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# Schlicht, Ekkehart: Selection Wages: An Illustration

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# Selection Wages: An Illustration

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## ABSTRACT

Offering higher wages may enable firms to attract more applicants and screen them more carefully. If firms compete in this way in the labor market, “selection wages” emerge. This note illustrates this wage-setting mechanism. Selection wages may engender unconventional results, such as a pre-tax wage compression induced by the introduction of a progressive wage tax.

*Keywords:* wage formation, efficiency wage, incentive wage, mobility, job-specific pay, wage-tax.

*Journal of Economic Literature classification:* J2, J3, J4, J5

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## S                    W

A firm that wants to hire a certain number of workers may offer a wage rate that attracts more applicants than needed in order to screen the applicants more carefully and hire only the best. If wages are set with regard to implementing a hiring standard, and thereby controlling the quality of the work force, we have “selection wages.” The frequently encountered practice of screening applicants suggests that the selection-wage mechanism may be of considerable empirical relevance.

Selection wages can be expected to emerge in a context characterized jointly by three features: First, a certain degree of labor heterogeneity; second, a certain degree of labor immobility; and third, job-specific pay.

Labor heterogeneity makes it worthwhile to screen applicants and to enlarge the pool of applicants by offering higher wages. This permits implementing a more demanding hiring standard. Labor immobility refers to the idea that workers have heterogeneous preferences over jobs, such that a firm cannot attract all the best workers by offering a wage slightly above the market wage. (With perfect mobility, there would be perfect sorting of workers, and selection wages would not emerge.) Job-specific pay prevents firms from offering higher wages to better applicants, which would complicate the argument. The assumption of job-specific pay can be defended empirically as providing a stylized description of a number of empirical compensation systems; and it can be defended theoretically by alluding to informational arguments. (The firm has private knowledge of the outcome of the performance test. There would be no reason to reveal to a worker that he was tested as outstandingly productive if this would imply having to pay him higher wages.)

The purpose of this note is to illustrate the selection-wage mechanism by means of a simple algebraic example. The argument may entail quite unconventional implications, such as an equalizing effect of a progressive wage tax on pre-tax wages, as will be sketched in the last section.

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The term is coined in analogy to P \_\_\_\_\_'s(\_\_\_\_\_) “incentive wages.” The idea itself goes back at least to the classic contributions by R \_\_\_\_\_ (\_\_\_\_\_,\_\_\_\_\_) and has been developed more recently in S \_\_\_\_\_ (\_\_\_\_\_) where further references and a discussion of some related efficiency-wage arguments may be found.

## A E

Consider an industry that produces a certain good by employing heterogeneous workers. A fraction  $q$  of workers—the *prolific* workers—have productivity 1. The others—the *mediocre* workers—have productivity  $x < 1$ . Average productivity in the industry is thus

$$a = q + (1 - q) \cdot x. \quad (1)$$

The average productivity of a firm's work force may deviate from average market productivity  $a$ , if the share of prolific workers in a firm differs from the market average. Denote the share of prolific workers enjoyed by the firm under consideration by  $\rho$ . The entailed productivity of the firm's work force is

$$\alpha = \rho + (1 - \rho) \cdot x. \quad (2)$$

The share of prolific workers in the firm's workforce  $\rho$  will depend in turn on the wage offer  $w$  the firm makes, as compared to the market wage rate  $W$ . If the firm pays above the market wage ( $w > W$ ), it will attract more prolific applicants and need hire only fewer mediocre workers. If the firm offers a wage below the market wage ( $w < W$ ), it will find fewer prolific applicants and has to hire more mediocre workers. This idea can be expressed by

$$\rho = q \cdot \left( 1 + \mu \cdot \log \left( \frac{w}{W} \right) \right) \quad (3)$$

where the constant  $1 > \mu > 0$  parametrizes *mobility*. (We exclude  $\mu \geq 1$  because it would be always optimal to pay maximum wages in this case, and the selection effect would not apply in any interesting way.)

