Since all product ideas (except φ^*) make positive profits, the expected discounted profits before knowing φ is given by,

$$\Pi_e = [1 - \Gamma(\varphi^*)] \bar{\Pi} = [1 - \Gamma(\varphi^*)] \int_{\varphi^*}^{\infty} \frac{\Pi(\varphi)}{1 - \Gamma(\varphi^*)} d\Gamma(\varphi) > 0,$$

where $1 - \Gamma(\varphi^*)$ equals the probability that an inventor will draw a φ high enough to be profitable. $\bar{\Pi}$ equals the average discounted profits of all product ideas available in the market.

Free entry of inventors ensures that new product ideas enter the market until the expected discounted profits before entering the market equal the fixed investment costs f_e , that is

$$[1 - \Gamma(\varphi^*)] \bar{\Pi} = \int_{\varphi^*}^{\infty} \Pi(\varphi) d\Gamma(\varphi) = f_e. \quad (8)$$

The zero cut-off profit condition in Equation (7) and the free entry condition in Equation (8) determine the number of active firms M in the product market¹¹ and the productivity φ^* of the firm with the lowest productivity in the economy.

¹⁰We separate inventors and firms, because otherwise we would have to model firm growth. When firms also invest, new firms that enter the market would only post \bar{v} vacancies due to convex vacancy creation costs. Hence, they would grow steadily up to the optimal steady-state level. This would change firms' position in the wage offer distribution over time and violate the rank preserving assumption that is needed to solve a dynamic version of the Burdett–Mortensen model (see Moscarini & Postel-Vinay, 2013).

¹¹The number of active firms M enter the zero cut-off profit condition in Equation (7) and the free entry condition in Equation (8) through the labour input $l(w(\varphi))$ given in Equation (15).

3.3 | Aggregation and steady-state conditions

Aggregate profits are used to finance new product ideas and thus the initial research and development costs of inventors, that is

$$M\delta\bar{\Pi} = f_e I_e. \tag{9}$$

I_e is the total mass of inventors who attempt entry and pay the fixed investment costs f_e each period. A large unbounded set of potential new product ideas ensures an unlimited supply of potential entrants into the market for new product ideas. Steady state requires that the flow into the pool of new product ideas is equal to the inflow of existing firms that want to buy new product ideas, that is

$$[1 - \Gamma(\varphi^*)] I_e = \delta M. \tag{10}$$

It is straightforward to show that the steady-state conditions of Equations (9) and (10) hold if the free entry condition in Equation (8) holds.

In steady-state in- and outflows to employment offset each other such that the distribution of employment over firms and the unemployment rate are stationary. Equating the flows in and out of unemployment gives the steady-state measure of the unemployed, that is

$$u = \frac{\chi + \phi}{\chi + \phi + \lambda}, \tag{11}$$

where χ denotes the quitting rate into unemployment, ϕ the rate at which workers exit the labour market and λ the rate at which unemployed workers find a job. Equating the inflow and outflow of workers earning less than w gives the steady-state wage earnings distribution $G(w)$, that is

$$\lambda F(w) u = G(w) (1 - u) [\chi + \phi + \lambda [1 - F(w)]] \tag{12}$$

$$\Rightarrow G(w) = \frac{(\chi + \phi) F(w)}{\chi + \phi + \lambda [1 - F(w)]}, \tag{13}$$

where $F(w)$ denotes the wage offer distribution across firms. Here, we assume that the wage offer distribution $F(w)$ is continuous.¹²

The steady-state size of a firm $l(w)$ is determined by the hiring and quitting rates at a firm that pays wage w , that is

$$l(w) = \frac{\eta(M) \bar{v} [u + (1 - u) G(w)]}{\chi + \phi + \lambda [1 - F(w)]}. \tag{14}$$

The number of recruited workers depends on the contact rate $\eta(M)$, the number of vacancies \bar{v} , and the probability that contacted workers are willing to work for the wage w . If the wages w exceed

¹²This requires low enough unemployment benefits z . The respective condition is derived in Appendix C. If unemployment benefits z and hence reservation wages are high enough, marginal profits of the least productive firm with \bar{v} vacancies could be negative. In this case, the least productive firm will want to lower the number of vacancies \bar{v} in order to increase its marginal profit. The respective equilibrium is covered by the general case of endogenous vacancy creation analyzed in our extension described in Holzner and Larch (2011). With endogenous vacancy creation mass points never exist.

unemployment benefit z , then all unemployed workers u and all workers employed at a lower wage will accept it, that is $(1 - u)G(w)$. The rate at which workers quit for a better-paying job is given by $\lambda[1 - F(w)]$. Substituting λ using the aggregate matching condition of Equation (1), u using the steady-state measure of unemployed workers in Equation (11), and $G(w)$ using Equation (13) allows us to write the steady-state labour force of a firm that pays a wage w as,

$$l(w) = \frac{\lambda(x + \phi)}{[x + \phi + \lambda[1 - F(w)]]^2 M}. \quad (15)$$

As in Burdett and Mortensen (1998), Equation (15) implies that the size of a firm's labour force $l(w)$ is increasing in the wage w , since firms with a high wage attract more workers from low paid jobs and lose fewer workers to high-paid jobs. Equation (15) also shows that a higher number of active firms M results in additional competition between firms and decreases the size of each firm's labour force.

4 | EQUILIBRIUM OF THE MODEL

4.1 | Equilibrium definition

The set $\{w(\varphi), j(\varphi), \Pi(\varphi), F(w(\varphi)), G(w(\varphi)), \varphi^*, u, M\}$ defines a steady-state equilibrium. Thus, in equilibrium firms maximise the steady-state profit flow of Equation (6) by choosing a wage offer $w(\varphi)$ and a number of export destinations $j(\varphi)$. Given the wage offer distribution $F(w(\varphi))$, the number of active firms M in the economy, the productivity distribution $\Gamma(\varphi)$, the cut-off productivity φ^* and the optimal search strategy of workers. Thus, the equilibrium optimality condition requires,

$$\begin{aligned} \delta\Pi(\varphi) &= Y(j(\varphi))[\varphi l(w(\varphi))]^p - w(\varphi)l(w(\varphi)) - f - j(\varphi)f_x, \\ \delta\Pi(\varphi) &\geq Y(j(\varphi'))[\varphi' l(w(\varphi'))]^p - w(\varphi')l(w(\varphi')) - f - j(\varphi')f_x. \end{aligned}$$

Furthermore, in the steady-state equilibrium, the unemployment rate u and the wage earnings distribution $G(w(\varphi))$ in Equations (11) and (13) have to be consistent with the wage offer distribution and steady-state turnover of workers.

Inventors enter the market for new product ideas if the quality of their idea exceeds the cut-off productivity, that is $\varphi \geq \varphi^*$, where φ^* is implicitly defined by Equation (7). The number of active firms M in the product market has to be such that the average profits of active firms are sufficient to finance the research and development costs f_e of potential inventors, that is, such that the inflow of new product ideas equals the number of products whose demand breaks down. The respective steady-state conditions in Equations (9) and (10) or, equivalently, in the free entry condition in Equation (8) must be satisfied in equilibrium.

4.2 | Firms' wage offer and export decision

Each firm chooses a wage $w(\varphi)$ and the number of export destinations $j(\varphi)$ that maximises its steady-state profit flow. In the presence of on-the-job search and convex vacancy creation costs, more productive firms pay higher wages. They are therefore able to poach workers from less productive firms. The following Proposition shows not only that the posted wage $w(\varphi)$ weakly increases with productivity φ as in Mortensen (1990) but also the number of export destinations $j(\varphi)$.

Proposition 1 *Wage offer and export decision* Take two firms with productivity φ and $\varphi' \in [\varphi^*, \infty)$ and assume $\varphi > \varphi'$, then $w(\varphi) \geq w(\varphi')$ and $j(\varphi) \geq j(\varphi')$.

Proof See Appendix B.

Proposition 1 implies that more productive firms pay higher wages like in Mortensen (1990). More productive firms have higher marginal revenues and therefore find it optimal to attract more workers by paying higher wages. Thus, the optimal position of a firm in the wage offer distribution $F(w(\varphi))$ is equivalent to its position in the productivity distribution of active firms, that is

$$F(w(\varphi)) = 1 - \left(\frac{\varphi^*}{\varphi}\right)^\gamma \text{ for all } \varphi \in [\varphi^*, \infty). \tag{16}$$

Since vacancy creation cost is such that all firms find it optimal to open \bar{v} vacancies, a firm's optimal position in the equilibrium wage offer distribution determines a firm's labour input. In other words, convex vacancy creation costs imply that firms find it too costly to freely adjust their labour input. This drives the second result of Proposition 1, that is that more productive firms choose to export to (weakly) more countries. Given that firms' cost minimising labour input and thus their optimal output is constrained by convex vacancy creation costs, they find it too costly to increase production to meet all foreign demand. They rather decide how many export markets to serve given the output that they produce. Given that more productive firms produce more output and given that they face monopolistic competition in each country, they can make more profits by serving more export markets.

Given that demand in all countries is the same, exporters are indifferent to which country to export. Thus, although the aggregate export volume is well defined, bilateral trade flows are not. Trade will still be balanced, but it could well be that some countries import more intermediate goods than they export. In this case, these countries must export more from their numeraire good q_0 to balance trade. Perfect competition of final good producers ensures that the numeraire good q_0 is always used in production.

We can define the export cut-off productivities φ_x^j for $j \in \{1, 2, \dots, n\}$ export destinations as follows. Firms with $\varphi \geq \varphi_x^j$ find it optimal to export to j or more countries while firms with $\varphi < \varphi_x^j$ will serve fewer than j foreign markets and the domestic market (or only the domestic market). The firm with the export cut-off productivity φ_x^j is indifferent between serving j export markets and the domestic market or serving $j - 1$ export markets and the domestic market, that is $\delta\Pi(w, j | \varphi_x^j) = \delta\Pi(w, j - 1 | \varphi_x^j)$. Using Equation (6), the export cut-off productivity can be written as follows,

$$[\varphi_x^j 1(w(\varphi_x^j))]^\rho = \frac{f_x}{[\Upsilon(j) - \Upsilon(j - 1)]}. \tag{17}$$

Equation (17) implies, in accordance with Proposition 1, that the number of export markets $j \leq n$ served by a firm is increasing in its productivity.

Given the export destination decision of firms as defined by the export cut-off productivities $\{\varphi_x^j\}_{j=1}^n$, we can derive the wage paid by a firm with productivity φ . Since a firm has to pay at least the reservation wage in order to attract workers, the least productive firm φ^* will offer a wage equal to unemployment benefits z . In Appendix D, we derive the optimal wage $w(\varphi)$ posted by a firm with productivity $\varphi > \varphi^*$, that is

$$w(\varphi) = \frac{1}{l(w(\varphi))} \left[\Upsilon(j(\varphi)) [\varphi l(w(\varphi))]^\rho - f - jf_x \right] - \frac{1}{l(w(\varphi))} \sum_{i=1}^{j(\varphi)+1} \left[\Upsilon(i-1) \int_{\varphi_x^{i-1}}^{\varphi_x^i} \frac{\rho}{\tilde{\varphi}} [\% \tilde{\varphi} l(w(\% \tilde{\varphi}))]^\rho d\tilde{\varphi} \right], \quad (18)$$

where $\varphi_x^0 = \varphi^*$, $\varphi_x^{j+1} = \varphi$ and $j \leq n$.

Note that profit maximisation ensures that the wage function does not jump upward at φ_x^j , that is, the support of the wage distribution is connected. To see this, suppose the opposite, that is, that the exporting firm with the lowest productivity φ_x^j would pay a wage $w(\varphi_x^j) = w(\varphi) + \Delta$, where $\Delta > 0$ denotes the jump at $w(\varphi)$ where productivity is given by $\varphi = \varphi_x^j - \varepsilon$ for any small $\varepsilon > 0$. The wage jump does not increase the number of workers of firm φ_x^j since it has the same position in the wage distribution as before. It is therefore optimal for the firm to pay a wage that is only slightly above $w(\varphi)$ and save the wage costs Δ per worker. Thus, the wage function is continuous on $[\varphi^*, \infty)$.

Multiplying Equation (18) by $l(w(\varphi))$ reveals that total wage payments are given by revenues (the first term on the right-hand side in brackets) minus fixed production and export costs minus the profits of a firm with productivity φ (the second line on the right-hand side).

4.3 | Firm entry decision

Free entry of potential inventors ensures that the expected discounted profits earned in the product market $[1 - \Gamma(\varphi^*)] \bar{\Pi}$ are used to finance the fixed investment costs f_e as stated in Equation (8). Substituting per period profits from Equation (6) and the optimal wage from Equation (18) implies the following *free entry condition* for product idea inventors,

$$f_e = \frac{1}{\delta} \int_{\varphi^*}^{\infty} \left[\sum_{i=1}^{j(\varphi)+1} \left[\Upsilon(i-1) \int_{\varphi_x^{i-1}}^{\varphi_x^i} \frac{\rho}{\tilde{\varphi}} [\tilde{\varphi} l(w(\tilde{\varphi}))]^\rho d\tilde{\varphi} \right] \right] d\Gamma(\varphi), \quad (19)$$

where $\varphi_x^0 = \varphi^*$, $\varphi_x^{j+1} = \varphi$ and $j \leq n$. The derivation of Equation (19) is given in Appendix E.

The expected discounted profits decrease with the number of active goods producers because the size of a firm's labour force $l(w(\varphi))$ is a decreasing function of the number of active firms M . At the same time, the expected discounted profits of an inventor increase if the cut-off productivity decreases because the likelihood of having a productivity draw that can be sold to a goods producer increases. Using the implicit function theorem, we show in Appendix F that the free entry condition defines a decreasing relation between the zero cut-off productivity φ^* and the number of active goods producers M in the market.¹³

A firm has to offer at least the level of unemployment benefits z in order to attract any worker. Given this lower bound of the support of the wage offer distribution $F(w(\varphi))$, the zero cut-off productivity firm φ^* employs $l(z) = l(w(\varphi^*))$ workers. Utilising the per period profits definition from Equation (6) and the expression for $l(w(\varphi^*))$ from Equation (15) implies that the *zero cut-off productivity* level φ^* is given by,

$$\left[\varphi^* \frac{\lambda(x + \phi)}{[x + \phi + \lambda]^2 M} \right]^\rho = z \frac{\lambda(x + \phi)}{[x + \phi + \lambda]^2 M} + f. \quad (20)$$

¹³We displayed on the x -axis the number of active firms M times the fixed number of vacancies \bar{v} posted by each firm in order to also illustrate (in Section 5) the effect of trade liberalisation with linear vacancy creation costs.

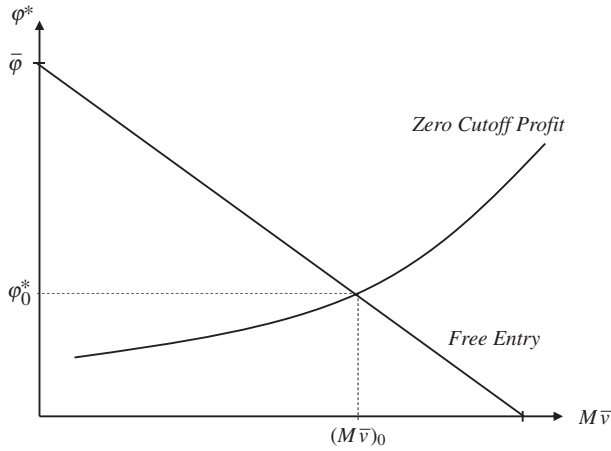


FIGURE 1 Number of total vacancies ($M\bar{v}$) and cut-off productivity

Since the zero cut-off productivity firm pays the reservation wage z , it will attract only unemployed workers and lose its current workers to all other firms that pay higher wages. Consequently, a higher number of active firms M increases the number of quits at the zero cut-off productivity firm and therefore reduces its steady-state labour input. This decreases the firm's net revenue. The firm will subsequently no longer be able to cover the wage payments or the fixed costs f . Thus, only more profitable firms will survive, which increases the zero cut-off productivity. Using the implicit function theorem, we show in Appendix F that the zero cut-off profit condition defines an increasing relation between the zero cut-off productivity φ^* and the number of active goods producers M in the market. Thus, the free entry condition and the zero profit condition determine a unique equilibrium, as shown in Figure 1.

4.4 | Linear vacancy creation costs

The previous analysis was based on the assumption that firms cannot expand their labour input by opening new vacancies. In this subsection, we allow firms to influence their contact rate by posting vacancies, as in Mortensen (2003), given linear vacancy creation costs cv . The contact rate of a firm with productivity φ depends on the number of vacancies $v(\varphi)$ it posts. Let \tilde{v} denote the average number of vacancies posted by all M active firms in the economy. Firms will choose the vacancies $v(\varphi)$ such that profits are maximised. The average number of vacancies must satisfy,

$$\tilde{v} = \int_{\varphi^*}^{\infty} \frac{v(\varphi)}{1 - \Gamma(\varphi^*)} d\Gamma(\varphi).$$

A firm with productivity φ chooses its wage w , its number of vacancies v , and its export destinations j such that per period profits are maximised, that is

$$\delta\Pi(\varphi) = \max_{w,v,j} \left[\Upsilon(j) [\varphi l(w, v)]^\rho - wl(w, v) - cv - f - jf_x \right] \tag{21}$$

$$\text{s.t. } l(w, v) = \frac{\lambda(x + \phi)v}{[x + \phi + \lambda[1 - F(w)]]^2 M\tilde{v}}. \tag{22}$$

The number of employees $l(w, v)$ working for a firm with productivity φ increases proportionally with the number of vacancies v , as in Mortensen (2003), and with the wage, as in Burdett and Mortensen (1998). Thus, firms can increase their labour input by increasing the wage and by opening more vacancies.

The free entry condition and the zero profit condition determine a unique equilibrium as shown in Figure 1. In Appendix G, we derive the free entry and the zero cut-off condition for firms with linear vacancy creation costs and prove the uniqueness of the equilibrium.

5 | TRADE LIBERALISATION

5.1 | Open economy

In the presence of convex vacancy creation costs and on-the-job search, more productive firms pay higher wages. They are therefore able to poach workers from less productive firms, which implies that the labour input of a firm increases with productivity. If we would not allow for on-the-job search firms could only hire unemployed workers. In this case, they will pay the reservation wage z . With linear vacancy creation costs and on-the-job search, wages also increase with productivity. The reason firms pay different wages is the same as in the simple Burdett–Mortensen model. To see this, suppose the contrary holds, that is, all firms paid the same wage. Then, each firm would have an incentive to deviate and offer a slightly higher wage, since it will then also be able to recruit workers employed at other firms at no extra cost, that is, without paying extra vacancy creation costs. The respective wage equation can be obtained by rearranging the optimality conditions for wages and vacancies (as shown in Appendix H),

$$w(\varphi) = c \left[\frac{v(\varphi^*)}{l(z, v(\varphi^*))} - \frac{v(\varphi)}{l(w(\varphi), v(\varphi))} \right] + z. \quad (23)$$

Furthermore, in a framework with linear vacancy creation costs labour input also increases with productivity.

Consider now the trade patterns that emerge under the different frameworks when a country opens up to trade. Proposition 1, together with the export cut-off Equation (17), implies that in a framework with convex vacancy creation costs, firms with very low productivity serve only the domestic market, firms with higher productivity export, and more productive firms export to more countries. The assumptions of convex vacancy creation costs and export fixed costs f_x are the driving forces behind this result. With convex vacancy creation costs and fixed costs to enter an additional export market, firms will find it optimal to serve only a limited number of countries and charge higher prices in them such that overall revenues are maximised. Formally, this can be seen by Equation (17), which shows that the number of export destinations j is increasing in the export cut-off productivity φ_x^j . If firms face linear vacancy creation costs, they will adjust their labour input such that marginal revenues from employing a worker are equal to the costs of vacancy creation, that is

$$\left[\rho Y(j(\varphi)) \varphi^\rho l(w(\varphi), v(\varphi))^{(\rho-1)} - w(\varphi) \right] \frac{l(w(\varphi), v(\varphi))}{v(\varphi)} = c. \quad (24)$$

Substituting wages using Equation (23) implies that firms increase their number of vacancies up to the point where marginal revenues are equalised across firms with different productivities, that is

$$\rho \Upsilon(j(\varphi)) \varphi^\rho l(w(\varphi), v(\varphi))^{(\rho-1)} = z + c \frac{v(\varphi^*)}{l(z, v(\varphi^*))}. \tag{25}$$

The reason for the constant marginal revenue condition is that with linear vacancy creation costs firms find it optimal to increase their labour input until marginal revenues are equal to the marginal costs of creating a vacancy. This constant marginal revenue condition allows us to determine the number of workers $l(w(\varphi), v(\varphi))$ employed by a firm with productivity φ , that is

$$l(w(\varphi), v(\varphi)) = \Upsilon(j(\varphi))^{1/(1-\rho)} \left[\frac{z}{\rho} + \frac{c}{\rho} \frac{v(\varphi^*)}{l(z, v(\varphi^*))} \right]^{-\frac{1}{1-\rho}} \varphi^{\frac{\rho}{1-\rho}}, \tag{26}$$

where $v(\varphi^*)/l(z, v(\varphi^*))$ can be substituted by noting that Equation (22) implies $l(z, v(\varphi^*)) = [v(\varphi^*) \lambda(x + \phi)] / [M\% \bar{v} [x + \phi + \lambda]^2]$.

Taking the profit Equation (21), substituting wage and vacancy creation costs using the optimality condition for vacancy posting in Equation (24), and substituting labour input $l(w(\varphi), v(\varphi))$ using Equation (26) implies that profits are linear in the number of export destinations, that is

$$\delta \Pi(\varphi, j) = (1 - \rho) \Upsilon(j)^{1/(1-\rho)} \left[\frac{z}{\rho} + \frac{c}{\rho} \frac{v(\varphi^*)}{l(z, v(\varphi^*))} \right]^{-\frac{\rho}{1-\rho}} \varphi^{\frac{\rho}{1-\rho}} - f - j f_x, \tag{27}$$

where $v(\varphi^*)/l(z, v(\varphi^*))$ can again be substituted by using Equation (22). We can now use the profit Equation (27) to obtain the export cut-off, that is $\delta \Pi(\varphi_x, j) = \delta \Pi(\varphi_x, j - 1)$. The export decision is independent of j , the number of export destinations, that is

$$\varphi_x = \frac{\tau}{\rho} \left[\frac{f_x}{(1 - \rho)} \right]^{\frac{1-\rho}{\rho}} \left[z + c \frac{v(\varphi^*)}{l(z, v(\varphi^*))} \right]. \tag{28}$$

Since the cost to adjust labour input to foreign demand is linear in output, firms that find it profitable to serve only one foreign market also find it profitable to serve all foreign markets. Thus, all exporting firms serve all n export markets. Proposition 2 summarises the trade patterns in the different frameworks.

Proposition 2 *Trade pattern* With convex vacancy creation costs, the number of export destinations j is weakly increasing in productivity. With linear vacancy creation costs, all exporting firms serve all n foreign countries.

Proof See Appendix I for the proof that the number of export destinations j increases with productivity in the framework with convex vacancy creation costs. The result for linear vacancy creation costs follows directly from Equation (28).

Proposition 2 shows the importance of convex vacancy creation costs for obtaining a realistic trade pattern, where the number of export destinations increases with firm productivity. If we assume a Pareto distribution for firm productivity, as it is common in the trade literature, then convex vacancy creation costs explain why most exporting firms sell to only one foreign market and why the number of firms that sell to multiple markets declines with the number of destinations, as documented by Arkolakis (2010) and Eaton et al. (2011).

Eaton et al. (2011) have also shown that the export strategies of observationally equivalent firms vary greatly across countries with similar characteristics. This can also be explained by convex vacancy creation costs because exporting firms that face convex vacancy creation costs do not find it optimal to adjust their production so as to serve all foreign markets. Thus, even if two export markets

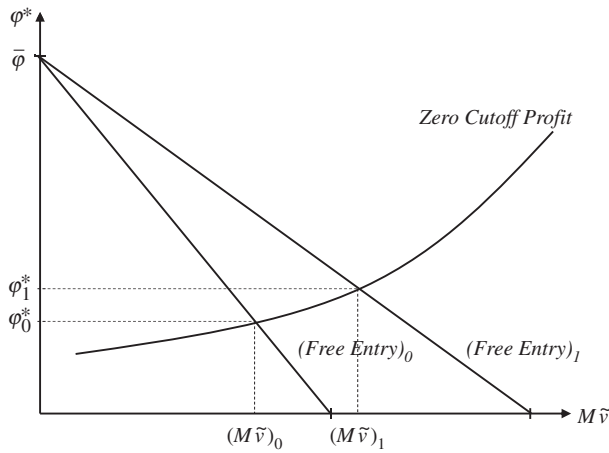


FIGURE 2 Number of total vacancies ($M\bar{v}$) and cut-off productivity if trade is liberalised

are similar and firms indifferent between them, firms might export to just one country and not the other, because they find it too costly to increase production to serve both countries.

5.2 | Comparative statics

Let us now investigate the effects of trade liberalisation in the form of lower shipping costs τ and lower exporting fixed costs f_x .¹⁴

Lower shipping costs τ and lower exporting fixed costs f_x directly increase exporting firms' profits. The associated increase in average profits rotates the free entry curve outward as shown in Figure 2. This is true irrespective of the type of vacancy creation cost as shown in Appendix J.

The zero cut-off profit condition in Equation (20) remains unchanged if trade is liberalised, since the firm with the lowest productivity is not affected as it pays the reservation wage z and serves only the domestic market. The outward rotation of the free entry condition shows that higher average profits caused by trade liberalisation trigger entry. In a framework with convex vacancy creation costs where the number of vacancies per firm is fixed at \bar{v} , entry occurs at the extensive margin, that is, the number of active firms M increases. In a framework with linear vacancy creation costs, entry can occur along the extensive and intensive margin. The outward rotation of the free entry condition alone only implies a higher number of vacancies $M\bar{v}$ in the economy. Given the increased number of vacancies in the economy in either framework, potential entrants realise that more vacancies are competing for the same number of workers, which implies that only more productive firms can survive and that the cut-off productivity φ^* has to increase.

To investigate the effect of trade liberalisation on the trade pattern consider the ratio of the export cut-off productivities to the zero profit cut-off productivity φ^* for the cases of convex and linear vacancy creation costs, respectively, that is

$$\left[\frac{\varphi^* l(z)}{\varphi_x^j l\left(w\left(\varphi_x^j\right)\right)} \right]^p = \frac{f}{f_x} [\Upsilon(j) - \Upsilon(j-1)], \quad (29)$$

¹⁴In Holzner and Larch (2011), a previous working paper version of this paper, we show in a calibration exercise that the same results hold for an exponential convex vacancy creation cost function if we switch from autarky to an open economy.

$$\frac{\varphi^*}{\varphi_x} = \frac{1}{\tau} \left[\frac{f}{f_x} \right]^{\frac{1-\rho}{\rho}} \tag{30}$$

These ratios φ^* / φ_x^j determine the fraction of firms that export to at least j countries in the frameworks with convex vacancy creation costs, that is $1 - \tilde{\Gamma}(\varphi_x^j)$. In the frameworks with linear vacancy creation costs, the fraction of firms that export ($1 - \tilde{\Gamma}(\varphi_x)$) increases with φ^* / φ_x . Trade liberalisation therefore increases the fraction of exporters in all frameworks. If vacancy creation costs are linear, all firms that start to export will export to all n export destinations, since linear vacancy creation costs allow them to expand their labour input at constant cost to meet all foreign demand. If vacancy creation costs are convex, less productive exporters export to fewer countries than more productive exporters. The reason for this trade pattern is that firms with convex vacancy creation costs find it optimal to serve only a limited number of countries. The following Proposition states the effect of trade liberalisation on trading patterns in the different frameworks.

Proposition 3 *A decrease in shipping costs τ or exporting fixed costs f_x increases the fraction of exporting firms. Furthermore*

1. if vacancy creation costs are linear, all new exporters serve all n foreign countries, but
2. if vacancy creation costs are convex, the fraction of domestic firms and firms that export to $j \in \{1, \dots, n - 1\}$ countries decreases, while the fraction of firms that export to n countries increases.

Proof See Appendix K.