Equations (1) and (3) imply

$$\alpha = x + q \cdot (1 - x) \left( 1 + \mu \cdot \log \left( \frac{w}{W} \right) \right). \quad (4)$$

The industry is composed of a number of firms. Each firm can employ a certain number of workers  $n$  and incurs some non-labor costs  $C$  which include normal profits. With productivity  $\alpha$ , a firm's production will be  $\alpha \cdot n$ . For a product price  $p$ , sales receipts will be  $p \cdot \alpha \cdot n$ . With a wage rate  $w$ , the firm incurs labor costs  $w \cdot n$ . Further, it has to cover non-labor costs  $C$ . The firm's profits will thus be equal to  $\Pi = p \cdot \alpha \cdot n - w \cdot n - C$ .

For the subsequent argument it is convenient to express profits of the typical firm in per-capita terms. Denoting per-capita non-labor costs by  $c = \frac{1}{n}C$ , these per-capita profits are given by

$$\begin{aligned}\pi &= p \cdot \alpha - w - c \\ &= p \cdot \left( x + q \cdot (1 - x) \left( 1 + \mu \cdot \log \left( \frac{w}{W} \right) \right) \right) - w - c.\end{aligned}\quad ( )$$

Consider now market equilibrium. As all firms are alike. All firms will pay the same wage rate  $w$  which can be identified with the market wage rate  $W$ . Equilibrium requires two things: First, per-capita profits must be zero. Otherwise there would be market entry or market exit, changing conditions of supply and demand. Second, it must be optimal for each firm to set its wage rate  $w$  equal to the market wage rate  $W$ . Else the market wage rate would change.

The zero-profit condition at  $w = W$  is equivalent to

$$p = \frac{W + c}{x + q \cdot (1 - x)}. \quad ( )$$

The conditions for a profit maximum with respect to  $w$  are

$$\frac{\partial \pi}{\partial w} = p \cdot q \cdot \mu \cdot (1 - x) \frac{1}{w} - 1 = 0 \quad ( )$$

$$\frac{\partial^2 \pi}{\partial w^2} = -p \cdot q \cdot \mu \cdot (1 - x) \frac{1}{w^2} < 0. \quad ( )$$

As the second-order condition (8) is always satisfied, the first-order condition (7) guarantees a profit maximum (if it exists at all). At  $w = W$ , equation ( ) implies

$$W = p \cdot q \cdot \mu \cdot (1 - x). \quad ( )$$

Equations ( ) and ( ) entail the equilibrium market wage rate

$$\bar{W} = \frac{\mu q (1 - x)}{x + q (1 - x) (1 - \mu)} \cdot c \quad ( )$$

and the equilibrium price

$$\bar{p} = \frac{c}{x + q (1 - x) (1 - \mu)}. \quad ( )$$

With the shorthand  $\Theta = \frac{1}{(1+q(1-x)(1-\mu))} > 0$  we find the derivatives

$$\frac{\partial \bar{W}}{\partial c} = \Theta \mu q (1 - x) > 0$$

$$\frac{\partial \bar{p}}{\partial c} = \Theta > 0$$

$$\frac{\partial \bar{W}}{\partial x} = -\Theta^2 \mu q c < 0$$

$$\frac{\partial \bar{p}}{\partial x} = -\Theta^2 (1 - q (1 - \mu)) c < 0$$

$$\frac{\partial \bar{W}}{\partial q} = -\Theta^2 \mu (1 - x) x c < 0$$

$$\frac{\partial \bar{p}}{\partial q} = -\Theta^2 (1 - \mu) (1 - x) c < 0$$

$$\frac{\partial \bar{W}}{\partial \mu} = \Theta^2 q (x + q (1 - x)) (1 - x) c > 0 \quad \frac{\partial \bar{p}}{\partial \mu} = \Theta^2 (1 - x) q c > 0$$

Hence in the little model, the equilibrium wage level and the equilibrium price level move always in the same direction. Both increase with increasing non-labor costs  $c$  and increasing mobility  $\mu$ , and both decrease with an increase in the productivity of the mediocre workers  $x$  and with an increase in the share of prolific workers  $q$ .