Comparing the effect of trade liberalisation with a linear vacancy creation cost function with the effect with a convex vacancy creation cost function again confirms that the framework with convex vacancy creation costs is better able to describe how trade liberalisation affects the trade pattern observed in reality. Part (ii) of Proposition 3, which characterises the impact of trade liberalisation in the framework with convex vacancy creation costs, predicts that trade liberalisation leads to an overproportional increase in the number of firms that export to n countries. The following Corollary shows that for the assumed vacancy creation cost function the tendency that more firms export to all n countries is not strong enough to eliminate the trade pattern that is typical for convex vacancy creation costs, that is, that the number of export destinations j is weakly increasing in firm productivity.¹⁵

Corollary 4 *If trade is fully liberalised, that is $f_x \rightarrow f$ and $\tau \rightarrow 1$, and*

1. if vacancy creation costs are linear, all firms export to all n foreign countries, but
2. if vacancy creation costs are convex, some firms serve only the domestic market and the number of export destinations j among exporters increases with productivity φ .

Proof See Appendix K.

The fact that not all firms export has implications for the effect of trade liberalisation on the wage distribution, as we will show below.

Wages increase when trade is liberalised (with the exception of those workers that earn the reservation wage). This follows from the fact that exporting firms profit from lower shipping costs or

¹⁵The proof is based on the vacancy creation cost function used in this paper. In Holzner and Larch (2011), we provide simulations which suggest that the trade pattern persists even with mild convex vacancy creation costs.

lower exporting fixed costs. This increases exporting firms' incentive to increase their labour input by offering higher wages in order to attract more workers. In equilibrium, less productive firms are also forced to increase their wages (if they are not forced out of the market) because the increased profit due to lower shipping or lower exporting fixed costs triggers firm entry and thereby increases firms' competition for workers.

Proposition 5 *A decrease in shipping costs τ or exporting fixed costs f_x increases all wages (except the reservation wage).*

Proof See Appendix L.

The fact that wages at the upper end of the wage distribution increase but the wage at the least productive firm does not (it pays a fixed reservation wage z) does not necessarily imply that wage inequality increases as the relative change of lower and higher wages could still be identical. We use Lorenz dominance to measure inequality. In order to be able to derive analytical results for the change in wage inequality, we normalise unemployment benefit to zero or unemployment benefits proportional to average wages. Lorenz dominance is consistent with lower inequality according to a wide class of inequality measures, most prominently the Gini coefficient. Take two wage distributions $G_0(w)$ and $G_1(w)$, then $G_0(w)$ Lorenz dominates $G_1(w)$ if and only if,

$$\frac{\int_0^G w(G'_0) dG'_0}{\int_0^1 w(G'_0) dG'_0} \geq \frac{\int_0^G w(G'_1) dG'_1}{\int_0^1 w(G'_1) dG'_1}, \quad (31)$$

for all quantiles $G \in [0, 1]$ and for some G with strict inequality. In the case of linear vacancy creation costs, trade liberalisation does not directly affect wages, but it does affect wages indirectly via the total number of vacancies $M\tilde{v}$. Since wages are linear in the total number of vacancies, as one can easily see by substituting $v(\cdot)/l(w(\cdot), v(\cdot))$ in Equation (23) using Equation (22), wages relative to the average wage $\int_0^1 w(G'_0) dG'_0$ remain unaffected by changes in $M\tilde{v}$ caused by trade liberalisation. Thus, trade liberalisation affects wages proportionally, which leaves inequality unchanged as one can see by looking at Equation (31).

Similarly, the indirect effect on wages via the number of active firms M drops out in our framework with convex vacancy creation costs. However, with convex vacancy creation costs wages are also directly affected by lower shipping costs or lower exporting fixed costs. The trade pattern implied by convex vacancy creation costs, that is that the number of export market served increases with productivity, implies that more productive firms will export to more countries and hence profit more from lower trade costs. They will therefore compete more for workers than firms selling to fewer destinations if trade is liberalised. This increases wages of firms exporting to many destinations more than wages of firms exporting to only a few destinations or even domestic firms. This causes wage inequality in our framework to react to trade liberalisation. With linear vacancy creation costs firms post-vacancies until marginal profits are equal to the constant cost of vacancy creation. This implies that all firms, irrespective of their export strategy, have the same marginal profit in equilibrium. Thus, wages only depend on aggregate labour market tightness. Trade liberalisation, which increases profits and vacancy creation, therefore increases all wages proportionally.¹⁶ This is summarised in the following Proposition.

¹⁶Helpman et al. (2010) find an effect of trade liberalisation on wage inequality although they assume linear vacancy creation costs. The reason for this is that in their paper worker heterogeneity and the associated screening mechanism lead to a non-proportional effect of trade liberalisation on wages.

Proposition 6 For $z = 0$ or unemployment benefits proportional to average wages, a decrease in shipping costs τ or exporting fixed costs f_x affects wage inequality (measured by Lorenz dominance) only in a framework with convex vacancy creation costs.

Proof See Appendix L.

With sufficiently high convex vacancy creation costs, which prevent the least productive firms from exporting, firms' profits in an open economy are more dispersed than under autarky. This causes wage inequality in an open economy to be higher than under autarky. Thus, with convex vacancy creation costs wage inequality increases if a country opens up to trade. If a country that already trades with other countries decreases trade costs, wage inequality can increase or decrease with further trade liberalisation. As more and more firms export, the profitability gaps and, hence, the wage gaps between exporting and non-exporting firms increases wage inequality at the lower end of the wage distribution. At the same time, more and more exporting firms will start to export to all n countries due to trade liberalisation, which decreases the profitability gaps and, hence, the wage gaps at the upper end of the wage distribution. It is unclear which effect dominates.

Corollary 7 For $z = 0$ or unemployment benefits proportional to average wages, wage inequality in an open economy (including free trade, i.e., $f_x \rightarrow f$ and $\tau \rightarrow 1$) is higher than under autarky (in a framework with convex vacancy creation costs).

Proof See Appendix L.

While Helpman et al. (2010) find an inverse u-shape relationship between trade liberalisation and wage inequality, Corollary 7 shows that such an inverse u-shape does not necessarily occur in our framework. We find that wage inequality under free trade, that is $f_x \rightarrow f$ and $\tau \rightarrow 1$, is higher than under autarky if vacancy creation costs are (sufficiently) convex. This complements the results by Helpman et al. (2010), who find that full trade liberalisation implies the same wage inequality as under autarky. The result in Helpman et al. (2010) is driven by the trade pattern that emerges in their framework. Like in our linear vacancy creation cost case, they find that all firms will export to all n foreign countries if trade is fully liberalised. Thus, all firms profit equally if trade is fully liberalised. This leaves wage inequality the same as in autarky. In our model with convex vacancy creation costs and on-the-job search, the pattern of trade in which the number of export destinations increases with productivity remains intact even if trade is fully liberalised. Thus, under free trade, domestic and exporting firms are affected differently by trade openness implying that wage inequality under free trade is always higher than under autarky.

6 | CONCLUSIONS

We introduce convex vacancy creation costs and on-the-job search into a Melitz-type new trade model with monopolistic competition and heterogeneous firms resulting in a simple and analytically tractable model that is in line with many empirical facts about both the trade and labour market side. To obtain realistic trade patterns, we need convex vacancy creation costs. To obtain a realistic wage distribution, we need on-the-job search. To get an effect of trade liberalisation on residual wage inequality, we need both.

The result that trade liberalisation affects wage inequality only in a model with convex vacancy creation costs follows from the fact that with linear vacancy creation costs, firms post-vacancies until marginal profits are equal to the constant cost of vacancy creation. This implies that all

firms—irrespective of their export strategy—have the same marginal profit in equilibrium. Thus, wages paid in equilibrium will not depend on a firm's export strategy, but only on aggregate labour market tightness. Trade liberalisation, which increases vacancy creation, therefore increases all wages proportionally, but leaves wage inequality unchanged, since all commonly used wage inequality measures (e.g. Gini coefficient) are invariant to proportional changes. With convex vacancy creation costs, marginal profits and thus wages depend on the number of destinations to which a firm exports. Residual wage inequality therefore increases with trade liberalisation. Even if trade is fully liberalised, not all firms will export and not all exporters will serve all markets and hence wage inequality is higher in an open economy than under autarky. With linear vacancy creation costs, however, free trade implies that all firms serve all markets. Their wages hence increase proportionally. Thus, wage inequality is the same as in autarky.

Our theoretical model suggests that studies employing structural estimation to evaluate the extent to which trade liberalisation explains the documented rise in wage inequality should not only capture firm-level employment and wage patterns but also the richness of firms' exporting behaviour.

ACKNOWLEDGEMENTS

We acknowledge useful comments from two anonymous reviewers and from participants at NOeG, May 2010, in Vienna, EALE/SOLE, June 2010, in London, EEA, August 2010, in Glasgow, at the CESifo Area Conference on Global Economy, February 2011, in Munich, at the Göttinger Workshop "Internationale Wirtschaftsbeziehung", March 2011, in Göttingen, at the Internal Seminar, Vrije Universiteit Amsterdam, March 2011, Amsterdam, at the workshop "Globalization and Labour Market Outcomes", June 2011, in Geneva, at ESEM, August 2011, in Oslo, at the 68th European Meeting of the Econometric Society, August 2014, in Toulouse, and at IFN Stockholm conference "Globalization and New Technology: Effects on Firms and Workers", 2017, Vaxholm.

REFERENCES

- Akerman, A., Helpman, E., Itskhoki, O., Muendler, M.-A., & Redding, S. (2013). Sources of wage inequality. *American Economic Review Papers & Proceedings*, 103(3), 214–219.
- Amiti, M., & Davis, D. R. (2012). Trade, firms, and wages: Theory and evidence. *Review of Economic Studies*, 79(1), 1–36.
- Arkolakis, C. (2010). Market penetration costs and the new consumers margin in international trade. *Journal of Political Economy*, 118(6), 1151–1199.
- Attanasio, O., Goldberg, P., & Pavcnik, N. (2004). Trade reforms and wage inequality in Columbia. *Journal of Development Economics*, 74(2), 331–366.
- Bellon, M. (2018). *Trade liberalization and inequality: A dynamic model with firm and worker heterogeneity*. Unpublished manuscript. Retrieved from <https://matthieubellon.com/docs/TradeLiberalizationInequalityDynamics2018.pdf>.
- Blatter, M., Muehleemann, S., & Schenker, S. (2012). The costs of hiring skilled workers. *European Economic Review*, 56(1), 20–35.
- Brecher, R. A. (1974). Optimal commercial policy for a minimum-wage economy. *Journal of International Economics*, 4(2), 139–149.
- Burdett, K., & Mortensen, D. (1998). Wage differentials, employer size, and unemployment. *International Economic Review*, 39(2), 257–273.
- Card, D., Heining, J., & Kline, P. (2013). Workplace heterogeneity and the rise of west German wage inequality. *The Quarterly Journal of Economics*, 128(3), 967–1015.
- Chaney, T. (2014). The network structure of international trade. *American Economic Review*, 104(11), 3600–3634.
- Coşar, A. K., Guner, N., & Tybout, J. (2016). Firm dynamics, job turnover, and wage distributions in an open economy. *American Economic Review*, 106(3), 625–663.

- Cuñat, A., & Melitz, M. (2010). A many-country, many-good model of labor market rigidities as a source of comparative advantage. *Journal of the European Economic Association*, 8(2–3), 434–441.
- Cuñat, A., & Melitz, M. (2012). Volatility, labor market exibility, and the pattern of comparative advantage. *Journal of the European Economic Association*, 10(2), 225–254.
- Davidson, C., Martin, L., & Matusz, S. (1999). Trade and search generated unemployment. *Journal of International Economics*, 48(2), 271–299.
- Davidson, C., & Matusz, S. J. (2004). *International trade and labor markets – theory, evidence, and policy implications*. W.E. Upjohn Institute for Employment Research.
- Davis, D. R. (1998). Does European unemployment prop up American wages? National labor markets and global trade. *American Economic Review*, 88(3), 478–494.
- Dutt, P., Mitra, D., & Ranjan, P. (2009). International trade and unemployment: Theory and cross-national evidence. *Journal of International Economics*, 78(1), 32–44.
- Eaton, J., Kortum, S., & Kramarz, F. (2011). An anatomy of international trade: Evidence from French firms. *Econometrica*, 79(5), 1453–1498.
- Eaton, J., Kortum, S., & Kramarz, F. (2015). *Firm-to-firm trade: Imports, exports, and the labor market*. Unpublished manuscript. Retrieved from <https://economics.yale.edu/sites/default/files/kortum-150218.pdf>.
- Egger, H., & Kreckemeier, U. (2012). Fairness, trade, and inequality. *Journal of International Economics*, 86(2), 184–196.
- Fajgelbaum, P. D. (2020). Labor market frictions, firm growth and international trade. *Review of Economic Studies*, 87(3), 1213–1260.
- Felbermayr, G., Impullitti, G., & Prat, J. (2018). Firm dynamics and residual inequality in open economies. *Journal of the European Economic Association*, 16(5), 1476–1539.
- Felbermayr, G., Prat, J., & Schmerer, H.-J. (2011a). Globalization and labor market outcomes: Wage bargaining, search frictions, and firm heterogeneity. *Journal of Economic Theory*, 146(1), 39–73.
- Felbermayr, G., Prat, J., & Schmerer, H.-J. (2011b). Trade and unemployment: What do the data say? *European Economic Review*, 55(4), 741–758.
- Hasan, R., Mitra, D., Ranjan, P., & Ahsan, R. N. (2012). Trade liberalization and unemployment: Theory and evidence from India. *Journal of Development Economics*, 97(2), 269–280.
- Helpman, E., & Itskhoki, O. (2010). Labor market rigidities, trade and unemployment. *Review of Economic Studies*, 77(3), 1100–1137.
- Helpman, E., Itskhoki, O., & Redding, S. (2010). Inequality and unemployment in a global economy. *Econometrica*, 78(4), 1239–1283.
- Holzner, C., & Larch, M. (2011). *Capacity constraining labor market frictions in a global economy* (CESifo Working Paper No. 3597).
- Holzner, C., & Launov, A. (2010). Search equilibrium and social and private returns to education. *European Economic Review*, 54(1), 39–59.
- Klepper, S. (1996). Entry, exit, growth, and innovation over the product life cycle. *American Economic Review*, 86(3), 562–583.
- Manning, A. (2006). A generalised model of monopsony. *The Economic Journal*, 116(508), 84–100.
- Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6), 1695–1725.
- Menezes-Filho, N. A., & Muendler, M. A. (2011). *Labor reallocation in response to trade reform* (NBER Working Paper No. 17372), August 2011.
- Merz, M., & Yashiv, E. (2007). Labor and the market value of the firm. *American Economic Review*, 97(4), 1419–1431.
- Morales, E., Sheu, G., & Zahler, A. (2019). Extended gravity. *The Review of Economic Studies*, 86(6), 2668–2712.
- Mortensen, D. (1990). Equilibrium wage distribution: A synthesis estimating structural models of unemployment and job duration. In J. Hartog, G. Ridder, & J. Theeuwes (Eds.), *Panel data and labor market studies*. North-Holland.
- Mortensen, D. (2003). *Wage dispersion – Why are similar workers paid differently?*. MIT Press, Massachusetts Institute of Technology.
- Mortensen, D. T. (2009). *Wage dispersion in the search and matching model with intra-firm bargaining* (NBER Working Paper No. 15033).
- Moscarini, G., & Postel-Vinay, F. (2013). Stochastic search equilibrium. *Review of Economic Studies*, 80(4), 1545–1581.

- Postel-Vinay, F., & Robin, J.-M. (2002). Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica*, 70(6), 2295–2350.
- Song, J., Price, D. J., Guvenen, F., Bloom, N., & von Wachter, T. (2018). Firming up inequality. *The Quarterly Journal of Economics*, 134(1), 1–50.
- Stijepic, D. (2016). Globalization, worker mobility and wage inequality. *Review of International Economics*, 25(1), 108–131.
- Suverato, D. (2014). *Export and the labor market: A dynamic model with on-the-job search* (Munich Discussion Paper No. 2014-20). Retrieved from <http://epub.uni-muenchen.de/20919/1/Suverato2014WP.pdf>.
- Vernon, R. (1966). International investment and international trade in the product cycle. *Quarterly Journal of Economics*, 80(2), 190–207.
- Yashiv, E. (2006). Evaluating the performance of the search and matching model. *European Economic Review*, 50, 909–936.

APPENDIX A

QUANTITIES SOLD IN THE DOMESTIC AND EACH EXPORT MARKET

An exporting firm that decided to serve j foreign countries maximises its profits by equalising marginal revenues across markets. Revenues of an exporting firm are given by,

$$\begin{aligned} R(\varphi) &= p_d(\varphi) q_d(\varphi) + j p_x(\varphi) \frac{q_x(\varphi)}{\tau} \\ &= p_d(\varphi) [q(\varphi) - j q_x(\varphi)] + j p_x(\varphi) \frac{q_x(\varphi)}{\tau} \\ &= [q(\varphi) - j q_x(\varphi)]^\rho + j \left[\frac{q_x(\varphi)}{\tau} \right]^\rho. \end{aligned}$$

Firms choose its domestic and export sells according to equalisation of marginal revenues,

$$\begin{aligned} \frac{\partial R(\varphi)}{\partial q_x(\varphi)} &= 0, \\ \rho j [q(\varphi) - j q_x(\varphi)]^{\rho-1} &= \rho j \frac{1}{\tau} \left[\frac{q_x(\varphi)}{\tau} \right]^{\rho-1}, \\ q(\varphi) - j q_x(\varphi) &= \frac{1}{\tau^{\rho/(\rho-1)}} q_x(\varphi), \\ \tau^{\rho/(\rho-1)} q(\varphi) &= [1 + j \tau^{\rho/(\rho-1)}] q_x(\varphi). \end{aligned}$$

Rearranging and using the fact that $q_d(\varphi) = q(\varphi) - j q_x(\varphi)$ implies Equations (4) and (5). The revenue of an exporting firm is given by,

$$\begin{aligned} R(\varphi) &= \left[q(\varphi) - j q(\varphi) \frac{\tau^{\rho/(\rho-1)}}{1 + j \tau^{\rho/(\rho-1)}} \right]^\rho + j \left[\frac{q(\varphi)}{\tau} \frac{\tau^{\rho/(\rho-1)}}{1 + j \tau^{\rho/(\rho-1)}} \right]^\rho \\ &= [1 + j \tau^{\rho/(\rho-1)}] \left[\frac{q(\varphi)}{1 + j \tau^{\rho/(\rho-1)}} \right]^\rho \\ &= [1 + j \tau^{\rho/(\rho-1)}]^{(1-\rho)} q(\varphi)^\rho \\ &= Y(j) q(\varphi)^\rho. \end{aligned}$$

APPENDIX B

PROOF OF PROPOSITION 1

Note, first that the steady-state labour input Equation (15) implies $l(w(\varphi)) > l(w(\varphi'))$ for $w(\varphi) > w(\varphi')$. The second step is to show that the profit function $\Pi(w, j|\varphi)$ is supermodular in φ , w and j . Supermodularity of the profit function $\Pi(w, j|\varphi)$ is given if for any $\{w, j\}$ and $\{w', j'\}$ with $\varphi > \varphi'$,

$$\Pi(\max[w, w'], \max[j, j'] | \varphi) + \Pi(\min[w, w'], \min[j, j'] | \varphi') \geq \Pi(w, j | \varphi) + \Pi(w', j' | \varphi').$$

Supermodularity for the profit function $\Pi(w, j|\varphi)$ follows from the fact that the same inequality holds for revenues due to the Cobb–Douglas structure of the revenue function, that is $Y(j)[\varphi l(w)]^\rho$, replaced $\left[1 + j\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)}$ by $Y(j)$. and because wages, production fixed costs and export fixed costs cancel out.

In the third and last step, we prove by contradiction that $\varphi > \varphi'$ implies $w(\varphi) \geq w(\varphi')$ and $j(\varphi) \geq j(\varphi')$. Suppose that for any $\{w(\varphi), j(\varphi)\}$ and $\{w'(\varphi'), j'(\varphi')\}$, where wages and the number of export destinations are chosen optimally, that is $w(\varphi)$ and $w(\varphi')$ and $j(\varphi)$ and $j(\varphi')$, that one of the following statements holds,

- (1) $w(\varphi) < w(\varphi')$ and $j(\varphi) \geq j(\varphi')$,
- (2) $w(\varphi) \geq w(\varphi')$ and $j(\varphi) < j(\varphi')$,
- (3) $w(\varphi) < w(\varphi')$ and $j(\varphi) < j(\varphi')$.

The following chain of inequalities for each statement (1) to (3) follows from the optimality condition required in equilibrium (first and third inequality) and supermodularity of the profit function (second inequality), that is

$$0 < \Pi(w(\varphi), j(\varphi) | \varphi) - \Pi(\max[w(\varphi), w'(\varphi')], \max[j(\varphi), j'(\varphi')] | \varphi) \leq \Pi(\min[w(\varphi), w'(\varphi')], \min[j(\varphi), j'(\varphi')] | \varphi') - \Pi(w'(\varphi'), j'(\varphi') | \varphi') < 0,$$

and gives the contradiction necessary to complete the proof.

APPENDIX C

ASSUMPTION ENSURING CONTINUOUS WAGE OFFER DISTRIBUTION

We first show that if we fix the number of vacancies $v = \bar{v}$ for all firms, wages might not increase with productivity at the lower end of the wage distribution. To see this suppose that firms with different productivities $\varphi > \varphi'$ pay, the same wage \tilde{w} . If \tilde{w} is optimal for φ' , that is $\Pi'_w(\% \tilde{w}, j(\varphi') | \varphi') = 0$, then $\varphi > \varphi'$ and $j(\varphi) \geq j(\varphi')$ implies,

$$\delta \frac{\partial \Pi(\% \tilde{w}, j(\varphi) | \varphi)}{\partial \tilde{w}} = [\Upsilon(j(\varphi)) \varphi^\rho \rho l(\% \tilde{w})^{(\rho-1)} - \tilde{w}] \frac{\partial l(\% \tilde{w})}{\partial \tilde{w}} - l(\% \tilde{w}) \geq 0.$$

Thus, the firm with productivity φ will optimally increase its wage, that is, $w(\varphi) > \tilde{w}$. If \tilde{w} is optimal for φ , that is $\Pi'_w(\% \tilde{w}, j(\varphi) | \varphi) = 0$, then again the first-order condition implies $\Pi'_w(\% \tilde{w}, j(\varphi') | \varphi') < 0$, since $\varphi' > \varphi$ and $j(\varphi') \leq j(\varphi)$. Thus, the firm with productivity φ' will choose a wage $w(\varphi') < \tilde{w}$ as long as the wage \tilde{w} is above the reservation wage, that is $\tilde{w} > z$. If $\tilde{w} = z$ both firms will post the same wage, that is wages are not strictly increasing in productivity.

Let us now derive the assumption, which ensures that wages strictly increase with productivity if all firms post $v = \bar{v}$ vacancies. The firm with the cut-off productivity that serves only the domestic market pays the reservation wage z , that is,

$$\Pi(z, 0 | \varphi^*) = 0 \Rightarrow [\varphi^* l(z)]^\rho = z l(z) + f.$$

We want a condition that ensures $\Pi'_w(z, 0 | \varphi') \geq 0$ for all $\varphi' \in [\varphi^*, \bar{\varphi}]$. Using the cut-off condition to substitute $[\varphi^* l(z)]^\rho$ in the first-order condition of firms paying the reservation wage gives,

$$\Pi'_w(z, 0 | \varphi^*) \geq 0 \Leftrightarrow [\rho f - (1 - \rho) z l(z)] \frac{1}{l(z)} \frac{\partial l(z)}{\partial z} - l(z) \geq 0.$$

Substituting $\frac{\partial l(z)}{\partial z} \frac{1}{l(z)} = \frac{2\lambda f(z)}{[x + \phi + \lambda]}$ and rearranging implies,

$$(1 - \rho) z l(z) + \frac{x + \phi + \lambda}{2\lambda f(z)} l(z) \leq \rho f.$$

If we substitute $f(z) = \gamma / \varphi^*$ using the fact that φ is Pareto distributed according to $F(w(\varphi)) \equiv 1 - (\varphi^* / \varphi)^\gamma$ and substituting $\varphi^* = (z l(z) + f)^{1/\rho} / l(z)$ using the zero profit condition implies

$$(1 - \rho) z l(z) + \frac{x + \phi + \lambda}{2\lambda \gamma} (z l(z) + f)^{1/\rho} \leq \rho f. \quad (32)$$

Note that the number of workers per firm cannot exceed one, that is, $l(z) \leq 1$, since the number of workers is normalised to one. Thus, any $z \leq \bar{z}$ satisfies inequality (32), where \bar{z} is defined as follows,

$$(1 - \rho) \bar{z} + \frac{x + \phi + \lambda}{2\lambda \gamma} (\bar{z} + f)^{1/\rho} = \rho f.$$

This can be seen as follows, that is,

$$(1 - \rho) z l(z) + \frac{x + \phi + \lambda}{2\lambda \gamma} (z l(z) + f)^{1/\rho} \leq (1 - \rho) \bar{z} + \frac{x + \phi + \lambda}{2\lambda \gamma} (\bar{z} + f)^{1/\rho}.$$

Thus, $z \leq \bar{z}$ is sufficient to ensure non-negative marginal profits if all firms post $v = \bar{v}$ vacancies, that is to ensure $\Pi'_w(z, 0 | \varphi') \geq 0$ for all $\varphi' \in [\varphi^*, \bar{\varphi}]$.

APPENDIX D

DERIVATION OF WAGE EQUATION (18)

The wage equation for exporting firms follows from the first-order condition and the equilibrium condition (16). The first-order condition implies,

$$1 = [\Upsilon(j(\varphi)) \varphi^\rho \rho l(w(\varphi))^{(\rho-1)} - w(\varphi)] \frac{\partial l(w(\varphi))}{\partial w(\varphi)} \frac{1}{l(w(\varphi))}.$$

Using the fact that wages increase with productivity implies that we can multiply the above equation with $\partial w(\varphi) / \partial \varphi$. This gives,

$$\frac{\partial w(\varphi)}{\partial \varphi} = [\Upsilon(j(\varphi)) \varphi^\rho \rho l(\varphi)^{(\rho-1)} - w(\varphi)] \frac{\partial l(\varphi)}{\partial \varphi} \frac{1}{l(\varphi)}. \tag{33}$$

Define,

$$T(\varphi) = \ln[l(\varphi)] \text{ and } T'(\varphi) = \frac{\partial l(\varphi)}{\partial \varphi} \frac{1}{l(\varphi)}.$$

Substitution simplifies the above differential equation to,

$$\begin{aligned} \frac{\partial w(\varphi)}{\partial \varphi} + w(\varphi) \frac{\partial l(\varphi)}{\partial \varphi} \frac{1}{l(\varphi)} &= [\Upsilon(j(\varphi)) \varphi^\rho \rho l(\varphi)^{(\rho-1)}] \frac{\partial l(\varphi)}{\partial \varphi} \frac{1}{l(\varphi)} \\ \frac{\partial w(\varphi)}{\partial \varphi} + w(\varphi) T'(\varphi) &= \Upsilon(j(\varphi)) \rho \varphi^\rho [e^{T(\varphi)}]^{(\rho-1)} T'(\varphi). \end{aligned}$$

Any solution to this differential equation has to satisfy,

$$w(\varphi) e^{T(\varphi)} = \int_{\varphi_x^j}^\varphi \Upsilon(j(\% \tilde{\varphi})) \rho [\% \tilde{\varphi} e^{T(\% \tilde{\varphi})}]^\rho T'(\% \tilde{\varphi}) d\tilde{\varphi} + A, \tag{34}$$

where A is the constant of integration. Note that,

$$\frac{d [\varphi e^{T(\varphi)}]^\rho}{d\varphi} = \rho [\varphi e^{T(\varphi)}]^\rho T'(\varphi) + \rho \varphi^{\rho-1} [e^{T(\varphi)}]^\rho.$$

The integral can thus be written as,

$$\begin{aligned} \int_{\varphi_x^j}^\varphi \rho [\% \tilde{\varphi} e^{T(\% \tilde{\varphi})}]^\rho T'(\% \tilde{\varphi}) d\tilde{\varphi} &= \int_{\varphi_x^j}^\varphi \left[\frac{d [\% \tilde{\varphi} e^{T(\% \tilde{\varphi})}]^\rho}{d\tilde{\varphi}} - \rho \tilde{\varphi}^{(\rho-1)} [e^{T(\% \tilde{\varphi})}]^\rho \right] d\tilde{\varphi} \\ &= [\varphi e^{T(\varphi)}]^\rho - \left[\varphi_x^j e^{T(\varphi_x^j)} \right]^\rho - \int_{\varphi_x^j}^\varphi \rho \tilde{\varphi}^{(\rho-1)} [e^{T(\% \tilde{\varphi})}]^\rho d\tilde{\varphi}. \end{aligned}$$