## S

First, consider stability of adjustment. Assume that the prevailing wage level  $W$  initially differs from the equilibrium wage level  $\bar{W}$ . By combining (6) and (7), we obtain the profit-maximizing wage level  $w$  for the typical firm as

$$w = \frac{\mu (1 - x) q}{x + (1 - x) q} (W + c)$$

which implies together with ( ) and ( )

$$w - W = -\frac{(x + (1 - \mu) (1 - x) q)}{x + (1 - x) q} (W - \bar{W}). \quad ( )$$

If the wage level is above the equilibrium wage level ( $W > \bar{W}$ ), each firm will set its wage  $w$  below the market wage level  $W$ . This drives the market wage level down until the equilibrium wage level is reached. Conversely, for  $W < \bar{W}$  the firms set  $w > W$ . This drives the wage level up to  $\bar{W}$ . This establishes stability of adjustment.

Further, the equilibrium would be unstable if the wage rate exceeds the marginal value product of a mediocre worker. If this were the case it would not be profitable for any firm to hire a mediocre worker, and leave all jobs unmanned that cannot be filled with prolific workers. This condition is  $p \cdot x > W$ . Together with ( ) and

( ) it can be equivalently stated as

$$x > \frac{\mu q}{1 - \mu q} \quad \text{or} \quad \mu q < \frac{x}{1 + x}. \quad ( )$$

If the productivity  $x$  of the mediocre workers is too low, it would not be worthwhile to employ them. If mobility is high, the equilibrium wage level  $\bar{W}$  would be high, and mediocre workers were too expensive to employ, and the same would hold true if the ratio of prolific workers in the work force were too high.

A third stability requirement would simply be that the equilibrium wage rate  $\bar{W}$  must not be below the reservation wages of both types of workers. Denote by  $R_0$  the reservation wage of the mediocre workers and by  $R_1$  the reservation wage of the prolific workers. This condition would simply read

$$\bar{W} \geq \min \{R_0, R_1\}. \quad ( )$$

For  $\bar{W} < R_1$  the equilibrium described above could not be maintained, and the market wage rate would have to be  $R_1$ , entailing a market price  $\frac{R_1 + c}{x + q \cdot (1 - x)}$ . Further, and to assure employment of the mediocre workers,  $p \cdot x > R_0$  has to be maintained, *etc.* In the following we assume that conditions ( ) and ( ) are satisfied.

$$\begin{array}{c} \mathbf{P} \\ \mathbf{T} \end{array}$$

The reaction of the wage level to parameter changes is fairly conventional. Yet the model delivers some non-standard results. To illustrate, consider progressive taxation of wage income. Denote the pre-tax wage by  $w$  and the associated after-tax wage by  $v$ , and denote by  $V$  the after-tax wage entailed by the pre-tax market wage rate  $W$ . The relation between these quantities is given by the tax function function  $\psi$  as

$$v = \psi(w) \quad , \quad V = \psi(W). \quad ( )$$

In presence of taxation, the sorting effects described in equation ( ) will depend now on the ratio of net wages  $\frac{v}{V}$  rather than gross wages  $\frac{w}{W}$ , and (4) is to be replaced by

$$\alpha = x + q \cdot (1 - x) \left( 1 + \mu \cdot \log \left( \frac{\psi(w)}{\psi(W)} \right) \right).$$

The zero-profit condition (6) remains unaffected, but the incentive condition (9) changes to

$$W = p \cdot q \cdot (1 - x) \cdot \mu \varepsilon \quad ( )$$

where

$$\varepsilon = \psi'(W) \cdot \frac{W}{\psi(W)}$$

gives the elasticity of net income in response to gross income changes. A proportionate tax would be characterized by  $\varepsilon = 1$ , and a progressive tax by  $\varepsilon < 1$ . Comparing ( ) and ( ) we note that  $\mu$  is replaced by  $\mu\varepsilon$ . The introduction of a progressive income tax is, in this model, equivalent to a reduction of mobility. In view of the effects of mobility on the equilibrium price and wage levels, as analyzed in section II, we find that the introduction of progressive taxation reduces pre-tax wages, as well as prices and after-tax wages. This contrasts with the result obtainable in a conventional setting where progressive taxation would increase pre-tax wages and prices, while reducing after-tax wages. In so far as allocational effects go along with such changes, it seems to be of relevance whether selection wages or market-clearing wages prevail.

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See S ( , IV) for a similar effect in a model where, in addition, the effect is welfare-enhancing.