Substituting into the wage Equation (34) gives,

$$w(\varphi) e^{T(\varphi)} = \Upsilon(j(\varphi)) \left[[\varphi e^{T(\varphi)}]^\rho - \left[\varphi_x^j e^{T(\varphi_x^j)} \right]^\rho - \int_{\varphi_x^j}^\varphi \frac{\rho}{\tilde{\varphi}} [\% \tilde{\varphi} e^{T(\% \tilde{\varphi})}]^\rho d\tilde{\varphi} \right] + A,$$

where,

$$\begin{aligned} A &= w(\varphi_x^j) e^{T(\varphi_x^j)} \\ &= \Upsilon(j(\varphi) - 1) \left[\left[\varphi_x^j e^{T(\varphi_x^j)} \right]^\rho - \left[\varphi_x^{j-1} e^{T(\varphi_x^{j-1})} \right]^\rho - \int_{\varphi_x^{j-1}}^{\varphi_x^j} \frac{\rho}{\tilde{\varphi}} [\% \tilde{\varphi} e^{T(\% \tilde{\varphi})}]^\rho d\tilde{\varphi} \right] \\ &\quad + w(\varphi_x^{j-1}) e^{T(\varphi_x^{j-1})}, \end{aligned}$$

and,

$$\begin{aligned} w(\varphi_x^1) e^{T(\varphi_x^1)} &= \left[\varphi_x^1 e^{T(\varphi_x^1)} \right]^\rho - [\varphi^* e^{T(\varphi^*)}]^\rho \\ &\quad - \int_{\varphi^*}^{\varphi_x^1} \frac{\rho}{\tilde{\varphi}} [\% \tilde{\varphi} e^{T(\% \tilde{\varphi})}]^\rho d\tilde{\varphi} + z e^{T(\varphi^*)}. \end{aligned}$$

Since A depends on the wage payments of those firms with export cut-off productivities $\varphi_x^j < \varphi$ we need to rewrite the wage equation as follows,

$$\begin{aligned} w(\varphi) e^{T(\varphi)} &= \Upsilon(j(\varphi)) \left[[\varphi e^{T(\varphi)}]^\rho - \left[\varphi_x^j e^{T(\varphi_x^j)} \right]^\rho - \int_{\varphi_x^j}^\varphi \frac{\rho}{\tilde{\varphi}} [\% \tilde{\varphi} e^{T(\% \tilde{\varphi})}]^\rho d\tilde{\varphi} \right] \\ &+ \Upsilon(j(\varphi) - 1) \left[\left[\varphi_x^j e^{T(\varphi_x^j)} \right]^\rho - \left[\varphi_x^{j-1} e^{T(\varphi_x^{j-1})} \right]^\rho - \int_{\varphi_x^{j-1}}^{\varphi_x^j} \frac{\rho}{\tilde{\varphi}} [\% \tilde{\varphi} e^{T(\% \tilde{\varphi})}]^\rho d\tilde{\varphi} \right] \\ &+ \dots + \\ &+ \left[\varphi_x^1 e^{T(\varphi_x^1)} \right]^\rho - [\varphi^* e^{T(\varphi^*)}]^\rho - \int_{\varphi^*}^{\varphi_x^1} \frac{\rho}{\tilde{\varphi}} [\% \tilde{\varphi} e^{T(\% \tilde{\varphi})}]^\rho d\tilde{\varphi}, \end{aligned} \quad (35)$$

or,

$$\begin{aligned} w(\varphi) e^{T(\varphi)} &= \Upsilon(j(\varphi)) \left[[\varphi e^{T(\varphi)}]^\rho - \int_{\varphi_x^j}^\varphi \frac{\rho}{\tilde{\varphi}} [\% \tilde{\varphi} e^{T(\% \tilde{\varphi})}]^\rho d\tilde{\varphi} \right] \\ &\quad - \sum_{i=1}^{j(\varphi)} [\Upsilon(i) - \Upsilon(i-1)] \left[\varphi_x^i e^{T(\varphi_x^i)} \right]^\rho \\ &\quad - \sum_{i=2}^{j(\varphi)} \left[\Upsilon(i-1) \int_{\varphi_x^{i-1}}^{\varphi_x^i} \frac{\rho}{\% \tilde{\varphi}} [\% \tilde{\varphi} e^{T(\% \tilde{\varphi})}]^\rho d\tilde{\varphi} \right] \\ &\quad - \left[[\varphi^* e^{T(\varphi^*)}]^\rho + \int_{\varphi^*}^{\varphi_x^1} \frac{\rho}{\tilde{\varphi}} [\% \tilde{\varphi} e^{T(\% \tilde{\varphi})}]^\rho d\tilde{\varphi} \right]. \end{aligned} \quad (36)$$

Substituting $e^{T(\varphi)} = l(w(\varphi))$, z by using the zero cut-off profit condition (20) and $\left[\varphi_x^j l(w(\varphi_x^j)) \right]^\rho$ using the export cut-off condition (17) gives,

$$\begin{aligned}
 w(\varphi)l(w(\varphi)) &= \Upsilon(j(\varphi))[\varphi l(w(\varphi))]^\rho - j(\varphi)f_x \\
 &- \Upsilon(j(\varphi)) \int_{\varphi_x^j}^{\varphi} \frac{\rho}{\tilde{\varphi}} [\% \tilde{\varphi} l(w(\% \tilde{\varphi}))]^\rho d\tilde{\varphi} \\
 &- \sum_{i=2}^{j(\varphi)} \left[\Upsilon(i-1) \int_{\varphi_x^{i-1}}^{\varphi_x^i} \frac{\rho}{\% \tilde{\varphi}} [\% \tilde{\varphi} l(w(\% \tilde{\varphi}))]^\rho d\tilde{\varphi} \right] \\
 &- \int_{\varphi^*}^{\varphi_x^1} \frac{\rho}{\tilde{\varphi}} [\% \tilde{\varphi} l(w(\% \tilde{\varphi}))]^\rho d\tilde{\varphi} - f.
 \end{aligned}$$

The wage Equation (18) follows immediately by defining $\xi^0 = \varphi^*$, $\xi^i = \varphi_x^i$ for $i \in \{1, 2, \dots, j(\varphi)\}$, $\xi^{j(\varphi)+1} = \varphi$ and $j \leq n$.

APPENDIX E

FREE ENTRY CONDITION

Rearranging the wage Equation (18) implies that profits of an exporting firm are given by,

$$\delta \Pi(\varphi) = \Upsilon(j(\varphi))[\varphi l(w(\varphi))]^\rho - w(\varphi)l(w(\varphi)) - f - j(\varphi)f_x \tag{37}$$

$$= \sum_{i=1}^{j(\varphi)+1} \left[\Upsilon(i-1) \int_{\xi^{i-1}}^{\xi^i} \frac{\rho}{\tilde{\varphi}} [\tilde{\varphi} l(w(\tilde{\varphi}))]^\rho d\tilde{\varphi} \right], \tag{38}$$

where $\xi^0 = \varphi^*$, $\xi^i = \varphi_x^i$ for $i \in \{1, 2, \dots, j(\varphi)\}$, $\xi^{j(\varphi)+1} = \varphi$ and $j(\varphi) \leq n$. Since free entry implies $f_e = \Pi_e = \int_{\varphi^*}^{\infty} \Pi(\varphi) \gamma(\varphi) d\varphi$, integrating over all firms with productivity $\varphi \in [\varphi^*, \infty)$ implies the free entry condition for an open economy as stated in Equation (19).

APPENDIX F

EXISTENCE PROOF

Applying the implicit function theorem to the free entry condition (8) implies,

$$\frac{d\varphi^*}{dM} = - \frac{\int_{\varphi^*}^{\infty} \frac{d\Pi(\varphi)}{dM} d\Gamma(\varphi)}{\int_{\varphi^*}^{\infty} \frac{d\Pi(\varphi)}{d\varphi^*} d\Gamma(\varphi) - \Pi(\varphi^*) \frac{d\Gamma(\varphi^*)}{d\varphi^*}} = - \frac{\int_{\varphi^*}^{\infty} \frac{d\Pi(\varphi)}{dM} d\Gamma(\varphi)}{\int_{\varphi^*}^{\infty} \frac{d\Pi(\varphi)}{d\varphi^*} d\Gamma(\varphi)} < 0,$$

since the derivative of the profits of a firm with productivity φ with respect to the zero productivity cut-off φ^* using Equation (37) is given by,

$$\begin{aligned}
 \delta \frac{d\Pi(\varphi)}{d\varphi^*} &= \sum_{i=1}^{j(\varphi)+1} \left[\Upsilon(i-1) \int_{\varphi_x^{i-1}}^{\varphi_x^i} \rho^2 [\% \tilde{\varphi} l(w(\% \tilde{\varphi}))]^\rho \frac{\partial l(w(\% \tilde{\varphi}))}{\partial \varphi^*} d\tilde{\varphi} \right] \\
 &- \sum_{i=1}^{j(\varphi)} \frac{\rho}{\varphi_x^i} f_x \frac{d\varphi_x^i}{d\varphi^*} - \frac{\rho}{\varphi^*} [\varphi^* l(w(\varphi^*))]^\rho < 0,
 \end{aligned} \tag{39}$$

where the second term follows from differentiating $\delta\Pi(\varphi)$ with respect to the export cut-off productivities φ_x^i and using the export cut-off condition (17) to substitute for $[\varphi_x^i l(w(\varphi_x^i))]^\rho$. The last term follows from noting that $\varphi_x^0 = \varphi^*$. Note that $d\varphi_x^i/d\varphi^*$ in the second term is positive, since applying the implicit function theorem to the export cut-off condition (17) implies,

$$\frac{d\varphi_x^i}{d\varphi^*} = - \frac{\rho [\varphi_x^i l(w(\varphi_x^i))]^{\rho-1} \varphi_x^i \frac{\partial l(w(\varphi_x^i))}{\partial \varphi^*}}{\rho [\varphi_x^i l(w(\varphi_x^i))]^{\rho-1} [l(w(\varphi_x^i)) + \varphi_x^i \frac{\partial l(w(\varphi_x^i))}{\partial \varphi_x^i}]} > 0,$$

since $\partial l(w(\varphi))/\partial \varphi > 0$ and $\partial l(w(\varphi_x^i))/\partial \varphi^* < 0$ as the position of a firm in the wage offer distribution decreases if φ^* increases. The same applies to $\partial l(w(\tilde{\varphi}))/\partial \varphi^* < 0$ in the first term of Equation (39). The derivative with respect to the number of total vacancies M is given by,

$$\begin{aligned} \delta \frac{d\Pi(\varphi)}{dM} &= \sum_{i=1}^{j(\varphi)+1} \left[\Upsilon(i-1) \int_{\varphi_x^{i-1}}^{\varphi_x^i} \rho^2 [\% \tilde{\varphi} l(w(\% \tilde{\varphi}))]^{\rho-1} \frac{\partial l(w(\% \tilde{\varphi}))}{\partial M} d\tilde{\varphi} \right] \\ &\quad - \sum_{i=1}^{j(\varphi)} \frac{\rho}{\varphi_x^i} f_x \frac{d\varphi_x^i}{dM} < 0, \end{aligned} \quad (40)$$

since $\partial l(w(\tilde{\varphi}))/\partial M < 0$ as one can easily verify from Equation (15). Note that, similar to the argument above, as $d\varphi_x^i/d\varphi^*$ in the second term is positive, one can easily verify from applying the implicit function theorem to the export cut-off condition (17) that $d\varphi_x^i/dM$ is positive. Thus, the free entry condition defines a decreasing relationship between the zero cut-off productivity φ^* and the number of active firms M in the market.

Applying the implicit function theorem to the zero cut-off profit condition (20) implies,

$$\frac{d\varphi^*}{dM} = \frac{\rho [\varphi^*]^\rho [l(z)]^\rho - zl(z)}{\rho [\varphi^*]^{\rho-1} [l(z)]^\rho} \frac{1}{M} > 0, \quad (41)$$

where the assumption ensuring a continuous wage offer distribution (see Appendix C), that is $\Pi_w'(z, 0|\varphi^*) \geq 0 \Rightarrow \rho [\varphi^*]^\rho [l(z)]^\rho > zl(z)$, ensures an increasing relation between the zero cut-off productivity φ^* and the number of active firms M in the market. Thus, a unique equilibrium exists if unemployment benefits z are low enough.

APPENDIX G

EXISTENCE PROOF FOR AN EQUILIBRIUM WITH LINEAR VACANCY CREATION COSTS

The zero cut-off productivity φ^* is defined by the zero profit condition of the least productive firm. Using the profit Equation (27), the zero profit condition can be stated as,

$$0 = (1 - \rho) \left[\frac{z}{\rho} + \frac{c}{\rho} \frac{v(\varphi^*)}{l(z, v(\varphi^*))} \right]^{-\frac{\rho}{1-\rho}} [\varphi^*]^{\frac{\rho}{1-\rho}} - f. \quad (42)$$

Using Equation (22) to replace $v(\varphi^*)/l(z, v(\varphi^*))$, it follows that the zero cut-off profit condition defines an increasing relationship between φ^* and $M\tilde{v}$.

In a framework without convex vacancy creation costs, the free entry condition is given by integrating the profit Equation (27) with $j(\varphi) = 0$ for non-exporting firms over the productivity range $[\varphi^*, \varphi_x]$ and $j(\varphi) = n$ for exporting firms over the productivity range $[\varphi_x, \infty)$ and by equating average profits to the entry costs f_e . Substituting $cv(\varphi^*)/l(z, v(\varphi^*))$ using the zero cut-off profit condition (42) implies,

$$\delta f_e = f \int_{\varphi^*}^{\varphi_x} \left[\frac{\varphi}{\varphi^*} \right]^{\frac{\rho}{1-\rho}} d\Gamma(\varphi) + f \left[1 + n\tau^{\frac{\rho}{\rho-1}} \right] \int_{\varphi_x}^{\infty} \left[\frac{\varphi}{\varphi^*} \right]^{\frac{\rho}{1-\rho}} d\Gamma(\varphi) - [1 - \Gamma(\varphi^*)]f - nf_x [1 - \Gamma(\varphi_x)]. \tag{43}$$

The free entry condition, which is independent of $M\bar{v}$, defines the zero cut-off productivity φ^* . Differentiating the right-hand side implies existence and uniqueness, that is

$$\frac{\partial \delta \Pi_e}{\partial \varphi^*} = -\frac{\rho}{1-\rho} \frac{1}{\varphi^*} [\delta f_e + [1 - \Gamma(\varphi^*)]f + [1 - \Gamma(\varphi_x)]nf_x] < 0.$$

The total number of vacancies in the economy $M\bar{v}$ is then given by the zero cut-off profit condition (42) where $v(\varphi^*)/l(z, v(\varphi^*))$ is given by (using again Equation (22)),

$$\frac{v(\varphi^*)}{l(z, v(\varphi^*))} = \frac{[\chi + \phi + \lambda]^2}{\lambda(\chi + \phi)} M\bar{v}.$$

APPENDIX H

DERIVATION OF WAGE EQUATION (23)

The optimality condition for vacancies (24) and wages,

$$\rho Y(j(\varphi)) \varphi^\rho l(w(\varphi), v(\varphi))^{(\rho-1)} - w(\varphi) = \frac{l(w(\varphi), v(\varphi)) \frac{\partial w(\varphi)}{\partial \varphi}}{\frac{\partial l(w(\varphi), v(\varphi))}{\partial w(\varphi)} \frac{\partial w(\varphi)}{\partial \varphi}},$$

imply the equality of marginal costs, that is

$$\left[\frac{\partial l(w(\varphi), v(\varphi))}{\partial w(\varphi)} \frac{\partial w(\varphi)}{\partial \varphi} \right] \frac{v(\varphi)}{l(w(\varphi), v(\varphi))} c = \frac{\partial w(\varphi)}{\partial \varphi} l(w(\varphi), v(\varphi)). \tag{44}$$

Similar to Appendix D, using Equation (22) we define,

$$\frac{l(w(\varphi), v(\varphi))}{v(\varphi)} = \frac{\lambda(\chi + \phi)}{M\bar{v}} e^{T(\varphi)}, \tag{45}$$

where

$$T(\varphi) = -\ln [[\chi + \phi + \lambda [1 - F(w(\varphi))]]^2],$$

$$T'(\varphi) = \frac{2\lambda f(w(\varphi))}{[\chi + \phi + \lambda [1 - F(w(\varphi))]]} \frac{\partial w(\varphi)}{\partial \varphi}.$$

Given the definition of $l(w(\varphi), v(\varphi))$ in Equation (22), that is

$$l(w(\varphi), v(\varphi)) = \frac{\lambda(x + \phi)v(\varphi)}{M\tilde{v}} \frac{1}{[x + \phi + \lambda[1 - F(w(\varphi))]]^2},$$

we get

$$\frac{\partial l(w(\varphi), v(\varphi))}{\partial w(\varphi)} = l(w(\varphi), v(\varphi)) \frac{2\lambda f'(w(\varphi))}{[x + \phi + \lambda[1 - F(w(\varphi))]]}.$$

Substituting this into Equation (44) using $T'_\varphi(\varphi)$ and Equation (45) allows us to write the differential equation for wages as follows,

$$\frac{\partial w(\varphi)}{\partial \varphi} = \frac{cM\tilde{v}}{\lambda(x + \phi)} \frac{T'_\varphi(\varphi)}{e^{T(\varphi)}}.$$

Integration gives,

$$\begin{aligned} w(\varphi) &= \frac{cM\tilde{v}}{\lambda(x + \phi)} \int_{\varphi^*}^{\varphi} \frac{T'_\varphi(\varphi) d\varphi}{e^{T(\varphi)}} + A \\ &= \frac{cM\tilde{v}}{\lambda(x + \phi)} [e^{-T(\varphi^*)} - e^{-T(\varphi)}] + A \\ &= c \left[\frac{v(\varphi^*)}{l(0, v(\varphi^*))} - \frac{v(\varphi)}{l(w(\varphi), v(\varphi))} \right] + z, \end{aligned} \quad (46)$$

where $A = z$ follows from $w(\varphi^*) = z$.

APPENDIX I

PROOF OF PROPOSITION 2

The export cut-off productivity φ_x^j is defined by $\delta\Pi(w, j|\varphi_x^j) = \delta\Pi(w, j-1|\varphi_x^j)$, where,

$$\delta\Pi(w, j|\varphi) = \Upsilon(j) [\varphi l(w(\varphi))]^\rho - w(\varphi) l(w(\varphi)) - f - jf_x.$$

Since profit maximisation implies that the wage is continuous at φ_x^j , and since the same wage implies that the number of workers employed by both types of firms are identical and given by $l(w(\varphi_x^j))$, we may write,

$$\begin{aligned} &\Upsilon(j) [\varphi_x^j l(w(\varphi_x^j))]^\rho - w(\varphi_x^j) l(w(\varphi_x^j)) - f - jf_x \\ &= \Upsilon(j-1) [\varphi_x^j l(w(\varphi_x^j))]^\rho - w(\varphi_x^j) l(w(\varphi_x^j)) - f - (j-1)f_x. \end{aligned}$$

Thus, the export cut-off condition (17) can be derived,

$$[\varphi_x^j l(w(\varphi_x^j))]^\rho = \frac{f_x}{[\Upsilon(j) - \Upsilon(j-1)]}.$$

The right-hand side of the last equation (and therefore the export cut-off productivity φ_x^j) is increasing in j , that is

$$= f_x \frac{[\varphi_x^j l(w(\varphi_x^j))]^\rho - [\varphi_x^{j-1} l(w(\varphi_x^{j-1}))]^\rho}{2Y(j-1) - Y(j-2) - Y(j)} > 0,$$

where the last inequality follows from Jensen's inequality, that is

$$\begin{aligned} \frac{1}{2}Y(j-2) + \frac{1}{2}Y(j) &= \frac{1}{2} \left[1 + (j-2) \tau^{\frac{\rho}{\rho-1}} \right]^{(1-\rho)} + \frac{1}{2} \left[1 + j \tau^{\frac{\rho}{\rho-1}} \right]^{(1-\rho)} \\ &< \left[\frac{1}{2} \left[1 + (j-2) \tau^{\frac{\rho}{\rho-1}} \right] + \frac{1}{2} \left[1 + j \tau^{\frac{\rho}{\rho-1}} \right] \right]^{(1-\rho)} \\ &= \left[1 + \left[\frac{1}{2}(j-2) + \frac{1}{2}j \right] \tau^{\frac{\rho}{\rho-1}} \right]^{(1-\rho)} \\ &= \left[1 + (j-1) \tau^{\frac{\rho}{\rho-1}} \right]^{(1-\rho)} \\ &= Y(j-1). \end{aligned}$$

APPENDIX J

THE EFFECTS OF TRADE LIBERALISATION ON φ^* BETA AND M (OR $M\bar{v}$ MTLDEV)

The free entry condition always guarantees that the zero cut-off productivity φ^* and the total number of vacancies $V \equiv M\bar{v}$ in the case of convex vacancy creation costs and $V \equiv M\bar{v}$ in the case of linear vacancy creation costs, adjust such that average profits are equal to the entry costs f_e . In order to show the effect of trade liberalisation, we can thus use the implicit function theorem, that is

$$\Pi_e = f_e \iff \frac{\partial \Pi_e}{\partial x} + \sum_{i=1}^n \frac{d\Pi_e}{d\varphi_x^i} \frac{d\varphi_x^i}{dx} + \frac{d\Pi_e}{d\varphi^*} \frac{d\varphi^*}{dx} + \frac{d\Pi_e}{dV} \frac{dV}{dx} = 0, \tag{47}$$

for every $x \in \{\tau, f_x\}$.

Convex vacancy creation costs

Note first that the zero cut-off profit condition (20) remains unchanged (compare Figure 2) if trade is liberalised. Thus, a change in the cut-off productivity φ^* in response to trade liberalisation must go along with a change in the total number of vacancies V in the same direction, since the zero cut-off profit (ZCP) condition defines an increasing relationship between φ^* and V as shown in Appendix F, that is since $d\varphi^*/dM|_{ZCP}$ and therefore $d\varphi^*/dV|_{ZCP} > 0$. Thus, the implicit function theorem implies,

$$\frac{d\varphi^*}{dx} = - \frac{\frac{\partial \Pi_e}{\partial x} + \sum_{i=1}^n \frac{d\Pi_e}{d\varphi_x^i} \frac{d\varphi_x^i}{dx}}{\frac{d\Pi_e}{d\varphi^*} + \frac{d\Pi_e}{dV} \frac{dV}{d\varphi^*} \Big|_{ZCP}} \text{ and } \frac{dV}{dx} = \frac{dV}{d\varphi^*} \Big|_{ZCP} \frac{d\varphi^*}{dx} \text{ for every } x \in \{\tau, f_x\}. \tag{48}$$

Average profits are given by,

$$\begin{aligned} \Pi_e &= \int_{\varphi^*}^{\infty} \Pi(\varphi) \gamma(\varphi) d\varphi \\ &= \frac{1}{\delta} \int_{\varphi^*}^{\infty} \left[\sum_{i=1}^{n+1} \left[\Upsilon(i-1) \int_{\varphi_x^{i-1}}^{\varphi_x^i} \frac{\rho}{\% \tilde{\varphi}} [\% \tilde{\varphi} l(w(\% \tilde{\varphi}))]^{\rho} d\tilde{\varphi} \right] \right] \gamma(\varphi) d\varphi, \end{aligned} \quad (49)$$

where $\varphi_x^{n+1} = \varphi$ and $\varphi_x^0 = \varphi^*$. If on-the-job search is possible $l(w(\varphi))$ is given by Equation (15). Before we apply the implicit function theorem, let us investigate the single derivatives.

In order to use the implicit function theorem as stated in Equation (48) consider the direct effect that the variables $x \in \{\tau, f_x\}$ have on a firm's profits. The shipping costs τ enter profits directly and exporting fixed costs f_x only enter through the export cut-off productivities φ_x^i , that is (using Equation (38)),

$$\begin{aligned} \delta \frac{\partial \Pi(\varphi)}{\partial \tau} &= - \sum_{i=1}^{j(\varphi)+1} \left[\rho(i-1) \tau^{\frac{1}{\rho-1}} \Upsilon(i-1)^{\frac{-\rho}{1-\rho}} \int_{\varphi_x^{i-1}}^{\varphi_x^i} \frac{\rho}{\tilde{\varphi}} [\% \tilde{\varphi} l(w(\% \tilde{\varphi}))]^{\rho} d\tilde{\varphi} \right] < 0, \\ \delta \frac{\partial \Pi(\varphi)}{\partial f_x} &= 0. \end{aligned}$$

Thus, integrating the respective terms over $[\varphi^*, \infty)$, weighted with $\gamma(\varphi)$, implies,

$$\frac{\partial \Pi_e}{\partial \tau} < 0, \text{ and } \frac{\partial \Pi_e}{\partial f_x} = 0. \quad (50)$$

The derivative of the profits of a firm with productivity φ with respect to the export cut-off productivities is given by,

$$\begin{aligned} \delta \sum_{i=1}^{j(\varphi)} \frac{d\Pi(\varphi)}{d\varphi_x^i} \frac{d\varphi_x^i}{dx} &= \sum_{i=1}^{j(\varphi)} [\Upsilon(i-1) - \Upsilon(i)] \frac{\rho}{\varphi_x^i} [\varphi_x^i l(w(\varphi_x^i))]^{\rho} \frac{d\varphi_x^i}{dx} \\ &= - \sum_{i=1}^{j(\varphi)} \frac{\rho}{\varphi_x^i} f_x \frac{d\varphi_x^i}{dx}, \end{aligned}$$

where we used the fact that $\varphi_x^{j+1} = \varphi$ and $\varphi_x^0 = \varphi^*$ are not export cut-off productivities and that $[\varphi_x^i l(w(\varphi_x^i))]^{\rho}$ in the second line can be substituted using the export cut-off condition (17). Applying the implicit function theorem to the export cut-off Equation (17) shows, that is

$$\begin{aligned} \frac{d\varphi_x^i}{df_x} &= \frac{\varphi_x^i}{f_x \rho \left[1 + \frac{\varphi_x^i}{l(w(\varphi_x^i))} \frac{\partial l(w(\varphi_x^i))}{\partial \varphi_x^i} \right]} > 0, \\ \frac{d\varphi_x^i}{d\tau} &= \frac{\varphi_x^i \tau^{\frac{1}{\rho-1}} \left[\Upsilon(i)^{\frac{-\rho}{1-\rho}} [i] - \Upsilon(i-1)^{\frac{-\rho}{1-\rho}} [i-1] \right]}{\left[\Upsilon(j) - \Upsilon(j-1) \right] \left[1 + \frac{\varphi_x^i}{l(w(\varphi_x^i))} \frac{\partial l(w(\varphi_x^i))}{\partial \varphi_x^i} \right]} \\ &> 0, \end{aligned}$$

where the last inequality follows from,

$$\begin{aligned} & Y(i)^{\frac{-\rho}{1-\rho}} [i] > Y(i-1)^{\frac{-\rho}{1-\rho}} [i-1], \\ & 1 + \frac{1}{i-1} > \left[1 + \frac{1}{\tau^{\rho/(1-\rho)} + (i-1)} \right]^{\rho} \text{ for any } \rho \in (0, 1), \end{aligned}$$

and noting that $Y(j) - Y(j-1) > 0$, as otherwise $[\varphi_x^j \cdot l(w(\varphi_x^j))]$ would be negative according to Equation (17).

Integrating the respective terms over $[\varphi^*, \infty)$, weighted with $\gamma(\varphi)$, implies,

$$\sum_{i=1}^n \frac{d\Pi_e}{d\varphi_x^i} \frac{d\varphi_x^i}{df_x} = \varphi^* \cdot \infty^* \int_{i(\varphi)=1}^{\tilde{\gamma}} \sum_{i(\varphi)=1}^{j(\varphi)} \frac{d\Pi(\varphi)}{d\varphi_x^{i(\varphi)}} \frac{d\varphi_x^{i(\varphi)}}{df_x} \gamma(\varphi) d\varphi < 0, \text{ and} \tag{51}$$

$$\sum_{i=1}^n \frac{d\Pi_e}{d\varphi_x^i} \frac{d\varphi_x^i}{d\tau} = \varphi^* \cdot \infty^* \int_{i(\varphi)=1}^{\tilde{\gamma}} \sum_{i(\varphi)=1}^{j(\varphi)} \frac{d\Pi(\varphi)}{d\varphi_x^{i(\varphi)}} \frac{d\varphi_x^{i(\varphi)}}{d\tau} \gamma(\varphi) d\varphi < 0. \tag{52}$$

The effect of a change in the potential trading partners n does not alter the export cut-offs as Equation (17) is independent of n .

The derivative of the profits of a firm with productivity φ with respect to the zero productivity cut-off φ^* and M (and therefore V) is given by Equations (39) and (40) in Appendix F, respectively. Integrating the respective derivatives over $[\varphi^*, \infty)$ weighted with $\gamma(\varphi)$ implies,

$$\frac{d\Pi_e}{d\varphi^*} + \frac{d\Pi_e}{dV} \frac{dV}{d\varphi^*} < 0. \tag{53}$$

Inserting Equations (50–53) into formula (48) implies that trade liberalisation ($\tau \downarrow, f_x \downarrow$) increases the zero cut-off productivity φ^* and thus also the total number of vacancies V in the economy.

Linear vacancy creation costs

We can use the free entry condition (43) to derive the effect of trade liberalisation on the cut-off productivity φ^* . Changing the variable of integration from φ to $\tilde{\Gamma}(\varphi) \equiv [\Gamma(\varphi) - \Gamma(\varphi^*)] / [1 - \Gamma(\varphi^*)] = 1 - (\varphi^* / \varphi)^\gamma$ implies the following equation,

$$\begin{aligned} \delta f_e = & f [1 - \Gamma(\varphi^*)] \int_0^{\tilde{\Gamma}(\varphi_x)} [1 - \tilde{\Gamma}]^{-\frac{1}{\gamma} \frac{\rho}{1-\rho}} d\tilde{\Gamma} \\ & + f [1 - \Gamma(\varphi^*)] \left[1 + n\tau^{\frac{\rho}{\rho-1}} \right] \int_{\tilde{\Gamma}(\varphi_x)}^1 [1 - \tilde{\Gamma}]^{-\frac{1}{\gamma} \frac{\rho}{1-\rho}} d\tilde{\Gamma} \\ & - [1 - \Gamma(\varphi^*)] f - [1 - \Gamma(\varphi^*)] [1 - \% \tilde{\Gamma}(\varphi_x)] n f_x. \end{aligned} \tag{54}$$

Taking the derivatives of the right-hand side of Equation (54) with respect to φ^* , f_x , and τ , that is

$$\begin{aligned} \frac{\partial \delta \Pi_e}{\partial \varphi^*} &= -\frac{\gamma(\varphi^*)}{[1-\Gamma(\varphi^*)]} \delta \Pi_e \\ &+ f[1-\Gamma(\varphi^*)] \frac{\partial \tilde{\Gamma}(\varphi_x)}{\partial \varphi^*} n \left[\frac{f_x}{f} - \tau^{\frac{\rho}{1-\rho}} [1-\% \tilde{\Gamma}(\varphi_x)]^{-\frac{1}{\gamma} \frac{\rho}{1-\rho}} \right] \\ &= -\frac{\gamma(\varphi^*)}{[1-\Gamma(\varphi^*)]} \delta \Pi_e < 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial \delta \Pi_e}{\partial f_x} &= -f[1-\Gamma(\varphi^*)] n \tau^{\frac{\rho}{1-\rho}} [1-\% \tilde{\Gamma}(\varphi_x)]^{-\frac{1}{\gamma} \frac{\rho}{1-\rho}} \frac{\partial \tilde{\Gamma}(\varphi_x)}{\partial f_x} \\ &+ [1-\Gamma(\varphi^*)] \frac{\partial \tilde{\Gamma}(\varphi_x)}{\partial f_x} n f_x \\ &- [1-\Gamma(\varphi^*)] [1-\% \tilde{\Gamma}(\varphi_x)] n \\ &= -[1-\Gamma(\varphi^*)] \frac{1}{\tau^\gamma} \left[\frac{f}{f_x} \right]^{\gamma \frac{1-\rho}{\rho}} n < 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial \delta \Pi_e}{\partial \tau} &= -f[1-\Gamma(\varphi^*)] n \tau^{\frac{\rho}{1-\rho}} [1-\% \tilde{\Gamma}(\varphi_x)]^{-\frac{1}{\gamma} \frac{\rho}{1-\rho}} \frac{\partial \tilde{\Gamma}(\varphi_x)}{\partial \tau} \\ &- f[1-\Gamma(\varphi^*)] \frac{\rho}{1-\rho} n \tau^{\frac{1}{1-\rho}} \int_{\Gamma(\varphi_x)}^1 [1-\% \tilde{\Gamma}]^{-\frac{1}{\gamma} \frac{\rho}{1-\rho}} d\tilde{\Gamma} \\ &+ [1-\Gamma(\varphi^*)] \frac{\partial \tilde{\Gamma}(\varphi_x)}{\partial \tau} n f_x \\ &= -f[1-\Gamma(\varphi^*)] \frac{\rho}{1-\rho} n \tau^{\frac{1}{1-\rho}} \int_{\tilde{\Gamma}(\varphi_x)}^1 [1-\% \tilde{\Gamma}]^{-\frac{1}{\gamma} \frac{\rho}{1-\rho}} d\tilde{\Gamma} < 0, \end{aligned}$$

since according to Equation (30) we know that,

$$[1-\tilde{\Gamma}(\varphi_x)]^{\frac{1}{\gamma}} = \frac{\varphi^*}{\varphi_x} = \frac{1}{\tau} \left[\frac{f}{f_x} \right]^{\frac{1-\rho}{\rho}}.$$

Using the implicit function theorem implies that the cut-off productivity φ^* increases as trade is liberalised, that is $f_x \downarrow$ and $\tau \downarrow$. The inequalities in the last two derivatives follow from,

$$\begin{aligned} \frac{\partial \tilde{\Gamma}(\varphi_x)}{\partial f_x} &= \frac{\partial}{\partial f_x} \left[1 - \left(\frac{\varphi^*}{\varphi_x} \right)^\gamma \right] \\ &= \gamma \frac{1-\rho}{\rho} \frac{1}{\tau^\gamma} \left[\frac{f}{f_x} \right]^{\gamma \frac{1-\rho}{\rho}} \frac{1}{f_x} > 0, \\ \frac{\partial \tilde{\Gamma}(\varphi_x)}{\partial \tau} &= \gamma \frac{1}{\tau} \frac{1}{\tau^\gamma} \left[\frac{f}{f_x} \right]^{\gamma \frac{1-\rho}{\rho}} > 0, \end{aligned}$$

where,

$$\left(\frac{\varphi^*}{\varphi_x} \right)^\gamma = \frac{1}{\tau^\gamma} \left[\frac{f}{f_x} \right]^{\gamma \frac{1-\rho}{\rho}},$$

is obtained by dividing the zero cut-off profit condition (42) by the export cut-off condition (28).

Thus, the increase in the total number of vacancies $M\tilde{v}$ follows from the fact that the zero cut-off profit condition (42), which remains unchanged if trade is liberalised, defines an increasing relationship between φ^* and $M\tilde{v}$.

APPENDIX K

PROOF OF PROPOSITION 3

Convex vacancy creation costs

Combining the export cut-off condition (17) with the zero profit condition (20) implies,

$$G = \left[\frac{\varphi^* l(z)}{\varphi_x^j l(w(\varphi_x^j))} \right]^\rho - \frac{f}{f_x} [\Upsilon(j) - \Upsilon(j-1)].$$

Using,

$$1 - \tilde{\Gamma}(\varphi_x^j) = \left(\frac{\varphi^*}{\varphi_x^j} \right)^\gamma,$$

and the labour input according to Equation (15) implies,

$$\left[\frac{\varphi^* l(z)}{\varphi_x^j l(w(\varphi_x^j))} \right]^\rho = [1 - \% \tilde{\Gamma}(\varphi_x^j)]^{\frac{\rho}{\gamma}} \left[\frac{x + \phi + \lambda [1 - \% \tilde{\Gamma}(\varphi_x^j)]}{x + \phi + \lambda} \right]^{2\rho}.$$

Differentiating G with respect to the fraction of firms that export to at least j countries $\tilde{\Gamma}(\varphi_x^j)$, and with respect to f_x and τ ,

$$\begin{aligned} G'_{\tilde{\Gamma}(\varphi_x^j)} &= -\frac{\rho}{\gamma} [1 - \% \tilde{\Gamma}(\varphi_x^j)]^{\frac{\rho-\gamma}{\gamma}} \left[\frac{x + \phi + \lambda [1 - \% \tilde{\Gamma}(\varphi_x^j)]}{x + \phi + \lambda} \right]^{2\rho} \\ &\quad - [1 - \% \tilde{\Gamma}(\varphi_x^j)]^{\frac{\rho}{\gamma}} 2\rho \left[\frac{x + \phi + \lambda [1 - \% \tilde{\Gamma}(\varphi_x^j)]}{x + \phi + \lambda} \right]^{2\rho-1} \frac{\lambda}{x + \phi + \lambda}. \\ &< 0, \\ G'_{f_x} &= \frac{f}{(f_x)^2} [\Upsilon(j) - \Upsilon(j-1)] > 0, \\ G'_\tau &= \frac{f}{f_x} \rho \tau^{\frac{1}{\rho-1}} [j \Upsilon(j)^{\frac{-\rho}{1-\rho}} - (j-1) \Upsilon(j-1)^{\frac{-\rho}{1-\rho}}] > 0, \end{aligned}$$

where the last inequality follows from,

$$j\Upsilon(j)^{\frac{-\rho}{1-\rho}} > (j-1)\Upsilon(j-1)^{\frac{-\rho}{1-\rho}},$$

$$1 + \frac{1}{j-1} > \left[1 + \frac{1}{\tau^{\rho/(1-\rho)} + (j-1)} \right]^\rho \text{ for any } \rho \in (0, 1).$$

The implicit function theorem therefore implies that the fraction of firms that export to at least j export destinations increases in response to trade liberalisation, that is $f_x \downarrow$ and $\tau \downarrow$. Let us now determine the derivatives $d^2\tilde{\Gamma}(\phi_x^j)/df_x dj$ and $d^2\tilde{\Gamma}(\phi_x^j)/d\tau dj$. Since $G'_{\tilde{\Gamma}(\phi_x^j)}$ is independent of j we get,

$$\frac{d^2\tilde{\Gamma}(\phi_x^j)}{df_x dj} = -\frac{G''_{f_x j}}{G'_{\tilde{\Gamma}(\phi_x^j)}} < 0 \text{ and } \frac{d^2\tilde{\Gamma}(\phi_x^j)}{d\tau dj} = -\frac{G''_{\tau j}}{G'_{\tilde{\Gamma}(\phi_x^j)}} < 0.$$

This implies that the fraction of firms exporting to j or less export destinations increases at an increasing rate if trade is liberalised and that the fraction of firms exporting to j or more export destinations increases at a decreasing rate. Put differently, the fraction of domestic firms exporting to $j \in \{1, \dots, n-1\}$ export destinations decrease with trade liberalisation, while the fraction of firms exporting to n destinations increases. The above results follow from,

$$G''_{f_x j} = \frac{f}{(f_x)^2} (1-\rho) \tau^{\frac{\rho}{\rho-1}} \left[\Upsilon(j)^{\frac{-\rho}{1-\rho}} - \Upsilon(j-1)^{\frac{-\rho}{1-\rho}} \right] < 0,$$

because

$$\begin{aligned} \Upsilon(j)^{\frac{-\rho}{1-\rho}} &< \Upsilon(j-1)^{\frac{-\rho}{1-\rho}} \\ \left[1 + j\tau^{\frac{\rho}{\rho-1}} \right]^{-\rho} &< \left[1 + (j-1)\tau^{\frac{\rho}{\rho-1}} \right]^{-\rho}, \\ 1 &< \left[1 + \frac{\tau^{\rho/(\rho-1)}}{1 + (j-1)\tau^{\rho/(\rho-1)}} \right]^\rho \text{ for any } \rho \in (0, 1), \end{aligned}$$

and from

$$\begin{aligned} G''_{\tau j} &= \frac{f}{f_x} \rho \tau^{\frac{1}{\rho-1}} \left[\left[1 + j\tau^{\frac{\rho}{\rho-1}} \right]^{-\rho} - \rho \tau^{\frac{\rho}{\rho-1}} j \left[1 + j\tau^{\frac{\rho}{\rho-1}} \right]^{-\rho-1} \right] \\ &\quad - \frac{f}{f_x} \rho \tau^{\frac{1}{\rho-1}} \left[\left[1 + (j-1)\tau^{\frac{\rho}{\rho-1}} \right]^{-\rho} - \rho \tau^{\frac{\rho}{\rho-1}} (j-1) \left[1 + (j-1)\tau^{\frac{\rho}{\rho-1}} \right]^{-\rho-1} \right] \\ &< 0, \end{aligned}$$

because

$$\begin{aligned} \left[1 + j\tau^{\frac{\rho}{\rho-1}} \right]^{-\rho} - \rho \tau^{\frac{\rho}{\rho-1}} j \left[1 + j\tau^{\frac{\rho}{\rho-1}} \right]^{-\rho-1} &< \left[1 + (j-1)\tau^{\frac{\rho}{\rho-1}} \right]^{-\rho} \\ &\quad - \rho \tau^{\frac{\rho}{\rho-1}} (j-1) \left[1 + (j-1)\tau^{\frac{\rho}{\rho-1}} \right]^{-\rho-1} \\ \left[1 + j\tau^{\frac{\rho}{\rho-1}} \right]^{-\rho} \left[1 - \rho \frac{j\tau^{\rho/(\rho-1)}}{1 + j\tau^{\rho/(\rho-1)}} \right] &< \left[1 + (j-1)\tau^{\frac{\rho}{\rho-1}} \right]^{-\rho} \\ &\quad \left[1 - \rho \frac{(j-1)\tau^{\rho/(\rho-1)}}{1 + (j-1)\tau^{\rho/(\rho-1)}} \right] \\ \frac{1 + (1-\rho)j\tau^{\rho/(\rho-1)}}{1 + (1-\rho)(j-1)\tau^{\rho/(\rho-1)}} &< \left[\frac{1 + j\tau^{\rho/(\rho-1)}}{1 + (j-1)\tau^{\rho/(\rho-1)}} \right]^{\rho+1} \text{ for any } \rho \in (0, 1). \end{aligned}$$

The latter follows because,

$$\frac{1 + (1 - \rho)j\tau^{\rho/(\rho-1)}}{1 + (1 - \rho)(j - 1)\tau^{\rho/(\rho-1)}} < \frac{1 + j\tau^{\rho/(\rho-1)}}{1 + (j - 1)\tau^{\rho/(\rho-1)}} < \left[\frac{1 + j\tau^{\rho/(\rho-1)}}{1 + (j - 1)\tau^{\rho/(\rho-1)}} \right]^{\rho+1}.$$

The trade pattern, which exists according to Proposition 2 in a framework with convex vacancy creation costs, also remains if trade is fully liberalised, that is $f_x \rightarrow f, \tau \rightarrow 1$. This follows from the fact that,

$$\begin{aligned} f_x f, \tau \lim_{f_x} \frac{f}{f_x} & \left[\left[1 + j\tau^{\frac{\rho}{\rho-1}} \right]^{(1-\rho)} - \left[1 + (j-1)\tau^{\frac{\rho}{\rho-1}} \right]^{(1-\rho)} \right] \\ & = \left[\left[1 + j \right]^{(1-\rho)} - \left[1 + (j-1) \right]^{(1-\rho)} \right]. \end{aligned}$$

Linear vacancy creation costs

Combining the export cut-off condition (28) with the zero profit condition (42) implies,

$$\frac{\varphi^*}{\varphi_x} = \frac{1}{\tau} \left[\frac{f}{f_x} \right]^{\frac{1-\rho}{\rho}}.$$

Thus, with linear vacancy creation costs the fraction of exporting firms $(\varphi^* / \varphi_x)^{\gamma}$ increases if trade is liberalised, that is $\tau \downarrow$ and $f_x \downarrow$. Furthermore, the model with linear vacancy creation costs predicts that all firms that start to export will export to all n destination countries. If trade is fully liberalised, that is $f_x \rightarrow f, \tau \rightarrow 1$, the model predicts that all firms will export to all countries. This follows from $\lim_{f_x \rightarrow f, \tau \rightarrow 1} (\varphi^* / \varphi_x) = 1$.

APPENDIX L

PROOF OF PROPOSITIONS 5 AND 6

Linear vacancy creation costs

Consider the wage Equation (23), that is

$$w(\varphi) = c \left[\frac{v(\varphi^*)}{l(z, v(\varphi^*))} - \frac{v(\varphi)}{l(w(\varphi), v(\varphi))} \right] + z.$$

Using Equations (13) and (16) we can write,

$$\tilde{\Gamma}(\varphi) = 1 - \left(\frac{\varphi^*}{\varphi} \right)^{\gamma} = \frac{(\chi + \phi + \lambda)G(w(\varphi))}{\chi + \phi + \lambda G(w(\varphi))},$$

which can be substituted into $v(\varphi) / l(w(\varphi), v(\varphi))$ given by Equation (22) to obtain,

$$\frac{v(\varphi)}{l(w(\varphi), v(\varphi))} = \left[\frac{\chi + \phi + \lambda}{\chi + \phi + \lambda G(w(\varphi))} \right]^2 \frac{(\chi + \phi)M\tilde{v}}{\lambda}.$$

Substituting into the wage equation allows us to write the wage as a function of the quantile of the wage earnings distribution G , that is

$$w(G) = c \left[1 - \left[\frac{x + \phi}{x + \phi + \lambda G} \right]^2 \right] \left[\frac{x + \phi + \lambda}{x + \phi} \right]^2 \frac{(x + \phi) M \bar{v}}{\lambda} + z.$$

Trade liberalisation ($\tau \downarrow, f_x \downarrow$) has only an indirect effect on wages via the number of vacancies $M \bar{v}$ in the economy. Since trade liberalisation increases the number of vacancies as shown in Appendix J, it follows that wages increase with trade liberalisation (except for the firm at the cut-off productivity, which pays the reservation wage). Furthermore, wages at higher quantiles increase more, since,

$$\frac{\partial^2 w(G)}{\partial (M \bar{v}) \partial G} = \left[\frac{2c(x + \phi)}{x + \phi + \lambda G} \right] \left[\frac{x + \phi + \lambda}{x + \phi + \lambda G} \right]^2 > 0.$$

In order to prove the effect on wage inequality, we use Lorenz dominance. In order to be able to derive analytical results, we normalise unemployment benefit to zero, that is $z = 0$, in the following. Wage inequality according to Lorenz dominance increases with trade liberalisation ($\tau \downarrow, f_x \downarrow$), if,

$$\frac{\partial \int_0^G w(G') dG'}{\partial x \int_0^1 w(G') dG'} > 0 \text{ for all } G \in [0, 1), \text{ where } x \in \{\tau, f_x\}.$$

Substituting $w(G)$ implies that the degree of wage inequality is independent of trade liberalisation, that is

$$\frac{\int_0^G w(G') dG'}{\int_0^1 w(G') dG'} = \frac{\int_0^G \left[1 - \left[\frac{x + \phi}{x + \phi + \lambda G'} \right]^2 \right] dG'}{\int_0^1 \left[1 - \left[\frac{x + \phi}{x + \phi + \lambda G'} \right]^2 \right] dG'}.$$

This also holds if we assume that unemployment benefits are proportional to the average wages, that is $z = b \int_0^1 w(G') dG'$,

$$\frac{\int_0^G w(G') dG'}{\int_0^1 w(G') dG'} = \frac{\int_0^G \left[1 - \left[\frac{x + \phi}{x + \phi + \lambda G'} \right]^2 \right] dG' - b \int_0^1 \left[1 - \left[\frac{x + \phi}{x + \phi + \lambda G'} \right]^2 \right] dG'}{\int_0^1 \left[1 - \left[\frac{x + \phi}{x + \phi + \lambda G'} \right]^2 \right] dG' - b \int_0^1 \left[1 - \left[\frac{x + \phi}{x + \phi + \lambda G'} \right]^2 \right] dG'}.$$

Convex vacancy creation costs

Let us start with taking the first-order condition for wages, that is

$$1 = \left[\Upsilon(j(\varphi)) \varphi^\rho \rho l(w(\varphi))^{(\rho-1)} - w(\varphi) \right] \frac{\partial l(w(\varphi))}{\partial w(\varphi)} \frac{1}{l(w(\varphi))}.$$

Using the fact that wages increase with G gives,

$$\frac{\partial w(G)}{\partial G} = \left[\Upsilon(j(G)) \varphi(G)^\rho \rho l(G)^{(\rho-1)} - w(G) \right] \frac{\partial l(G)}{\partial G} \frac{1}{l(G)}.$$

Define,

$$T(G) = \ln [l(G)] \text{ and } T'(G) = \frac{\partial l(G)}{\partial G} \frac{1}{l(G)}.$$

Substitution simplifies the above differential equation to,

$$\frac{\partial w(G)}{\partial G} e^{T(\varphi)} + w(G) e^{T(\varphi)} T'(G) = \Upsilon(j(G)) \varphi(G)^\rho \rho l(G)^\rho T'(G).$$

Any solution to this differential equation has to satisfy,

$$w(G) e^{T(\varphi)} = \int_{G_x^i}^G \Upsilon(j(G)) \varphi(G')^\rho \rho l(G')^\rho T'(G') dG' + A,$$

$$\text{or } w(G) = \frac{1}{l(G)} \sum_{i=0}^{j(G)} \int_{G_x^i}^{G_x^{i+1}} \Upsilon(i) \rho [\varphi(G') l(G')]^\rho T'(G') dG' + z, \tag{55}$$

where $G_x^{j(G)+1} = G$ and $G_x^0 = 0$ and $w(0) = z$.

We also know that using Equations (13), (15) and (16) we can write,

$$l(G) = \frac{\lambda}{x + \phi} \left[\frac{x + \phi + \lambda G}{x + \phi + \lambda} \right]^2 \frac{1}{M},$$

$$T'_G(G) = \frac{2\lambda}{x + \phi + \lambda G}.$$

Using the Pareto distribution $F(G) = 1 - (\varphi^* / \varphi)^\gamma$ we get,

$$\varphi(G) = \varphi^* \left(\frac{(x + \phi)(1 - G)}{x + \phi + \lambda G} \right)^{-\frac{1}{\gamma}}.$$

The quantile G_x^i at which firms export to i countries is implicitly defined by combining the export cut-off condition (17) with the zero profit condition (20), that is

$$H = \frac{(x + \phi)^{2\rho + \frac{\rho}{\gamma}} (1 - G_x^i)^{\frac{\rho}{\gamma}}}{(x + \phi + \lambda G_x^i)^{2\rho + \frac{\rho}{\gamma}}} - \frac{f}{f_x} [\Upsilon(i) - \Upsilon(i - 1)] = 0.$$

For later reference note that the implicit function theorem implies,

$$\frac{dG_x^i}{df_x} = - \frac{\partial H / \partial f_x}{\partial H / \partial G_x^i} > 0,$$

$$\frac{dG_x^i}{d\tau} = -\frac{\partial H/\partial\tau}{\partial H/\partial G_x^i} > 0,$$

since

$$\frac{\partial H}{\partial G_x^i} = -\frac{(x + \phi)^{2\rho + \frac{\rho}{\gamma}} \frac{\rho}{\gamma} (1 - G_x^i)^{-1} + (2\rho + \frac{\rho}{\gamma}) \lambda (x + \phi + \lambda G_x^i)^{-1}}{(x + \phi + \lambda G_x^i)^{2\rho + \frac{\rho}{\gamma}}} (1 - G_x^i)^{\frac{\rho}{\gamma}} < 0,$$

$$\frac{\partial H}{\partial f_x} = \frac{f}{(f_x)^2} [\Upsilon(i) - \Upsilon(i-1)] > 0,$$

$$\frac{\partial H}{\partial\tau} = \frac{f}{f_x} \rho \tau^{\frac{1}{\rho-1}} \left[i \Upsilon(i)^{\frac{-\rho}{1-\rho}} - (i-1) \Upsilon(i-1)^{\frac{-\rho}{1-\rho}} \right] > 0,$$

since

$$i \left[1 + i \tau^{\rho/(\rho-1)} \right]^{-\rho} > (i-1) \left[1 + (i-1) \tau^{\rho/(\rho-1)} \right]^{-\rho},$$

$$1 + \frac{1}{i-1} > \left[1 + \frac{1}{\tau^{\rho/(1-\rho)} + (i-1)} \right]^{\rho} \text{ for any } \rho \in (0, 1).$$

Differentiating Equation (55) implies that wages increase with trade liberalisation ($\tau \downarrow, f_x \downarrow$),

$$\begin{aligned} \frac{\partial w(G)}{\partial f_x} &= \frac{(1-\rho)}{M} \frac{\partial M}{\partial f_x} w(G) \\ &\quad - \frac{1}{l(G)} \left(\sum_{i=1}^{j(G)} [\Upsilon(i) - \Upsilon(i-1)] \rho [\varphi(G_x^i) l(G_x^i)]^{\rho} T'(G_x^i) \frac{\partial G_x^i}{\partial f_x} \right) \\ &< 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial w(G)}{\partial\tau} &= \frac{(1-\rho)}{M} \frac{\partial M}{\partial\tau} w(G) \\ &\quad - \frac{1}{l(G)} \sum_{i=0}^{j(G)} \int_{G_x^i}^{G_x^{i+1}} \left[\rho i \tau^{\frac{1}{\rho-1}} \right] \Upsilon(i)^{\frac{-\rho}{1-\rho}} \rho [\varphi(G^i) l(G^i)]^{\rho} T'(G^i) dG^i \\ &\quad - \frac{1}{l(G)} \left(\sum_{i=1}^{j(G)} [\Upsilon(i) - \Upsilon(i-1)] \rho [\varphi(G_x^i) l(G_x^i)]^{\rho} T'(G_x^i) \frac{\partial G_x^i}{\partial\tau} \right) \\ &< 0, \end{aligned}$$

since the number of active firms M decreases with f_x and τ as shown in Appendix J. Thus, all wages increase except for the reservation wage z .

In order to prove the effect on wage inequality, we use Lorenz dominance. In order to be able to derive analytical results, we normalise unemployment benefit to zero, that is $z = 0$, in the following. Wage inequality according to Lorenz dominance increases with trade liberalisation ($\tau \downarrow, f_x \downarrow$), if,

$$\frac{\partial \int_0^G w(\% \tilde{G}) d\tilde{G}}{\partial x \int_0^1 w(\% \tilde{G}) d\tilde{G}} > 0 \text{ for all } G \in [0, 1), \text{ where } x \in \{\tau, f_x\}.$$

Since we can write

$$\begin{aligned}
 w(G) &= \frac{1}{l(G)} \sum_{i=0}^{j(G)} \int_{G_x^i}^{G_x^{i+1}} \left[1 + i\tau^{\frac{\rho}{\rho-1}}\right]^{(1-\rho)} \rho [\varphi(G')l(G')]^\rho T'(G') dG' \\
 &= \rho [\varphi^*]^\rho M^{(1-\rho)} (x + \phi)^{\left(1 - \rho - \frac{\rho}{\gamma}\right)} [x + \phi + \lambda]^{2(1-\rho)} (\lambda)^{(\rho-1)} R(G),
 \end{aligned}$$

where

$$R(G) = \frac{2\lambda}{[x + \phi + \lambda G]^2} \sum_{i=0}^{j(G)} \int_{G_x^i}^{G_x^{i+1}} \Upsilon(i)(1 - G')^{-\frac{\rho}{\gamma}} (x + \phi + \lambda G')^{2\rho - 1 + \frac{\rho}{\gamma}} dG'$$

The Lorenz dominance criterion is therefore given by

$$\frac{\int_0^G R(\% \tilde{G}) d\tilde{G}}{\int_0^1 R(\% \tilde{G}) d\tilde{G}}.$$

Since $R(G)$ changes with trade liberalisation (not proportionally), we know that trade liberalisation affects wage inequality. This also holds if we assume that unemployment benefits are proportional to the average wages, that is $z = b \int_0^1 w(G') dG'$,

$$\frac{\int_0^G R(\% \tilde{G}) d\tilde{G} - b \int_0^1 R(\% \tilde{G}) d\tilde{G}}{\int_0^1 R(\% \tilde{G}) d\tilde{G} - b \int_0^1 R(\% \tilde{G}) d\tilde{G}}.$$

Under autarky we have,

$$R_a(G) = \frac{2\lambda}{[x + \phi + \lambda G]^2} \int_0^G (1 - G')^{-\frac{\rho}{\gamma}} (x + \phi + \lambda G')^{2\rho - 1 + \frac{\rho}{\gamma}} dG'.$$

This implies $R(G) \geq R_a(G)$ for all G and $R(G) > R_a(G)$ for all $G \geq G_x^1$. This implies that wage inequality in an open economy (even under free trade) is higher than under autarky.